Leaky LMS algorithm based low complexity Adaptive Noise Cancellation

***Abstract*—Neurodegenerative conditions are widespread, marked by a gradual worsening of symptoms that progressively hinder daily activities and diminish overall well-being. Parkinson's disease is one of the neurodegenerative condition that affects movement and often leads to speech difficulties. Those with the condition may notice their voice becoming quieter or their speech becoming slurred, making communication increasingly challenging over time. Parkinson's disease has impacted over 15 million people all around the world. These speech related neurological diseases are often diagnosed using acoustic signals like speech, ultrasound and PCG signal. Often existing environmental noises in the signal, leads to less accurate diagnosis. To mitigate the impact of noise, researchers often employ signal processing techniques. The proposed Leaky LMS algorithm has low complexity compared to the existing methods.**

***Index Terms*—Least Mean Square, Cancellation of adaptive Noise, Leaky LMS.**

I. Introduction

One of the key areas where the signals are becoming increasingly important is in the timely identification of diseases [1]. Parkinson's disease is one of the neurodegenerative condition that harms coordination, mental health and movement with symptoms such as shaking of the body, muscle stiffness, and slowed movements. But they are often not noticeable until the disease has advanced significantly. Parkinson's disease can be detected in early stages with the help of the person's speech, as it often begins with speech disorders such as slurred speech, soft or weak voice, monotonicity, and decreased articulation. Speech signal analysis is a useful diagnostic technique in this case for early detection. Often, the speech input recorded for analysis is accompanied by external background noises. For accurate detection, it is crucial to de-noise the signal for further analysis. To address this challenge, researchers have developed signal processing techniques that can effectively isolate and remove noise from acoustic signals [2], enhancing the reliability and diagnostic accuracy [3].

LMS (Least Mean Square) adaptive filter is a powerful denoising algorithm for ANC [4]. These adaptive filters work by iteratively adjusting their coefficients to reduce the difference between the filtered signal and the target signal [5]. Though there are many existing techniques such as Wavelet transform , Empirical Mode Decomposition(EMD) and Deep Learning for denoising applications, these methods are expensive and complex. LMS algorithm is the less complex and provides good convergence [6]. LMS algorithm has been employed in many fields for various applications such as biomedical signal processing [7], noise cancellation [8], channel equalization [9], and speech processing [10]. The LMS filter is extensively utilized in a variety of applications, especially for de-noising tasks. One significant drawback of the LMS algorithm is the drifting problem, as discussed in [11]. This issue arises when the algorithm generates parameter estimates that become unbounded, even though the input sequence remains bounded. As a result, the LMS weight update may diverge if the input sequence is not proper.

Leaky LMS is the variation of the LMS algorithm in which leakage factor is introduced. The leakage parameter is crucial in this technique as it improves the stability of the filter and mitigates the weight divergence issue which was present LMS filter. The leakage parameter helps prevent the filter coefficients from diverging, which can cause instability in the filter. It keeps the weights under control, preventing them from becoming excessively large, especially when there is significant noise in the input signal. The Leaky LMS algorithm offers improved MSE performance over the traditional LMS by better handling non-stationary Gaussian data as seen in [12]. By incorporating a leak parameter, it allows for faster convergence and better stability. Tuning the leak parameter helps balance the rate of convergence and the steady-state error, optimizing overall performance. Hence, Leaky LMS algorithm is more capable of handling non-stationary environments making it more effective for real-time adaptive filtering tasks. This is particularly useful in clinical applications [13], where accurate analysis of speech signals is crucial for articular conditions. In case of Speech de-noising, by analyzing the statistical characteristics of the noise and the speech signal, the Leaky LMS filter can effectively remove the noise while preserving the important features of the underlying vocal sounds. Implementation of the Leaky LMS in Multistage setup obtains more efficient results compared to other models. In this paper, denoising the speech signal is implemented using Multistage Leaky Least Mean Square (LLMS) adaptive filter to achieve the result.

II. PROPOSED MULTISTAGE LEAKY LMS FILTER

A signal mixed with interference, represented as $d(m)$ forms the fundamental input signal passed on to the ANC. $d_{1LLMS}(m) = s(m) + l(m)$, as per the given situation, $s(m)$ depicts the noise free signal, while $l(m)$ depicts the introduced interference. It is essential to focus on the fact that $s(m)$ and $l(m)$ do not exhibit correlation based on time. On the other hand, the signal input to the filter is $x_1(m)$, which exhibits a relationship with the noise signal over time is $l(m)$. A precise duplicate to the signal with noise is generated and described as:

$$d_{1LLMS}(m) = s(m) + l(m) \tag{1}$$

$$x_1(m) = l^{'}(m). \tag{2}$$

During starting phase, output derived by LLMS algorithm are described as:

$$e_{1LLMS}(m) = d_{1LLMS}(m) - y_{1LLMS}(m) \tag{3}$$

$$\begin{aligned} y_{1LLMS}(m) &= \mathbf{w}_1^T(m)\mathbf{x}_1(m) \\ &= \mathbf{w}_1^T(m)\mathbf{l}^{'}(m) \\ &= \hat{l}(m), \end{aligned} \tag{4}$$

The weights are updated according to the following steps:

$$w_1(m+1) = (1-\gamma\mu)w_1(m) + e_{1LLMS}(m)x_1(m) \tag{5}$$

$$= (1-\gamma\mu)w_1(m) + [d_{1LLMS}(m) - x_T(m)w_1(m)]x(m) \tag{6}$$

γ is leakage component brought into the equation. Once the leakage coefficient is applied, any undamped modes are completely eliminated, and the existing filter coefficients are also proportionally reduced to zero in the case of either $x_1(m)$ or $e_{1LLMS}(m)$ is zero. μ represents the step-size of Leaky LMS filter.
Equation (1) and equation (4) are to be put in place of equation (3) to get

$$\begin{aligned} e_{1LLMS}(m) &= s(m) + l(m) - \hat{l}(m) \\ &= s(m) + \Delta l(m) \end{aligned} \tag{7}$$

Substituting equations for the second stage

$$\begin{aligned} d_{2LLMS}(m) &= s(m) + l(m) - \hat{l}(m) \\ &= s(m) + \Delta l(m). \end{aligned} \tag{8}$$

To filter out the noise $\Delta l(m)$ from $d_{2LLMS}(m)$, It is necessary to use a reference signal associated with $\Delta l(m)$ at the stage II filter (i.e)

$$x_2(m) = x_1(m) - y_{1LLMS}(m) \tag{9}$$

Substituting equations (2) and (4) in (9) we get

$$\begin{aligned} x_2(m) &= l^{'}(m) - \hat{l}(m) \\ &= \Delta l^{'}(m) \end{aligned} \tag{10}$$

Therefore, at stage II, the resulting error is:

$$e_{2LLMS}(m) = d_{2LLMS}(m) - y_{2LLMS}(m) \tag{11}$$

where

$$\begin{aligned} y_{2LLMS}(m) &= \mathbf{w}_2^T(m)\mathbf{x}_2(m) \\ &= \mathbf{w}_2^T(m)\Delta\mathbf{l}^{'}(m) \\ &= \Delta\hat{l}(m) \end{aligned} \tag{12}$$

The Stage II update for Leaky LMS filter coefficients is described by:

$$w_2(m+1) = (1-\gamma\mu)w_2(m) + e_{2LLMS}(m)x_2(m) \tag{13}$$

$$= (1-\gamma\mu)w_1(m) + [d_{2LLMS}(m) - x_T(m)w_2(m)]x(m) \tag{14}$$

Substituting equations (8) and (12) in (11) we get

$$\begin{aligned} e_{2LLMS}(m) &= s(m) + \Delta l(m) - \Delta\hat{l}(m) \\ &= s(m) + \delta l(m), \end{aligned} \tag{15}$$

As per the above equations, the primary input signal $d_i(m)$ and the signal from the reference input $x_i(m)$ going to next stage of the filter is based on the inaccuracy in the previous stage $e_{i-1}(m)$ and disparity between the input signal and the previous stage filter's output $x_{i-1}(m) - y_{i-1}(m)$ correspondingly. The connection between the reference noise $l^{'}(m)$ and the output inaccuracy of the i^{th} stage i.e $e_i(m)$ controls automated addition of additional stages. It is not necessary to add more stages if the association between errors $e_i(m)$ and $l^{'}(m)$ is very low i.e (≈ 0). The absence of correlation between $e_i(m)$ and $l^{'}(m)$ is the optimal condition needed for the ANC to enter into its final stage. This is nearly impossible to accomplish, though, thus a little amount of noise is still present at the output. Additionally, the step size varies according to the input signal's autocorrelation. In final stage M, the inputs to the filter are

$$d_{MLLMS}(m) = e_{MLLMS-1}(m) = s(m) + \rho l(m), \tag{16}$$

and

$$x_M(m) = x_{M-1}(m) - y_{M-1}(m) = \rho l^{'}(m). \tag{17}$$

and the outputs are

$$\begin{aligned} y_{MLLMS}(m) &= \mathbf{w}_M^T(m)\mathbf{x}_M(m) \\ &= \mathbf{w}_M^T(m)\rho\mathbf{l}^{'}(m) = \rho\hat{l}(m). \end{aligned} \tag{18}$$

With LLMS, the filter's weights are adjusted as

$$w_M(m+1) = (1-\gamma\mu)w_M(m) + e_{MLLMS}(m)x_M(m) \tag{19}$$

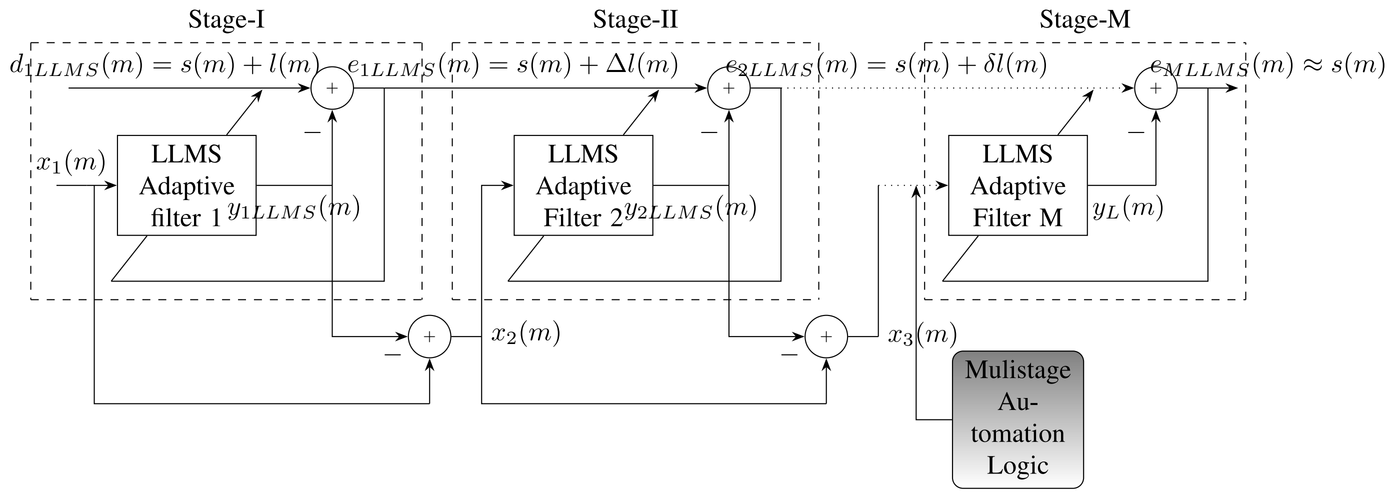

Fig. 1. A design illustration of the suggested LLMS filter model with multiple stages

$$= (1-\gamma\mu)w_M(m)+[d_{MLLMS}(m)-x_T(m)w_M(m)]x(m) \tag{20}$$

The leakage factor, which enhances the LLMS's sensitivity by rapidly reacting to unexpected shifts in the input signal while holding onto some recollection of previous updates, is its specialty. Better stability can be obtained with a leaky LMS algorithm, particularly when working with speech signals that are noisy or poorly conditioned.

III. Regulation of Stages Based on Correlation Coefficient

In a multi-stage Leaky LMS adaptive filter, computation of resulting ANC stages is dependent on the relationship between the reference noise signal $l^{'}(m)$ collected from the input noise signal and the error obtained from every stage, depicted as $e_i(m)$ indicating estimated clean signal. It is important to highlight the distinct nature of clean noise free signal $s(m)$ and the signal to which noise is added $l(m)$. Consequently, we deduce that reference signal for noise $l^{'}(m)$,that is independent of $s(m)$, is nearly the same as $l(m)$. However, $s(m)$ is connected to the error obtained $e_i(m)$ at every step. The selection of adding more stages is based on the correlation among $e_i(m)$ and $l^{'}(m)$ at every stage.More precisely, a strong association between $l^{'}(m)$ and $e_i(m)$ is an undesirable sign of a substantial amount of noise in the output error signal. In these cases, an additional stage is regarded as required; otherwise, the ideal stage has been completed. The correlation between $e_i(m)$ and $l^{'}(m)$ can be estimated and calculated as

$$\rho_{e_i l'} = \frac{Cov(e_i, l^{'})}{\sigma_{e_i}\sigma_{l'}} \tag{21}$$

where $\rho_{e_i l'}$ is the correlation coefficient.The standard deviation and covariance of the reference noise and output error are represented as σ_{e_i}, $\sigma_{l'}$, and $Cov(e_i, l^{'})$, respectively.The correlation coefficient $\rho_{e_i l'}$ is compared to a minimum threshold $\rho_{threshold}$ in order to identify which stage is right. Stages are added till $\rho_{e_i l'}$ is greater than $\rho_{threshold}$; when that threshold is reached, stage addition pauses.

IV. Results and Discussion

The suggested adaptive filter's capability to reduce noise in speech signal is trained by Gaussian noise reference signal taken from Noise Database and Biomedical sources. It is observed in the figure 2, it is observed that for normal speech signal impacted by noise source of 0dB SNR, the suggested filter model shows good denoising performance. Figure 3 represents the accurate denoising capability of the suggested multistage Leaky LMS filter output with respect to abnormal speech signal with Gaussian noise introduced, having an SNR of 5 dB. Figure 4 depicts the Output comparison between different stages of Leaky LMS Filter output for normal speech audio signal with Gaussian noise introduced, having an SNR of 5 dB. Likewise, the next figure, figure 5 represents the output comparison between different stages of Leaky LMS Filter output for abnormal speech signal with added noise of input SNR 0dB. Table 1 shows the functionality of the Multistage Leaky LMS Filter algorithm with respect to normal speech signal distorted with the Gaussian noise of different input SNR's. Table 2 displays the functionality of the Multistage Leaky LMS Filter algorithm with respect to abnormal speech audio signal mixed with Gaussian noise at various input SNR levels. Table 3 shows the Output SNR comparison, Mean Square error and Correlation coefficients between different stages of Leaky LMS

Filter output with respect to normal speech audio signal with Gaussian noise introduced, having an SNR of 5 dB. Table 4 shows the Output SNR comparison, Mean Square error and Correlation coefficients between different stages of Leaky LMS Filter output with respect to abnormal speech signal with Gaussian noise introduced, having an SNR of 5 dB.

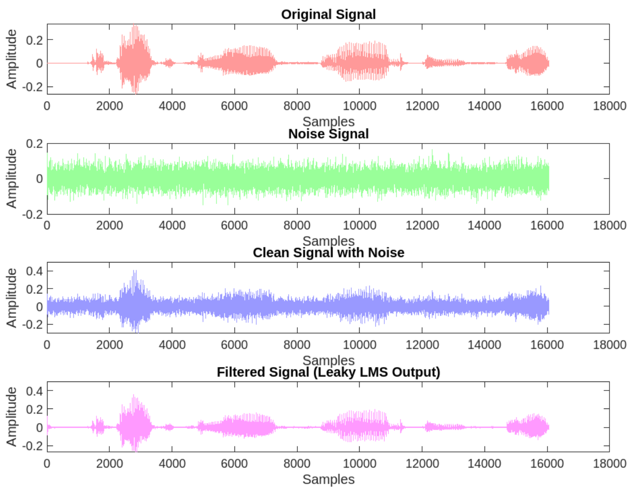

Fig. 2. Noise removal from a standard speech signal affected by Gaussian noise (i) Pure speech waveform (ii) Gaussian Noise Signal input SNR=0db (iii) Clean Speech signal+ Gaussian Noise signal (iv) Multistage Leaky LMS filter output

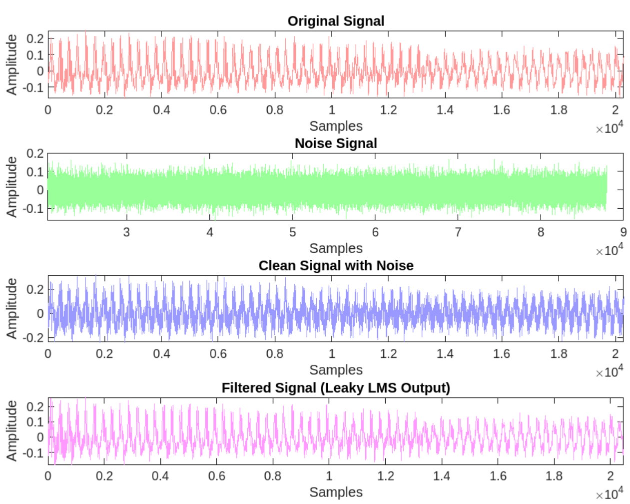

Fig. 3. De-noising an abnormal speech signal influenced by Gaussian noise (i) Clean abnormal speech waveform (ii) Gaussian Noise Signal input SNR=5db (iii) Abnormal Noisy speech signal affected by Gaussian interference (iv) Multistage Leaky LMS filter output

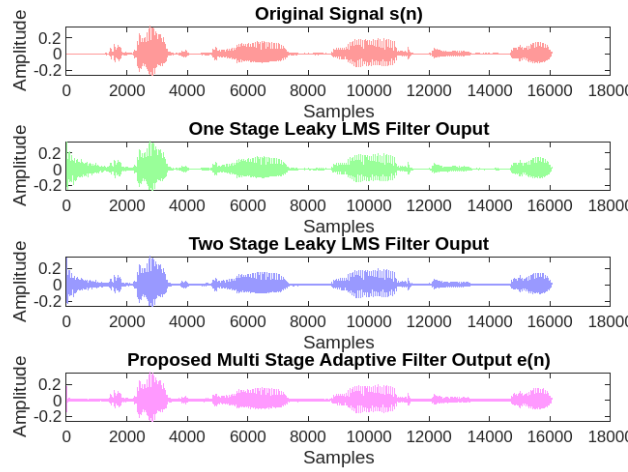

Fig. 4. Noise removal from a standard speech signal affected by Gaussian noise having an SNR of 5 dB (i) Noise free speech signal (ii) One stage LLMS AF output (iii) Two stage LLMS AF output (iv) Multistage LLMS AF output

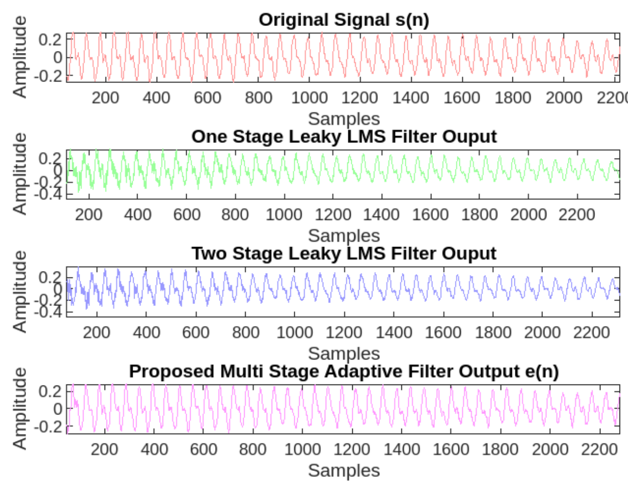

Fig. 5. Noise removal from a abnormal speech signal corrupted by the Gaussian noise 0dB (a) Original clean signal (b) One stage Leaky LMS filter output (c) One stage Leaky LMS filter output (d) Multistage Leaky LMS filter output

Input SNR(dB)	MSE	SNR(dB)
Gaussian -5dB	8.9203E-05	13.1141
Gaussian -0dB	8.8843E-05	13.1619
Gaussian +5dB	8.9489E-05	13.1559

TABLE I

EVALUATION OF THE LLMS FILTER FOR NOISE REMOVAL IN NORMAL SPEECH SIGNALS WITH GAUSSIAN INTERFERENCE

V. CONCLUSION

In this study, the proposed method demonstrates that the Multistage Leaky Least Mean Square algorithm exceeds the performance of existing techniques. It provides more accurate results while reducing complexity, making it a more efficient solution. Additionally, the algorithm achieves a higher Signal-to-Noise Ratio (SNR), which enhances its ability to detect Parkinson's disease at earlier stages. This approach not only improves diag-

Input SNR(dB)	MSE	SNR(dB)
Gaussian -5dB	9.0439E-05	20.7001
Gaussian -0dB	9.0824E-05	20.1752
Gaussian +5dB	8.8307E-05	20.8540

TABLE II
EVALUATION OF LLMS FILTER FOR ABNORMAL SPEECH WAVEFORM WITH GAUSSIAN INTERFERENCE AT INPUT SNR= 5DB

Input SNR(dB)	Output SNR(dB)	MSE	Corr.(ρ)
One Stage LLMS	18.8009	4.6221E-05	0.12049
Two Stage LLMS	20.7654	8.9174E-05	0.084928
Multistage LLMS	31.1837	6.7207E-06	0.026668

TABLE III
EVALUATION OF LLMS FILTER FOR NORMAL SPEECH WAVEFORM WITH GAUSSIAN INTERFERENCE AT SNR=0DB

Input SNR(dB)	Output SNR(dB)	MSE	Corr.(ρ)
One Stage LLMS	18.2045	3.6029E-05	0.018966
Two Stage LLMS	20.7654	3.6054E-05	0.084928
Multistage LLMS	23.8524	3.6081E-05	0.018899

TABLE IV
EVALUATION OF LLMS FILTER FOR ABNORMAL NORMAL SPEECH SIGNAL WITH GAUSSIAN NOISE = 0DB

nostic accuracy but also simplifies the detection process, offering a more reliable and effective way to identify the disease.

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