INTRODUCTION TO CONTROL SYSTEMS:

* When a number of elements or components are connected in a sequence to perform a specific function, the group thus formed is called a system.
* In a system when the output quantity is controlled by varying the input quantity, the system is called control system.
* The output quantity is called controlled variable or response and input quantity is called command signal or excitation.

Open Loop System:

* Any physical system which does not automatically correct the variation in its output is called an open loop system.
* Control systems in which output quantity has no effect upon the input quantity are called open loop control systems (ie) the output is not fedback to input for correction.

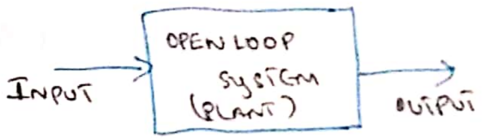


FIG: OPEN LOOP SYSTEM

* In open loop system, output can be varied by varying the input. But due to external disturbances the system output may change.
* when output changes due to disturbances, it is not followed by changes in input to correct output.
* So input has to be changed manually.

Closed loop System:

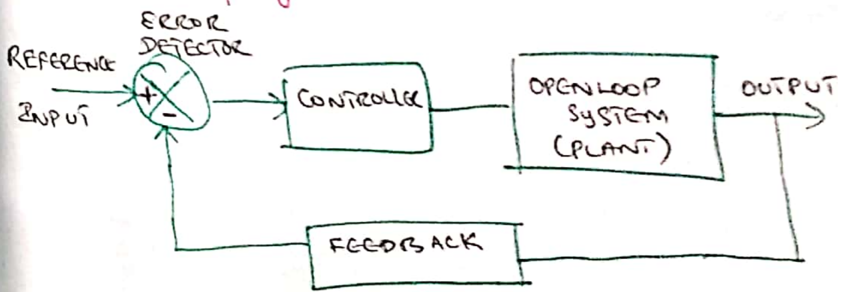


FIG: CLOSED LOOP SYSTEM.

* Control systems in which the output has an effect upon the input quantity so as to maintain the desired output value are called closed loop systems.
* The open loop system can be modified as closed loop system by providing a feedback.

* The provision of feedback automatically corrects the changes in output due to disturbances.
* So CLCS is called automatic control systems which consists of an

error detector, a controller, plant (OLCS) and feedback path elements.

Advantages & Disadvantages of Open loop systems:

① OLCS are simple and economical
② Easier to construct
③ Stable

① OLCS are inaccurate & unreliable
② Changes in O/p due to external disturbance are not corrected automatically.

Advantages & Disadvantages of Closed loop systems:

① Accurate even in presence of nonlinearities
② Sensitivity small so system stable
③ Less affected by noise.

① Complex & costlier
② Feedback leads to oscillatory response
③ FB Reduces overall gain
④ Stability is less.

Examples of CS: ① Temp. Control System ② Traffic control System ③ Numerical control System ④ Position CS

MODELING & MATHEMATICAL DESCRIPTION OF SYSTEMS IN TIME & FREQ DOMAIN:

* The mathematical model of a control system constitutes a set of differential eqns.
* The response of the system can be studied by solving diff. eqns for various I/p conditions.
* A control system is considered to be a linear time invariant system for mathematical analysis.
* The analysis is done using transfer function of the system

$$\text{Transfer function} = \left.\frac{\text{Laplace Transform of O/p}}{\text{Laplace transform of I/p}}\right|_{\text{with zero initial conditions}}$$

Types of Control Systems:

Mechanical Systems — Translational, Rotational

Electric Systems

Pneumatic Systems

Hydraulic Systems

Mechanical Translational Systems:

* Three basic elements of MTS are (i) Mass (ii) Spring (iii) dash pot. based on three phenomena which occur in Mechanical systems

* The weight of the mechanical system is called mass and is concentrated at the center of the body.

* The elastic deformation of the body is called spring.

* The friction existing in the mechanical system is called dash-pot.

* When a force is applied to a translational system it is opposed by opposing forces due to mass, friction & elasticity.

* The force acting on a mechanical body is governed by Newton's second law of motion.

* Newton's second law states that the sum of applied forces is equal to the sum of opposing forces on a body.

* The list of symbols used in MTS are

x – Displacement in m.

v – $\frac{dx}{dt}$ – velocity in m/sec.

a – $\frac{dv}{dt} = \frac{d^2x}{dt^2}$ – Accelation in m/sec^2

f – Applied force in Newton

f_m – Opposing force offered by mass of body in Newtons

f_k – Opposing force offered by elasticity of body (Spring) in Newtons.

f_b – Opposing force offered by friction of body (dash pot) in Newtons

M – Mass in kg.

K – Stiffness of spring N/m.

B – Viscous friction coefficient N-sec/m.

Force balance equations of ideal elements:

* Negligible friction and elasticity
* When force is applied the mass offers an opposing force proportional to accelaration of body

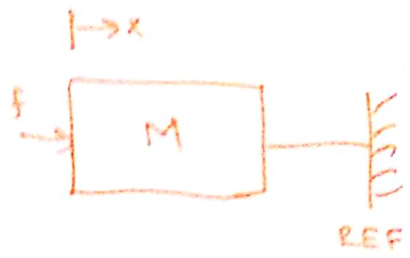


FIG: IDEAL MASS ELE

f - Applied force

f_m - Opposing force due to mass

$$f_m \propto \frac{d^2x}{dt^2} \quad \text{or} \quad f_m = M\frac{d^2x}{dt^2}$$

By Newton's second law

$$f = f_m = M\frac{d^2x}{dt^2}$$

* Consider an ideal frictional element dashpot with negligible mass and elasticity

f - Applied force

f_b - Opposing force due to friction

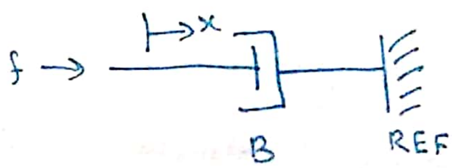


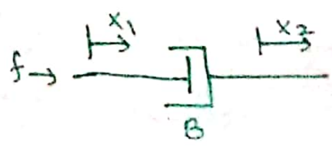
FIG: IDEAL DASHPOT WITH ONE END FIXED TO REF.

$$f_b \propto \frac{dx}{dt} \quad \text{or} \quad f_b = B\frac{dx}{dt}$$

$$f = f_b = B\frac{dx}{dt}$$

* When dashpot has displacement on both sides

$$f_b \propto \frac{d}{dt}(x_1 - x_2) \; ; \; f_b = B\frac{d}{dt}(x_1 - x_2)$$



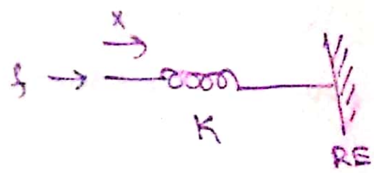
$$f = f_b = B\frac{d}{dt}(x_1 - x_2)$$

FIG: IDEAL DASHPOT WITH DISPLACEMENT ON BOTH SIDES/ENDS

* Consider an ideal elastic element spring with negligible mass and friction

f - Applied force

f_k - Opposing force due to elasticity



$$f_k \propto x \quad \text{or} \quad f_k = Kx \; ; \; f = f_k = Kx$$

FIG: IDEAL SPRING WITH ONE END FIXED TO REF.

· When spring has displacement at both ends

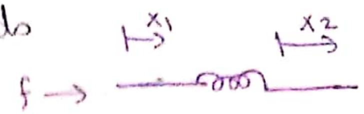


FIG: IDEAL SPRING WITH DISP AT BOTH ENDS

$$f_k \propto (x_1 - x_2)$$

$$f_k = K(x_1 - x_2)$$

$$f = f_k = K(x_1 - x_2).$$

Laplace transform of

$$x(t) = L(x(t)) = X(s)$$

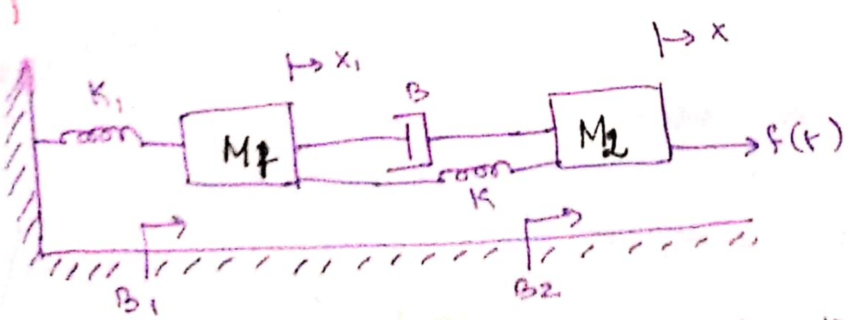
$$\frac{dx(t)}{dt} = L\left(\frac{dx(t)}{dt}\right) = sX(s)$$

$$\frac{d^2x(t)}{dt^2} = L\left(\frac{d^2x(t)}{dt^2}\right) = s^2X(s)$$

* Guidelines for TF of M/S:

1. Force balance equations at the nodes in a system. Mass elts are usually nodes
2. Linear displacement of masses (nodes) are $x_1, x_2 \ldots$ The first derivative is velocity and second derivative is accelaration.
3. Draw the free body diagrams.
4. For each free body diagram write a differential equation by equating sum of applied forces to sum of opposing forces.
5. Take laplace transform of differential equations to convert to algebraic equations obtain transfer function.

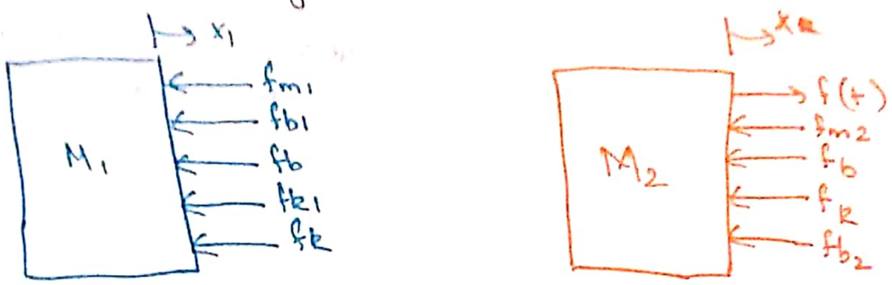
Example 1:



* In the above system f(t) is the input and displacement x is the output.

$$L(f(t)) = F(s) \quad ; \quad L(x) = X(s) \quad ; \quad TF = \frac{X(s)}{F(s)}$$

* The system has two nodes M1 and M2.
* The free body diagram of M1 & M2 are given as follows



$f_{m1} = M_1 \frac{d^2 x_1(t)}{dt^2}$; $f_{b1} = B_1 \frac{dx_1}{dt}$; $f_{k1} = K_1 x_1$

$f_b = B \frac{d}{dt}(x_1 - x)$; $f_k = K(x_1 - x)$

By Newton's second law

$$f_{m1} + f_{b1} + f_b + f_{k1} + f_k = 0.$$

$$M_1 \frac{d^2 x_1(t)}{dt^2} + B_1 \frac{dx_1}{dt} + K_1 x_1 + B \frac{d}{dt}(x_1 - x) + K(x_1 - x) = 0.$$

Taking Laplace on both sides

$$M_1 s^2 X_1(s) + B_1 s X_1(s) + K_1 X_1(s) + Bs(X_1(s) - X(s)) + K(X_1(s) - X(s)) = 0.$$

$$X_1(s)\left[M_1 s^2 + (B_1 + B)s + (K_1 + K)\right] = X(s)\left[Bs + K\right].$$

$$X_1(s) = \frac{X(s)\left[Bs + K\right]}{M_1 s^2 + (B_1 + B)s + K_1 + K}$$

$f_{m2} = M_2 \frac{d^2 x}{dt^2}$; $f_{b2} = B_2 \frac{dx}{dt}$

$f_b = B \frac{d}{dt}(x - x_1)$; $f_k = K(x - x_1)$

$$f_{m2} + f_b + f_k + f_{b2} = f(t)$$

$$M_2 \frac{d^2 x}{dt^2} + B_2 \frac{dx}{dt} + B \frac{d}{dt}(x - x_1) + K(x - x_1) = f(t)$$

Taking Laplace on both sides

$$M_2 s^2 X(s) + B_2 s X(s) + Bs\left[X(s) - X_1(s)\right] + K\left[X(s) - X_1(s)\right] = F(s)$$

$$X(s)\left[M_2 s^2 + (B_2 + B)s + K\right] - X(s)\frac{(Bs + K)^2}{M_1 s^2 + (B_1 + B)s + K_1 + K} = F(s)$$

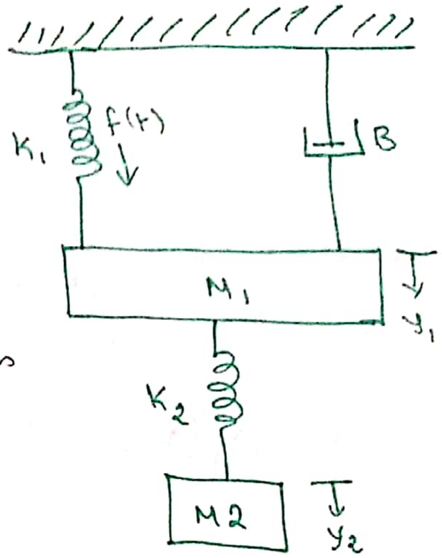
$$\frac{X(s)}{F(s)} = \frac{M_1 s^2 + (B_1 + B)s + (K_1 + K)}{\left[M_1 s^2 + (B_1 + B)s + K_1 + K\right] \times \left[M_2 s^2 + (B_2 + B)s + K - (Bs + K)^2\right]}$$

The transfer function of system is

$$\boxed{\frac{X(s)}{F(s)} = \left[\frac{M_1 s^2 + (B_1 + B)s + K_1 + K}{\left[M_1 s^2 + (B_1 + B)s + K_1 + K\right]\left[M_2 s^2 + (B_2 + B)s + K - (Bs + K)^2\right]}\right]}$$

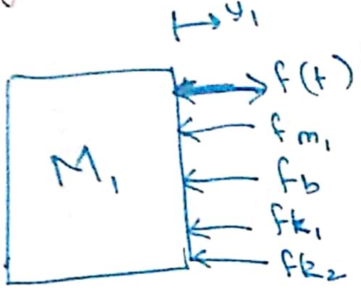
Example 2:

* The transfer function for this system is



$$TF = \frac{Y_2(s)}{F(s)}$$

* The system has two nodes M_1 & $M2$
* The free body diagram for Mass M_1 is given as



$$f_{m_1} = M_1 \frac{d^2 y_1}{dt^2} \qquad f_{k_1} = K_1 y_1$$

$$f_b = B \frac{dy_1}{dt} \qquad f_{k_2} = K_2 (y_1 - y_2)$$

By newton's second law

$$f_{m_1} + f_b + f_{k_1} + f_{k_2} = f(t)$$

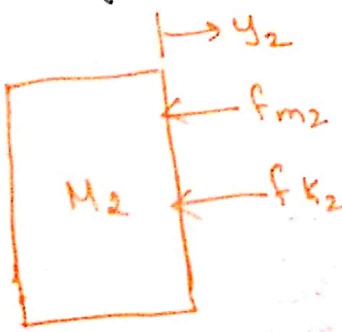
$$M_1 \frac{d^2 y_1}{dt^2} + B \frac{dy_1}{dt} + K_1 y_1 + K_2 (y_1 - y_2) = f(t) \quad \text{—— (1)}$$

Taking laplace on both sides of (1)

$$M_1 s^2 Y_1(s) + Bs\, Y_1(s) + K_1 Y_1(s) + K_2 (Y_1(s) - Y_2(s)) = F(s).$$

$$Y_1(s)\left[M_1 s^2 + Bs + K_1 + K_2\right] - K_2 Y_2(s) = F(s) \quad \text{—— (2)}$$

* The free body diagram for mass M_2 is given as



$$f_{m_2} = M_2 \frac{d^2 y_2}{dt^2} \qquad f_{k_2} = K_2 (y_2 - y_1)$$

By newton's second law

$$f_{m_2} + f_{k_2} = 0.$$

$$M_2 \frac{d^2 y_2}{dt^2} + K_2 (y_2 - y_1) = 0.$$

Taking laplace on both sides

$$M_2 s^2 Y_2(s) + K_2 Y_2(s) - K_2 Y_1(s) = 0$$

$$Y_2(s)(M_2 s^2 + K_2) - K_2 Y_1(s) = 0 \quad \text{—— (3)}$$

(7)

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$$Y_1(s) = Y_2(s)\left[\frac{M_2 s^2 + K_2}{K_2}\right]. \quad \text{--- } (4)$$

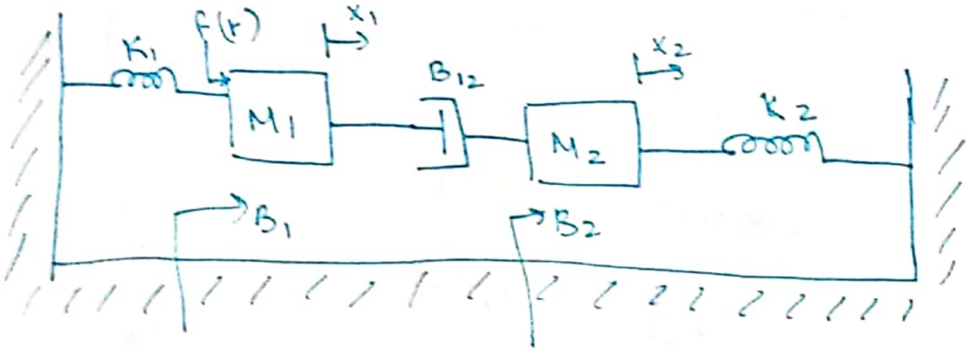
Subs $Y_1(s)$ in (4) into (2)

$$Y_2(s)\left[\frac{M_2 s^2 + K_2}{K_2}\right]\left[M_1 s^2 + Bs + K_1 + K_2\right] - K_2 Y_2(s) = F(s)$$

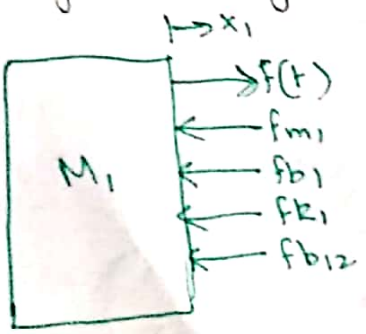
$$Y_2(s)\left[\frac{(M_2 s^2 + K_2)(M_1 s^2 + Bs + K_1 + K_2)}{K_2} - K_2\right] = F(s)$$

$$\boxed{\frac{Y_2(s)}{F(s)} = \frac{K_2}{(M_2 s^2 + K_2)(M_1 s^2 + Bs + K_1 + K_2) - K_2^2}}$$

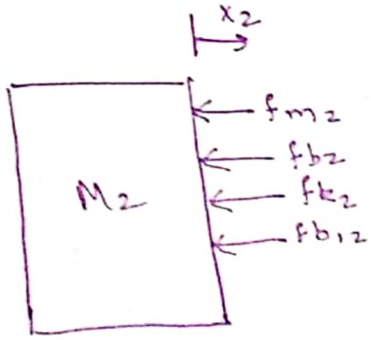
Example 3



* The transfer function for above system is $\frac{X_1(s)}{F(s)}$ & $\frac{X_2(s)}{F(s)}$
* There are two nodes M_1 & M_2
* The free body diagrams are



FBD1



FBD2

Sol FBD 1 of M_1

$f_{m_1} + f_{b_1} + f_{b_{12}} + f_{k_1} = f(t)$

$f_{m_1} = M_1 \frac{d^2 x_1}{dt^2}$; $f_{b_1} = B_1 \frac{dx_1}{dt}$; $f_{b_{12}} = B_{12} \frac{d}{dt}(x_1 - x_2)$; $f_{k_1} = K_1 x_1$

$$M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B_{12} \frac{d}{dt}(x_1 - x_2) + K_1 x_1 = f(t)$$

$$M_1 s^2 X_1(s) + B_1 s X_1(s) + B_{12} s \left(X_1(s) - X_2(s)\right) + K_1 X_1(s) = F(s)$$

$$X_1(s)\left[M_1 s^2 + B_1 s + B_{12} s + K_1\right] - B_{12} s X_2(s) = F(s) \quad \text{—— (1)}$$

Sol FBD 2 of M_2

$f_{m_2} + f_{b_2} + f_{b_{12}} + f_{k_2} = 0.$

$$M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{dx_2}{dt} + B_{12} \frac{d}{dt}(x_2 - x_1) + K_2 x_2 = 0.$$

$$M_2 s^2 X_2(s) + B_2 s X_2(s) + B_{12} s \left(X_2(s) - X_1(s)\right) + K_2 X_2(s) = 0$$

$$(M_2 s^2 + B_2 s + B_{12} s + K_2) X_2(s) - B_{12} s X_1(s) = 0.$$

$$X_2(s) = \frac{B_{12} s (X_1(s))}{(M_2 s^2 + B_2 s + B_{12} s + K_2)} \quad \text{—— (2)}$$

Substituting (2) in (1)

$$X_1(s)\left[M_1 s^2 + (B_1 + B_{12}) s + K_1\right] - \frac{B_{12} s \, B_{12} s \, X_1(s)}{M_2 s^2 + B_2 s + B_{12} s + K_2} = F(s)$$

$$X_1(s)\left[\frac{(M_1 s^2 + (B_1 + B_{12}) s + K_1)(M_2 s^2 + (B_2 + B_{12}) s + K_2) - B_{12} B_{12} s^2}{M_2 s^2 + (B_2 + B_{12}) s + K_2}\right] = F(s)$$

$$\boxed{\frac{X_1(s)}{F(s)} = \left[\frac{M_2 s^2 + (B_2 + B_{12}) s + K_2}{(M_1 s^2 + (B_1 + B_{12}) s + K_1)(M_2 s^2 + (B_2 + B_{12}) s + K_2) - (B_{12} s)^2}\right]} \quad \text{—— (3)}$$

From eqn (2)

$$X_1(s) = \frac{X_2(s)\left[M_2 s^2 + s(B_2+B_{12}) + K_2\right]}{B_{12}\, s} \quad \text{--- (4)}$$

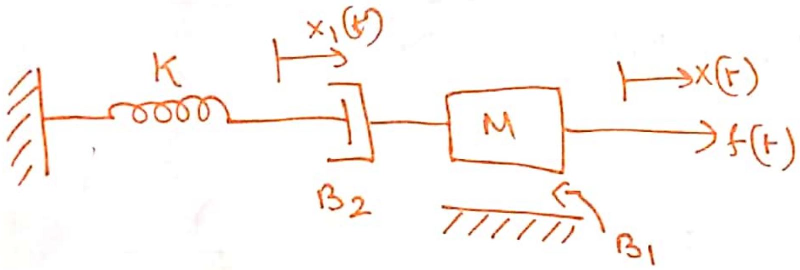
Subs (4) in (1)

$$\frac{(M_2 s^2 + s(B_2+B_{12}) + K_2)(M_1 s^2 + (B_1+B_{12})s + K_1)\, X_2(s)}{B_{12}\, s} - B_{12} s\, X_2(s) = F(s)$$

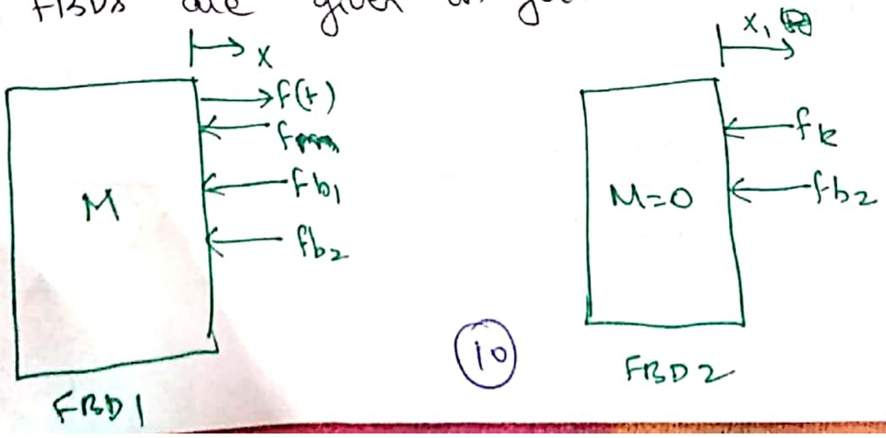
$$X_2(s)\left[\frac{(M_2 s^2 + s(B_2+B_{12})+K_2)(M_1 s^2+(B_1+B_{12})s+K_1) - (B_{12}s)^2}{B_{12}\, s}\right] = F(s)$$

$$\boxed{\frac{X_2(s)}{F(s)} = \frac{B_{12}\, s}{(M_2 s^2+s(B_2+B_{12})+K_2)(M_1 s^2+(B_1+B_{12})s+K_1) - (B_{12}s)^2}} \quad \text{--- (5)}$$

Example (4):



* There is one node M and another node which is the meeting point of spring & dash pot.
* The FBDs are given as follows



For FBD 1

$$f_m = M\frac{d^2x}{dt^2} \quad ; \quad fb_1 = B_1\frac{dx}{dt} \quad ; \quad fb_2 = B_2\frac{d(x(t) - x_1(t))}{dt}$$

$$f_m + fb_1 + fb_2 = f(t)$$

$$M\frac{d^2x}{dt^2} + B_1\frac{dx}{dt} + B_2\frac{d}{dt}(x - x_1) = f(t)$$

$$Ms^2X(s) + B_1 s X(s) + B_2 s(X(s) - X_1(s)) = F(s)$$

$$X(s)\left[Ms^2 + (B_1 + B_2)s\right] - B_2 s X_1(s) = F(s) \quad \text{—— (1)}$$

For FBD 2

$$f_k = K x_1 \quad ; \quad fb_2 = B_2\frac{d}{dt}(x_1 - x)$$

$$f_k + fb_2 = 0.$$

$$K X_1(s) + B_2 s X_1(s) - B_2 s X(s) = 0.$$

$$X_1(s)\left[B_2 s + K\right] = B_2 s X(s).$$

$$X(s) = \left[\frac{B_2 s + K}{B_2 s}\right] X_1(s) \quad \text{—— (2)}$$

$$X_1(s) = \left[\frac{B_2 s}{B_2 s + K}\right] X(s) \quad \text{—— (3).}$$

Subs (3) in (1)

$$X(s)\left[Ms^2 + (B_1 + B_2)s\right] - \left(\frac{(B_2 s) B_2 s}{B_2 s + K}\right) X(s) = F(s)$$

$$X(s)\left[\frac{\left[Ms^2 + (B_1 + B_2)s\right]\left[B_2 s + K\right] - B_2 B_2 s^2}{B_2 s + K}\right] = F(s)$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{B_2 s + K}{\left[Ms^2 + (B_1 + B_2)s\right]\left[B_2 s + K\right] - (B_2 s)^2}} \quad \text{—— (4)}$$

11

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SAKS (2) Pin (4)

Mechanical Rotational Systems:

* The model of MRS is obtained using 3 elements (i) moment of Inertia (J) of mass (ii) dash pot with rotational frictional coefficient (B) (iii) torsional spring with stiffness (K).

* Moment of Inertia of a system is concentrated at the center of gravity of the body

* When a torque is applied to a MRS, it is opposed by opposing torque due to J, B & K.

* The list of symbols in MRS are

θ – Angular displacement, rad. — x

$\frac{d\theta}{dt}$ – Angular velocity, rad/sec — $\frac{dx}{dt}$

$\frac{d^2\theta}{dt^2}$ – Angular acceleration, rad/sec^2 – $\frac{d^2x}{dt^2}$

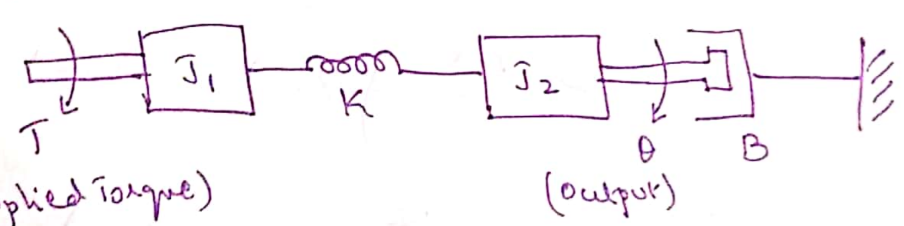
T – Applied torque, N-m — $f(t)$

J – Moment of Inertia, Kg-m^2/rad. – f_m

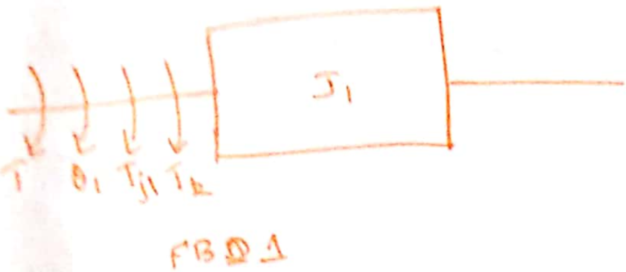
B – Rotational frictional component, N-m/(rad/sec) → f_b

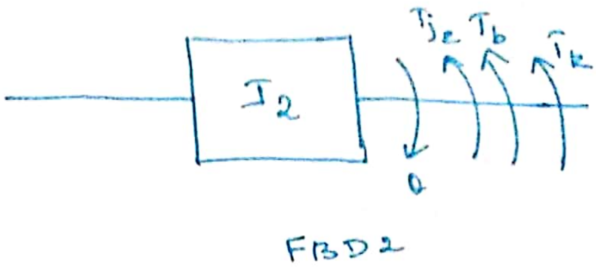
K – Stiffness of spring, N-m/rad — f_k

Example (1):



* There are two nodes J_1 & J_2 so the free body diagrams of both are given as follows





For FBD 1 the differential equations are given as

$$T_{j1} = J_1 \frac{d^2\theta_1}{dt^2} \qquad T_k = K(\theta_1 - \theta)$$

$$T_{j1} + T_k \Rightarrow J_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta) = T$$

$$J_1 s^2 \theta_1(s) + K(\theta_1(s) - \theta(s)) = T(s)$$

$$(J_1 s^2 + K)\,\theta_1(s) - K\theta(s) = T(s) \quad \text{——— (1)}$$

For FBD 2 the differential equations are given as

$$T_{j2} = J_2 \frac{d^2\theta}{dt^2} \;;\quad T_b = B\frac{d\theta}{dt} \;;\quad T_k = K(\theta - \theta_1)$$

$$J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K[\theta - \theta_1] = 0.$$

$$J_2 s^2 \theta(s) + Bs\,\theta(s) + K(\theta(s) - \theta_1(s)) = 0$$

$$\theta(s)[J_2 s^2 + Bs + K] - K\theta_1(s) = 0 \quad \text{——— (2)}$$

Transfer function is $\frac{\theta(s)}{T(s)}$ So. from (2)

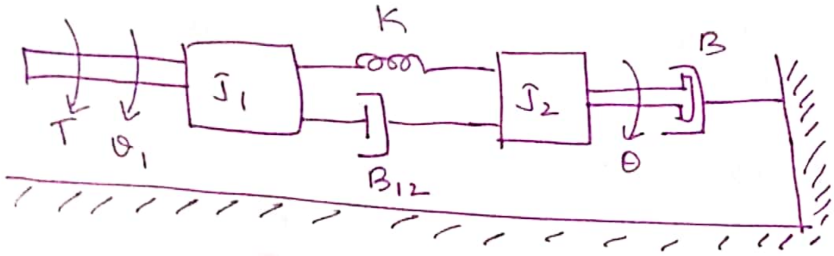
$$\theta_1(s) = \frac{(J_2 s^2 + Bs + K)\,\theta(s)}{K.} \quad \text{——— (3)}$$

Subs (3) in (1)

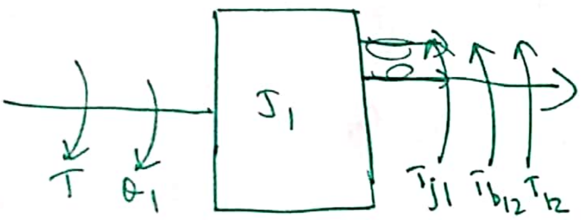
$$\left[\frac{(J_1 s^2 + K)(J_2 s^2 + Bs + K)}{K} - K\right]\theta(s) = T(s) \quad \text{— (4)}$$

$$\boxed{\frac{\theta(s)}{T(s)} = \frac{K}{(J_1 s^2 + K)(J_2 s^2 + Bs + K) - K^2} \quad \text{— (5)}}$$

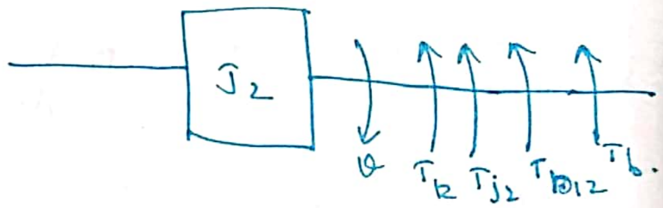
Example (6):



* There are two nodes J_1 & J_2 and the FBDs are given as



FBD 1



$T_{j1} + T_{b12} + T_{k2} = T$.

$T_{j2} + T_{b12} + T_b + T_{k2} = 0$.

$$\theta_1(s)\left[J_1 s^2 + sB_{12} + K\right] - \theta(s)\left[B_{12}s + K\right] = T(s) \quad \text{— (1)}$$

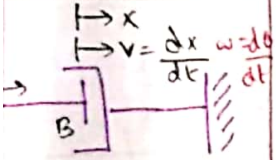
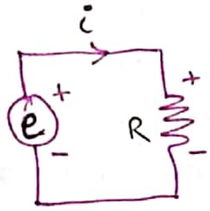
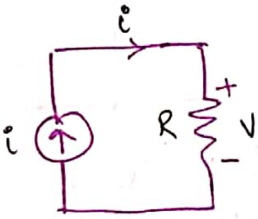
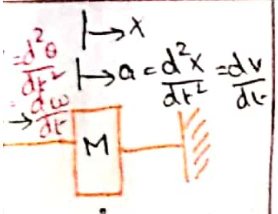
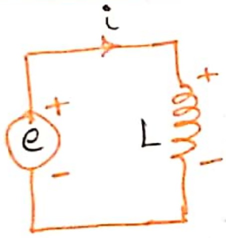
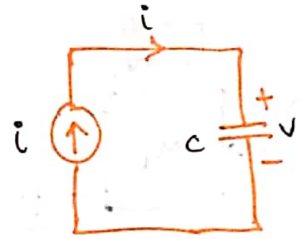
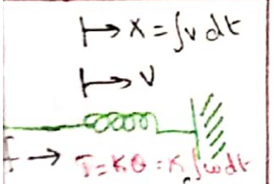
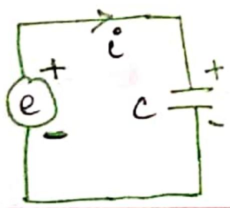
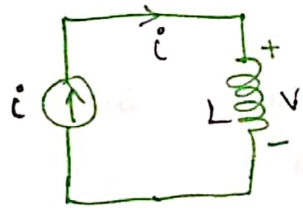
$$\theta(s)\left[J_2 s^2 + s(B_{12} + B) + K\right] - \theta_1(s)\left[B_{12}s + K\right] = 0. \quad \text{— (2)}$$

$$\theta_1(s) = \left[\frac{J_2 s^2 + s(B_{12} + B) + K}{B_{12}s + K}\right]\theta(s) \quad \text{— (3)}$$

$$\boxed{\frac{\theta(s)}{T(s)} = \frac{sB_{12} + K}{(J_2 s^2 + s(B_{12} + B) + K)(J_1 s^2 + sB_{12} + K) - (sB_{12} + K)^2} \quad \text{— (4)}}$$

Electric Systems :

- Models of electrical systems can be obtained using resistor, capacitor and inductor
- The electric network elements are R, L, C, voltage or current source.
- The differential equations for electric circuits can be formed using Kirchoff's voltage or current laws.

MECHANICAL SYSTEM	ELECTRICAL SYSTEM (VOLTAGE)	ELECTRICAL SYSTEM (CURRENT
INPUT : FORCE OUTPUT : VELOCITY	INPUT : VOLTAGE SOURCE OUTPUT : CURRENT THROUGH THE ELEMENT	INPUT : CURRENT SOURCE OUTPUT : VOLTAGE ACROSS THE ELEMENT.
 $f = B\frac{dx}{dt} = Bv$ $T = B\,d\theta/dt = B\omega$	 $e = Ri$	 $i = \frac{1}{R}V$
 $f = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$ $T = J\frac{d^2\theta}{dt^2} = J\,d\omega/dt$	 $e = L\frac{di}{dt}$	 $i = C\frac{dv}{dt}$
 $T = K\theta = K\int \omega\,dt$ $f = Kx = K\int v\,dt$	 $e = \frac{1}{C}\int i\,dt$	 $i = \frac{1}{L}\int v\,dt$

* To obtain electrical analogous of mechanical systems

1) In electrical systems the elts in series have same current and elts in parallel have same voltage, likewise in mechanical systems elts of same velocity are said to be in series and elts of same force are said to be in ||el.

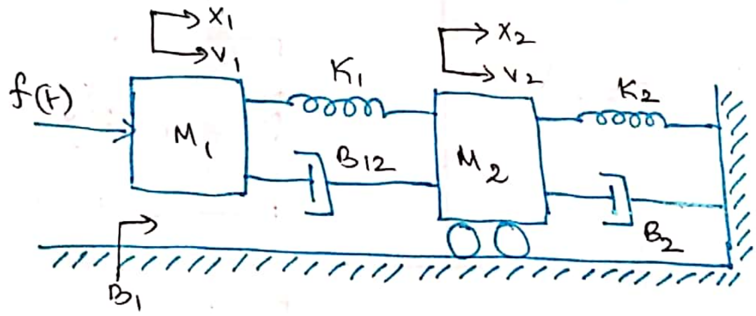
2) Each node in mech system corresponds to closed loop/node in electrical system.

3) The number of meshes/nodes in electrical system is equal to no. of nodes in mech system.

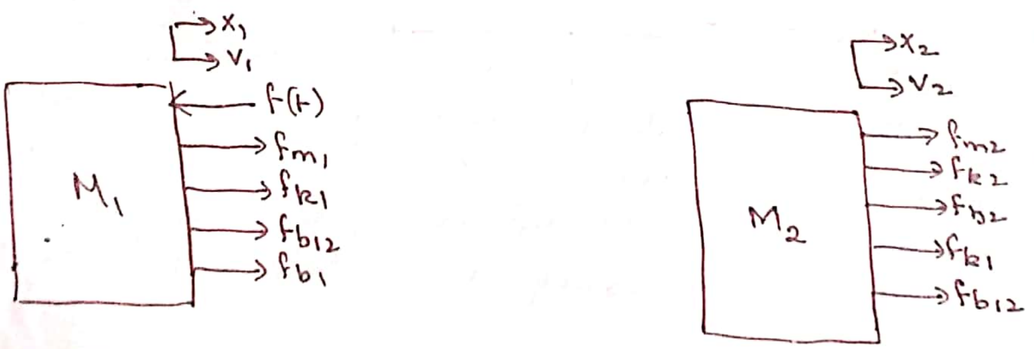
4) The element connected between two nodes in mechanical system is represented as common element between two meshes/nodes in electrical system

ITEM	MECHANICAL SYSTEM	ELECTRICAL SYSTEM (MESH BASIS SYSTEM)	ELECTRICAL SYSTEM (NODE BASIS SYSTEM)
INDEPENDANT VARIABLE INPUT	FORCE F TORQUE, T	VOLTAGE, e	CURRENT, i
DEPENDENT VARIABLE OUTPUT	VELOCITY, V ANGULAR VELOCITY, ω	CURRENT, i	VOLTAGE, ϕ
	DISPLACEMENT, X ANGULAR DISPLACEMENT, θ	CHARGE, q	FLUX, ϕ
DISSIPATIVE ELEMENT	FRICTIONAL/ROTATIONAL COEFFICIENT OF DASH POT, B	RESISTANCE, R	CONDUCTANCE $G=\frac{1}{R}$
STORAGE ELEMENT	MASS, M MOMENT OF INERTIA, J	INDUCTANCE, L	CAPACITANCE, C
	STIFFNESS OF SPRING, K	INVERSE OF CAPACITANCE, 1/C	INVERSE OF INDUCTANCE, 1/L
PHYSICAL LAW	NEWTON'S SECOND LAW $\Sigma F = 0$ $\Sigma T = 0$	KIRCHOFF'S VOLTAGE LAW $\Sigma V = 0$	KIRCHOFF'S CURRENT LAW $\Sigma i = 0$
CHANGING THE LEVEL OF INDEPENDENT VARIABLE.	LEVER GEAR $\frac{f_1}{f_2} = \frac{l_1}{l_2}$; $\frac{T_1}{T_2} = \frac{n_1}{n_2}$	TRANSFORMER $\frac{e_1}{e_2} = \frac{N_1}{N_2}$	TRANSFORMER $\frac{i_1}{i_2} = \frac{N_2}{N_1}$

Example (1): WRITE THE DIFFERENTIAL EQUATIONS GOVERNING THE MECHANICAL SYSTEM SHOWN BELOW. DRAW THE FORCE - VOLTAGE & FORCE-CURRENT ELECTRICAL ANALOGOUS CIRCUITS AND VERIFY BY WRITING MESH & NODE EQUATIONS.



* There are two nodes M_1 and M_2. The free body diagrams are given as follows



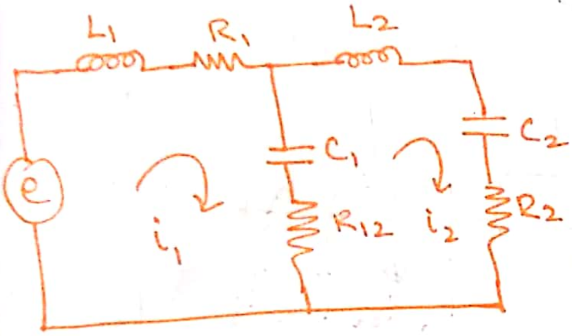
* The force balance equations are

$$f(t) = f_{m1} + f_{k1} + f_{b12} + f_{b1} \quad \text{—— (1)}$$

$$0 = f_{m2} + f_{k2} + f_{b2} + f_{k1} + f_{b12} \quad \text{—— (2)}$$

Force Voltage analogous circuit:

M_1	L_1	K_2	$1/C_2$
M_2	L_2	B_2	R_2
K_1	$1/C_1$ — Common	V_1	i_1
B_{12}	R_{12} — Common	V_2	i_2
B_1	R_1	$f(t)$	e

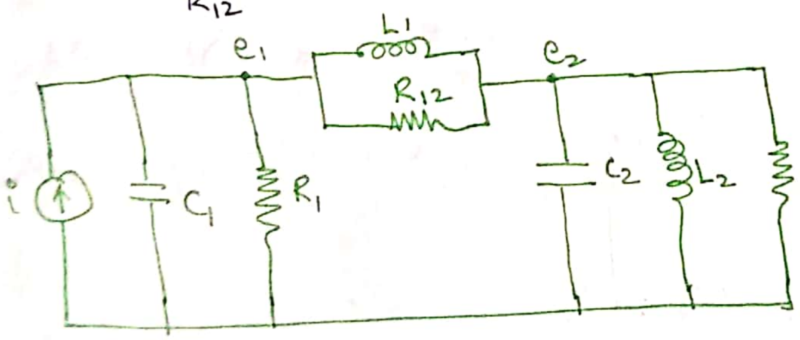


$$e = L_1 \frac{d}{dt} i_1 + \frac{1}{C_1}\int (i_1 - i_2)\,dt + R_{12}(i_1 - i_2) + R_1 i_1 \quad \text{— (7)}$$

$$0 = L_2 \frac{d}{dt} i_2 + \frac{1}{C_1}\int (i_2 - i_1)\,dt + R_{12}(i_2 - i_1) + \frac{1}{C_2}\int i_2\,dt + R_2 i_2 \quad \text{— (8)}$$

Force current analogous circuit:

M_1	C_1	B_1	$\frac{1}{R_1}$
M_2	C_2	B_2	$1/R_2$
K_1	$\frac{1}{L_1}$ — Common	V_1	e_1
K_2	$\frac{1}{L_2}$	V_2	e_2
B_{12}	$\frac{1}{R_{12}}$ Common.	$f(t)$	i



$$i = C_1 \frac{de_1}{dt} + \frac{1}{L_1}\int (e_1 - e_2)\,dt + \frac{1}{R_{12}}(e_1 - e_2) + \frac{1}{R_1} e_1 \quad \text{— (9)}$$

$$0 = C_2 \frac{de_2}{dt} + \frac{1}{L_2}\int e_2\,dt + \frac{1}{R_2} e_2 + \frac{1}{R_{12}}(e_2 - e_1) + \frac{1}{L_1}\int (e_2 - e_1)\,dt \quad \text{— (10)}$$

Force balance equations of mechanical system:

$$f(t) = M_1 \frac{d^2 x_1}{dt^2} + K_1(x_1 - x_2) + B_{12}\frac{d}{dt}(x_1 - x_2) + B_1 \frac{d}{dt} x_1 \quad \text{— (3)}$$

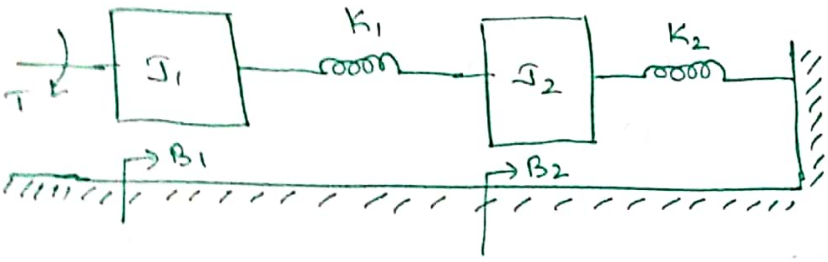
$$0 = M_2 \frac{d^2 x_2}{dt^2} + K_2(x_2) + B_2 \frac{d}{dt} x_2 + B_{12}\frac{d}{dt}(x_2 - x_1) + K_1(x_2 - x_1) \quad \text{— (4)}$$

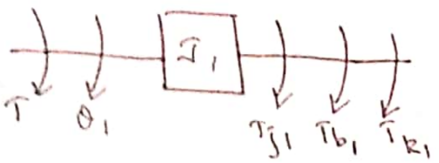
In terms of velocity

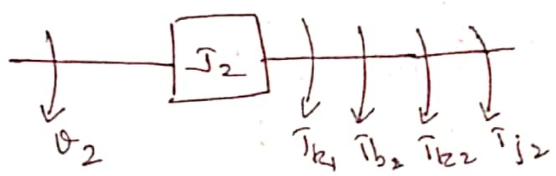
$$f(t) = M_1 \frac{d}{dt} V_1 + K_1 \int (V_1 - V_2)\,dt + B_{12}(V_1 - V_2) + B_1 V_1 \quad \text{— (5)}$$

$$0 = M_2 \frac{d}{dt} V_2 + K_2 \int V_2\,dt + B_2 V_2 + B_{12}(V_2 - V_1) + K_1 \int (V_2 - V_1)\,dt \quad \text{— (6)}$$

EXAMPLE (2): Write the differential equations governing the mechanical rotational system. Draw the torque-voltage and torque-current electrical analogous circuits and verify by writing mesh and node equations.

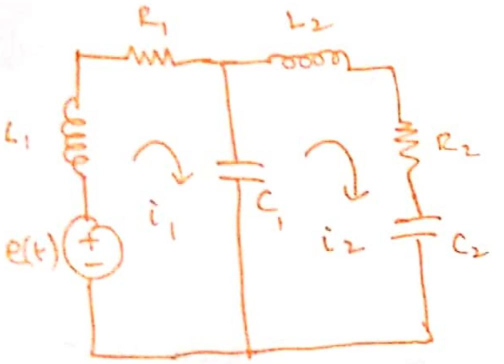


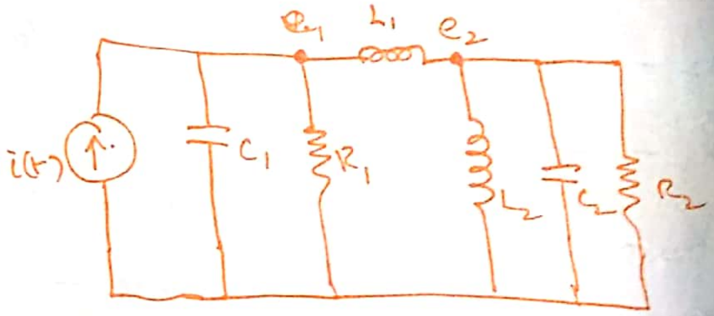




$$T = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + K_1(\theta_1 - \theta_2)$$

$$0 = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + K_2(\theta_2) + K_1(\theta_2 - \theta_1)$$





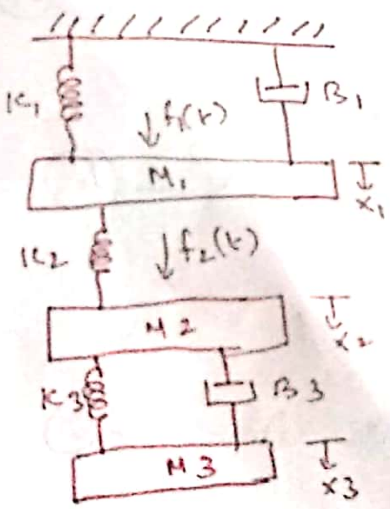
$$L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int(i_1 - i_2) = e(t)$$

$$L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt + \frac{1}{C_1}\int(i_2 - i_1)\,dt = 0$$

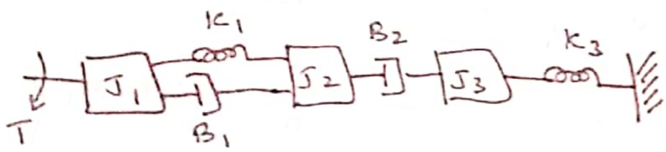
$$C_1 \frac{de_1}{dt} + \frac{1}{R_1} e_1 + \frac{1}{L_1}\int(e_1 - e_2)\,dt = i(t)$$

$$C_2 \frac{de_2}{dt} + \frac{1}{R_2} e_2 + \frac{1}{L_2}\int e_2\,dt + \frac{1}{L_1}\int(e_2 - e_1)\,dt = 0$$

EXAMPLE (3):



EXAMPLE (4):



CLOSED LOOP SYSTEMS, BLOCK DIAGRAM ALGEBRA, SIGNAL FLOW GRAPHS:

BLOCK DIAGRAMS

A control system may consist of a number of components.

A block diagram of a system is a pictorial representation of the functions performed by each component and of the flow of signals.

The elements of block diagram are block, branch point and summing pt.

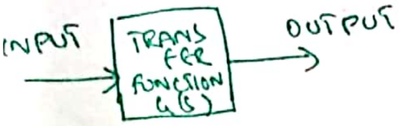


FIG: BLOCK.

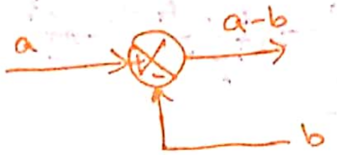


FIG: SUMMING POINT

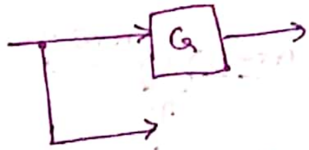
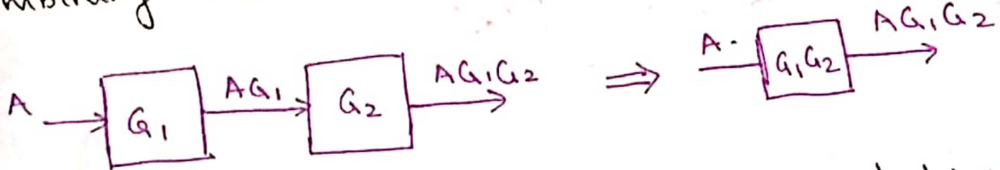


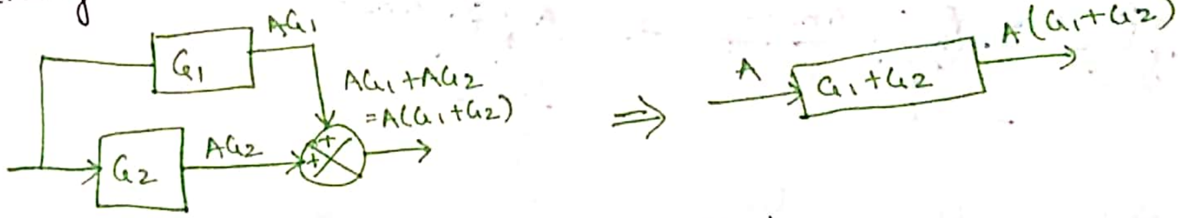
FIG: BRANCH POINT.

BLOCK DIAGRAM REDUCTION

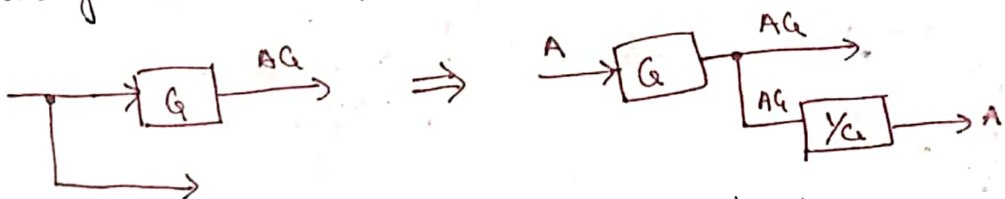
1) Combining the blocks in cascade.



2) Combining Parallel blocks (or combining feed forward paths)

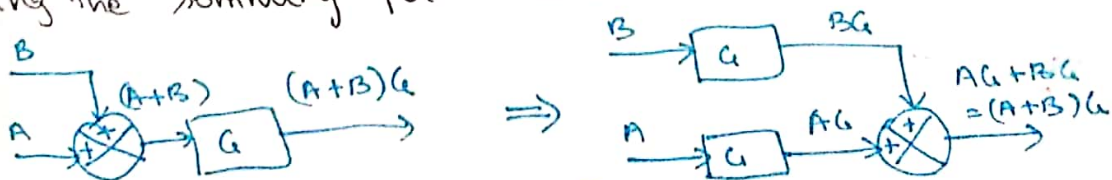


3) Moving the branch point ahead of the block.

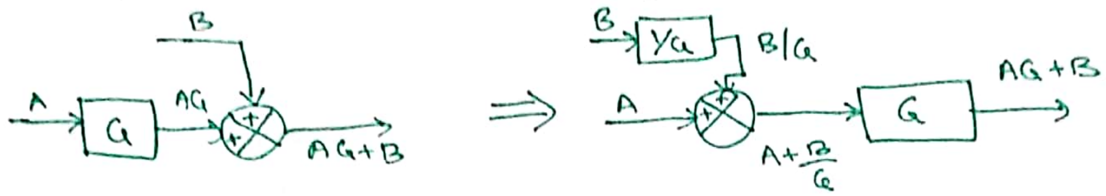


4) Moving the branch point before the block.

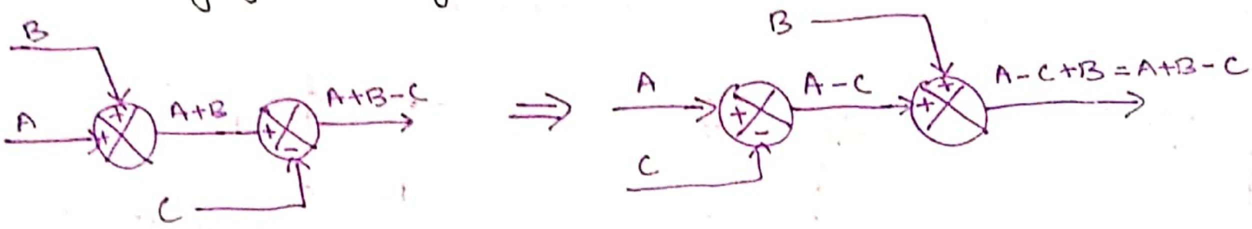
5) Moving the summing point ahead of the block.



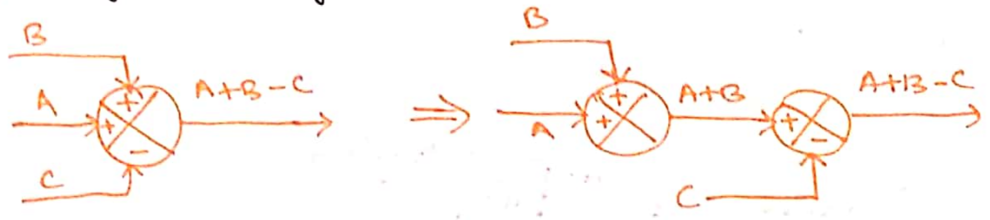
(6) Moving the summing point before the block.



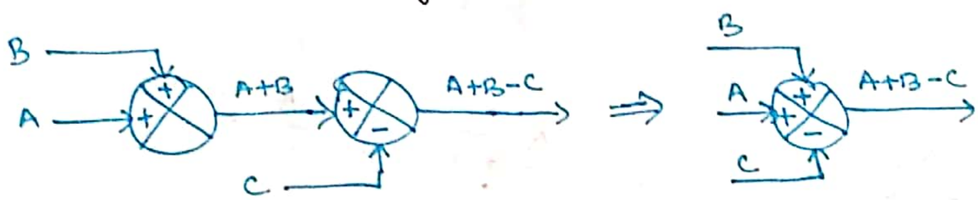
(7) Interchanging summing point.



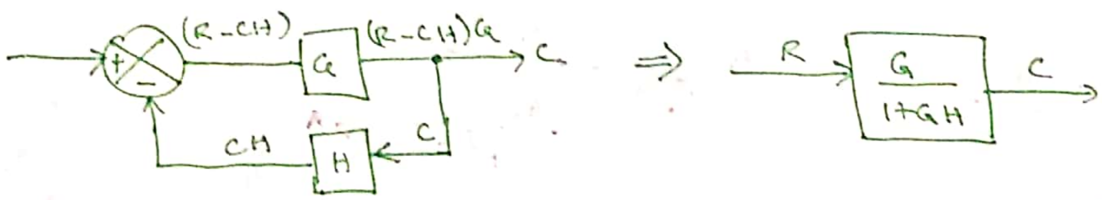
(8) Splitting summing points



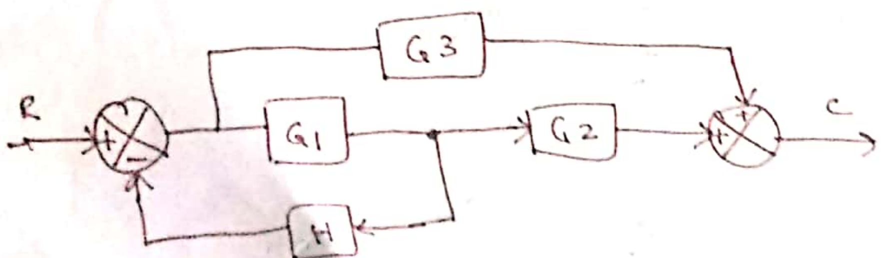
(9) Combining summing points



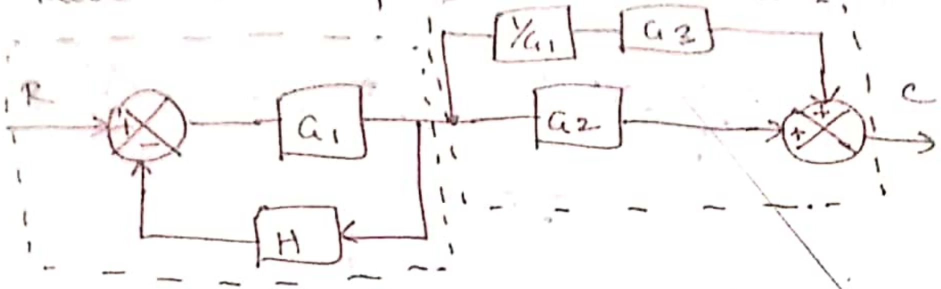
(10) Elimination of feedback loop



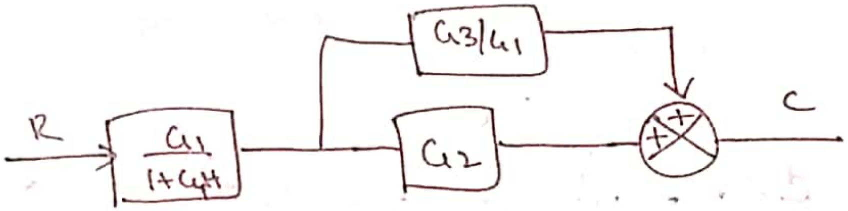
EXAMPLE (1): Reduce the block diagram shown below



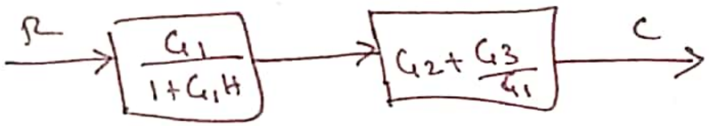
Step 1 : Move branch point G_3 ahead of block G_1 :



Step 2 : Eliminate feedback path and cascade block are combined



Step 3 : Combining parallel blocks



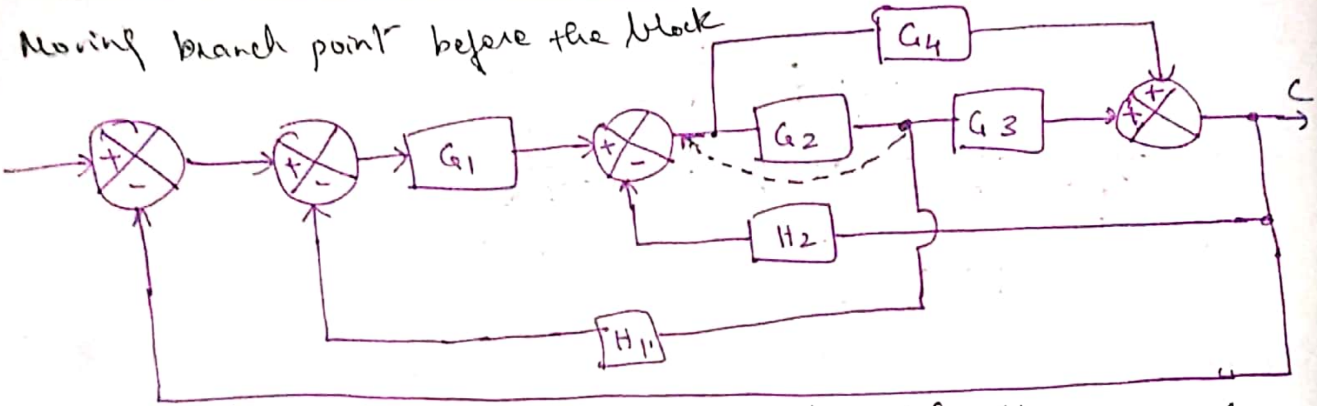
The transfer function is given as

$$\frac{C}{R} = \left(\frac{G_1}{1+G_1H}\right)\left(G_2 + \frac{G_3}{G_1}\right) = \left(\frac{\cancel{G_1}}{1+G_1H}\right)\left(\frac{G_2G_1+G_3}{\cancel{G_1}}\right)$$

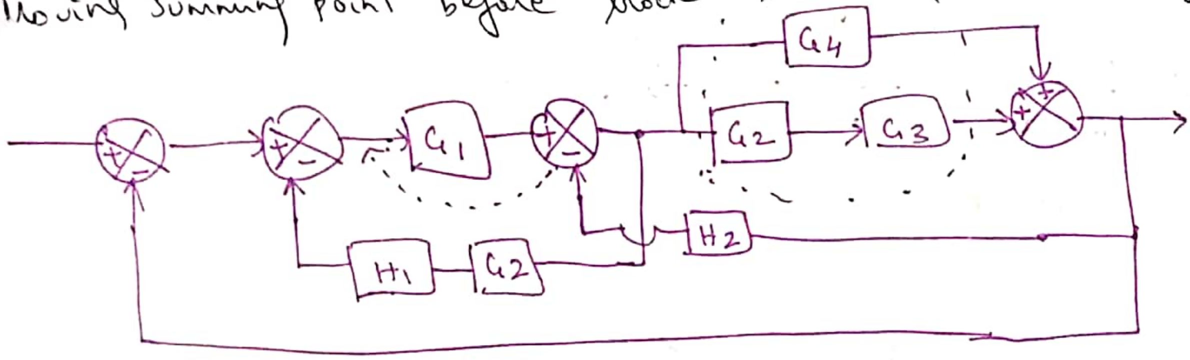
$$\boxed{\frac{C}{R} = \frac{G_2G_1+G_3}{1+G_1H}}$$

Scanned with CamScanner

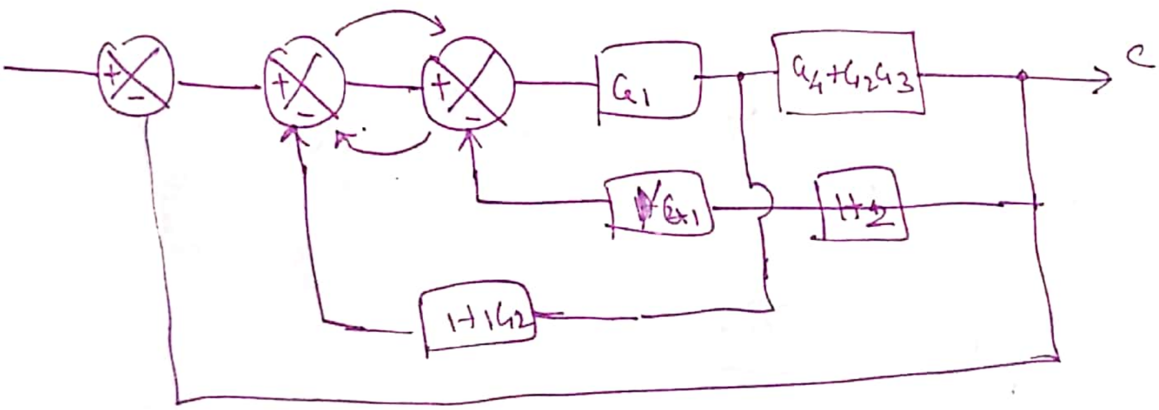
Moving branch point before the block



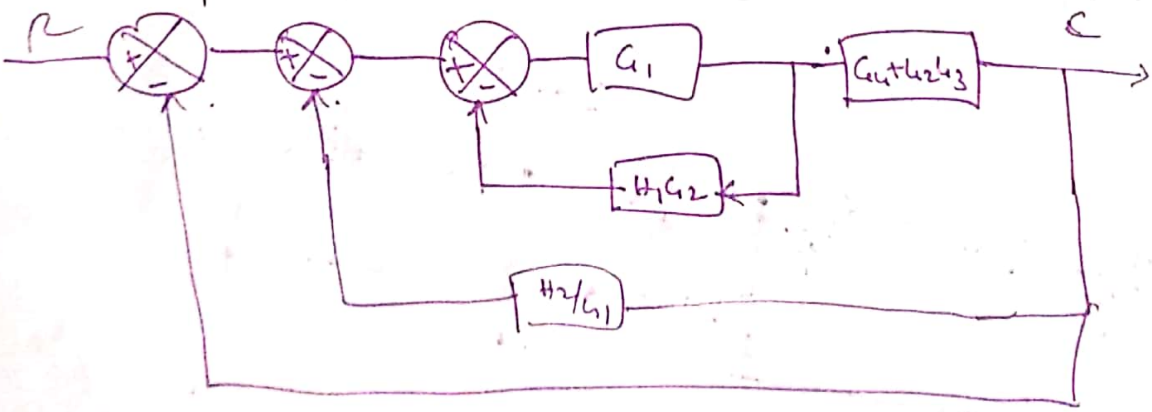
Moving Summing point before block & reducing ~~feedback~~ loop (G4 G2 G3)

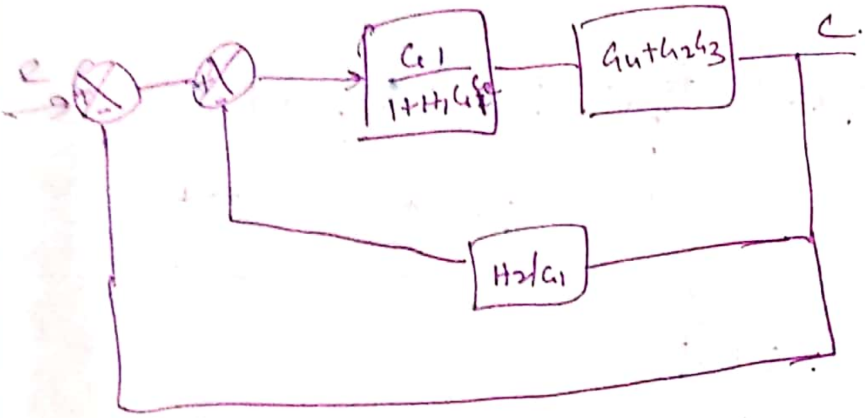


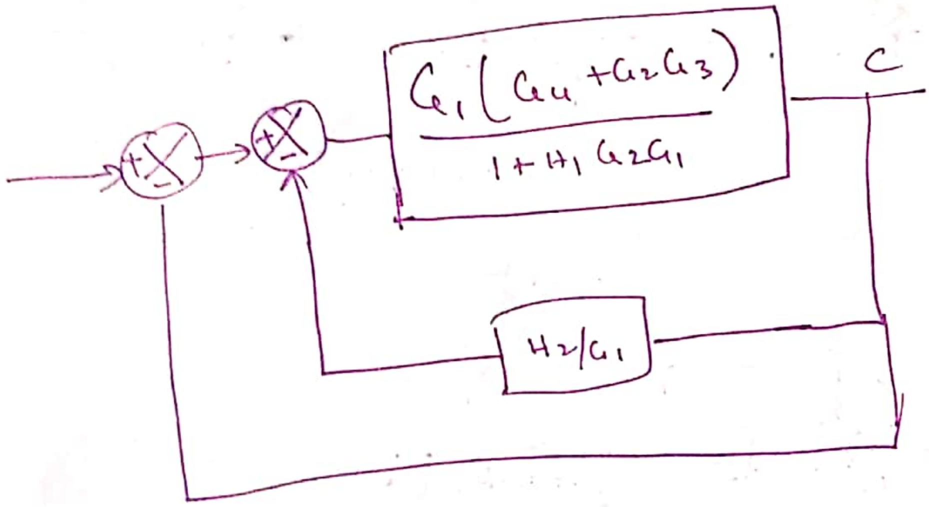
Interchanging summing points

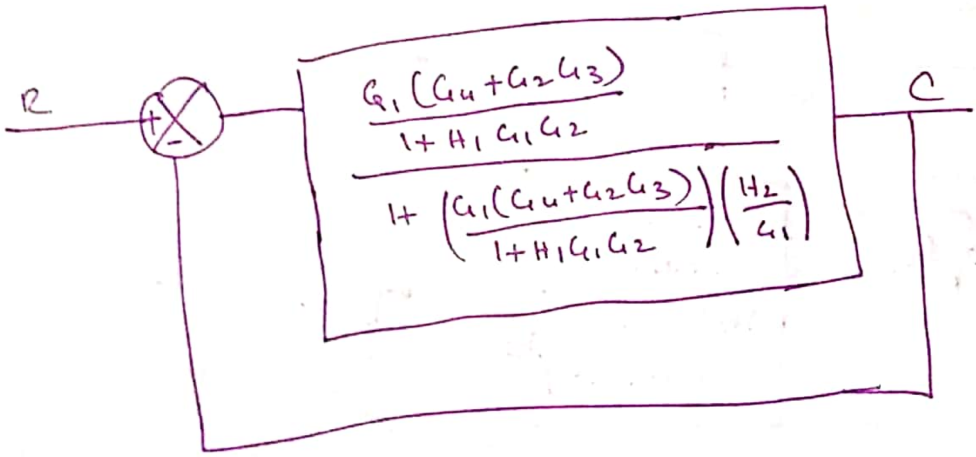


Reducing 3 feedback loops.



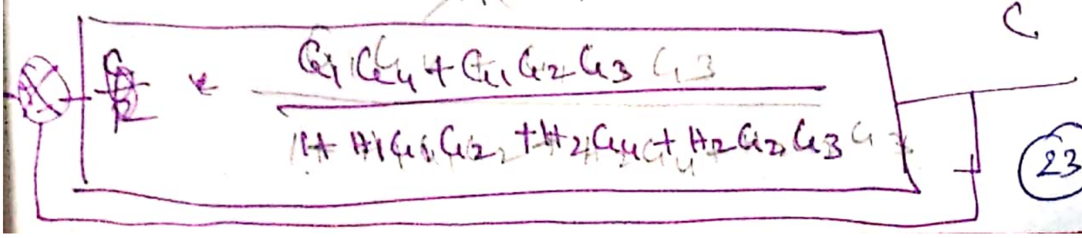




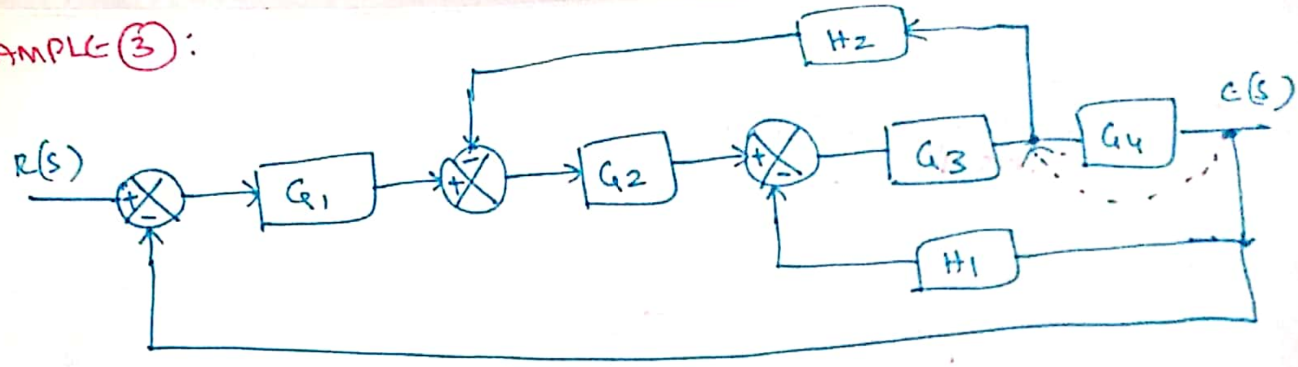


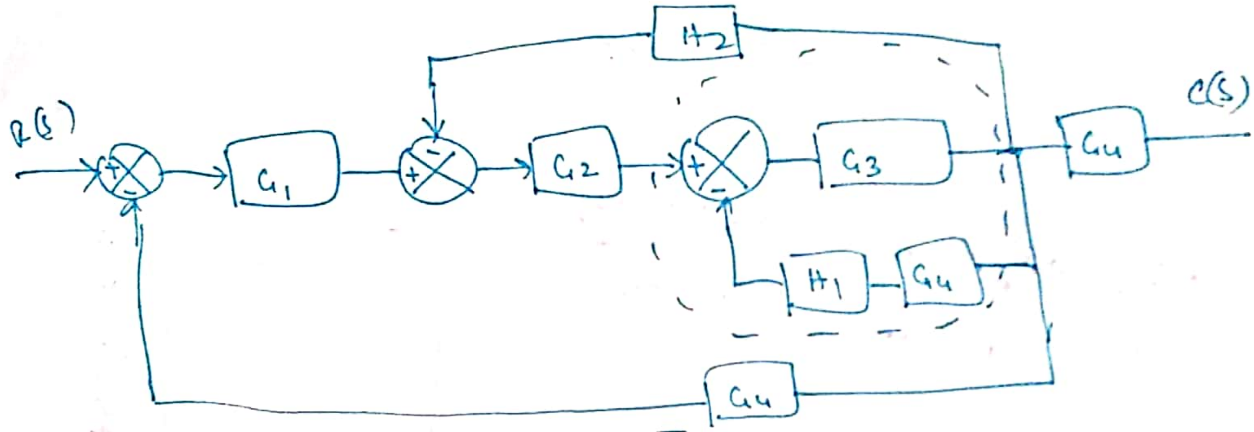
$$\frac{C}{R} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2H_1 + G_2G_3H_2 + H_2G_4 + G_1G_2G_3 + G_1G_4}$$

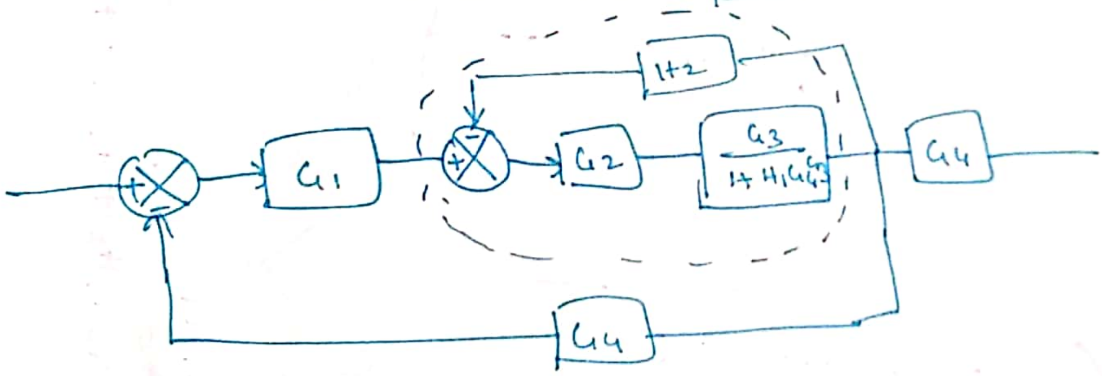
$$\frac{\dfrac{G_1(G_4+G_2G_3)}{1+H_1G_1G_2}}{\dfrac{G_1(1+H_1G_1G_2)+G_1(G_4+G_2G_3)H_2}{G_1(1+H_1G_1G_2)}}$$

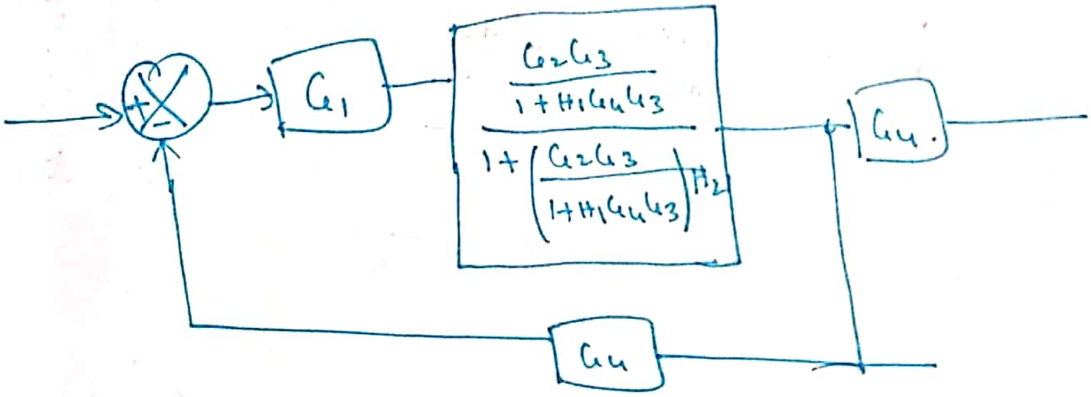


EXAMPLE-3:









$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4}}$$

SIGNAL FLOW GRAPH:

* The signal flow graph is used to represent the control system graphically and was developed by S. J. Mason.
* The signal flow graph depicts the flow of signals from one point of a system to another and gives relationships among the signals.
* Signal flow graph consists of a network with nodes connected by directed branches.
* Each branch acts as a signal multiplier. Each branch has a gain or transmittance.
* Mason's gain formula can be directly used to find the transfer function of the system.

Mason's gain formula states that the overall gain of the system (TF) as

$$\boxed{\text{OVERALL GAIN, } T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K}$$

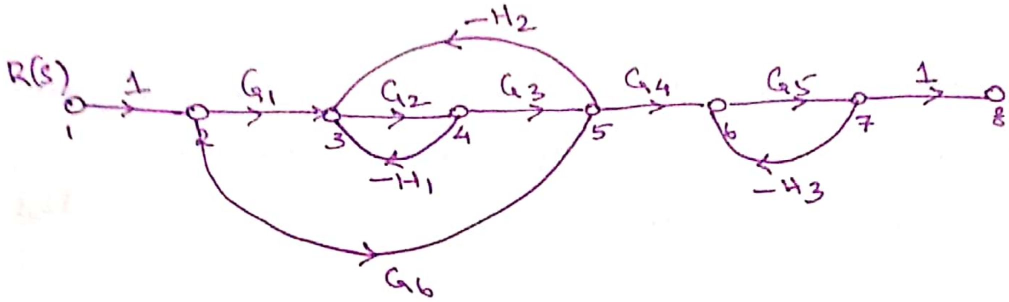
$T = T(s)$ – Transfer function of the system

P_K – Forward path gain of K^{th} forward path

$\Delta = 1 -$ (Sum of individual loop gains) + (Sum of gain products of all possible combinations of 2 nontouching loops) – (Sum of gain products of all possible combination of 3 nontouching loops) + – ....

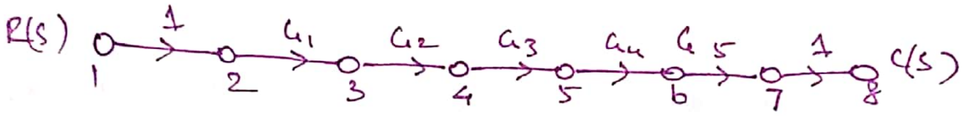
$\Delta_K = \Delta$ for that part of the graph which is not touching K^{th} forward path.

EXAMPLE (1): Find the TF of the system whose SFG is given.

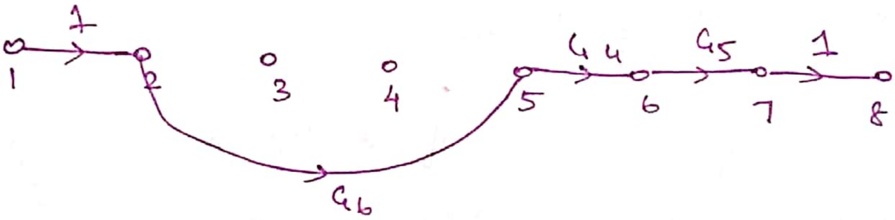


Solution:

I. Forward Path Gains: There are two forward paths $K=2$



FORWARD PATH 1

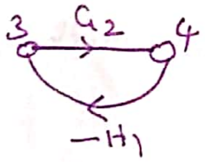


FORWARD PATH 2

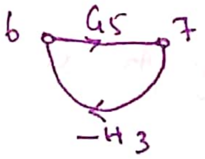
Gain of FP 1 $- G_1 G_2 G_3 G_4 G_5 = P_1$

Gain of FP 2 $- G_6 G_4 G_5 = P_2$

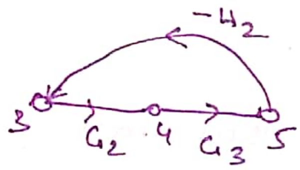
II. Individual Loop Gain:



Loop 1



Loop 2



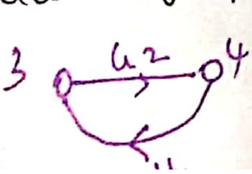
Loop 3.

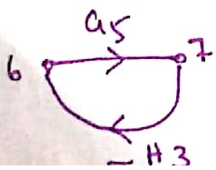
Loop gain of loop 1 $= P_{11} = -G_2 H_1$

Loop gain of loop 2 $= P_{21} = -G_5 H_3$

Loop gain of loop 3 $= P_{31} = -G_2 G_3 H_2$

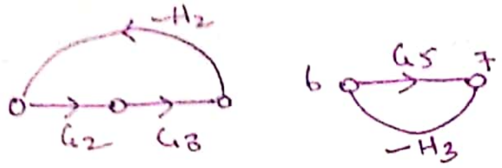
III. Gain of products of two non touching loops





First pair of two nontouching loops

$P_{12} = G_2 G_5 H_1 H_3$



Second pair of nontouching loops

$P_{22} = G_2 G_3 G_5 H_2 H_3$

iv Calculation of Δ and Δ_K.

$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (P_{12} + P_{22})$$

$$= 1 - (-G_2H_1 - G_2G_3H_2 - G_5H_3) + (G_2G_5H_1H_3 + G_2G_3G_5H_2H_3)$$

$$= 1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3$$

$\Delta_1 = 1$ [Since there is no nontouching loop with 1st forward path]

$$\Delta_2 = 1 - P_{11}$$

$$= 1 + G_2H_1$$

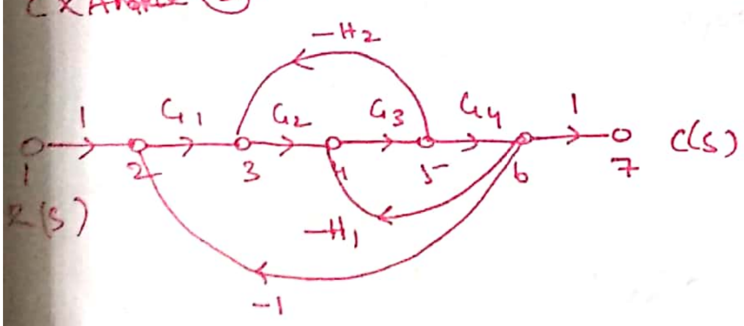
v Transfer function T

$$T = \frac{1}{\Delta} \sum_K P_K \Delta_K$$

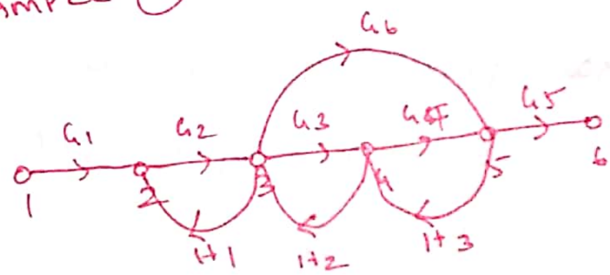
$$= \frac{1}{\Delta} [P_1 \Delta_1 + P_2 \Delta_2]. \quad [\because K = 2]$$

$$\boxed{T = \frac{[G_1 G_2 G_3 G_4 G_5 \times 1] + [G_6 G_4 G_5][1 + G_2 H_1]}{[1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3]}}$$

EXAMPLE (2):



EXAMPLE (3):



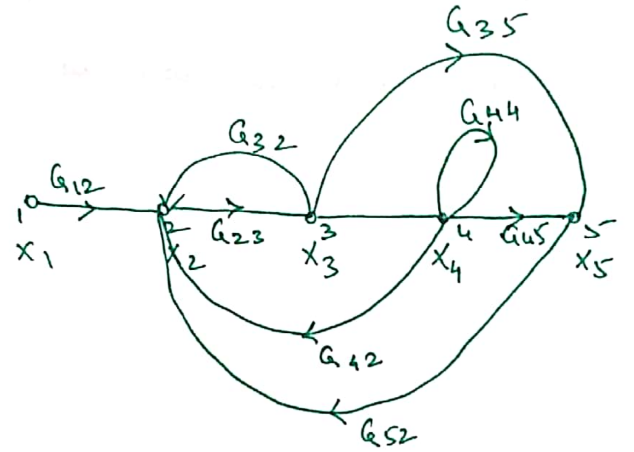
EXAMPLE (4): The system is described by the following set of algebraic equations [Gopal pg. No. 86)

$$X_2 = G_{12}X_1 + G_{32}X_3 + G_{42}X_4 + G_{52}X_5$$

$$X_3 = G_{23}X_2$$

$$X_4 = G_{34}X_3 + G_{44}X_4$$

$$X_5 = G_{35}X_3 + G_{45}X_4.$$



Forward path gain : P_1 , P_2

$P_1 \rightarrow G_{12}G_{23}G_{34}G_{45}$; $P_2 \rightarrow G_{12}G_{23}G_{35}$

Individual loop gain :

$P_{11} - G_{23}G_{34}G_{42}$

$P_{21} - G_{23}G_{32}$

$P_{31} - G_{44}$

$P_{41} - G_{23}G_{34}G_{45}G_{52}$

$P_{51} - G_{23}G_{35}G_{52}$

Gain products of two nontouching loops :

$$P_{12} = P_{21} \cdot P_{31} = G_{23}G_{32}G_{44}$$

$$P_{22} = P_{51} \cdot P_{31} = G_{23}G_{35}G_{44}G_{52}$$

Calculation of Δ_k

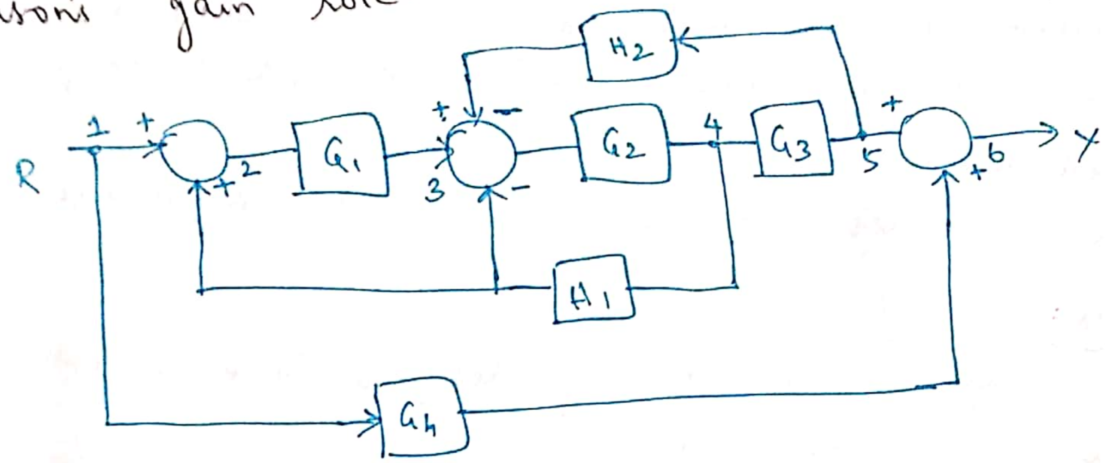
$$\Delta_1 = 1 \;;\quad \Delta_2 = 1 - P_{31}$$

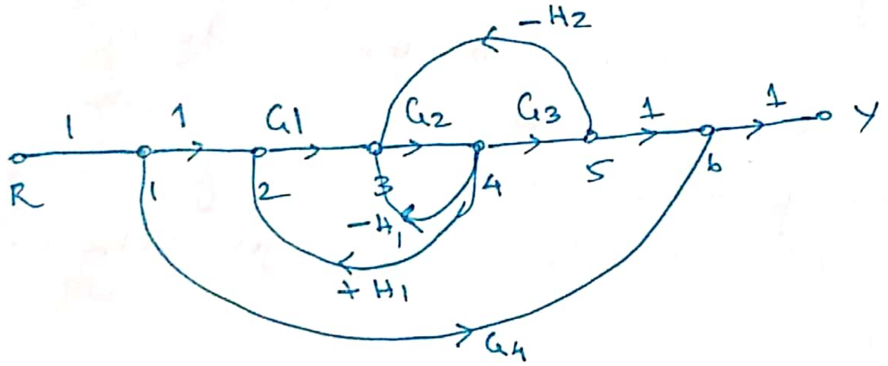
Calculation of Δ

$$\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41} + P_{51}) + (P_{12} + P_{22})$$

$$M = \frac{1}{\Delta}\sum_{k=1}^{N=2} \Delta_k P_k = \frac{G_{12}G_{23}G_{34}G_{45} + G_{12}G_{23}G_{35}(1 - G_{44})}{1 - G_{23}G_{34}G_{42} - G_{23}G_{32} - G_{44} - G_{23}G_{34}G_{45}G_{52} - G_{23}G_{35}G_{52} + G_{23}G_{32}G_{44} + G_{23}G_{35}G_{44}G_{52}}$$

EXAMPLE (5) : Convert the block diagram shown to a signal flow graph and obtain the input output transfer function using Mason's gain rule.





Forward path gain : $P_1 = G_1 G_2 G_3$

$P_2 = G_4$.

Individual loop gain : $P_{11} = -G_2 H_1$

$P_{21} = G_1 G_2 H_1$

$P_{31} = -G_2 G_3 H_2$

Calculation of Δ_K

$\Delta_1 = 1$

$\Delta_2 = 1 - (P_{11} + P_{21} + P_{31})$

Calculation of Δ

$\Delta = 1 - (P_{11} + P_{21} + P_{31})$

$\Delta = 1 + G_2 H_1 + G_2 G_3 H_2 - G_1 G_2 H_1$

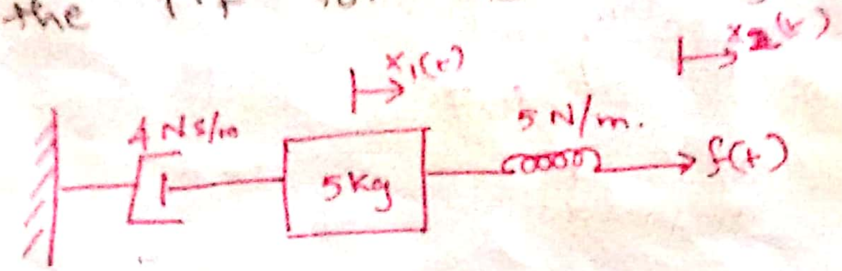
Calculation of Transfer function

$$M = \frac{Y(s)}{R(s)} = \frac{G_1 G_2 G_3 + G_4 (1 + G_2 H_1 - G_1 G_2 H_1 + G_2 G_3 H_2)}{1 + G_2 H_1 + G_2 G_3 H_2 - G_1 G_2 H_1}$$

$$\boxed{M = G_4 + \frac{G_1 G_2 G_3}{1 + G_2 H_1 + G_2 G_3 H_2 - G_1 G_2 H_1}}$$

UNIT-I - ADDITIONAL PROBLEMS

① Find the T.F for the following MTS system:



$$5\frac{d^2x_1}{dt^2} + 4\frac{dx_1}{dt} + 5(x_1 - x_2) = 0 \quad \text{— ①}$$

$$5x_2(t) - 5x_1(t) = f(t) \quad \text{— ②}$$

Laplace transform of above equations give

$$5s^2 X_1(s) + 4sX_1(s) + 5X_1(s) - 5X_2(s) = 0 \quad \text{— ③}$$

$$(5s^2 + 4s + 5)X_1(s) - 5X_2(s) = 0$$

$$(5s^2 + 4s + 5)X_1(s) = 5X_2(s)$$

$$X_1(s) = \frac{5}{(5s^2+4s+5)} X_2(s) \quad \text{— ④}$$

$$5X_2(s) - 5X_1(s) = F(s) \quad \text{— ⑤}$$

Substitute $X_1(s)$ from ④ into ⑤

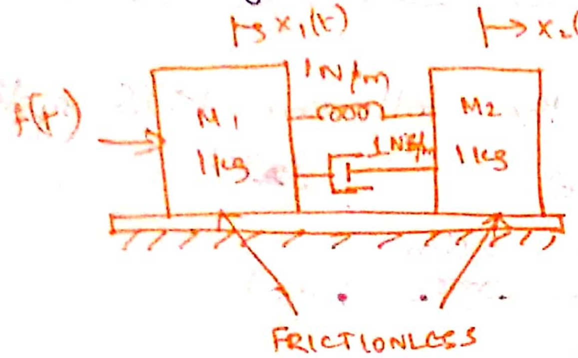
$$5X_2(s) - 5 \times \frac{5X_2(s)}{(5s^2+4s+5)} = F(s)$$

$$\left[5 - \frac{25}{5s^2+4s+5}\right] X_2(s) = F(s)$$

$$\frac{X_2(s)}{F(s)} = \frac{1}{\left[\frac{5(5s^2+4s+5) - 25}{5s^2+4s+5}\right]}$$

$$\frac{X_2(s)}{F(s)} = \frac{5s^2+4s+5}{25s^2+20s+\cancel{25}-\cancel{25}} = \frac{5s^2+4s+5}{s(25s+20)} = \frac{5s^2+4s+5}{5s(5s+4)}$$

(2) Find the T.F for the M.T.S shown below



$$1\frac{d^2 x_1(t)}{dt^2} + 1\left(x_1(t) - x_2(t)\right) + 1\left(\frac{d}{dt}x_1(t) - \frac{d}{dt}x_2(t)\right) = f(t) \quad \text{— (1)}$$

$$1\frac{d^2 x_2(t)}{dt^2} + 1\left(x_2(t) - x_1(t)\right) + 1\left(\frac{d}{dt}x_2(t) - \frac{d}{dt}x_1(t)\right) = 0 \quad \text{— (2)}$$

Laplace Transform of above two equations give

$$s^2 X_1(s) + X_1(s) - X_2(s) + s X_1(s) - s(X_2(s)) = F(s) \quad \text{— (3)}$$

$$s^2 X_2(s) + X_2(s) - X_1(s) + s(X_2(s)) - s(X_1(s)) = 0 \quad \text{— (4)}$$

From (4) rearranging we get

$$X_2(s)\left[s^2 + 1 + s\right] - X_1(s)\left[1 + s\right] = 0. \quad \text{— (5)}$$

$$X_2(s)\left[s^2 + s + 1\right] = X_1(s)\left[s+1\right]$$

$$\frac{X_2(s)\left[s^2+s+1\right]}{(s+1)} = X_1(s) \quad \text{— (6)}$$

Subs (6) in (3) we get

$$(s^2 + s + 1)\, X_1(s) - X_2(s)(s+1) = F(s)$$

$$(s^2 + s + 1) \times \frac{(s^2+s+1)}{(s+1)} X_2(s) - X_2(s)(s+1) = F(s)$$

$$X_2(s)\left[\frac{(s^2+s+1)^2}{(s+1)} - (s+1)\right] = F(s)$$

$s^2 = a$

$(s+1) = b$

$(a+b)^2 = (s^2)^2 + (s+1)^2 + 2s^2(s+1)$

$$\Rightarrow \frac{(s^2)^2 + \cancel{(s+1)^2} + 2s^2(s+1) - \cancel{(s+1)^2}}{(s+1)}$$

$$= \frac{(s^2)^2 + 2s^2(s+1)}{(s+1)}$$

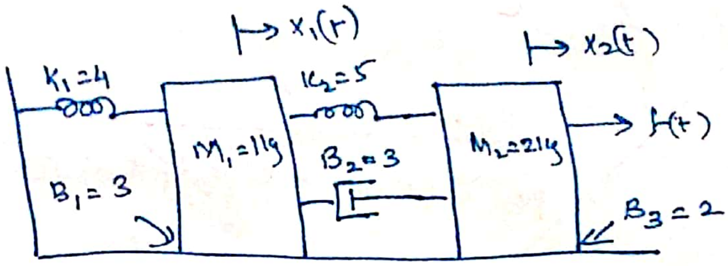
$$\frac{(s^2)^2 + 2s^2(s+1)}{(s+1)} \Rightarrow (s^2)\left[\frac{s^2+2(s+1)}{(s+1)}\right]$$

$$\Rightarrow \frac{s^2(s^2+2s+2)}{(s+1)}$$

$$\frac{F(s)}{X_2(s)} \Rightarrow \frac{s^2(s^2+2s+2)}{(s+1)}$$

$$\boxed{\frac{X_2(s)}{F(s)} = \frac{s+1}{s^2(s^2+2s+2)}}$$

③ for the MTS shown below find T.F. & $\frac{X_1(s)}{F(s)}$.



$$1\frac{d^2 x_1(t)}{dt^2} + 4x_1(t) + 3\frac{d}{dt}x_1(t) + 5(x_1(t) - x_2(t)) + 3\left(\frac{d}{dt}x_1(t) - \frac{d}{dt}x_2(t)\right) = 0 \quad \text{— ①}$$

$$2\frac{d^2 x_2(t)}{dt^2} + 2\frac{d}{dt}x_2(t) + 5(x_2(t) - x_1(t)) + 3\left(\frac{d}{dt}x_2(t) - \frac{d}{dt}x_1(t)\right) = F(t) \quad \text{— ②}$$

$$s^2 X_1(s) + 4X_1(s) + 3s(X_1(s)) - 5X_2(s) + 5X_1(s) + 3sX_1(s) - 3sX_2(s) = 0. \quad \text{— ③}$$

$$2s^2 X_2(s) + 2sX_2(s) + 5X_2(s) - 5X_1(s) + 3sX_2(s) - 3sX_1(s) = F(s) \quad \text{— ④}$$

From ③ & rearranging

$$X_1(s)[s^2 + 4 + 3s + 5 + 3s] - X_2(s)[5 + 3s] = 0$$

$$X_1(s) = \frac{X_2(s)[5+3s]}{[s^2+6s+9]} \quad \text{— ⑤}$$

Subs $X_1(s)$ in ④

$$(2s^2 + 2s + 5 + 3s)X_2(s) - 5X_1(s) - 3sX_1(s) = F(s)$$

$$X_2(s)[2s^2 + 5s + 5] - X_1(s)[5+3s] = F(s)$$

$$X_2(s)\left[2s^2 + 5s + 5\right] - \frac{X_2(s)[5+3s]^2}{s^2+6s+9} = F(s)$$

$$X_2(s)\left[2s^2 + 5s + 5\right] - (5+3s)^2 / s^2+6s+9 = F(s)$$

$$\frac{(2s^2+5s+5)(s^2+6s+9) - (5+3s)^2}{(s^2+6s+9)} = \frac{F(s)}{X_2(s)}$$

$$\frac{X_2(s)}{F(s)} = \frac{(s^2+6s+9)(2s^2+5s+5) - (25+9s^2+30s)}{s^2+6s+9}$$

$$= \frac{2s^4+12s^3+18s^2+5s^3+30s^2+45s+5s^2+30s+45 - 25 - 9s^2 - 30s}{s^2+6s+9}$$

$$\boxed{\frac{X_2(s)}{F(s)} = \frac{2s^4+17s^3+44s^2+45s+20}{s^2+6s+9}}$$