Maths Assignment - 2

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ECE - A
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Part - A

1) How many distinct four digit integ-ers integers can one make from the digits 1, 3, 3, 7, 7 & 8?

A) For four digit formation, using these digits, we have three cases,

Case (i):- All the digits in four digit integer are distinct,

that is $= 4! = 24$

Case (ii):- Exactly two digits are same in the four digit number,

that is $= \cancel{2} \times 3 \times \frac{4!}{\cancel{2}!} = 3 \times 4! = 24 \times 3 = \underline{72}$

Case (iii):- Numbers that only consist of 3 & 7, they are,

$\frac{4!}{2! \times 2!} \Rightarrow \frac{{}^{2}\cancel{4} \times 3 \times \cancel{2} \times \cancel{1}}{\cancel{2} \times \cancel{1} \times \cancel{2} \times 1} \Rightarrow 3 \times 2 = \underline{6}.$

$\therefore$ final answer $= 24 + 72 + 6 = 30 + 72 = \underline{102}$ integers ~~integers~~

2) How many positive integers not exceeding 1000 that are divisible by 7 (or) 11?

A) Let us take two sets A & B.

A = set of numbers that are divisible by 7,

B = set of integers that are divisible by 11,

$\therefore n(A)$ = no. of integers that are divisible by 7.

$\Rightarrow \frac{1000}{7} = 142.85 \simeq \underline{142}$

$\therefore n(B)$ = no. of integers that are divisible by 11,

$\Rightarrow \frac{1000}{11} = 90.9 \simeq \underline{90}$

$n(A) = 142$ & $n(B) = 90$.

we have to find $n(A \cap B)$ since, thee are some comm -on in the set A & set B. So, we have to exclude the -m, by osing $n(A \cap B)$.

$n(A \cap B)$ = numbers that are divisible by both 7 & 11

$$\Rightarrow \frac{1000}{7 \times 11} \Rightarrow \frac{1000}{77} = 12.98 \cong 12$$

So, we have formula,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B) = 142 + 90 - 12$$

$$n(A \cup B) = 130 + 90$$

$$n(A \cup B) = 220$$

$\therefore$ There are ~~2220~~ 220 integers below 1000 that are divisble by 7 (or) 11.

3) If $^nP_4 = 20\,^nP_3$; find "n"?

A) we have, $^nP_r = \frac{n!}{(n-r)!}$

So, $$^nP_4 = \frac{n!}{(n-4)!} \Rightarrow \frac{n \times (n-1) \times (n-2) \times (n-3) \times \cancel{(n-4)!}}{\cancel{(n-4)!}}$$

$$^nP_4 = n \times (n-1) \times (n-2) \times (n-3).$$

Similarly, $$^nP_3 = n \times (n-1) \times (n-2).$$

$$\Rightarrow {}^nP_4 = 20\,^nP_3$$

$$\cancel{n} \times \cancel{(n-1)} \times \cancel{(n-2)} \times (n-3) = \cancel{n} \times \cancel{(n-1)} \times \cancel{(n-2)} \times 20.$$

$$n - 3 = 20.$$

$$\underline{n = 23}$$

4) If there are 5 points inside a square of side length 2, prove that two of the points with in a distance of $\sqrt{2}$ each other.

A) we have a 2x2 square, we want to split into 4 squares of 1x1 cm, simply by Joining the centres of opp. sides, This creates a small grid.

Now, we have to insert 5 points in this 4 squares

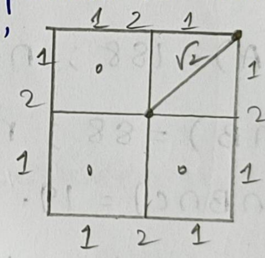
Now, we have to apply pigeon hole principle,

so, $\left\lfloor \frac{5-1}{4} \right\rfloor + 1 = \left\lfloor \frac{4}{4} \right\rfloor + 1 = 1+1 = \underline{\underline{2}}$.

So, one square must end up containg atleast two points.

Now, we know that given 5 points are inside a square of side length 2, so, two points will lie in 1x1 square.

∴ The max. distance b/w the two points is the diagonal of the square side length "1"



⇒ That is equal to $\underline{\sqrt{2}}$

Hence proved.

5) which positive integers less than 30, that are relatively prime?

A) For the relatively prime to 30, we have to find greatest common divisor (gcd) for all prime number which is less than 30. means, 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Now,

$\gcd(1,30) = 1$ ✓ $\quad$ $\gcd(5,30) \neq 1$ ✗

$\gcd(2,30) \neq 1$ ✗ $\quad$ $\gcd(7,30) = 1$ ✓

$\gcd(3,30) \neq 1$ ✗ $\quad$ $\gcd(11,30) = 1$ ✓

$\gcd(13, 30) = 1$ ✓ $\gcd(23,30) = 1$ ✓
$\gcd(17,30) = 1$ ✓ $\gcd(29,30) = 1$ ✓
$\gcd(19, 30) = 1$ ✓

$\therefore$ 1, 7, 11, 13, 17, 19, 23, 29 are the positive intgers that are less than 30, relatively prime to 30.

Part - B

6) 250 students in an engineering college,
188 → Fortan; 100 → C; 35 → Java.
88 → Fortan & C; 23 → C & Java; 29 → fortan & Java.
19 → Fortan, C, Java... How many people didn't take any of there coursers.

A) A → fortan; B → C; C → Java.

$n(A) = 188$; $n(B) = 100$; $n(C) = 35$.

$n(A \cap B) = 88$; $n(B \cap C) = 23$; $n(A \cap C) = 29$.

$n(A \cap B \cap C) = 19$.

$$\therefore n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C).$$

$$n(A \cup B \cup C) = 188 + 100 + 35 - 88 - 23 - 29 + 19 = 202$$

$\therefore$ no. of. Students took atleast one course = 202

$\therefore$ no. of. Students who didn't take any course = 250 - 202 = 48

7) A round table conference is to be held b/w 10 delegates from 10 Countries. In how many ways they Can be seated if,

(i) Two particular delegates are always together

(ii) Two particular delegates are either side of the chairperson.

A) (i) The two particular delegates who wish to sit together be treated as one unit,

So, we have 9 delegates.

In circular permutation, it is equal to

$$(9-1)! = 8!$$

And After this, the two delegates can be permutated b/w themselves, so it will become,

$$= 2 \times 8!$$
$$= 2 \times 40320 = \underline{80640}.$$

(ii) Let the person be arranged in between two particular delegates in $^8C_1$ ways.

Reaming arrangements Can be done in, $(10-3+1-1)!$

$\Rightarrow 7!$ ways.

Two particular delegates can interchange among themselves,

$\therefore$ Total ways are $= {}^8C_1 \times 7! \times 2!$ $[\because \underline{{}^8C_1 \times 7! = 8!}]$.

$= 2 \times 8!$

8) Find the integer m & n such that, $28844m + 15712n = 4$.

A) $28844 = 1 \times 15712 + 13{,}132$

$15712 = 1 \times 13132 + 2580$.

$13132 = 5 \times 2580 + 232$

$2580 = 11 \times 232 + 28$.

$232 = 8 \times 28 + 8$.

$28 = 3 \times 8 + \underline{4}$.

$8 = 4 \times 2 + 0$

$\therefore \gcd(28844, 15712) = 4$.

$\Rightarrow 4 = 28 - (3 \times 8)$.

$4 = 28 - (3 \times (232 - 8 \times 28))$.

$= 28 - (3 \times 232 - 8 \times 3 \times 28)$.

$\Rightarrow 28 + 8 \times 3 \times 28 - 3 \times 232$

$\Rightarrow 28(1 + 24) - 3 \times 232$

$\Rightarrow 28(25) - 3 \times 232$

$\Rightarrow 25 \times 28 - 3 \times 232$

$\Rightarrow 25(2580 - 11 \times 232) - 3 \times (232)$

$\Rightarrow 25 \times 2580 - 278(232)$

$\Rightarrow 25(2580) - 278(13132 - 5 \times 2580)$.

$\Rightarrow 25(2580) - 278(13132) + 1390(2580)$.

$\Rightarrow 1415(2580) - 278(13132)$

$\Rightarrow 1415(15712 - 13132) - 278(13132)$

$\Rightarrow 1415(15712) - 1693(13132)$

$\Rightarrow 1415(15712) - 1693(28844 - 15712)$.

$\Rightarrow 1415(15712) - 1693(28844) + 1693(15712)$

$\Rightarrow 3108(15712) - 1693(28844)$.

$\therefore \underline{\underline{m = -1693}}$ and $\underline{\underline{n = 3108}}$.

9) Using Euclid's algorithm find gcd of 12345 & 54321.

A) So,

$$54321 = 4 \times 12345 + 4941$$

$$12345 = 2 \times 4941 + 2463.$$

$$4941 = 2 \times 2463 + 15.$$

$$2463 = 164 \times 15 + 3 \rightarrow \text{last Non-zero remainder}$$

$$15 = 5 \times 3 + \underline{0}.$$

$\therefore$ GCD of 12345 & 54321 is "3".

Since "3" is the last non-zero remainder.

10) If $2^n - 1$ is a prime number, then S.T "n" is prime

A) Suppose, 2^{n-1} to be prime,

Let it be "p"

$$P = \cancel{2^{n-1}}\ 2^n - 1$$

Assume, $n = xy$ & n is not prime $(x, y > 0)$.

$$P = 2^{xy} - 1$$

$$P = (2^x)^y - 1$$

$$P = \frac{(2^x)^y - 1}{2^x - 1} \times (2^x - 1). \rightarrow (1).$$

So this can be written as,

$$1 + 2^x + (2^x)^2 + (2^x)^3 + \dots + (2^x)^{y-1}$$

$$= \frac{(2^x)^y - 1}{2^x - 1}$$

$\left[\because a + ar + ar^2 + \dots ar^n \Rightarrow \frac{a(r^n - 1)}{r - 1}\right).$

Since, $a = 1$

$1 + r + r^2 + \dots r^{n-1} = \frac{r^n - 1}{r - 1}\Big].$

sub the above in eq^n ①.

$$p = (1 + 2^x + (2^x)^2 + \cdots + (2^x)^{y-1}) * (2^x - 1). \rightarrow ②$$

from eq^n ②, it is clear that "p" can be written as a product of two numbers. This implies that "p" is not a prime number, which is contradiction to our assumption.

$\therefore$ our assumption "n" is not a prime number is wrong... Thus we can say $(2^n - 1)$ is prime & then "n" is also prime.

Hence proved.