18/1/23

Radio Detection And Ranging (RADAR)

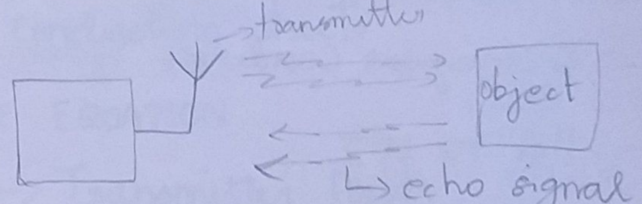
Book : Introduction to RADAR System → Merril I Skolnik

5th unit : Elements of Electronic Navigation → N. S. Nagarajan

Radar Engineering (G.S.N. Raju).

UNIT 1 - Introduction to RADAR Equations

It is an electromagnetic system for the detection and location of objects. An elementary form of RADAR consist of transmitting antenna emitting electromagnetic radiation.



Range (R) $R = \frac{CT_R}{2}$.

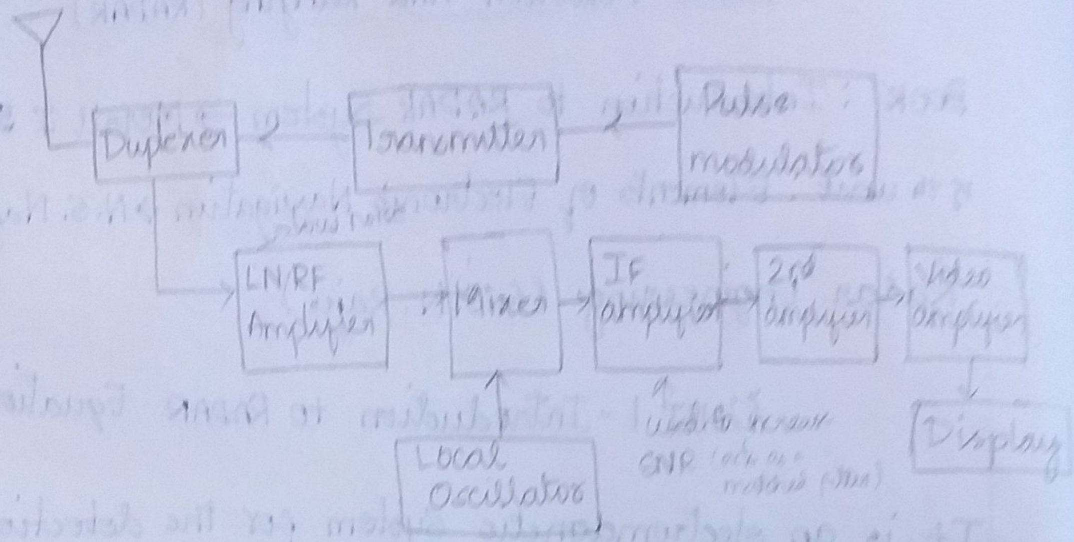
T_R → Round trip time

c → Speed of light $3 \times 10^8 m/s$

In terms of freq $R = \frac{c}{2f}$

Nature of signal transmitted in RADAR is pulsed

If the duration btwn 2 pulses is short the second pulse might reach before the first pulse results in multiple time round echoes.



Duplexer acts as a switch. It will be enabled according to the transmission & reception.

→ Wave guide unit
ATR, TR

→ Polar coordinate

PPI → Planned Position Indicator

A Scope → Rectangular Coordinate System

B Scope → ||el to PPI

Band Designation	Freq. Range.
HF	3 - 30 MHz
VHF	30-300 MHz
UHF	300-1000 MHz
L	1 GHz - 2 GHz
S	2 GHz - 4 GHz
C	4 GHz - 8 GHz
X → used in labs	8 GHz - 12 GHz
Ku → K upper	12 GHz - 18 GHz
K	18 GHz - 27 GHz
Ka → K above	27 GHz - 40 GHz
mm	40 GHz - 300 GHz

Application OF RADAR.

→ Ship-based RADAR used as navigational aid.

→ ship Safety

→ Aircraft navigation

→ Remote Sensing

→ Law enforcement, Military

Limitations of Basic Radar.

→ It cannot recognize the color of the objects

→ It cannot resolve the objects at short distance

→ It cannot see the objects placed behind the conducting sheets.

RANGE EQUATION

P_t → Transmitter Power

R → Range of target

$$\text{Power density at 'R'} = \frac{P_t}{4\pi R^2} \quad \text{—— ①}$$

$$\text{Gain } G = \frac{\text{Max Power density radiated by a directive antenna}}{\text{Power density radiated by a loss less isotrophic antenna with the same Power i/P.}}$$

Power density from directive antenna

$$= \frac{P_t G}{4\pi R^2} \quad \text{—— ②}$$

Reradiated power back at Radar

$$= \frac{P_t G}{4\pi R^2} \times \frac{\sigma}{4\pi R^2} \quad \text{—— (3)}$$

$\sigma \rightarrow$ Radar Cross Sectional (or) target Cross Sectional

Received signal Power.

$$P_r = \frac{P_t G \sigma}{(4\pi)^2 R^4} \times A_e \quad \text{—— (4)}$$

$A_e \rightarrow$ Effective Aperture area of the antenna

When $R = R_{max}$, $P_r = S_{min}$

$$S_{min} = \frac{P_t G \sigma A_e}{(4\pi)^2 R_{max}^4} \quad \text{—— (5)}$$

$$R_{max} = \left[\frac{P_t G \sigma A_e}{(4\pi)^2 S_{min}}\right]^{1/4} \quad \text{—— (6)}$$

→ Basic form of Radar range

We know that,

$$G = \frac{4\pi A_e}{\lambda^2} \quad \text{—— (7)}$$

$$A_e = \frac{G \lambda^2}{4\pi}$$

3 diff forms of Radar Range eqns

$$R_{max} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^2 S_{min}}\right]^{1/4} \quad \text{—— (8)}$$

Sub (7) in (6);

$$R_{max} = \left[\frac{P_t A_e^2 \sigma}{4\pi \lambda^2 S_{min}}\right]^{1/4} \quad \text{—— (9)}$$

Numericals :-

1. If a Radar has a maximum range of 500 km then find the required Pulse Repetition freq (PRF) to obtain unambiguous reception.

$R = 500\ Km$

$$R = \frac{c}{2f_p}$$

$$f_p = \frac{3 \times 10^8}{2 \times 500 \times 10^3}$$

$$= \frac{3 \times 10^5}{1000} = 3 \times 10^{5-3}$$

$$f_p = 3 \times 10^2 = 300\ Hz.$$

2. A Radar operates at 10GHz and peak power of 500 KW. Its minimum receivable power is 0.1 PW its antenna has a capture area of 5 sq.m and the radar cross section of the target is $20\ m^2$. Find max range of radar.

$$freq,\ f = 10\ GHz = 10 \times 10^9\ Hz$$

$$\text{peak power } P_t = 500\ KW = 500 \times 10^3\ W$$

$$\sigma = 20\ m^2$$

$$A_e = 5\ m^2$$

$$S_{min} = 0.1 \times 10^{-12}\ W.$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10 \times 10^9} = 3 \times 10^{8-10}$$

$$= 3 \times 10^{-2} = 0.03 = 30\ mm$$

$$R_{max} = \left[\frac{P_t A_e^2 \sigma}{4 \pi \lambda^2 S_{min}}\right]^{1/4} = \left[\frac{500 \times 10^3 \times 5 \times 5 \times 20 \times 10^{12}}{4 \times 3.14 \times 30 \times 30 \times 10^{-3} \times 0.1 \times 10^{3}}\right]^{1/4}$$

$$= \cancel{1.24 \times 10^5\ m}\ .\ 685.4\ Km$$

Receiver Noise

$$R_{max} = \left[\frac{P_t G \sigma A_e}{(4\pi)\lambda^2 S_{min}}\right]$$

Available thermal noise power $= KTB_n$

$$B_n = \int_0^{\infty} |H(F)|^2 / |H(F_0)|^2$$

$H(F)$ = Frequency response of IF Amplifier

$H(F_0)$ = Maximum frequency response.

Noise Figure, $F_N = \frac{(SNR)_i}{(SNR)_o}$

$$= \frac{S_i/N_i}{S_o/N_o} \quad \text{—— (1)}$$

$$= \frac{N_o}{K T_o B_n G_a}$$

G → available gain

B_N → Bandwidth of Noise

T_o → Absolute temperature

K → Boltzman Constant

N_o → Noise Output

from (1)

$$S_i = \frac{S_o N_i F_N}{N_o}$$

$$= \frac{S_o F_N K T_o B_n}{N_o}$$

$$S_{min} = \frac{K T_o B_n F_N S_o}{N_o}$$

S_{min} → minimum received Signal (input) s.

N_i → input noise (or) Therman noise $= K T_o B_n$

$$R_{max}^4 = \frac{P_t G \sigma A_e N_o}{4\pi \lambda^2 K T_o B_n F_N S_o}$$ in the form of SNR.

$$R_{max}^4 = \frac{P_t G \sigma A_e}{(4\pi)^2 K T_o B_n F_N \left(\frac{S_o}{N_o}\right)_{max}}$$

INTEGRATION OF RADAR PULSES. (used to improve the detection)

$$n_B = \frac{\theta_B f_P}{\theta_S}$$

θ_B → Antenna beam width

θ_S → Scanning rate

f_P → pulse repetition frequency

$n_B \rightarrow$ no of pulses returned from a point target

$$n_B = \frac{\theta_B f_P}{6 \times \omega_m}$$ $\omega_m \rightarrow$ angular frequency

Integrator can be installed before or after 2^{nd} detector. If it is placed before or 2^{nd} detector it is called predetection Integrator. If it is placed after 2^{nd} detector it is called post detection integrator

→ Pre detection will improve SNR

→ Post detection reduces SNR.

Post detection is easier in terms of System Complexity

$$E_i(n) = \frac{(S/N)_1}{n(S/N)_n}$$

where $E_i(n) \rightarrow$ Efficiency of n pulses

$n(E_i)_n \rightarrow$ Integration improvement factor.

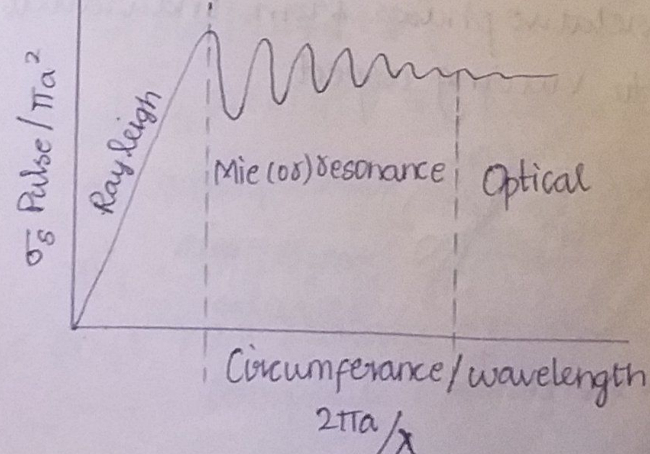
$$R^4_{max} = \frac{P_t G A_e \sigma_i \eta_i E_i(n)}{4\pi \lambda^2 K T_0 B_n F_N (S/N)_i}$$

RADAR CROSS SECTION OF TARGET

$$\sigma = \frac{\text{Power reflected towards Source / unit Solid angle}}{\text{incident power density} / 4\pi}$$

$$= 4\pi R^2 \frac{|E_r|^2}{|E_i|^2}$$ $E_r \rightarrow$ electric field strength of the echo signal back at radar.

$E_i \rightarrow$ incident electric field strength.



$\sigma \rightarrow$ dimension of object compared to λ $\quad \sigma \propto f^4$ (or) $1/\lambda^4$

$\lambda \gg$ dimension $\rightarrow$ Rayleigh region $\rightarrow$ rain, fog, smoke
Simple

$\lambda \ll$ dimension $\rightarrow$ Optical region $\rightarrow$ Complex targets $\quad$ $a \rightarrow$ radius of Sphere

Scattering affects cross section with respect to change in f or aspect angle

$\sigma \rightarrow$ more affected by shape of object than by projected area

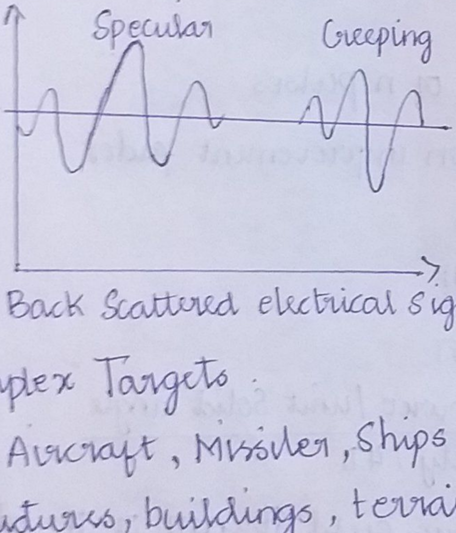
Sphere

$\frac{2\pi a}{\lambda} \ll 1 \rightarrow \sigma \propto f^4$

$\frac{2\pi a}{\lambda} \gg 1 \rightarrow \sigma$ approaches physical area of sphere (πa^2)

Changes in σ occurs with changing freq.

two waves Specular, Creeping.



Back Scattered electrical signal.

Complex Targets:

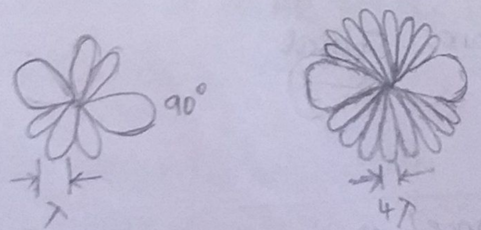
$\rightarrow$ Aircraft, Missiles, Ships, ground vehicles, fabricated structures, buildings, terrain.

$\rightarrow \sigma$ varies with respect to viewing aspect & freq

$\rightarrow$ multiple individual scatteres causes change in σ

$\rightarrow$ Each individual scatteres $\rightarrow$ has A & phase.

$\rightarrow$ Change in relative phases from individual Scatters $\rightarrow$ w.r to viewing aspect.



target with 2 scatters

$$\frac{\sigma_c}{\sigma_0} = 2\left[H \cos\left(\frac{4\pi l}{\lambda} \sin\theta\right)\right]$$

$\sigma_0 \rightarrow$ cross section of each scatters

$l \rightarrow$ Seperation

$\theta \rightarrow$ Viewing angle wr to the normal of the line joining the scatters.

As $\lambda \uparrow$, Scattering lobes become narrow.

Jet Aircraft

→ Nose-on aspect.

→ Reflection from jet engines and in takes ducts

→ Compressor blades → modulate the echo signal.

2. At angles few degree of the nose

→ σ decreases

3. Vicinity of Broadside.

→ Fuse lage & Engine nacelles → source of back scatter

→ Vertical stabilizers contribute to echoes.

4. Above the plane of the wing

→ Backscatter from corner of wing and fuselage

→ Strong echo from vertical fin and horizontal stabilizer.

CROSSSECTION FLUCTUATIONS.

$$S_r(t) = \sum_{i=1}^{N} a_i \sin(2\pi f t + \phi_i)$$

$$= A \sin(2\pi f_t + \phi)$$

$$A = \left[\left(\sum_i a_i \sin\phi_i\right)^2 + \left(\sum_i a_i \cos\phi_i\right)^2\right]^{1/2}$$

$a_i \rightarrow$ amplitude of i^{th} point scattering er.

$$\phi_i = 2\pi f T_i$$

Phase shift $> 2\pi$ radians → amplitude & phase of composite echo signal → cross section fluctuation

→ Pdf of the cross section of the target.
↳ Swerling

SWERLING TARGET MODEL.

Case (i) : Echo pulse have constant amplitude throughout a scan.

→ Scan to Scan fluctuators (slow) / Rayleigh Scatterers

→ many independent Scatterers

$$P(\sigma) = \frac{1}{\sigma_{av}} \exp\left[-\frac{\sigma}{\sigma_{av}}\right], \quad \sigma \geq 0.$$

Case (ii) :→ Fluctuation independent from pulse to pul

→ fast fluctuations. or pulse to pulse.

$$P(\sigma) = \frac{1}{\sigma_{av}} \exp\left[-\frac{\sigma}{\sigma_{av}}\right], \quad \sigma \geq 0.$$

Case (iii) : Large scatters with n no of Small Scatter

$$P(\sigma) = \frac{4\sigma}{\sigma_{av}^2} \exp\left[-\frac{2\sigma}{\sigma_{av}}\right] \quad \sigma \geq 0.$$

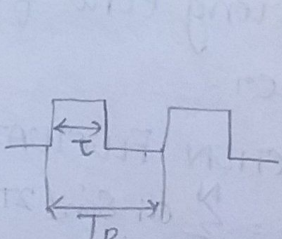
case (iv) : fluctuations pulse to pulse.

$$P(\sigma) = \frac{4\sigma}{\sigma_{av}^2} \exp\left[-\frac{2\sigma}{\sigma_{av}}\right] \quad \sigma \geq 0.$$

TRANSMITTER POWER :-

P_t → Peak Power

P_{av} → average Power.



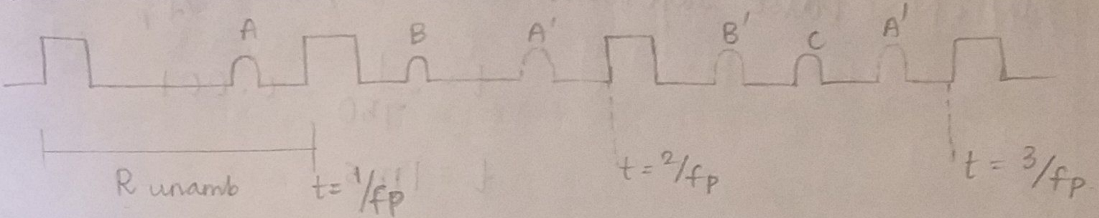
$$P_{av} = \frac{P_t \tau}{T_p} = P_t \tau f_p.$$

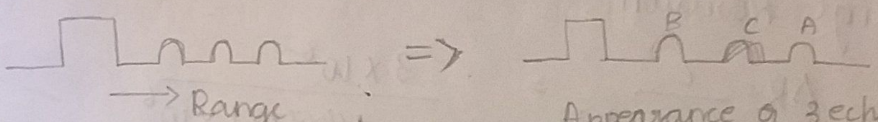
$$\frac{P_{av}}{P_t} = \frac{\tau}{T_p} \text{ or } \tau f_p \to \text{duty cycle}$$

type of waveform
pulse width
types of Tx-er used
whether pulse compr is used

$$R_{max}^4 = \frac{P_{av} G A_e \sigma n E_i(n)}{(4\pi)^2 k T_0 F_n (B_n \tau)(S/N) f_p}$$

PULSE REPETITION FREQUENCY





appearance of 3 echoes on A Scope

Appearance of 3 echoes with changing prf in A Scope display

PRF_1, R_1, R_{unamb_1}

$$R_{true} = R_1 \text{ or } (R_1 + R_{unamb_1}) \text{ or } (R_1 + 2R_{unamb_1})$$

PRF_2, R_2, R_{unamb_2}

$$R_{true} = R_2 \text{ or } (R_2 + R_{unamb_2}) \text{ or } (R_2 + 2R_{unamb_2})$$

NUMERICALS.

1. The unambiguous range of radar is 200 km it has bandwidth of 1 MHz.
   a) find PRF, pats
   b) Pulse repetition interval
   c) range resolution
   d) Pulse width.

a)
$$R_{unamb} = \frac{c}{2f_p}$$

$$200 \times 10^3 = \frac{3 \times 10^8}{2 \times f_p}$$

$$f_p = \frac{3 \times 10^{\cancel{8}3}}{2 \times 2\cancel{0}\cancel{0} \times \cancel{10^3}} = \frac{3 \times 10^3}{4} = 0.75 \times 10^3$$

$$f_p = 750\,Hz.$$

b) Pulse repetition interval, $t = 1/f_p$.

$$t = \frac{1}{750} = 0.001$$

$$t = 1ms$$

c) Range Resolution

$$\Delta R = \frac{c}{2\ \text{Bandwith}} = \frac{3 \times 10^8}{2 \times \cancel{100} \times \cancel{10} 1 \times 10^6}$$

$$= \frac{3 \times 100}{2}$$

$$= 3 \times 50 = 150\ m$$

d) Pulse width $= \frac{2\Delta R}{c}$

$$= \frac{2 \times 150}{3 \times 10^8} = 1\ \mu s.$$

2. If a pulse Radar operates with a pulse width of 2 μs & PRf of 800 Hz. find maximum unambiguous range.

$$R_{unam} = \frac{c}{2 f_p}$$

$$= \frac{3 \times 10^{\cancel{8} 6}}{2 \times 80\cancel{0}} = 0.188 \times 10^6 =$$

$$R_{unamb} = 187.5\ Km$$

3. A pulse radar has peak power $P_t = 5\ KW$. The required avg power is 1 KW. Determine the duty cycle

$$\text{duty cycle} = \frac{P_{avg}}{P_t} = \frac{1 \times 10^3}{5 \times 10^3}$$

$$= 0.2 = 20\%.$$

DETECTION OF SIGNAL IN NOISE.

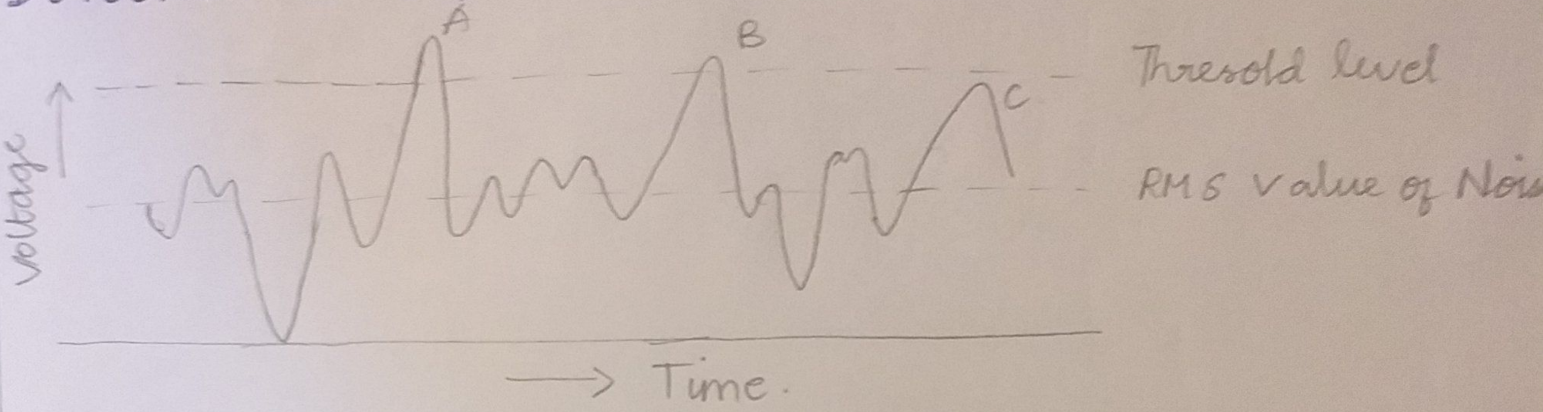


Fig : Envelope of the Radar receiver output as a function of time.

Maximum Detectable Signal :

- weakest signal detected by receiver. (Smin)

False alarm :

→ thresold too low → noise exceeds it.

Missed Detection

→ Thresold too high → weak signal might not exceed it.