unit - I

Set theory

Basic concept & notations :-

Definition : A set is a well defined collection of objects.
↳ group

Eg :- A set of all +ve integers
A set of states in India.
collection of 5 Best Indian Actors is not a set. It doesnot vary from one person to another

* Objects of a set are called members or elements of the set.

A set is written with all elements enclosed between curly brackets { }.

* Notation :-

sets are denoted by A, B, C .....
Elements of the sets are denoted by a, b, c .... x, y, z ....
If A is any set & 'a' is any element of A. it is denoted by $a \in A$. If b is not in A.

It is denoted by $b \notin A$.

* **Representation of a set :-**

Sets are represented in two ways namely

(i) Roaster or Tabular Form.

(ii) Set - builder notation.

Roaster Form :-

* All of the elements of the set are listed if possible seperated by commas & enclosed with in braces $\{ \}$.

Eg :- Set of all vowels in English alphabets.

i.e. $V = \{a, e, i, o, u\}$

* set of Even positive integers less than or equal to 10. $E = \{2, 4, 6, 8, 10\}$

Set - Builder Form :-

We define the elements of the set by specifying a property that they have in common.

Eg: $V = \{x \mid x$ is a vowel in English Alphabets$\}$.

We use this notation when it is not possible to list all the elements of the set, Eg

Eg :- The set of even integers is denoted by $2Z = \{x \mid x = 2n, n \in Z\}$

N - The set of all natural nos.

$= \{1, 2, 3, 4 \ldots$

Z = The set of all integers

$= \{ \ldots -3, -2, -1, 0, 1, 2 \ldots \}$

Z^+ = set of all +ve integers

Z^- = " " " −ve integers.

Q = Set of all Rational numbers. $\{p/q \mid p \in Z, q \in Z, q \neq 0\}$

R = " " " Real numbers

W = " " " whole numbers.

Definition 3 :-

Cardinality of a set :- or Size of set

Let A be any set. The number of distinct (different) elements in A is called the cardinality of A denoted by $|A|$ or $\#A$.

Eg :- $A = \{a, b, c\}$

$n(A) = 3$.

Definition :-

A set which contains no element is called the null set & is denoted by ϕ or $\{ \}$ or Empty set.

It is clear that $|\phi|$ is zero.

* A set which has only one element is called a singleton set.

Eg : $A = \{a\}$ is a singleton set.

Definition :-

A set which has cardinality n where $n \in Z^+$ is called a finite set.

* Any set that is not finite is called an infinite set.

Eg :- $B = \{2x \mid x \in Z^+\}$

$= \{2, 4, 6, 8 \ldots\}$ is an infinite set.

Set of all days in a week.
It has 7 elements & hence this set is finite.

Definition :-

A set A is ~~called~~ said to be a subset of set B if each element of A is an element of B.

It is written as $A \subseteq B$

between 1 & 100.

Eg:- Set of all even positive integers between 1 & 100 is a

* Every set is a subset of itself.

  i.e. $A \subseteq A$.

* Empty set is a subset of each set.

  $\phi \subseteq A$.

If A is not a subset of B i.e. if $A \not\subseteq B$, atleast one element of A does not belong to B.

Eg :- $A = \{1, 2, 3\}$

$B \subseteq \{1,2,3\}$ $B = \{1, 2, 3, 4, 5\}$ proper subset.

Every element of A is also an element of B. Hence A is a subset of B. $A \subseteq B$. (Improper subsets)

proper subset :-

It is defined as subset of but not equal to A is a proper subset of set B. if $\forall$ element of A is an element of set B & $A \subset B$ ⇒ A is contained in B otherwise, it is called Improper subset. then there is atleast one element in B which is not in A.

* If $A \subseteq B$, B is called the super set of A

Equality of sets :-

Two sets A & B are equal iff $A \subseteq B$ & $B \subseteq A$. [if both have same elements]

Eg : $A = \{a, b, c\}$ $B = \{c, b, a\}$ or $\{a, b, c\}$

order of elements of a set does not matter.

Power sets :-

Given any set A, the collection of all subsets of A is called the power set of A denoted by
$P(A)$ or 2^A.

The cardinality of a power set of a set A of cardinality n is 2^n.

Ex :- $A = \{1, 2, 3\}$

$$P(A) = \{ \{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$$

Note :- The power set of empty set
$|P(\phi)| = 2^0 = 1$.

* If $|A| = n$, then $P(A)$ has 2^n elements.

Equivalent set :-

~~Set Operations~~ :-

If the cardinalities of two sets are same, they are called equivalent sets.

Overlapping sets :-

Two sets that have atleast one common element are called overlappings sets.

Set operations :-

Universal set :-

A set U is called a universal set if U is the superset of all the sets.

$U = \{1, 2, 3, 4, 5, 6, \}$

$A = \{1, 2, 3\}$ $B = \{4, 5, 6\}$ $C = \{2, 6\}$

clearly U is the superset of all sets A, B, C & hence U is a universal set.

Union of two sets :-

The union of two sets A & B denoted by $A \cup B$ is the set of all elements that belong to A or B or both.

ie $A \cup B = \{x \mid x \in A$ or $x \in B\}$

Eg :- If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$

$C = \{3, 4, 5\}$

then $A \cup B = \{1, 2, 3, 4\}$, $B \cup C = \{2, 3, 4, 5\}$

$C \cup A = \{1, 2, 3, 4, 5\}$

Intersection :-

The intersection of two sets A & B denoted by $A \cap B$ is the set of all elements that belong to both A & B.

i.e. $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

Eg : $A = \{1, 2, 3\}$ $B = \{2, 3, 4\}$

$A \cap B = \{2, 3\}$

Two sets are disjoint, if they have no elements in common

i.e. $A \cap B = \{\phi\}$.

Eg : $A = \{a, b, c\}$ $B = \{1, 2, 3\}$

$A \cap B = \{\ \}$.

complement :-

If U is the universal set & A is any set, then the complement of A denoted by A^c or $\bar{A}$ or A' is the set of all elements which belongs to U but not belong to A.

i.e. $A^c = \{x \mid x \in U \text{ and } x \notin A\}$.

Eg :- $U = \{1, 2, 3, 4, 5\}$

$A = \{1, 3, 5\}$.

then $\bar{A} = \{2, 4\}$.

Difference of A & B :-

If A & B any two sets, then the set of element that belong to A but do not belong to B is called the difference of A & B or relative complement of B w.r. to A & is denoted by A − B or A/B.
or B with respect to A.

i.e. $A - B = \{x \mid x \in A \text{ and } x \notin B\}$.

Eg :- $A = \{1, 2, 3\}$ $B = \{1, 3, 5, 7\}$

$A - B = \{2\}$

$B - A = \{5, 7\}$.

$\therefore A - B \neq B - A$.

ordered pair :-

An ordered pair consists of two objects in a given fixed order. An ordered pair is not a set consisting of two elements. We shall denote an ordered pair by (x, y).

Cartesian product :-

If A & B are sets, the set of all ordered pairs whose first component belongs to A & second component belong to B is called the cartesian product of A & B is denoted by

$A \times B = \{(a,b) \mid a \in A \ \& \ b \in B\}$.

Eg :- If $A = \{a, b, c\}$ & $B = \{1, 2\}$

$A \times B = \{(a,1)\ (a,2), (b,1)\ (b,2)\ (c,1)$
$(c,2)\}$.

$B \times A = \{(1,a)\ (1,b)\ (1,c)\ (2,a)\ (2,b)\ (2,c)\}$

$A \times B \neq B \times A$ unless $A = \phi$
$A \times B = B \times A$ iff $A = B$ | $B = \phi$.

Symmetric Difference :-

If A & B are any two sets the set of elements that belong to A or B not to both is called the symmetric difference of A & B denoted by $A \oplus B$ or $A \triangle B$.
(or) $(A \cup B) - (A \cap B)$

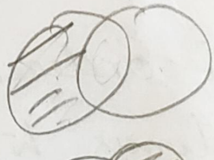
$A \oplus B = (A - B) \cup (B - A)$.

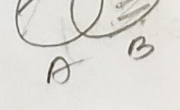
Ex :- Let $A = \{1, 2, 3\}$ $B = \{1, 3, 5, 7\}$

$A - B = \{2\}$. $B - A = \{5, 7\}$

$A \oplus B = (A - B) \cup (B - A)$

$= \{2, 5, 7\}$.





Problems :-

(1) Let us use the set builder notation to establish the identity

$A \cap B = B \cap A$

$A \cap B = \{x \mid x \in A \cap B\}$

$= \{x \mid x \in A \text{ and } x \in B\}$

$= \{x \mid x \in B \text{ \& } x \in A\}$

$= \{x \mid x \in B \cap A\}$

$= B \cap A$

(2) Let us use the set builder notation to establish the identity

$\overline{A \cap B} = \bar{A} \cup \bar{B}$

$x \in \overline{A \cap B} = \{x \mid x \notin A \cap B\}$

$= \{x \mid x \notin A \text{ and } x \notin B\}$

$\{x \mid x \notin A \text{ or } x \in \bar{B}\}$

$\{x \mid x \in \bar{A} \text{ or } x \in \bar{B}\}$

$\{x \mid x \in \bar{A} \text{ or } x \in \bar{B}\}$

$\{x \mid x \in \bar{A}$

$\{x \in \bar{A} \cup \bar{B}$ — (1)

$= \bar{A} \cup \bar{B}$

$\therefore (\overline{A \cup B}) \subset A' \cap B'$ — (1)

let $x \in A' \cap B'$

$\Rightarrow x \in A' \;\&\; x \in B'$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \notin A \cup B$

$\Rightarrow x \in \overline{A \cup B}$

*Note :- The dual of any statement is obtained by replacing $\cup$ by $\cap$, $\cap$ by $\cup$, ϕ by Uni, U by ϕ.

(3) Prove that $(A-C) \cap (C-B) = \varphi$ analytic
$(A \setminus C) \cap (C \setminus B) = \phi$
where A, B & C are sets.

$$(A-C) \cap (C-B) = \{x \mid x \in A \text{ and } x \notin C \text{ and } x \in C \text{ and } x \notin B\}$$

$$= \{x \mid x \in A \text{ and } (x \in C \text{ and } x \in \bar{C}) \text{ and } x \in \bar{B}\}$$

$$= \{x \mid (x \in A \text{ and } x \in \phi) \text{ and } x \in \bar{B}\}$$

$$= \{x \mid x \in A \cap \varphi \text{ and } x \in \bar{B}\}$$

$$= \{x \mid x \in \varphi \text{ and } x \in \bar{B}\}$$

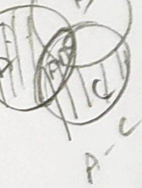
$$= \{x \mid x \in \varphi \cap \bar{B}\} = \phi.$$

<u>using</u> <u>Identities</u> :-

$$(A-C) \cap (C-B) = (A \cap \bar{C}) \cap (C \cap \bar{B}) \qquad [(A \cap \bar{C}) \cap C] \cap \bar{B}$$

A.L $\qquad [A \cap (\bar{C} \cap C)] \cap \bar{B}$

$$= [A \cap (\bar{C} \cap C)] \cap \bar{B}$$



$$= [A \cap \phi] \cap \bar{B}$$ compliment law. $\quad (A \cap \phi) \cap \bar{B}$

$$= \phi \cap \bar{B}$$ Domination law $\quad \phi \cap \bar{B} = \phi$

"

$$= \phi$$

(4). Prove that $A-(B \cap C) = (A-B) \cup (A-C)$ where A, B, & C are sets. analytically

$$A-(B \cap C) = \{x \mid x \in A \text{ and } x \notin B \cap C\}$$

$= \{x \mid x \in A \text{ and } (x \in (B \cap C)^c\}$

$= \{x \mid x \in A \text{ and } \{x \in B^c \cup x \in C^c\}\}$ De morgans

$= \{x \mid x \in A \text{ and } \{x \in B^c \text{ or } x \in C^c\}\}$

$= \{x \mid (x \in A \text{ and } x \notin B\} \text{ or } (x \in A \text{ and } x \notin C\}$

$= \{x \mid x \in (A-B) \text{ or } x \in (A-C)\}$

$= \{x \mid x \in (A-B) \cup (A-C)$

$= (A-B) \cup (A-C)$

Identities :-

$A - (B \cap C) = A \cap (B \cap C)^c$

$= A \cap (B^c \cup C^c)$ Demorgan's law.

$= (A \cap B^c) \cup (A \cap C^c)$ Distributive law

$= (A-B) \cup (A-C)$

(6) If A, B, C are sets then prove analytically that

$A \cap (B-C) = (A \cap B) - (A \cap C)$

$A \cap (B-C) = \{x \mid x \in A \text{ and } x \in B-C\}$

$= \{x \mid x \in A \text{ and } (x \in B \text{ and } x \notin C\}$

$= \{x \mid x \in A \text{ and} (x \in B \text{ and } x \in \bar{C})$

$= \{x \mid x \in A \cap B \cap \bar{C}\}$

$= A \cap B \cap \bar{C}$ — (1)

R.H.S $(A \cap B) - (A \cap C)$

$= \{x \mid x \in A \cap B \text{ and } x \notin A \cap C\}$

$= \{x \mid x \in A \cap B \text{ and } x \in (A \cap C)^c$

$= \{x \mid x \in A \cap B \text{ and } x \in (\bar{A} \cup \bar{C})\}$

$= \{x \mid x \in A \cap B \text{ and } (x \in \bar{A} \text{ or } x \in \bar{C})\}$

$= \{x \mid x \in (A \cap B) \text{ and } x \in \bar{A}] \text{ or}$

$\{x \in A \cap B \text{ and } x \in \bar{C}\}$

$= \{x \mid x \in \underbrace{A \cap B \cap \bar{A}}) \text{ or } x \in (A \cap B \cap \bar{C})$

$A \cap \bar{A} \cap B$

$\phi \cap B = \phi$

$= \{x \mid x \in \phi \text{ or } x \in (A \cap B \cap \bar{C})$

$= \{x \mid x \in \phi \cup (A \cap B \cap \bar{C})\}$

$= \{x \mid x \in (A \cap B \cap \bar{C}\}$

$= A \cap B \cap \bar{C}$ — (2)

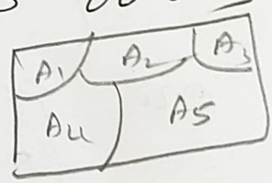
From ① & ② we get

$A \cap (B - C) = (A \cap B) - (A \cap C)$.

Partition of a set :-

If S is a non empty set, a collection of disjoint non-empty subsets of S whose union is S is called a partition of S.

Inotherwords, the collection of subsets of A_i, is a partition of S iff, for each i,



(i) $A_i \neq \varphi$

(ii) $A_i \cap A_j = \varphi$ for $i \neq j$

(iii) $\cup A_i = S$ where $\cup A_i$ represents the union of the subsets of A_i for all i.

Eg :- $A = \{1, 2, 3 \ldots 10\}$

$A_1 = \{1, 3, 5\}$, $A_2 = \{2, 4, 6, 8\}$

$A_3 = \{7, 9\}$. $A_4 = \{10\}$

Then A_1, A_2, A_3, A_4 form a partition ~~of~~ A.

② $A = \{a, b, c, d, e, f, g, h\}$

Consider the following subsets

$A_1 = \{a, b, c, d\}$, $A_2 = \{a, c, e, f, g, h\}$

$A_3 = \{a, c, e, g\}$ $A_4 = \{b, d\}$ $A_5 = \{f, h\}$

then $\{A_1, A_2\}$ is not a partition

since $A_1 \cap A_2 \neq \phi$.

$\{A_1, A_5\}$ is not a partition. Since

$e \notin A_1$ & $e \notin A_5$.

The collection $P = \{A_3, A_4, A_5\}$ is

a partition of A

one probable partitioning $\{a\}$, $\{b, c, d\}$

$\{e, f, g, h\}$

Another probable " $\{a, b\}$, $\{c, d\}$

$\{e, f, g, h\}$.

<u>Minsets</u> :- Let $\{B_1, B_2 \ldots B_n\}$ be a collection of subsets of a set A, the minset or minterm generated by $B_1, B_2 \ldots B_n$

Let $\{B_1, B_2 \ldots B_n\}$ be a set of subsets of a set A. A set of the form $D_1 \cap D_2 \cap \ldots \cap D_m$ where each D_i may be either D_i or D_i^c is called a minset or minterm generated by $B_1, B_2 \ldots B_n$. is of the form $D_1 \cap D_2 \cap \ldots \cap D_n$ when each $D_i = B_i$ or B_i^c

① Let $U = \{1, 2, \ldots 10\}$

$A = \{2, 4, 6\}$ $B = \{3, 5, 7, 9\}$

Find the min term & max term with respect to B.

$B^C = \{1, 2, 4, 6, 8, 10\}$

$A^C = \{1, 3, 5, 7, 8, 9, 10\}$

Soln D_1: $A \cap B = \{\ \}$

D_2: $A \cap B^C = \{2, 4, 6\}$

D_3: $A^C \cap B = \{3, 5, 7, 9\}$

D_4 $A^C \cap B^C = \{1, 8, 10\}$

$\therefore$ A_1, A_2, A_3, A_4 are the possible minterm

Note ① Union of minterms is the set A & they are disjoint (ie $\cup D_i = U$, $D_i \cap D_j = \phi$)

$\therefore$ set of all minterms of A is the partition of A.

② Dual of minset is maxset.

max term is 2^n.

Max term :-

$A \cup B = \{2, 3, 4, 5, 6, 7, 9\}$

$A^C \cup B = \{1, 3, 5, 7, 8, 9, 10\}$

$A \cup B^C = \{1, 2, 4, 6, 8, 10\}$

$A^C \cup B^C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Note :- If n subsets of a set are given, then the number of minterm or

(2) $A = \{1,2,3,4,5,6,7,8,9\}$. Find the minsets generated by $B_1 = \{5,6,7\}$ $B_2 = \{2,4,5,9\}$ $B_3 = \{3,4,5,6,8,9\}$

Soln

$B_1 \cap B_2 \cap B_3 = \{5\}$

$B_1^c \cap B_2 \cap B_3 = \{4,9\}$

$B_1 \cap B_2^c \cap B_3 = \{6\}$

$B_1 \cap B_2 \cap B_3^c = \{\ \}$

$B_1^c \cap B_2^c \cap B_3 = \{3,8\}$

$B_1 \cap B_2^c \cap B_3^c = \{7\}$

$B_1^c \cap B_2 \cap B_3^c = \{2\}$

$B_1^c \cap B_2^c \cap B_3^c = \{1\}$

$B_1^c = \{1,2,3,4,8,9\}$

$B_2^c = \{1,3,6,7,8\}$

$B_3^c = \{1,2,7\}$

The above sets are the minsets generated by B_1, B_2, B_3.

Since the minsets are mutually disjoint & their union is the given set A. ∴ The minset form a partition of A.

(3) Let $A = \{1,2,3,4,5,6\}$ $B_1 = \{1,3,5\}$ $B_2 = \{1,2,3\}$. Find the max sets generated by B_1, B_2.

Soln

$B_1^c = \{2,4,6\}$, $B_2^c = \{4,5,6\}$

$B_1 \cup B_2^c = \{1,3,4,5,6\}$, $B_1^c \cup B_2 = \{1,2,3,4,6\}$

$B_1 \cup B_2 = \{1,2,3,5\}$ $B_1^c \cup B_2^c = \{2,4,5,6\}$

Note The set of maxsets does not constitute a partition of A.