Problem 2.1 Antenna Look angles

A geostationary satellite is located at 90^0 W. Calculate the azimuth angle for an earth station antenna at latitude 35^0N and longitude 100^0 W. Also find the range and antenna elevation angle.

Solution

Given quantities are

$\phi_{ss} = -90^0$ [∵ 90^0 west]

$\phi_E = -100^0$ [∵ 100^0 west]

$\lambda_E = 35^0$

Step 1: Find angle 'B'

$$B = \phi_E - \phi_{ss}$$
$$= -100^0 + 90^0$$
$$= -10^0 < 0$$

Step 2: Find angle 'b'

$$b = \text{arc cos}(\cos B \cos \lambda_E)$$
$$= \text{arc cos}(0.8067)$$
$$= \cos^{-1}(0.8067)$$
$$\doteq 36.23^0$$

Step 3: Find angle 'A'

$$A = \text{arc sin}\left(\frac{\sin|B|}{\sin b}\right)$$
$$= \text{arc sin}(0.2938)$$
$$= 17.1^0$$

Step 4: Find azimuth angle 'A_Z'

Here $\lambda E > 0$ and $B < 0$

According to figure 2.3 c

$$A_Z = 180^0 - A$$
$$\boxed{A_Z = 162.9^0}$$

Step 5: Find range 'd'

Values to be remembered:
• Radius of the orbit $a_{GSO} = 42164$ km
• Equatorial radius of earth $a_E = 6378$ km
• Average radius of earth $R = 6371$ km
• Earth's geocentric gravitational constant $\mu = 3.986005 \times 10^{14}$ m^3/s^2

$$d = \sqrt{R^2 + a_{GSO}^2 - 2Ra_{GSO}\cos b}$$
$$= \sqrt{6371^2 + 42164^2 - 2\times 6371\times 42164\times \cos 36.23^0}$$
$$= 37215.8 \text{ km}$$

$$\boxed{d \cong 37215 \text{ km}}$$

Step 6: Find elevation angle '$E\ell$'

$$E\ell = \text{arc cos}\left(\frac{a_{GSO}}{d}\sin b\right)$$
$$= \text{arc cos}\left(\frac{42164}{37215}\sin 36.23^0\right)$$
$$= \cos^{-1}(0.6696)$$
$$\boxed{E\ell \cong 48^0}$$

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Problem 2.2 Antenna Look Angles

An earth station is located at latitude 35^0 N and longitude 100^0 W Calculate the antenna look angles for a satellite at 67^0 W

Solution

Given data

$$\lambda_E = 35^0$$

$$\phi_E = 100^0\ W = -100^0$$

$$\phi_{ss} = 67^0\ W = -67^0$$

Step 1: Find angle 'B'

$$B = \phi_E - \phi_{ss}$$
$$= -100^0 + 67^0$$
$$= -33^0 < 0$$

Step 2: Find angle 'b'

$$b = \text{arc cos}\,(\cos B \cos \lambda_E)$$
$$= \cos^{-1}(0.671)$$
$$\simeq 47.86^0$$

Step 3: Find angle 'A'

$$A = \text{arc sin}\left(\frac{\sin|B|}{\sin b}\right)$$
$$= \sin^{-1}(0.735)$$
$$= 47.27^0$$

Step 4: Find azimuth angle 'A_Z'

Here, $\lambda_E > 0$ and $B < 0$

$$\therefore\ A_Z = 180^0 - A$$
$$= 180^0 - 47.27^0$$
$$\boxed{A_Z = 132.73^0}$$

Step 5: *Find range 'd'*

$$d = \sqrt{R^2 + a_{GSO}^2 - 2Ra_{GSO}\cos b}$$
$$= \sqrt{6371^2 + 42164^2 - 2\times 6371 \times 42164 \times \cos 47.86^0}$$
$$\simeq 38182\ \text{km}$$

Step 6: Find elevation angle '$E\ell$'

$$E\ell = \text{arc cos}\left(\frac{a_{GSO}}{d}\sin b\right)$$
$$= \cos^{-1}\left(\frac{42164}{38182}\sin 47.86^0\right)$$
$$\boxed{E\ell \simeq 35^0}$$

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Problem 2.3 Antenna Look Angles

An earth station is located at latitude 12^0 S and longitude 52^0 W. Calculate the antenna look angles for a satellite at 70^0 W.

Solution

Given data

$$\lambda_E = 12^0\ S = -12$$

$$\phi_E = 52^0\ W = -52^0$$

$$\phi_{ss} = 70^0\ W = -70^0$$

Step 1: Find angle 'B'

$$B = \phi_E - \phi_{ss}$$

$$= -52^0 + 70^0$$

$$= 18^0 > 0$$

Step 2: Find angle 'b'

$$b = \arccos(\cos B \cos \lambda_E)$$

$$= \cos^{-1}(\cos 18 . \cos(-12))$$

$$= 21.5^0$$

Step 3: Find angle 'A'

$$A = \arcsin\left(\frac{\sin|B|}{\sin b}\right)$$

$$= \sin^{-1}\left(\frac{\sin 18}{\sin 21.5}\right)$$

$$= 57.5^0$$

Step 4: Find azimuth angle 'A_Z'

Here, $\lambda_E < 0$ and $B > 0$

$$\therefore A_Z = 360^0 - A$$

$$= 360^0 - 57.5^0$$

$$\boxed{A_Z = 302.5^0}$$

Step 5: Find range 'd'

$$d = \sqrt{R^2 + a_{GSO}^2 - 2Ra_{GSO}\cos b}$$

$$= \sqrt{6371^2 + 42164^2 - 2\times 6371\times 42164\times \cos 21.5^0}$$

$$= 36311.46$$

$$\approx 36311\text{km}$$

Step 6: Find elevation angle '$E\ell$'

$$E\ell = \arccos\left(\frac{a_{GSO}}{d}\sin b\right)$$

$$= \cos^{-1}\left(\frac{42164}{36311}\sin 21.5^0\right)$$

$$\boxed{E\ell \simeq 65^0}$$

Problem 2.4 Antenna Look Angles

An earth station is located at latitude 35^0 N and longitude 65^0 E. Calculate the antenna look angles for a satellite at 19^0 E.

Solution

Given data

$$\lambda_E = 35^0$$

$$\phi_E = 65^0$$

$$\phi_s = 19^0$$

Step 1: Find angle 'B'

$$B = 65^0 - 19^0$$

$$= 46^0 > 0$$

Step 2: Find angle 'b'

$$b = \text{arc} \cos(\cos B \cos \lambda_E)$$

$$= \cos^{-1}(\cos 46 . \cos 35)$$

$$= 55.32^0$$

Step 3: Find angle 'A'

$$A = \text{arc}\sin\left(\frac{\sin|B|}{\sin b}\right)$$

$$= \sin^{-1}\left(\frac{\sin 46}{\sin 55.32}\right)$$

$$= 61^0$$

Step 4: Find azimuth angle 'A_Z'

Here, $\lambda_E > 0$ and $B > 0$

$$\therefore \ A_Z = 180^0 + A$$

$$= 180^0 + 61^0$$

$$A_Z = 241^0$$

Step 5: Find range 'd'

$$d = \sqrt{R^2 + a_{GSO}^2 - 2Ra_{GSO}\cos b}$$

$$= \sqrt{6371^2 + 42164^2 - 2 \times 6371 \times 42164 \times \cos 55.32^0}$$

$$\approx 38893 \text{ km}$$

Step 6: Find elevation angle 'El'

$$E\ell = \text{arc}\cos\left(\frac{a_{GSO}}{d}\sin b\right)$$

$$= \cos^{-1}\left(\frac{42164}{38893}\sin 55.32^0\right)$$

$$\boxed{E\ell \simeq 27^0}$$

Problem 2.5 **Angle of Tilt**

Determine the angle of tilt required for a polar mount used with an earth station at latitude 49^0 north. Assume a spherical earth of mean radius 6371 km, and ignore earth station altitude.

Solution

Given data

$\lambda_E = 49^0$

$R = 6371$ km

$\therefore\ b = \lambda_E = 49^0$ **[for due south situation]**

$a_{GSO} = 42164$ km

Step 1: Find elevation angle $E\ell$

$$\text{Range } d = \sqrt{R^2 + a_{GSO}^2 - 2Ra_{GSO}\cos b}$$

$$= \sqrt{6371^2 + 42164^2 - 2\times 6371\times 42164\times \cos 49^0}$$

$$\cong 38287 \text{ km}$$

$$\therefore\ E\ell = \arccos\left(\frac{a_{GSO}}{d}\sin b\right)$$

$$= \cos^{-1}\left(\frac{42164}{37215}\sin 49^0\right)$$

$$\boxed{E\ell \cong 33.8^0}$$

Step 2: Find angle of tilt δ

$$\delta = 90^0 - E\ell_0 - \lambda_E$$

$$= 90^0 - 33.8^0 - 49^0$$

$$\boxed{\delta \cong 7^0}$$

Problem 2.6 Angle of Tilt

An earth station is located at latitude 35^0N. Assuming a polar mount antenna is used, calculate the angle of tilt.

Solution

Given data

$\lambda_E = 35^0$

$\therefore\ b = \lambda_E = 35^0$

Also, we know that R = 6371 km, $a_{GSO} = 42164$ km

Step 1: *Find range 'd'*

$$= \sqrt{6371^2 + 42164^2 - 2 \times 6371 \times 42164 \times \cos 35^0}$$

$$\simeq 37125 \text{ km}$$

Step 2: *Find elevation angle 'El'*

$$= \cos^{-1}\left(\frac{42164}{37125}\sin 35^0\right)$$

$$El \simeq 49.4^0$$

Step 3: *Find angle of tilt 'δ'*

$$\delta = 90^0 - El_0 - \lambda_E$$

$$= 90^0 - 49.4^0 - 35^0$$

$$= 5.6^0$$

$$\boxed{\delta \simeq 6^0}$$

roblem 2.7 Limits of Visibility

Determine the limits of visibility for an earth station situated at mean sea level, at latitude 48.42° north and longitude 89.26° W. Assume a minimum angle of elevation of 5°.

olution

Given data

$\lambda_E = 48.42^0$

$\phi_E = -89.26^0$ [∴ 89.26° west]

$El_{min} = 5^0$

and, we know that

$a_{GSO} = 42164$ km

$R = 6371$ km

Step 1: Find subtended angle 'S'

$$S = \arcsin\left(\frac{R}{a_{GSO}} \sin \sigma_{min}\right)$$

$$\sigma_{min} = 90^0 + El_{min}$$

$$= 90^0 + 5^0$$

$$= 95^0$$

$$\therefore \quad S = \arcsin\left(\frac{6371}{42164} \sin 95^0\right)$$

$$= \sin^{-1}\left(\frac{6371}{42164} \sin 95^0\right)$$

$$= 8.66^0$$

Step 2: Find angle 'b'

$$b = 180 - \sigma_{min} - S$$

$$= 180 - 95^0 - 8.66^0$$

$$= 76.34^0$$

Step 3: Find angle 'B'

$$B = \arccos\left(\frac{\cos b}{\cos \lambda_E}\right)$$

$$= \arccos\left(\frac{\cos 76.34^0}{\cos 48.42^0}\right)$$

$$= \cos^{-1}\left(\frac{\cos 76.34^0}{\cos 48.42^0}\right)$$

$$= 69.15^0$$

Step 4: Find satellite limits

$$\phi_{ss} = \phi_E \pm B$$

$$\phi_E + B = -89.26^0 + 69.15^0 = -20^0 \text{ approx}$$

$$\phi_E - B = -89.26^0 - 69.15^0 = -158^0 \text{ approx}$$

∴ The satellite limit east of the earth station is at -20^0 approx

The satellite limit west of the earth station is at -158^0 approx