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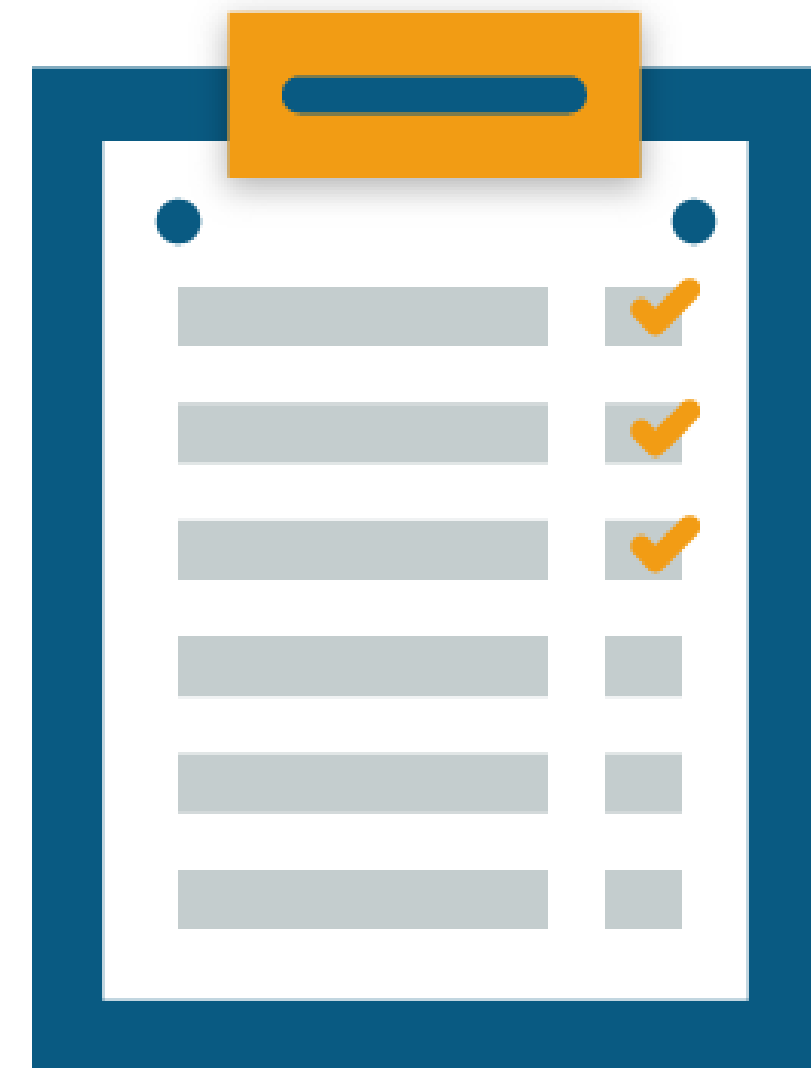
Module 2 – Probability

Part I

Topics

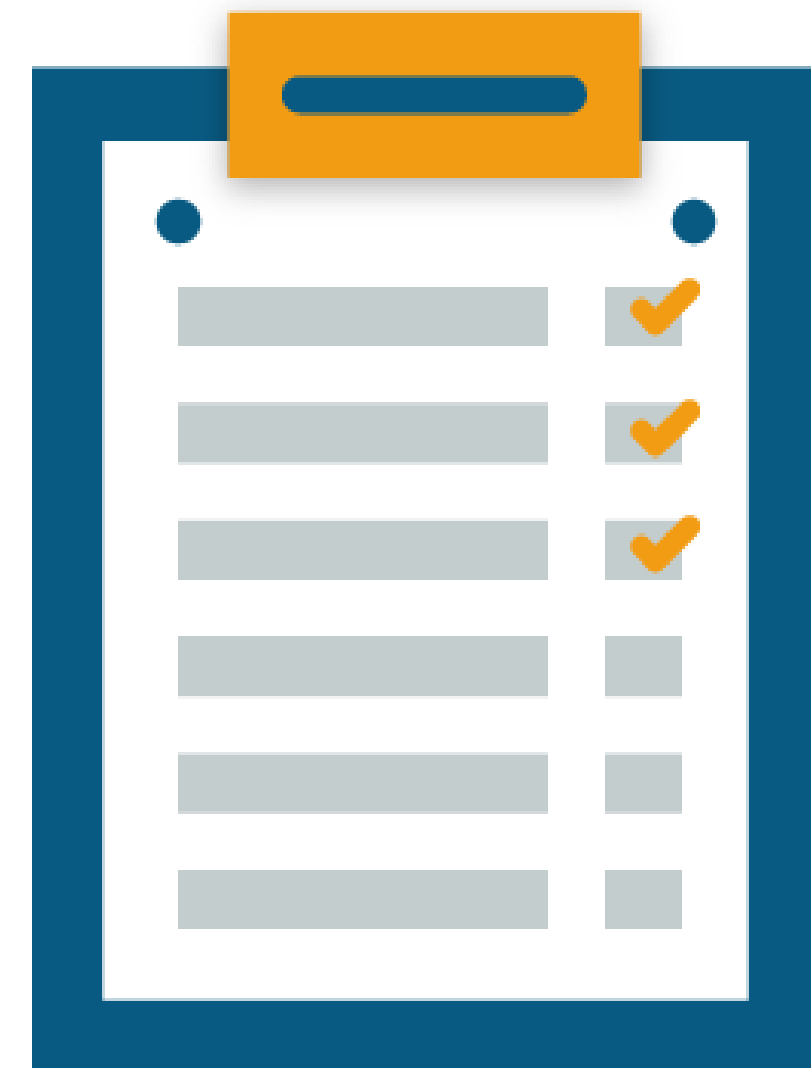
Following are the topics covered in this module:

- Why Use Probability?
- What is Probability?
- Rules of probability
- Types of Probability
 - Marginal Probability
 - Joint Probability



Objectives

- Understand the concepts of Probability
- Examine the rules of Probability
- Implement Joint and Marginal Probability



Video 1 – Introduction to Probability

Video 1 Summary

What is Probability?

The probability of an event represents the *frequency* or *chance* that the event will happen



What is the probability of scoring a **six** on the one roll of a single die?

- For notation, if A is an event, $P(A)$ is the probability of the occurrence of the event
- We can define the actual probability of an event, A , as follows:

$$P(A) = \frac{\text{number of ways } A \text{ occurs}}{\text{size of sample space}} \longrightarrow \frac{1}{6} \rightarrow \text{six occurs only once}$$

$\frac{1}{6} \rightarrow \text{die has 6 faces}$

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Why Use Probability?



Helps in measuring uncertainty



Helps in evaluating the chances of success of an event



Probability Distribution helps in analyzing the future outcome



To classify data in machine learning, the concept of probability is required



Logistic Regression and Naïve Bayes are based on the concept of probability

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Rules of Probability

- For any Event A , $P(A^c) = 1 - P(A)$

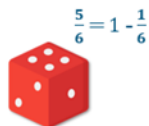


$$\frac{1}{2} = 1 - \frac{1}{2}$$

If event is *getting a head*, then its complement is *tail*

Complement of an Event are all outcomes that are **NOT** the event

If event is *scoring a 6*, then its complement is *1, 2, 3, 4, 5*



$$\frac{5}{6} = 1 - \frac{1}{6}$$



If event is *Weekend (Sat, Sun)*, then its complement is *(Mon, Tues, Wed, Thurs, Fri)*

$$\frac{5}{7} = 1 - \frac{2}{7}$$

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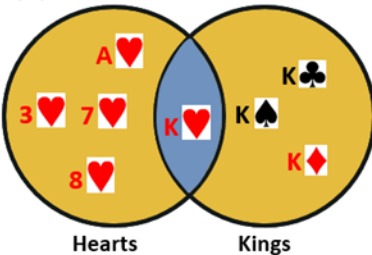
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Rules of Probability

- For any two Events A and B :

$$P(A \cup B) = P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$

For example,



When two events are mutually inclusive or joint it is possible for them to happen together

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Rules of Probability

- When A and B are *Disjoint Events*, $P(A \cap B) = 0$,

$$\text{Hence } P(A \cup B) = P(A) + P(B)$$

For example,



Aces

Kings

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Aka **Mutually Exclusive Events**
When two events are mutually exclusive or disjoint it is impossible for them to happen together

Rules of Probability

- Product Rule for Independent Events:**

$$P(A \cap B) = P(A) \times P(B)$$

For example,



What is the probability of getting two heads on two consecutive coin flips?

- Probability of getting a head on the first flip: $P(A) = \frac{1}{2}$
- Probability of getting a head on the second flip: $P(B) = \frac{1}{2}$
- $P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

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ALICE'S QUESTION

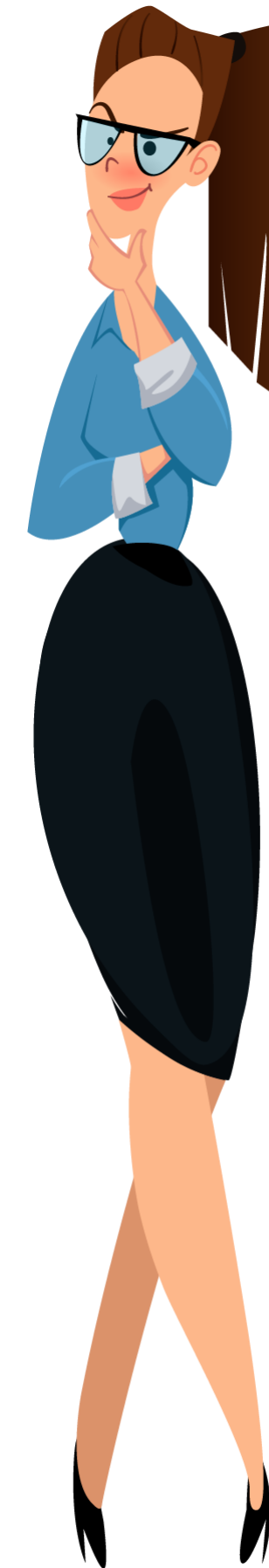
1. Three identical dice are rolled. The probability that same number will appear on each of them?

A> $\frac{1}{6}$

B> $\frac{1}{36}$

C> $\frac{1}{216}$

D> None of the above



1. Three identical dice are rolled. The probability that same number will appear on each of them?

A> $\frac{1}{6}$

B> $\frac{1}{36}$

C> $\frac{1}{216}$

D> None of the above

Explanation: $S=6 \times 6 \times 6=216$

$$E: (1, 1, 1), (2, 2, 2), \dots, (6, 6, 6) = 6$$

$$P(E) = \frac{6}{216} = \frac{1}{36}$$

2. The probabilities of a student getting A, B, and C grade are

$\frac{1}{10}$, $\frac{7}{12}$ and $\frac{1}{6}$. What will be the probability that he will get D grade?

A> $\frac{51}{60}$

B> $\frac{10}{12}$

C> $\frac{9}{60}$

D> None of the above



2. The probabilities of a student getting A, B, and C grade are

$\frac{1}{10}$, $\frac{7}{12}$ and $\frac{1}{6}$. What will be the probability that he will get D grade?

A> $\frac{51}{60}$

B> $\frac{10}{12}$

C> $\frac{9}{60}$

D> None of the above

Explanation: $P(A) + P(B) + P(C) + P(D) = 1$

$$\Rightarrow P(D) = 1 - P(A) - P(B) - P(C)$$

3. What will be the probability that a leap year selected will have 53 Mondays?

A> $\frac{4}{53}$

B> $\frac{1}{7}$

C> $\frac{2}{7}$

D> None of the above



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3. What will be the probability that a leap year selected will have 53 Mondays?

A> $\frac{4}{53}$

B> $\frac{1}{7}$

C> $\frac{2}{7}$

D> None of the above

Explanation: A leap year has 52 weeks and 2 days.

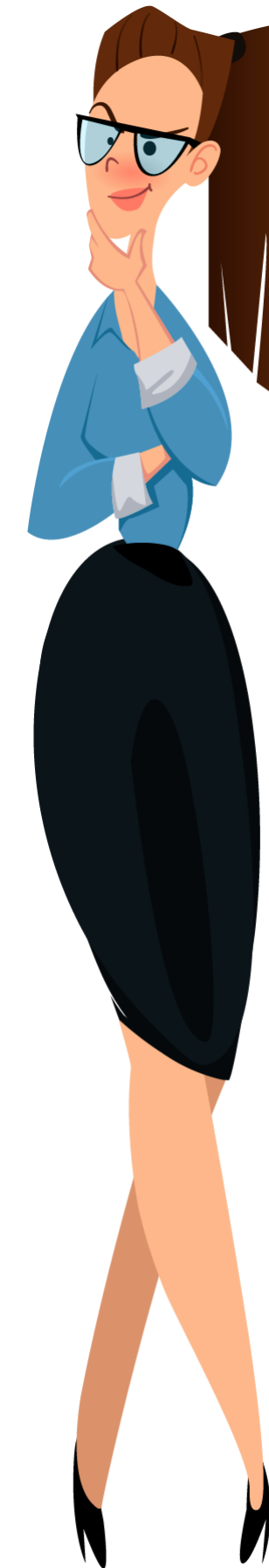
For remaining two days, sample space will be:

[(Sun+Mon) , (Mon+Tue) , (Tue+Wed) , (Wed+Thu) , (Thu+Fri) , (Fri+Sat) , (Sat+Sun)]

P (E) : $\frac{2}{7}$

4. From a deck of 52 cards . If A=card is of diamond, B=card is an ace and $A \cap B$: card is ace of diamond, then events A and B are:

- A> Independent
- B> Dependent
- C> Mutually Exclusive
- D> None of the above





4. From a deck of 52 cards . If A =card is of diamond, B =card is an ace and $A \cap B$: card is ace of diamond, then events A and B are:

- A> Independent
- B> Dependent
- C> Mutually Exclusive
- D> None of the above

Explanation: Events A and B will be dependent.

Case Study Scenario: EnCare Bank

Case Study Scenario: EnCare Bank

EnCare bank is a multinational bank serving millions of customers. Recently, they noticed that a significant number of their customers have stopped taking their term deposit schemes.



Foremost Objective

OBJECTIVE

Find term deposit subscription status based on the customer data.



Data Collection

The Data Acquisition team at EnCare Bank compiled the data for all the customers.
Below is a sample of the data:

gender	age	Job	marital	education	default	housing	loan	contact	term_deposit
female	56	housemaid	married	basic.4y	no	no	no	telephone	no
male	57	services	married	high.school	yes	no	no	telephone	no
female	37	services	married	high.school	no	yes	no	telephone	no
female	40	admin.	married	basic.6y	no	no	no	telephone	no
female	56	services	Married	high.school	no	no	yes	telephone	no
male	44	services	Married	basic.6y	yes	no	no	telephone	no
female	27	housemaid	Married	high.school	no	yes	no	telephone	no
male	42	admin.	married	basic.6y	no	no	no	telephone	no

Tasks to be Performed

Question Type A

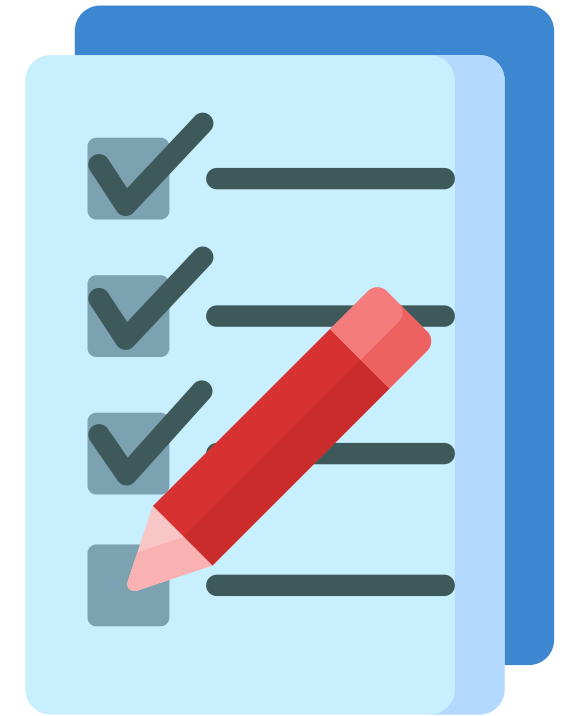
- Find the probability to that a customer will take term deposit
- Find the probability that a customer has at least basic 9 years of education
- Find the probability that a customer is a technician

Question Type B

- Find the probability for a male customer who has already taken term deposit
- Probability that a customer is married and has housing as well
- Probability that a customer has university degree and no loan

Question Type C

- Given that probability that customer has taken term deposit, find the probability that she is a female.
- Given that selected customer has defaulted in past, find the probability that he/she has a blue collar job.



Types of Probability



Video 2 – Marginal and Joint Probability

Video 2 Summary

Marginal Probability

Marginal Probability is the probability of occurrence of a single case

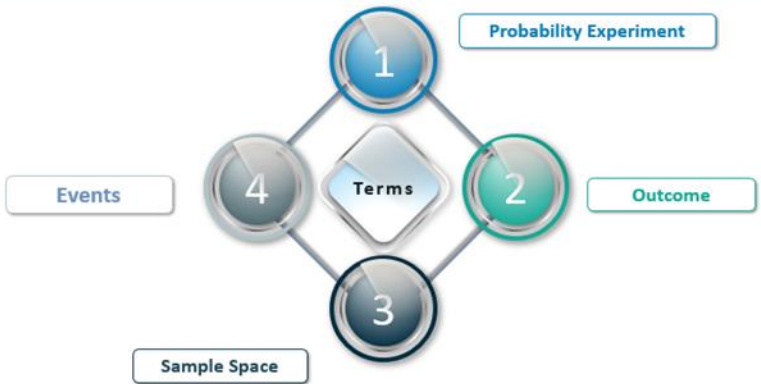
EXAMPLES:

- What will be the probability that a customer has no housing?
- What will be the probability that a customer is illiterate?
- What will be the probability that a customer has a telephone?

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Associated Terminologies



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Joint Probability

Joint Probability is the measure of two events happening at the same time

Joint Probability is given by: $P(A \cap B) = P(A) \times P(B)$ → The probability of events A and B i.e., $P(A \text{ and } B)$ occurring is the probability of intersection of A and B

EXAMPLES:

- What will be the probability that the customer is single and has a term deposit?
- What will be the probability that a customer is male and has at most basic 6 year education?
- What will be the probability that a customer is retired and is divorced?

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Intersection of Events

Event A: Customer is single

Event B: Customer has term deposit

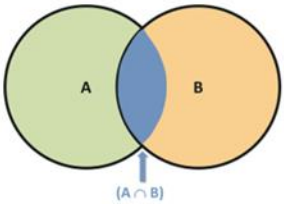
marital
married
single
married
single
divorced
single
divorced
single



term_deposit
no
yes
yes
yes
yes
yes
no
no



✓
✓
✓



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Union of Events

Event A: Customers with basic 4 years education

Event B: Customers with basic 6 years education

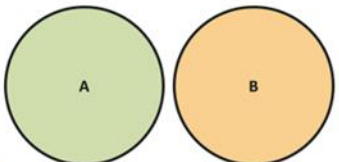
education
basic.4y
high.school
high.school
basic.6y
high.school
basic.6y
high.school
basic.6y



education
basic.4y
high.school
high.school
basic.6y
high.school
basic.6y
high.school
basic.6y



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Marginal Probability

Example 1: No Housing

Probability
Experiment 1

What will be the probability that a customer has no housing?

Event A: Customer has no housing

A (number of ways A occurs): 18622

S (size of sample space): 41188

$$P(A) = \frac{\text{number of ways A occurs}}{\text{size of sample space}} = \frac{18622}{41188} = 0.452$$

Housing (Outcomes)

Yes	22566
No	18622
Total	41188

Probability that a randomly chosen customer has no housing is **0.452**

Example 2: Customer is Illiterate

Probability Experiment 2

What will be the probability that a customer is illiterate?

Event A: Customer is illiterate

A (number of ways A occurs): 18

S (size of sample space): 41188

$$P(A) = \frac{\text{number of ways A occurs}}{\text{size of sample space}} = \frac{18}{41188} = 0.00043$$

Education (Outcomes)

Professional Course	5243
University degree	12168
High School	9515
Basic 9y	6045
Basic 6y	2292
Basic 4y	4176
Unknown	1731
Illiterate	18
Total	41188

Probability that a randomly chosen customer is illiterate is **0.0043**

Example 3: Customer has Telephone

Probability
Experiment 3

What will be the probability that a customer has a telephone?

Event A: Customer has a telephone

A (number of ways A occurs): 15044

S (size of sample space): 41188

$$P(A) = \frac{\text{number of ways A occurs}}{\text{size of sample space}} = \frac{15044}{41188} = 0.365$$

Contact (Outcomes)

Cellular	26144
Telephone	15044
Total	41188

Probability that a randomly chosen customer has a telephone is **0.365**

ALICE'S QUESTION

1. The probability of picking a 4 from a deck of cards?

A> $\frac{1}{26}$

B> $\frac{1}{13}$

C> $\frac{1}{52}$

D> None of the above



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1. The probability of picking a 4 from a deck of cards?

A> $\frac{1}{26}$

B> $\frac{1}{13}$

C> $\frac{1}{52}$

D> None of the above

Explanation: Total no. of cards in a deck = 52

Total no. of 4s in a deck = 4

$$P(E) = \frac{4}{52} = \frac{1}{13}$$

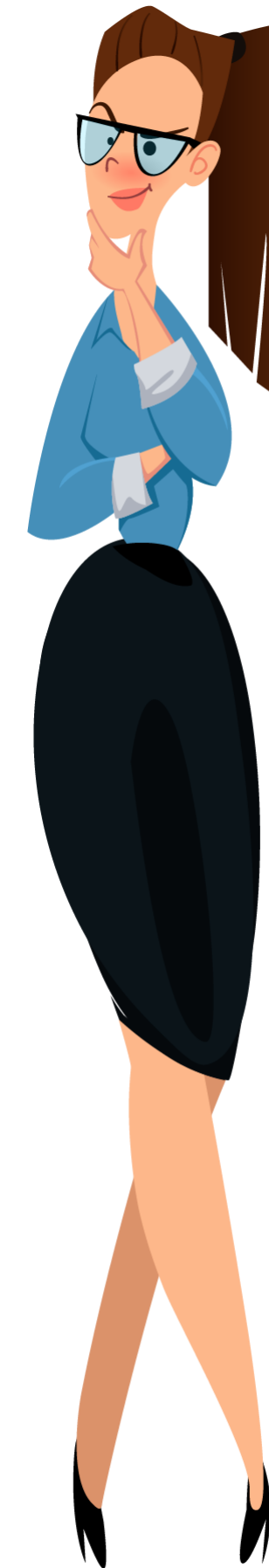
2. Probability of getting a number less than or equal to 8 at the roll of two dice?

A> $\frac{7}{11}$

B> $\frac{2}{3}$

C> $\frac{1}{6}$

D> None of the above



2. Probability of getting a number less than or equal to 8 at the roll of two dice?

A> $\frac{7}{11}$

B> $\frac{2}{3}$

C> $\frac{1}{6}$

D> None of the above

Explanation: Total no. of outcomes at rolling two dice: 11 `[2,3,4,5,6,7,8,9,10,11,12]`

P(E) : $7/11$ `[2,3,4,5,6,7,8]`

Joint Probability

Example 1: Single and has Term Deposit

Probability
Experiment 1

What will be the probability that the customer is single and has a term deposit?

Event A: Customer is single

Event B: Customer has term deposit

marital	term_deposit		
	no	yes	All
divorced	4136	476	4612
married	22396	2532	24928
single	9948	1620	11568
unknown	68	12	80
All	36548	4640	41188

Example 1: Single and has Term Deposit (Cont.)

Probability
Experiment 1

What will be the probability that the customer is single and has a term deposit?

Event A: Customer is single

Event B: Customer has term deposit

Event occurrence:

A: 11568 B: 4640 $(A \cap B) : 1620$ S: 41188

$$P(A) = \frac{11568}{41188} = 0.28$$

$$P(B) = \frac{4640}{41188} = 0.11$$

$$P(A \cap B) = \frac{1620}{41188} = 0.039$$

Probability that a customer is single and has term deposit is **0.039**

Example 2: Male and at most Basic 6 Year Education

Probability
Experiment 2

What will the probability that a customer is male
and has at most basic 6 year education?

Event A: Customer is male

Event B: Customer has at most basic 6 year education

education	gender		
	female	male	All
basic.4y	2104	2072	4176
basic.6y	1169	1123	2292
basic.9y	3092	2953	6045
high.school	4735	4780	9515
illiterate	7	11	18
professional.course	2647	2596	5243
university.degree	6142	6026	12168
unknown	902	829	1731
All	20798	20390	41188

Example 2: Male and at most Basic 6 Year Education

Probability Experiment 2

What will the probability that a customer is male and has at most basic 6 year education?

Event A: Customer is male

Event B: Customer has at most basic 6 year education

Event occurrence:

A: 20390 B: 4176+2292 = 6468 (A ∩ B) : 2072+1123 = 3195 S: 41188

$$P(A) = \frac{20390}{41188} = 0.495$$

$$P(B) = \frac{6468}{41188} = 0.157$$

$$P(A \cap B) = \frac{3195}{41188} = 0.078$$

Probability that a customer is male and has at most basic 6 year education is **0.078**

Example 3: Retired and Divorced

Probability
Experiment 3

What will be the probability that a customer is retired
and is divorced?

Event A: Customer is retired

Event B: Customer is divorced

marital	job												
	admin.	blue-collar	entrepreneur	house maid	management	retired	self-employed	services	student	technician	unemployed	unknown	All
divorced	1280	728	179	161	331	348	133	532	9	774	124	13	4612
married	5253	6687	1071	777	2089	1274	904	2294	41	3670	634	234	24928
single	3875	1825	203	119	501	93	379	1137	824	2287	251	74	11568
unknown	14	14	3	3	3	5	5	6	1	12	5	9	80
All	10422	9254	1456	1060	2924	1720	1421	3969	875	6743	1014	330	41188

Example 3: Retired and Divorced (Cont.)

Probability
Experiment 3

What will be the probability that a customer is retired and is divorced?

Event A: Customer is retired

Event B: Customer is divorced

Event occurrence:

A: 1720 B: 4612 $(A \cap B) : 348$ S: 41188

$$P(A) = \frac{1720}{41188} = 0.0416$$

$$P(B) = \frac{4612}{41188} = 0.112$$

$$P(A \cap B) = \frac{348}{41188} = 0.0084$$

Probability that a customer is retired and divorced is **0.0084**

ALICE'S QUESTION

1. Out of 500 people, 120 males and 75 females play Football; 100 males and 25 females play Rugby; 50 males and 130 females play other sports. What is the probability of being a male and liking Rugby?

- A> 0.2106
- B> 0.54
- C> 0.135
- D> None of the above



1. Out of 500 people, 120 males and 75 females play Football; 100 males and 25 females play Rugby; 50 males and 130 females play other sports. What is the probability of being a male and liking Rugby?

A> 0.2106

B> 0.54

C> 0.135

D> None of the above

Explanation: $P(\text{male}) = \frac{120+100+50}{500} = \frac{270}{500} = 0.54$

$P(\text{rugby}) = \frac{100+25}{500} = \frac{125}{500} = 0.25 \rightarrow P(\text{male AND rugby}) = 0.54 \times 0.25 = 0.135$

ALICE'S QUESTION

2. Out of 300 people, 180 people smoke. Also, from this group of 300 people, 66 people have lung cancer; 34 have asthma; 200 people have no disease. What is the probability that a person is a smoker and has lung cancer?

- A> 0.125
- B> 0.132
- C> 0.22
- D> None of the above



2. Out of 300 people, 180 people smoke. Also, from this group of 300 people, 66 people have lung cancer; 34 have asthma; 200 people have no disease. What is the probability that a person is a smoker and has lung cancer?

A> 0.125

B> 0.132

C> 0.22

D> None of the above

Explanation: $P(\text{smoker}) = \frac{180}{300} = 0.6$

$P(\text{lung cancer}) = \frac{66}{300} = 0.22 \rightarrow P(\text{smoker AND lung cancer}) = 0.6 \times 0.22 = 0.132$



EnCare Bank Demo on Marginal and Joint Probability

Check out the demo file on the LMS

In-Class Practice Session - I

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You have been provided with a dataset on Marvel Comics characters. Find out the following:

1. Calculate the probability of a character being male
2. Calculate the probability of a character being female with red hair

Download the dataset from the LMS.

[Click Here!](#) To Go To Google Co-Lab



Questions



FEEDBACK



Thank You



For more information please visit our website