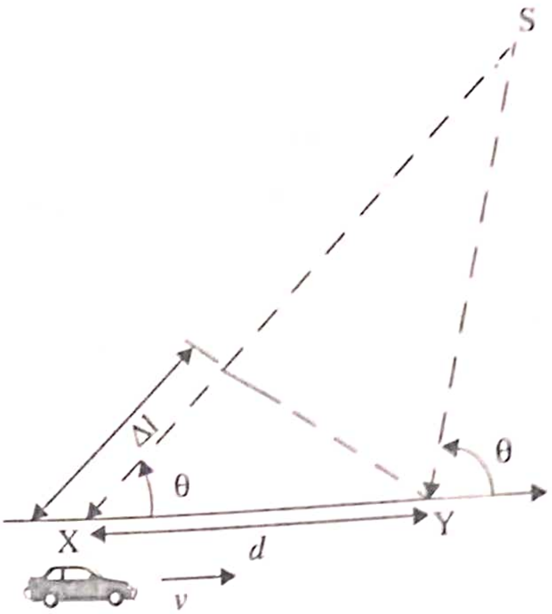


Figure 5.1 Illustration of Doppler effect.

Example 5.1

In the U.S. digital cellular system, if $f_c = 900$ MHz and the mobile velocity is 70 km/hr, calculate the received carrier frequency if the mobile (a) directly toward the transmitter (Positive Doppler Shift), (b) directly away from the transmitter (Negative Doppler Shift), and (c) in a direction perpendicular to the direction of the arrival of the transmitted signal.

Solution

Given:

Carrier frequency $f_c = 900$ MHz

Therefore, wavelength $\lambda = c/f_c = \dfrac{3\times 10^8}{900\times 10^6} = 1/3\,\text{m} = 0.33$ m

Vehicle speed, $v = 70\times 1000/60\times 60 = 19.44$ m/s

(a) The vehicle is moving directly toward the transmitter.

The received frequency is

$$f = f_c + f_d = 900\times 10^6 + \frac{19.44}{0.33} = 900.0000589 \text{ MHz}$$

(b) The vehicle is moving directly away from the transmitter.

The received frequency is given by

$$f = f_c - f_d = 900\times 10^6 - \frac{19.44}{0.33} = 899.9999411 \text{ MHz}$$

(c) The vehicle is moving perpendicular to the angle of arrival of the transmitted signal.

In this case, $\theta = 90°$, $\cos\theta = 0$, and there is no Doppler shift.

The received signal frequency is the same as the transmitted frequency of 900 MHz.

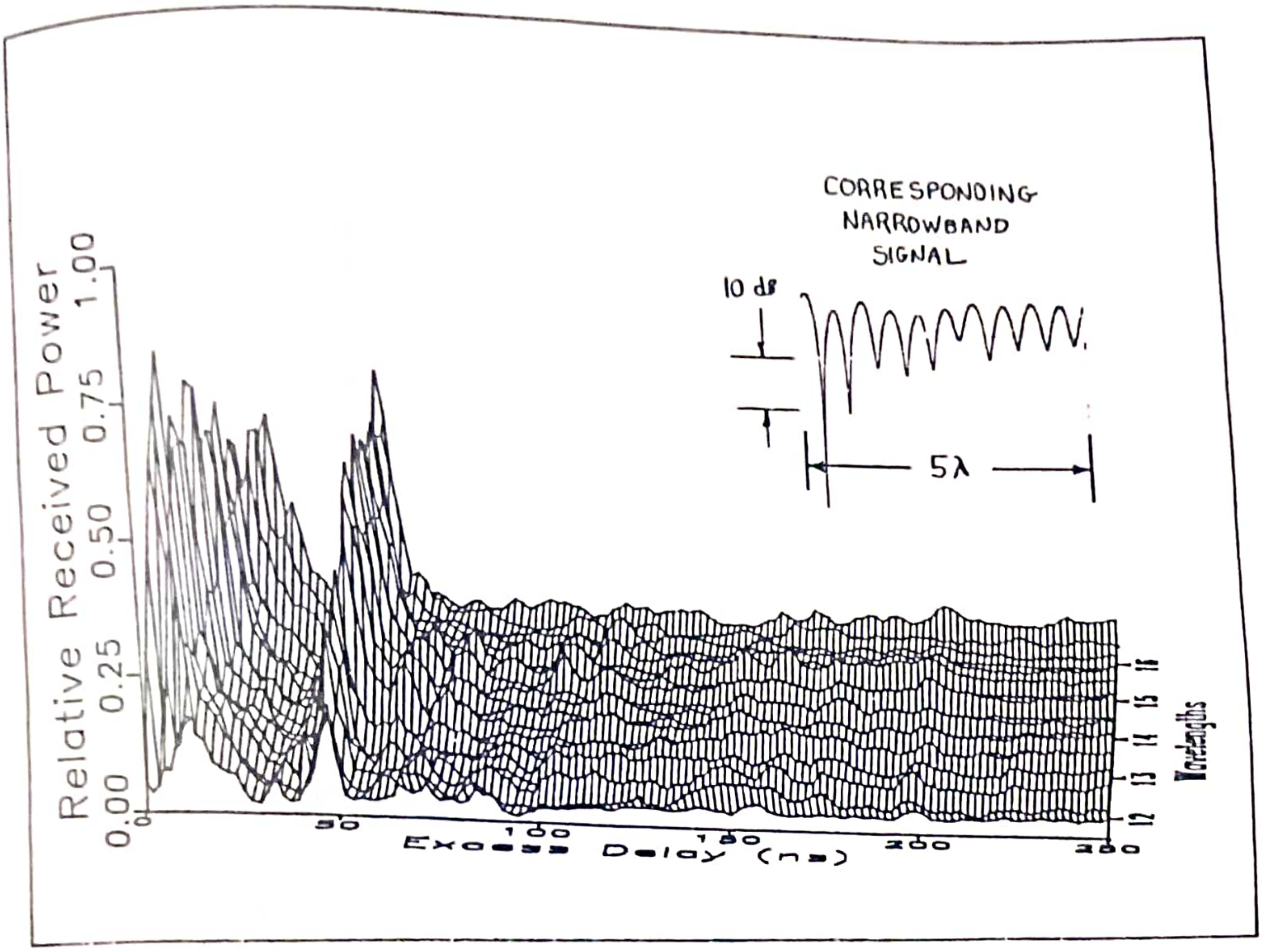


Figure 5.5 Measured wideband and narrowband received signals over a 5λ (0.375 m) measurement track inside a building. Carrier frequency is 4 GHz. Wideband power is computed using Equation (5.19), which can be thought of as the area under the power delay profile. The axis into the page is distance (wavelengths) instead of time.

Figure 5.5 illustrates actual indoor radio channel measurements made simultaneously with a wideband probing pulse having $T_{bb} = 10$ ns, and a CW transmitter. The carrier frequency was 4 GHz. It can be seen that the CW signal undergoes rapid fades, whereas the wideband measurements change little over the 5λ measurement track. However, the local average received powers of both signals were measured to be virtually identical [Haw91].

Example 5.2

An urban RF radio channels are modelled on SIRCIM and SMRCIM statistical channel models (described in Section 5.7.6) with excess delays as large as 150 μs and microcellular channels with excess delays no larger than 4 μs. If the multiple path bin is selected at 70. Calculate (a) $\Delta\tau$, (b) the maximum bandwidth which two models can accurately represent, and (c) if the indoor channel model with excess delays as large as 500 ns exists, calculate the values of (a) and (b).

Solution

The maximum excess delay of the channel model is given by $\tau_N = N\Delta\tau$.

(a) Given, for $\tau_N = 150\ \mu s$, and $N = 70$, $\Delta\tau = \tau_N/N = 2.14\ \mu s$.

(b) The maximum bandwidth that the SMRCIM model can accurately represent is equal to $2/\Delta\tau = 2/2.14\ \mu s = 0.933$ MHz.

For the SMRCIM urban microcell model, $\tau_N = 4\ \mu s$, $\Delta\tau = \tau_N/N = 57.1$ ns. The maximum RF bandwidth that can be represented is

$$2/\Delta\tau = 2/57.1 \text{ ns} = 35 \text{ MHz}.$$

(c) Similarly, for indoor channels, $\Delta\tau = \dfrac{500 \times 10^{-9}}{70} = 7.14$ ns.

The maximum RF bandwidth for the indoor channel model is

$$2/\Delta\tau = 2/7.14 \text{ ns} = 280 \text{ MHz}.$$

Example 5.3

Assume a mobile traveling at a velocity of 10 m/s receives two multipath components at a carrier frequency of 1000 MHz. The first component is assumed to arrive at $\tau = 0$ with an initial phase of 0° and a power of −70 dBm, and the second component which is 3 dB weaker than the first component is assumed to arrive at $\tau = 1\ \mu s$, also with an initial phase of 0°. If the mobile moves directly toward the direction of arrival of the first component and directly away from the direction of arrival of the second component, compute the narrowband instantaneous power at time intervals of 0.1 s from 0 s to 0.5 s. Compute the average narrowband power received over this observation interval. Compare average narrowband and wideband received powers over the interval, assuming the amplitudes of the two multipath components do not fade over the local area.

Solution

Given $v = 10$ m/s, time intervals of 0.1 s correspond to spatial intervals of 1 m. The carrier frequency is given to be 1000 MHz, hence the wavelength of the signal is

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1000 \times 10^6} = 0.3 \text{ m}$$

The narrowband instantaneous power can be computed using Equation (5.21).

Note −70 dBm = 100 pW. At time $t = 0$, the phases of both multipath components are 0°, hence the narrowband instantaneous power is equal to

$$|r(t)|^2 = \left|\sum_{i=0}^{N-1} a_i \exp(j\theta_i(t, \tau))\right|^2$$

$$= \left|\sqrt{100 \text{ pW}} \times \exp(0) + \sqrt{50 \text{ pW}} \times \exp(0)\right|^2 = 291 \text{ pW}$$

Now, as the mobile moves, the phase of the two multipath components changes in opposite directions.

At $t = 0.1$ s, the phase of the first component is

$$\theta_i = \frac{2\pi d}{\lambda} = \frac{2\pi vt}{\lambda} = \frac{2\pi \times 10(\text{m/s}) \times 0.1\ \text{s}}{0.3\ \text{m}}$$

$$= 20.94\ \text{rad} = 2.09\ \text{rad} = 120^\circ$$

Since the mobile moves toward the direction of arrival of the first component, and away from the direction of arrival of the second component, θ_1 is positive, and θ_2 is negative.

Therefore, at $t = 0.1$ s, $\theta_1 = 120^\circ$, and $\theta_2 = -120^\circ$, and the instantaneous power is equal to

$$|r(t)|^2 = \left|\sum_{i=0}^{N-1} a_i \exp(j\theta_i(t,\tau))\right|^2$$

$$= |\sqrt{100\ \text{pW}} \times \exp(j120^\circ) + \sqrt{50\ \text{pW}} \times \exp(-j120^\circ)|^2 = 79.3\ \text{pW}$$

Similarly, at $t = 0.2$ s, $\theta_1 = 240^\circ$, and $\theta_2 = -240^\circ$, and the instantaneous power is equal to

$$|r(t)|^2 = \left|\sum_{i=0}^{N-1} a_i \exp(j\theta_i(t,\tau))\right|^2$$

$$= |\sqrt{100\ \text{pW}} \times \exp(j240^\circ) + \sqrt{50\ \text{pW}} \times \exp(-j240^\circ)|^2 = 79.3\ \text{pW}$$

Similarly, at $t = 0.3$ s, $\theta_1 = 360^\circ = 0^\circ$, and $\theta_2 = -360^\circ = 0^\circ$, and the instantaneous power is equal to

$$|r(t)|^2 = \left|\sum_{i=0}^{N-1} a_i \exp(j\theta_i(t,\tau))\right|^2$$

$$= |\sqrt{100\ \text{pW}} \times \exp(j0^\circ) + \sqrt{50\ \text{pW}} \times \exp(-j0^\circ)|^2 = 291\ \text{pW}$$

It follows that at $t = 0.4$ s, $|r(t)|^2 = 79.3$pW, and at $t = 0.5$ s, $|r(t)|^2 = 79.3$ pW.

The average narrowband received power is equal to

$$\frac{(2)(291) + (4)(79.3)}{6}\ \text{pW} = 149\ \text{pW}$$

Using Equation (5.19), the wideband power is given by

$$E_{a,\theta}[P_{W,B}] = E_{a,\theta}\left[\sum_{i=0}^{N-1} |a_i \exp(j\theta_i)|^2\right] \approx \sum_{i=0}^{N-1} \overline{a_i^2}$$

$$E_{a,\theta}[P_{W,B}] = 100\ \text{pW} + 50\ \text{pW} = 150\ \text{pW}$$

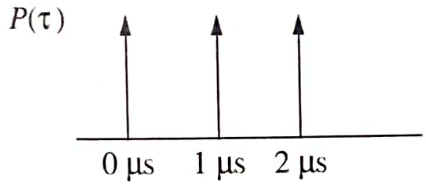
As can be seen, the narrowband and wideband received power are virtually identical when averaged over 0.5 s (or 5 m). While the CW signal fades over the observation interval, the wideband signal power remains constant over the same spatial interval.

In practice, values for $\bar{\tau}$, $\overline{\tau^2}$, and σ_τ depend on the choice of noise threshold used to process $P(\tau)$. The noise threshold is used to differentiate between received multipath components and thermal noise. If the noise threshold is set too low, then noise will be processed as multipath, thus giving rise to values of $\bar{\tau}$, $\overline{\tau^2}$, and σ_τ that are artificially high.

It should be noted that the power delay profile and the magnitude frequency response (the spectral response) of a mobile radio channel are related through the Fourier transform. It is therefore possible to obtain an equivalent description of the channel in the frequency domain using its frequency response characteristics. Analogous to the delay spread parameters in the time domain, *coherence bandwidth* is used to characterize the channel in the frequency domain. The rms delay spread and coherence bandwidth are inversely proportional to one another, although their exact relationship is a function of the exact multipath structure.

Example 5.4

Compute the rms delay spread for the following power delay profile:



(a) Calculate the rms delay spread for the given figure.

(b) If BPSK modulation is used, what is the maximum bit rate that can be sent through the channel without needing an equalizer?

Solution

(a) $\bar{\tau} = \dfrac{[(1)(0) + (1)(1) + (1)(2)]}{1+1} = 1.5\ \mu s$

$$\overline{\tau^2} = \frac{(1)(0)^2 + (1)(1)^2 + (1)(2)^2}{1+1} = \frac{5}{2} = 2.5\ \mu$$

$$\sigma_\tau = \sqrt{[(2.5) - (1.5)^2]} = \sqrt{2.5 - 2.25} = 0.5\ \mu s$$

(b) $\dfrac{\sigma_\tau}{T_s} \le 0.1$

or, $T_s \ge \dfrac{\sigma_\tau}{0.1}$

or, $T_s \ge 5\ \mu s$

Maximum bit rate, $R_s = 1/T_S = 0.2 \times 10^6 = 200$ ksps

$R_b = 200$ kbps

5.4.2 Coherence Bandwidth

While the delay spread is a natural phenomenon caused by reflected and scattered propagation paths in the radio channel, the coherence bandwidth, B_c, is a defined relation derived from the rms delay spread. Coherence bandwidth is a statistical measure of the range of frequencies over which the channel can be considered "flat" (i.e., a channel which passes all spectral components with approximately equal gain and linear phase). In other words, coherence bandwidth is the range of frequencies over which two frequency components have a strong potential for amplitude correlation. Two sinusoids with frequency separation greater than B_c are affected quite differently by the channel. If the coherence bandwidth is defined as the bandwidth over which the frequency correlation function is above 0.9, then the coherence bandwidth is approximately [Lee89b]

$$B_c \approx \frac{1}{50\sigma_\tau} \tag{5.38}$$

If the definition is relaxed so that the frequency correlation function is above 0.5, then the coherence bandwidth is approximately

$$B_c \approx \frac{1}{5\sigma_\tau} \tag{5.39}$$

It is important to note that an exact relationship between coherence bandwidth and rms delay spread is a funtion of specific channel impulse responses and applied signals, and Equations (5.38) and (5.39) are "ball park estimates." In general, spectral analysis techniques and simulation are required to determine the exact impact that time varying multipath has on a particular transmitted signal [Chu87], [Fun93], [Ste94]. For this reason, accurate multipath channel models must be used in the design of specific modems for wireless applications [Rap91a], [Woe94].

Example 5.5

Calculate the mean excess delay, rms delay spread, and the maximum excess delay (10 dB) for the multipath profile given in the figure below. Estimate the 50% coherence bandwidth of the channel. Would this channel be suitable for AMPS or GSM service without the use of an equalizer?

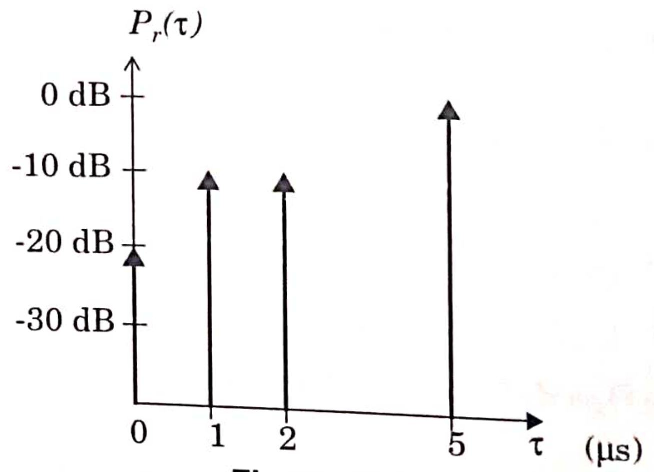


Figure E5.5

Solution
Using the definition of maximum excess delay (10 dB), it can be seen that $\tau_{10\,dB}$ is 5 μs. The rms delay spread for the given multipath profile can be obtained using Equations (5.35)–(5.37). The delays of each profile are measured relative to the first detectable signal. The mean excess delay for the given profile is

$$\bar{\tau} = \frac{(1)(5) + (0.1)(1) + (0.1)(2) + (0.01)(0)}{[0.01 + 0.1 + 0.1 + 1]} = 4.38\ \mu s$$

The second moment for the given power delay profile can be calculated as

$$\overline{\tau^2} = \frac{(1)(5)^2 + (0.1)(1)^2 + (0.1)(2)^2 + (0.01)(0)}{1.21} = 21.07\ \mu s^2$$

Therefore the rms delay spread is $\sigma_\tau = \sqrt{21.07 - (4.38)^2} = 1.37\ \mu s$

The coherence bandwidth is found from Equation (5.39) to be

$$B_c \approx \frac{1}{5\sigma_\tau} = \frac{1}{5(1.37\mu s)} = 146\ \text{kHz}$$

Since B_c is greater than 30 kHz, AMPS will work without an equalizer. However, GSM requires 200 kHz bandwidth which exceeds B_c, thus an equalizer would be needed for this channel.

5.4.3 Doppler Spread and Coherence Time

Delay spread and coherence bandwidth are parameters which describe the time dispersive nature of the channel in a local area. However, they do not offer information about the time varying nature of the channel caused by either relative motion between the mobile and base station, or by movement of objects in the channel. *Doppler spread* and *coherence time* are parameters which describe the time varying nature of the channel in a small-scale region.

Doppler spread B_D is a measure of the spectral broadening caused by the time rate of change of the mobile radio channel and is defined as the range of frequencies over which the received Doppler spectrum is essentially non-zero. When a pure sinusoidal tone of frequency f_c is transmitted, the received signal spectrum, called the Doppler spectrum, will have components in the range $f_c - f_d$ to $f_c + f_d$, where f_d is the Doppler shift. The amount of spectral broadening depends on f_d which is a function of the relative velocity of the mobile, and the angle θ between the direction of motion of the mobile and direction of arrival of the scattered waves. *If the baseband signal bandwidth is much greater than B_D, the effects of Doppler spread are negligible at the receiver.* This is a *slow* fading channel.

Coherence time T_c is the time domain dual of Doppler spread and is used to characterize the time varying nature of the frequency dispersiveness of the channel in the time domain. The Doppler spread and coherence time are inversely proportional to one another. That is,

$$T_C \approx \frac{1}{f_m} \qquad (5.40.a)$$

Coherence time is actually a statistical measure of the time duration over which the channel impulse response is essentially invariant, and quantifies the similarity of the channel response at different times. In other words, coherence time is the time duration over which two received signals have a strong potential for amplitude correlation. If the reciprocal bandwidth of the baseband signal is greater than the coherence time of the channel, then the channel will change during the transmission of the baseband message, thus causing distortion at the receiver. If the coherence time is defined as the time over which the time correlation function is above 0.5, then the coherence time is approximately [Ste94]

$$T_C \approx \frac{9}{16\pi f_m} \tag{5.40.b}$$

where f_m is the maximum Doppler shift given by $f_m = v/\lambda$. In practice, (5.40.a) suggests a time duration during which a Rayleigh fading signal may fluctuate wildly, and (5.40.b) is often too restrictive. A popular rule of thumb for modern digital communications is to define the coherence time as the geometric mean of Equations (5.40.a) and (5.40.b). That is,

$$T_C = \sqrt{\frac{9}{16\pi f_m^2}} = \frac{0.423}{f_m} \tag{5.40.c}$$

The definition of coherence time implies that two signals arriving with a time separation greater than T_C are affected differently by the channel. For example, for a vehicle traveling 60 mph using a 900 MHz carrier, a conservative value of T_C can be shown to be 2.22 ms from Equation (5.40.b). If a digital transmission system is used, then as long as the symbol rate is greater than $1/T_C = 454$ bps, the channel will not cause distortion due to motion (however, distortion could result from multipath time delay spread, depending on the channel impulse response). Using the practical formula of (5.40.c), $T_C = 6.77$ ms and the symbol rate must exceed 150 bits/s in order to avoid distortion due to frequency dispersion.

Example 5.6

Determine the proper spatial sampling interval required to make small-scale propagation measurements which assume that consecutive samples are highly correlated in time. How many samples will be required over 10 m travel distance if $f_c = 1900$ MHz and $v = 50$ m/s. How long would it take to make these measurements, assuming they could be made in real time from a moving vehicle? What is the Doppler spread B_D for the channel?

Solution

For correlation, ensure that the time between samples is equal to $T_C/2$, and use the smallest value of T_C for conservative design.

Using Equation (5.40.b)

$$T_C \approx \frac{9}{16\pi f_m} = \frac{9\lambda}{16\pi v} = \frac{9c}{16\pi v f_c} = \frac{9\times 3\times 10^8}{16\times 3.14\times 50\times 1900\times 10^6}$$

$$T_C = 565\mu s$$

Taking time samples at less than half T_C, at 282.5 μs corresponds to a spatial sampling interval of

$$\Delta x = \frac{vT_C}{2} = \frac{50 \times 565\ \mu s}{2} = 0.014125\ \text{m} = 1.41\ \text{cm}$$

Therefore, the number of samples required over a 10 m travel distance is

$$N_x = \frac{10}{\Delta x} = \frac{10}{0.014125} = 708\ \text{samples}$$

The time taken to make this measurement is equal to $\frac{10\ \text{m}}{50\ \text{m/s}} = 0.2\ \text{s}$

The Doppler spread is $B_D = f_m = \frac{vf_c}{c} = \frac{50 \times 1900 \times 10^6}{3 \times 10^8} = 316.66\text{Hz}$

5.5 Types of Small-Scale Fading

Section 5.3 demonstrated that the type of fading experienced by a signal propagating through a mobile radio channel depends on the nature of the transmitted signal with respect to the characteristics of the channel. Depending on the relation between the signal parameters (such as bandwidth, symbol period, etc.) and the channel parameters (such as rms delay spread and Doppler spread), different transmitted signals will undergo different types of fading. The time dispersion and frequency dispersion mechanisms in a mobile radio channel lead to four possible distinct effects, which are manifested depending on the nature of the transmitted signal, the channel, and the velocity. While multipath delay spread leads to *time dispersion* and *frequency selective fading*, Doppler spread leads to *frequency dispersion* and *time selective fading*. The two propagation mechanisms are independent of one another. Figure 5.11 shows a tree of the four different types of fading.

5.5.1 Fading Effects Due to Multipath Time Delay Spread

Time dispersion due to multipath causes the transmitted signal to undergo either flat or frequency selective fading.

5.5.1.1 Flat fading

If the mobile radio channel has a constant gain and linear phase response over a bandwidth which is greater than the bandwidth of the transmitted signal, then the received signal will undergo *flat fading*. This type of fading is historically the most common type of fading described in the technical literature. In flat fading, the multipath structure of the channel is such that the spectral characteristics of the transmitted signal are preserved at the receiver. However the strength of the received signal changes with time, due to fluctuations in the gain of the channel caused by multipath. The characteristics of a flat fading channel are illustrated in Figure 5.12.

It can be seen from Figure 5.12 that if the channel gain changes over time, a change of amplitude occurs in the received signal. Over time, the received signal $r(t)$ varies in gain, but the spectrum of the transmission is preserved. In a flat fading channel, the reciprocal bandwidth