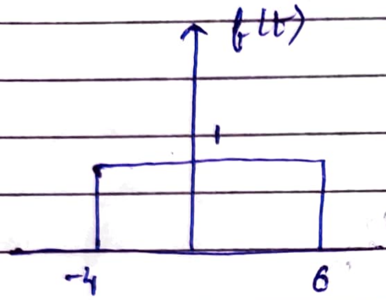
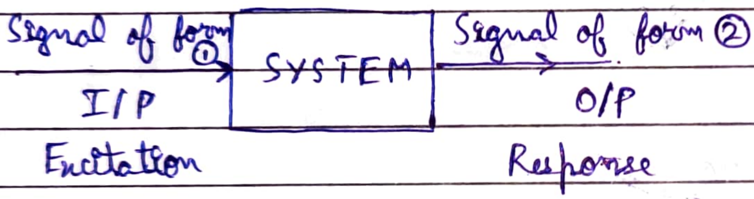
→ Intro to Signals & Systems

Physical quantity that contains information

Signals are expressed mathematically as function of indep. variable which is usually 'time'.

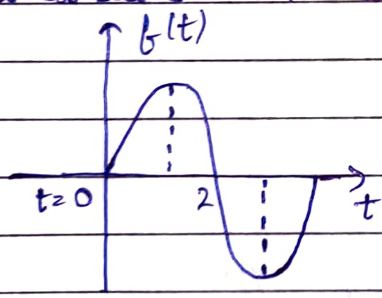


$$f(t) = \begin{cases} 1 & -4 \le t \le 6 \\ 0 & \text{otherwise} \end{cases}$$

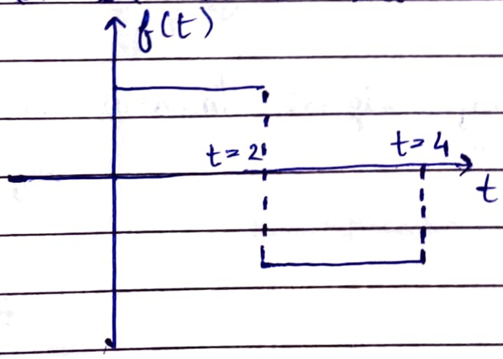


→ Basic types of signals

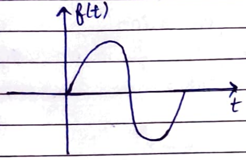
① Continuous in time & continuous in value signals

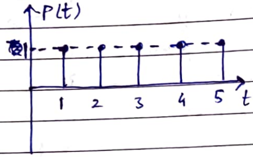


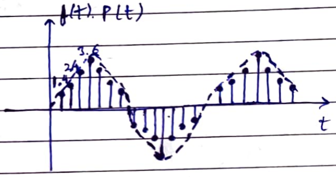
② C. in time but discrete in value



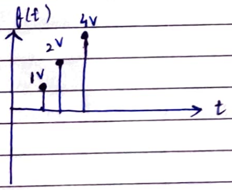
③ C. in value but discrete in time





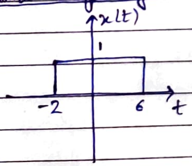


④ D in time & D in value



→ Operations on signals

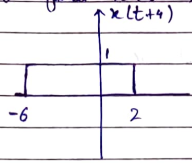
① Time shifting



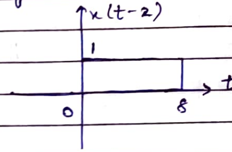
To find $x(t \pm t_0)$ — $+$ → Advance, $-$ → Delay; t_0 → Any const.

Delay : Shift signal towards right
Advance : " " " left
} by t_0

Q) To find $x(t+4)$



Q) To find $x(t-2)$



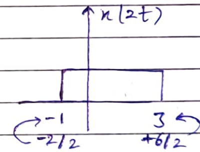
② Time scaling operation

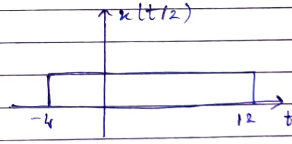
To find $x(\alpha t)$

α ↑ Scaling factor

$\alpha > 1 \rightarrow$ Signal compression
$\alpha < 1 \rightarrow$ Signal expansion

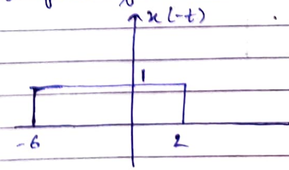
Divide the existing limits by 'α'

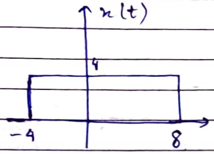




③ Time reversal / folding operation

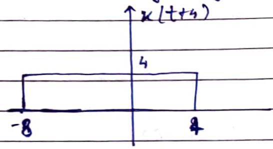
To find $x(-t)$

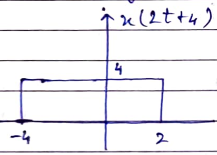


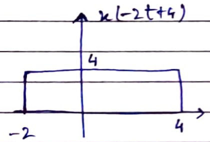


Find $x(-2t+4)$

M-1 : Moving right to left

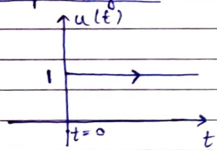






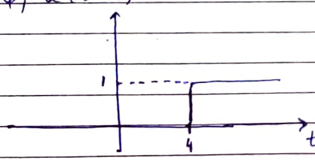
→ Elementary signals

① Unit step signal (Heaviside step function).

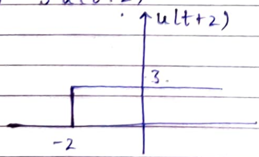


$u(t) = 1 \quad t > 0$
$\quad\;\; = 0 \quad t < 0$

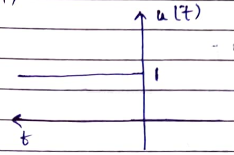
Q) $u(t-4)$



Q) $3u(t+2)$

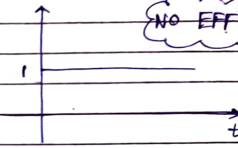


Q) $u(-t)$

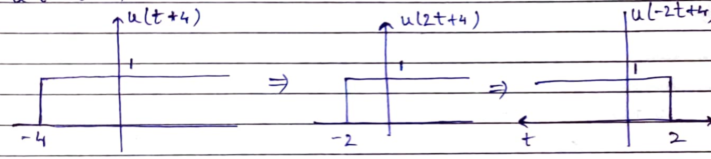


Q) $u(2t) = u(3t) = u(4t)$

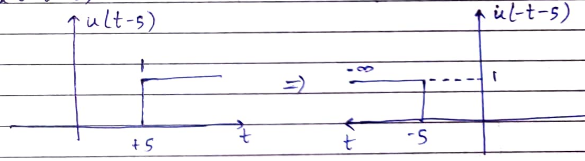
NO EFFECT



Q) $u(-2t+4)$



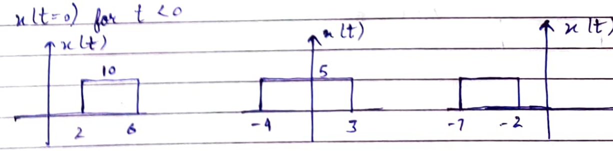
Q) $u(-t-5)$

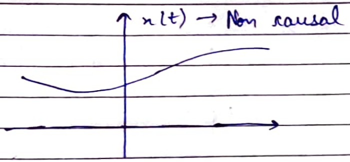


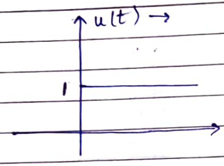
Signals

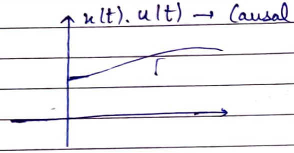
Causal | Non-Causal | Anti-Causal

$x(t=0)$ for $t<0$

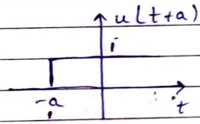


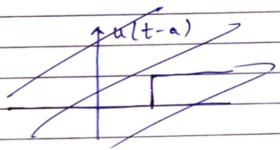


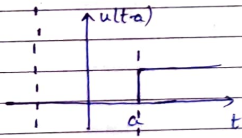


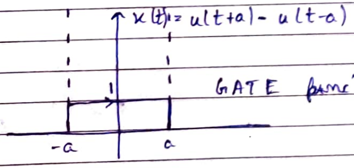


q) $x(t) = u(t+a) - u(t-a)$



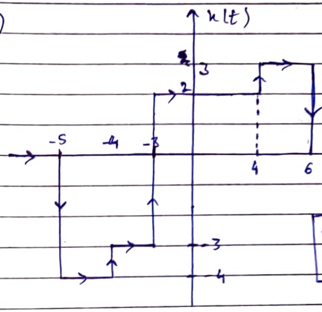






GATE func^n

q)

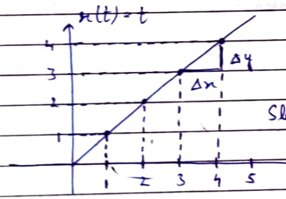


$$\boxed{x(t) = -4u(t+5) + u(t+4) + 5u(t+3) + u(t-4) - 3u(t-6)}$$

② Ramp signal

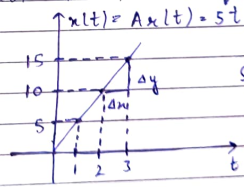
$x(t) = t \qquad t \geq 0$

$\;\;\;\;\;\; = 0 \qquad \text{otherwise}$



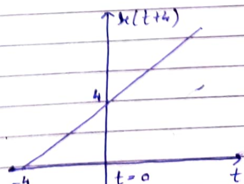
$\text{Slope} = \frac{3-2}{3-2} =$

$x(t) = A\,x(t) = At \quad \text{for} \quad t \geq 0$



$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{15-10}{3-2} = 5$

$$\boxed{x(t) = t.\,u(t)}$$



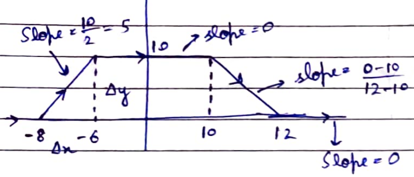
$\text{Slope} = \frac{\Delta y}{\Delta x} = 1$

$\Delta y = \Delta x$

$\Delta y = 0 - (-4)$

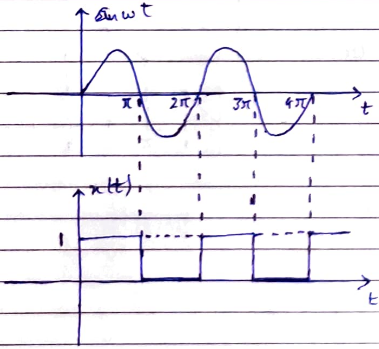
$\boxed{\Delta y = 4}$

$$\boxed{x(t) = 5x(t+8) - 5x(t+6) - 5x(t-10) + 5x(t-12)}$$



Q) $u(t) = 1 \quad t > 0$
$\quad\;\; = 0 \quad t < 0$

To draw $x(t) = u(\sin \omega t) = 1 \quad \sin \omega t > 0$
$\qquad\qquad\qquad\qquad\qquad\;\; = 0 \quad \sin \omega t < 0$



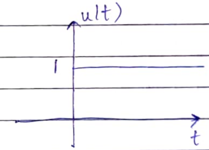
→ Basic Signals

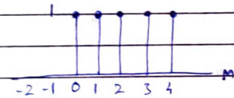
① Unit step signal
② Impulse function
③ Signum func"
④ Exponential func"
⑤ Unit Ramp signal
⑥ Parabolic signal
⑦ Rectangular signal
⑧ Triangular signal
⑨ Sinusoidal "
⑩ Sinc func"
⑪ Sampling func"

→ Unit Step Signal

$u(t)$ & $u(n)$

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \qquad u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$





Properties

1) $[u(t)]^n = u(t)$

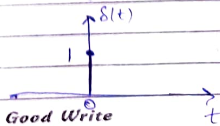
2) $u(at) = u(t)$

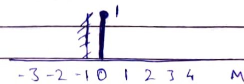
$u(at - t_0) = u[a(t - t_0/a)]$
$\qquad\qquad = u(t - t_0/a)$

$$\boxed{\begin{aligned} u(2t-4) &= u(t - 4/2) \\ &= u(t-2) \end{aligned}}$$

→ Impulse function ($\delta(t)$)

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases} \qquad \delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$





Properties

1) $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

2) $\delta(n-k) = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases}$

3) $\delta(n) = u(n) - u(n-1)$

4) $f(t)\,\delta(t) = f(0)$

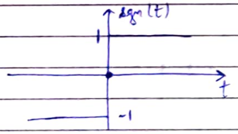
5) $\delta(t-t_0)\, f(t) = f(t_0)$

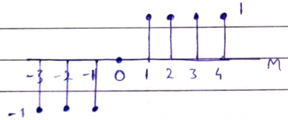
6) $\delta(kt) = \frac{1}{|k|}\, \delta(t)$

7) $\delta(-t) = \delta(t) \rightarrow$ Even func^n

→ Signum function ($sgn(t)$, $sgn(n)$)

$sgn(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$ $\qquad sgn(n) = \begin{cases} 1 & n > 0 \\ 0 & n = 0 \\ -1 & n < 0 \end{cases}$





Relation b/w $u(t)$ & $sgn(t)$

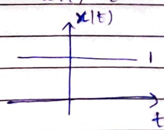
$\boxed{sgn(t) = 2u(t) - 1}$

→ Exponential signal [$x(t) = e^{\alpha t}$]

Shape of exponential depends upon α

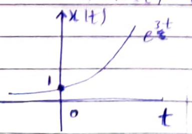
i) When $\alpha = 0$

$x(t) = e^{0 \cdot t} = 1$



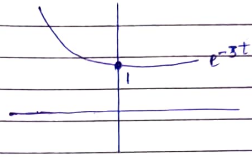
ii) When $\alpha > 0$

$\alpha = 3 \Rightarrow x(t) = e^{3t}$

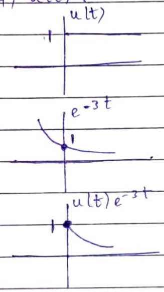


iii) When $\alpha < 0$

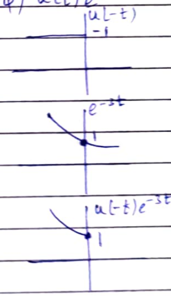
$\alpha = -3 \Rightarrow x(t) = e^{-3t}$



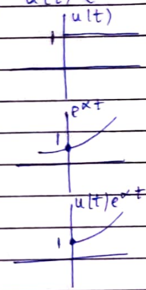
Q) $u(t)\, e^{-3t}$



Q) $u(-t)\, e^{-3t}$

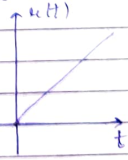


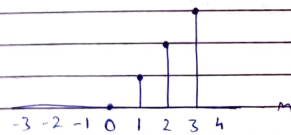
Q) $u(t)\, e^{\alpha t}$



→ Unit Ramp signal [$r(t)$ or $r(n)$]

$r(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$ $\qquad r(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$





$\boxed{\int u(t) = r(t)}$

$\boxed{u(t) = \frac{d}{dt} r(t)}$

$\int \delta(t) = u(t)$

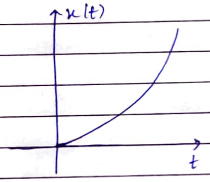
$\iint \delta(t) = r(t)$

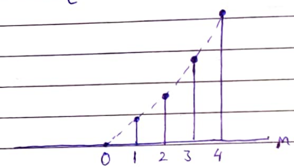
$\delta(t) = \frac{d^2}{dt^2} r(t)$

→ **Unit Parabolic signal**

$$x(t) = \begin{cases} \frac{t^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$x(n) = \begin{cases} \frac{n^2}{2} & n \geq 0 \\ 0 & n < 0 \end{cases}$$





$$\int r(t) = \int t = \frac{t^2}{2} = x(t)$$

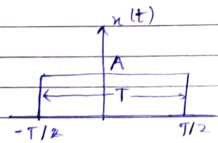
$$\iint u(t) = \frac{t^2}{2} = x(t)$$

$$\iiint \delta(t) = x(t)$$

$$\boxed{\begin{array}{l} x(t) = \int r(t) = \iint u(t) = \iiint \delta(t) \\ r(t) = \frac{d}{dt} x(t) \quad u(t) = \frac{d^2}{dt} x(t) \quad \delta(t) = \frac{d^3}{dt} x(t) \end{array}}$$

→ **Rectangular pulse** $[x(t)]$

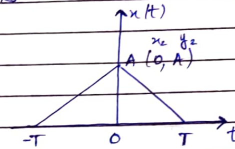
$$x(t) = A\,\text{rect}\left(\frac{t}{T}\right)$$



A = amplitude of rect.
T = Period of rect.

→ **Triangular signal** $[x(t)]$

$$x(t) = A\left(1 - \frac{|t|}{T}\right)$$



$(-T, 0)$ — x_1 y_1 $\qquad$ $(T, 0)$ — x_1 y_1

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \cancel{y}\,\frac{A - 0}{0 - (-T)}(x - (-T))$$

$$y = \frac{A}{T}(x + T)$$

$$x(t) = \frac{A}{T}(t + T) = \frac{A}{\cancel{T}}\left(\cancel{T}\left(1 + \frac{t}{T}\right)\right)$$

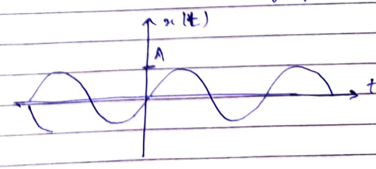
$$x(t) = A\left(1 + \frac{t}{T}\right)$$

Similarly $x(t) = A\left(1 - \frac{t}{T}\right)$

→ **Sinusoidal Signal** $[x(t)\ \&\ x(n)]$

$$x(t) = A\cos(\omega_0 t + \phi) \quad \text{OR} \quad A\sin(\omega_0 t + \phi)$$

A → Amplitude; ω_0 → Fundamental freq.; t → Time period; ϕ → Phase shift



$$\omega_0 = 2\pi f$$

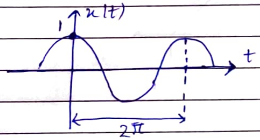
$$f = \frac{\omega_0}{2\pi}$$

$$\boxed{T = \frac{1}{f} = \frac{2\pi}{\omega_0}}$$

Q) $x(t) = \cos t$

$A = 1, \quad \omega_0 = 1$

$T = \frac{2\pi}{\omega_0} = 2\pi$

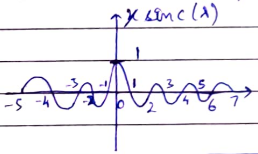


→ Sinc & sampling functions

Sinc

- Denoted with $\text{sinc}(\lambda)$
- Normalized funcⁿ
- $\text{sinc}(\lambda) = \frac{\sin \pi\lambda}{\pi\lambda}$

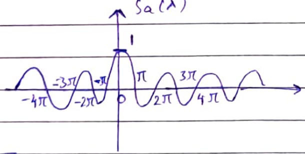
$= 0 \quad \lambda = \pm 1, \pm 2, \ldots$



Sampling

- Denoted with $Sa(\lambda)$
- Unnormalized funcⁿ sinc
- $Sa(\lambda) = \frac{\sin\lambda}{\lambda}$

$= 0 \quad \lambda = \pm\pi, \pm 2\pi, \ldots$



$$\boxed{\text{sinc}(0) = Sa(0) = 1}$$

→ Classification of Signals

1) Continuous time & discrete time signals
2) Even & Odd signals
3) Periodic & Aperiodic signals
4) Energy signals or power signals.
5) Deterministic & Non-deterministic signals (Random)
6) Real & imaginary signals

- For discrete time signals time is discrete amplitude is continuous
- For digital signal both amplitude & time are discrete

→ Even & Odd signals

$x(t) = x(-t) \qquad x(-t) = -x(t)$

$x(t) = \underbrace{x_e(t)}_{\text{Even}} + \underbrace{x_o(t)}_{\text{Odd}}$

$x(-t) = x_e(t) - x_o(t)$

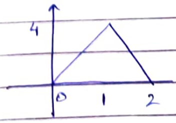
$x(t) + x(-t) = 2x_e(t)$

$$\boxed{x_e(t) = \frac{x(t) + x(-t)}{2}}$$

$x(t) - x(-t) = 2x_o(t)$

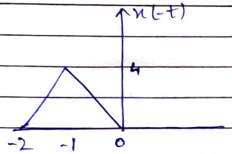
$$\boxed{x_o(t) = \frac{x(t) - x(-t)}{2}}$$

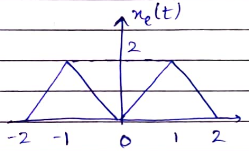
Q) Find $x_e(t)$ & $x_o(t)$ of signal.



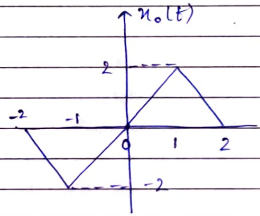
$x_e(t) = \frac{x(t) + x(-t)}{2}$

$x_o(t) = \frac{x(t) - x(-t)}{2}$





$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$



$x_o(t) = [x(t) - x(-t)]$

Q) Trace the even & odd component of signal

$x(t) = \underset{E}{\cos t} + \underset{O}{\sin t} + \underset{O}{\cos t \sin t} + \underset{E}{\sin^2 t}$ $\sin^2 t = \frac{1-\cos 2t}{2} \to E$

$x_e(t) = \frac{x(t) + x(-t)}{2}$ $x_e(t) = \cos t + \sin^2 t$

$x_o(t) = \frac{x(t) - x(-t)}{2}$ $x_o(t) = \sin t + \cos t \sin t$

→ Checking of periodicity

Method - 1 : Ratio

Consider two Signals $x_1(t)$ & $x_2(t)$ with periods T_1 & T_2 resp. when they are summed the resultant signal is said to be periodic when $\frac{T_1}{T_2}$ = Rational no.

Q) $x(t) = \cos 4t + \sin \pi t$

$\omega_1 = 4$ $\omega_2 = \pi$

$T_1 = \frac{2\pi}{4}$ $T_2 = \frac{2\pi}{\pi}$

$T_1 = \frac{\pi}{2}$ $T_2 = 2$

$\frac{T_1}{T_2} = \frac{\pi/2}{2} = \frac{\pi}{4} \to$ Aperiodic

Q) $x(t) = \cos 2\pi t + \sin 4\pi t$

$\omega_1 = 2\pi$ $\omega_2 = 4\pi$

$T_1 = \frac{2\pi}{2\pi}$ $T_2 = \frac{2\pi}{4\pi}$

$T_1 = 1$ $T_2 = \frac{1}{2}$

$\frac{T_1}{T_2} = 2 \to$ Periodic

Method 2 : GCD

If GCD of freq. is possible then it is said to be periodic otherwise aperiodic.

→ Energy & Power Signals

Total energy transmitted to load $E = \int_{-T}^{T} x^2(t)\,dt$ [Finite duraⁿ]

A signal is said to be an power / Energy signal if the total Energy / power transmitted is finite
$0 < E < \infty$ OR $0 < P < \infty$

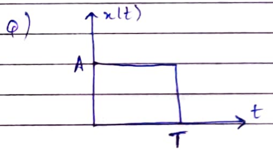
$E = \lim_{T\to\infty} \int_{-T}^{T} x^2(t)\,dt$ [Infinite duraⁿ]

Average Power $P = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt$ (Finite duraⁿ)

$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt$ (Infinite ")

Power of Energy signal $= 0$
Energy " power " $= \infty$

Q)



$x(t) = \begin{cases} A & 0 < t < T \\ 0 & \text{else} \end{cases}$

$E = \int_{-T}^{T} x^2(t)\,dt$

$= \int_{0}^{T} A^2\,dt$

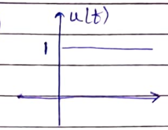
$= A^2 \int_0^T dt$

$= A^2 [T - 0]$

$= A^2 T$ Joules → Finite

Energy signal ✓

Q)



$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$ → Infinite duraⁿ

$E = \lim_{T\to\infty} \int_{-T}^{T} x^2(t)\,dt = \lim_{T\to\infty} \int_0^{\infty} 1^2\,dt$

$= \lim_{T\to\infty} [T]_0^{\infty} = \infty$ → Infinite

Not an energy signal

$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\,dt$

$= \lim_{T\to\infty} \frac{1}{2T} \int_0^T 1\,dt$

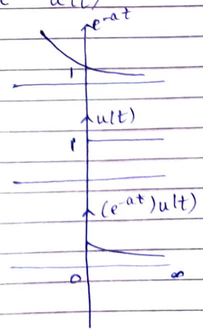
$= \lim_{T\to\infty} \frac{1}{2T} [t]_0^T$

$= \lim_{T\to\infty} \frac{1}{2\cancel{T}} [\cancel{T} - 0]$

$= \frac{1}{2} = 0.5$ W

Power signal ✓

Q) $e^{-at} u(t)$



$E = \lim_{T\to\infty} \int_{-T}^{T} x^2(t)\,dt$

$= \lim_{T\to\infty} \int_0^{\infty} [e^{-at} \underbrace{u(t)}_{1}]^2\,dt$

$= \lim_{T\to\infty} \int_0^{\infty} [e^{-at}]^2\,dt$

$= \lim_{T\to\infty} \int_0^{T} [e^{-2at}]\,dt$

$= \lim_{T\to\infty} \frac{e^{-2at}}{-2a} - \frac{e^0}{-2a}$

$= \frac{e^{-\infty}}{-2a} + \frac{e^0}{2a} = \frac{1}{2a}$ → Finite

Energy signal ✓

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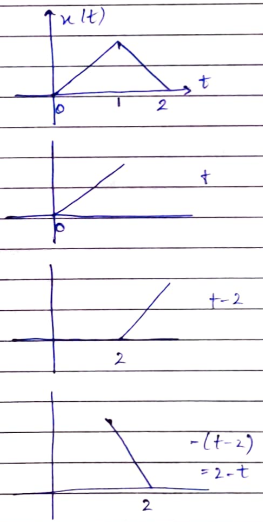
Q) If energy of given signal $x(t) = e^{-5t}u(t)$ is $1/10$ J. Then energy of time scaled version of signal $x(2t)$ is

If Energy of $x(t) = E$
then " " $x(at) = \frac{E}{a}$

Energy of $x(t) = 1/10$
" " $x(2t) = \frac{1/10}{2} = \frac{1}{20}$ J

Q)



$E = \int_{-T}^{T} x^2(t)\,dt$

$= \int_0^1 x^2(t)\,dt + \int_1^2 x^2(t)\,dt$

$= \int_0^1 t^2\,dt + \int_1^2 -(t-2)^2\,dt$

$= \left[\frac{t^3}{3}\right]_0^1 + \int_1^2 (4+t^2-4t)\,dt$

$= \frac{2}{3}$ J

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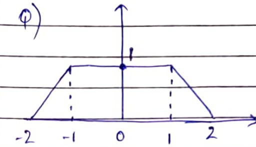
Q)



$E = \int_{-T}^{T} x^2(t)\,dt$

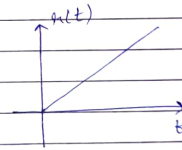
$= \int_{-2}^{2} x^2(t)\,dt$

$= \int_{-2}^{-1} (t+2)^2\,dt + \int_{-1}^{1} 1^2\,dt + \int_1^2 (2-t)^2\,dt$

$= \frac{8}{3}$ J

Q) Check energy & power of ramp signal.

$x(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$



$E = \lim_{T\to\infty} \int_{-T}^{T} x^2(t)\,dt$

$= \lim_{T\to\infty} \int_0^T x^2(t)\,dt$

$= \lim_{T\to\infty} \int_0^T t^2\,dt$

$= \lim_{T\to\infty} \frac{T^3}{3}$

$E = \infty$

Energy ✗

$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\,dt$

$= \lim_{T\to\infty} \frac{1}{2\cancel{T}} \times \frac{T^{\cancel{3}\,2}}{3}$

$P = \infty$

Neither energy nor power signal

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→ Classification of signals

1) Linear & Non-linear system
2) Time Variant & Time invariant system
3) ~~LTI~~ LTV & LTI systems
4) Static & Dynamic systems
5) Causal & Non-Causal
6) Invertible & Non-invertible systems
7) Stable & Unstable system.

→ Linear & Non-linear system

A system is said to be linear if it satisfies the superposⁿ principle. Consider a system with input $x_1(t)$, $x_2(t)$ & output $y_1(t)$ & $y_2(t)$

$$T[a_1x_1(t) + a_2x_2(t)] = a_1T[x_1(t)] + a_2T[x_2(t)]$$

Q) $y(t) = x^2(t)$

$T[x_1(t)] = x_1^2(t)$
$T[x_2(t)] = x_2^2(t)$ } → $a_1x_1^2(t) + a_2x_2^2(t)$ (Individual response)

$\neq$

$T[a_1x_1(t) + a_2x_2(t)] = [a_1x_1(t) + a_2x_2(t)]^2$ (Common response)

Non-linear system

Q) $y(t) = x(t)$

$T[x_1(t)] = x_1(t)$
$T[x_2(t)] = x_2(t)$ } → $a_1x_1(t) + a_2x_2(t)$

$\parallel$

$T[a_1x_1(t) + a_2x_2(t)] = a_1x_1(t) + a_2x_2(t)$

Linear system

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→ Time Variant & Time Invariant

A system is said to be time variant if its input, output characteristics changes with time otherwise said to be time invariant.

Condition for time invariance is $\boxed{y(n,k) = y(n-k)}$
where $y(n,k) = T[x(n-k)]$

Q) $y(n) = x(n) + \cancel{x(2n)}\ x(n-2)$

$y(n,k) = T[x(n-k)] = x(n-k) + x(n-2-k)$

$y(n-k) = x(n-k) + x(n-k-2)$

$y(n,k) = y(n-k)$

∴ Time invariant

Q) $y(n) = x(n) + n(x(n-3))$

$y(n,k) = T[x(n-k)] = x(n-k) + \cancel{(n-k)}^{n}\, x(n-3-k)$

$y(n-k) = x(n-k) + (n-k)x(n-3-k)$

$y(n,k) \neq y(n-k)$

Time variant

→ Linear time variant & Linear time invariant

A system is said to be LTV when it satisfies both linearity & time variance.
A system is said to be LTI when it satisfies both linearity & time invariance.

Q) $y(n) = nx^2(n)$

$y_1(t) = T[x_1(t)] = n\,x_1^2(t)$
$y_2(t) = T[x_2(t)] = n\,x_2^2(t)$ } → $a_1nx_1^2(t) + a_2nx_2^2(t)$

$\neq$

$T[a_1x_1(t) + a_2x_2(t)] = n[a_1x_1(t) + a_2x_2(t)]^2$

Non-linear

→ Static & Dynamic system

Q) $y(n) = x(n)$

$y(0) = x(0) \rightarrow$ Present

Static system

Q) $y(n) = 2x^2(t)$

$y(-2) = \cancel{2(-2)^2}\ 2x^2(-2)$ → Present

Static

Q) $y(n) = x(n) + x(n-1)$

$y(1) = x(1) + x(1-1)$

$= x(1) + x(0)$ ↳ Past

Dynamic

Q) $y(t) = x(t) + x(t+3)$

$y(-1) = x(-1) + x(-1+3)$

$y(-1) = x(-1) + x(2)$

Present ↳ Future

Dynamic

→ Causal & Non-Causal system

A system dependent on present & past inputs & not on future input → Causal

A system dependent on present, past & future inputs → Non Causal

Q) $y(n) = x(n) + \frac{1}{x(n-1)}$

$y(1) = x(1) + \frac{1}{x(0)}$

Present, Past

Causal

Q) $y(t) = 2x(t) + \frac{1}{x^2(t)}$

$y(0) = \cancel{2t}\ 2x(0) + \frac{1}{x^2(0)}$

Causal

Q) $y(n) = x(n) + \frac{1}{2x(n+1)}$

$y(0) = x(0) + \frac{1}{2x(1)}$

Non-Causal

All static systems are Causal

All Causal systems may not be static

All non-causal systems are dynamic

Vice-Versa is not true

→ Invertible & Non-invertible system

If input appears at the output → Invertible

If $y(t) \neq x(t)$ → Non-invertible

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$

$y(t) = x(t) \cdot h(t)$

→ Stable & Unstable system

When system produces bounded output for a bounded input → Stable

Q) $y(n) = x^2(n)$

$x(n) \rightarrow \boxed{\quad} \rightarrow y(n) = x^2(n)$

Let $x(n) = u(n)$

$y(n) = [u(n)]^2 = u(n)$ → Stable

Q) $y(t) = 2x^2(t)$

Let $\cancel{y(t}\ x(t) = u(t)$

$y(t) = 2u^2(t) = 2u(t)$ → Stable

Q) $y(t) = \int x(t)$

Let $x(t) = u(t)$

$y(t) = \int u(t) = r(t)$ → Unstable

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Q) $x(t) = 4\cos \pi t + 5\sin 2\pi t + 2\sin 3\pi t$

$\omega_{0_1} = \pi \qquad \omega_{0_2} = 2\pi \qquad \omega_{0_3} = 3\pi$

$T_1 = \frac{2\pi}{\pi} = 2 \qquad T_2 = 1 \qquad T_3 = \frac{2}{3}$

$\cancel{\frac{T_1}{T_2} = \frac{2\pi}{\pi} = 2} \qquad \frac{T_1}{T_2} = \frac{2}{1} = 2 \qquad \frac{T_2}{T_3} = \frac{1}{2/3} = \frac{3}{2}$

$\therefore x(t)$ is periodic

$T = \text{LCM of } (T_1, T_2, T_3)$

$= \text{LCM of } \left(\frac{2}{1}, \frac{1}{1}, \frac{2}{3}\right) \qquad \left[\frac{\text{LCM of Num.}}{\text{HCF of Den.}}\right]$

$= \frac{2}{1} = 2$

Q) $x(t) = 4 + \cos^2 4\pi t$

$x(t) = 4 + \left[\frac{1+\cos 8\pi t}{2}\right]$

$= 4 + \frac{1}{2} + \frac{1}{2}\cos 8\pi t$

$= \frac{9}{2} + \frac{1}{2}\cos 8\pi t$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \cos^2\theta - 1 + \cos^2\theta$
$= 2\cos^2\theta - 1$
$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\omega_0 = 8\pi$

$T = \frac{2\pi}{8\pi} = \frac{1}{4} = 0.25\ s$

→ Effect of time scaling on periodicity & period

If $x(t)$, $T = T_1$
$x(\alpha t)$, $T = \frac{T_1}{\alpha}$

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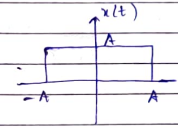
→ Effect of time scaling on energy of a signal

Consider an energy signal existing for $-4 < t < 4$ with $E_x = 100$ J

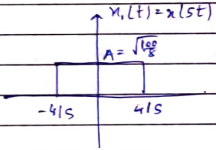
If $x_1(t) = x(5t)$, find $E_{x_1}(t) = ?$

$E_x(t) = \int_{-4}^{4} A^2\, dt = 100$ (Given)



$= A^2 \times 8 = 100$

$\Rightarrow A = \sqrt{\frac{100}{8}}$



$x_1(t) = x(5t)$

$E_{x_1}(t) = \int_{-4/5}^{4/5} \left(\sqrt{\frac{100}{8}}\right)^2 dt$

$= 20$ J

$\boxed{\begin{array}{l}\text{Energy of } x(t) = E \\ \text{Energy of } x(\alpha t) = \frac{E}{\alpha}\end{array}}$

→ Effect of time scaling on power of a signal

No effect

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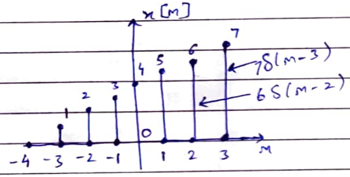
→ Representation of discrete time signals

1) Tabular form

$x[n] = \{1, 2, 3, 4, 5, 6, 7\}$



2) Impulse representation

$x[n] = \{1, 2, 3, 4, 5, 6, 7\}$

$x[n] = \delta(n+3) + 2\delta(n+2) + 3\delta(n+1) + 4\delta(n) + 5\delta(n-1) + 6\delta(n-2) + 7\delta(n-3)$

→ Time scaling on discrete time signals

To find $x[\alpha n]$ ← Scaling factor

$\alpha > 1 \rightarrow$ Decimation in time

$\alpha < 1 \rightarrow$ ~~Imp~~ Interpolation

Q) $x[n] = \cancel{\{1, 2, 3, 4, 5, 6, 7\}}$ $\{2, 5, 8, 7, 9, 12, 14, 18, 20\}$ (n: $-4, -3, -2, -1, n=0, 1, 2, 3, 4$)

To find $x_1[n] = x[2n]$

$x_1[n] = x[2n] = \{9, 14, 20, 8, 2\}$ (2 values skipped)

$x_2[n] = x[3n] = \{\cancel{6}\,5, 9, 18\}$ (3 " ")

~~$x_1[0] =$~~

Interpolation $[\alpha < 1]$

$x[n] = \{1, 2, 3, 4, 5, 6, 7\}$

$x_1[n] = x\left[\frac{n}{\alpha}\right]$

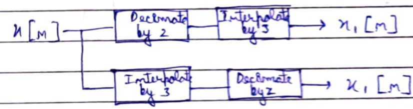
$(\alpha - 1)$ zeros will be appended b/w each two values of $x[n]$

Q) $x[n] = \{5, 9, 3, 12, 14, 18\}$

$x\left[\frac{n}{3}\right] = \{5, 0, 0, 9, 0, 0, 3, 0, 0, 12, 0, 0, 14, 0, 0, 18\}$

Q) $x[n] = \{3, 5, 6, 8, 9\}$ ($n=0$ at 5)

To find $x_1[n] = x\left[\frac{2}{3}n\right]$



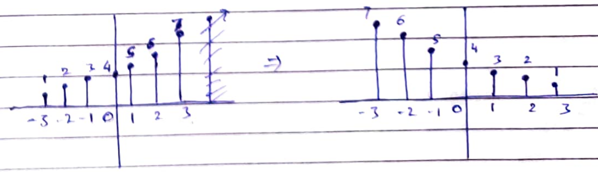
$x_1[2n] = \{5, 8\}$

$x_1\left[\frac{2n}{3}\right] = \{5, 0, 0, 8\} = x_1[n]$

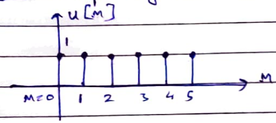
→ Time reversal operation

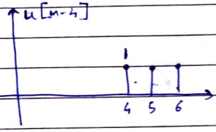
$x[n] = \{1, 2, 3, 4, 5, 6, 7\}$

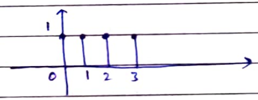
$x[-n] = \{7, 6, 5, 4, 3, 2, 1\}$



→ Unit step signal







→ Unit Impulse / Sample / Delta function

$$\frac{d}{dt}u(t) = \delta(t)$$

$$\int_{-\infty}^{t}\delta(\tau)\,d\tau = u(t)$$

$$\int_{-\infty}^{+\infty}\delta(\tau)\,d\tau = 1$$

→ Relation b/w impulse & step signal

① $u[n]-u[n-1] = \delta[n]$

② $\sum_{k=-\infty}^{n}\delta[k] = u[n]$

for $n<0$ $\sum_{k=-\infty}^{n}\delta[k] = 0$

for $n\geq 0$ $\sum_{k=-\infty}^{n}\delta[k] = 1$

→ Properties of delta function

① $\delta[\alpha n] = \delta[n]$

② $x[n].\delta[n] = x[0].\delta[n]$

$x[n].\delta[n-n_0] = x[n_0]\,\delta[n-n_0]$

③ $\sum_{n=-\infty}^{\infty} x[n].\delta[n] = x[0]$

$\sum_{n=-\infty}^{\infty} x[n].\delta[n-n_0] = x[n_0]$

Even & Odd sequence

$x[-n] = x[n]$ → Even

$x[-n] = -x[n]$ → Odd

$x[n] = \{2, 2, \underset{\uparrow}{1}, 3, 2\}$ → Even

$x[n] = \{-2, -3, \underset{\uparrow}{0}, 3, 2\}$ → Odd

$x[-n] = \{2, 3, 0, -3, -2\}$ → Odd

Any D.T sequence can never be odd if it is having any non-zero value at origin

$$\boxed{x_e[n] = \frac{x[n]+x[-n]}{2}}$$

$$\boxed{x_o[n] = \frac{x[n]-x[-n]}{2}}$$

Q) $y(n) = x(n).x(n-1)$

For i/p $u(n)$

$y(n) = u(n).u(n-1)$

For i/p $-u(n)$

$y(n) = \{-u(n)\}\{-u(n-1)\}$

$= u(n).u(n-1)$

∴ Non-invertible

Q) $y(n) = \begin{cases} x(n+2) & n\geq 0 \\ x(n) & n<-1 \end{cases}$

For $x(n) = \delta(n)$

$y(n) = \delta(n+2)$ for $n\geq 0$

$= 0$

For $x(n) = 2\delta(n)$

$y(n) = 2.\delta(n+2)$ for $n\geq 0$

$= 0$

∴ Non-invertible

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Q) $y(t) = \int_{-\infty}^{t} x(t).\ \sin 2t\ dt$

$x(t) = \sin 2t$

$y(t) = \int_{-\infty}^{t} \sin^2 2t\ dt = \int_{-\infty}^{t} \frac{1-\cos 4t}{2}\ dt$

$= \frac{1}{2}\left[\int_{-\infty}^{t} dt - \int_{-\infty}^{t} \cos 4t\ dt\right]$

$\therefore$ Unstable

→ Invertible & Non-invertible systems

If i/p produces distinct o/p → Invertible

If i/p can be determined by observing the o/p.

Q) $y(t) = 4x(t)$

$x(t)$	$y(t) = 4x(t)$
0	0
-1	-4
+1	+4

$\therefore$ Invertible system.

Q) $y(t) = x^2(t)$

$x(t)$	$y = x^2(t)$
0	0
+1	1
-1	1
2	4
-2	4

$\therefore$ Non-invertible system

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→ Test Signals

$u(t)$	$u(n)$
$-u(t)$	$-u(n)$
$\delta(t)$	$\delta(n)$
$-\delta(t)$	$-\delta(n)$
$2u(t)$	$2u(n)$
$2\delta(t)$	$2\delta(n)$

Q) $y(t) = \sin t.\ x(t)$

For $x(t) = \delta(t)$

$y(t) = \sin t.\ \delta(t)$

$= \sin 0.\ \delta(t) = 0$

$\therefore$ Non-invertible system

For $x(t) = 4\delta(t)$

$y(t) = \sin t.4\delta(t) = 0$

$[x(t).\delta(t) = x(0).\delta(t)]$
$= 0$

Q) $y(n) = x(n).\ x(n-1)$

For i/p $u(n)$

$y(n) = u(n).\ u(n-1)$

For i/p $-u(n)$

$y(n) = \{-u(n)\}\{-u(n-1)\}$

$= u(n).\ u(n-1)$

$\therefore$ Non-invertible system

Q) $y(n) = \begin{cases} x(n+2) & n \geq 0 \\ x(n) & n < -1 \end{cases}$

For $x(n) = \delta(n)$

$y(n) = \delta(n+2)$ for $n \geq 0$

$= 0$

For $x(n) = 2\delta(n)$

$y(n) = 2\delta(n+2)$ for $n \geq 0$

$= 0$

$\therefore$ Non-invertible system

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→ Convolution

Continuous TS — Convolution integral

Discrete TS — Convolution summation

It is used to find o/p of any LTI system for any arbitrary i/p provided impulse response of the system is known.

$$x(t) \rightarrow \boxed{\begin{array}{c} LTI \\ h(t) \end{array}} \xrightarrow{y(t)}$$

$$y(t) = x(t) * h(t)$$

→ Continuous time Convolution

$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau).\, h(t-\tau)\, d\tau$$

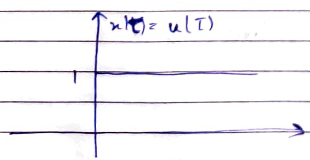
$$h(t-\tau) = h(-\tau + t)$$

Q) $x(t) = u(t)$, $h(t) = u(t)$

Find o/p $y(t) = x(t) * h(t)$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau).\, h(t-\tau)\, d\tau$$

$$x(t) = u(t) \Rightarrow x(\tau) = u(\tau)$$



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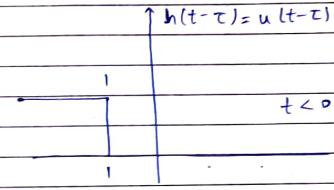
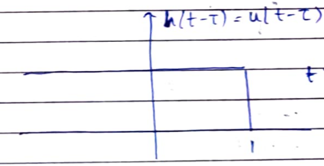
$$h(t) = u(t) \Rightarrow \cancel{h(\tau) = u(\tau)}$$

$$h(t-\tau) = u(t-\tau) = u(-\tau + t)$$





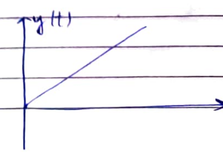
$$y(t) = \int_{-\infty}^{\infty} u(\tau)\, u(t-\tau)$$

$y(t) = 0$ for $t < 0$ as no common area exists

For $t \geq 0$

$$y(t) = \int_0^t 1.1\, d\tau = [\tau]_0^t = t$$

$$y(t) = 0 \qquad t < 0$$
$$y(t) = t \qquad t \geq 0$$
$$y(t) = t.\, u(t) = r(t)$$



$\boxed{u(t) * u(t) = r(t)}$

Q) $x(t)=u(t) \longrightarrow \boxed{h(t)=\delta(t)} \longrightarrow y(t)$

$$y(t) = x(t) * \cancel{x}h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$x(t)=u(t)$ $\qquad h(t)=\delta(t)$

$x(\tau)=u(\tau)$ $\qquad \cancel{h(\tau)}\ h(t-\tau)=\delta(t-\tau)$

$$y(t) = \int_{-\infty}^{\infty} u(\tau).\, \cancel{h}\,\underbrace{\delta(t-\tau)}_{=0}\, d\tau$$

$$= u(\tau)\Big|_{\tau=t} = u(t)$$

$\boxed{u(t) * \delta(t) = u(t)}$

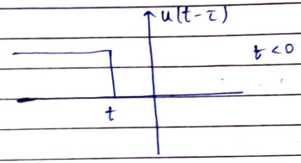
Q) $x(t)=e^{t}u(-t)$ $\qquad h(t)=u(t)$

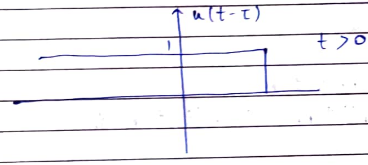
$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$x(t)=e^{t}u(-t)$ $\qquad h(t)=u(t)$

$x(\tau)=e^{\tau}u(-\tau)$ $\qquad h(t-\tau)=u(t-\tau)$

$$y(t) = \int_{-\infty}^{\infty} e^{\tau}\underbrace{u(-\tau)}_{-\infty<\tau<0}.\,\underbrace{u(t-\tau)}\, d\tau$$





For $t > 0$

$$y(t) = \int_{-\infty}^{t} e^{\tau}.\, d\tau = [e^{\tau}]_{-\infty}^{t} = e^{t} - 0 = e^{t}$$

For $t > 0$

$$y(t) = \int_{-\infty}^{0} e^{\tau}.\, d\tau = [e^{\tau}]_{-\infty}^{0} = 1 - 0 = 1$$

$$y(t) = \begin{cases} e^{t} & t<0 \\ 1 & t \geq 0 \end{cases}$$

→ Properties of Convolution

① Commutative property

$$y(t) = x(t) * h(t) = h(t) * x(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau$$

2) Associative property

$$y(t) = \{x_1(t) * x_2(t)\} * x_3(t)$$
$$= x_1(t) * [x_2(t) * x_3(t)]$$

3) Distributive property

$$y(t) = x(t) * [h_1(t) \pm h_2(t)]$$
$$= x(t) * h_1(t) \pm x(t) * h_2(t)$$

4) Property based on linearity

$$y(t) = x(t) * h(t)$$
$$x'(t) * h(t) = y'(t)$$
$$x(t) * h'(t) = y'(t)$$
$$x'(t) * h'(t) = y''(t)$$

5) Property based on time invariancy

$$x(t) * h(t) = y(t)$$

$$\boxed{\begin{array}{l} x(t-t_0) * h(t) = y(t-t_0) \\ x(t) * h(t-t_0) = y(t-t_0) \\ x(t-t_0) * h(t-t_0) = y(t-2t_0) \end{array}}$$

Ex- $u(t) * u(t-4) = \cancel{x}\, r(t-4)$
$u(t-2) * u(t-4) = r(t-7)$
$u(t-3) * \delta(t+4) = u(t+1)$

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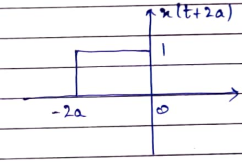
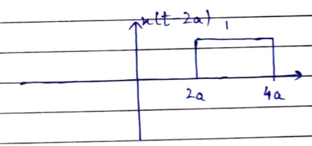
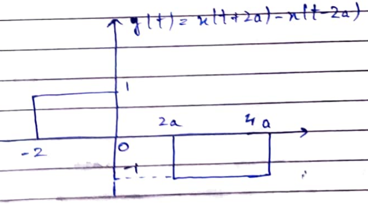
Q) $x(t) = 1 \quad 0 \le t \le 2a$
$h(t) = \delta(t+2a) - \delta(t-2a)$
$y(t) = x(t) * h(t) = ?$

$$y(t) = x(t) * [\delta(t+2a) - \delta(t-2a)]$$
$$= [x(t) * \delta(t+2a)] - [x(t) * \delta(t-2a)]$$
$$y(t) = x(t+2a) - x(t-2a)$$

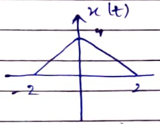


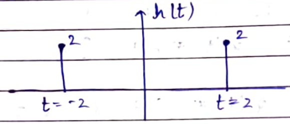




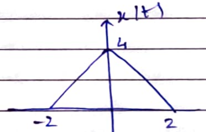
Good Write

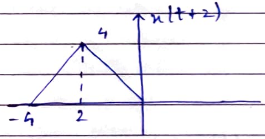
Q) $x(t) = 4\,\text{tri.}(t/2)$ $\qquad h(t) = 2\delta(t+2) + 2\delta(t-2)$

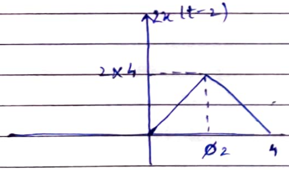


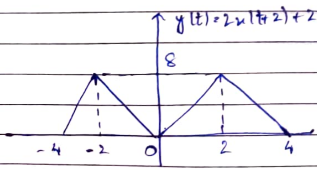


$$y(t) = x(t) * h(t)$$
$$= x(t) * [2\delta(t+2) + 2\delta(t-2)]$$
$$= 2x(t+2) + 2x(t-2)$$









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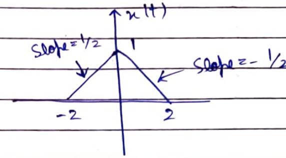
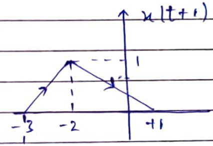
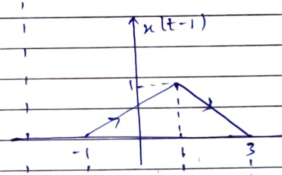
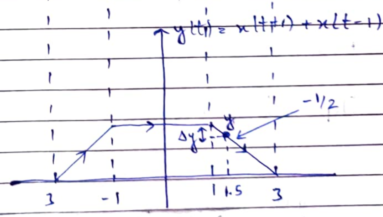
Q) $y(t) = x(t+1) + x(t-1)$









Find $y(t)\big|_{t=1.5}$

① $\Delta x = 1.5 - 1$

$\Delta y = ?$

$\text{Slope} = \dfrac{\Delta y}{\Delta x} = -\dfrac{1}{2}$

$\Delta y = -\dfrac{1}{4} = -0.25$

$y = 1 + (-0.25)$

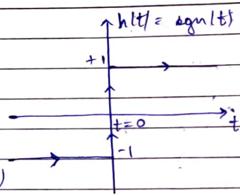
$\boxed{y = 0.75}$

Q) $x(t) = u(t+a) - u(t-a)$

$h(t) = \mathrm{sgn}(t)$

$x(t) * h(t) = y(t)$

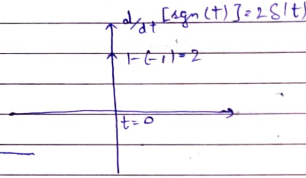
$x(t) * h'(t) = y'(t)$

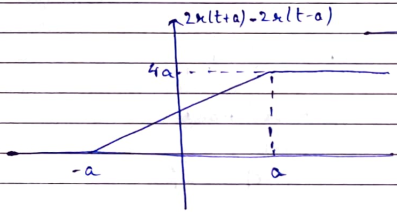


$y'(t) = [u(t+a) - u(t-a)] * 2\delta(t)$

$y'(t) = 2u(t+a) - 2u(t-a)$

$\boxed{y(t) = 2r(t+a) - 2r(t-a)}$





$p(t) = \frac{t^2}{2} u(t)$ $\qquad \frac{d}{dt}$ [Parabolic] = Ramp

$r(t) = t.u(t)$ $\qquad \frac{d}{dt}$ [Ramp] = Step

$u(t) = 1.u(t)$ $\qquad \frac{d}{dt}$ [Step] = Impulse

Q) $y(t) = u(t-2) * \delta(4-t)$

$= u(t-2) * \delta(t-4)$

$y(t) = u(t-6)$

Q) $y(t) = u(t) * \delta(3-4t)$

$= \delta\{-(4t-3)\} = \delta(4t-3)$

$= \delta\left\{4\left(t-\frac{3}{4}\right)\right\}$

$= \frac{1}{4}\delta\left\{t-\frac{3}{4}\right\}$

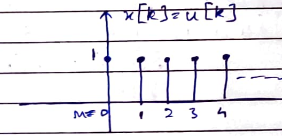
$= u(t) * \frac{1}{4}\delta(t-3/4)$

$y(t) = \frac{1}{4} u(t-3/4)$

→ Discrete time convolution

Q) $x[m] = u[m]$

$h[m] = u[m]$

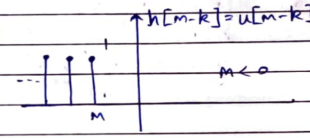


$y[m] = x[m] * h[m]$

$x[m] = u[m] \Rightarrow x[k] = u[k]$

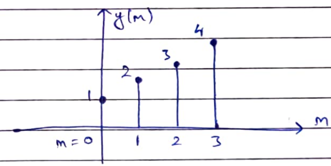
$h[m] = u[m] \Rightarrow h[m-k] = u[m-k]$

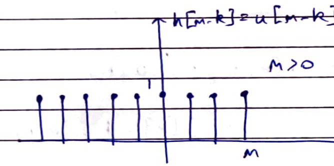
$= u[-k+m]$



$y[m] = 0, \quad m<0$

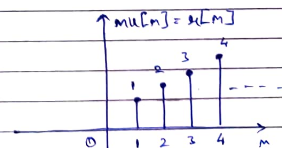
$y[m] = m+1, \quad m \geq 0$





$y[m] = (m+1)u[m]$: $u[m] * u[m] = (m+1)u[m]$

$= r[m+1]$



1) $x[n] * \delta[n] = x[n]$
2) $x[n-1] * \delta[n+3] = x[n+2]$
3) $\{u[n-1] - u[n-2]\} * \{u[n-1] - u[n-2]\}$
$\delta[n-1] * \delta[n-1]$
$\delta[n-2]$

Q) $x[n] = (0.5)^n u[n]$
$h[n] = \delta[n+2] + 0.5\,\delta[n+1]$
$y[n] = (0.5)^n u[n] * \{\delta[n+2] + 0.5\,\delta[n+1]\}$
$= (0.5)^{n+2} u[n+2] + 0.5\,(0.5)^{n+1} u[n+1]$
$= (0.5)^{n+2} u[n+2] + 0.5^{n+2} u[n+1]$
$= (0.5)^{n+2} \{u[n+2] + u[n+1]\}$

Q) $x[n] = \left(\frac{1}{2}\right)^n u[n]$
$h[n] = \delta[n] - \frac{1}{2}\delta[n-1]$
$y[n] = x[n] * \{\delta[n] - 1/2\,\delta[n-1]\}$
$= x[n] - \frac{1}{2} x[n-1]$
$= \left(\frac{1}{2}\right)^n u[n] - \frac{1}{2}\left(\frac{1}{2}\right)^{n-1} u[n-1]$
$= \left(\frac{1}{2}\right)^n \{u[n] - u[n-1]\}$
$= \left(\frac{1}{2}\right)^n \delta[n]$ $_{=0}$
$= \left(\frac{1}{2}\right)^0 \cdot \delta[n]$
$\boxed{y[n] = \delta[n]}$

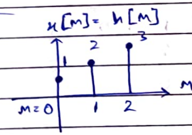
Q) Three LTI systems are described by their impulse responses as
$h_1[n] = \left(\frac{1}{2}\right)^n u[n]$
$h_2[n] = u[n+3]$
$h_3[n] = \delta[n] - \delta[n-1]$
If the systems are connected in cascade, then the impulse response of equivalent system will be

$\boxed{h_1[n] = \left(\frac{1}{2}\right)^n u[n]}$ — $\boxed{h_2[n] = u[n+3]}$ — $\boxed{\delta[n] - \delta[n-1]}$

$h[n] = h_1[n] * \{h_2[n] * h_3[n]\}$ —— (1)
$h_2[n] * h_3[n] = u[n+3] * \{\delta[n] - \delta[n+1]\}$
$= u[n+3] - u[n+2] = \delta[n+3]$
$h[n] = \left(\frac{1}{2}\right)^n u[n] * \delta[n+3]$
$h[n] = \left(\frac{1}{2}\right)^{n+3} u[n+3]$

→ Convolution of sequences

Q) $x[n] = \{\underset{\uparrow}{1}, 2, 3\}$
$h[n] = \{\underset{\uparrow}{1}, 2, 3\}$



$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

$x[k] = \{\underset{\uparrow}{1}, 2, 3\}$
$h[k] = \{\underset{\uparrow}{1}, 2, 3\}$
$h[-k] = \{3, 2, \underset{\uparrow}{1}, 0, 0\}$

$y[0] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[-k] = 1$

$y[1] = \sum_{k=-\infty}^{\infty} x[k]\, h[1-k]$

$x[k] = \{0, \underset{\uparrow}{1}, 2, 3\}$

$h[1-k] = \{3, \underset{\uparrow}{2}, 1, 0\}$

$y[1] = 0+2+2+0 = 4$

$y[2] = \sum_{k=-\infty}^{\infty} x[k]\, h[2-k]$

$x[k] = \{\underset{\uparrow}{1}, 2, 3\}$

$h[2-k] = \{\underset{\uparrow}{3}, 2, 1\}$

$y[2] = 3+4+3 = 10$

$y[-1] = \sum_{k=-\infty}^{\infty} x[k]\, h[-1-k]$

$x[k] = \{\underset{\uparrow}{1}, 2, 3\}$

$h[-1-k] = \{3, 2, 1, \underset{\uparrow}{0}, 0, 0\}$

$y[-1] = 0$

Matrix method

h[n] \ x[n]	1	2	3
1	1	2	3
2	2	4	6
3	3	6	9

$y[n] = \{1, 4, 10, 12, 9\}$

Q) $x[n] = \{1, \underset{\uparrow}{2}, 3, 4\}$

$h[n] = \{1, \underset{\uparrow}{2}, 2, 1\}$

h[n] \ x[n]	0	1	2	3	4
1	0	1	2	3	4
2	0	2	4	6	8
→ 2	0	2	4	6	8
1	0	1	2	3	4
0	0	0	0	0	0

y[-4], y[-3], y[-2], y[-1], y[0], y[1], y[2], y[3], y[4]

$\boxed{y[n] = \{\cancel{0}, 1, 4, 9, 15, 16, 4\}}$

Q) $y[n] = g[n] * h[n]$

$h[n] = \left(\frac{1}{2}\right)^n u[n] = \{\underset{\uparrow}{1}, 1/2, 1/4, 1/8, ----\}$

$g[n] \rightarrow$ Causal sequence $\Rightarrow \{\underset{\uparrow}{g(0)}, g(1), g(2), ----\}$

$y[0] = 1, \quad y[1] = 1/2$

$g[1] = ?$

g[n] \ h[n]	1	1/2	1/4	1/8 ----
→ g(0)	g(0)	1/2 g(0)	1/4 g(0)	1/8 g(0)
g(1)	g(1)	1/2 g(1)	1/4 g(1)	1/8 g(1)
g(2)	g(2)	1/2 g(2)	1/4 g(2)	1/8 g(2)
⋮				

$y[0] = g[0] = 1$

$y[1] = g[1] + \frac{1}{2} g[0]$

$\frac{1}{2} = g[1] + \frac{1}{2} \times 1$

$\boxed{g[1] = 0}$

Q) $x[n] = \alpha^n u[n]$

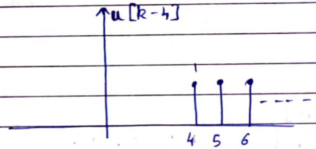
$h[n] = \left(\frac{-1}{2}\right)^n u[n-4]$

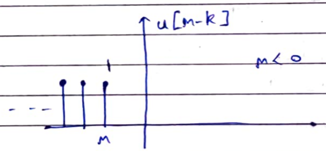
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

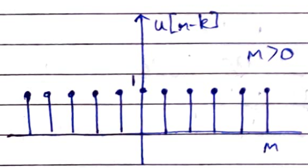
$$= \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

$x[n-k] = (\alpha)^{n-k} u[n-k]$

$h[k] = \left(\frac{-1}{2}\right)^k u[k-4]$







$$y[n] = \sum_{k=-\infty}^{\infty} \left(\frac{-1}{2}\right)^k \underbrace{u[k-4]}_{n \geq 4} . (\alpha)^{n-k} \underbrace{u[n-k]}_{-\infty \leq k < n}$$

$y[n] = 0 \qquad n < 0$

$$y[n] = \sum_{k=4}^{n} \left(\frac{-1}{2}\right)^k (\alpha)^n . (\alpha)^{-k}$$

$$= (\alpha)^n \sum_{k=4}^{n} \left(\frac{-1}{2\alpha}\right)^k$$

Let $k-4 = m$

$k = m+4$

When $k = 4,\ m = 0$

$k = n,\ m = n-4$

$$(\alpha)^n \sum_{m=0}^{n-4} \left(\frac{-1}{2\alpha}\right)^{m+4}$$

$$= (\alpha)^n . \left(\frac{-1}{2\alpha}\right)^4 \sum_{m=0}^{n-4} \left(\frac{-1}{2\alpha}\right)^m$$

$$= (\alpha)^{n-4} \left(\frac{-1}{2}\right)^4 \left[\frac{1-(-1/2\alpha)^{n-4+1}}{1-(-1/2\alpha)}\right]$$

$$= +\frac{1}{16} (\alpha)^{n-1} \left[\frac{1-(-1/2\alpha)^{n-3}}{1+1/2\alpha}\right]$$

→ Continuous Fourier Time Series

↳ Only for periodic signals

$e^{j\omega_0 t}$ ↑ Freq. $\qquad e^{j2\omega_0 t}$ ↑ Freq. $\qquad e^{j3\omega_0 t}$ ↑ Freq.

$$x(t) = A_1 e^{j\omega_0 t} + A_2 e^{j2\omega_0 t} + A_3 e^{j3\omega_0 t} + \dots$$

Synthesis equation

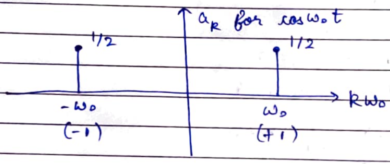
$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}}$$

Analysis equation

$$\boxed{a_k = \frac{1}{T} \int_0^T x(t). e^{-jk\omega_0 t}\, dt}$$

$$x(t) = \cos\omega_0 t = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} = \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t}$$

$a_1 = \frac{1}{2}$ $\quad a_{-1} = \frac{1}{2}$



$$x(t) = \sin\omega_0 t = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} = \frac{1}{2j}e^{j\omega_0 t} - \frac{1}{2j}e^{-j\omega_0 t}$$

$a_1 = \frac{1}{2j}$ $\quad a_{-1} = \frac{-1}{2j}$

Q) Find magnitude & phase spectra of

$$x(t) = 4 + \underbrace{2\cos\frac{2\pi}{3}t}_{T_1} + \underbrace{4\sin\frac{5\pi}{3}t}_{T_2}$$

$\omega_{0_1} = \frac{2\pi}{3}$ $\quad \omega_{0_2} = \frac{5\pi}{3}$

$T_1 = \frac{2\pi}{(2\pi/3)}$ $\quad T_2 = \frac{2\pi}{(5\pi/3)}$

$T_1 = 3$ $\quad T_2 = \frac{6}{5}$

$$T = LCM\left(\frac{3}{1}, \frac{6}{5}\right)$$

$$= \frac{6}{1}$$

$\boxed{T = 6\,s}$ $\quad \boxed{\omega_0 = \frac{2\pi}{T} = \frac{\pi}{3}\ \text{rad/s}}$

$2\cos\frac{2\pi}{3}t \rightarrow$ Represents 2^{nd} harmonic

$4\sin\frac{5\pi}{3}t \rightarrow$ " 5^{th} harmonic

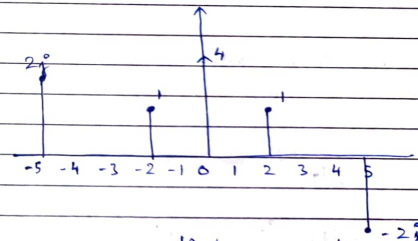
$$x(t) = 4.e^{j0.\pi/3 t} + 2\left[\frac{e^{j2\times\pi/3 t} + e^{-j2\times\pi/3 t}}{2}\right] + 4\left[\frac{e^{j5\times\pi/3 t} + e^{-j5\times\pi/3 t}}{2j}\right]$$

$$= 4.e^{j0\pi/3 t} + e^{j2\times\pi/3 t} + e^{-j2\times\pi/3 t} + \frac{2}{j}e^{j5\pi/3 t} - \frac{2}{j}e^{-j5\pi/3 t}$$

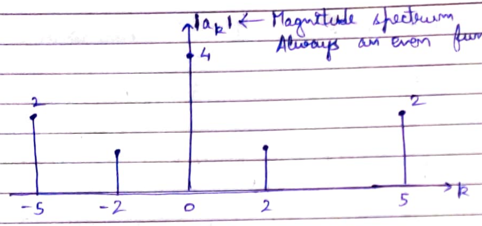
$a_0 = \cancel{0}\,4$

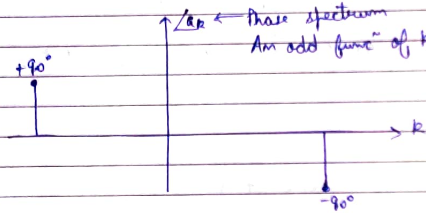
$a_2 = 1$ $\quad a_{-2} = 1$

$a_5 = -2j$ $\quad a_{-5} = +2j$



a_k	Values	$\lvert a_k \rvert$ Magnitude	$\angle a_k$ Phase angles
a_0	4	4	$0°$
a_2	1	1	$0°$
a_{-2}	1	1	$0°$
a_5	$-2j$	2	$-90°$
a_{-5}	$2j$	2	$+90°$





Q) Consider a periodic signal with T=8 sec & Fourier series coefficients given as:-

$a_2 = 2$ $\quad$ $a_3 = 8j$

$a_{-2} = 2$ $\quad$ $a_{-3} = -8j$

Find the signal $x(t)$

Synthesis equation $x(t) = \sum_{k=-\infty}^{\infty} a_k \cdot e^{jk\omega_0 t}$

$T = 8$ sec

$\omega_0 = \frac{2\pi}{8} = \frac{\pi}{4}$ rad/s

$x(t) = a_2 \cdot e^{j2\omega_0 t} + a_{-2} e^{-j2\omega_0 t} + a_3 e^{j3\omega_0 t} + a_{-3} e^{-j3\omega_0 t}$

$= 2e^{j\pi/2\, t} + 2e^{-j\pi/2\, t} + 8j e^{j3\pi/4\, t} - 8j e^{-j\frac{3\pi}{4} t}$

$= 2\times 2\left[\frac{e^{j\pi/2\, t} + 2e^{-j\pi/2\, t}}{2}\right] + 8j\left[\frac{e^{j3\pi/4\, t} - e^{-j3\pi/4\, t}}{2j}\right] \times 2j$

$$\boxed{x(t) = 4\cos\frac{\pi}{2}t - 16\sin\frac{3\pi}{4}t}$$

→ Properties of Fourier series

① Linearity property

$x(t) \xleftrightarrow{F.S} a_k$

$y(t) \xleftrightarrow{F.S} b_k$

$z(t) = \alpha x(t) + \beta y(t) \longleftrightarrow c_k$

$$\boxed{c_k = \alpha \cdot a_k + \beta b_k}$$

Q) $x(t) = \cos\omega_0 t$ $\quad$ $y(t) = \sin\omega_0 t$

$a_1 = \frac{1}{2}$ $\quad$ $a_{-1} = \frac{1}{2}$ $\quad$ $b_1 = \frac{1}{2j}$ $\quad$ $b_{-1} = \frac{-1}{2j}$

$z(t) = x(t) + y(t) = \cos\omega_0 t + \sin\omega_0 t$

$= \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t} + \frac{1}{2j}e^{j\omega_0 t} - \frac{1}{2j}e^{-j\omega_0 t}$

$= \underbrace{\left[\frac{1}{2} + \frac{1}{2j}\right]}_{c_1 = a_1 + b_1} e^{j\omega_0 t} + \underbrace{\left[\frac{1}{2} - \frac{1}{2j}\right]}_{c_{-1} = a_{-1} + b_{-1}} e^{-j\omega_0 t}$

② Time shifting property

$x(t) \xleftrightarrow{F.S} a_k$

$x(t - t_0) \xleftrightarrow{F.S} b_k$

$$\boxed{b_k = e^{-jk\omega_0 t} \cdot a_k}$$

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$x(t+t_0) \longleftrightarrow e^{jk\omega_0 t_0} \cdot a_k$

$b_k = e^{-j\omega_0 k t_0} \cdot a_k$

$b_k = |e^{-jk\omega_0 t_0}| \cdot |a_k|$

$\boxed{|b_k| = |a_k|}$

$\angle b_k = \angle e^{-jk\omega_0 t_0} + \angle a_k$

$\boxed{\angle b_k = \angle a_k - k\omega_0 t_0}$

Q) $x(t) \longleftrightarrow a_k$

$x_1(t) = x(t+t_0) + x(t-t_0) \longleftrightarrow b_k$

Find b_k in terms of a_k

$x(t) \longleftrightarrow a_k$

$x(t+t_0) \longleftrightarrow e^{jk\omega_0 t_0} \cdot a_k$

$x(t-t_0) \longleftrightarrow e^{-jk\omega_0 t_0} \cdot a_k$

$\therefore$ using linearity property

$b_k = e^{jk\omega_0 t_0} \cdot a_k + e^{-jk\omega_0 t_0} a_k$

$= a_k \dfrac{[e^{jk\omega_0 t_0} + e^{-jk\omega_0 t_0}]}{2} \times 2$

$\boxed{b_k = 2\cos k\omega_0 t_0 \cdot a_k}$

③ Time reversal property

$x(t) \xleftrightarrow{F.S} a_k$

$x(-t) \xleftrightarrow{F.S} b_k$

$\boxed{b_k = a_{-k}}$

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Case-I - If $x(t)$ is even signal

$x(-t) = x(t)$

$\boxed{b_k = +a_{-k} = a_k}$

Case-II - If $x(t)$ is odd signal

$x(-t) = -x(t)$

$x(t) \longleftrightarrow a_k$

$x(-t) = -x(t) \longleftrightarrow -a_k$

$\boxed{b_k = a_{-k} = -a_k}$

Q) $x(t) \longleftrightarrow a_k$

$x_1(t) = x(1-t) + x(t-1) \longleftrightarrow b_k$

Find b_k in terms of a_k

$x(t) \longleftrightarrow a_k$

$x(t+1) \longleftrightarrow e^{jk\omega_0 t} \cdot a_k$

$x(-t+1) \longleftrightarrow e^{-jk\omega_0} \cdot a_{-k}$ —— ①

$x(t) \longleftrightarrow a_k$

$x(t-1) \longleftrightarrow e^{-jk\omega_0} \cdot a_k$ —— ②

$b_k = e^{-jk\omega_0} \cdot a_{-k} + e^{-jk\omega_0} \cdot a_k$

$\boxed{b_k = e^{-jk\omega_0}[a_{-k} + a_k]}$ —— ③

④ Time scaling property

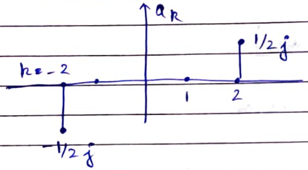
$x(t) \xleftrightarrow{F.S} a_k$

$x(\alpha t) \xleftrightarrow{F.S} b_k$

$\boxed{b_k = a_k}$

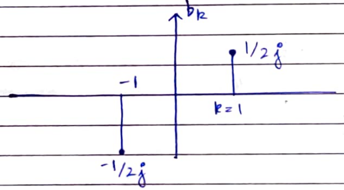
Q) $x(t) = \sin 2\omega_0 t$ $\quad \omega_0$ = Fundamental freq.

$a_2 = \frac{1}{2j} \quad a_{-2} = \frac{-1}{2j}$



$x_1(t) = x(t/2) = \sin\left(2\omega_0\left(\frac{t}{2}\right)\right) = \sin \omega_0 t$

$b_1 = \frac{1}{2j} \quad b_{-1} = \frac{1}{2j}$



⑤ Differentiation property

$x(t) \xleftrightarrow{F.S} a_k$

$\frac{d}{dt} x(t) \xleftrightarrow{F.S} b_k$

$\boxed{b_k = jk\omega_0 . a_k}$

$|b_k| = |jk\omega_0| . |a_k| = k\omega_0 |a_k|$

$\angle b_k = \angle jk\omega_0 + \angle a_k = \angle a_k + 90^\circ$

⑥ Integration property

$x(t) \xleftrightarrow{F.S} a_k$

$\int_{-\infty}^{t} x(\tau)\, d\tau \longleftrightarrow b_k$

$|b_k| = \frac{|a_k|}{k\omega_0}$

$\boxed{b_k = \frac{1}{jk\omega_0} a_k}$

$\angle b_k = \angle a_k - \frac{\pi}{2}$

⑦ Frequency shifting property

$x(t) \xleftrightarrow{F.S} a_k$

Integer

$e^{jm\omega_0 t} \longleftrightarrow b_k$

then $\boxed{b_k = a_{k-m}}$

$e^{-jm\omega_0 t} . x(t) \longleftrightarrow a_{k+m}$

⑧ Multiplication property

$x(t) \longleftrightarrow a_k$

$y(t) \longleftrightarrow b_k$

$z(t) = x(t).y(t) \longleftrightarrow C_k$

$\boxed{C_k = a_k * b_k}$

Discrete convolution

→ Discrete time fourier series

$x(N+m) = x(m)$

$x[m] = \sum_{k=0}^{N-1} c(k) e^{j\frac{2\pi k m}{N}}$

$x(t) = \sum_{k=-\infty}^{\infty} e^{jk\omega_0 m}$

$C_k = \frac{1}{T}\int x(t) e^{-jk\omega_0 t} dt$

$c(k) = \frac{1}{N}\sum_{m=0}^{N-1} x[m] . e^{-j\frac{2\pi k m}{N}}$

Periodicity property of $c(k)$

$k \to k+N$

$c(k+N) = \frac{1}{N}\sum_{m=0}^{N-1} x[m] e^{-j\frac{2\pi(k+N)m}{N}}$

$\boxed{c(k+N) = c(k)}$

→ Properties of DTFS

① Linearity property

$$\alpha x_1[n] + \beta x_2[n] \longleftrightarrow \alpha C_1(k) + \beta C_2(k)$$

② Time shifting property

$$x[n] \longleftrightarrow c(k)$$
$$x[n-n_0] \longleftrightarrow e^{-j\frac{2\pi k}{N} \cdot n_0} \cdot c(k)$$

③ Time scaling

$$x[n] \longleftrightarrow c(k)$$
$$x[\alpha n] \longleftrightarrow c(k) \cdot \alpha$$

④ Convolution theorem

$$\underset{\substack{\uparrow \\ \text{Periodic convolution}}}{x_1[n] * x_2[n]} \longleftrightarrow N \cdot C_1(k) \cdot C_2(k)$$

$$y[n] = x_1[n] * x_2[n] = \sum_{m=0}^{N-1} x_1(m)\, x_2(n-m)$$

⑤ Multiplication theorem

$$x_1[n] \cdot x_2[n] \longleftrightarrow C_1(k) * C_2(k)$$

⑥ Frequency shifting theorem

$$x[n] \longleftrightarrow c(k)$$
$$e^{j\frac{2\pi}{N} k_0 \cdot n} \cdot x[n] \longleftrightarrow c(k-k_0)$$

⑦ Symmetric properties

x[n] Real	Img	R.E	R.O	I.E	I.O
c(k) Conj	CAS	RE	IO	IE	RO

Q) $x[n] = \cos \frac{n\pi}{3}$

$$C(k) = \frac{1}{N} \sum_{n=0}^{N-1} x[n] \cdot e^{-j\frac{2\pi k n}{N}}$$

$$x[n] = \sum_{k=0}^{N-1} c(k) \cdot e^{j\frac{2\pi k n}{N}}$$

$$x[n] = \cos \frac{\pi}{3} \cdot n$$

$$\omega_0 = \frac{\pi}{3} \qquad N = \frac{2\pi}{\omega_0} \cdot m$$

$$= \left(\frac{2\cancel{\pi} \times 3}{\cancel{\pi}}\right) m$$

$$\boxed{N = 6}$$

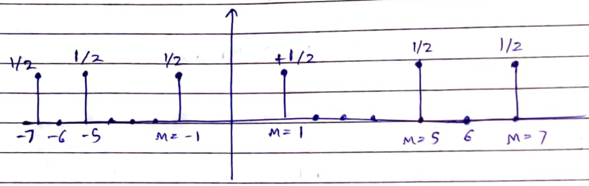
$$x[n] = \sum_{k=-3}^{2} C(k) \cdot e^{j\frac{2\pi k}{6}}$$

$$x[n] = C(-3)e^{j\frac{\pi}{3}(-3)} + C(-2)e^{j\pi/3(-2)} + C(-1)e^{j\pi/3(-1)}$$
$$+ C(0) + C(1)e^{j\pi/3(1)} + C(2)e^{j\pi/3(2)} \longleftarrow ①$$

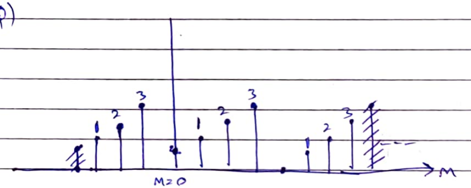
$$x[n] = \cos \frac{\pi}{3} n = \frac{1}{2} e^{j\pi/3\, n} + \frac{1}{2} e^{-j\pi/3\, n} \longleftarrow ②$$

Comparing ① & ②

$C(-1) = \frac{1}{2}$, $C(1) = \frac{1}{2}$, $C(-3) = C(-2) = C(0) = C(2) = 0$



Q)



$$C(k) = \frac{1}{N}\sum_{M=0}^{N-1} x[M].\, e^{\frac{-j2\pi kM}{N}}$$

$N=4$

$$c(k) = \frac{1}{4}\left[e^{-j\pi/2 k} + 2e^{-j\pi k} + 3e^{-j\pi k3/2}\right]$$

$$= \frac{1}{4}\left[\left(e^{-j\pi/2}\right)^k + 2\left(e^{-j\pi}\right)^k + 3\left(e^{-j3\pi/2}\right)^k\right]$$

$$e^{-j\pi/2} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} = -1$$

$$e^{-j\pi} = \cos\pi - j\sin\pi = 1$$

$$e^{-j3\pi/2} = \cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2} = +j$$

$$c(k) = \frac{1}{4}\left[(-j)^k + 2(-1)^k + 3(j)^k\right]$$

→ Parseval's theorem (CTFS)

Total avg. power in any periodic signal is equal to sum of avg. power contained in all of its harmonic components.

$$P_x = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x(t)|^2 dt = \sum_{k=-\infty}^{\infty}|a_k|^2$$

Q) $x(t) = \cos w_0 t$

Power $= \frac{A^2}{2} = \frac{1}{2}$ W

$a_1 = \frac{1}{2}$ $\quad a_{-1} = \frac{1}{2}$

$$\sum_{k=-\infty}^{\infty}|a_k|^2 = |a_1|^2 + |a_{-1}|^2$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}\text{ W}$$

$$\boxed{a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t).\, e^{-jkw_0 t}\, dt}$$

→ Parseval's theorem (DTFS)

$$\cancel{\frac{1}{N}\sum_{M=-N}^{N}}$$

$$\boxed{\frac{1}{N}\sum_{M=0}^{N-1}|x(m)|^2 = \sum_{k=0}^{N-1}|c(k)|^2}$$

→ Impulse response

$$\delta(t) \longrightarrow \boxed{\text{LTI system}} \longrightarrow h(t)$$

$h(t)$ is fixed

$\downarrow$ LT

$H(s)$ is fixed

1) $$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

ILT $\uparrow \downarrow$ LT

$$Y(s) = \frac{X(s)}{s} \Rightarrow \frac{Y(s)}{X(s)} = \frac{1}{s} = H(s)$$

$\downarrow$ ILT

$$h(t) = u(t)$$

2) $y(t) = ?$

LT: $h(t) =$ ______

$$H(s) = \frac{Y(s)}{X(s)} \Rightarrow Y(s) = H(s) \cdot X(s)$$

$\downarrow$ ILT

$$y(t) = h(t) * x(t)$$

$\uparrow$

Convolution