Example 3.1

A spectrum of 30 MHz is allocated to a wireless FDD cellular system which uses two 25 kHz simplex channels to provide full duplex voice and control channels, compute the number of channels available per cell if a system uses (a) four-cell reuse, (b) seven-cell reuse, and (c) 12-cell reuse. If 1 MHz of the allocated spectrum is dedicated to control channels, determine an equitable distribution of control channels and voice channels in each cell for each of the three systems.

Solution

Given:

Total bandwidth = 30 MHz

Channel bandwidth = 25 kHz × 2 simplex channels = 50 kHz/duplex channel

Total available channels = 30,000/50 = 600 channels

(a) For $N = 4$,

total number of channels available per cell = 600/4 ≈ 150 channels.

(b) For $N = 7$,

total number of channels available per cell = 600/7 ≈ 85 channels.

(c) For $N = 12$,

total number of channels available per cell = 600/12 ≈ 50 channels.

A 1 MHz spectrum for control channels implies that there are 1000/50 = 20 control channels out of the 600 channels available. To evenly distribute the control and voice channels, simply allocate the same number of voice channels in each cell wherever possible.

(a) For $N = 4$, we can have 5 control channels and 145 voice channels per cell. In practice, however, each cell only needs a single control channel (the control channels have a greater reuse distance than the voice channels). Thus, 1 control channel and 145 voice channels would be assigned to each cell.

(b) Total number of voice channels for $N = 7$, $(600 - 20)/7 = 82$ voice channels are to be assigned to each cell approximately, 4 cells with 3 control channels and 82 voice channels, and 3 cells with 2 control channels are to be assigned along with 83 voice channels.

Note: there is no fixed distribution of control channel as control channel has longer reuse distance than voice channel.

(c) For $N = 12$, we can have eight cells with two control channels and 48 voice channels, and four cells with one control channel and 49 voice channels each. In an actual system, each cell would have 1 control channel, 8 cells would have 48 voice channels, and 4 cells would have 49 voice channels.

Example 3.2

For given path loss exponent (a) $n = 4$ and (b) $n = 3$, find the frequency reuse factor and the cluster size that should be used for maximum capacity. The signal-to-interference ratio of 15 dB is minimum required for satisfactory forward channel performance of a cellular system. There are six co-channel cells in the first tier, and all of them are at the same distance from the mobile. Use suitable approximations.

Solution

(a) $n = 4$

First, let us consider a seven-cell reuse pattern.

Using Equation (3.4), Frequency reuse factor, $Q = D/R = \sqrt{(3\ N)} = \sqrt{21} =$ 4.583.

Using Equation (3.9), the signal-to-noise interference ratio is given by

$$S/I = (1/6) \times (4.583)^4 = 75.3 = 18.66 \text{ dB}$$

Since this is greater than the minimum required S/I, $N = 7$ can be used.

(b) $n = 3$

First, let us consider a seven-cell reuse pattern.

Using Equation (3.9), the signal-to-interference ratio is given by

$$S/I = (1/6) \times (4.583)^3 = 16.04 = 12.05 \text{ dB}$$

Since this is less than the minimum required S/I, we need to use a larger N.

Using Equation (3.3), the next possible value of N is 12, ($i = j = 2$).

The corresponding co-channel ratio is given by Equation (3.4) as

$$D/R = 6.0$$

Using Equation (3.3), the signal-to-interference ratio is given by

$$S/I = (1/6) \times (6)^3 = 36 = 15.56 \text{ dB}$$

Since this is greater than the minimum required S/I, $N = 12$ is used.

Example 3.5

An urban area has a population of two million residents. Three competing trunked mobile networks (systems A, B, and C) provide cellular service in this area. System A has 394 cells with 19 channels each, system B has 98 cells with 57 channels each, and system C has 49 cells, each with 100 channels. Find the number of users that can be supported at 2% blocking if each user averages two calls per hour at an average call duration of three minutes. Assuming that all three trunked systems are operated at maximum capacity, compute the percentage market penetration of each cellular provider.

Solution

System A

Given:

Probability of blocking = 2% = 0.02

Number of channels per cell used in the system, $C = 19$

Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 19$, from the Erlang B chart, the total carried traffic, A, is obtained as 12 Erlangs.

Therefore, the number of users that can be supported per cell is

$U = A/A_u = 12/0.1 = 120$

Since there are 394 cells, the total number of subscribers that can be supported by System A is equal to $120 \times 394 = 47280$

System B

Given:

Probability of blocking = 2% = 0.02

Number of channels per cell used in the system, $C = 57$

Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 57$, from the Erlang B chart, the total carried traffic, A, is obtained as 45 Erlangs.

Therefore, the number of users that can be supported per cell is
$U = A/A_u = 45/0.1 = 450$
Since there are 98 cells, the total number of subscribers that can be supported by System B is equal to $450 \times 98 = 44{,}100$

System C
Given:
Probability of blocking = 2% = 0.02
Number of channels per cell used in the system, $C = 100$
Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 100$, from the Erlang B chart, the total carried traffic, A, is obtained as 88 Erlangs.
Therefore, the number of users that can be supported per cell is
$U = A/A_u = 88/0.1 = 880$
Since there are 49 cells, the total number of subscribers that can be supported by System C is equal to $880 \times 49 = 43{,}120$
Therefore, total number of cellular subscribers that can be supported by these three systems are $47{,}280 + 44{,}100 + 43{,}120 = 134{,}500$ users.
Since there are two million residents in the given urban area and the total number of cellular subscribers in System A is equal to 47280, the percentage market penetration is equal to
$47{,}280/2{,}000{,}000 = 2.36\%$
Similarly, market penetration of System B is equal to
$44{,}100/2{,}000{,}000 = 2.205\%$
and the market penetration of System C is equal to
$43{,}120/2{,}000{,}000 = 2.156\%$
The market penetration of the three systems combined is equal to
$134{,}500/2{,}000{,}000 = 6.725\%$

Example 3.6

A city with a coverage area of 1500 sq km is covered with a 12-cell system each with a radius of 1.387 km. If the total spectrum allocated is 28.5 MHz with a full duplex channel bandwidth of 25 MHz. Assume a GOS of 0.02 for an Erlang B system is specified and the offered traffic per user is 0.03 Erlangs, compute (a) the number of cells in the service area, (b) the number of channels per cell, (c) traffic intensity of each cell, (d) the maximum carried traffic, (e) the total number of users that can be served for 2% GOS, (f) the number of mobiles per unique channel (where it is understood that channels are reused), and (g) the theoretical maximum number of users that could be served at one time by the system.

For $GOS = 0.02$ and $C = 57$, from the Erlang B chart, the total carried traffic, A, is obtained as 45 Erlangs.

Therefore, the number of users that can be supported per cell is

$U = A/A_u = 45/0.1 = 450$

Since there are 98 cells, the total number of subscribers that can be supported by System B is equal to $450 \times 98 = 44{,}100$

System C

Given:

Probability of blocking = 2% = 0.02

Number of channels per cell used in the system, $C = 100$

Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 100$, from the Erlang B chart, the total carried traffic, A, is obtained as 88 Erlangs.

Therefore, the number of users that can be supported per cell is

$U = A/A_u = 88/0.1 = 880$

Since there are 49 cells, the total number of subscribers that can be supported by System C is equal to $880 \times 49 = 43{,}120$

Therefore, total number of cellular subscribers that can be supported by these three systems are 47,280 + 44,100 + 43,120 = 134,500 users.

Since there are two million residents in the given urban area and the total number of cellular subscribers in System A is equal to 47280, the percentage market penetration is equal to

47,280/2,000,000 = 2.36%

Similarly, market penetration of System B is equal to

44,100/2,000,000 = 2.205%

and the market penetration of System C is equal to

43,120/2,000,000 = 2.156%

The market penetration of the three systems combined is equal to

134,500/2,000,000 = 6.725%

Example 3.6

A city with a coverage area of 1500 sq km is covered with a 12-cell system each with a radius of 1.387 km. If the total spectrum allocated is 28.5 MHz with a full duplex channel bandwidth of 25 MHz. Assume a GOS of 0.02 for an Erlang B system is specified and the offered traffic per user is 0.03 Erlangs, compute (a) the number of cells in the service area, (b) the number of channels per cell, (c) traffic intensity of each cell, (d) the maximum carried traffic, (e) the total number of users that can be served for 2% GOS, (f) the number of mobiles per unique channel (where it is understood that channels are reused), and (g) the theoretical maximum number of users that could be served at one time by the system.

Solution

(a) Given:

Total city coverage area = 500 sq km and cell radius, $R = 1.387$ miles
The area of a cell (hexagon) can be shown to be $2.5981R^2$, thus each cell covers $2.5981(1.387)^2 = 5$ sq km.
Hence, the total number of cells are $N_c = 500/5 = 100$ cells.

(b) The total number of channels per cell (C)
= allocated spectrum/(channel width × frequency reuse factor)
= 28500/(25 × 12) = 95 channels/cell

(c) Given:
$C = 95$ and $GOS = 0.02$
From the Erlang B chart, we have
traffic intensity per cell $A = 84$ Erlangs/cell

(d) Maximum carried traffic = number of cells × traffic intensity per cell
= 100 × 84 = 8400 Erlangs.

(e) Given traffic per user = 0.03 Erlangs
Total number of users = Total traffic/traffic per user
= 8400/0.03 = 2,80,000 users.

(f) Number of mobiles per channel = number of users/number of channels
= 280000/1140 = 245 mobiles/channel.

(g) The theoretical maximum number of served mobiles is the number of available channels in the system (all channels occupied)
$= C \times N_C = 95 \times 100 = 9500$ users, which is 3.4% of the customer base.

Example 3.7

A hexagonal cell within a four-cell system has a radius of 1.387 km. A total of 60 channels are used within the entire system. If the load per user is 0.029 Erlangs, and λ = 1 call/hour, compute the following for an Erlang C system that has a 5% probability of a delayed call:

(a) How many users per square kilometer will this system support?
(b) What is the probability that a delayed call will have to wait for more than 10 s?
(c) What is the probability that a call will be delayed for more than 10 seconds?

Solution

Given:

Cell radius, $R = 1.387$ km
Area covered per cell is $2.598 \times (1.387)^2 = 5$ sq km
Number of cells per cluster = 4
Total number of channels = 60
Therefore, number of channels per cell = 60 / 4 = 15 channels.

(a) From Erlang C chart, for 5% probability of delay with $C = 15$, traffic intensity = 9.0 Erlangs.

Therefore, number of users = total traffic intensity/traffic per user

$= 9.0/0.029 = 310$ users

$= 310$ users/5 sq km $= 62$ users/sq km

(b) Given $\lambda = 1$, holding time

$H = A_u/\lambda = 0.029$ hour $= 104.4$ seconds.

The probability that a delayed call will have to wait longer than 10 s is

$$Pr[delay > t|delay] = \exp(-(C - A)t/H)$$

$$= \exp(-(15 - 9.0)10/104.4) = 56.29\%$$

(c) Given $Pr[delay > 0] = 5\% = 0.05$

Probability that a call is delayed more than 10 seconds,

$$Pr[delay > 10] = Pr[delay > 0]Pr[delay > t|delay]$$

$$= 0.05 \times 0.5629 = 2.81\%$$

Example 3.8

Consider Figure 3.9. Assume each base station uses 60 channels, regardless of cell size. If each original cell has a radius of 1 km and each microcell has a radius of 0.5 km, find the number of channels contained in a 3 km by 3 km square centered around A under the following conditions: (a) without the use of microcells; (b) when the lettered microcells as shown in Figure 3.9 are used; and (c) if all the original base stations are replaced by microcells. Assume cells on the edge of the square to be contained within the square.

Solution

(a) without the use of microcells:

A cell radius of 1 km implies that the sides of the larger hexagons are also 1 km in length. To cover the 3 km by 3 km square centered around base station A, we need to cover 1.5 km (1.5 times the hexagon radius) toward the right, left, top, and bottom of base station A. This is shown in Figure 3.9. From Figure 3.9, we see that this area contains five base stations. Since each base station has 60 channels, the total number of channels without cell splitting is equal to $5 \times 60 = 300$ channels.

(b) with the use of the microcells as shown in Figure 3.9:

In Figure 3.9, the base station A is surrounded by six microcells. Therefore, the total number of base stations in the square area under study is equal to $5 + 6 = 11$. Since each base station has 60 channels, the total number of channels will be equal to $11 \times 60 = 660$ channels. This is a 2.2 times increase in capacity when compared to case (a).

(c) if all the base stations are replaced by microcells:

From Figure 3.9, we see there are a total of $5 + 12 = 17$ base stations in the square region under study. Since each base station has 60 channels, the total number of channels will be equal to $17 \times 60 = 1020$ channels. This is a 3.4 times increase in capacity compared to case (a).

Theoretically, if all cells were microcells having half the radius of the original cell, the capacity increase would approach four.

Example 3.9

Consider a cellular system in which an average call lasts two minutes, and the probability of blocking is to be no more than 1%. Assume that every subscriber makes one call per hour, on average. If there are a total of 395 traffic channels for a seven-cell reuse system, there will be about 57 traffic channels per cell. Assume that blocked calls are cleared so the blocking is described by the Erlang B distribution. From the Erlang B distribution, it can be found that the unsectored system may handle 44.2 Erlangs or 1326 calls per hour.

Now employing 120° sectoring, there are only 19 channels per antenna sector (57/3 antennas). For the same probability of blocking and average call length, it can be found from the Erlang B distribution that each sector can handle 11.2 Erlangs or 336 calls per hour. Since each cell consists of three sectors, this provides a cell capacity of $3 \times 336 = 1008$ calls per hour, which amounts to a 24% decrease when compared to the unsectored case. Thus, sectoring decreases the trunking efficiency while improving the S/I for each user in the system.

It can be found that using 60° sectors improves the S/I even more. In this case, the number of first tier interferers is reduced from six to only one. This results in $S/I = 29$ dB for a seven-cell system and enables four-cell reuse. Of course, using six sectors per cell reduces the trunking efficiency and increases the number of necessary handoffs even more. If the unsectored system is compared to the six sector case, the degradation in trunking efficiency can be shown to be 44%. (The proof of this is left as an exercise.)