Principle of inclusion – Exclusion :-

If A & B are finite subsets of a finite universal set U, then $|A \cup B| = |A| + |B| - |A \cap B|$ where $|A|$ denotes the cardinality of (number of distinct elements in) the set A.

This principle can be extended to a finite number of finite sets $A_1, A_2 \ldots A_n$ as follows,

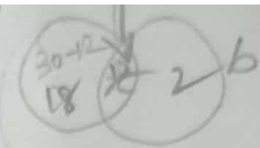
$$|A_1 \cup A_2 \cup \cdots \cup A_n| = \sum_i |A_i| - \sum_{i<j} |A_i \cap A_j| + \sum_{i<j<k} |A_i \cap A_j \cap A_k| - \cdots + (-1)^{n+1} |A_1 \cap A_2 \cap \cdots \cap A_n|$$

where the first sum is over all i, the second sum is over all pairs i, j with i < j, the third sum is over all triples i, j, k with i < j < k & so on.

$A \cup B = |A| + |B| - |A \cap B|$

$= 30 + 14 - 32$

$\begin{array}{r} 44 \\ -32 \\ \hline 12 \end{array}$



$A \cup B = 32$

$|A| = 30$

$|B| = 14$

Consider three finite sets A, B & C then

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$$

Problems :-

Find the number of integers between 1 & 250 both inclusive that are not divisible by any of the integers 2, 3, 5 & 7.

Soln

Let A, B, C, D be the sets of integers that lie between 1 & 250 & that are divisible by 2, 3, 5 & 7 resp. The elements of A are 2, 4, 6 ... 250

$\therefore |A| = \left\lfloor \frac{250}{2} \right\rfloor$

$= 125$

|||ly $|B| = \left\lfloor \frac{250}{3} \right\rfloor = 83$

$|C| = \left\lfloor \frac{250}{5} \right\rfloor = 50$

$$|D| = \left\lfloor \frac{250}{7} \right\rfloor = 35.$$

$$|A \cap B| = \left\lfloor \frac{250}{6} \right\rfloor = 41$$

$\therefore$ The set of integers between 1 & 250 which are divisible by 2 & 3 is the same as that which is divisible by 6, since 2 & 3 are relatively prime numbers.

$$|A \cap C| = \left\lfloor \frac{250}{10} \right\rfloor = 25$$

$$|A \cap D| = \left\lfloor \frac{250}{14} \right\rfloor = 17$$

$$|B \cap C| = \left\lfloor \frac{250}{15} \right\rfloor = 16$$

$$|B \cap D| = \left\lfloor \frac{250}{21} \right\rfloor = 11$$

$$|C \cap D| = \left\lfloor \frac{250}{35} \right\rfloor = 7$$

$$|A \cap B \cap C| = \left\lfloor \frac{250}{30} \right\rfloor = 8$$

$$|A \cap B \cap D| = \left\lfloor \frac{250}{42} \right\rfloor = 5$$

$$|A \cap C \cap D| = \left\lfloor \frac{250}{70} \right\rfloor = 3$$

$$|A \cap C \cap D| = \left\lfloor \frac{250}{105} \right\rfloor = 2$$

$$|A \cap B \cap C \cap D| = \left\lfloor \frac{250}{210} \right\rfloor = 1$$

By the principle of inclusion - exclusion the number of integers between 1 & 250 that are divisible by atleast one of 2, 3, 5 & 7 is given by

$$|A \cup B \cup C \cup D| = |A|+|B|+|C|+|D| - |A\cap B| - |A\cap C|$$
$$- |A\cap D| - |B\cap C| - |B\cap D| - |C\cap D|$$
$$+ |A\cap B\cap C| + |A\cap B\cap D| + |A\cap C\cap D|$$
$$|B\cap C\cap D| \cancel{+} |A\cap B\cap C\cap D|$$
$$= 125+83+50+35 - (41+25+17+16+11+7)$$
$$+ (8+5+3+2) - 1$$

$$= 293 - 117 + 18 - 1 \quad = 193$$

$\therefore$ Number of integers between 1 & 250 that are not divisible by any of the integers 2, 3, 5 & 7

= Total number of integers

$- |A \cup B \cup C \cup D|$

$= 250 - 193 = 57.$

120 — 35 pair are two.

52 wine, 75 mixed drinks, 62 iced tea, 20

2. In a Survey of 120 passengers, an airline found that 52 enjoyed wine with their meals, 75 enjoyed mixed drinks & 62 enjoyed iced tea. 35 enjoyed any given pair of these beverages & 20 passengers enjoyed all of them. Find the number of passengers who enjoyed

(i) only tea.

(ii) only one of the three

(iii) Exactly two of the three beverges

(iv) None of the drinks.

Soln

Let T, M, W denote the set of passengers enjoying iced tea, Mixed drink, wine resp.

$52+62+75-35-35-35+20$

Given $|T \cap M \cap W| = 20$

105

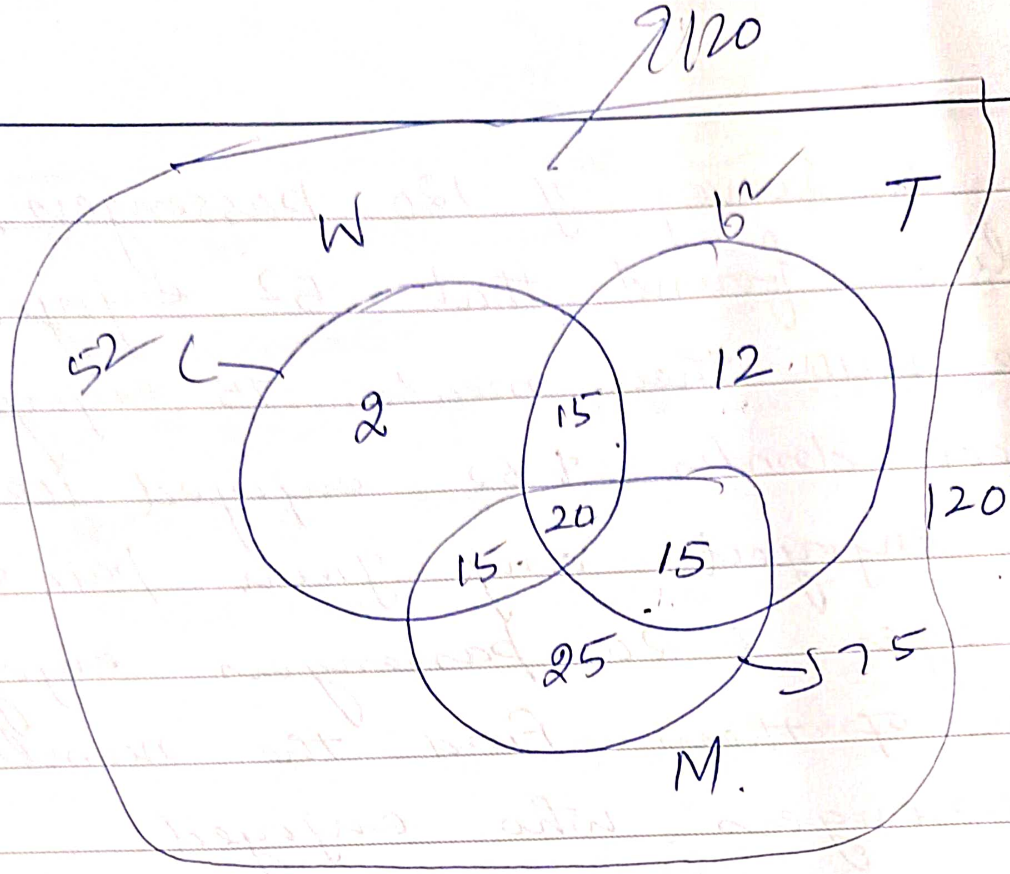
$|W| = 52 \quad |T| = 62 \quad |M| = 75$

114

$|T \cap M| = 35, \quad |M \cap W| = 35, \quad |T \cap W| = 35$

Number of passengers who enjoyed

only tea = ~~52~~ 12



62 − 50
12

(ii) Number of people who enjoyed only one of the three drinks

= 2+12+25 = 39

(iii) Exactly two of drinks = 15+15+15

= 45.

(iv) None of the drinks.

= 120 − 104.

= 16

62
52
75

either A or B or both

A∪B → atleast one of two events happen

A∩B → both A & B happens

$m \cap \bar{p} = m - p.$

(3) A survey of 150 college students reveals that :

83 own cars

97 own Bikes

28 own Motorcycles,

53 own a car & a bike,

14 own a car & a motorcycle,

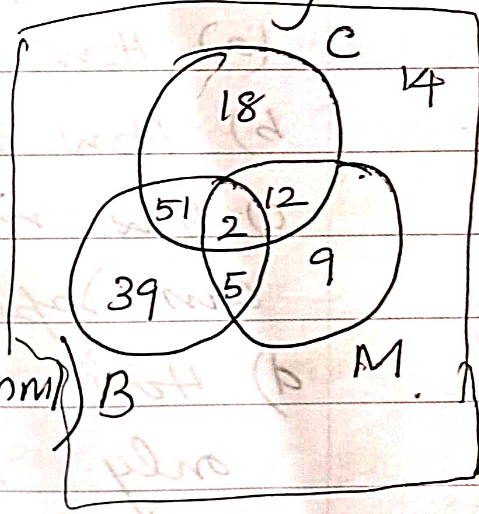
7 own a bike & a motorcycle,

2 own all three.

$39 + 51 + 7 = 97$

(i) How many own a bike & nothing else?

150



$|B - (C \cup M)|$

$= |B| - |B \cap (C \cup M)|$ → Distributive

$= |B| - |(B \cap C) \cup (B \cap M)|$

$= |B| - (|B \cap C| + |B \cap M| - |B \cap C \cap M|)$

$= 97 - (53 + 7 - 2)$

$= 39.$

b) How many students do not own any of the three?

$= 150 - |C \cup B \cup M|$

$= 150 - (83 + 97 + 28 - 53 - 14 - 7 + 2)$

$= 150 - 136 = 14.$

In a class of 80 students, the girls & boys are in the ratio 3 : 5. The students can speak only Hindi or only English or both Hindi & English. The number of boys & the number of girls who can speak only Hindi are equal & each of them is 40% of the total number of girls. 10% of the girls can speak both the languages & 58% of the boys can speak only English.

(a) How many girls can speak English

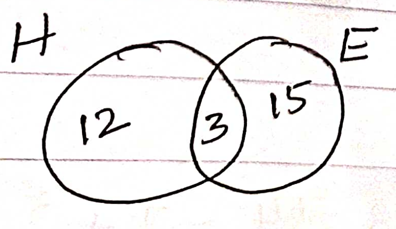
b) How many boys can " Hindi ?

c) How many students (boys & girls together) can speak both the languages ?

d) How many boys can speak either only Hindi or only English.

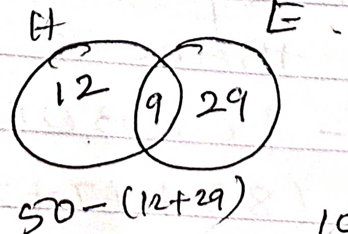
Girls

30



boys

50



$50-(12+29)$

$50-41$

$\frac{10}{100} \times 30$

80 students

3 : 5 – 8 parts

$\frac{40}{100} \times 30$

$\frac{58}{100} \times 50$

29.

a) $3+15 = 18$

b) $12+9 = 21$

c) $3+9 = 12$

d) $12+29 = 41.$

(5) There are 250 students in an engineering college. Of these 188 have taken a course in Fortran, 100 have taken a course in C & 35 have taken a course in Java. Further 88 have taken courses in both Fortran & C, 23 have taken courses in both C & Java & 29 have taken courses in both Fortran & Java. If 19 of these students have taken all the three courses, how many of them have not taken a course in any of these three programming languages?

Solution

F – Fortran C – C-languages

J – Java

$|F| = 188$, $|C| = 100$ $|J| = 35$

$|F \cap C| = 88$ $|C \cap J| = 23$, $|F \cap J| = 29$

$|F \cap C \cap J| = 19$.

$$|F \cup C \cup J| = |F| + |C| + |J| - |F \cap C| - |C \cap J| - |J \cap F| + |F \cap C \cap J|$$

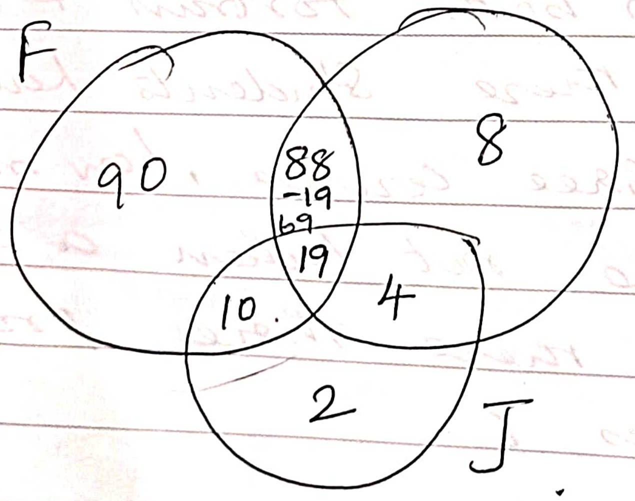
$= 202$ members have taken atleast one course.

Not taken any of the 3 courses is

$= 250 - 202$

$= 48$.

using Venn diagram



$$\begin{array}{r} 79 \\ 19 \\ \hline 98 \end{array}$$

$$\begin{array}{r} 188 \\ 98 \\ \hline 90 \end{array}$$

$= 250 - 202 = 48$.

(6) A total of 1232 students have taken a course in Tamil, 879 have taken a course in English & 114 have taken a course in Hindi. Further 103 have taken courses in both Tamil & English, 23 have taken courses in both Tamil & Hindi & 14 have taken courses in both English & Hindi. If 2092 students have taken atleast one of Tamil, English & Hindi, how many students have taken a course in all the three languages?

soln

$|T| = 1232$ $\quad |T \cap E| = 103$

$|E| = 879$ $\quad |T \cap H| = 23$

$|H| = 114$ $\quad |E \cap H| = 14.$

$|T \cup E \cup H| = 2092$ $\quad |T \cap E \cap H| = ?$

$$|T \cup E \cup H| = |T| + |E| + |H| - |T \cap E| - |T \cap H| - |E \cap H| + |T \cap E \cap H|$$

$$\text{ie } |T \cap E \cap H| = |T \cup E \cup H| - (|T| + |E| + |H|) + |T \cap E| + |T \cap H| + |E \cap H|$$

$$= 2092 - (2225) + 140.$$

$$= 7.$$

~~32~~ ~~57~~

42 68

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$(A_1 \cup A_2) \cap A_3 = (A_1 \cap A_3) \cup (A_2 \cap A_3)$$

$$|A_1 \cup A_2 \cup A_3| = |(A_1 \cup A_2) \cup A_3|$$

$$= |A_1 \cup A_2| + |A_3| - |(A_1 \cup A_2) \cap A_3|$$

$$= |A_1| + |A_2| - |A_1 \cap A_2| + |A_3| - |(A_1 \cap A_3) \cup (A_2 \cap A_3)|$$

$$= |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - (|A_1 \cap A_3| - |A_2 \cap A_3|)$$

$$+ |A_1 \cap A_3 \cap A_2 \cap A_3|$$

$$= |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_2 \cap A_3|$$

$$- |A_1 \cap A_3| + |A_1 \cap A_2 \cap A_3$$