realization of digital filters:-

* for a recasive realization the current output y(n) is a function of passed outputs, past and present input this corresponds to Inifite Impluse respons (IIR) digital filters.

* for non-recasive realization the current output y(n) is a function of pass and present input this corresponds to finite impluse respons (FIR) digital filters.

IIR filter can be realize using
* direct form I realization
* direct form II "
* cascaded form "
* parallel form "

direct form I realization:-

general derivation of direct form I

let us consider LTI (linear time inverent) recasive system discred by the following different equation

$$y(n) = -\sum_{K=1}^{N} a_K\, y(n-K) + \sum_{K=0}^{N} b_K\, x(n-K)$$

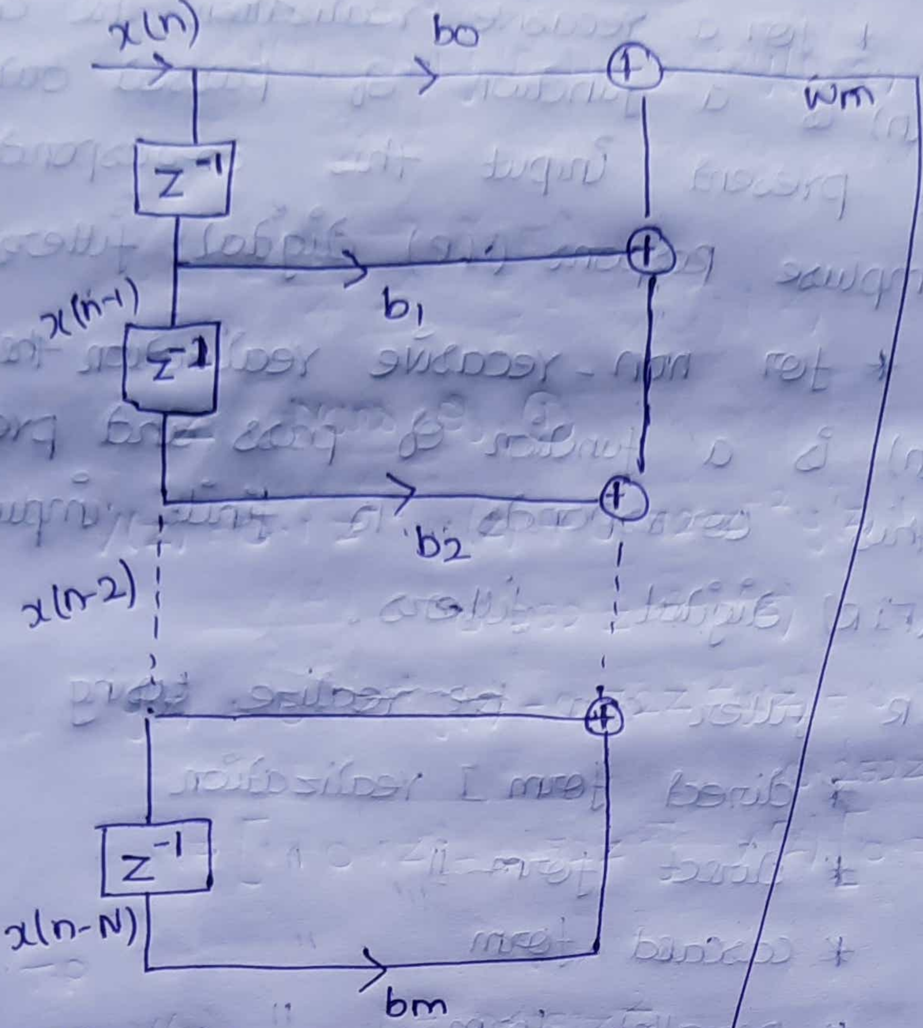
$$= -a_1 y(n-1) - a_2 y(n-2) - \cdots a_N y(n-N)$$

$$- b_0 x(n) - b_1 x(n-1) \cdots b_N x(n-N)$$

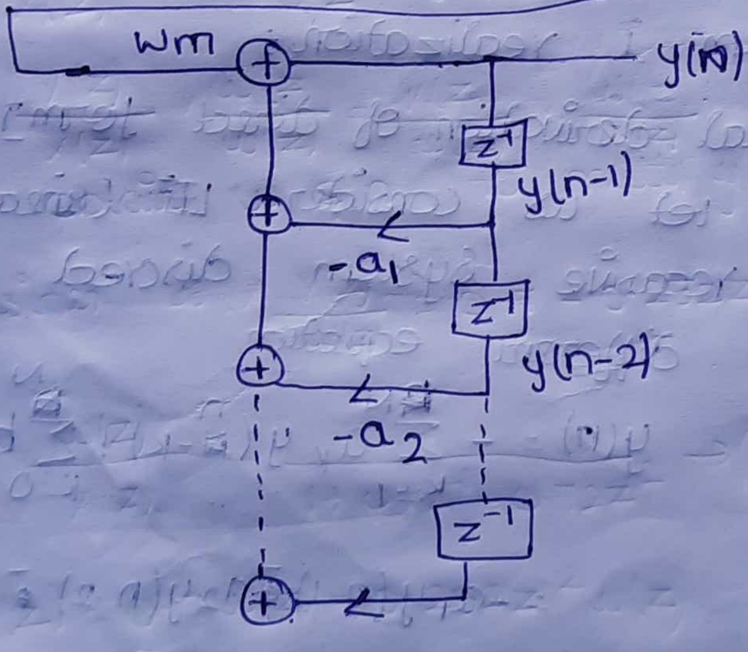
let $w_m = b_0 x(n) + b_1 x(n-1) \ldots b_m x(n-m)$] — ①

$$y(n) = -a_1 y(n-1) + a_2 y(n-2) \ldots a_N y(x-N) + w_m \quad (2)$$

Realization of equ (1)



Realization of equ (2) :-



final direct form I realization is :-

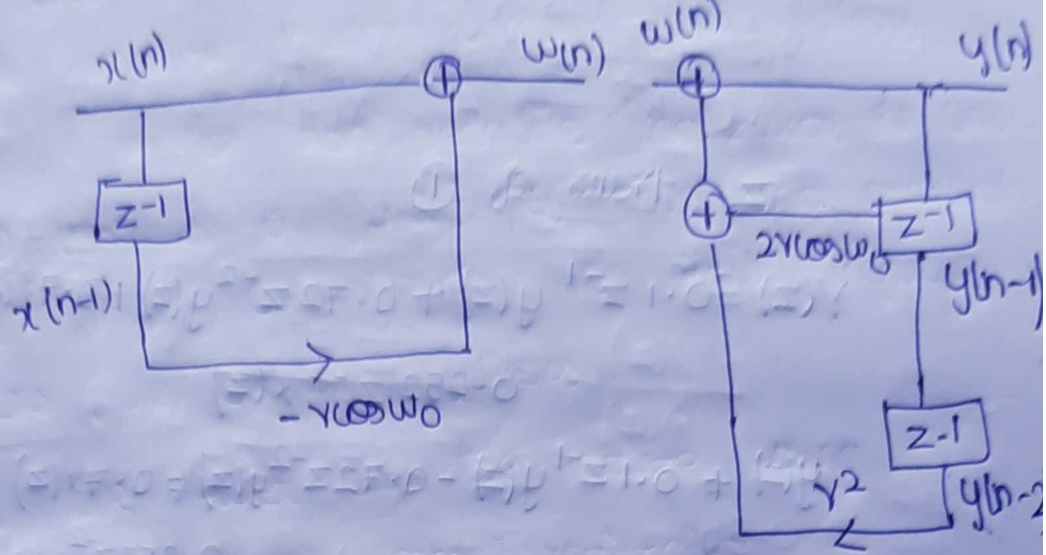
1. Realise the second order digital filter

$y(n) = 2r\cos\omega_0\, y(n-1) - r^2 y(n-2) + x(n) - r\cos\omega_0\, x(n-1)$

using direct form I.

$w(n) = x(n) - r\cos\omega_0\, x(n-1) \rightarrow ①$

$y(n) = 2r\cos\omega_0\, y(n-1) - r^2 y(n-2) + w(n) \rightarrow ②$



2. $y(x) = 0.5\, y(n-1) - 0.25\, y(n-2) + x(n) + 0.4\, x(n-1)$

Direct form II:

1. Determine direct form II realization for the following

$y(n) = 0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2) \rightarrow ①$

$x(n) \overset{Z}{\longleftrightarrow} X(z)$

$x(n-k) \overset{Z}{\longleftrightarrow} z^{-k} X(z)$

Z trans of ①

$Y(z) = 0.1z^{-1}Y(z) + 0.72z^{-2}Y(z) + 0.7X(z) - 0.252z^{-2}X(z)$

$Y(z) + 0.1z^{-1}Y(z) - 0.72z^{-2}Y(z) = 0.7X(z) - 0.252z^{-2}X(z)$

$Y(z)[1 + 0.1z^{-1} - 0.72z^{-2}] = X(z)[0.7 - 0.252z^{-2}]$

T.P

$$\frac{Y(z)}{X(z)} = \frac{0.7 - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

$$\frac{Y(z)}{X(z)} \times \frac{W(z)}{W(z)} = \frac{Y(z)}{W(z)} \times \frac{W(z)}{X(z)} = \frac{0.7 - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

To obtain W & n to multiply & divide

consider $\frac{Y(z)}{W(z)} = 0.7 - 0.252z^{-2} \rightarrow ②$

$$\frac{W(z)}{X(z)} = \frac{1}{1 + 0.1z^{-1} - 0.72z^{-2}} \rightarrow ③$$

$Y(z) = 0.7W(z) - 0.252z^{-2}W(z)$

inverse Z trans

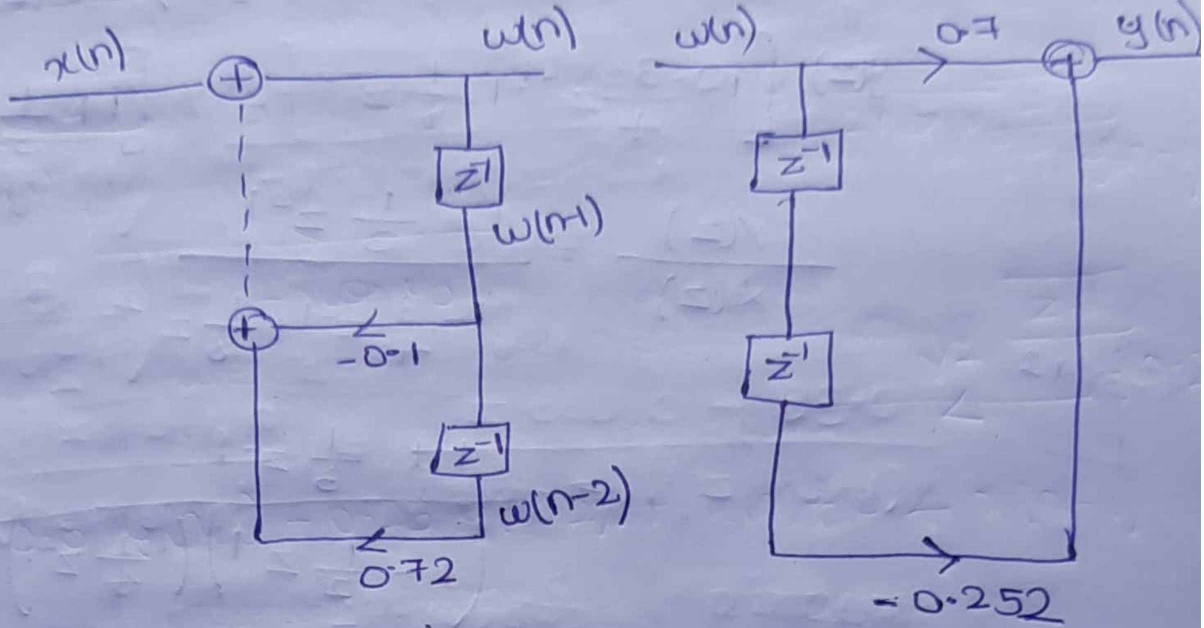
$y(n) = 0.7w(n) - 0.252w(n-2) \rightarrow ④$

$W(z) + 0.1z^{-1} - 0.72z^{-2}W(z) = X(z)$

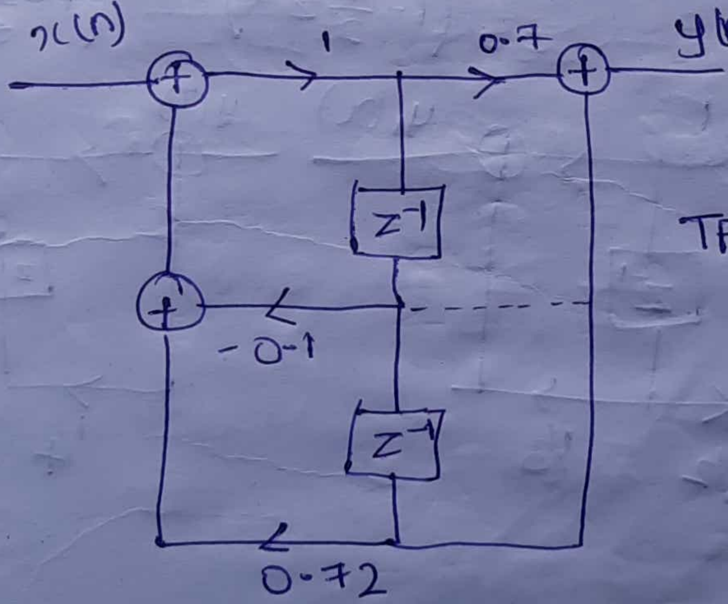
$\omega(z) = x(z) - 0.1 z^{-1}\omega(z) + 0.72 z^{-2}\omega(z)$

$\omega(n) = x(n) - 0.1\,\omega(n-1) + 0.72\,\omega(n-2)$ → (5)

Direct form II realization is represented as



Since the input to delay elements in the both the structure and the same we can compare the delay elements.



$TF = \frac{Y(z)}{X(z)}$

$= \frac{0.7 - 0.252 z^{-2}}{1 + 0.1 z^{-1} - 0.72 z^{-2}}$

$= -0.5 z^{-1}$

2. realise the system of the different equation using cascade form

$$y(x) = \frac{3}{4} y(x-1) - \frac{1}{2} y(n-2) + x(n) + \frac{1}{3} x(n-1)$$

$$Y(z) = \frac{3}{4} z^{-1} Y(z) - \frac{1}{8} z^{-2} Y(z) + X(z) + \frac{1}{3} z^{-1} X(z)$$

$$Y(z) - \frac{3}{4} z^{-1} Y(z) + \frac{1}{8} z^{-2} Y(z) = X(z) + \frac{1}{3} z^{-1} X(z)$$

$$Y(z)\left[1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}\right] = X\left[1 + \frac{1}{3} z^{-1}\right]$$

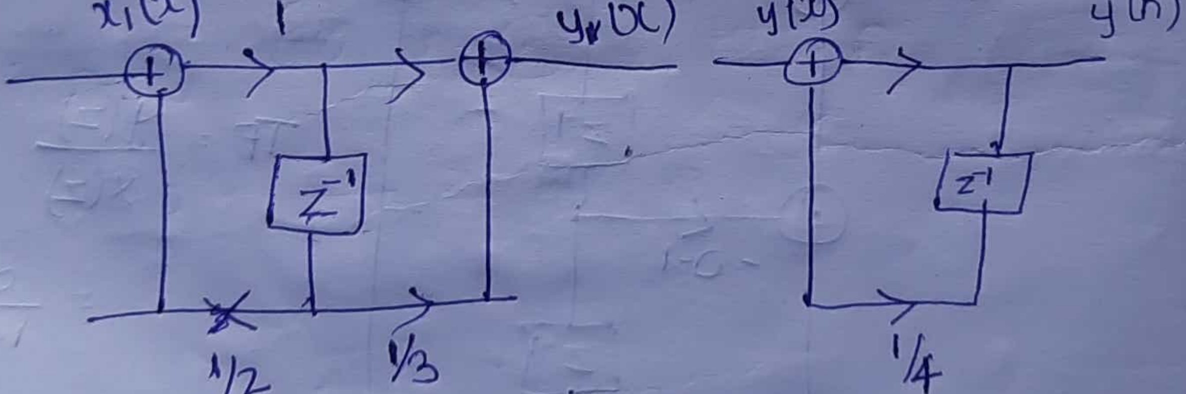
$$\frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{3} z^{-1}}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

$x^2 - 3x + 2$

$$= \frac{1 + \frac{1}{3} z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(-\frac{1}{4} z^{-1}\right)}$$

$$= H_1(z)\, H_2(z)$$

$$= \frac{1 + \frac{1}{3} z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)} \times \frac{1}{\left(1 - \frac{1}{4} z^{-1}\right)}$$



3. $y(x) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$

$$\cancel{\frac{Y(z)}{X(z)} = \frac{0.7 + 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}}}$$

~~Any fraction can be written as Q + R~~

$$Y(z) = -0.1z^{-1}Y(z) + 0.72z^{-2}Y(z) + 0.7X(z) - 0.252z^{-2}X(z)$$

$$Y(z) + 0.1z^{-1}Y(z) - 0.72z^{-2}Y(z) = 0.7X(z) - 0.252z^{-2}X(z)$$

$$Y(z)\left[1 + 0.1z^{-1} - 0.72z^{-2}\right] = X(z)\left[0.7 - 0.252z^{-2}\right]$$

$$\frac{Y(z)}{X(z)} = \frac{0.7 - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

$$3\overline{\smash{)}7}^{\,2}$$
$$\underline{6}$$
$$\underline{1}$$

$\frac{7}{3}$ can be written as $Q + \frac{\text{Rem}}{\text{Divisor}}$

$= 2 + \frac{1}{3}$

$$-0.72z^{-2} + 0.1z^{-1} + 1 \overline{\smash{)}\,-0.252z^{-2} + 0.7}^{\;0.35 \; (Q)}$$

$-0.252z^{-2} \qquad -+$

$\underline{\qquad + 0.035 + 0.35}$

(R) $\quad \underline{-0.035z^{-1} + 0.35}$

$$Q + \frac{R}{D}$$

$$\therefore \frac{Y(z)}{X(z)} = 0.35 + \frac{0.35 - 0.035z^{-1}}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

$$1 + 0.1z^{-1} - 0.72z^{-2}$$

$$1 + \frac{1}{10}z^{-1} - \frac{72}{100}z^{-2}$$

$$= 1 - \frac{8}{10}z^{-1} + \frac{9}{10}z^{-1} - \frac{72z^{-2}}{100}$$

$$\frac{-72}{100} = \frac{-8}{10} + \frac{9}{10}$$

$$= 1\left(1 - \frac{8}{10}z^{-1}\right) + \frac{9}{10}z^{-1}\left(1 - \frac{8}{10}z^{-1}\right)$$

$$= 1\left(1 + \frac{9}{10}z^{-1}\right) \cdot \left(1 - \frac{8}{10}z^{-1}\right)$$

$$\frac{0.35 - 0.035z^{-1}}{(1+0.9z^{-1})(1-0.8z^{-1})} = \left(\frac{A}{(1+0.9z^{-1})}\right) + \left(\frac{B}{1-0.8z^{-1}}\right)$$

$$\frac{0.35 - 0.035z^{-1}}{(1+0.9z^{-1})(1-\cancel{4}+z^{-1})} = \frac{A(1-0.8z^{-1}) + B(1+0.9z^{-1})}{(1+0.9z^{-1})(1-\cancel{0.1}z^{-1})}$$

equating constants:

$$0.22 = A + B \rightarrow (2)$$

equating half of z^{-1}

$$-0.035 = -0.8A + 0.9A \rightarrow (3)$$

$(2) \Rightarrow B = 0.35 - f \cdot (4)$

(4) in (3) + find A

DFT & IDFT:

DFT is given by

$$x(k) = \sum_{n=0}^{n-1} x(n) e^{-\frac{j2\pi nk}{N}}, \quad x = 0, 1 \ldots N-1$$

IDFT is

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{j\frac{2\pi nk}{N}}, \quad n = 0, 1, \ldots N-1$$

1. Find the DFT of the sequence $x(n) = \{1, 1, 0, 0\}$ and find IDFT of $y(k) = \{1, 0, 1, 0\}$

i) $$x(k) = \sum_{n=0}^{n-1} x(n) e^{-\frac{j2\pi nk}{N}}, \quad x = 0, 1 \ \ N-1 \Rightarrow 4$$

$$x(k) = \sum_{n=0}^{n-1} x(n) e^{-\frac{j2\pi nk}{4}}$$

$$x(k) = \sum_{n=0}^{3} x(n) e^{-\frac{j\pi nk}{2}}, \quad k = 0, 1, 2, 3$$

$k = 0$

$$x(0) = \sum_{n=0}^{3} x(n) \cdot 1$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1 + 1 + 0 + 0$$

$$= 2$$

$k = 1$

$e^{j\theta} = \cos\theta + j\sin\theta$

$e^{-j\theta} = \cos\theta - j\sin\theta$

$$x(1) = \sum_{n=0}^{3} x(n) \cdot e^{-\frac{j\pi n}{2}}$$

$$= \left[x(0) e^{-j\pi/2} + x(1) e^{-j\pi} + 0 + 0\right]$$

$$= e^{-j\pi/2} + e^{-j\pi}$$

$$= 1 + e^{-j\pi/2}$$

$$= 1 + \left[\underbrace{\cos \pi/2}_{0} - \underbrace{j\sin \pi/2}_{1}\right]$$

$$= 1 - j$$

$k=2$

$$x(2) = \sum_{n=0}^{3} x(n) e^{-j\pi n}$$

$$= [x(0)e^{0} + x(1)e^{-j\pi}]$$

$$= 1 + [\underbrace{\cos\pi}_{-1} - j\underbrace{\sin\pi}_{0}]$$

$$= 1 - 1 = 0$$

$k=3$

$$x(3) = \sum_{n=0}^{3} x(n) e^{-3/2 j\pi n}$$

$$= x(0)e^{0} + x(1) e^{-3/2 j\pi}$$

$$= 1 + \underbrace{\cos\frac{3\pi}{2}}_{0} - j\underbrace{\sin\frac{3\pi}{2}}_{-1}$$

$$= 1 + j$$

$$x(k) = \{2, 1-j, 0, 1+j\}$$

2. IDFT of $y(k) = \{1, 0, 1, 0\}$

$$y(k) = \frac{1}{N}\sum_{k=0}^{N-1} y(k) e^{j\frac{2\pi nk}{N}}, \; n = 0, 1, \ldots N-1$$

$$= \frac{1}{4}\sum_{k=0}^{3} y(k) e^{j\frac{\pi nk}{2}}, \; n = 0, 1, 2, 3$$

i) $n=0$

$$y(0) = \frac{1}{4}\sum_{k=0}^{3} y(k) (1)$$

$$= \frac{1}{4}[y(0) + y(1) + y(2) + y(3)]$$

$$= \frac{1}{4}[1 + 0 + 1 + 0]$$

$$= \frac{1}{4}(2) = \frac{2}{4} = 0.5$$

ii) $n=1$

$$y(1) = \frac{1}{4}\sum_{k=1}^{3} y(k)\, e^{\frac{j\pi k}{2}}$$

$$= \frac{1}{4}\left[y(0)e^{0} + y(1)e^{j\pi/2} + y(2)e^{j\pi} + y(3)e^{j3\pi/2}\right]$$

$$= \frac{1}{4}\left[1 + [\underbrace{\cos\pi}_{-1} + j\underbrace{\sin\pi}_{0}]\right]$$

$$= \frac{1}{4}\left[1 + (-1)\right]$$

$$= \frac{1}{4}(0)$$

$$= 0$$

iii) $n=2$

$$y(2) = \frac{1}{4}\sum_{k=1}^{3} y(k)\, e^{j\pi k}$$

$$= \frac{1}{4}\left[y(0)e^{0} + y(1)e^{j\pi} + y(2)e^{j2\pi} + y(3)\right.$$

$$= \frac{1}{4}\left[1 + e^{j2\pi}\right]$$

$$= \frac{1}{4}\left[1 + \cos 2\pi + j\sin 2\pi\right]$$

$$= \frac{1}{4}\left[1 + 1 + 0\right] = 0.5$$

iv) $n=3$

$$y(3) = \frac{1}{4}\sum_{k=1}^{3} y(k)\, e^{\frac{3j\pi k}{2}}$$

$$= \frac{1}{4}\left[y(0)e^{0} + y(1)e^{j3\pi/2} + y(2)e^{j3\pi} + y(3)\, e^{j9\pi/2}\right]$$

$$= \frac{1}{4}\left[1 + 1\,\underbrace{\cos 3\pi}_{-1} + j\underbrace{\sin 3\pi}_{0}\right]$$

$$= \frac{1}{4}\left[1 - 1 + 0\right]$$

$= 0$

$y(n) = \{0.5, 0, 0.5, 0\}$

3. Find DFT of $x(n) = 1$ for $0 \le n \le 2$ plot $|X(k)|$ & $\angle X(k)$

$= 2$ for o.w

i) $N = 4$

ii) $N = 8$

$|X(k)|$ & $\angle X(k)$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}}, \quad k = 0, 1, \ldots N-1$$

$$X(k) = \sum_{n=0}^{3} x(n) e^{-\frac{j\pi kn}{2}}, \quad k = 0, 1, 2, 3$$

$$X(0) = \sum_{n=0}^{3} x(n)$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1 + 1 + 1 + 0$$

$$= 3$$

$$X(1) = \sum_{n=0}^{3} x(n) e^{-j\frac{\pi}{2}n}$$

$$= x(0)e^{0} + x(1)e^{-j\pi/4} + x(2)e^{-j\pi} + 0$$

$$= 1 + \overset{\to 0}{\cos \pi/2} - \overset{\to 1}{j\sin \pi/2} + \overset{\to -1}{\cos \pi} - \overset{\to 0}{j\sin \pi}$$

$$= 1 - j - 1$$

$$= -j$$

iii) $$X(2) = \sum_{n=0}^{3} x(n) e^{-j\pi n}$$

$$= x(0)e^{0} + x(1)e^{-j\pi} + x(2)e^{-j2\pi}$$

$$= 1 + \overset{\to -1}{\cos \pi} - \overset{\to 0}{j\sin \pi} + \overset{\to 1}{\cos 2\pi} - \overset{\to 0}{j\sin 2\pi}$$

$= 1 - 1 + 1$

$= 1$

iii) $X(3) = \sum_{n=0}^{3} x(n) e^{-j\frac{3\pi n}{2}}$

$= x(0)e^{0} + x(1)e^{-j3\pi/2} + x(2)e^{-j3\pi} + 0$

$= 1 + \overset{\to 0}{\cos\frac{3\pi}{2}} - j\overset{\to -1}{\sin 3\pi/2} + \overset{\to -1}{\cos 3\pi} - j\underbrace{\sin 3\pi}_{0}$

$= 1 + j - 1$

$= j$

$X(k) = \{3, -j, 1, j\}$

$\boxed{a+jb} \Rightarrow |X(k)| = \sqrt{a^2+b^2}$

$\angle X(k) = \tan^{-1}(b/a)$

θ	$\sin\theta$	$\cos\theta$
0	0	0
$\pi/6$	$1/2$	$\sqrt{3}/2$
$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$
$\pi/3$	$\sqrt{3}/2$	$1/2$
$\pi/2$	1	0

$\tan(-\theta) = -\tan\theta$

$\tan 0 = 0$

$\tan \pi/6 = 1/\sqrt{3}$

$\tan \pi/4 = 1$

$\tan \pi/3 = \sqrt{3}$

$\tan \pi/2 = \infty$

i) $X(0) = 3 + 0j$

$|X(0)| = \sqrt{3^2 + 0^2} = 3$

$\angle X(0) = \tan^{-1}\left(\frac{0}{3}\right)$

$= \tan^{-1} 0 = 0$

ii) $X(1) = 0 - j$

$|X(1)| = \sqrt{0+1} = 1$

$$\angle x(1) = \tan^{-1}\left(\frac{-1}{2}\right)$$

$$= -\tan^{-1}(\infty)$$

$$= -\pi/2$$

iii) $x(2) = 1$

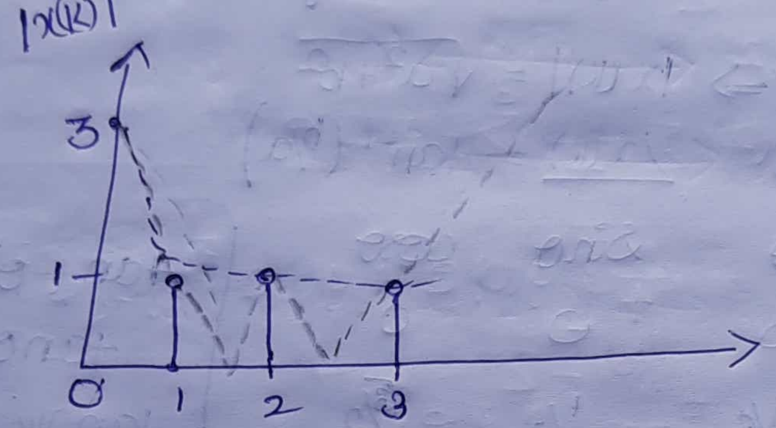
$$\angle x(2) = \tan^{-1} 0 = 0$$

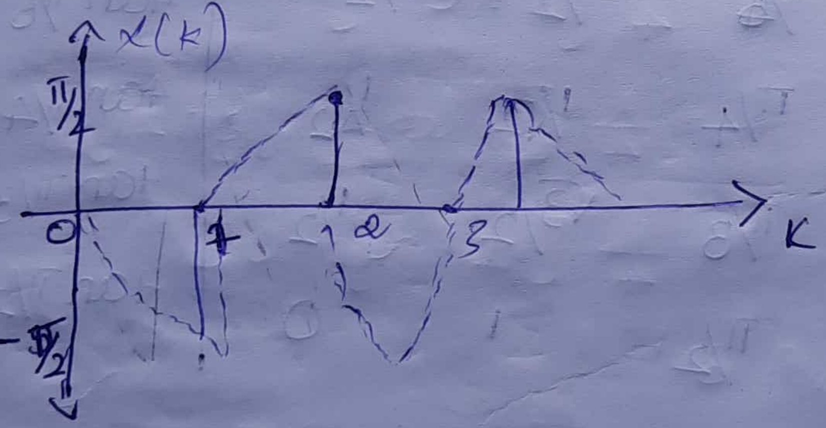
iv) $|x(3)| = 1$

$$\angle x(3) = \tan^{-1}\infty = \pi/2$$

$$|x(k)| = \{3, 1, 1, 1\}$$

$$\angle x(k) = \{0, -\pi/2, 0, \pi/2\}$$





4.

$N = 8$

$x(n) = \{1, 1, 1, 0, 0, 0, 0, 0\}$

$$X(k) = \sum_{n=0}^{7} x(n) e^{\frac{-j n \pi k}{4}} \quad , k = 0 \cdots 7$$

$$X(0) = \sum_{n=0}^{7} x(n) e^{0}$$

$$= x(0) + x(1) + x(2) + x(3) + x(4) + x(5) + x(6) + x(7)$$

$$= 1 + 1 + 1 + 0 + 0 + 0 + 0 + 0$$

$$= 3$$

$$X(1) = \sum_{n=0}^{7} x(n) e^{-j\frac{\pi n}{4}}$$

$$= x(0)e^{0} + x(1)e^{-j\pi/4} + x(2)e^{-j\pi/2} + x(3)e^{-j3\pi/4}$$

$$= 1 + e^{-j\pi/4} + e^{-j\pi/2} + 0$$

$$= 1 + \cos\pi/4 - j\sin\pi/4 + \cos\pi/2 - j s$$

$$= 1 + \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} + 0 - j$$

$$= 1 + \frac{1}{\sqrt{2}} - j\left(1 + \frac{1}{\sqrt{2}}\right)$$

Fourier transform pairs:

DFT and IDFT and combine are called as fourier transform pairs.

That is

$$x(n) \xleftrightarrow{DFT} x(k)$$

properties of DFT

i) periodicity

if signal repeat after consider x(n) to be discrete signal.

i) $x(n+N) = x(n)$

ii) $x(K+N) = x(k)$

proof:

i) $x(n) = \frac{1}{N} \sum_{K=0}^{n-1} x(K) e^{\frac{j2\pi K}{N}}$

put $n = n+N$

$$x(n+N) = \frac{1}{N} \sum_{K=0}^{N-1} x(k) e^{\frac{-j2\pi(n+N)K}{N}}$$

$$x(n+N) = \frac{1}{N} \sum_{K=0}^{N-1} x(K) e^{\frac{j2\pi K}{N}} e^{\frac{j2\pi NK}{N}}$$

consider

$$e^{j2\pi K} = (e^{j2\pi})^K$$

$$= (\cos 2\pi + j\sin 2\pi)^K$$

$$x(n+N) = \frac{1}{N} \sum_{K=0}^{N-1} x(k) e^{\frac{j2\pi K}{N}}$$

$$= x(n)$$

ii) $x(k) = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi nk}{N}}$

$$x(k+N) = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi n(k+N)}{N}}$$

$$= \underbrace{\sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi nk}{N}}}_{x(k)}\; e^{\frac{-j2\pi nN}{N}}$$

$$(e^{-j2\pi})^n$$

$$= \cos 2\pi - j\sin 2\pi$$

$$= (1)^n = 1$$

$$= x(k)$$

ii) Linearity:-

If $x_1(n) \xleftrightarrow{DFT} x_1(k)$

$x_2(n) \xleftrightarrow{DFT} x_2(k)$

then

$$a_1 x_1(n) + a_2 x_2(n) \xleftrightarrow{DFT} a_1 x_1(k) + a_2 x_2(k)$$

proof:-

$$DFT[x(n)] = x(k)$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi nk}{N}}$$

$$DFT[a_1 x_1(n) + a_2 x_2(n)] = \sum_{n=0}^{N-1} [a_1 x_1(n) + a_2 x_2(n)]\, e^{\frac{-j2\pi nk}{N}}$$

$$= \sum_{n=0}^{N-1} a_1 x_1(n) e^{\frac{-j2\pi nk}{N}} + \sum_{n=0}^{N-1} a_2 x_2(n) e^{\frac{-j2\pi nk}{N}}$$

$$= a_1 x_1(k) + a_2 x_2(k)$$

iii) conjugate symmetry:

If $DFT[x(n)] = X(K)$, then

$X(-K) = X^*(K)$

Proof:

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi K}{N}}$$

$$X(-K) = \sum_{n=0}^{N-1} x(n) e^{\frac{+j2\pi K}{N}}$$

$$= X^*(K)$$

convolution:

* linear convolution
* circular convolution

linear convolution	circular convolution
* $y(n) = x_1(n) * x_2(n)$	* $y(n) = x_1(n) ⊛ x_2(n)$
* $y(n) = \sum_{n=-\infty}^{\infty} x_1(K) x_2(n-K)$	* $y(n) = \sum_{K=0}^{N-1} x_1(K) x_2(n-K)$
* Length of $x_1(n) = L$, $x_2(n) = M$, $y(n) = L+M-1$	* length of $y(n) = \text{Max}(L, M)$

What is zero padding & uses

If a sequence $x(n)$ has length L then to find N point DFT (where N is greater than L) we must add (N - L) zeros to the sequence $x(n)$. This is called padding.

uses:-

* better display of frequency spectrum
* with zero padding, DFT can be used in linear filtering.

circular convolution:-

procedure:-

* Draw to concentric circule
* mark all the point of $x_1(n)$ in outer circule in anticlock wise direction.
* Mark all points of $x_2(n)$ starting from same point in the inner circule in clock wise direction.
* multiply the corresponding point & add
* Rotate the inner circle in anti clock wise direction & repeat the same procedure until the inner

circle reaches its original position.

1. Find circular convolution of $x_1(n) = \{1, -1, -2, 3, -1\}$ $x_2(n) = \{1, 2, 3\}$

Soln:

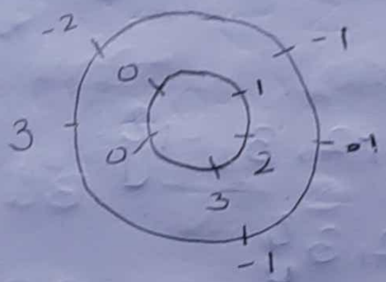
$x_2(n) = \{1, 2, 3, 0, 0\}$

$y(0) = 1(1) + 0(-1) + 0(-2) + 3(3) + 2(-1)$

$= 1 + 9 - 2$

$= 10 - 2$

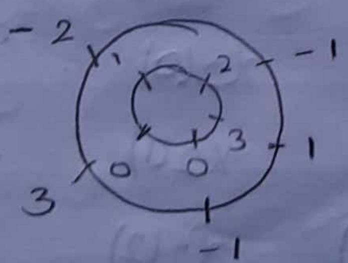
$= 8$



$y(1) = 2(1) + 1(-1) + 0(-2) + 0(3) + 3(-1)$

$= 2 - 1 - 3$

$= -2$



$y(2) = 3(1) + 2(-1) + 1(-2) + 0(3) + 0(-1)$

$= 3 - 2 - 2$

$= -1$

$y(3) = 0(1) + 3(-1) + 2(-2) + 1(3) + 0(-1)$

$= -3 - 4 + 3$

$= -4$

$y(4) = 0(1) + 0(-1) + 3(-2) + 2(3) + 1(-1)$

$= -6 + 6 - 1$

$= -1$

$y(n) = \{8, -2, -1, -4, -1\}$

convolution:

$x_1(n) = \{1, -1, -2, 3, -1\}$

$x_2(n) = \{1, 2, 3\}$

$x_2(n) = \{1, 2, 3, 0, 0\}$

$$\begin{bmatrix} x_2(0) & x_2(4) & x_2(3) & x_2(2) & x_2(1) \\ x_2(1) & x_2(0) & x_2(4) & x_2(3) & x_2(2) \\ x_2(2) & x_2(1) & x_2(0) & x_2(4) & x_2(3) \\ x_2(3) & x_2(2) & x_2(1) & x_2(0) & x_2(4) \\ x_2(4) & x_2(3) & x_2(2) & x_2(1) & x_2(0) \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \\ x_1(4) \end{bmatrix} = \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}$$

Fast fourier tr

FFT

DFT with red

computational

-9 divided

comparision

DFT & FFT ÷

5m

$$= \begin{bmatrix} 1 & 0 & 0 & 3 & 2 \\ 2 & 1 & 0 & 0 & 3 \\ 3 & 2 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -2 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 1+0+0+9-2 \\ 2-1+0+0-3 \\ 3-2-2+0 \\ 0-3-4+3 \\ -6+6-1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ -2 \\ -1 \\ -4 \\ -1 \end{bmatrix}$$

Fast fourier transform: (FFT)

FFT is a meathod for computing DFT with reduced number of calculations. Th[e] computational efficiency is achived by empl[oying] divided & concwed & achived.

-9

comparision of no. of computations in direc[t]

DFT & FFT:

(3m)

No. of points N, N=2	Direct computing: complex addition N(N-1)	Direct computing: complex multiplication N^2	RADIX-2 FFT: complex addition $N \log_2 N$	RADIX-2 FFT: complex multiplication $N/2 \log_2 N$
$4 = 2^2$	$4(4-1) = 12$	$4^2 = 16$	$4 \log_2 2^2 = 4(2) = 8$	$\frac{4}{2} \log_2 2^2 = 2(2) = 4$
$8 = 2^3$	$8(8-1) = 56$	$8^2 = 64$	$8 \log_2 2^3 = 24$	$\frac{8}{2} \log_2 2^3 = 12$
$16 = 2^4$	$16(16-1) = 240$	$16^2 = 256$	$16 \log_2 2^4 = 64$	$\frac{16}{2} \log_2 2^4 = 32$
$32 = 2^5$	$32(32-1) = 992$	$32^2 = 1024$	$32 \log_2 2^5 = 160$	$\frac{32}{2} \log_2 2^5 = 80$
$64 = 2^6$	$64(64-1) = 4032$	$64^2 = 4096$	$64 \log_2 2^6 = 384$	$\frac{64}{2} \log_2 2^6 = 192$

FFT

(DIT) - desimation in time → i/p → bit reversal order

desimation in frequency (DIF) → output bit removal order

Normal				Bit reversal			
0	0	0	0	0	0	0	0
1	0	0	1	1	0	0	4
2	0	1	0	0	1	0	2
3	0	1	1	1	1	0	6
4	1	0	0	0	0	1	1
5	1	0	1	1	0	1	5
6	1	1	0	0	1	1	3
7	1	1	1	1	1	1	7

Twiddle or face factor:

$$x(k) = \sum_{n=0}^{N-1} x(n) e^{-\frac{j2\pi nk}{N}}, \text{ for } k=0,1 \ldots N-1$$

$W_N \rightarrow$ Twiddle factor $= e^{-\frac{j2\pi}{N}}$

$$x(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk}, \; k=0,1 \ldots N-1$$

properties of twiddle factor:

1m or 2m

i) symmetry:

$$W_N^{k+\frac{N}{2}} = -W_N^{k}$$

ii) periodicity

$$W_N^{k+N} = W_N^{k}$$

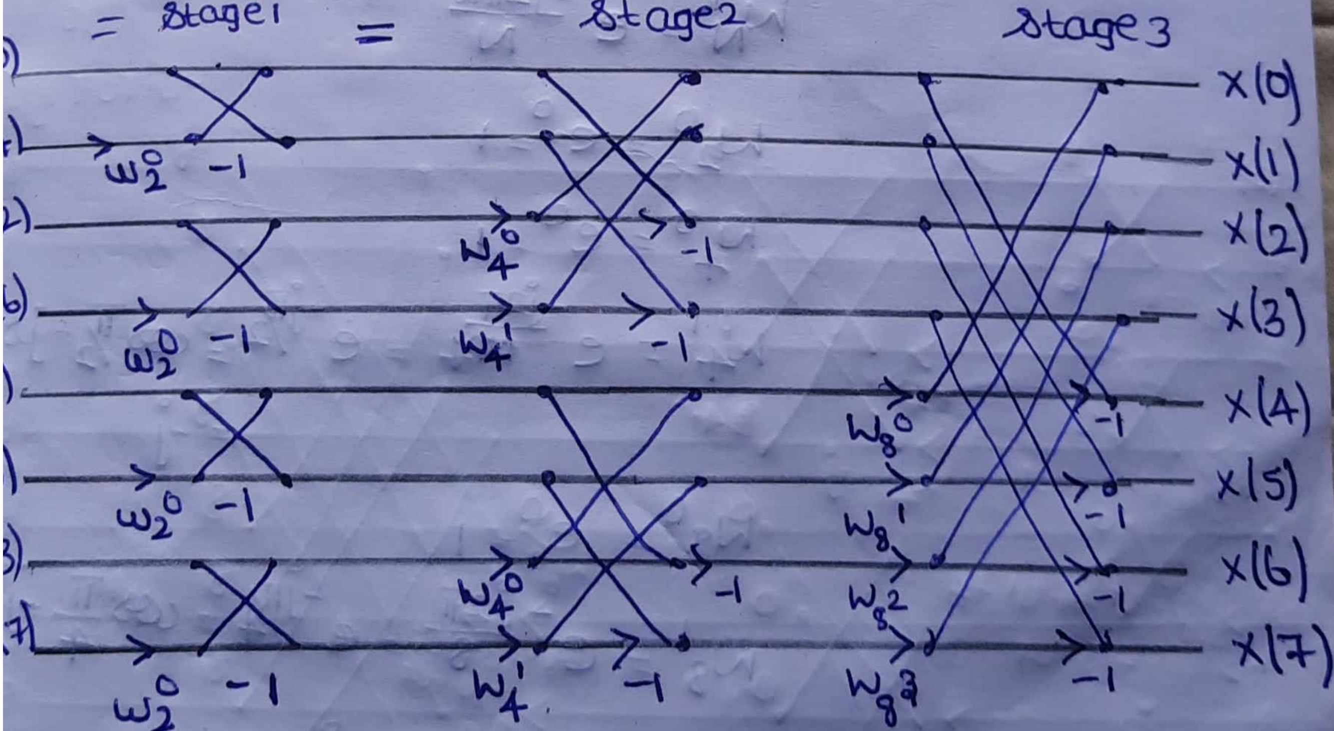
8-point DFT, $(N=8)$

$$W^{12} = W^{8+4} = W^{4}$$

$$W^{6} = W^{4+2} = -W^{2}$$

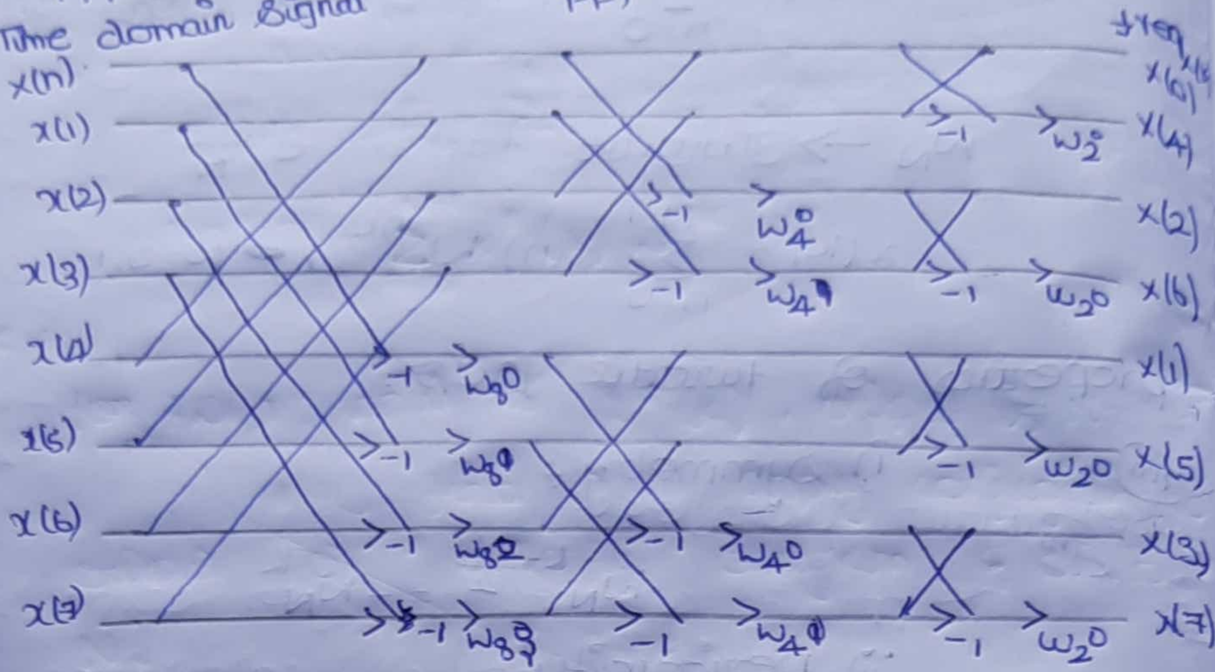
X:

Butterfly diagram for 8-point DFT using radix-2 DIT FFT



Butterfly diagram of 8 point DFT using radix to

DIF FFT



1. Compute 8 point DFT of the sequence

$$x(n) = \begin{cases} 1 ; & 0 \le n < 7 \\ 0 ; & \text{other wise} \end{cases}$$ using DIT & DIF algorithm

soln:- $W_2^0, W_4^0, W_4^1, W_8^0, W_8^1, W_8^2, W_8^3$

$$W_N = e^{\frac{-j2\pi}{N}}$$

$$W_N^k = e^{\frac{-j2\pi k}{N}}$$

$$W_2^0 = e^0 = 1$$

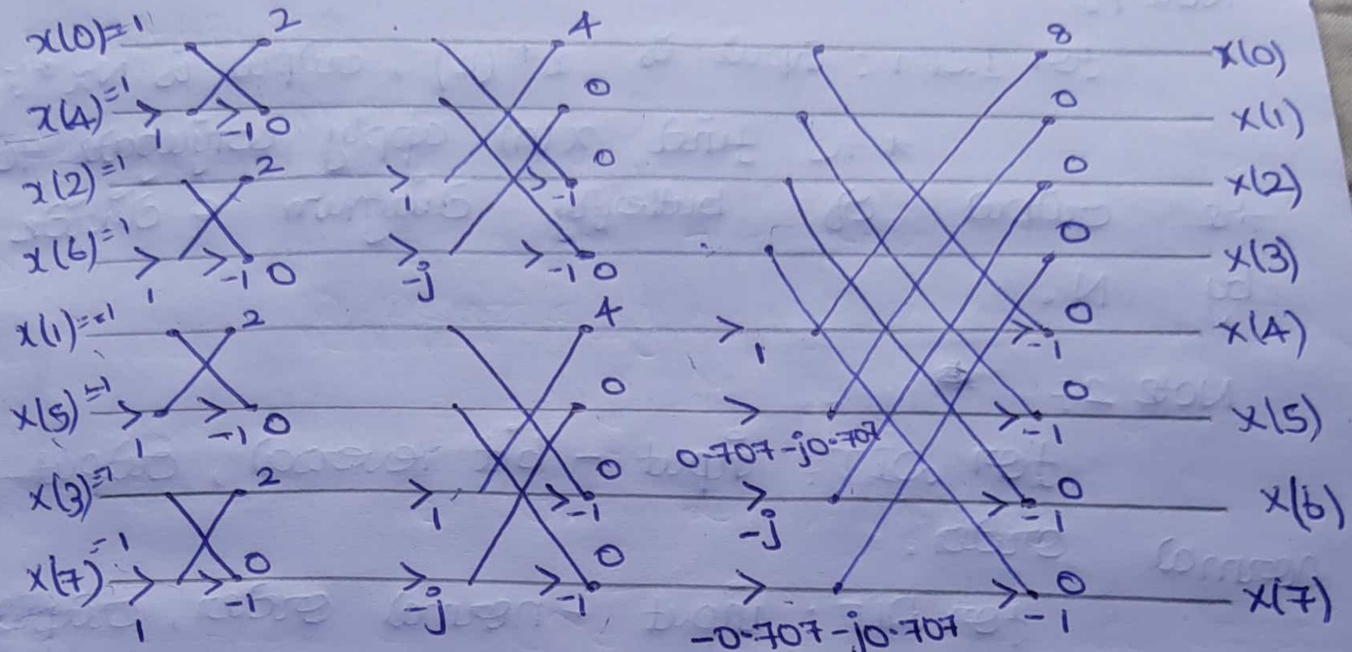
$$W_4^0 = e^0 = 1$$

$$W_4^1 = e^{\frac{-j2\pi}{4}} = e^{-j\pi/2} = \cos \pi/2 - j\sin \pi/2$$
$$= -j$$

$$W_8^0 = e^0 = 1$$

$$W_8^1 = e^{-\frac{j2\pi}{8}} = e^{-j\pi/4} = \cos\frac{\pi}{4} - j\sin\frac{\pi}{4}$$
$$= 0.707 - j0.707$$

$$W_8^2 = e^{\frac{-j2\pi 2}{8}}$$
$$= e^{-j\pi/2}$$
$$= \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} = -j$$
$$W_8^3 = e^{\frac{-j2\pi 3}{8}}$$
$$= -0.707 - j0.707$$



$$X(k) = \{8, 0, 0, 0, 0, 0, 0, 0\}$$

IDFT :- $= x(n) = \frac{1}{N} \sum_{n=0}^{N-1} x(k) W^{-nk}$

DFT :- $x(k) = \sum_{n=0}^{N-1} x(n) W^{nk}$

$$x^*(n) = \frac{1}{N} \sum_{n=0}^{N-1} x^*(k) W^{nk}$$

$$Nx^*(n) = \sum_{n=0}^{N-1} x^*(k) W^{nk}$$

Note 1 :-

for IDFT : input is $x^*(k)$, output is $Nx^*(n)$

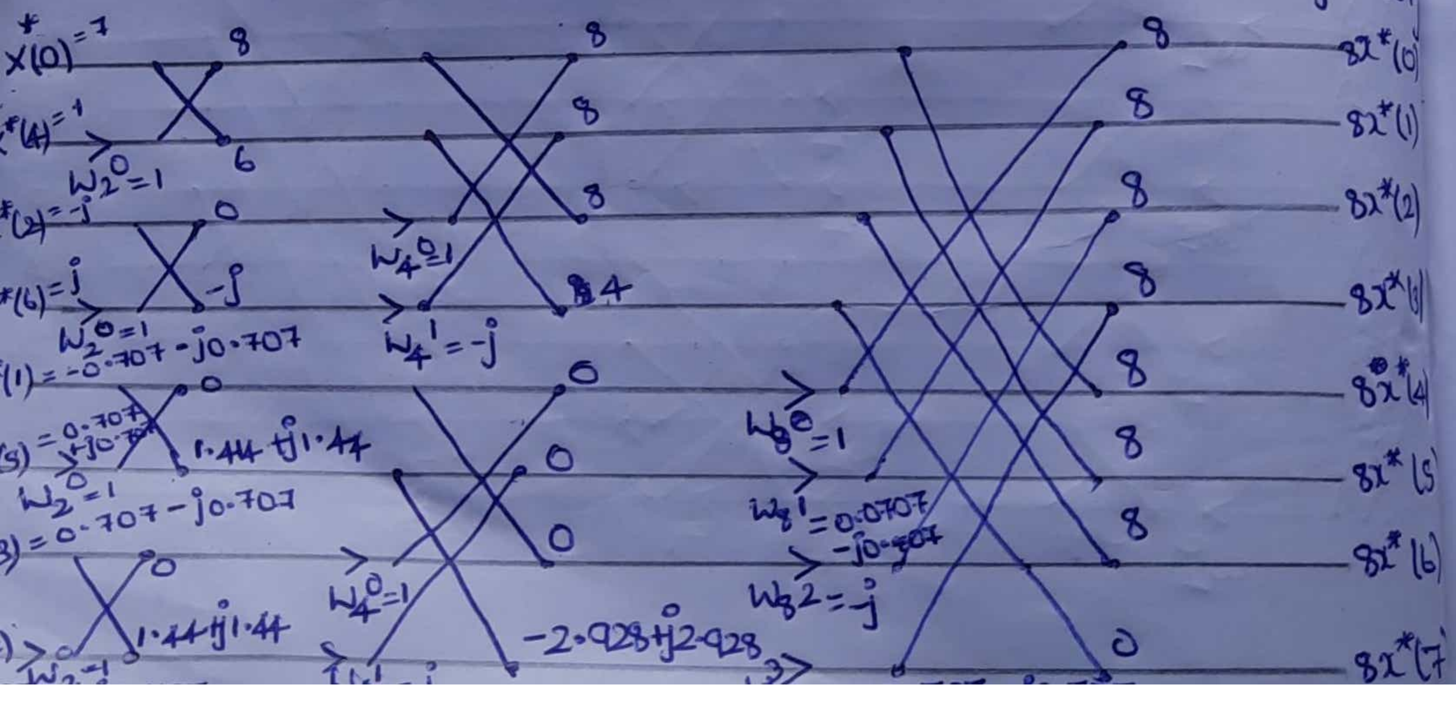
* to find $x(n)$ apply conjugate to the output of butterfly diagram & divide by N.

Note 2 :-

for DIT : input - bit reversal, output normal order.

for DIF : input - normal order, output - bit reversal order.

1. compute IDFT of the sequence $x(k) = \{7, -0.707 - j0.707, 0.707 - j0.707, 1, 0.707 + j0.707, j, -0.707 + j0.707\}$



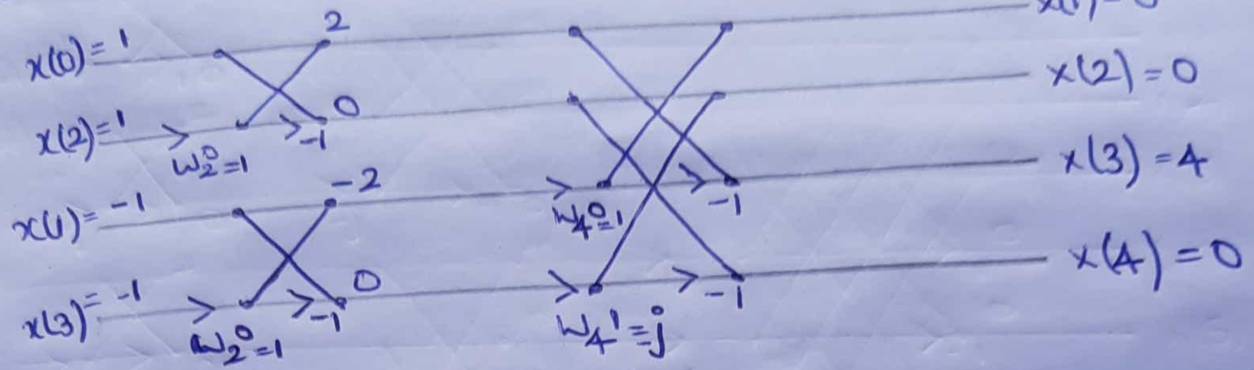
$N_x^*(n) = \{8,8,8,8,8,8,8,0\}$

$x^2(n) = \{1,1,1,1,1,1,1,0\}$

$x(n) = \{1,1,1,1,1,1,1,0\}$

2. compute DFT of a sequence $x(n) = \{\overset{0}{1}, \overset{1}{-1}, \overset{2}{1}, \overset{3}{-1}\}$ using DIT algorithm & DIF algorithm.

sol:-



$X(K) = \{0, 0, 4, 0\}$