# Assignment

# Digital and Analog Communication System

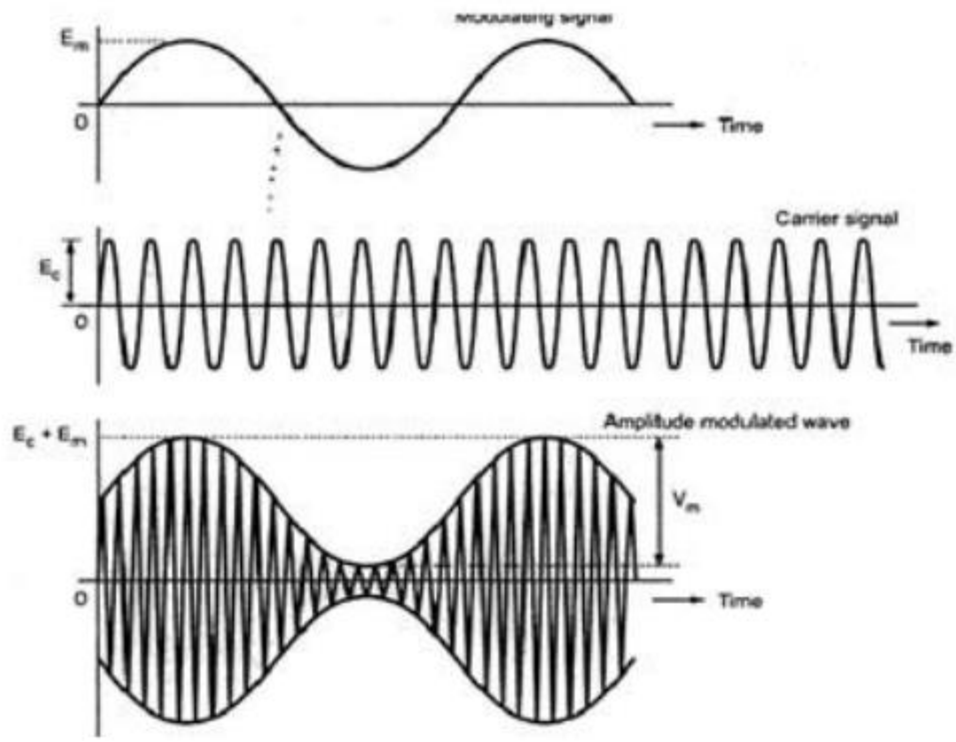
1. With suitable diagram and mathematical formulation discuss about amplitude modulation. Derive the expression for modulation index in terms of amplitude of an AM wave and show that the bandwidth of AM signal is twice of the maximum frequency of modulating signal.
   An audio frequency signal 10 sin $2\pi\times500t$ is used to amplitude modulate a carrier of 50 sin $2\pi\times10^5t$. Calculate,
   (i) Modulation index
   (ii) Sideband frequency
   (iii) Amplitude of each sideband frequencies
   (iv) Bandwidth required
   (v) Total power delivered to the load of $600\Omega$

**Answer:**

**Amplitude Modulation**

Amplitude Modulation is the changing the amplitude of the carrier signal with respect to the instantaneous change in message signal.

The amplitude modulated wave form, its envelope and its frequency spectrum and bandwidth.



Let us represent the modulating signal by $e_m$ and it is given as,

$$e_m = E_m \sin \omega_m t \qquad \text{... (1.2.1)}$$

and carrier signal can be represented by $e_c$ as,

$$e_c = E_c \sin \omega_c t \qquad \text{... (1.2.2)}$$

Here $E_m$ is maximum amplitude of modulating signal

$E_c$ is maximum amplitude of carrier signal

$\omega_m$ is frequency of modulating signal

and$\omega_c$ is frequency of carrier signal.

Using the above mathematical expressions for modulating and carrier signals, we can create a new mathematical expression for the complete modulated wave. It is given as,

$$E_{AM} = E_c + e_m$$

$$= E_c + E_m \sin \omega_m t \quad \text{by putting } e_m \text{ from equation (1.2.1)}$$

$\therefore$ The instantaneous value of the amplitude modulated wave can be given as,

$$e_{AM} = E_{AM} \sin \theta$$

$$= E_{AM} \sin \omega_c t$$

$$\therefore \quad \boxed{e_{AM} = (E_c + E_m \sin \omega_m t) \sin \omega_c t} \qquad \text{... (1.2.3)}$$

This is an equation of AM wave.

### Modulation Index and Percent Modulation

The ratio of maximum amplitude of modulating signal to maximum amplitude of carrier signal is called modulation index. i.e.,

$$\boxed{\text{Modulation index, } m = \frac{E_m}{E_c}} \qquad \text{... (1.2.4)}$$

Value of $E_m$ must be less than value of $E_c$ to avoid any distortion in the modulated signal. Hence maximum value of modulation index will be equal to 1 when $E_m = E_c$. Minimum value will be zero. If modulation index is higher than 1, then it is called *over modulation*. Data is lost in such case. When modulation index is expressed in percentage, it is also called percentage modulation.

**Calculation of modulation index from AM waveform :**

Fig. 1.2.2 shows the AM waveform. This is also called time domain representation of AM signal.

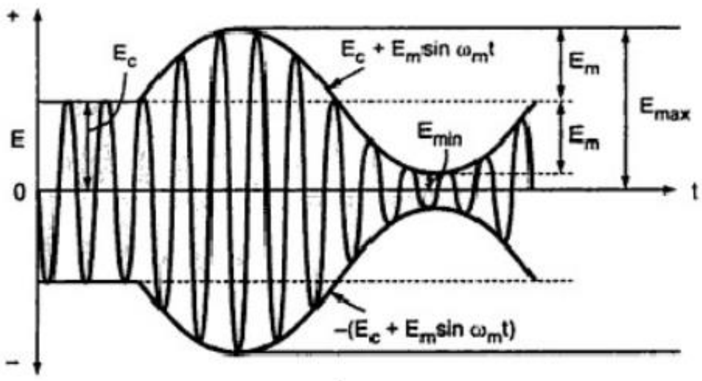



**Fig. 1.2.2 AM wave**

It is clear from the above signal that the modulating signal rides upon the carrier signal. From above figure we can write,

$$E_m = \frac{E_{max} - E_{min}}{2} \quad \text{... (1.2.5)}$$

and

$$E_c = E_{max} - E_m \quad \text{... (1.2.6)}$$

$$= E_{max} - \frac{E_{max} - E_{min}}{2} \text{ by putting for } E_m \text{ from equation (1.2.5)}$$

$$= \frac{E_{max} + E_{min}}{2} \quad \text{... (1.2.7)}$$

Taking the ratio of equation (1.2.5) and above equation,

$$m = \frac{E_m}{E_c} = \frac{\dfrac{E_{max} - E_{min}}{2}}{\dfrac{E_{max} + E_{min}}{2}}$$

$$\therefore \quad \boxed{m = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}} \quad \text{... (1.2.8)}$$

This equation gives the technique of calculating modulation index from AM wave.

### Frequency Spectrum and Bandwidth

The modulated carrier has new signals at different frequencies, called side frequencies or sidebands. They occur above and below the carrier frequency.

i.e.

$$f_{USB} = f_c + f_m$$

$$f_{LSB} = f_c - f_m$$

Here $f_c$ is carrier frequency and

$f_m$ is modulating signal frequency

$f_{LSB}$ is lower sideband frequency

Consider the expression of AM wave given by equation (1.2.3), i.e.,

$$e_{AM} = (E_c + E_m \sin \omega_m t) \sin \omega_c t \qquad \text{... (1.2.9)}$$

We know that $m = \frac{E_m}{E_c}$ from equation (1.2.4). Hence we have $E_m = m E_c$. Putting this value of $E_m$ in above equation we get,

$$\begin{aligned} e_{AM} &= (E_c + m E_c \sin \omega_m t) \sin \omega_c t \\ &= E_c (1 + m \sin \omega_m t) \sin \omega_c t \\ &= E_c \sin \omega_c t + m E_c \sin \omega_m t \sin \omega_c t \qquad \text{... (1.2.10)} \end{aligned}$$

We know that $\sin(A)\sin(B) = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$. Applying this result to last term in above equation we get,

$$\begin{aligned} e_{AM} = {} & E_c \sin \omega_c t + \frac{m E_c}{2} \cos(\omega_c - \omega_m) t \\ & - \frac{m E_c}{2} \cos(\omega_c + \omega_m) t \qquad \text{... (1.2.11)} \end{aligned}$$

In the above equation, the first term represents unmodulated carrier, the second term represents lower sideband and last term represents upper sideband. Note that $\omega_c = 2\pi f_c$ and $\omega_m = 2\pi f_m$. Hence above equation can also be written as,

$$\begin{aligned} e_{AM} = {} & E_c \sin 2\pi f_c t + \frac{m E_c}{2} \cos 2\pi (f_c - f_m) t \\ & - \frac{m E_c}{2} \cos 2\pi (f_c + f_m) t \qquad \text{... (1.2.12)} \\ = {} & E_c \sin 2\pi f_c t + \frac{m E_c}{2} \cos 2\pi f_{LSB} t + \frac{m E_c}{2} \cos 2\pi f_{USB} t \qquad \text{... (1.2.13)} \end{aligned}$$

From this equation we can prepare the frequency spectrum of AM wave as shown below in Fig. 1.2.3.

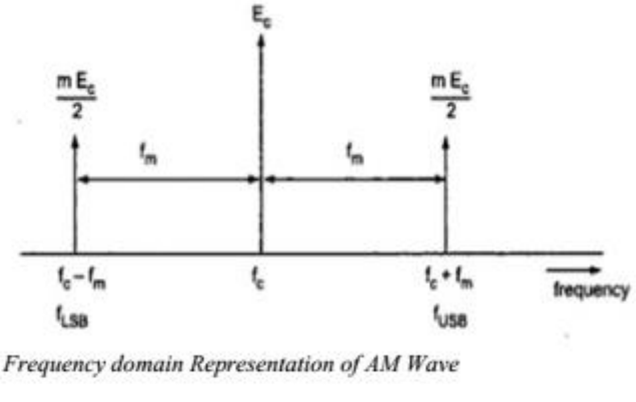



*Fig:1.2.3 : Frequency domain Representation of AM Wave*

This contains full carrier and both the sidebands, hence it is also called Double Sideband Full Carrier (DSBFC) system. We will be discussing this system, its modulation circuits and transmitters next, in this section.

We know that bandwidth of the signal can be obtained by taking the difference between highest and lowest frequencies. From above figure we can obtain bandwidth of AM wave as,

$$BW = f_{USB} - f_{LSB}$$

$$= (f_c + f_m) - (f_c - f_m)$$

$$\therefore \quad \boxed{BW = 2 f_m} \qquad \dots (1.2.14)$$

Thus bandwidth of AM signal is twice of the maximum frequency of modulating signal.

**Amplitude Modulation of Power distribution:**

AM signal has three components : Unmodulated carrier, lower sideband and upper sideband. Hence total power of AM wave is the sum of carrier power $P_c$ and powers in the two sidebands $P_{USB}$ and $P_{LSB}$. i.e.,

$$P_{Total} = P_c + P_{USB} + P_{LSB}$$

$$= \frac{E_{carr}^2}{R} + \frac{E_{LSB}^2}{R} + \frac{E_{USB}^2}{R} \qquad \dots (1.2.15)$$

***AM Power Distribution:***

Here all the three voltages are rms values and $R$ is characteristic impedence of antenna in which the power is dissipated. The carrier power is,

$$P_c = \frac{E_{carr}^2}{R} = \frac{\left(E_c / \sqrt{2}\right)^2}{R}$$

$$= \frac{E_c^2}{2R} \qquad \dots (1.2.16)$$

The power of upper and lower sidebands is same. i.e.,

$$P_{LSB} = P_{USB} = \frac{E_{SB}^2}{R} \qquad \text{Here } E_{SB} \text{ is rms voltage of sidebands.}$$

From equation (1.2.13) we know that the peak amplitude of both the sidebands is $\frac{m E_c}{2}$. Hence,

$$E_{SB} = \frac{m E_c / 2}{\sqrt{2}}$$

$$\therefore \quad P_{LSB} = P_{USB} = \left(\frac{m E_c / 2}{\sqrt{2}}\right)^2 \times \frac{1}{R}$$

$$= \frac{m^2 E_c^2}{8R} \qquad \dots (1.2.17)$$

Hence the total power (equation 1.2.15) becomes,

$$P_{Total} = \frac{E_c^2}{2R} + \frac{m^2 E_c^2}{8R} + \frac{m^2 E_c^2}{8R}$$

$$= \frac{E_c^2}{2R}\left[1+\frac{m^2}{4}+\frac{m^2}{4}\right]$$

$$\boxed{P_{Total} = P_c\left(1+\frac{m^2}{2}\right)} \qquad \text{... (1.2.19)}$$

$$\frac{P_{Total}}{P_c} = 1+\frac{m^2}{2} \qquad \text{... (1.2.20)}$$

*This equation relates total power of AM wave to carrier power, Maximum Value of modulation index, m=1 to avoid distortion. At this value of modulation index, Ptotal = 1.5 Pc. From the above equation we have*

$$\frac{m^2}{2} = \frac{P_{Total}}{P_c} - 1$$

$$\therefore \qquad m = \sqrt{2\left(\frac{P_{total}}{P_c}-1\right)}$$

**Solution :** (i) The given modulating signal is $e_m = 10\sin 2\pi\times 500t$. Hence, $E_m = 10$. The given carrier signal is $e_c = 50\sin 2\pi\times 10^5\, t$, hence, $E_c = 50$. Therefore modulation index will be,

$$m = \frac{E_m}{E_c} = \frac{10}{50} = 0.2 \quad \text{or} \quad 20\%$$

(ii) From the given equations,

$\omega_m = 2\pi\times 500,$ Hence $f_m = 500\,Hz$

And $\omega_c = 2\pi\times 10^5,$ Hence $f_c = 10^5\ Hz$ or 100 kHz

We know that $f_{USB} = f_c + f_m = 100\,kHz + 500\,Hz = 100.5\,kHz$

and $P_{LSB} = f_c - f_m = 100\,kHz - 500\,Hz = 99.5\,kHz.$

(iii) From equation (1.2.13) we know that the amplitudes of upper and lower sidebands is given as,

Amplitude of upper and lower sidebands $= \frac{mE_c}{2} = \frac{0.2\times 50}{2} = 5V$

(iv) Bandwidth of AM wave is given by equation(1.2.10) as,

$$BW \text{ of } AM = 2f_m = 2\times 500\,Hz = 1\,kHz$$

(v) Total power delivered to the load is given by equation (1.2.18) as

$$P_{total} = \frac{E_c^2}{2R}\left(1+\frac{m^2}{2}\right) = \frac{50^2}{2\times 600}\left(1+\frac{(0.2)^2}{2}\right)$$

$$\therefore \qquad = 2.125 \text{ watts}$$

2. With suitable diagram, discuss various pulse modulation techniques, such as, Pulse Amplitude Modulation (PAM), Pulse Width Modulation (PWM) and Pulse Position Modulation (PPM) along with their comparison. Discuss about the aliasing technique of signal regeneration at the receiver.

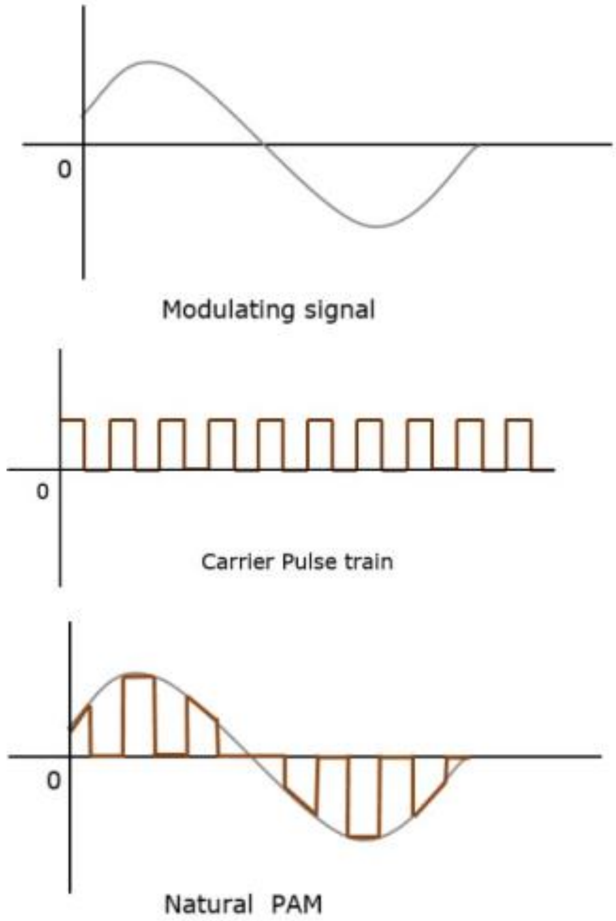
**Answer:**

**PULSE MODULATION**

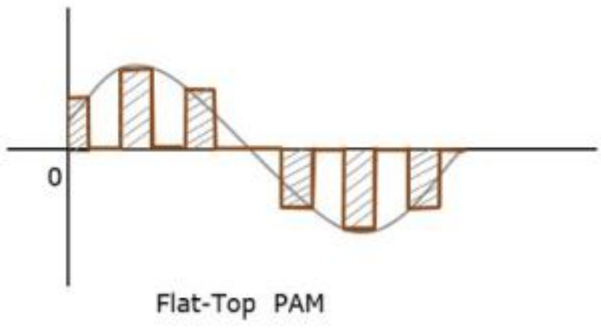
Pulse modulation is further divided into analog and digital modulation. The analog modulation techniques are mainly classified into Pulse Amplitude Modulation, Pulse Duration Modulation/Pulse Width Modulation, and Pulse Position Modulation. Pulse Amplitude Modulation (PAM) is an analog modulating scheme in which the amplitude of the pulse carrier varies proportional to the instantaneous amplitude of the message signal.

The pulse amplitude modulated signal, will follow the amplitude of the original signal, as the signal traces out the path of the whole wave. In natural PAM, a signal sampled at the Nyquist rate is reconstructed, by passing it through an efficient **Low Pass Frequency (LPF)** with exact cutoff frequency

The following figures explain the Pulse Amplitude Modulation




Though the PAM signal is passed through an LPF, it cannot recover the signal without distortion. Hence to avoid this noise, flat-top sampling is done as shown in the following figure.



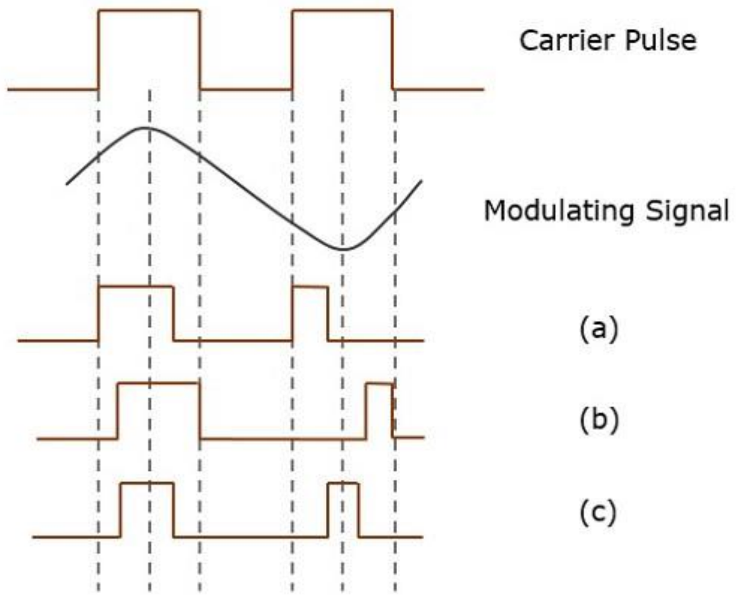

**Flat-top sampling** is the process in which sampled signal can be represented in pulses for which the amplitude of the signal cannot be changed with respect to the analog signal, to be sampled. The tops of amplitude remain flat. This process simplifies the circuit design.

## Pulse Width Modulation

Pulse Width Modulation (PWM) or Pulse Duration Modulation (PDM) or Pulse Time Modulation (PTM) is an analog modulating scheme in which the duration or width or time of the pulse carrier varies proportional to the instantaneous amplitude of the message signal.

The width of the pulse varies in this method, but the amplitude of the signal remains constant. Amplitude limiters are used to make the amplitude of the signal constant. These circuits clip off the amplitude, to a desired level and hence the noise is limited.

The following figures explain the types of Pulse Width Modulations.




There are three variations of PWM. They are –

- The leading edge of the pulse being constant, the trailing edge varies according to the message signal.
- The trailing edge of the pulse being constant, the leading edge varies according to the message signal.

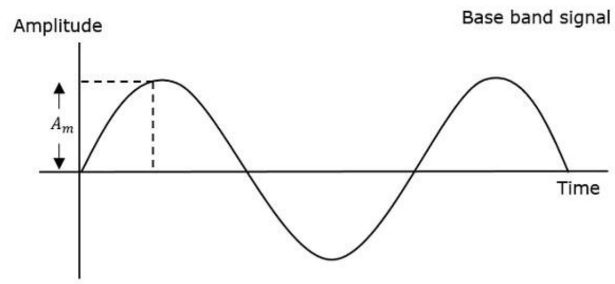
- The centre of the pulse being constant, the leading edge and the trailing edge varies according to the message signal.

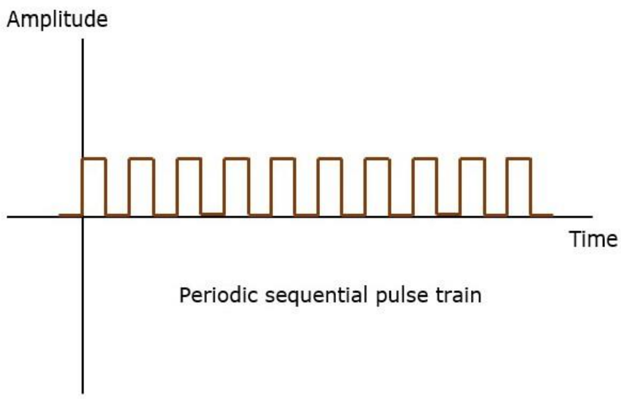
These three types are shown in the above given figure, with timing slots.

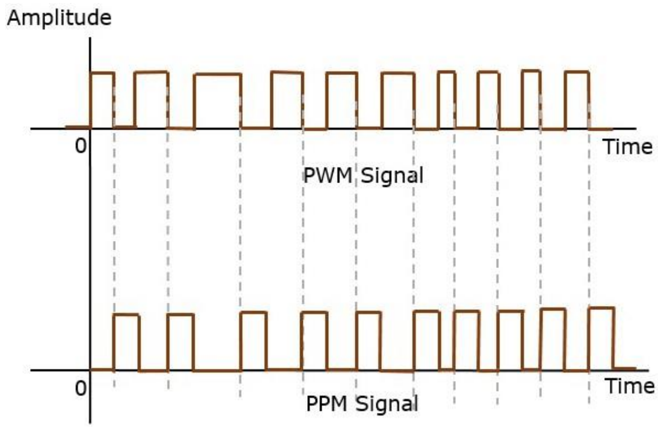
**Pulse Position Modulation**

**Pulse Position Modulation (PPM)** is an analog modulating scheme in which the amplitude and width of the pulses are kept constant, while the position of each pulse, with reference to the position of a reference pulse varies according to the instantaneous sampled value of the message signal.

The transmitter has to send synchronizing pulses (or simply sync pulses) to keep the transmitter and receiver in synchronism. These sync pulses help maintain the position of the pulses. The following figures explain the Pulse Position Modulation.



Pulse position modulation is done in accordance with the pulse width modulated signal. Each trailing of the pulse width modulated signal becomes the starting point for pulses in PPM signal. Hence, the position of these pulses is proportional to the width of the PWM pulses.

**Advantage**

As the amplitude and width are constant, the power handled is also constant.

**Disadvantage**

The synchronization between transmitter and receiver is a must. Comparison between PAM, PWM, and PPM. The comparison between the above modulation processes is presented in a single table.

| PAM | PWM | PPM |
|---|---|---|
| Amplitude is varied | Width is varied | Position is varied |
| Bandwidth depends on the width of the pulse | Bandwidth depends on the rise time of the pulse | Bandwidth depends on the rise time of the pulse |
| Instantaneous transmitter power varies with the amplitude of the pulses | Instantaneous transmitter power varies with the amplitude and width of the pulses | Instantaneous transmitter power remains constant with the width of the pulses |
| System complexity is high | System complexity is low | System complexity is low |
| Noise interference is high | Noise interference is low | Noise interference is low |
| It is similar to amplitude modulation | It is similar to frequency modulation | It is similar to phase modulation |

# Signal and system & DSP

1. Compute the signal power and signal energy for the discrete time signal.

$x(n) = e^{j10n}u(n)$

Energy $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

$= \sum_{n=-\infty}^{\infty} |e^{j10n}u(n)|^2$

$= \sum_{n=-\infty}^{\infty} |e^{j20n}| = \sum_{n=0}^{\infty} 1 = \infty$

$\therefore E = \infty$

Power $P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-\infty}^{\infty} |x(n)|^2$

$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} |e^{j20n}|$

$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1 = \lim_{N\to\infty} \frac{1}{2N+1}(N+1)$

$P = \lim_{N\to\infty} \frac{1+1/N}{2+1/N} = 1/2 \, W$

For given signal $E=\infty$ and $P = 1/2 \, W$

$\therefore$ $x(n)$ is Power signal

2. Given that x(t) has the Fourier transform X(ω), express the Fourier transforms of the signal listed below in terms of X(ω)

i) $x_1(t) = x(1-t) + x(-1-t)$

ii) $x_2(t) = x(3t-6)$

iii) $x_3(t) = \frac{d^2}{dt^2}x(t-1)$

a) Using the time-shifting property,

$$x(t+1) \longleftrightarrow X(\omega)\, e^{j\omega}$$

$$x(t-1) \longleftrightarrow X(\omega)\, e^{-j\omega}$$

For given signal

$$x(-t+1) \longleftrightarrow X(-\omega)\, e^{-j\omega}$$

$$x(-t-1) \longleftrightarrow X(-\omega)\, e^{j\omega}$$

$$\therefore\ x(1-t) + x(-1-t) \longleftrightarrow X(-\omega)\, e^{-j\omega} + X(-\omega)\, e^{j\omega}$$

$$x(1-t) + x(-1-t) \longleftrightarrow 2X(-\omega)\cos\omega$$

$$\therefore\ F[x_1(t)] = 2X(-\omega)\cos\omega$$

b) Using time shifting property $x(t-6) \longleftrightarrow X(\omega)\, e^{-j6\omega}$

$$x_2(t) = x(3t-6) \longleftrightarrow X_2(\omega) = \frac{1}{3} X\left(\frac{\omega}{3}\right) e^{-j6\omega/3}$$

$$F[x_2(t)] = \frac{1}{3} X\left(\frac{\omega}{3}\right) e^{-j2\omega}$$

c) Using differentiation in time property $\frac{d}{dt}x(t) \longleftrightarrow j\omega X(\omega)$

$$\frac{d^2x(t)}{dt^2} = (j\omega)^2 X(\omega)$$

$$x_3(t) = \frac{d^2x(t-1)}{dt^2} \longleftrightarrow X_3(\omega) = (j\omega)^2 X(\omega)\, e^{-j\omega}$$

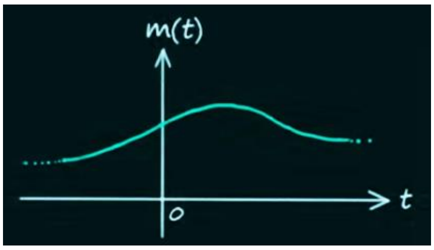
$$F[x_3(t)] = -\omega^2 X(\omega)\, e^{-j\omega}$$

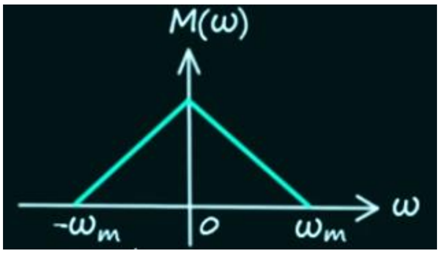
# DSP

1. Explain sampling theorem and derive the condition for perfect recovery of the signal

   Sampling is done to convert continuous time signal to discrete time signal. Shanon sampling theorem states that a signal can be represented in its samples and can be recovered back when sampling frequency is greater than or equal to twice of maximum frequency component present in the signal($\omega_s \geq 2\omega_m$)
   This sampling frequency equal to twice the maximum signal frequency is termed as Nyquist frequency or Nyquist rate.

Consider a band limited continuous signal *m(t)*

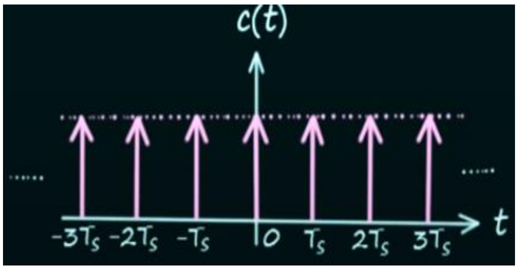


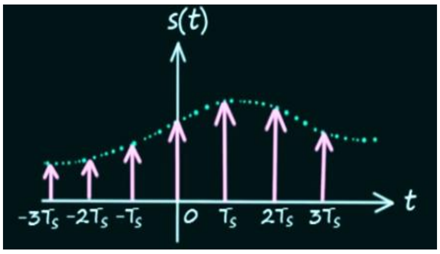

Message signal m(t)

spectrum of *m(t)* is given by *M(ω)*

Consider a train of impulse c(t) $C(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$




Sampled signal s(t) is obtained by multiplying m(t) and c(t) $s(t) = m(t)\ c(t)$




$s(t) = m(t)\ c(t)$

$s(t) \Leftrightarrow s(\omega)$

$$s(\omega) = \frac{1}{2\pi}\left[M(\omega) * C(\omega)\right]$$

$$C(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$C(\omega) = \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)$$

$$S(\omega) = \frac{1}{2\pi}\left[ M(\omega) * \ \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s) \right]$$

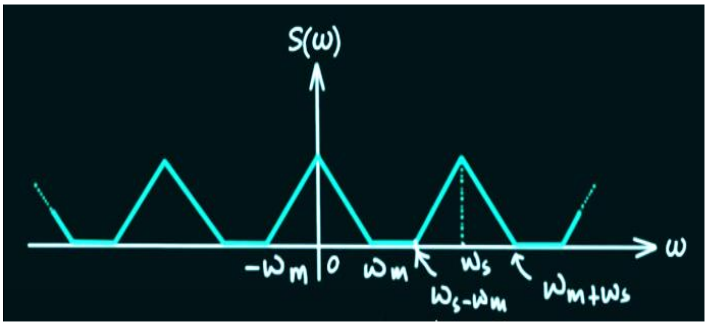
$$= \frac{\omega_s}{2\pi}\left[ M(\omega) * \ \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s) \right]$$

$$= \frac{1}{T_s}\left[ \sum_{n=-\infty}^{\infty} M(\omega) \ * \ \delta(\omega - n\omega_s) \right]$$

Since $x(t) * \delta(t - t_1) = x(t - t_1)$

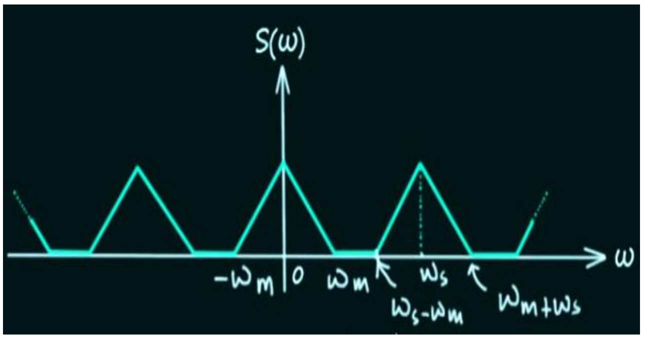
$$S(\omega) = \frac{1}{T_s}\left[ \sum_{n=-\infty}^{\infty} M(\omega - n\omega_s) \right]$$

$$S(\omega) = \frac{1}{T_s}[\ldots + M(\omega + \omega_s) + M(\omega) + M(\omega - \omega_s) + \ldots]$$

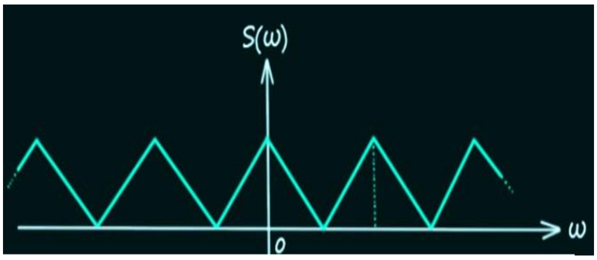



Three types of sampling condition

1. OVERSAMPLING $\omega_s - \omega_m > \omega_m \Rightarrow \omega_s > 2\omega_m$




2. NYQUIST SAMPLING $\omega_s - \omega_m = \omega_m \Rightarrow \omega_s = 2\omega_m$




3. UNDER SAMPLING $\omega_s < 2\omega_m$

4. what is a module.

A. A module contains one or more segments or partial segments. A module has a name assigned by the user. The module definitions the determine the scope of local symbols. An object file contains one or more modules. A module may be thought of as a "file" in many instances.

## Long type questions

1. Design a µ controller system using 8051, interface the external Rom of size 4KX8.

A. Given memory size = 4k

K = 1024 Bytes

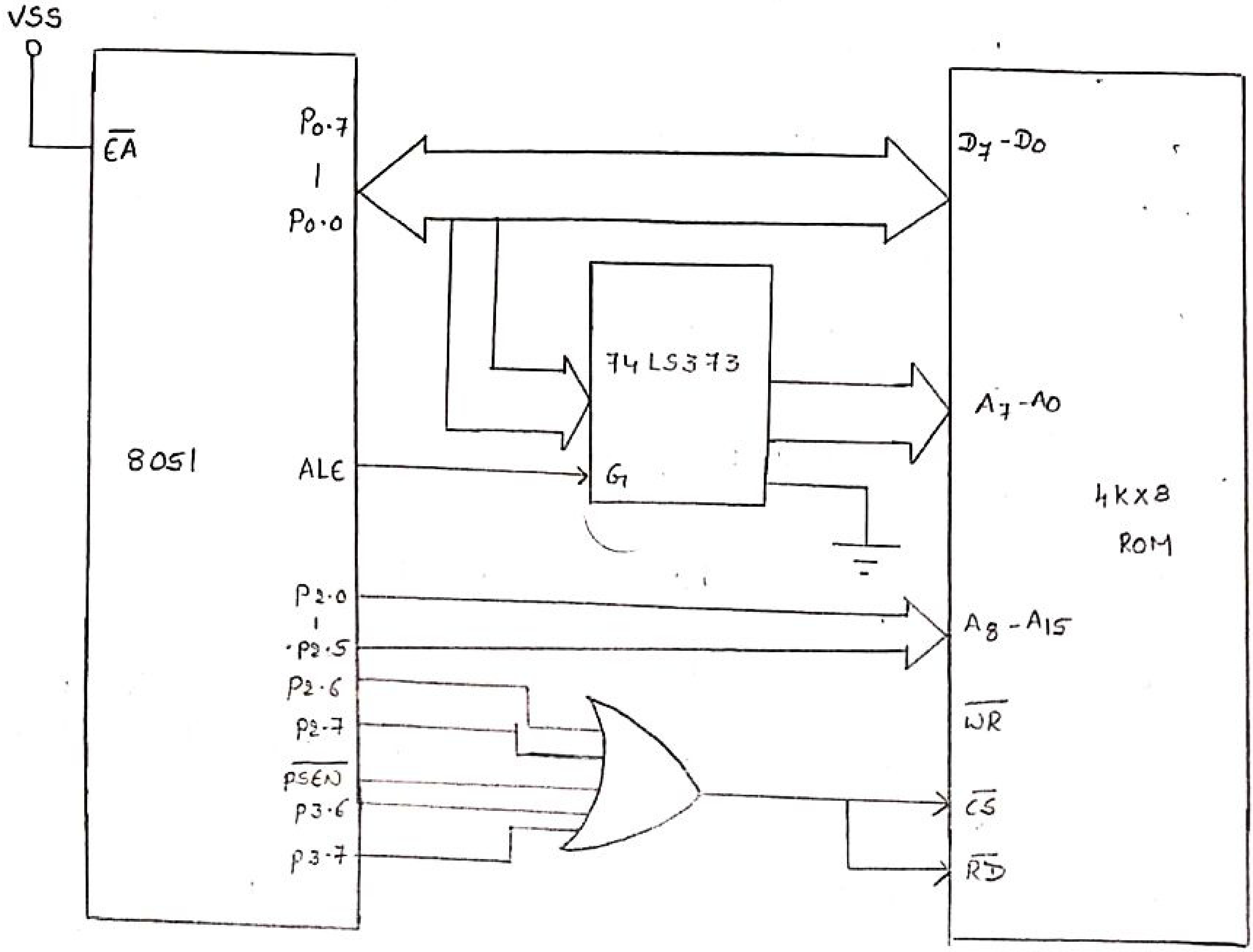
$2^n = 4k$

n = 12 is the Address lines

Here n = 12 ∴ $A_0$ to $A_{11}$ address lines are required.

Address Decoding (Memory Map) for 4kX8 RAM

| Address | $A_{15}$ | $A_{14}$ | $A_{13}$ | $A_{12}$ | $A_{11}$ | $A_{10}$ | $A_9$ | $A_8$ | $A_7$ | $A_6$ | $A_5$ | $A_4$ | $A_3$ | $A_2$ | $A_1$ | $A_0$ | Hex address |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| starting | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0000H |
| end | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0FFFH |




4K×8 memory (Rom) interfacing to µC 8051.

2. Design a µ controller system using 8051, 16k bytes of ROM and 32k bytes of RAM. Interface the memory such that starting address for ROM is 0000H and RAM is 8000H.

A. Given, memory size - ROM : 16 k

that means $2^n = 16k$

n address lines

Here n=14 ∴ Address lines are required ($A_0$ to $A_{13}$).

A14, A15, PSEN → ORed → $\overline{CS}$

when low - Rom is selected.

memory size - RAM: 32K

that means $2^n = 32K$

n address lines

Here $n = 15$ ∴ $A_0$ to $A_{14}$ address lines are required.

$A_{15}$ → inverted (Not Gate) → $\overline{CS}$

when high RAM is selected.

$\overline{PSEN}$ is used as chip select pin ROM

$\overline{RD}$ is used as read control signal pin } for RAM selection.

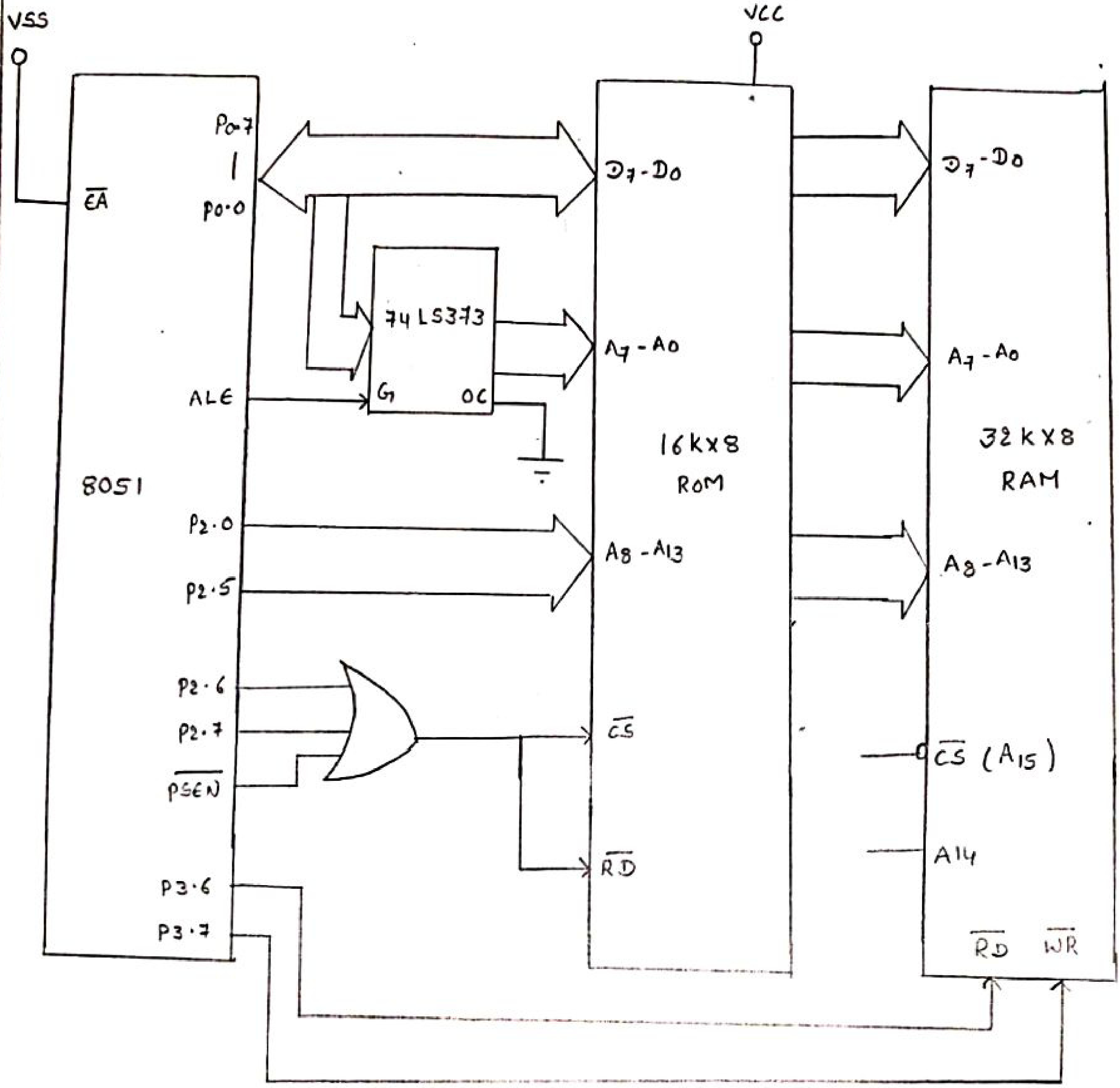
$\overline{WR}$ is used as write control signal pin

Address Decoding (memory Map) for 16KX8 ROM.

| Address | $A_{15}$ | $A_{14}$ | $A_{13}$ | $A_{12}$ | $A_{11}$ | $A_{10}$ | $A_9$ | $A_8$ | $A_7$ | $A_6$ | $A_5$ | $A_4$ | $A_3$ | $A_2$ | $A_1$ | $A_0$ | Hex address |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| starting | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0000H |
| end | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 3FFFH. |

Address Decoding (memory map) for 32KX8 RAM.

| Address | $A_{15}$ | $A_{14}$ | $A_{13}$ | $A_{12}$ | $A_{11}$ | $A_{10}$ | $A_9$ | $A_8$ | $A_7$ | $A_6$ | $A_5$ | $A_4$ | $A_3$ | $A_2$ | $A_1$ | $A_0$ | Hex address |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| starting | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8000H |
| end | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | FFFFH |




16KX8 (Rom) and 32KX8(RAM) memory interfacing to μC 8051.