

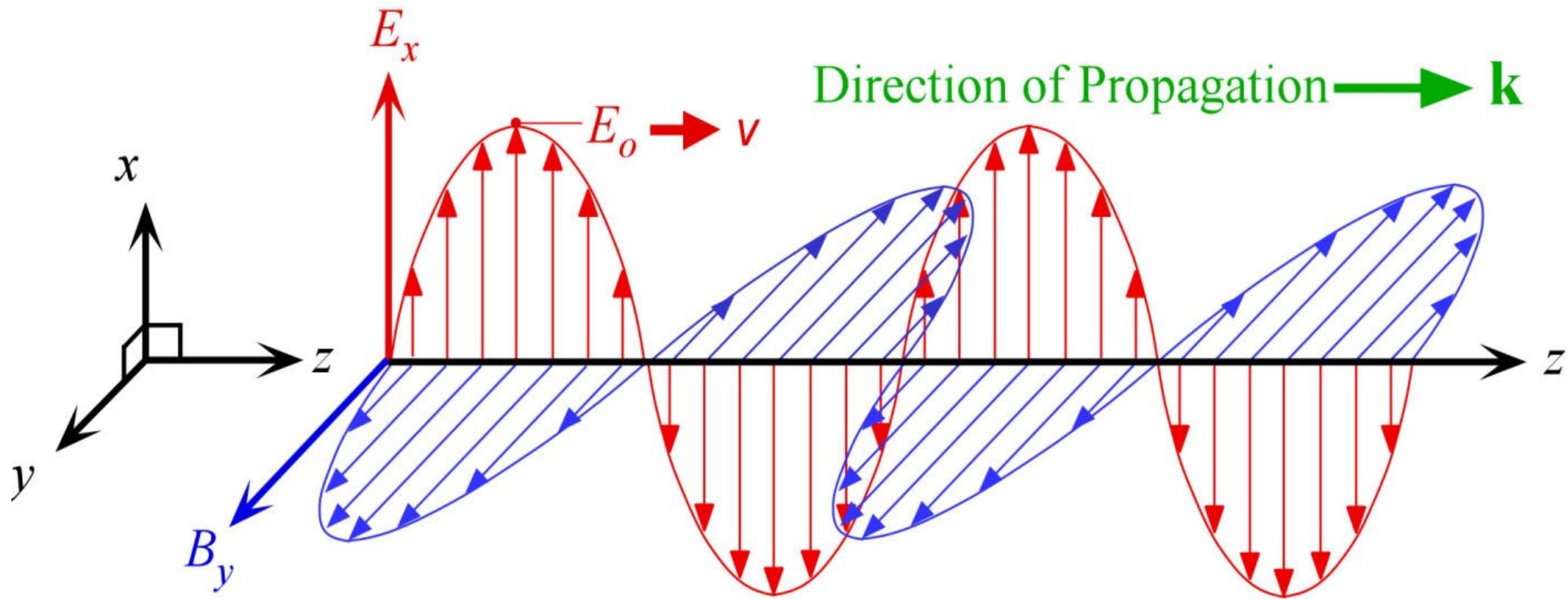
# **18ECE322T - OPTOELECTRONICS**

## **UNIT – 1 - WAVE NATURE OF LIGHT AND SEMICONDUCTOR OPTICS**

Light Waves in a Homogeneous Medium- Plane Electromagnetic Wave, Maxwell's Wave Equation - Refractive Index and Dispersion- Sellmeier Equation and Diamond, Cauchy Equation and Diamond - Polarization of Light - Snell's Law and Total Internal Reflection - Reflection and Refraction - Superposition and Interference of Waves - Diffraction Principles- Fraunhofer Diffraction, Diffraction Grating

Overview of Semiconductors - Interaction of Photons With Charge Carriers - Hole Pair Formation and Recombination - Absorption in Semiconductors - Effect of Electric Field on Absorption - Absorption in Quantum Wells - Radiation in Semiconductors - Heterojunctions

# Light is an electromagnetic wave



An electromagnetic wave is a traveling wave that has time-varying electric and magnetic fields that are perpendicular to each other and the direction of propagation  $z$ .

$$E_x = E_o \cos(\omega t - kz + \varphi_o)$$

$E_x$  = Electric field along  $x$  at position  $z$  at time  $t$

$k$  = **Propagation constant** =  $2\pi/\lambda$

$\lambda$  = Wavelength

$\omega$  = Angular frequency =  $2\pi\nu$  ( $\nu$  = frequency)

$E_o$  = Amplitude of the wave

$\varphi_o$  = Phase constant; at  $t = 0$  and  $z = 0$ ,  $E_x$  may or may not necessarily be zero depending on the choice of origin.

$(\omega t - kz + \varphi_o) = \varphi$  = **Phase of the wave**

This is a **monochromatic plane wave** of *infinite extent* traveling in the positive  $z$  direction.

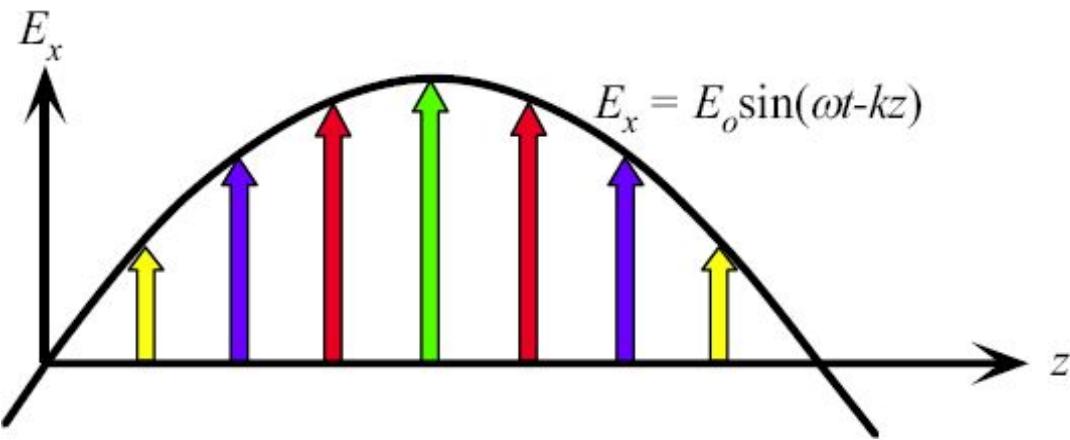
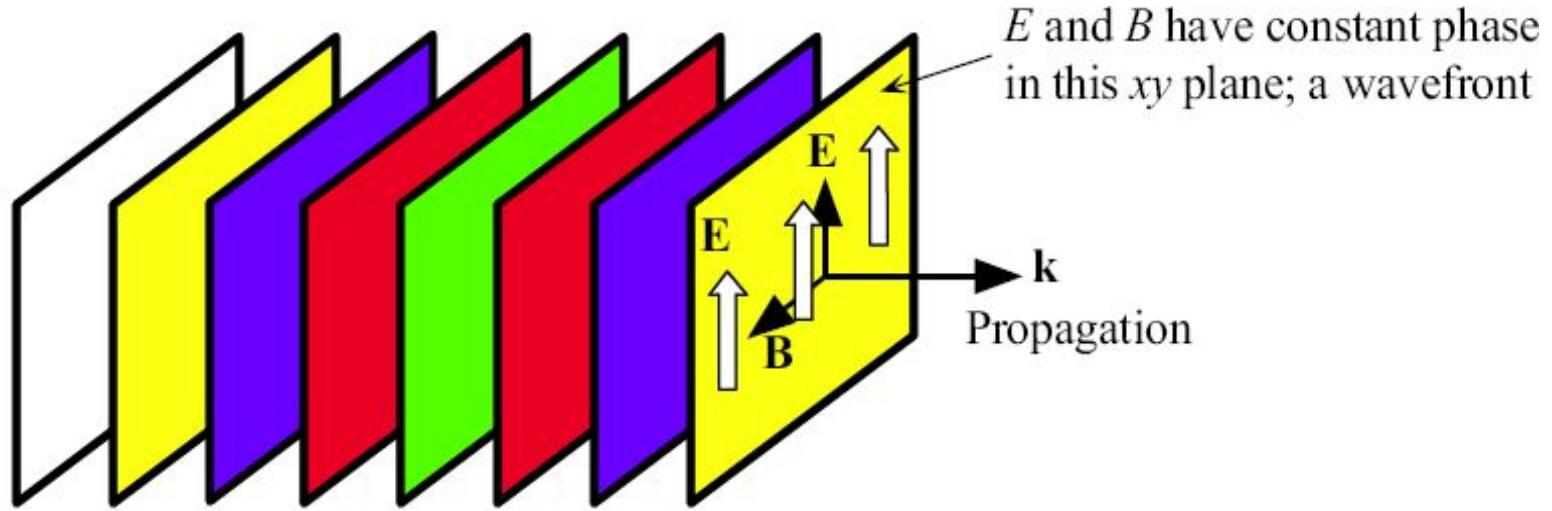
# Wavefront

A surface over which the phase of a wave is constant is referred to as a **wavefront**

A **wavefront** of a plane wave is a plane **perpendicular** to the direction of propagation

The interaction of a light wave with a nonconducting medium (conductivity = 0) uses the electric field component  $E_x$  rather than  $B_y$ .

**Optical field** refers to the electric field  $E_x$ .



A plane EM wave traveling along  $z$ , has the same  $E_x$  (or  $B_y$ ) at any point in a given  $xy$  plane. All electric field vectors in a given  $xy$  plane are therefore in phase. The  $xy$  planes are of infinite extent in the  $x$  and  $y$  directions.

# Phase Velocity

The time and space evolution of a given phase  $\varphi$ , for example that corresponding to a maximum field is described by

$$\varphi = \omega t - kz + \varphi_o = \text{constant}$$

During a time interval  $\delta t$ , this constant phase (and hence the maximum field) moves a distance  $\delta z$ . The phase velocity of this wave is therefore  $\delta z/\delta t$ . The **phase velocity**  $v$  is

$$v = \frac{\delta z}{\delta t} = \frac{\omega}{k} = v\lambda$$

## Phase change over a distance $\Delta z$

$$\varphi = \omega t - kz +$$

$$\varphi_o$$

$$\Delta\varphi = k\Delta z$$

The **phase difference** between two points separated by  $\Delta z$  is simply  $k\Delta z$

since  $\omega t$  is the same for each point

If this phase difference is 0 or multiples of  $2\pi$  then the two points are in phase. Thus, the phase difference  $\Delta\varphi$  can be expressed as  $k\Delta z$  or  $2\pi\Delta z/\lambda$

# Exponential Notation

Recall that

$$\cos\varphi = \operatorname{Re}[\exp(j\varphi)]$$

where  $\operatorname{Re}$  refers to the real part. We then need to take the real part of any complex result at the end of calculations. Thus,

$$E_x(z,t) = \operatorname{\textbf{Re}}[E_o \exp(j\varphi_o) \exp j(\omega t - kz)]$$

or

$$E_x(z,t) = \operatorname{\textbf{Re}}[E_c \exp j(\omega t - kz)]$$

where  $E_c = E_o \exp(j\varphi_o)$  is a complex number that represents the amplitude of the wave and includes the constant phase information  $\varphi_o$ .

# Wave Vector or Propagation Vector

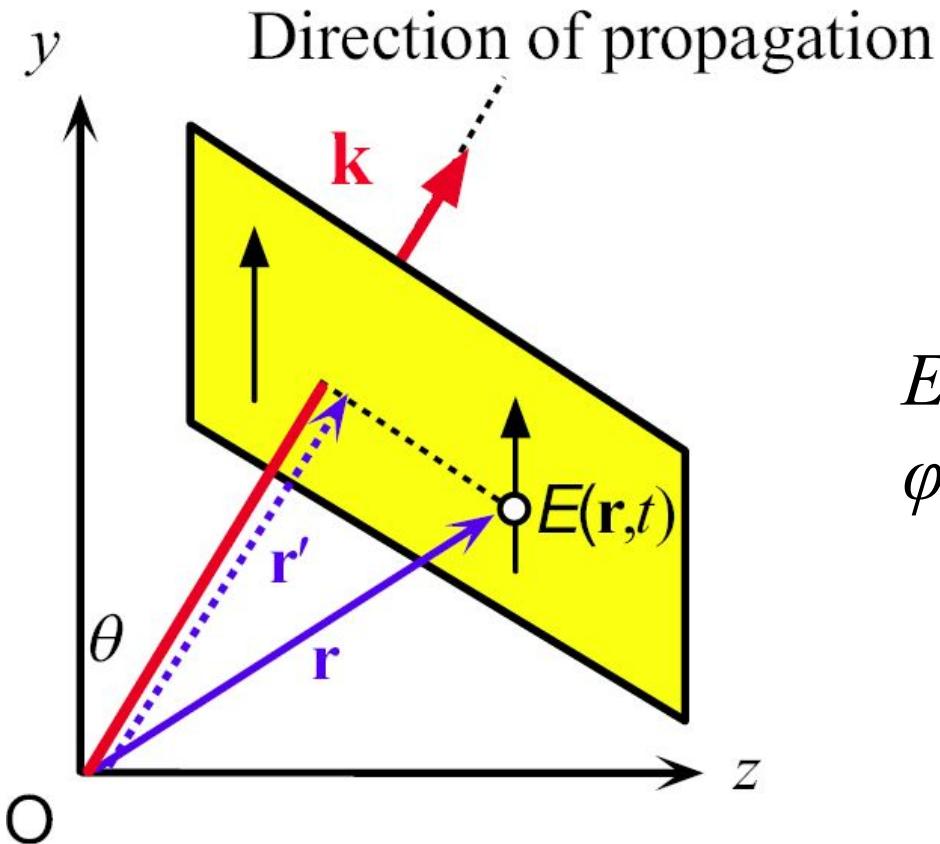
Direction of propagation is indicated with a vector  $\mathbf{k}$ , called the **wave vector**, whose magnitude is the *propagation constant*,  $k = 2\pi/\lambda$ .  $\mathbf{k}$  is *perpendicular* to constant phase planes.

When the electromagnetic (EM) wave is propagating along some arbitrary direction  $\mathbf{k}$ , then the electric field  $E(\mathbf{r},t)$  at a point  $\mathbf{r}$  on a plane perpendicular to  $\mathbf{k}$  is

$$E(\mathbf{r},t) = E_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \phi_0)$$

If propagation is along  $z$ ,  $\mathbf{k} \cdot \mathbf{r}$  becomes  $kz$ . In general, if  $\mathbf{k}$  has components  $k_x$ ,  $k_y$  and  $k_z$  along  $x$ ,  $y$  and  $z$ , then from the definition of the dot product,  $\mathbf{k} \cdot \mathbf{r} = k_x x + k_y y + k_z z$ .

# Wave Vector $\mathbf{k}$



$$E(\mathbf{r}, t) = E_o \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \phi_0)$$

A traveling plane EM wave along a direction  $\mathbf{k}$

# Maxwell's Wave Equation

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0$$

= A plane wave is a solution of Maxwell's wave equation

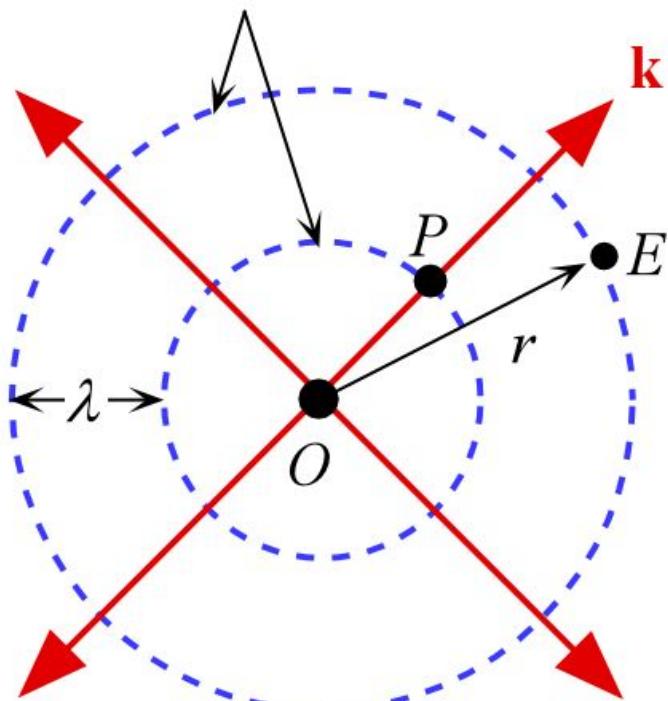
$$E_x = E_o \cos(\omega t - kz + \phi_o)$$

Substitute into Maxwell's Equation to show that this is a solution.

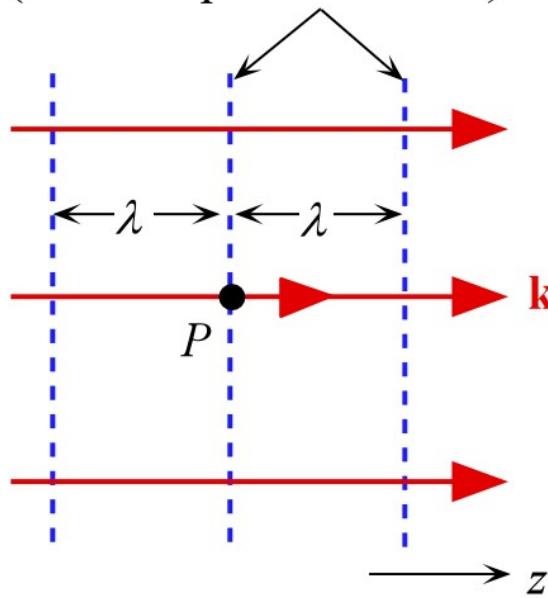
# Spherical Wave

$$E = \frac{A}{r} \cos(\omega t - kr)$$

Wavefronts

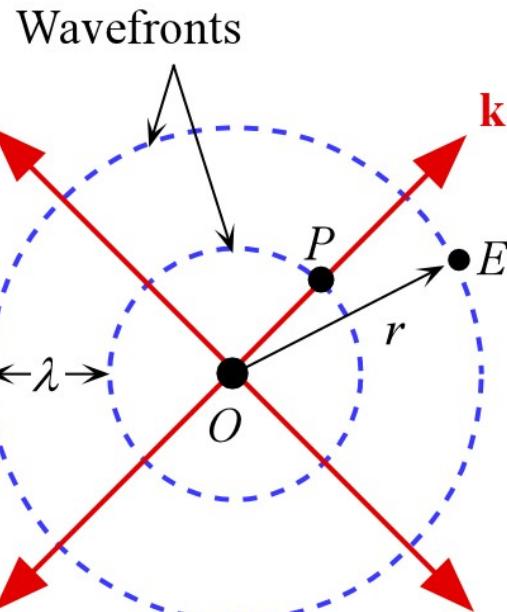


Wavefronts  
(constant phase surfaces)



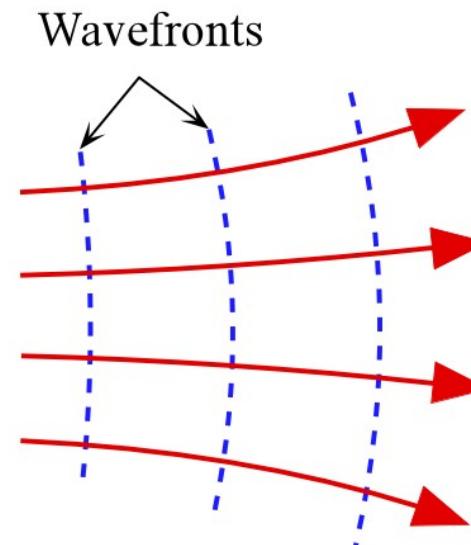
A perfect plane wave

(a)



A perfect spherical wave

(b)



A divergent beam

(c)

## Examples of possible EM waves

**Optical divergence** refers to the angular separation of wave vectors on a given wavefront.

# Refractive Index

When an EM wave is traveling in a dielectric medium, the oscillating electric field **polarizes** the molecules of the medium at the frequency of the wave

The **stronger** is the interaction between the field and the dipoles, the **slower** is the propagation of the wave

# Refractive Index

**TABLE 1.1** Low-frequency (LF) relative permittivity  $\epsilon_r(\text{LF})$  and refractive index  $n$

Material	$\epsilon_r(\text{LF})$	$[\epsilon_r(\text{LF})]^{1/2}$	$n$ (at $\lambda$ )	Comment
Si	11.9	3.44	3.45 (at 2.15 $\mu\text{m}$ )	Electronic bond polarization up to optical frequencies
Diamond	5.7	2.39	2.41 (at 590 nm)	Electronic bond polarization up to UV light
GaAs	13.1	3.62	3.30 (at 5 $\mu\text{m}$ )	Ionic polarization contributes to $\epsilon_r(\text{LF})$
$\text{SiO}_2$	3.84	2.00	1.46 (at 600 nm)	Ionic polarization contributes to $\epsilon_r(\text{LF})$
Water	80	8.9	1.33 (at 600 nm)	Dipolar polarization contributes to $\epsilon_r(\text{LF})$ , which is large

## Maxwell's Wave Equation in an isotropic medium

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} - \frac{\partial^2 E}{\partial t^2} = \epsilon \epsilon_r \mu \frac{\partial^2 E}{\partial t^2} = 0$$

=

A plane wave is a solution of Maxwell's wave equation

$$E_x = E_o \cos(\omega t - kz + \phi_o)$$

The phase velocity of this plane wave in the medium is given by

$$V = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon_o \epsilon_r \mu_o}}$$

The phase velocity in vacuum is

$$C = \frac{\omega}{k_o} = \frac{1}{\sqrt{\epsilon_o \mu_o}}$$

## Phase Velocity and $\alpha_r$

The relative permittivity  $\epsilon_r$  measures the ease with which the medium becomes polarized and hence it indicates the extent of interaction between the field and the induced dipoles.

For an EM wave traveling in a nonmagnetic dielectric medium of relative permittivity  $\epsilon_r$ , the phase velocity  $V$  is given by

$$V = \frac{1}{\sqrt{\epsilon_r \epsilon_0 \mu_0}}$$

# Refractive Index $n$

Phase Velocity and  $\epsilon_r$

$$V = \frac{1}{\sqrt{\epsilon_r \epsilon_o \mu_o}}$$

Refractive index  $n$   
definition

$$n = \frac{c}{V} = \sqrt{\epsilon_r}$$

# Optical frequencies

Typical frequencies that are involved in optoelectronic devices are in the infrared (including far infrared), visible, and UV, and we generically refer to these frequencies as **optical frequencies**

Somewhat arbitrary range:

**Roughly  $10^{12}$  Hz to  $10^{16}$  Hz**

# Low frequency (LF) relative permittivity $\epsilon_r$ (LF) and refractive index $n$ .

**TABLE 1.1** Low-frequency (LF) relative permittivity  $\epsilon_r$ (LF) and refractive index  $n$

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# Refractive Index and Propagation Constant

$k_o$  Free-space propagation constant (wave vector)

$k_o = 2\pi/\lambda_o$

$\lambda_o$  Free-space wavelength

$k$  Propagation constant (wave vector) in the medium

$\lambda$  Wavelength in the medium

$$n = \frac{k}{k_o}$$

In noncrystalline materials such as glasses and liquids, the material structure is the same in all directions and  $n$  does not depend on the direction. The refractive index is then **isotropic**

# Refractive Index and Wavelength

It is customary to drop the subscript  $o$  on  $k$  and  $\lambda$

$$k_{\text{medium}} = nk$$

medium

In free space

$$\lambda_{\text{medium}} = \lambda / n$$

medium

# Refractive Index and Isotropy

Crystals, in general, have nonisotropic, or **anisotropic**, properties

Typically **noncrystalline solids** such as glasses and liquids, and cubic crystals are **optically isotropic**; they possess only one refractive index for all directions

# ***n* depends on the wavelength $\lambda$**

Dispersion relation:  $n = n(\lambda)$

The simplest electronic polarization gives

$$n^2 = 1 + \left( \frac{N_{\text{at}} Z e^2}{\epsilon_0 m_e} \right) \left( \frac{\lambda_o}{2\pi c} \right)^2 \frac{\lambda^2}{\lambda^2 - \lambda_o^2}$$

$N_{\text{at}}$  = Number of atoms per unit volume  
 $Z$  = Number of electrons in the atom (atomic number)

$\lambda_o$  = A “resonant frequency”

## Sellmeier Equation

$$n^2 = 1 + \frac{A_1 \lambda^2}{\lambda^2 - \lambda_1^2} + \frac{A_2 \lambda^2}{\lambda^2 - \lambda_2^2} + \frac{A_3 \lambda^2}{\lambda^2 - \lambda_3^2}$$

***n* depends on the wavelength  $\lambda$**

Cauchy dispersion relation

$$n = n(v)$$

$$n = n_{-2}(hv)^{-2} + n_0 + n_2(hv)^2 + n_4(hv)^4$$

# $n$ depends on the wavelength $\lambda$

**TABLE 1.2** Sellmeier and Cauchy coefficients

Sellmeier	$A_1$	$A_2$	$A_3$	$\lambda_1$ (μm)	$\lambda_2$ (μm)	$\lambda_3$ (μm)
SiO <sub>2</sub> (fused silica)	0.696749	0.408218	0.890815	0.0690660	0.115662	9.900559
86.5%SiO <sub>2</sub> -13.5%GeO <sub>2</sub>	0.711040	0.451885	0.704048	0.0642700	0.129408	9.425478
GeO <sub>2</sub>	0.80686642	0.71815848	0.85416831	0.068972606	0.15396605	11.841931
Sapphire	1.023798	1.058264	5.280792	0.0614482	0.110700	17.92656
Diamond	0.3306	4.3356	–	0.1750	0.1060	–

Cauchy	Range of $h\nu$ (eV)	$n_{-2}$ (eV <sup>2</sup> )	$n_0$	$n_2$ (eV <sup>-2</sup> )	$n_4$ (eV <sup>-4</sup> )
Diamond	0.05–5.47	$-1.07 \times 10^{-5}$	2.378	$8.01 \times 10^{-3}$	$1.04 \times 10^{-4}$
Silicon	0.002–1.08	$-2.04 \times 10^{-8}$	3.4189	$8.15 \times 10^{-2}$	$1.25 \times 10^{-2}$
Germanium	0.002–0.75	$-1.0 \times 10^{-8}$	4.003	$2.2 \times 10^{-1}$	$1.4 \times 10^{-1}$

Source: Sellmeier coefficients combined from various sources. Cauchy coefficients from D. Y. Smith *et al.*, *J. Phys. CM*, 13, 3883, 2001.

## EXAMPLE 1.2.1 Sellmeier equation and diamond

Using the Sellmeier coefficients for diamond in Table 1.2, calculate its refractive index at 610 nm (red light) and compare with the experimental quoted value of 2.415 to three decimal places.

### Solution

The Sellmeier dispersion relation for diamond is

$$n^2 = 1 + \frac{0.3306\lambda^2}{\lambda^2 - 175 \text{ nm}^2} + \frac{4.3356\lambda^2}{\lambda^2 - 106 \text{ nm}^2}$$

$$n^2 = 1 + \frac{0.3306(610 \text{ nm})^2}{(610 \text{ nm})^2 - (175 \text{ nm})^2} + \frac{4.3356(610 \text{ nm})^2}{(610 \text{ nm})^2 - (106 \text{ nm})^2} = 5.8308$$

So that

$$n = 2.4147$$

which is 2.415 to three decimal places and matches the experimental value.

## EXAMPLE 1.2.2 Cauchy equation and diamond

Using the Cauchy coefficients for diamond in Table 1.2, calculate the refractive index at 610 nm.

### Solution

At  $\lambda = 610$  nm, the photon energy is

$$h\nu = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34} \text{ J s})(2.998 \times 10^8 \text{ m s}^{-1})}{(610 \times 10^{-9} \text{ m})} \times \frac{1}{1.602 \times 10^{-19} \text{ eV}^{-1}} = 2.0325 \text{ eV}$$

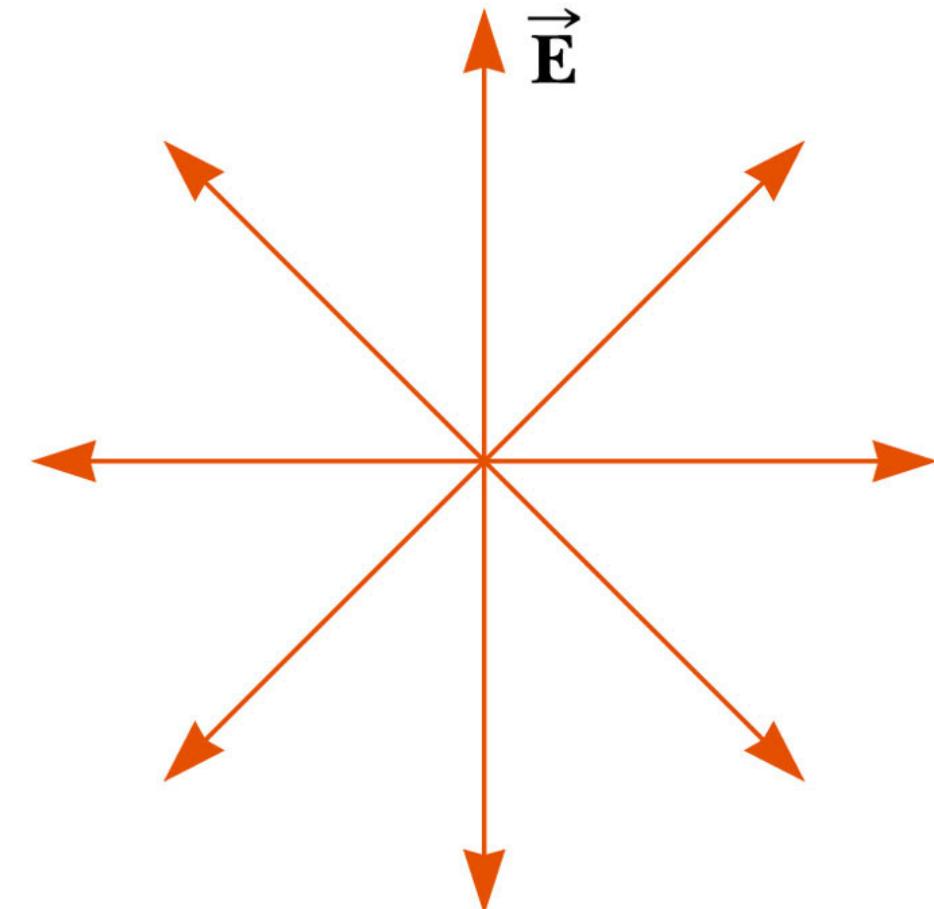
Using the Cauchy dispersion relation for diamond with coefficients from Table 1.2,

$$\begin{aligned} n &= n_{-2}(h\nu)^{-2} + n_0 + n_2(h\nu)^2 + n_4(h\nu)^4 \\ &= (-1.07 \times 10^{-5})(2.0325)^{-2} + 2.378 + (8.01 \times 10^{-3})(2.0325)^2 \\ &\quad + (1.04 \times 10^{-4})(2.0325)^4 \\ &= 2.4140 \end{aligned}$$

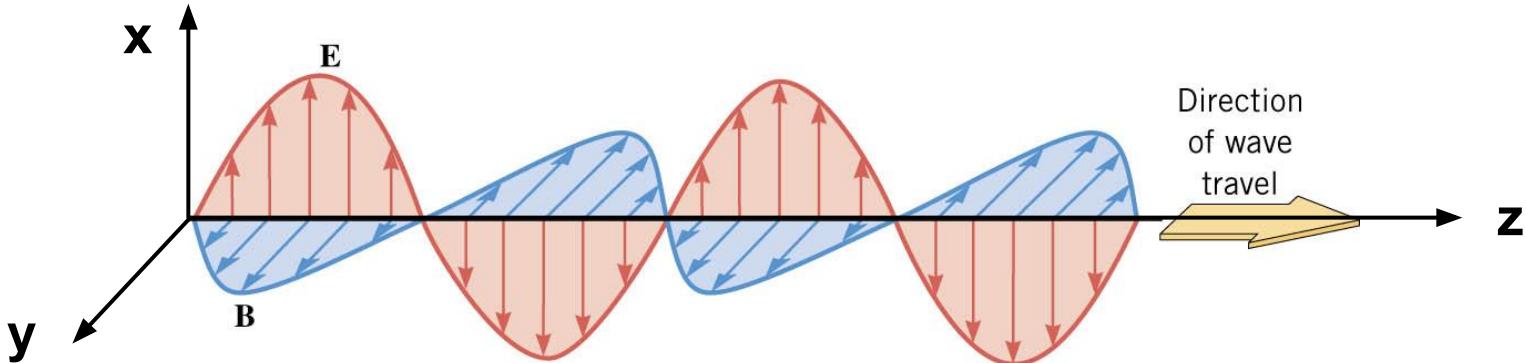
which is slightly different than the value calculated in Example 1.2.1; one reason for the discrepancy is due to the Cauchy coefficients quoted in Table 1.2 being applicable over a wider wavelength range at the expense of some accuracy. Although both dispersion relations have four parameters,  $A_1$ ,  $A_2$ ,  $\lambda_1$ ,  $\lambda_2$  for Sellmeier and  $n_{-2}$ ,  $n_0$ ,  $n_2$ ,  $n_4$  for Cauchy, the functional forms are different.

# Polarization of Light Waves

- Each atom produces a wave with its own orientation of  $\vec{E}$
- All directions of the electric field vector are equally possible and lie in a plane perpendicular to the direction of propagation
- This is an unpolarized wave



# Polarization



- Light is an electromagnetic wave with the electric and magnetic field having very specific orientations
- A light wave in which the electric field always vibrates along one direction is called a *linearly polarized* wave
- The direction of polarization is the axis along which the electric field vibrates
- In the diagram above, the wave polarization is x

# Polarization

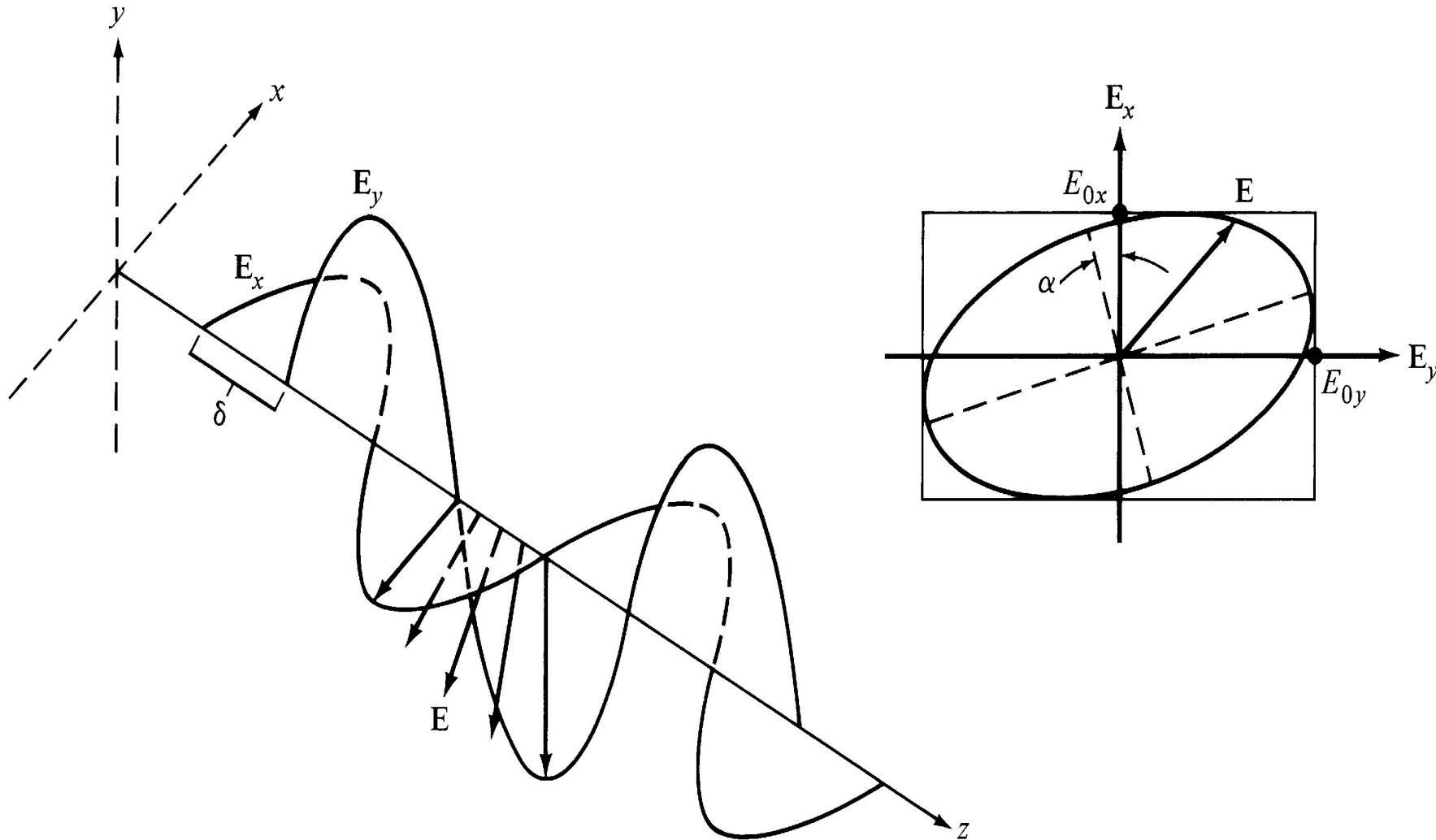
- the direction in which the electric field vibrates is the direction of polarization
- with *polarized* light the electric field always vibrates in one direction
- ordinary light is *unpolarized* so that the electric field is randomly oriented about the direction of travel

- a transverse wave is linearly polarized with its vibrations always along one direction
- a linearly polarized wave can pass through a slit that is parallel to the vibration direction
- the wave cannot pass through a slit that is perpendicular to the vibration direction

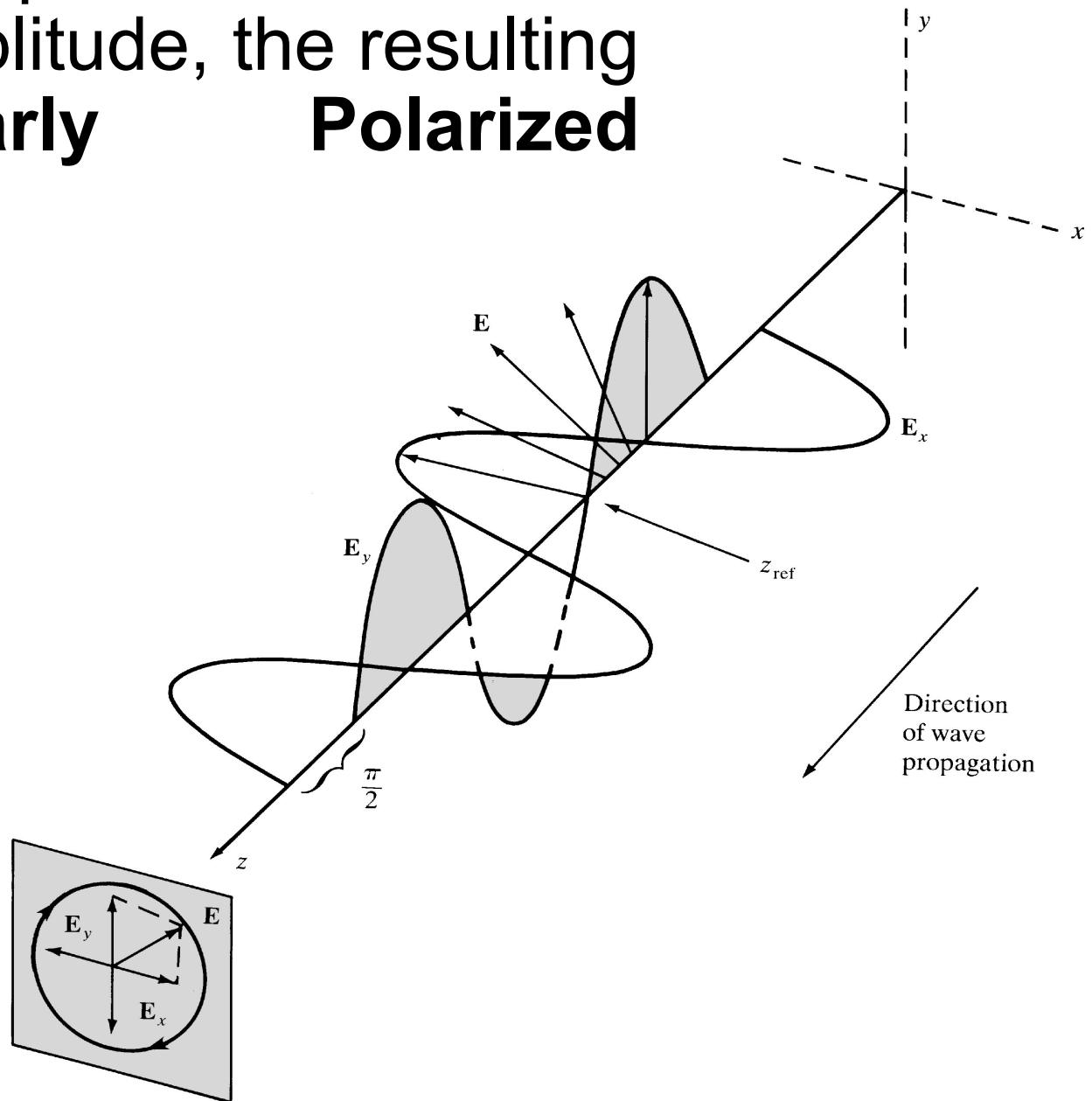
# Polarization

- **Polarization** of a plane wave is the orientation of the oscillations of the  $\underline{E}$  field; perpendicular to the direction of propagation
- For a simple harmonic wave, the electric vector in orthogonal directions may have:
  - Different amplitude
  - Different phase
- The resulting wave is
  - Linearly, elliptically or circularly polarized

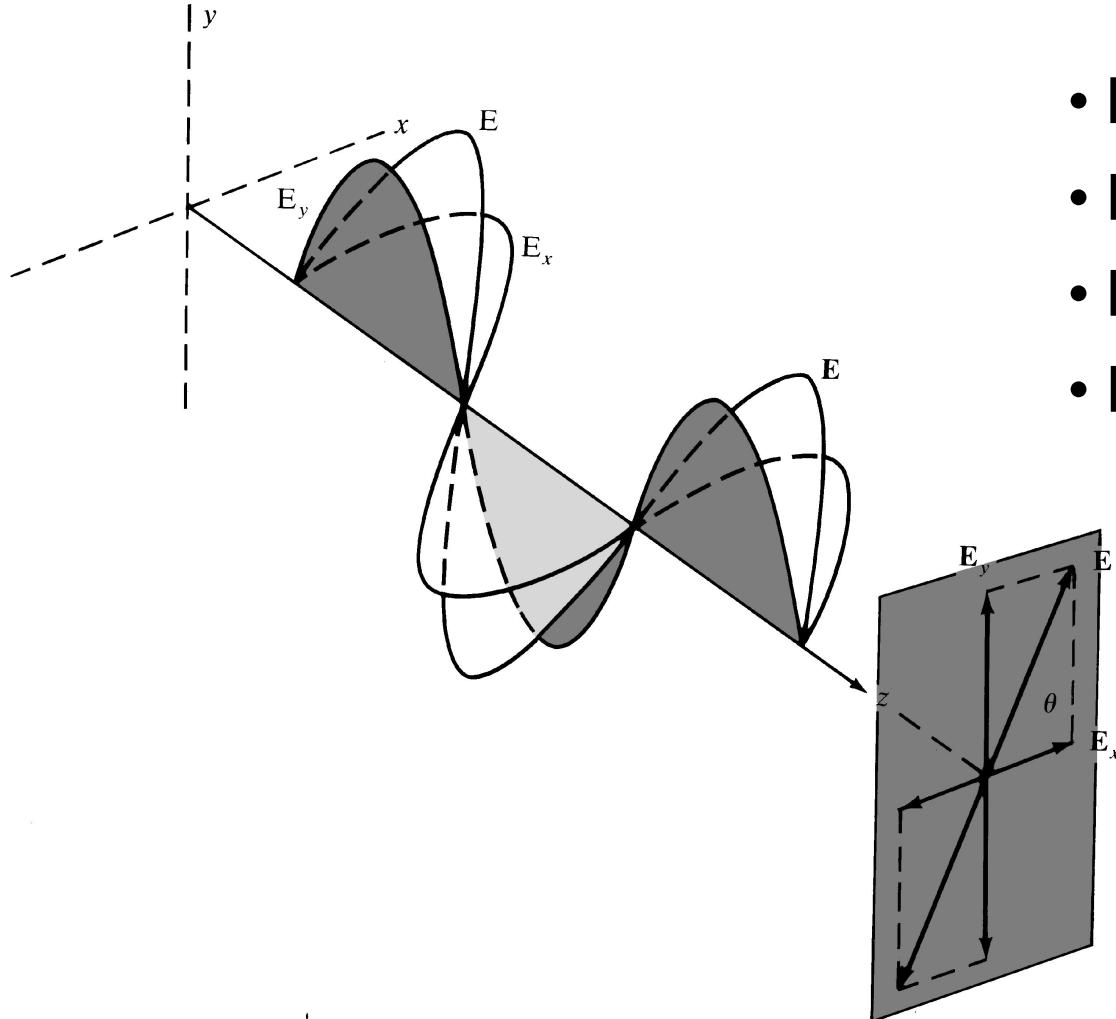
When the orthogonal components have different phase and amplitude, resulting wave is **Elliptically Polarized** (General Case)



When the orthogonal components have  $90^\circ$  phase shift and equal amplitude, the resulting wave is **Circularly Polarized**  
(Special Case)



When the orthogonal components have zero phase shift,  
resulting wave is **Linearly Polarized**



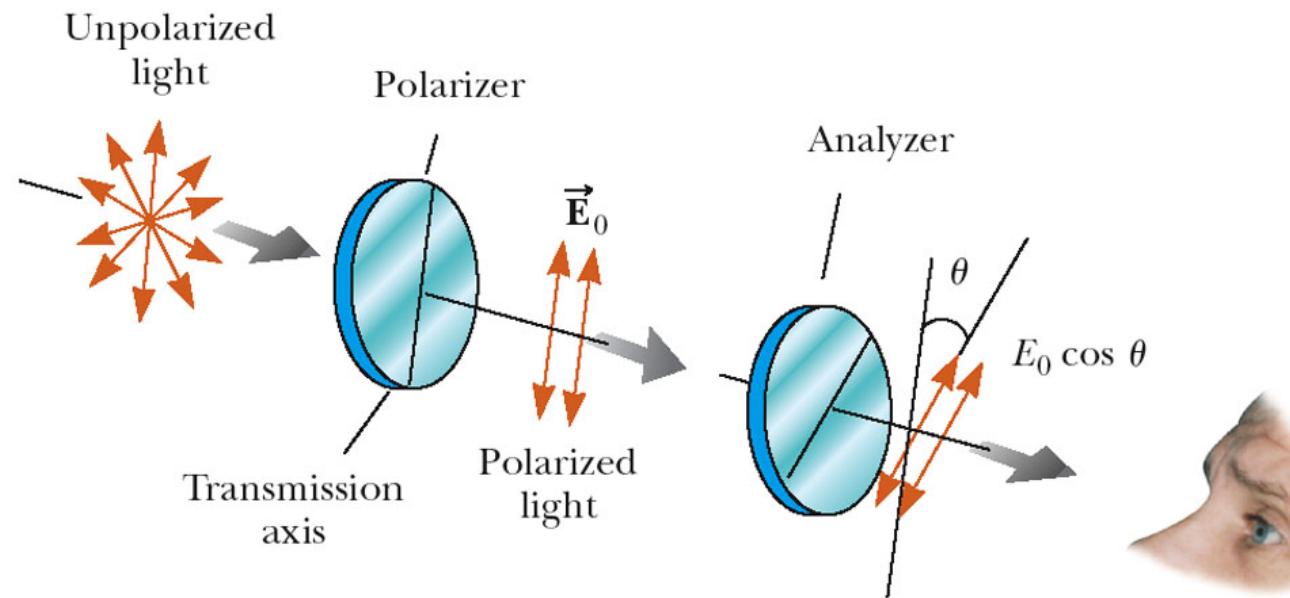
- More useful
- Emitted by lasers
- Polarization control is possible
- Horizontal and vertical polarizations

# Polarization of Light, cont

- A wave is said to be *linearly polarized* if the resultant electric field vibrates in the same direction at all times at a particular point
- Polarization can be obtained from an unpolarized beam by
  - **selective absorption**
  - **reflection**



# Polarization by Selective Absorption



- The most common technique for polarizing light
- Uses a material that transmits waves whose electric field vectors in the plane are parallel to a certain direction and absorbs waves whose electric field vectors are perpendicular to that direction

## Selective Absorption, cont

- E. H. Land discovered a material that polarizes light through selective absorption
  - He called the material **Polaroid**
  - The molecules readily absorb light whose electric field vector is perpendicular to their lengths and transmit light whose electric field vector is parallel to their lengths

## Selective Absorption, final

- The intensity of the polarized beam transmitted through the second polarizing sheet (the analyzer) varies as

- $I = I_o \cos^2 \theta$ 
  - $I_o$  is the intensity of the polarized wave incident on the analyzer
  - This is known as **Malus' Law** and applies to any two polarizing materials whose transmission axes are at an angle of  $\theta$  to each other

# Polarization by Reflection

- When an unpolarized light beam is reflected from a surface, the reflected light is
  - Completely polarized
  - Partially polarized
  - Unpolarized
- It depends on the angle of incidence
  - If the angle is  $0^\circ$  or  $90^\circ$ , the reflected beam is unpolarized
  - For angles between this, there is some degree of polarization
  - For one particular angle, the beam is completely polarized

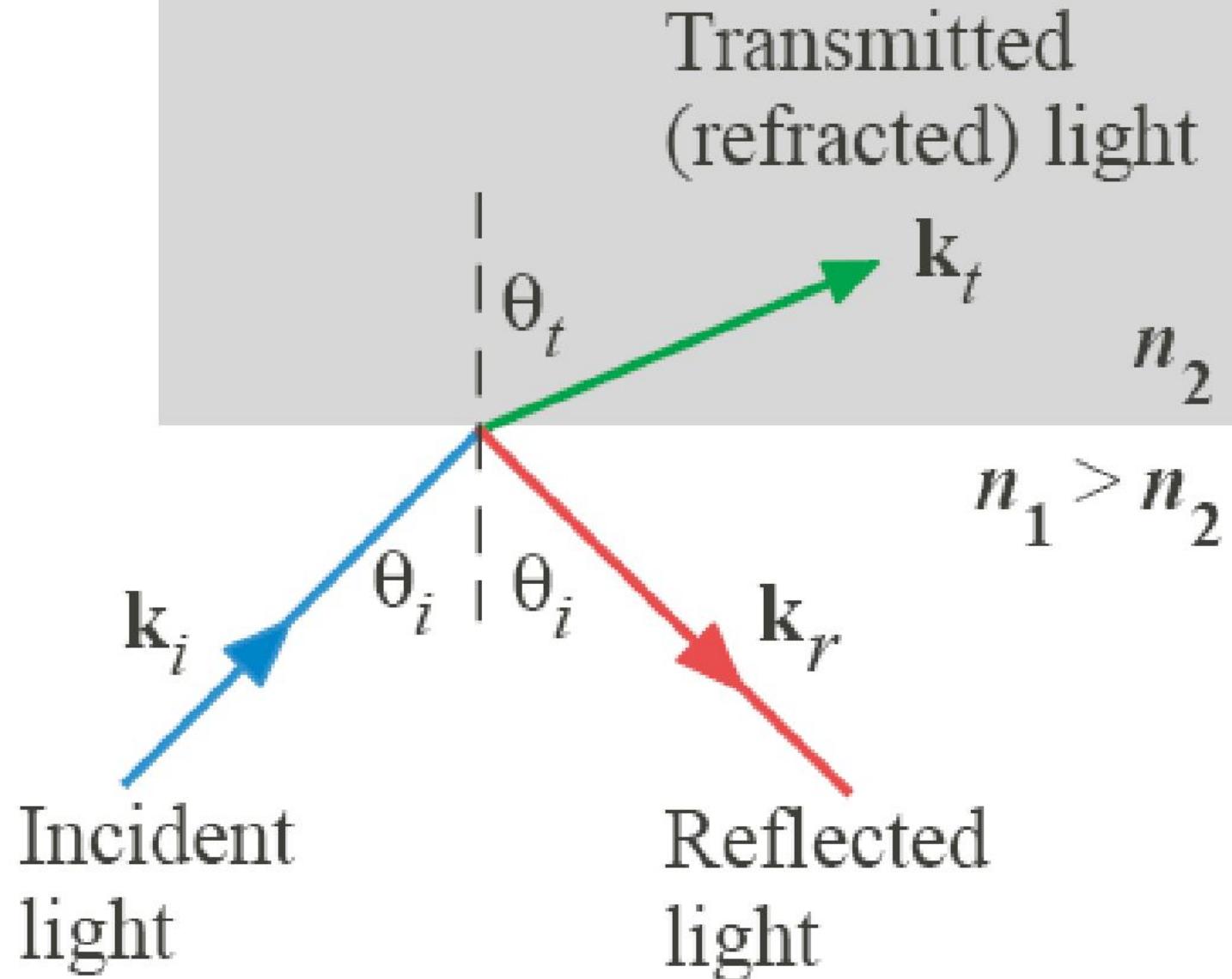
# Polarization by Reflection, cont

- The angle of incidence for which the reflected beam is completely polarized is called the *polarizing angle*,  $\theta_p$
- Brewster's Law relates the polarizing angle to the index of refraction for the material

$$n = \frac{\sin \theta_p}{\cos \theta_p} = \tan \theta_p$$

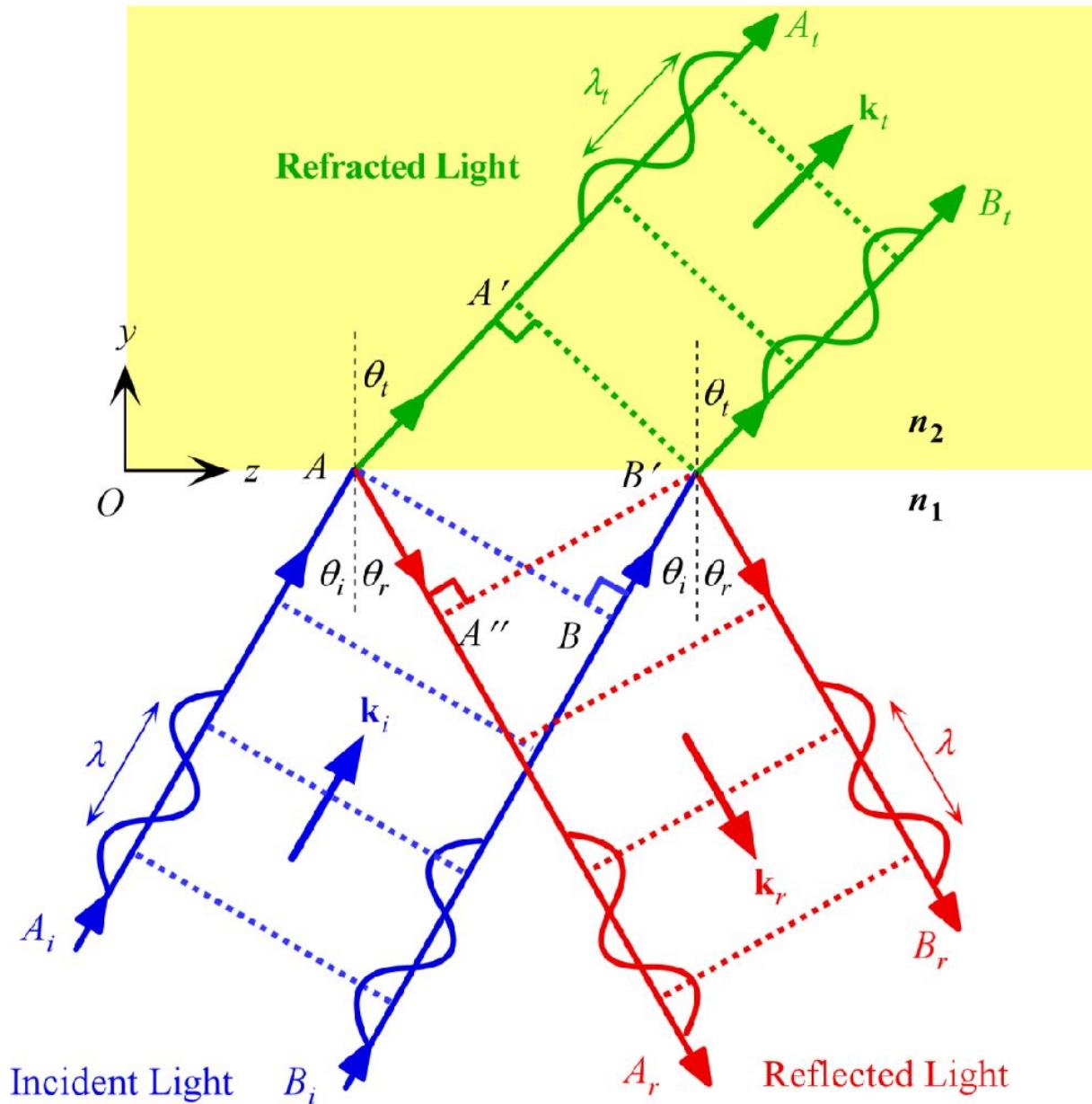
- $\theta_p$  may also be called Brewster's Angle

# Snell's Law



$$\frac{\sin \theta_i}{\sin \theta_t} = \frac{n_2}{n_1}$$

# Derivation of Snell's Law



A light wave traveling in a medium with a greater refractive index ( $n_1 > n_2$ ) suffers reflection and refraction at the boundary. ( $\lambda_t$  is slightly longer than  $\lambda$ .)

## Snell's Law

We can use constructive interference to show that there can only be one reflected wave which occurs at an angle equal to the incidence angle. The two waves along  $A_i$  and  $B_i$  are in phase.

When these waves are reflected to become waves  $A_r$  and  $B_r$  then they must still be in phase, otherwise they will interfere destructively and destroy each other. The only way the two waves can stay in phase is if  $\theta_r = \theta_i$ . All other angles lead to the waves  $A_r$  and  $B_r$  being out of phase and interfering destructively.

- Unless the two waves at  $A'$  and  $B'$  still have the same phase, there will be no transmitted wave.  $A'$  and  $B'$  points on the front are only in phase for one particular transmitted angle,  $\theta_t$ .
- It takes time  $t$  for the phase at  $B$  on wave  $B_i$  to reach  $B'$

$$\mathbf{B}B' = v_1 t = ct/n_1$$

- During this time  $t$ , the phase  $A$  has progressed to  $A'$

$$\mathbf{A}A' = v_2 t = ct/n_2$$

- $A'$  and  $B'$  belong to the same front just like  $A$  and  $B$  so that  $AB$  is perpendicular to  $\mathbf{k}_i$  in medium 1 and  $A'B'$  is perpendicular to  $\mathbf{k}_t$  in medium 2. From geometrical considerations,

$$\mathbf{A}B' = \mathbf{B}B'/\sin \theta_i \text{ and } \mathbf{A}B' = AA'/\sin \theta_t$$

$$AB' = \frac{v_1 t}{\sin \theta_i} = \frac{v_2 t}{\sin \theta_t}$$

$$\frac{\sin \theta_i}{\sin \theta_t} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

$$n \sin \theta = \text{constant}$$

This is **Snell's law** which relates the angles of incidence and refraction to the refractive indices of the media.

# Critical Angle

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

When  $n_1 > n_2$  then obviously the transmitted angle is greater than the incidence angle as apparent in the figure. When the refraction angle  $\theta_t$  reaches  $90^\circ$ , the incidence angle is called the **critical angle**  $\theta_c$  which is given by

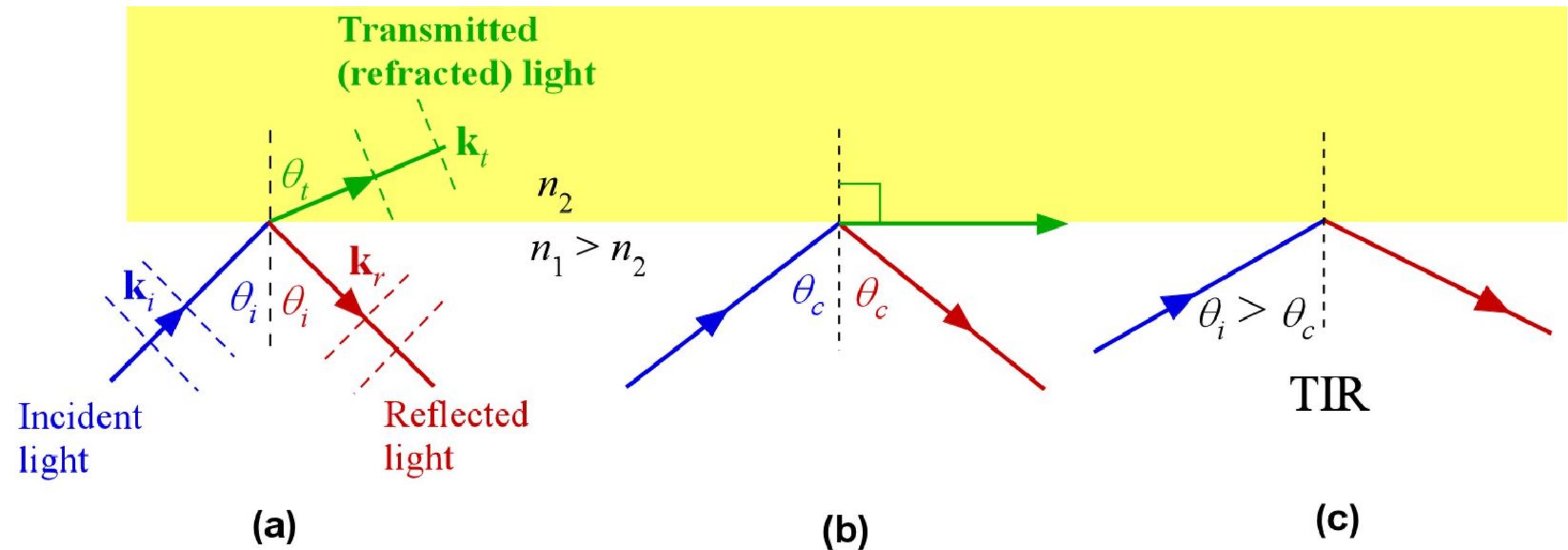
$$\sin \theta_c = \frac{n_2}{n_1}$$

# Total Internal Reflection (TIR)

$$\sin \theta_c = \frac{n_2}{n_1}$$

- When the incidence angle  $\theta_i$  exceeds  $\theta_c$  then there is no transmitted wave but only a reflected wave.
- The latter phenomenon is called **total internal reflection** (TIR).
- TIR phenomenon that leads to the propagation of waves in a dielectric medium surrounded by a medium of smaller refractive index as in **optical waveguides**, e.g. **optical fibers**.
  
- Although Snell's law for  $\theta_i > \theta_c$  shows that  $\sin \theta_t > 1$  and hence  $\theta_t$  is an "imaginary" angle of refraction, there is however an attenuated wave called the **evanescent wave**.

# Total Internal Reflection (TIR)

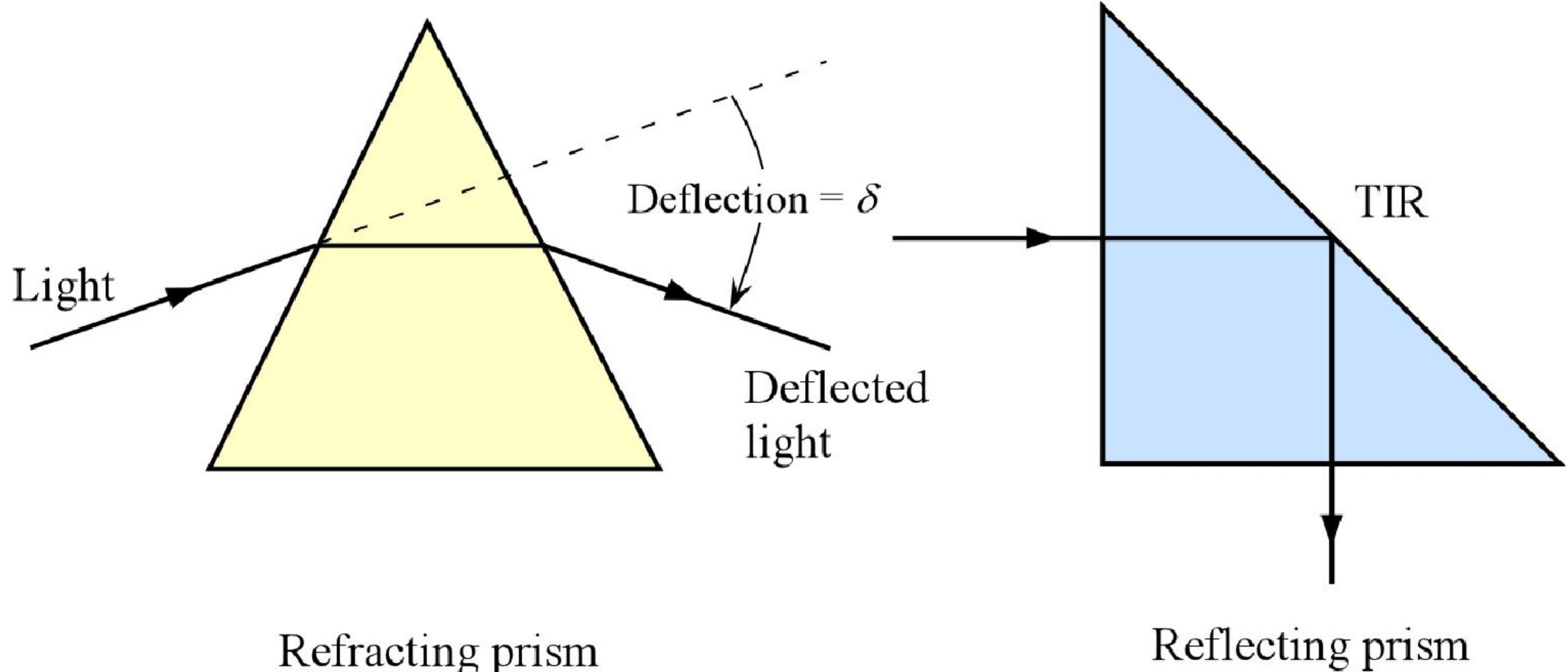


Light wave traveling in a more dense medium strikes a less dense medium.

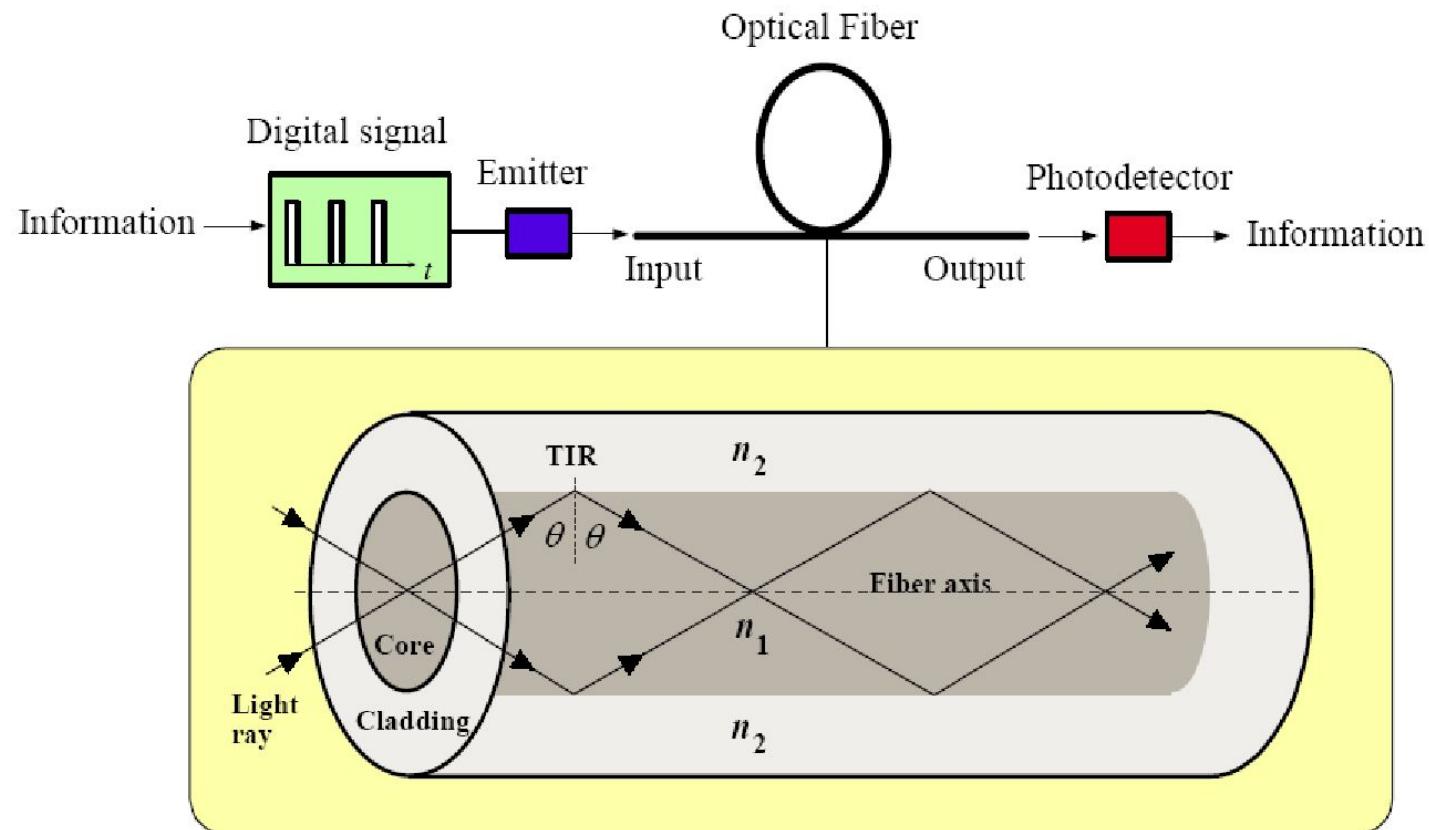
Depending on the incidence angle with respect to  $\theta_c$ , which is determined by the ratio of the refractive indices, the wave may be transmitted (refracted) or reflected.

(a)  $\theta_i < \theta_c$  (b)  $\theta_i = \theta_c$  (c)  $\theta_i > \theta_c$  and total internal reflection (TIR).

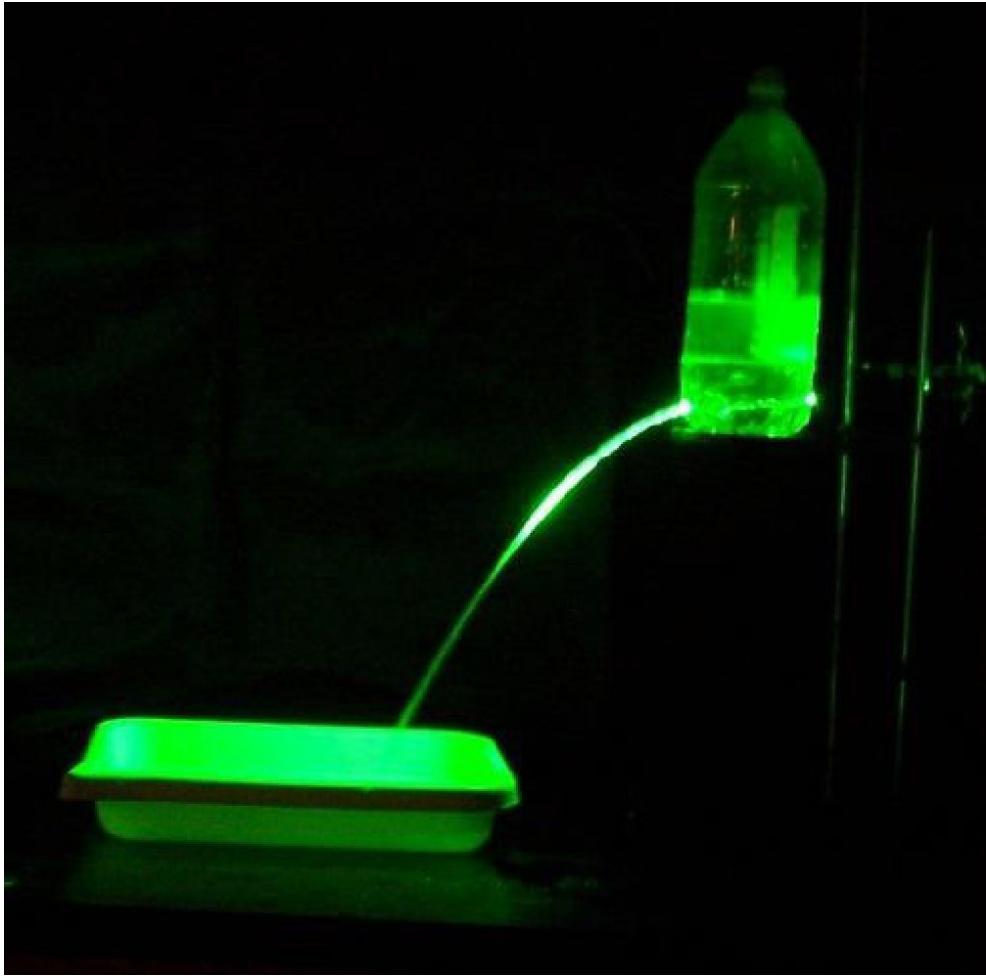
# Prisms



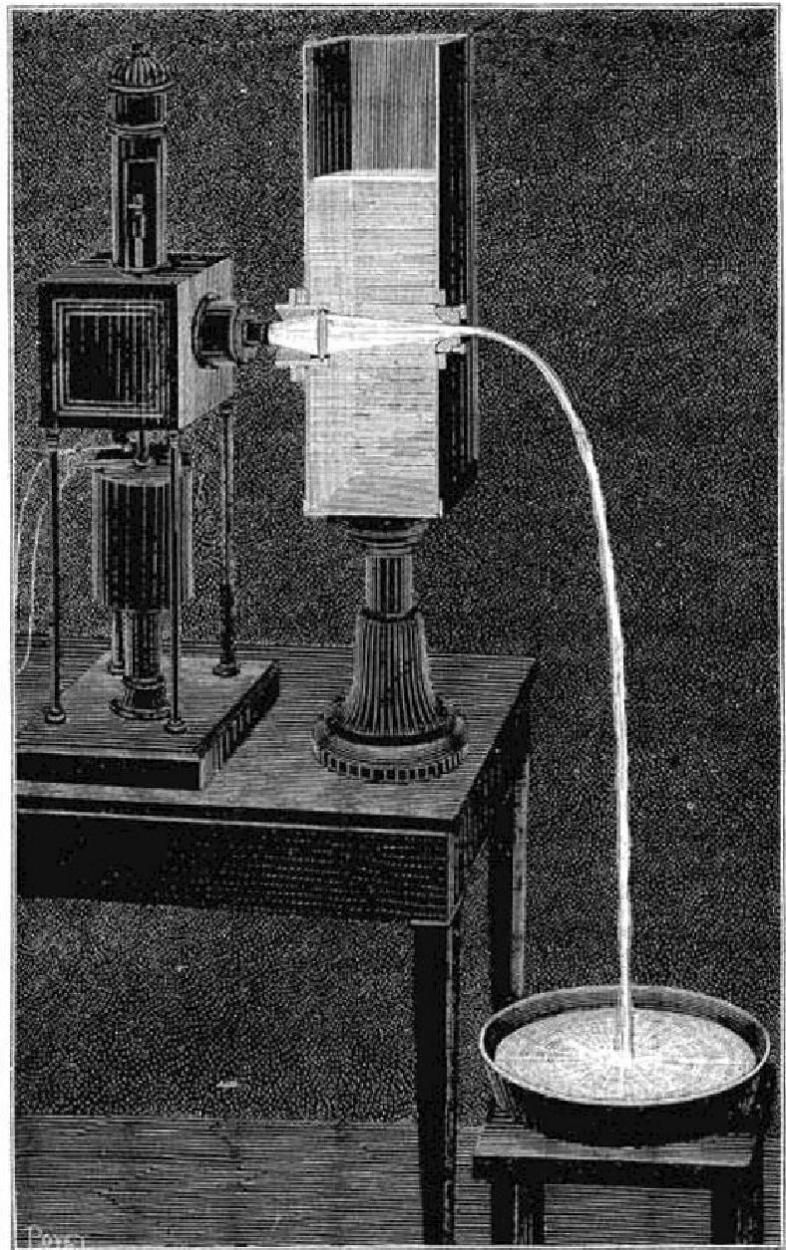
# Light travels by total internal reflection in optical fibers



An optical fiber link for transmitting digital information in communications. The fiber core has a higher refractive index so that the light travels along the fiber inside the fiber core by total internal reflection at the core-cladding interface.

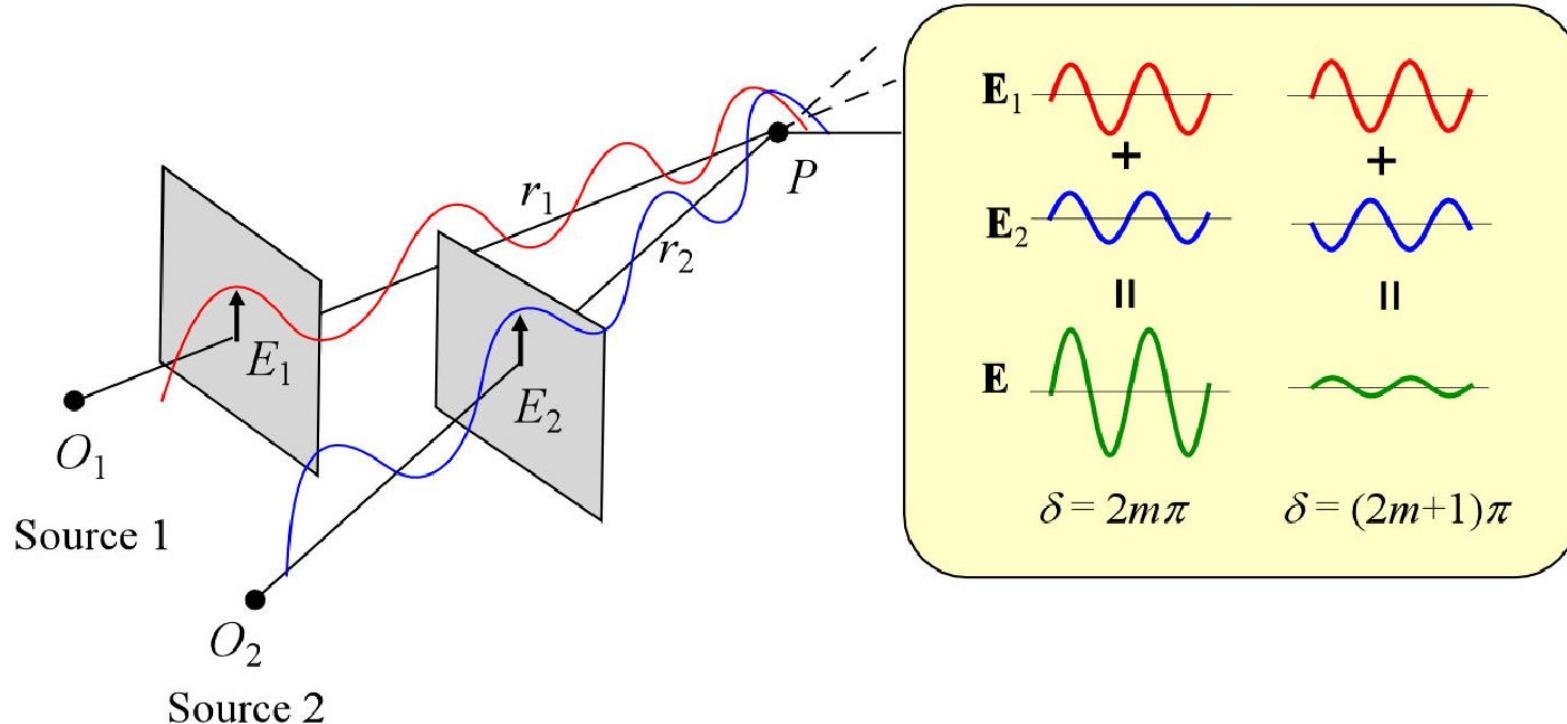


A small hole is made in a plastic bottle full of water to generate a water jet. When the hole is illuminated with a laser beam (from a green laser pointer), the light is guided by total internal reflections along the jet to the tray. The light guiding by a water jet was first demonstrated by Jean-Daniel Colladan, a Swiss scientist (Water with air bubbles was used to increase the visibility of light. Air bubbles scatter light.)



# **Superposition and Interference of waves**

# Interference

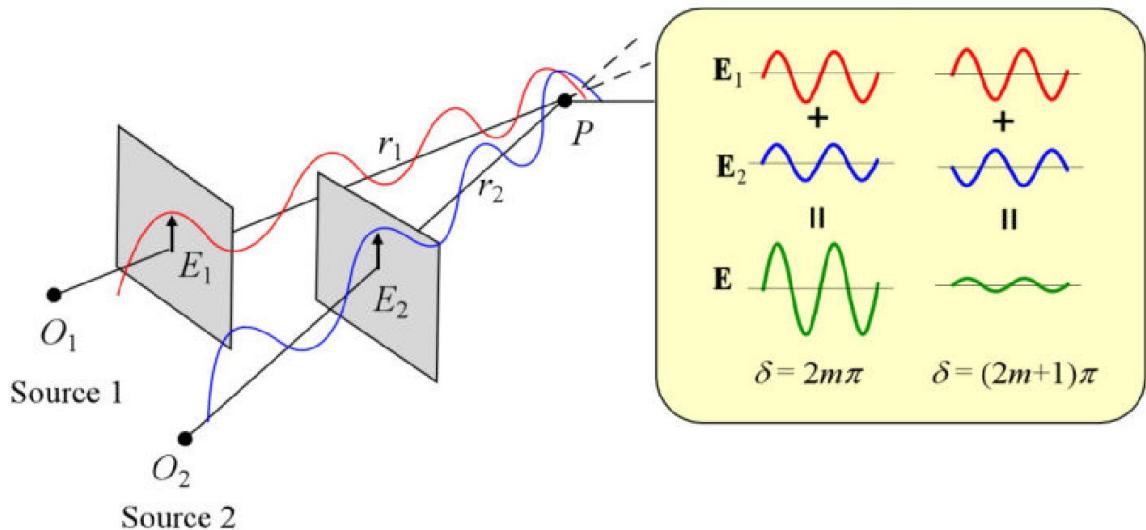


$$\mathbf{E}_1 = \mathbf{E}_{o1} \sin(\omega t - kr_1 - \varphi_1) \text{ and } \mathbf{E}_2 = \mathbf{E}_{o2} \sin(\omega t - kr_2 - \varphi_2)$$

Interference results in  $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$

$$\overline{\mathbf{E} \cdot \mathbf{E}} = \overline{(\mathbf{E}_1 + \mathbf{E}_2) \cdot (\mathbf{E}_1 + \mathbf{E}_2)} = \overline{\mathbf{E}_1^2} + \overline{\mathbf{E}_2^2} + 2\overline{\mathbf{E}_1 \mathbf{E}_2}$$

# Interference



Resultant intensity  $I$  is

$$I = I_1 + I_2 + 2(I_1 I_2)^{1/2} \cos \delta$$

$$\delta = k(r_2 - r_1) + (\varphi_2 - \varphi_1)$$

Phase difference due to optical path difference

**Constructive interference**

$$I_{\max} = I_1 + I_2 + 2(I_1 I_2)^{1/2}$$

If the interfering beams have equal irradiances, then

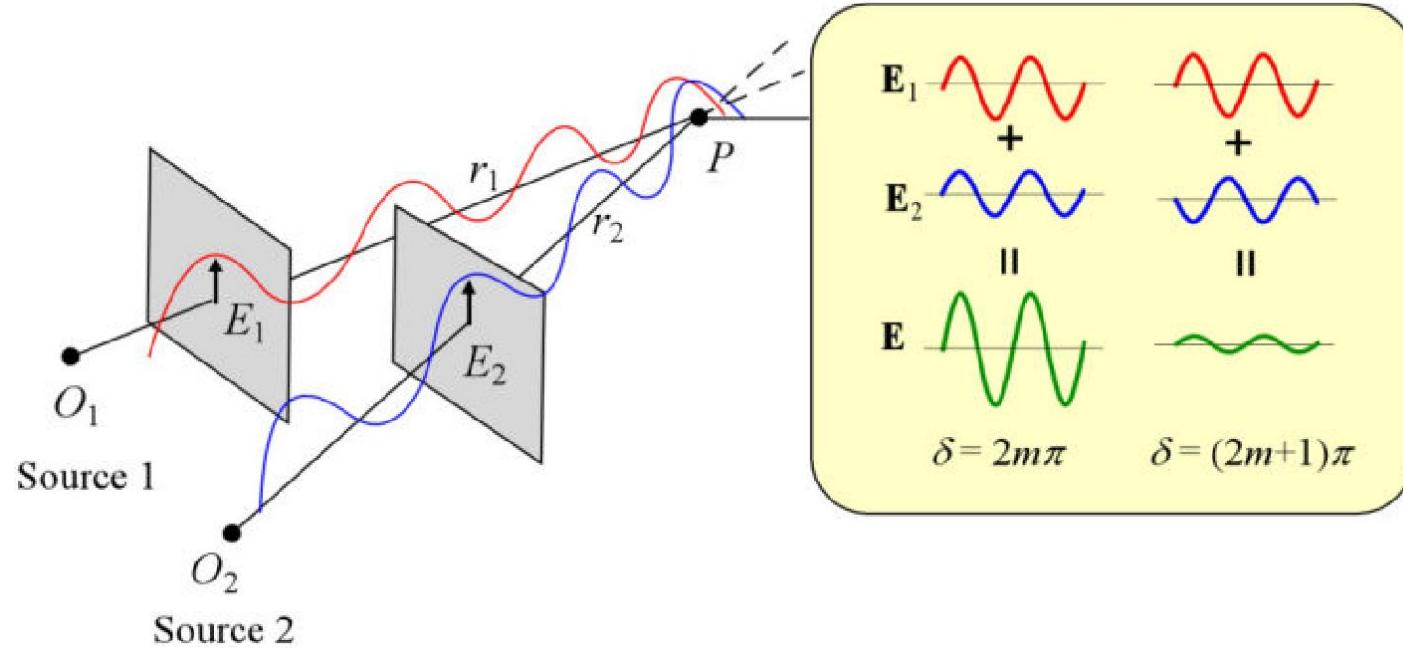
$$I_{\max} = 4I_1$$

**Destructive interference**

$$I_{\min} = I_1 + I_2 - 2(I_1 I_2)^{1/2}$$

$$I_{\min} = 0$$

# Interference between coherent waves



Resultant intensity  $I$  is

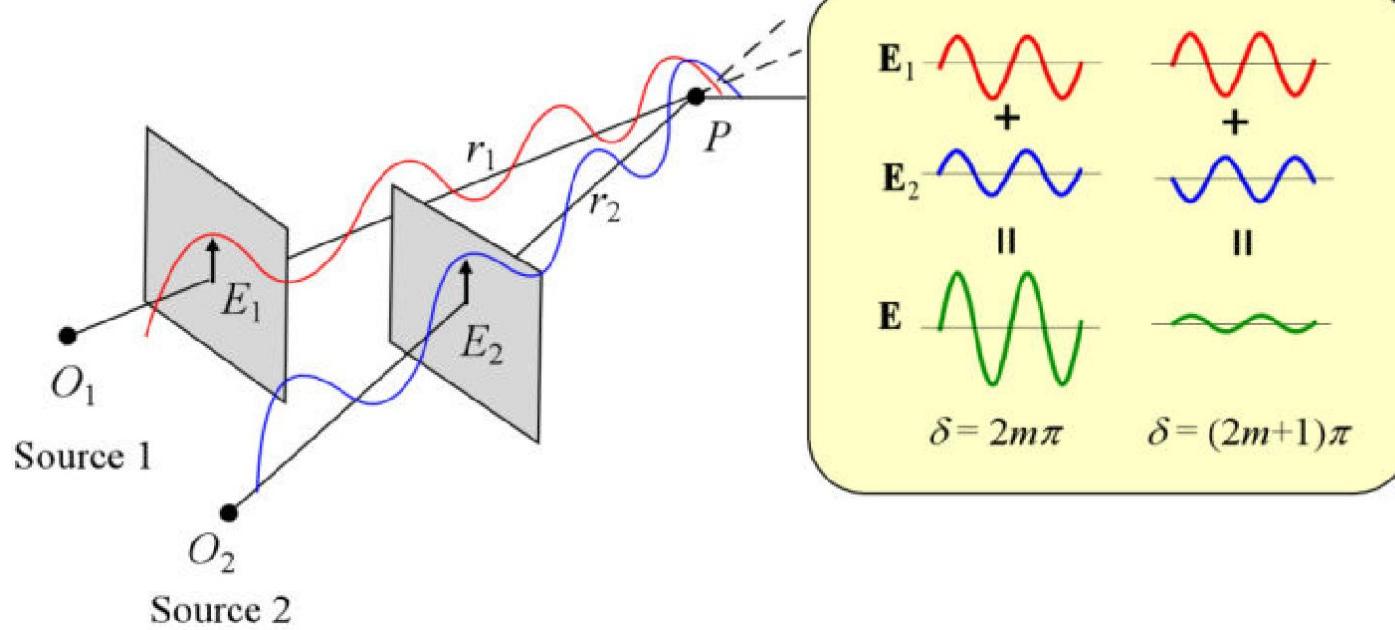
$$I = I_1 + I_2 + 2(I_1 I_2)^{1/2} \cos \delta$$

$$\delta = k(r_2 - r_1) + (\varphi_2 - \varphi_1)$$

## Interference between incoherent waves

$$I = I_1 + \\ I_2$$

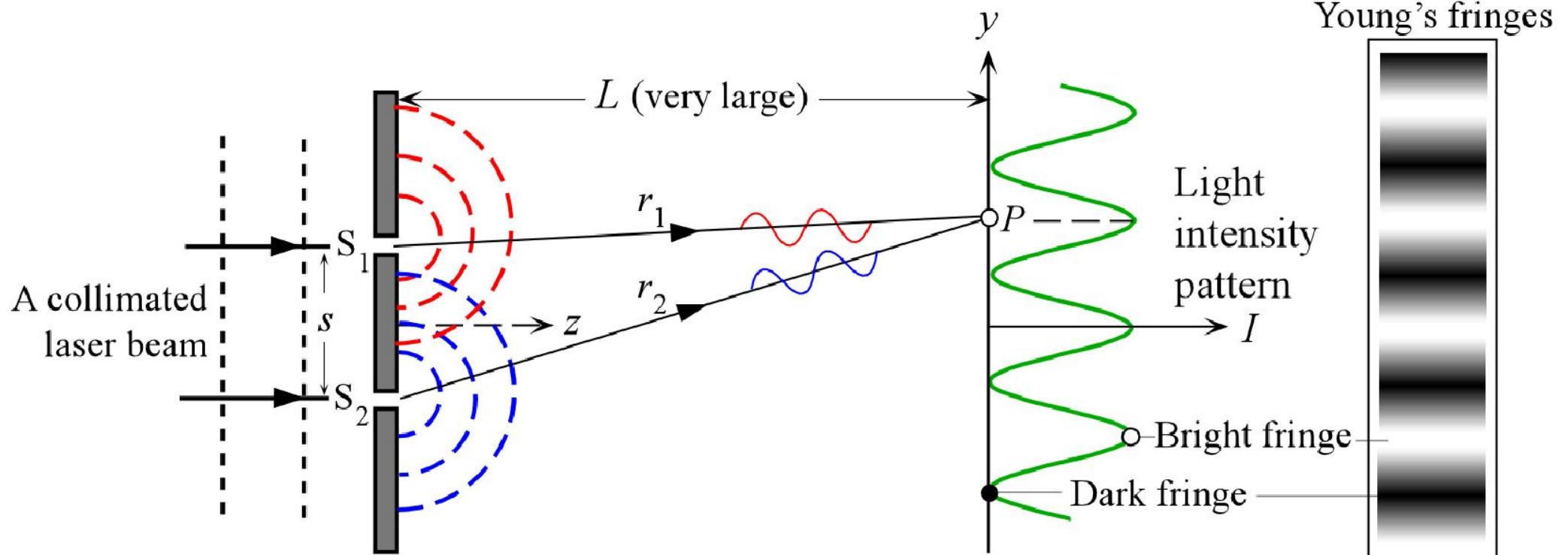
# Interference between coherent waves



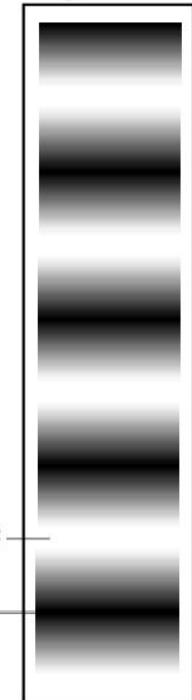
Resultant intensity  $I$  is

$$I = I_1 + I_2 + 2(I_1 I_2)^{1/2} \cos \delta$$

$$\delta = k(r_2 - r_1) + (\phi_2 - \phi_1)$$



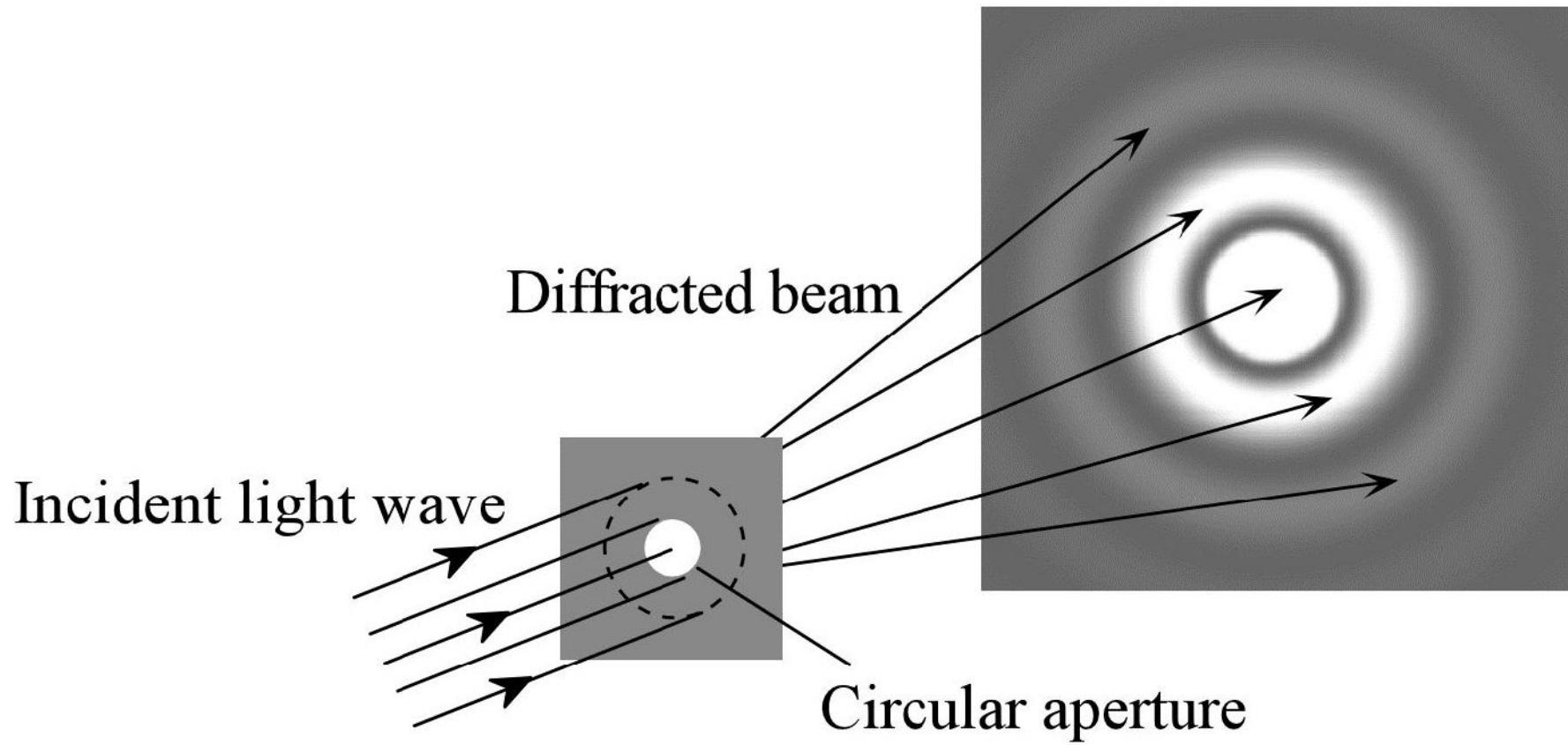
Young's fringes



# **Diffraction Principles- Fraunhofer diffraction, Diffraction Grating**

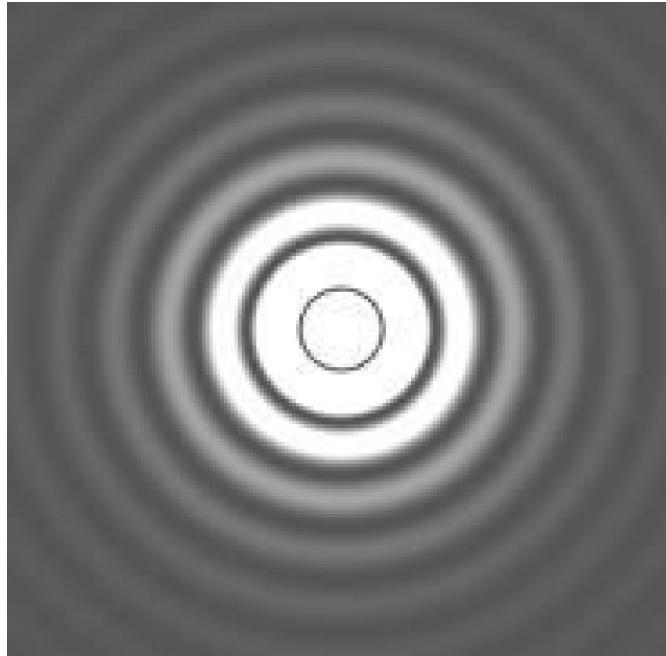
# Diffraction

Light intensity pattern



A light beam incident on a small circular aperture becomes diffracted and its light intensity pattern after passing through the aperture is a diffraction pattern with circular bright rings (called Airy rings). If the screen is far away from the aperture, this would be a Fraunhofer diffraction pattern.

# Diffraction from a Circular Aperture



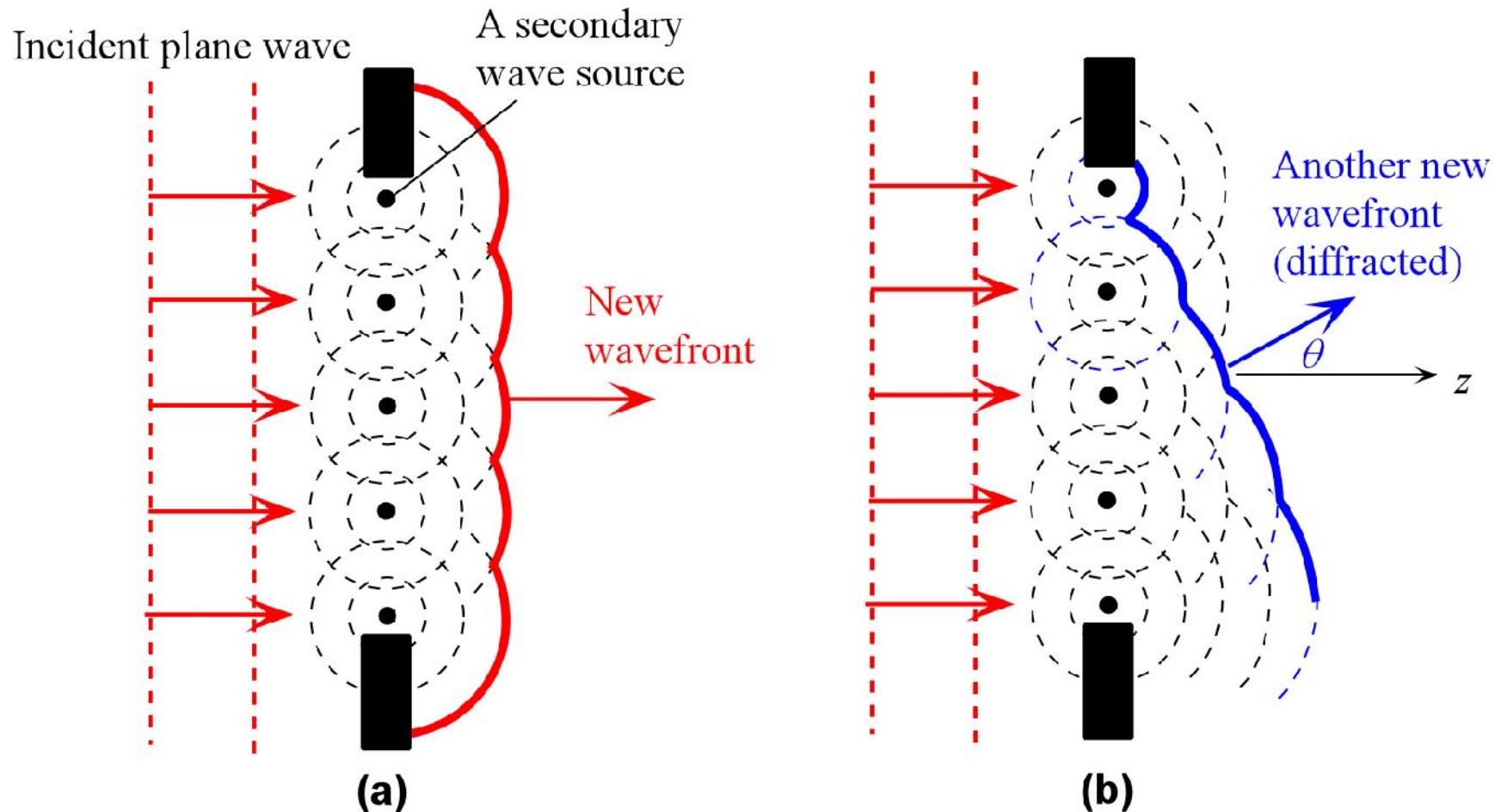
A light beam incident on a small circular aperture becomes diffracted and its light intensity pattern after passing through the aperture is a diffraction pattern with circular bright rings (called Airy rings). If the screen is far away from the aperture, this would be a Fraunhofer diffraction pattern.

# Diffraction

## Huygens-Fresnel principle

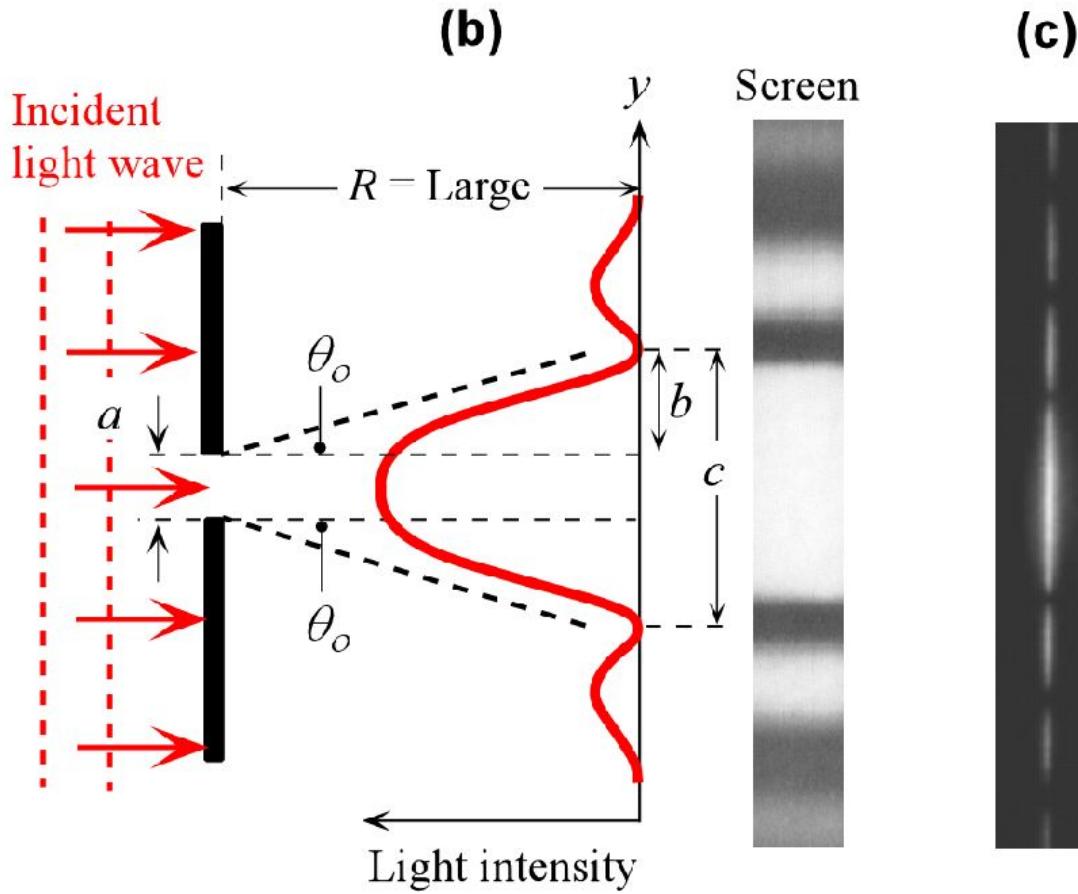
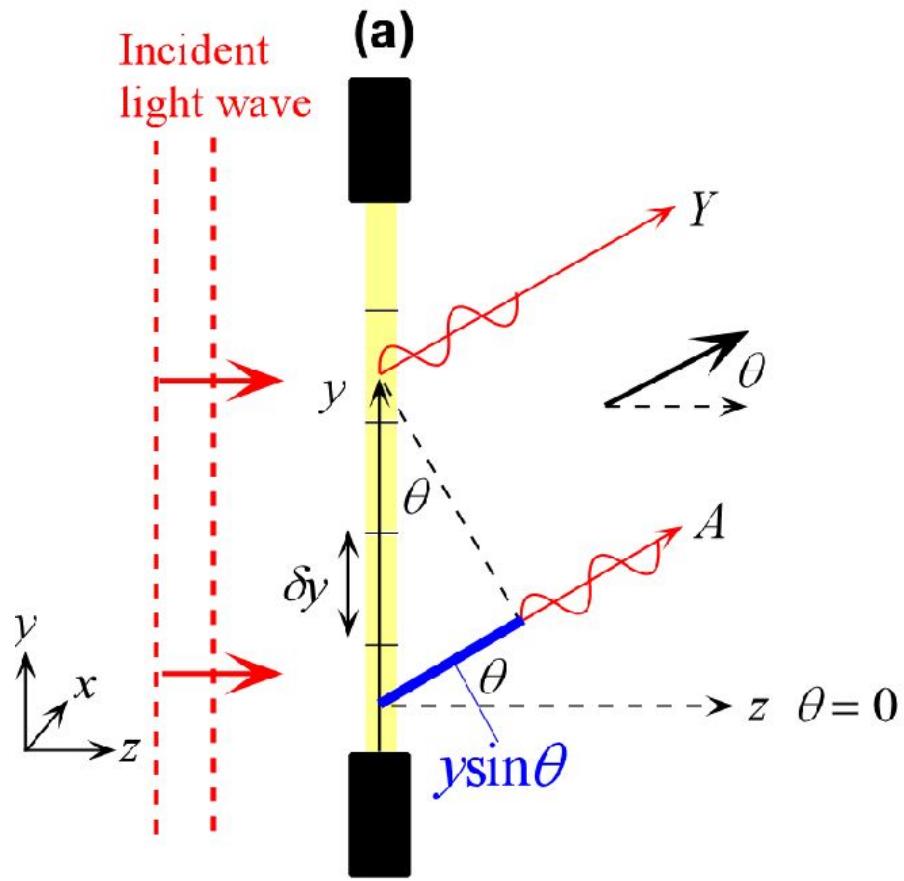
*Every unobstructed point of a wave front, at a given instant in time, serves as a source of spherical secondary waves (with the same frequency as that of the primary wave). The amplitude of the optical field at any point beyond is the superposition of all these wavelets (considering their amplitudes and relative phases)*

# Diffraction



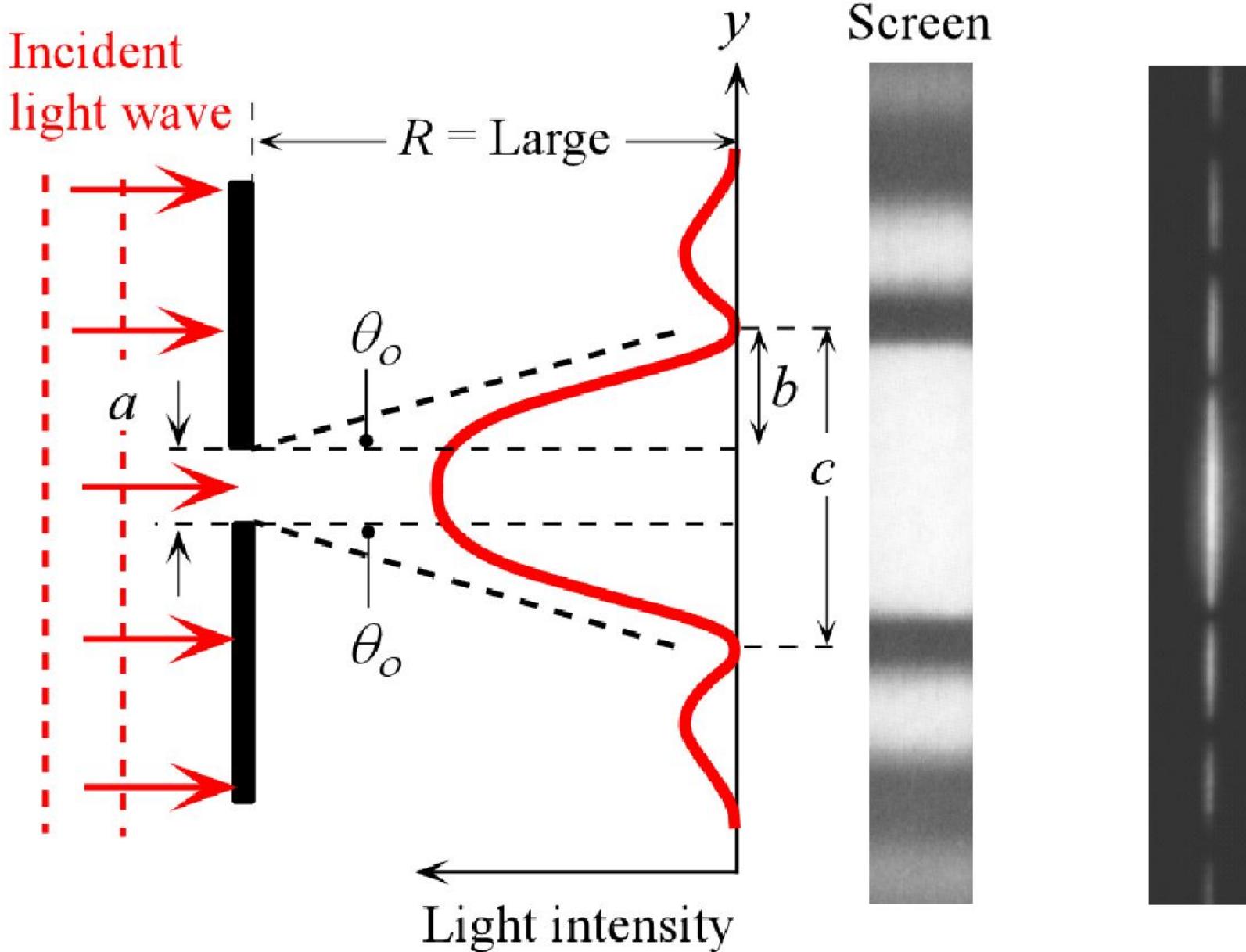
- (a) Huygens-Fresnel principle states that each point in the aperture becomes a source of secondary waves (spherical waves). The spherical wave fronts are separated by  $\lambda$ . The new wavefront is the envelope of all these spherical wavefronts.
- (b) Another possible wavefront occurs at an angle  $\theta$  to the z-direction which is a diffracted wave.

# Diffraction from a Single Slit



- (a) The aperture has a finite width  $a$  along  $y$ , but it is very long along  $x$  so that it is a one-dimensional slit. The aperture is divided into  $N$  number of point sources each occupying  $\delta y$  with amplitude proportional to  $\delta y$  since the slit is excited by a plane electromagnetic wave.
- (b) The intensity distribution in the received light at the screen *far away* from the aperture: *the diffraction pattern*. Note that the slit is very long along  $x$  and there is no diffraction along this dimension.
- (c) Diffraction pattern obtained by using a laser beam from a pointer incident on a single slit.

# Diffraction from a Single Slit



# Diffraction from a Single Slit

$$\delta E \propto (\delta y) \exp(-jk_y \sin \theta)$$

$$E(\theta) = C \int_{y=0}^{y=a} \delta y \exp(-jk_y \sin \theta)$$

$$E(\theta) = \frac{Ce^{-j\frac{1}{2}ka \sin \theta}}{\frac{1}{2}ka \sin \theta} a \sin(\frac{1}{2}ka \sin \theta)$$

$$I(\theta) = \left[ \frac{C'a \sin(\frac{1}{2}ka \sin \theta)}{\frac{1}{2}ka \sin \theta} \right]^2 = I(0) \text{sinc}^2(\beta)$$

$$\beta = \frac{1}{2}ka \sin \theta$$

# Diffraction from a Single Slit

$$I(\theta) = I(0) \left[ \frac{\sin(\frac{1}{2}ka \sin \theta)}{\frac{1}{2}ka \sin \theta} \right]^2 = I(0) \left[ \frac{\sin \beta}{\beta} \right]^2 = I(0) \text{sinc}^2(\beta)$$

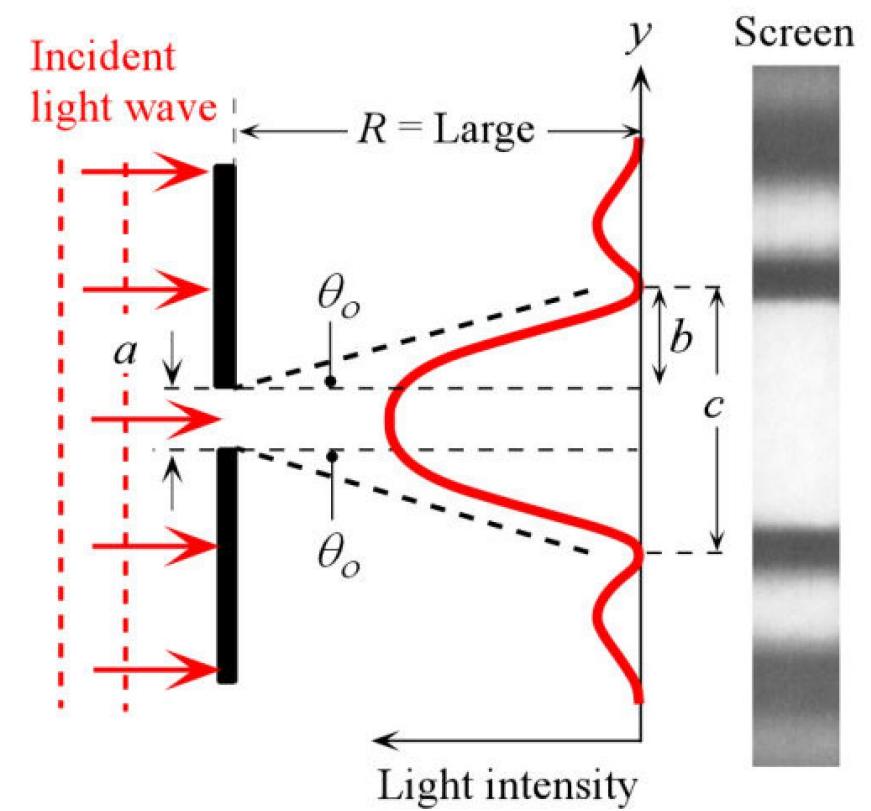
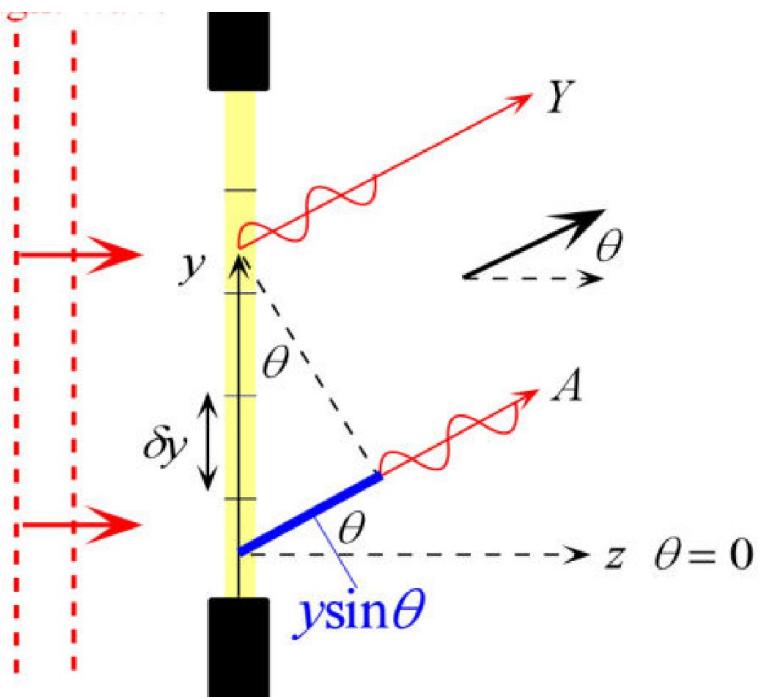
$$\beta = \frac{1}{2}ka \sin \theta$$

Zero intensity when  $I(\theta) = 0$

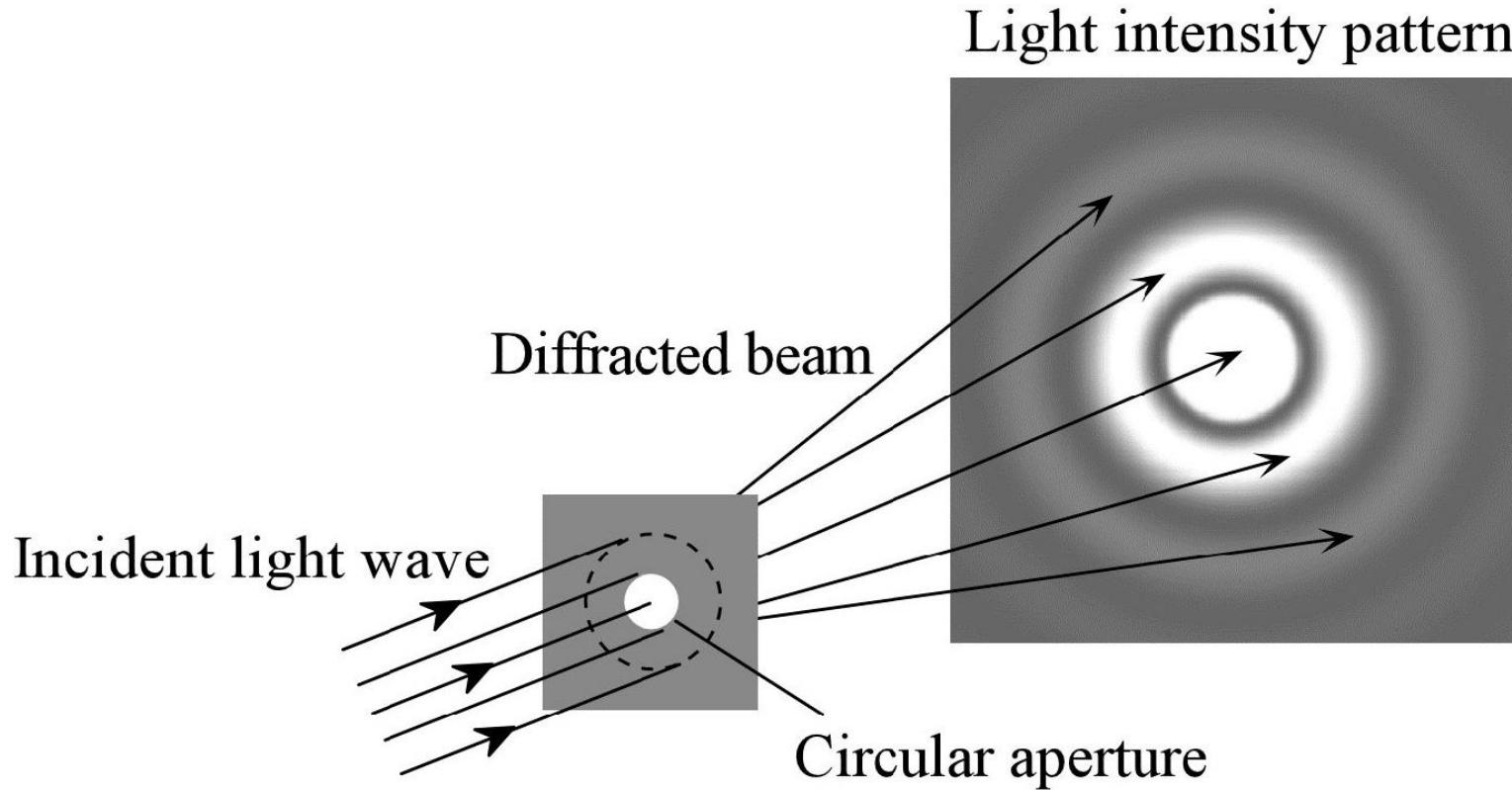
$$\sin \theta = \frac{m\lambda}{a}$$

Divergence

$$\Delta \theta = 2\theta_o \approx \frac{2\lambda}{a}$$



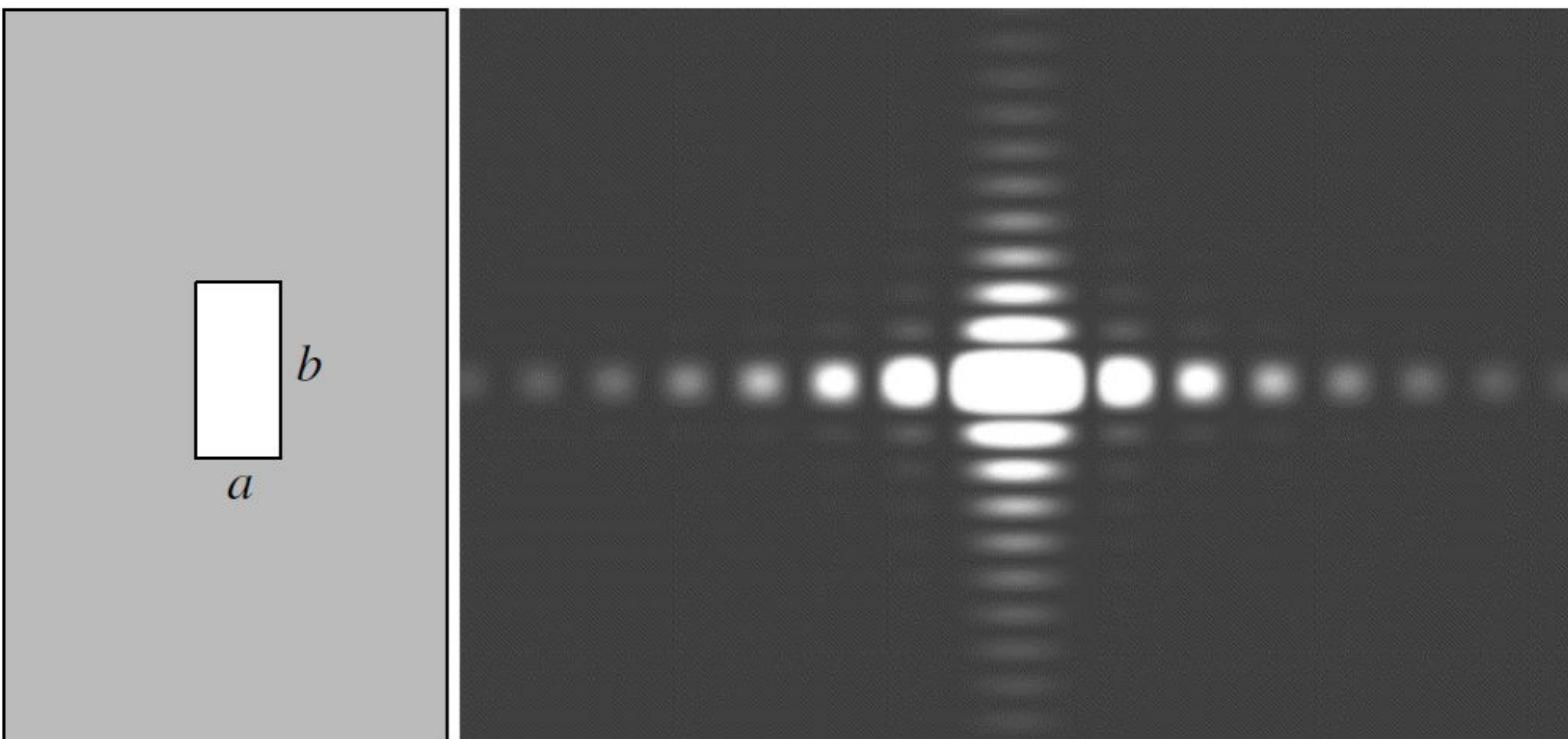
# Diffraction from a Circular Aperture



$$\sin \theta_o = 1.22 \frac{\lambda}{D}$$

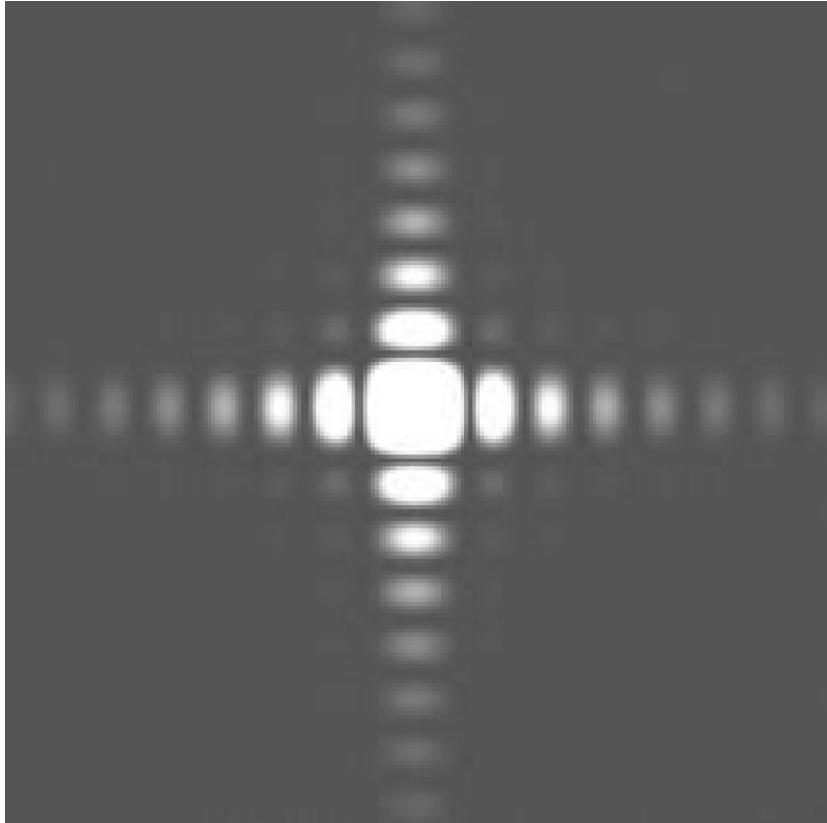
Diameter of aperture

# Diffraction from a Rectangular Aperture



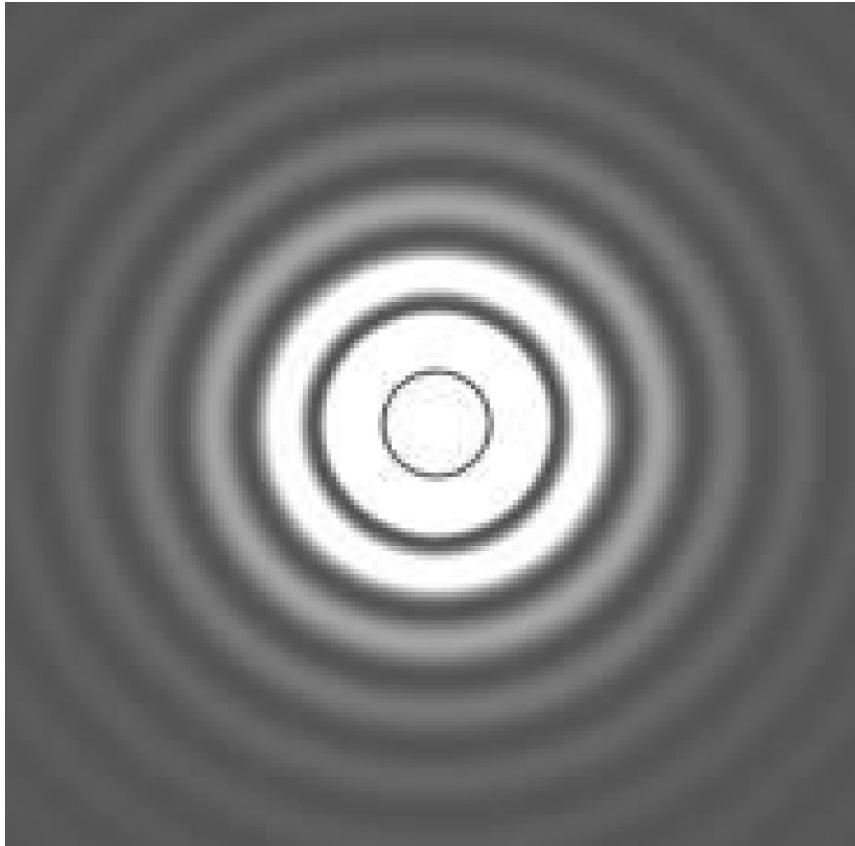
The rectangular aperture of dimensions  $a \times b$  on the left gives the diffraction pattern on the right. ( $b$  is twice  $a$ )

# Diffraction from a Square Aperture



Diffraction pattern far away from a square aperture. The image has been overexposed to capture the faint side lobes

# Diffraction from a Circular Aperture



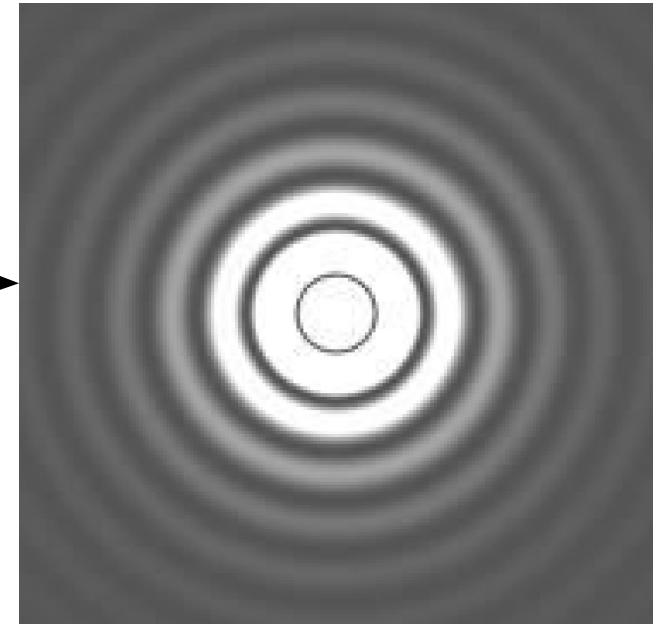
Diffraction pattern far away from a circular aperture. The image has been overexposed to capture the faint outer rings

# Diffraction from a Circular Aperture

Intensity distribution

$$I(\gamma) = I_o \left( \frac{2J_1(\gamma)}{\gamma} \right)^2 \longrightarrow$$

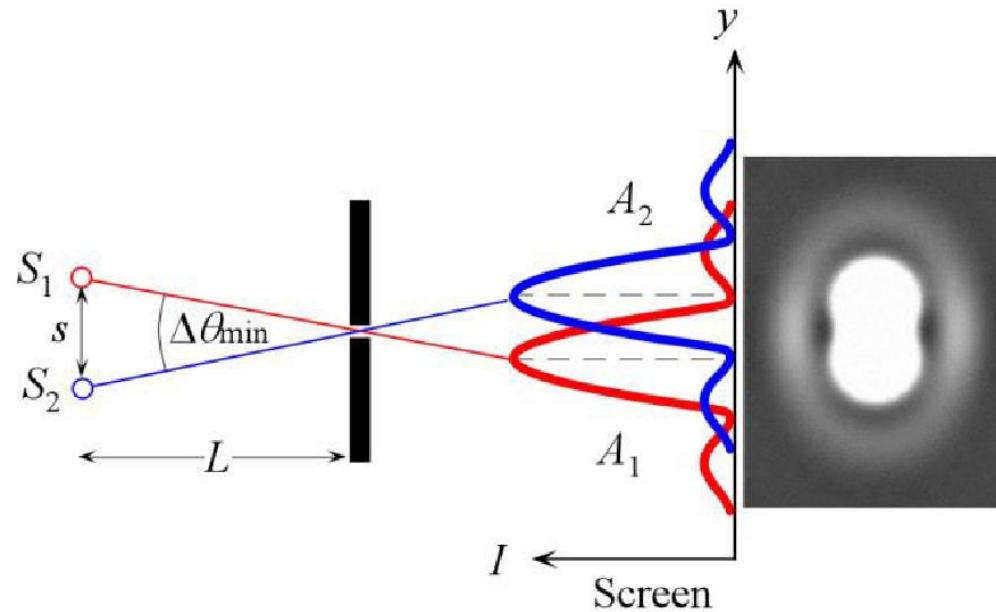
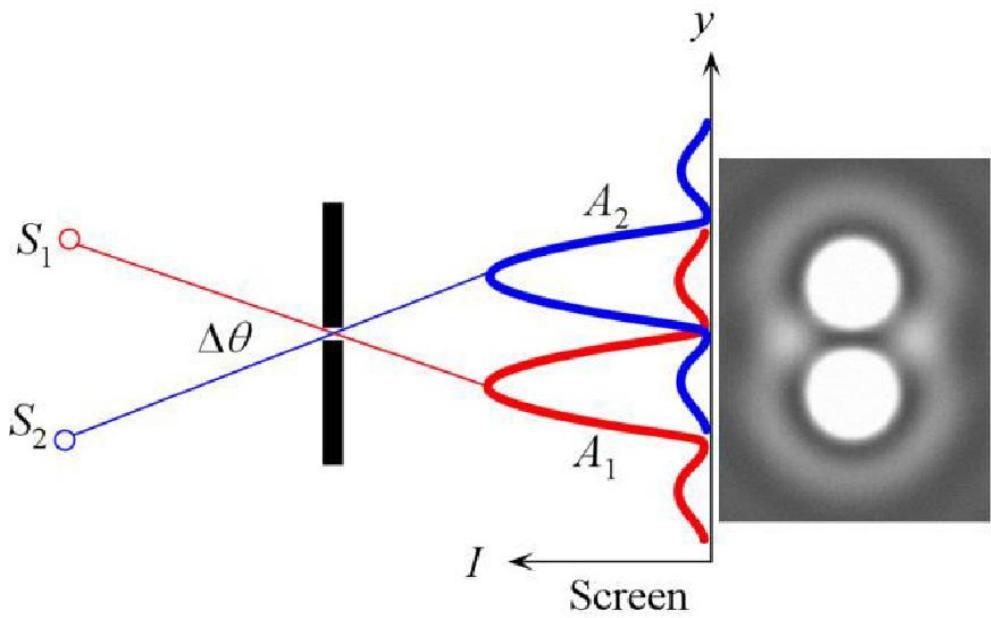
$$\gamma = (1/2)kD\sin\theta \quad k = 2\pi/\lambda$$



$$J_1(\gamma) = \frac{1}{\pi} \int_0^\pi \cos(\alpha - \gamma \sin \alpha) d\alpha$$

↑  
Bessel function (first kind,  
first order)

# Rayleigh Criterion



Resolution of imaging systems is limited by diffraction effects. As points  $S_1$  and  $S_2$  get closer, eventually the Airy patterns overlap so much that the resolution is lost. The Rayleigh criterion allows the minimum angular separation of two of the point sources to be determined.

$$\sin(\Delta\theta_{\min}) = 1.22 \frac{\lambda}{D}$$

# Rayleigh Criterion

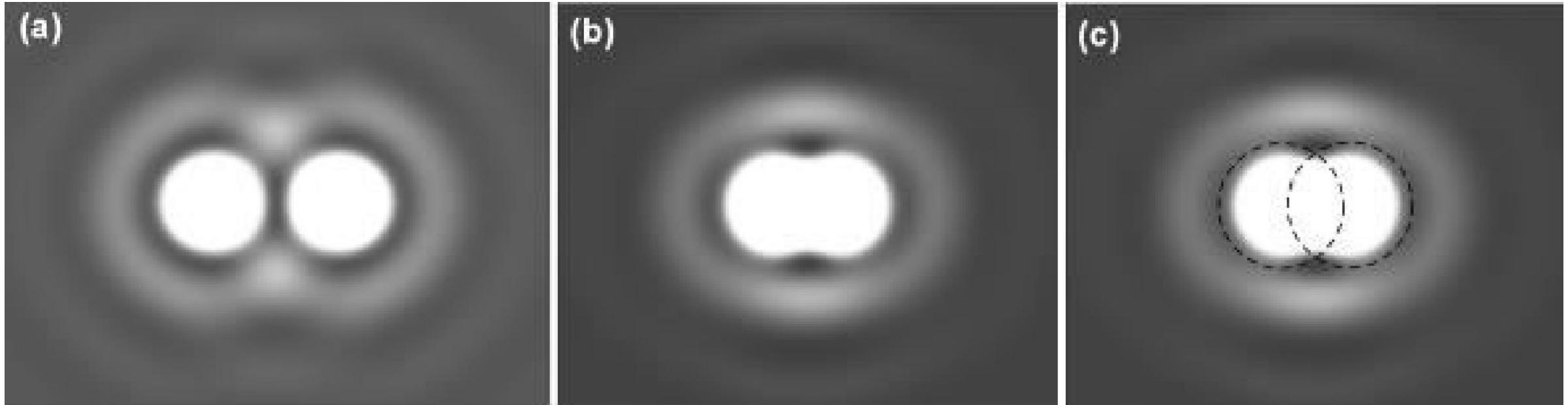
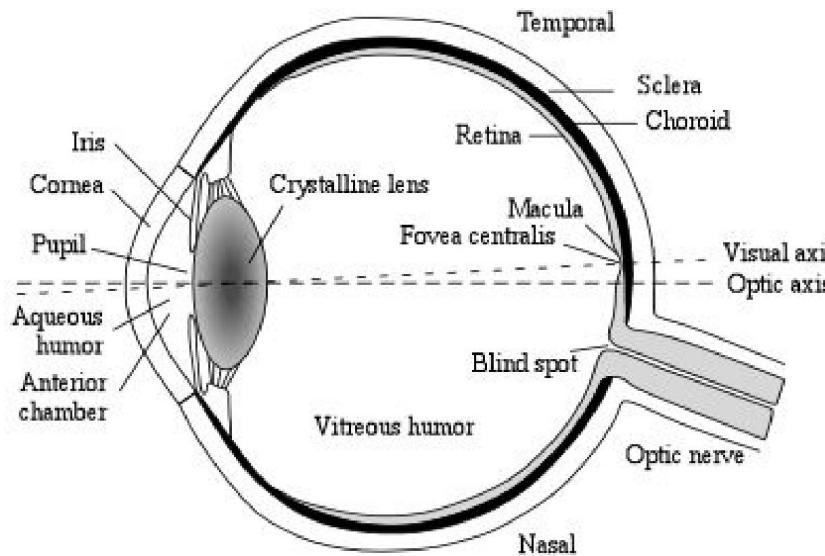


Image of two point sources captured through a small circular aperture. (a) The two points are fully resolved since the diffraction patterns of the two sources are sufficiently separated. (b) The two images near the Rayleigh limit of resolution. (c) The first dark ring through the center of the bright Airy disk of the other pattern. (Approximate.)

## Resolution of the Human Eye



The human eye has a pupil diameter of about 2 mm. What would be the minimum angular separation of two points under a green light of 550 nm and their minimum separation if the two objects are 30 cm from the eye?

The image will be diffraction pattern in the eye, and is a result of waves in this medium.

If the refractive index  $n \approx 1.33$  (water) in the eye, then

$$\sin(\Delta\theta_{\min}) = 1.22 \frac{\lambda}{nD} = 1.22 \frac{(550 \times 10^{-9} \text{ m})}{(1.33)(2 \times 10^{-3} \text{ m})}$$

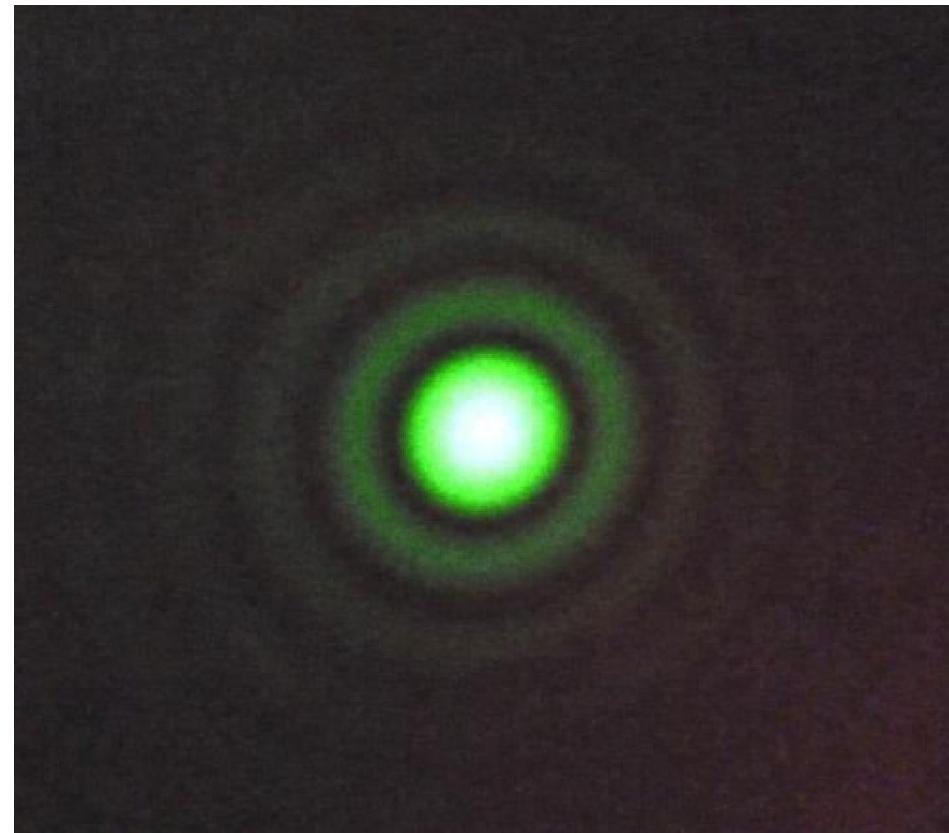
$$\Delta\theta_{\min} = 0.0145^\circ$$

Their minimum separation  $s$  would be

$$\begin{aligned}s &= 2L\tan(\Delta\theta_{\min}/2) = 2(300 \text{ mm})\tan(0.0145^\circ/2) \\ &= 0.076 \text{ mm} = 76 \text{ micron}\end{aligned}$$

which is about the thickness of a human hair

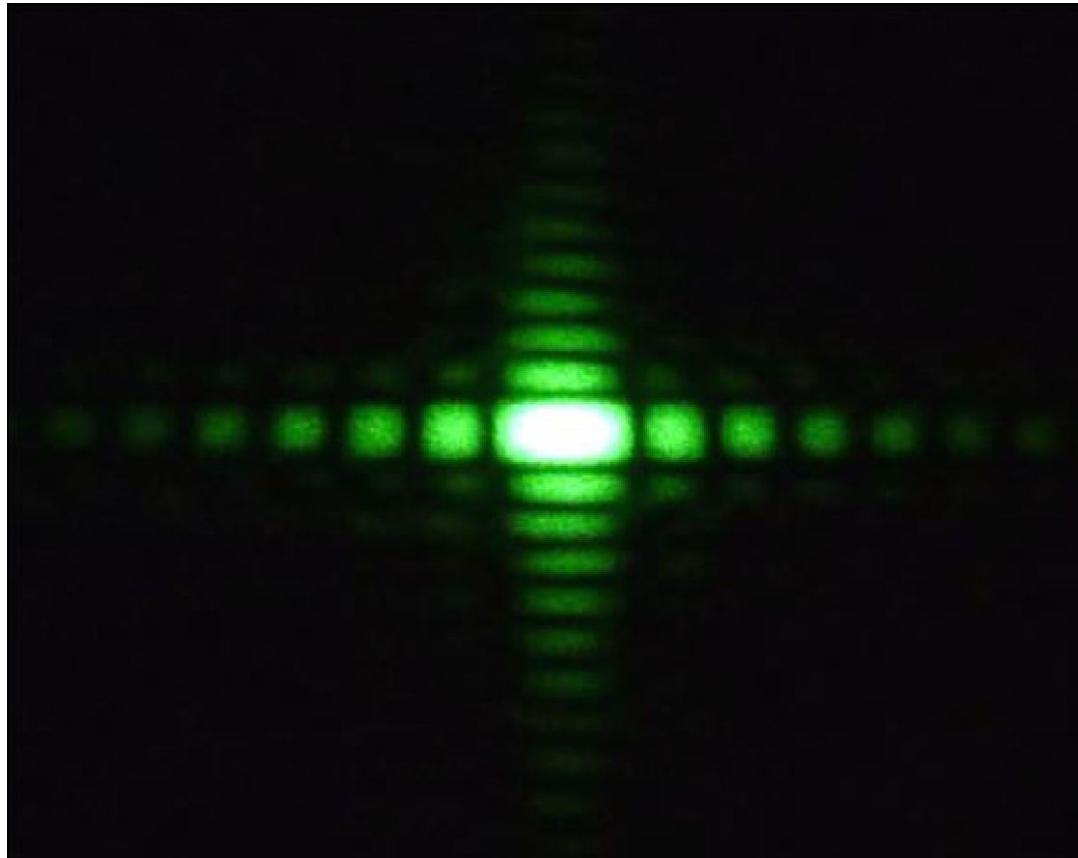
# Experimental Diffraction Patterns



**Diffraction from a circular aperture with a diameter of  $30 \mu\text{m}$**

**Green laser pointer used at a wavelength of  $532 \text{ nm}$**

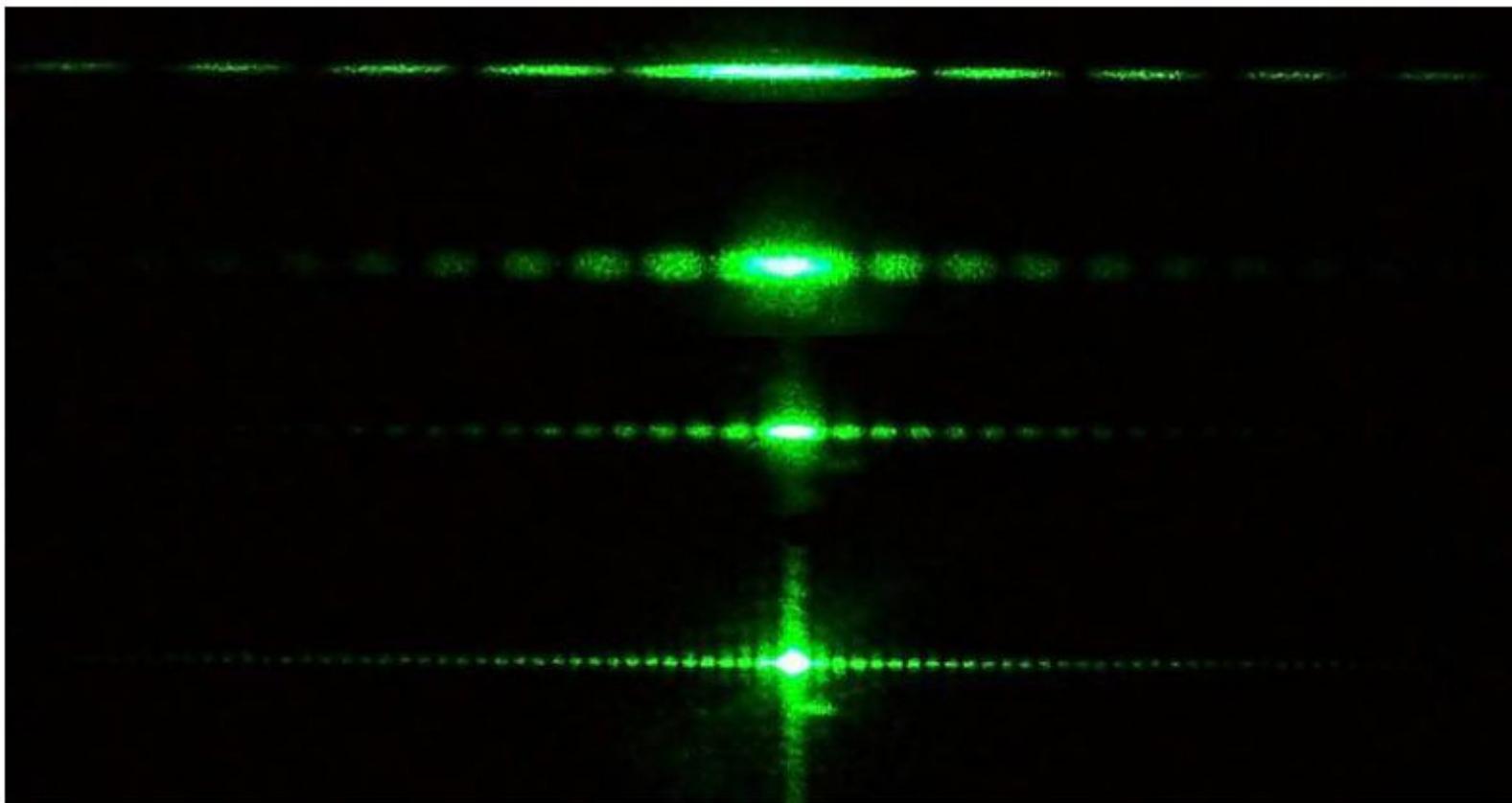
# Experimental Diffraction Patterns



Crossed slits  $200 \times 100 \mu\text{m}$

Green laser pointer used at a wavelength of  $532 \text{ nm}$

# Experimental Diffraction Patterns



$a = 20 \mu\text{m}$

$a = 40 \mu\text{m}$

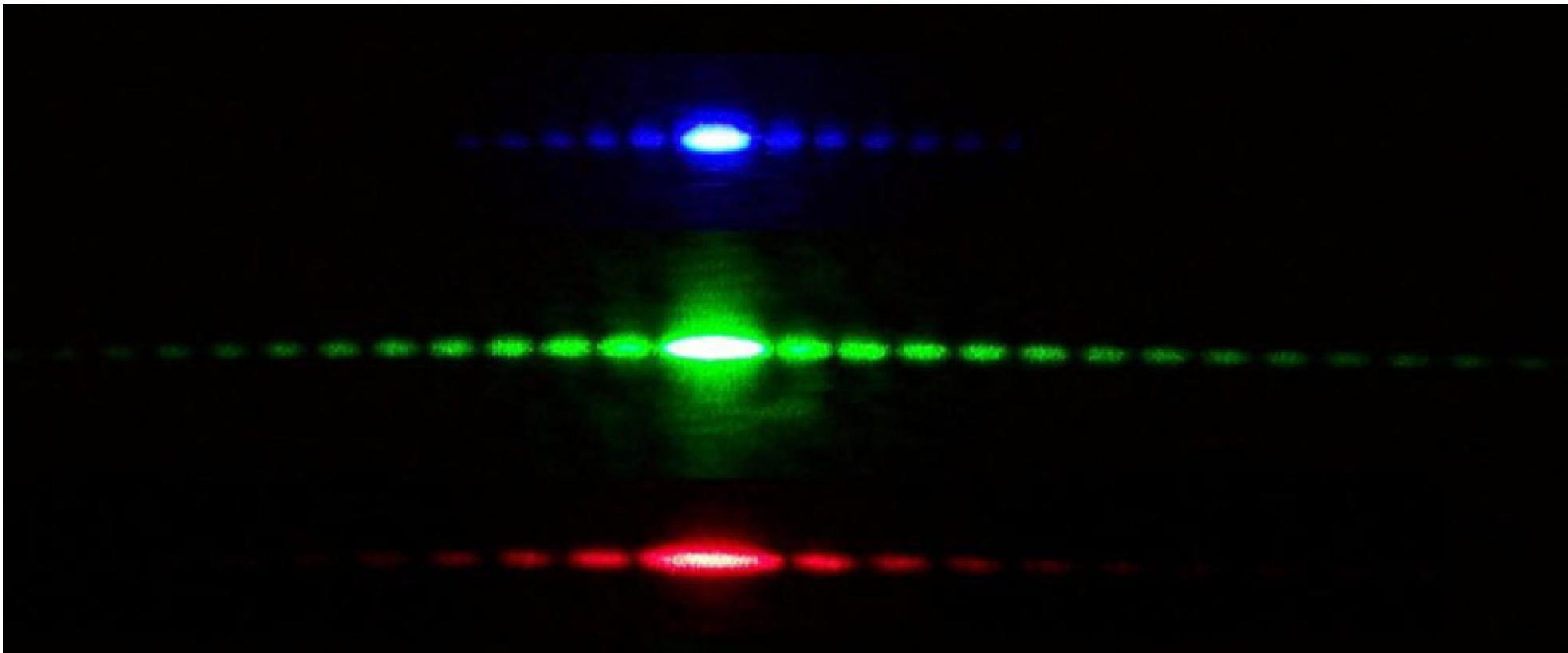
$a = 80 \mu\text{m}$

$a = 160 \mu\text{m}$

Single slit with a width  $a$

Green laser pointer used at a wavelength of 532 nm

# Experimental Diffraction Patterns



Single slit with a width 100  $\mu\text{m}$

Blue = 402 nm

Green = 532 nm

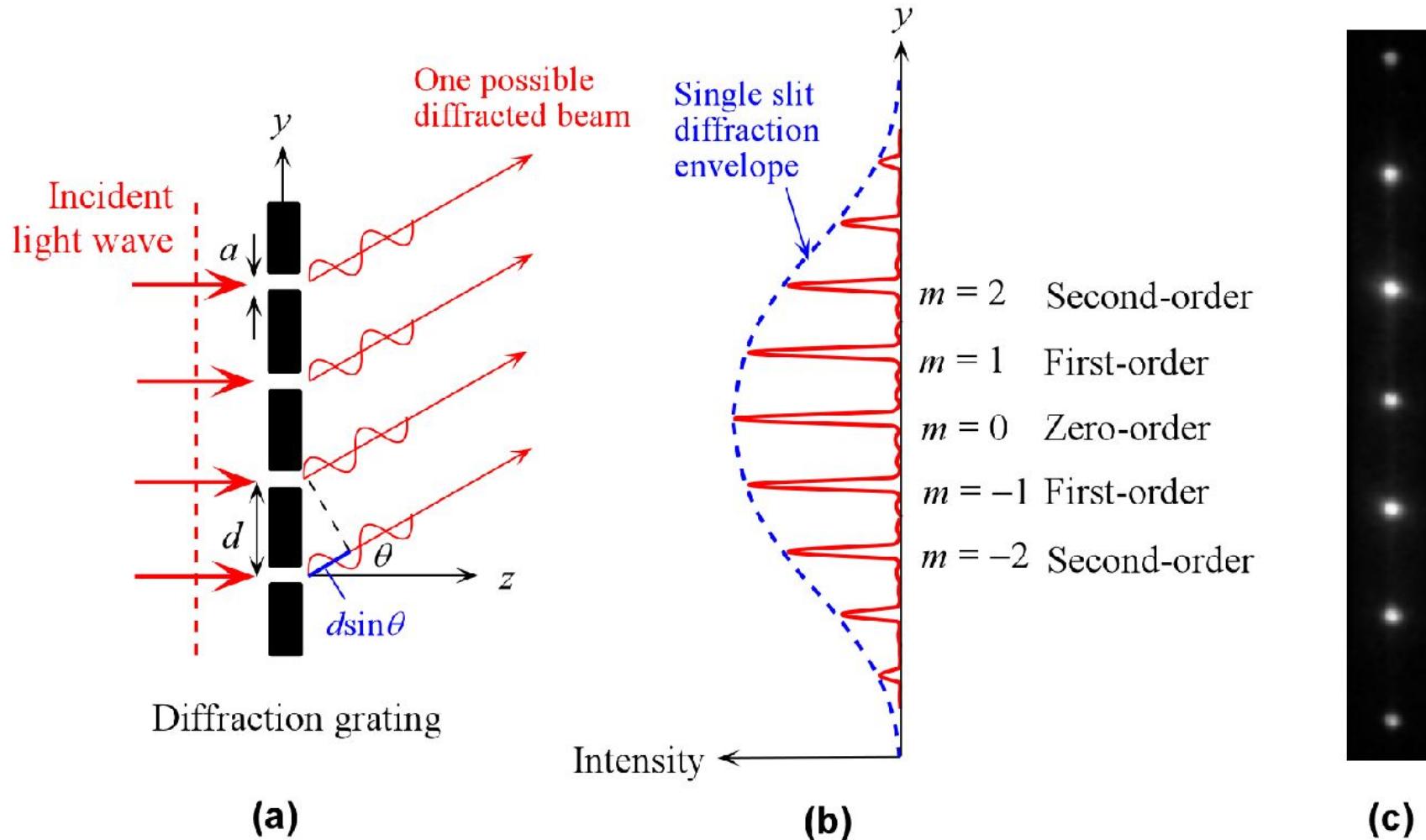
Red = 670 nm

Why does the central bright lobe get larger with increasing wavelength?

Answer

$$\Delta\theta = 2\theta_o \approx \frac{2\lambda}{a}$$

# Diffraction Grating

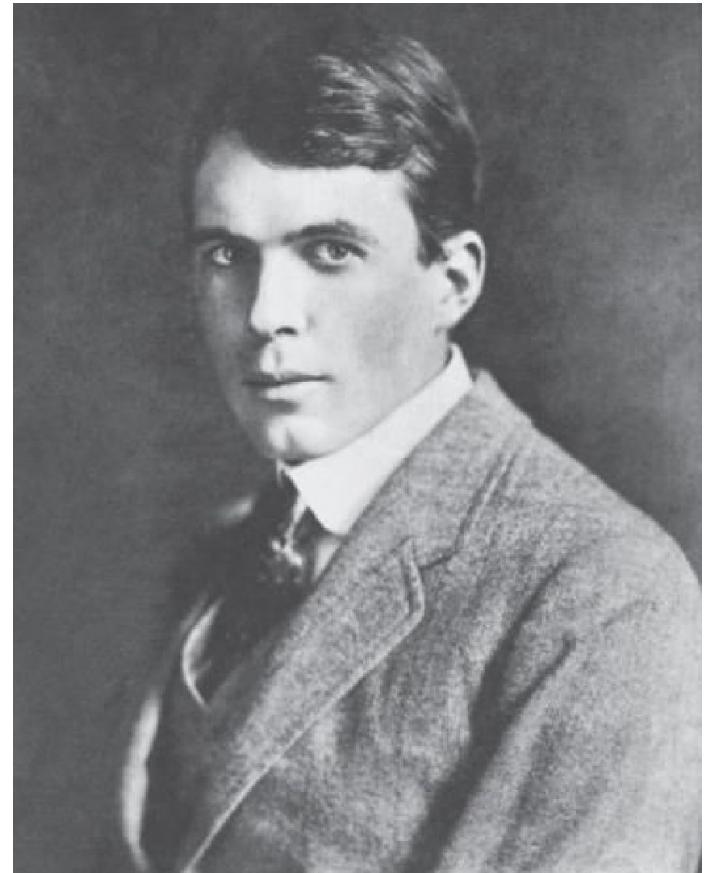


- (a) A diffraction grating with  $N$  slits in an opaque screen. Slit periodicity is  $d$  and slit width is  $a$ ;  $a \ll d$ .
- (b) The far-field diffracted light pattern. There are distinct, that is diffracted, beams in certain directions
- (c) Diffraction pattern obtained by shining a beam from a red laser pointer onto a diffraction grating. The finite size of the laser beam results in the dot pattern. (The wavelength was 670 nm, red, and the grating has 2000 lines per inch.)

# Diffraction Grating

Bragg diffraction condition  
Normal incidence

$$d \sin \theta = m\lambda ; m = 0, \pm 1, \pm 2, \dots$$



William Lawrence Bragg (1890-1971), Australian-born British physicist, won the Nobel prize with his father William Henry Bragg for his "famous equation" when he was only 25 years old

# Diffraction Gratings

Bragg diffraction condition  
Normal incidence

$$d \sin \theta = m\lambda ; m = 0, \pm 1, \pm 2, \dots$$

Oblique incidence

$$d(\sin \theta_m - \sin \theta_i) = m\lambda ; m = 0, \pm 1, \pm 2,$$

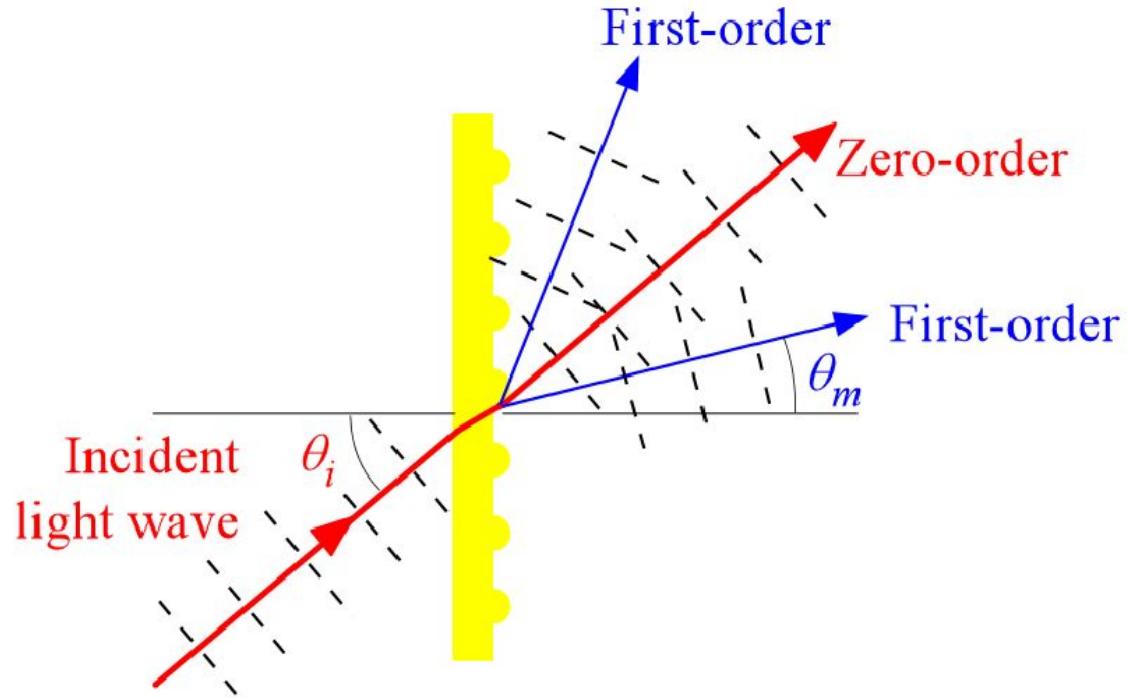
$$I(y) = I_o \left[ \frac{\sin(\frac{1}{2} k_y a)}{\frac{1}{2} k_y a} \right]^2 \left[ \frac{\sin(\frac{1}{2} N k_y d)}{N \sin(\frac{1}{2} k_y d)} \right]^2$$

Diffraction from a single slit

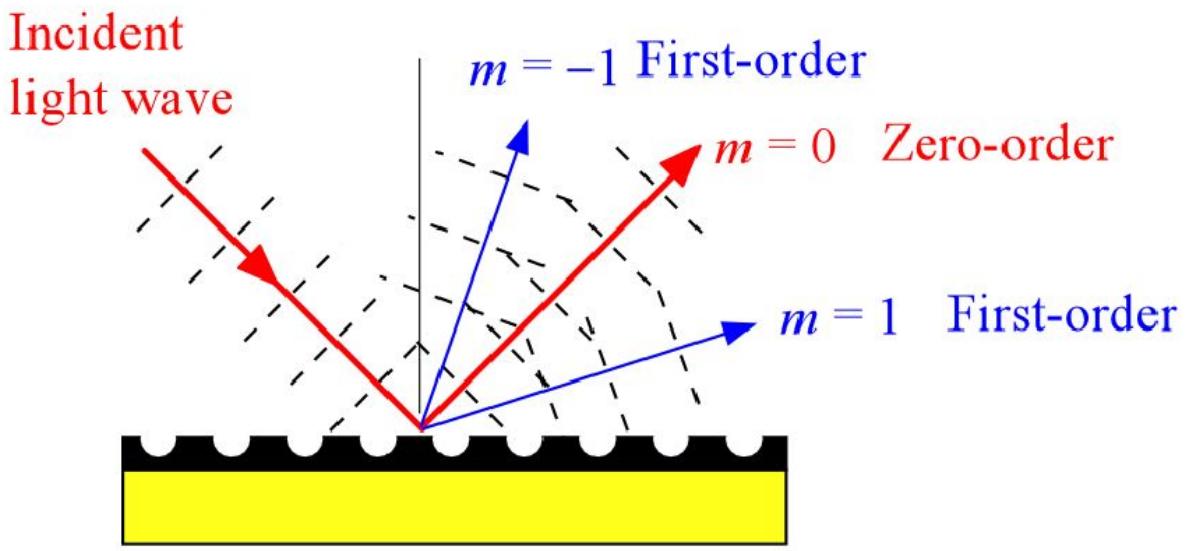
$$k_y = (2\pi/\lambda) \sin \theta$$

Diffraction from  $N$  slits

# Diffraction Gratings



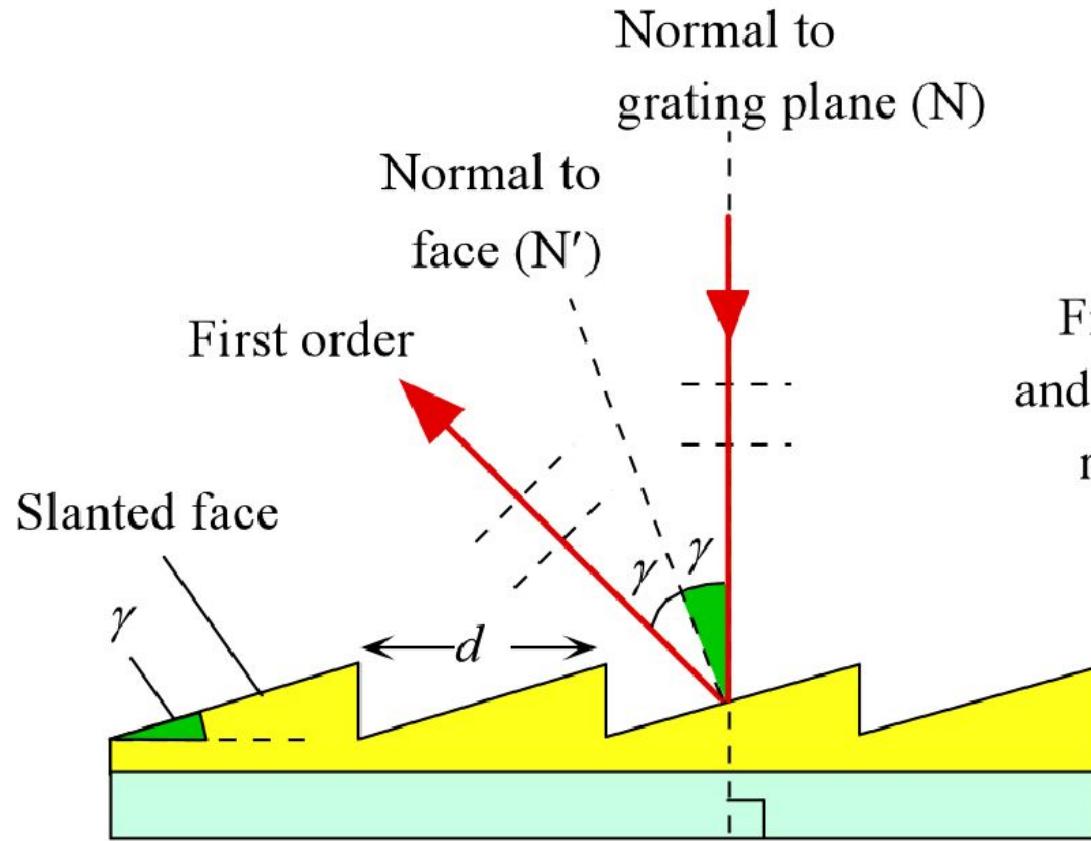
(a) Transmission grating



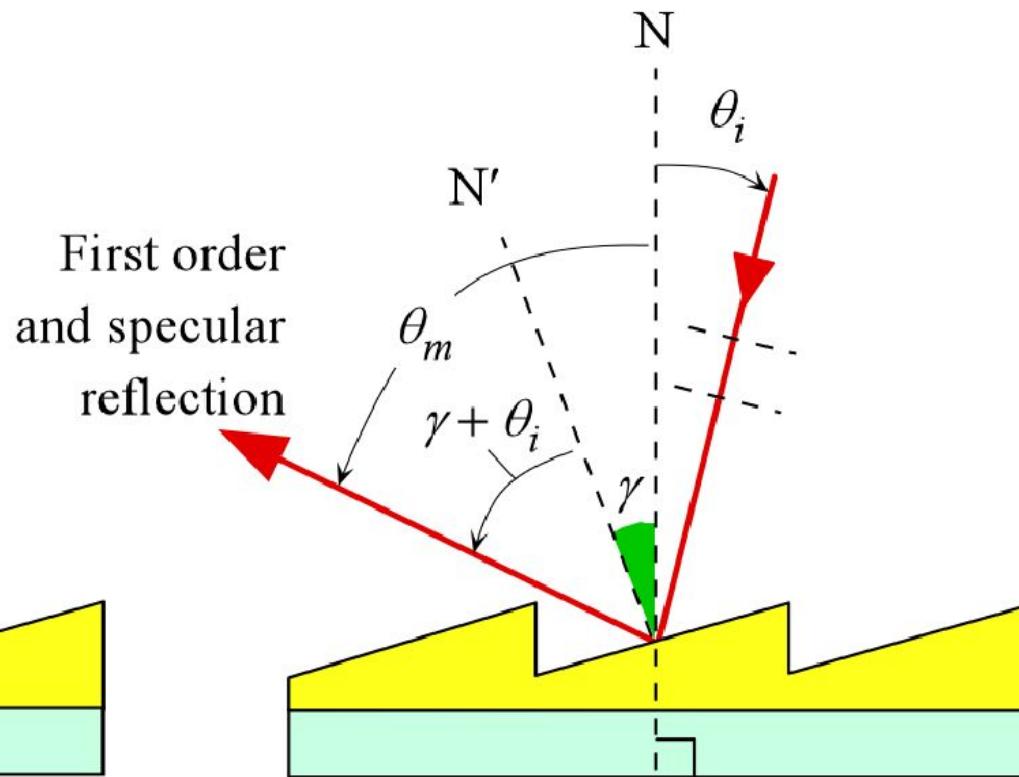
(b) Reflection grating

- (a) Ruled periodic parallel scratches on a glass serve as a *transmission grating*. (The glass plate is assumed to be very thin.)
- (b) A *reflection grating*. An incident light beam results in various "diffracted" beams. The zero-order diffracted beam is the normal reflected beam with an angle of reflection equal to the angle of incidence.

(a)



(b)



(a) A blazed grating. Triangular grooves have been cut into the surface with a periodicity  $d$ . The side of a triangular groove make an angle  $\gamma$  to the plane of the diffraction angle. For normal incidence, the angle of diffraction must be  $2\gamma$  to place the specular reflection on the diffracted beam.

(b) When the incident beam is not normal, the specular reflection will coincides with the diffracted beam, when  $(\gamma + \theta_i) + \gamma = \theta_m$

## Example: A reflection grating

Consider a reflection grating with a period  $d$  that is 10  $\mu\text{m}$ . Find the diffracted beams if a collimated light wave of wavelength 1550 nm is incident on the grating at an angle of  $45^\circ$  to its normal. What should be the blazing angle  $\gamma$  if we were to use the blazed grating with the same periodicity? What happens to the diffracted beams if the periodicity is reduced to 2  $\mu\text{m}$ ?

### Solution:

Put,  $m = 0$  to find the zero-order diffraction,  $\theta_0 = 45^\circ$ , as expected.

The general Bragg diffraction condition is

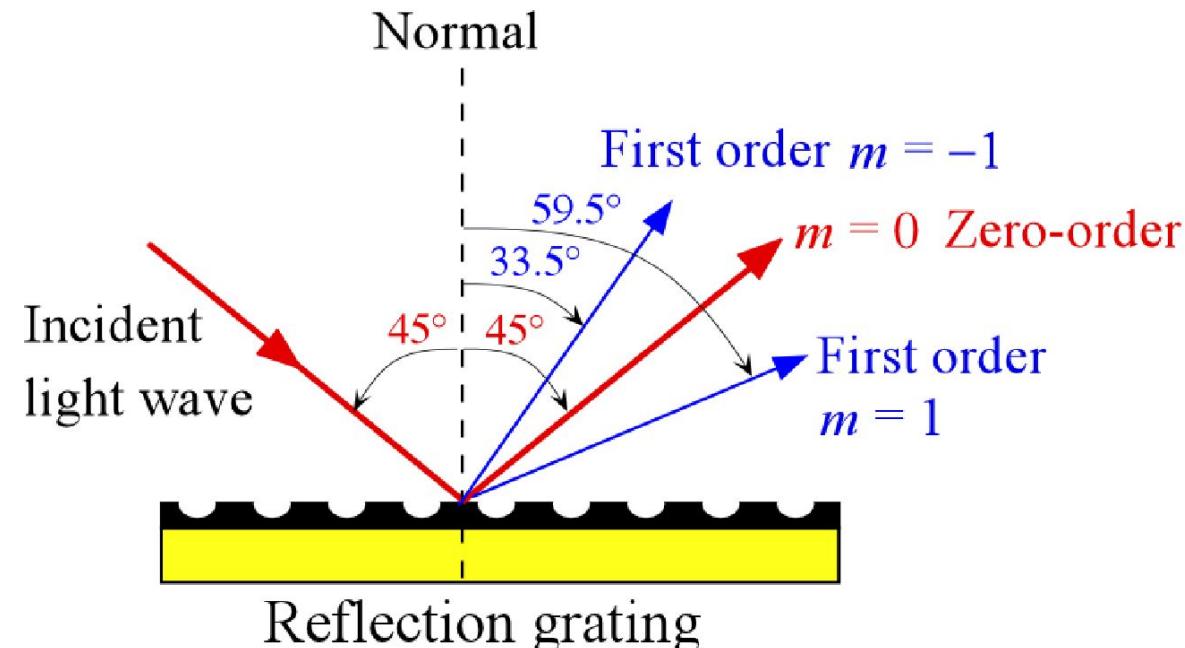
$$d(\sin\theta_m - \sin\theta_i) = m\lambda.$$

$$\therefore (10 \mu\text{m})(\sin\theta_m - \sin(45^\circ)) = (+1)(1.55 \mu\text{m})$$

$$\therefore (10 \mu\text{m})(\sin\theta_m - \sin(45^\circ)) = (-1)(1.55 \mu\text{m})$$

Solving these two equations, we find

$$\theta_m = 59.6^\circ \text{ for } m = 1 \text{ and } \theta_m = 33.5^\circ \text{ for } m = -1$$



## Example: A reflection grating

The secular reflection from the grooved surface coincides with the  $m^{\text{th}}$  order diffraction when  $2\gamma = \theta_m - \theta_i$   
 $\therefore \gamma = (1/2)(\theta_m - \theta_i) = (1/2)(59.6^\circ - 45^\circ) = 7.3^\circ$

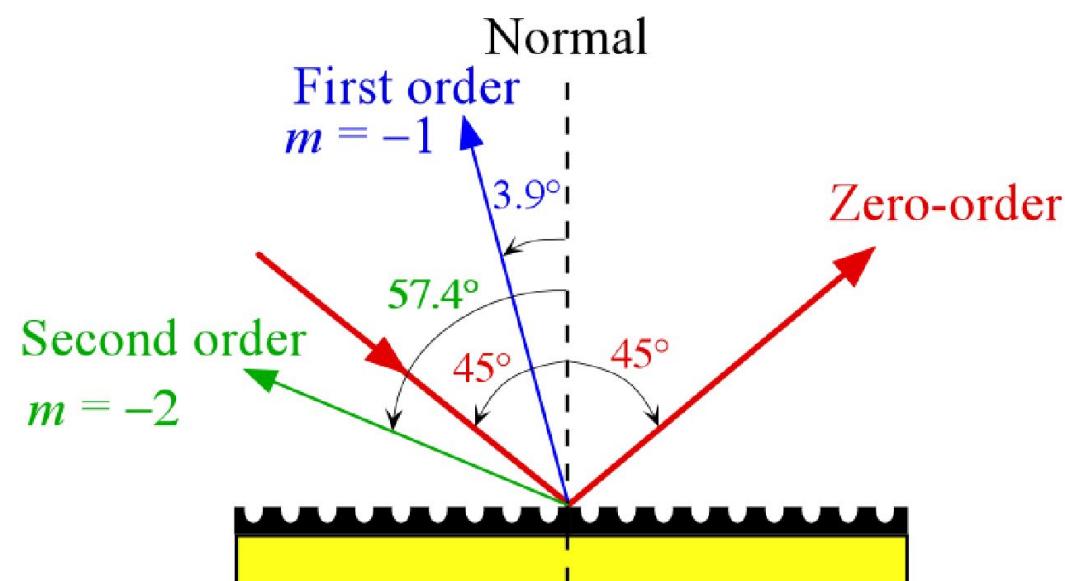
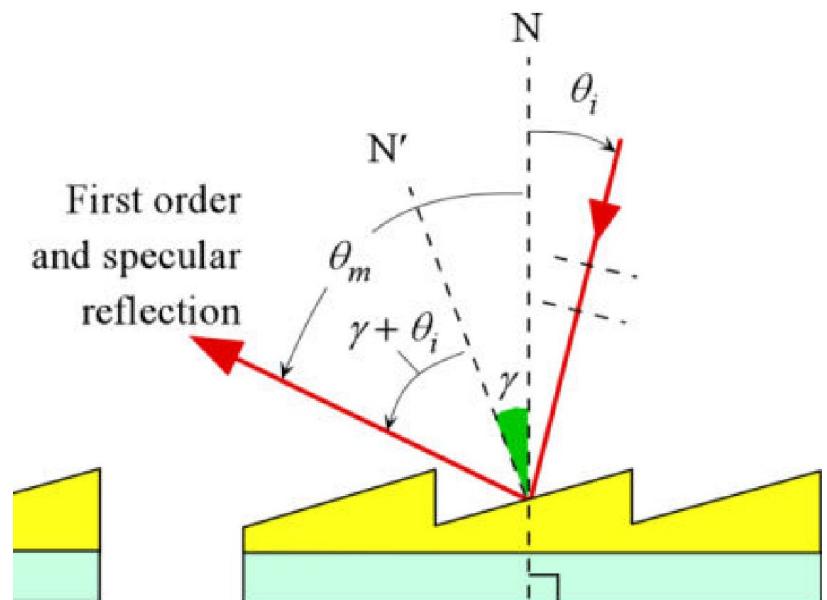
Suppose that we reduce  $d$  to 2  $\mu\text{m}$

Recalculating the above we find

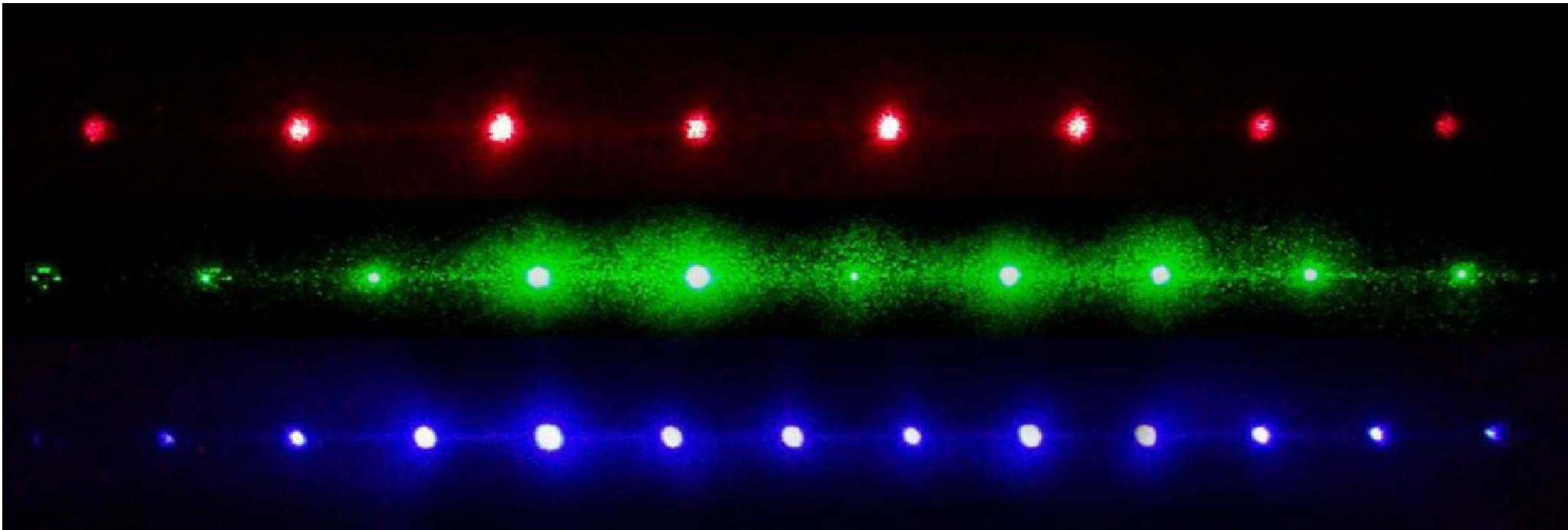
$\theta_m = -3.9^\circ$  for  $m = -1$  and imaginary for  $m = +1$ .

Further, for  $m = -2$ , there is a second order diffraction beam at  $\theta_m = -57.4^\circ$ .

If we increase the angle of incidence, for example,  $\theta_i = 85^\circ$  on the first grating, the diffraction angle for  $m = -1$  increases from  $33.5^\circ$  to  $57.2^\circ$  and the other diffraction peak ( $m = 1$ ) disappears



# Experiments with Diffraction Gratings



**Diffraction grating**

(2000 lines/inch)

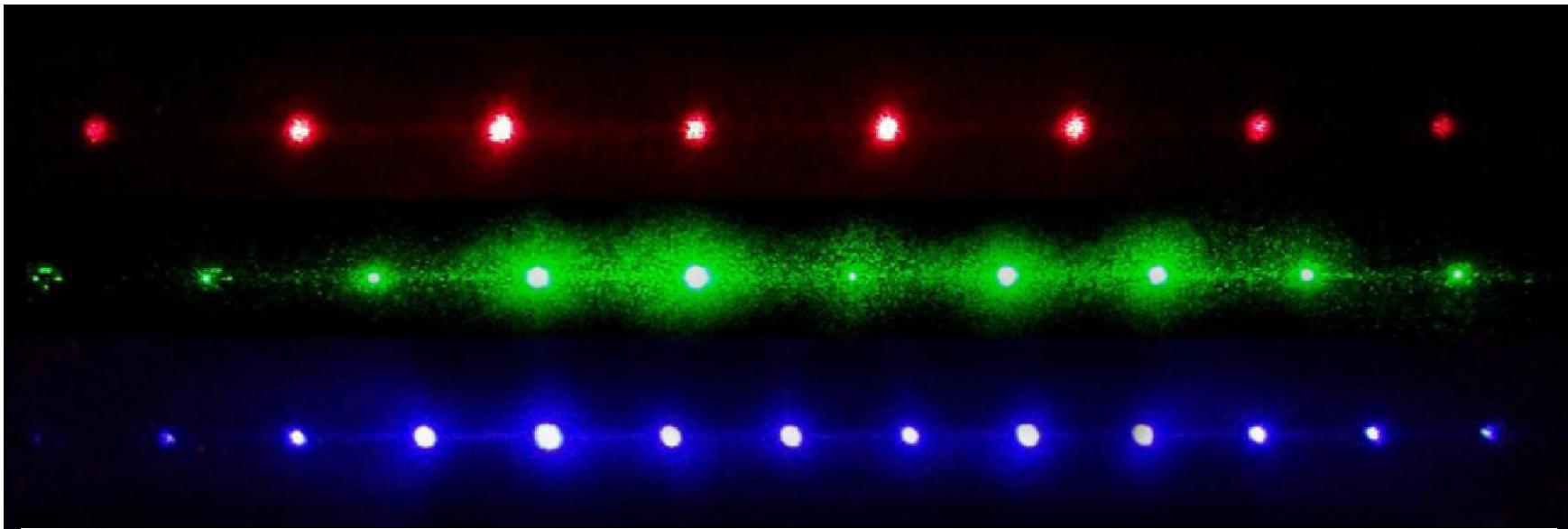
**Blue = 402 nm**

**Green = 532 nm**

**Red = 670 nm**

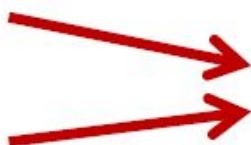
**Why do the diffraction spots become further separated as you increase the wavelength?**

## ANSWER



$$d \sin \theta_1 = m\lambda$$

$$d \sin \theta_2 = (m+1)\lambda$$



$$d(\sin \theta_2 - \sin \theta_1) = \lambda$$

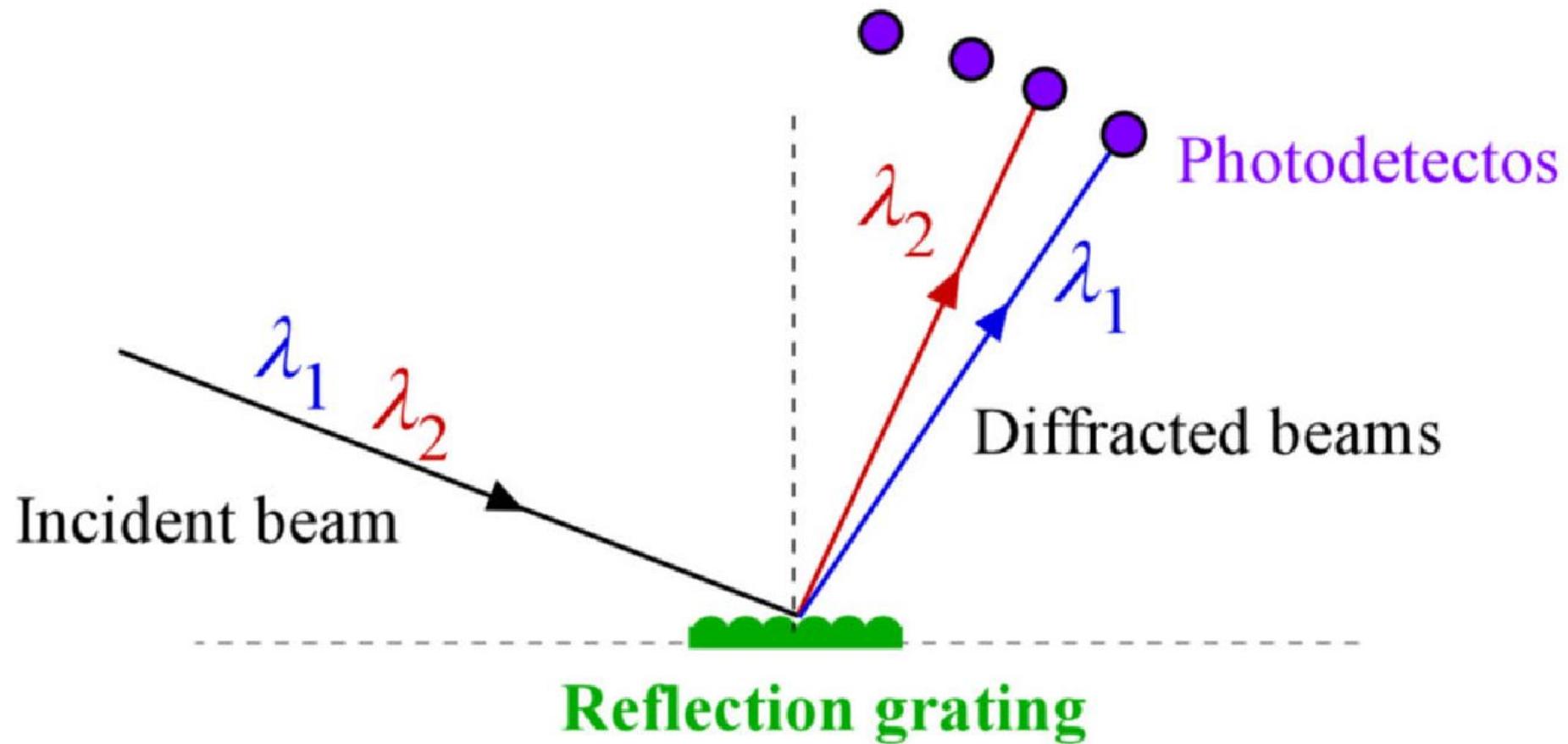
$$\downarrow$$

$$\sin \theta \approx \theta$$

Angular separation of spots

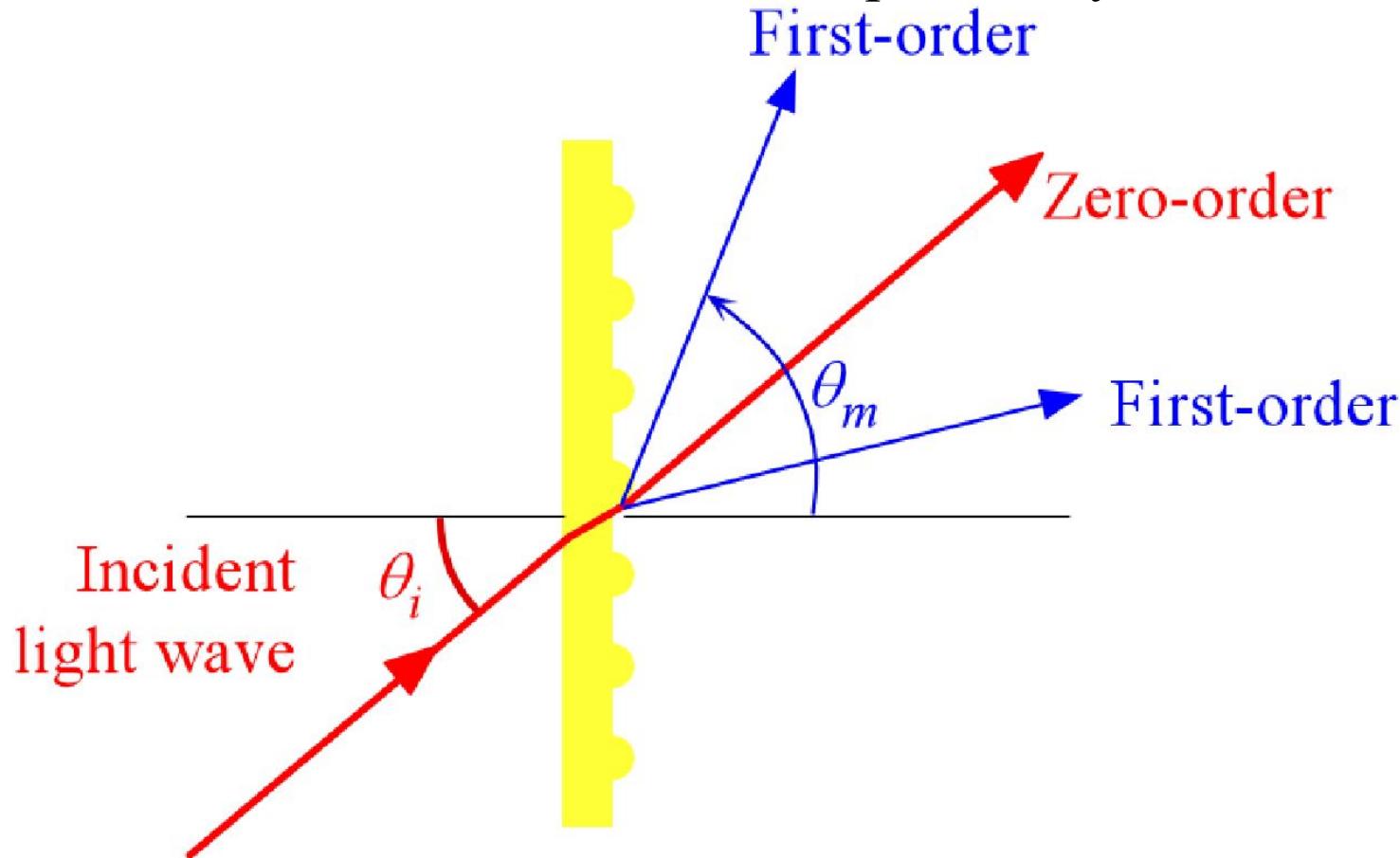
$$\Delta\theta \approx \theta_2 - \theta_1 = \frac{\lambda}{d}$$

We can separate wavelengths by using a diffraction grating  
Useful in Wavelength Division Multiplexing



## Example on Wavelength Separation by Diffraction

A transmission diffraction grating has a periodicity of 3  $\mu\text{m}$ . The angle of incidence is  $30^\circ$  with respect to the normal to the diffraction grating. What is the angular separation of the two wavelength components at 1550 nm and 1540 nm, separated by 10 nm?



$$d(\sin \theta_m - \sin \theta_i) = m\lambda$$

## Example on Wavelength Separation

$\theta_i = 45^\circ$ . Periodicity =  $d = 3 \text{ } \mu\text{m}$

$$d(\sin \theta_m - \sin \theta_i) = m\lambda.$$

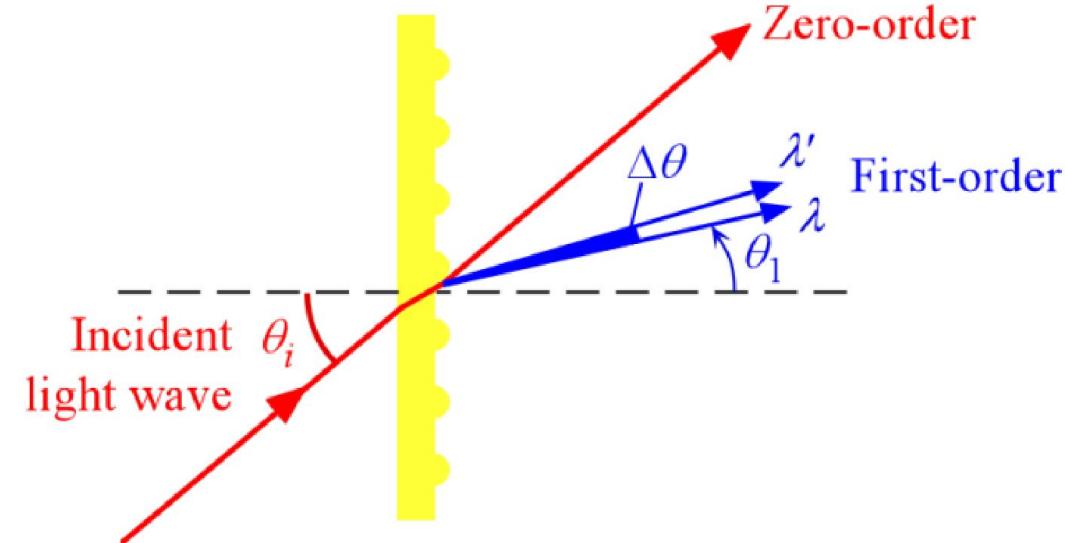
$d = 3 \text{ } \mu\text{m}$ ,  $\lambda = 1.550 \text{ } \mu\text{m}$ ,  $\theta_i = 45^\circ$ , and calculate the diffraction angle  $\theta_m$  for  $m = -1$

$$(3 \text{ } \mu\text{m})[\sin \theta_1 - \sin(45^\circ)] = (-1)(1.550 \text{ } \mu\text{m})$$

$$\therefore \theta_1 = 10.978^\circ$$

A  $\lambda' = 1.540 \text{ } \mu\text{m}$ , examining the same order,  $m = -1$ , we find  $\theta_1' = 11.173^\circ$

$$\therefore \Delta\theta_1 = 11.173^\circ - 10.978^\circ = 0.20^\circ$$



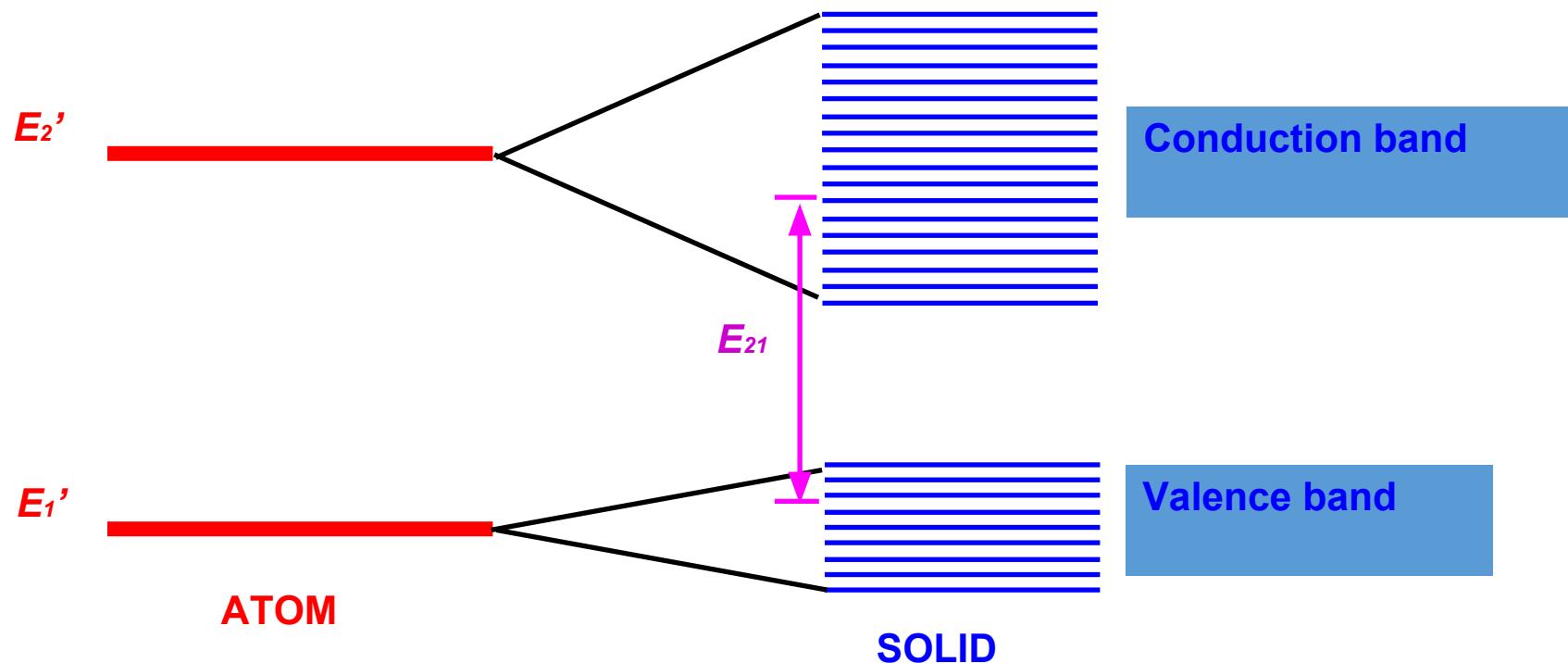
Note,  $m = 1$  gives a complex angle and should be neglected.

# Semiconductors

- A semiconductor is a solid material that has electrical conductivity in between a conductor and an insulator.
- Semiconductors can be used as optical detectors, sources (light-emitting diodes and lasers), amplifiers, waveguides, modulators, sensors, and nonlinear optical elements.

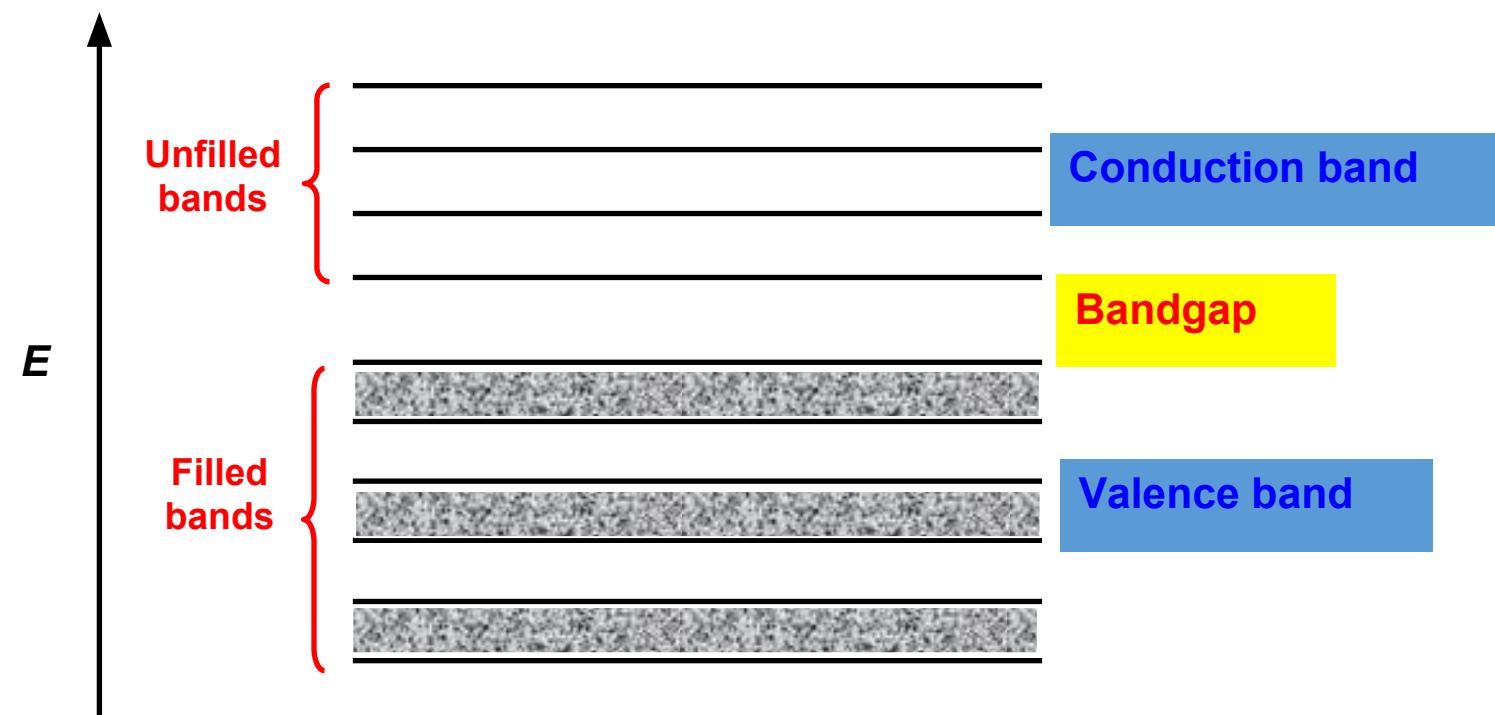
# Energy bands in semiconductors

The solution of the Schrödinger equation for the electron energy in the periodic potential created by the atoms in a crystal lattice, results in a splitting of the atomic energy levels and the formation of energy bands.

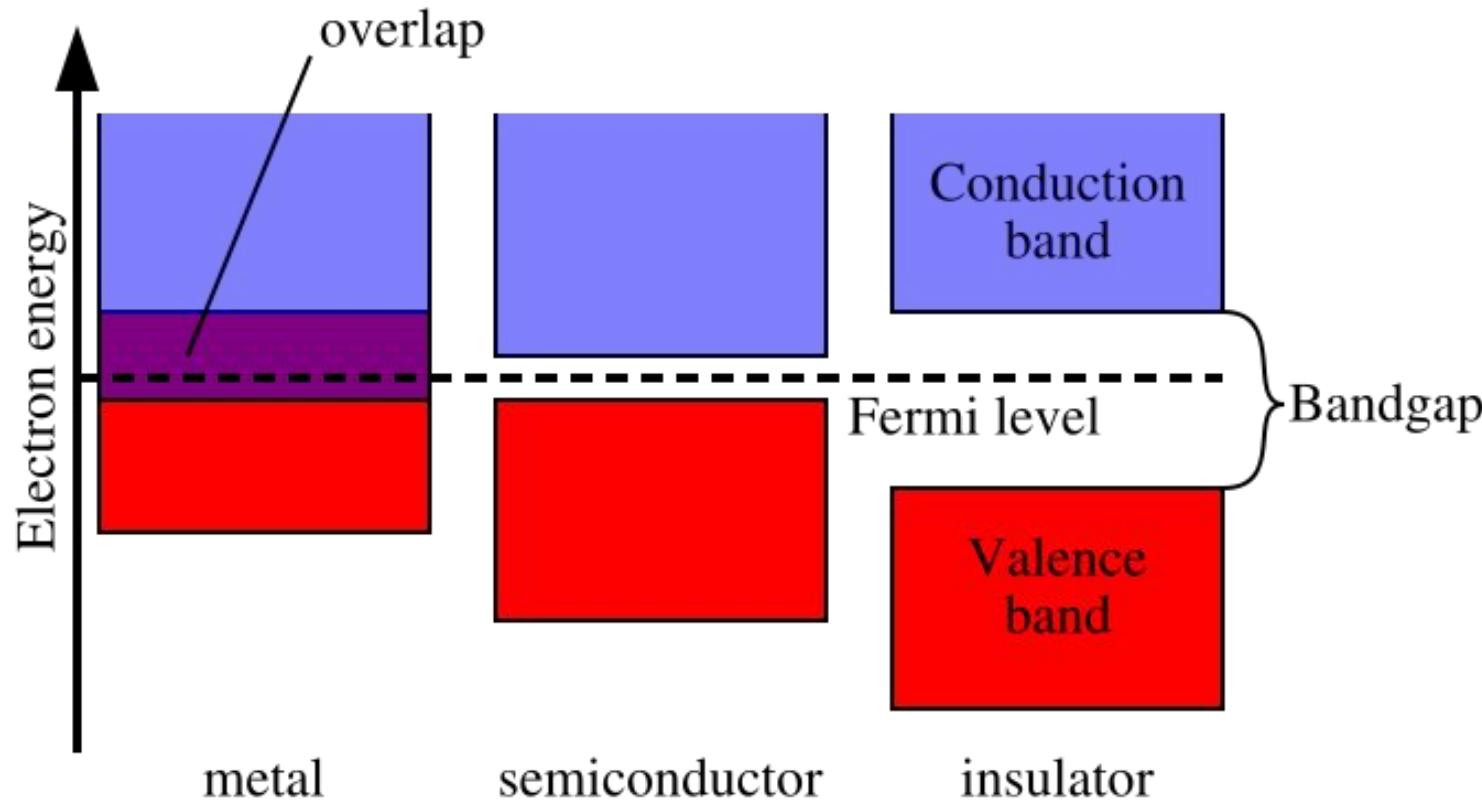


# Energy bands in semiconductors

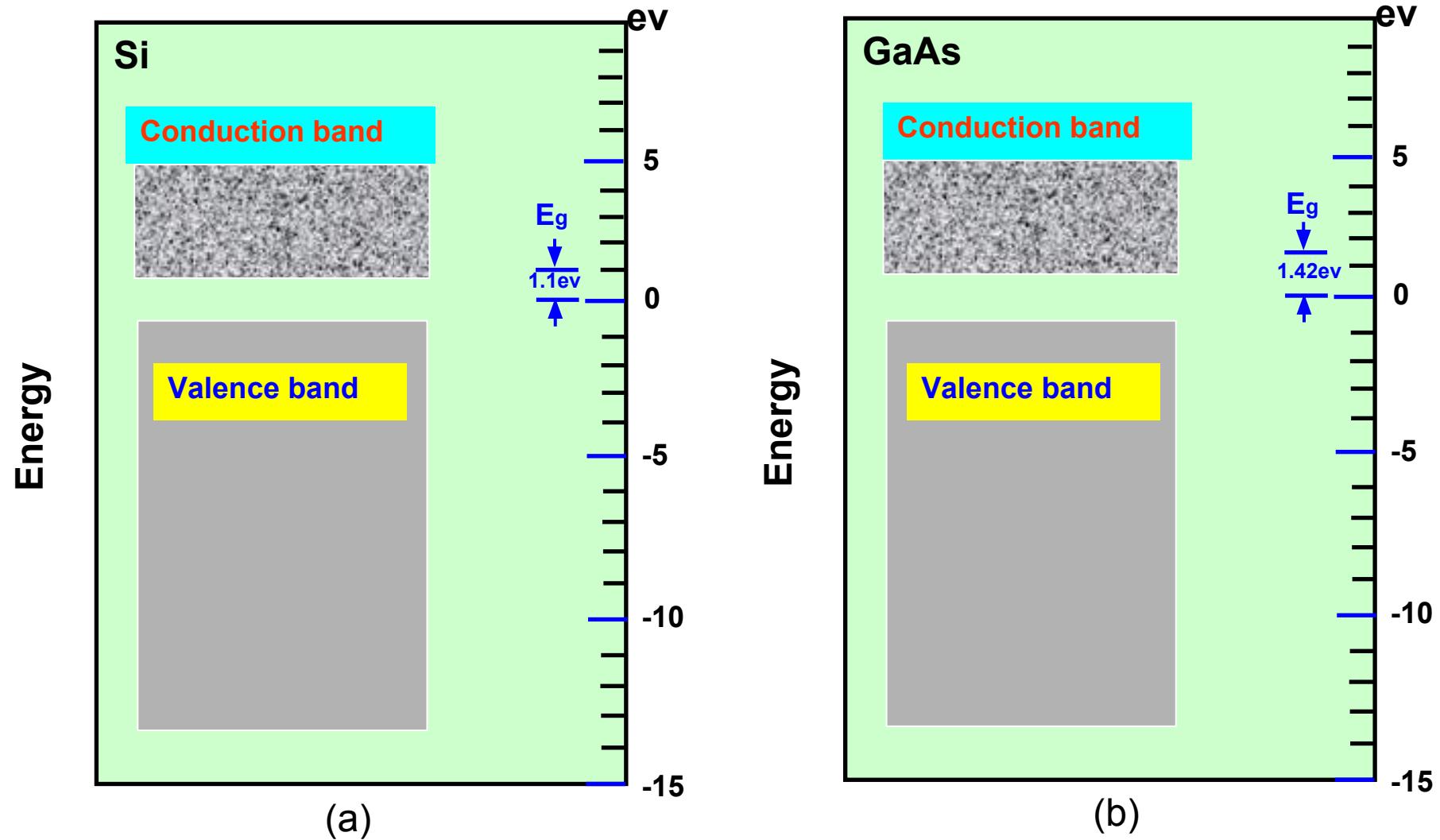
- Each band contains a large number of finely separated discrete energy levels that can be approximated as a continuum.
- The valence and conduction bands are separated by a “forbidden” energy gap of width  $E_g$  → **bandgap energy**



# Energy bands in semiconductors



Materials with a large energy gap ( $>3\text{eV}$ ) are insulators, those for which the gap is small or nonexistent are conductors, semiconductors have gaps roughly in the range 0.1 to 3 eV

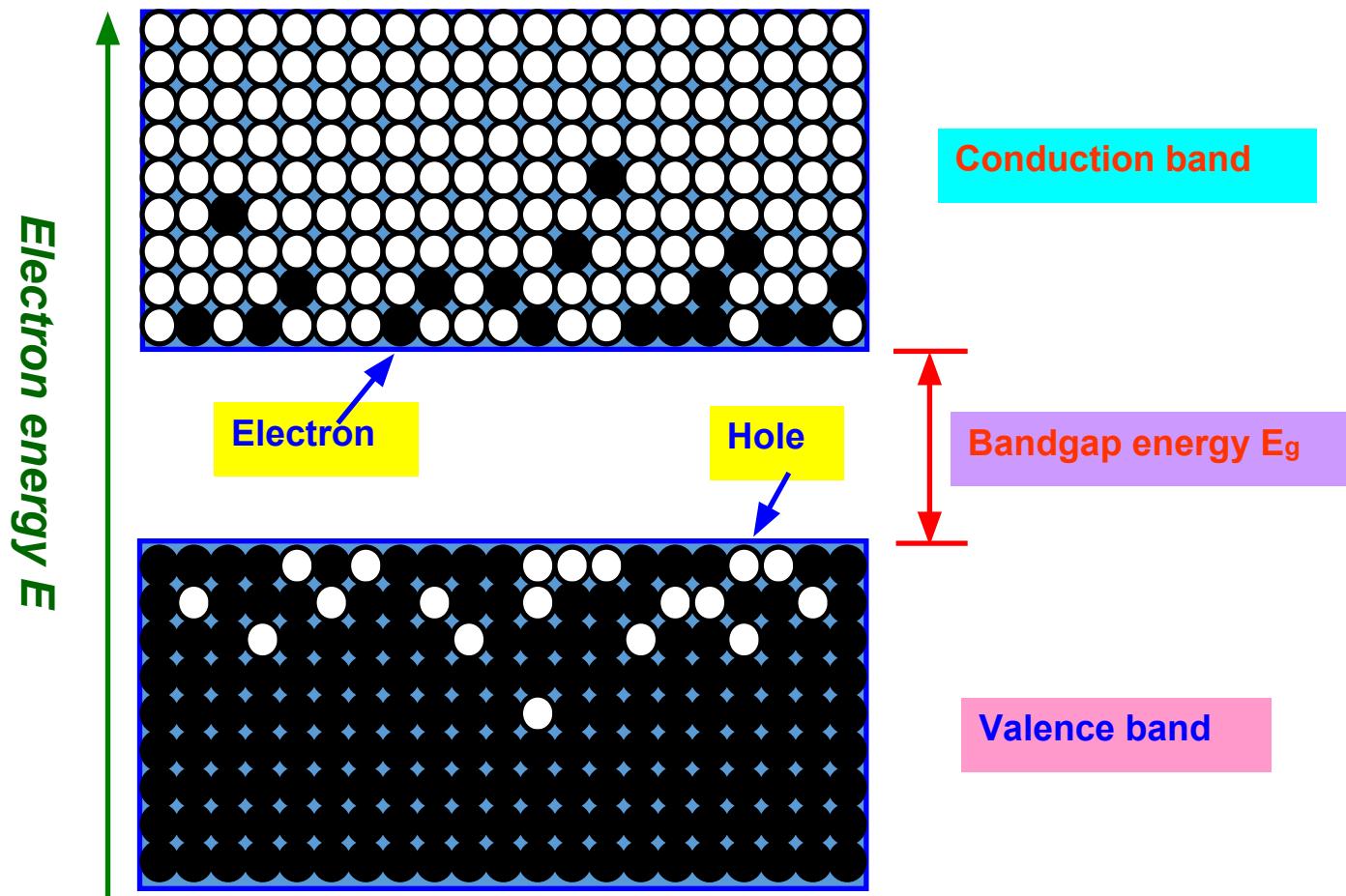


**Figure 15.1-1** Energy bands: (a) in Si, and (b) in GaAs

## Electrons and holes

- In the absence of thermal excitations, the valence band is completely filled and the conduction band is completely empty. Thus, the material cannot conduct electricity.
- As the temperature increases, some electrons will be thermally excited into the empty conduction band, result in the creation of a free electron in the conduction band and a free hole in the valence band.

# Electrons and holes



Electrons in the conduction band and holes in the valence band at  $T > 0$ .

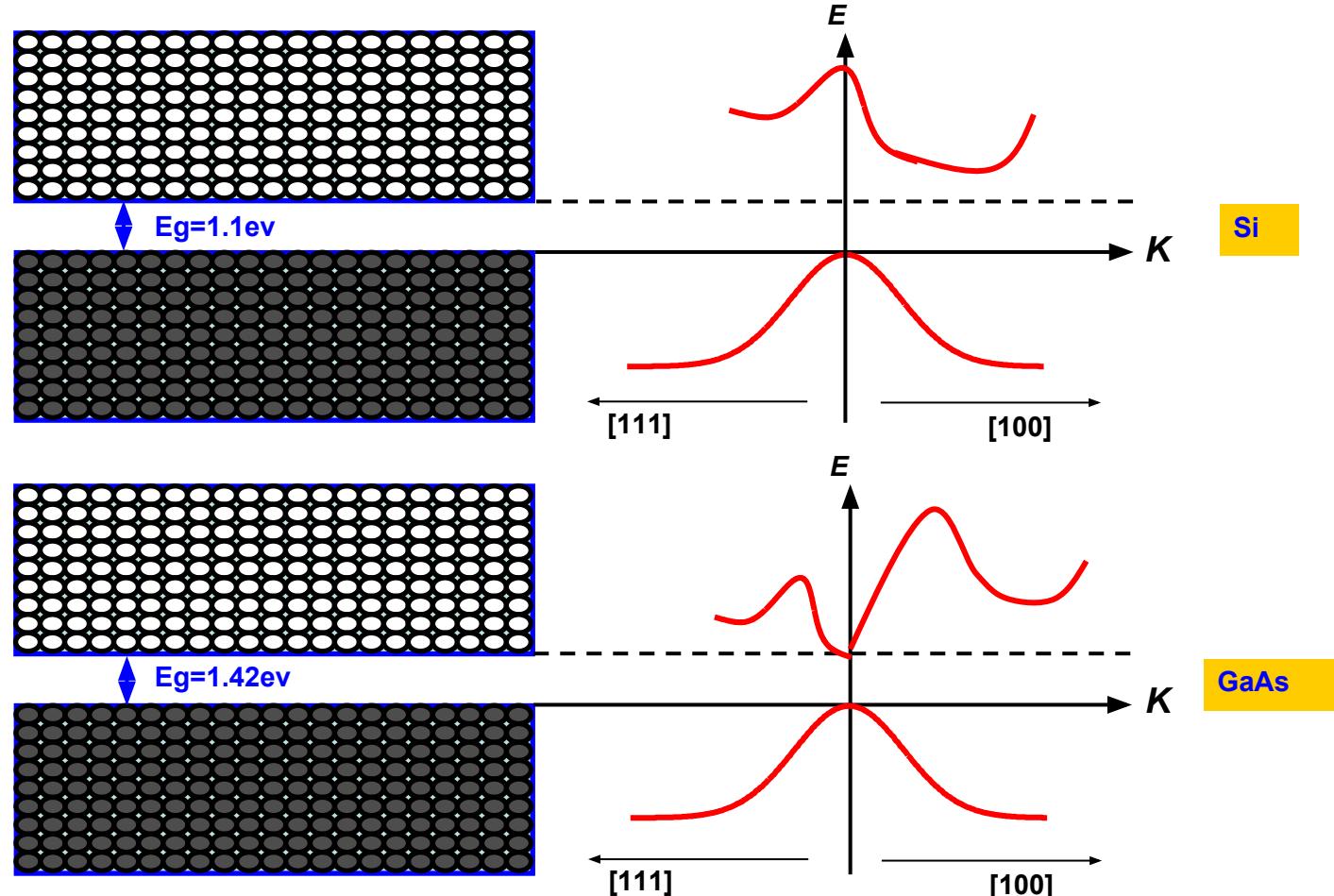
# Energy-momentum relations

$$E = \frac{p^2}{2m_0} = \frac{\hbar^2 k^2}{2m_0}$$

$p$  is the magnitude of the momentum

$$k = \frac{p}{\hbar}$$

$k$  is the magnitude of the wave vector associated with the electron's wave function  
 $m_0$  is the electron mass



**Figure 15.1-3** Cross section of the  $E$ - $K$  function for Si and GaAs along the crystal directions [111] and [100].

The energy of an electron in the conduction band depends not only on the magnitude of its momentum, but also on the direction in which it is traveling in the crystal

# Effective mass

Near the bottom of the conduction band, the  $E$ - $k$  relation may be approximated by the parabola

$$E = E_c + \frac{\hbar^2 k^2}{2m_c}$$

$$E = E_v - \frac{\hbar^2 k^2}{2m_v}$$

$E_c, E_v$ : the energy at the bottom of the conduction band and at the top of the valence band

$m_c, m_v$ : **effective mass** of the electron in the conduction band and the hole in the valence band

The **effective mass** depends on the crystal orientation and the particular band under consideration

# Effective mass

Typical ratios of the average effective masses to the mass of the free electron mass

	$m_c/m_0$	$m_v/m_0$
<b>Si</b>	<b>0.33</b>	<b>0.5</b>
<b>GaAs</b>	<b>0.07</b>	<b>0.5</b>

# Direct- and indirect-gap semiconductors

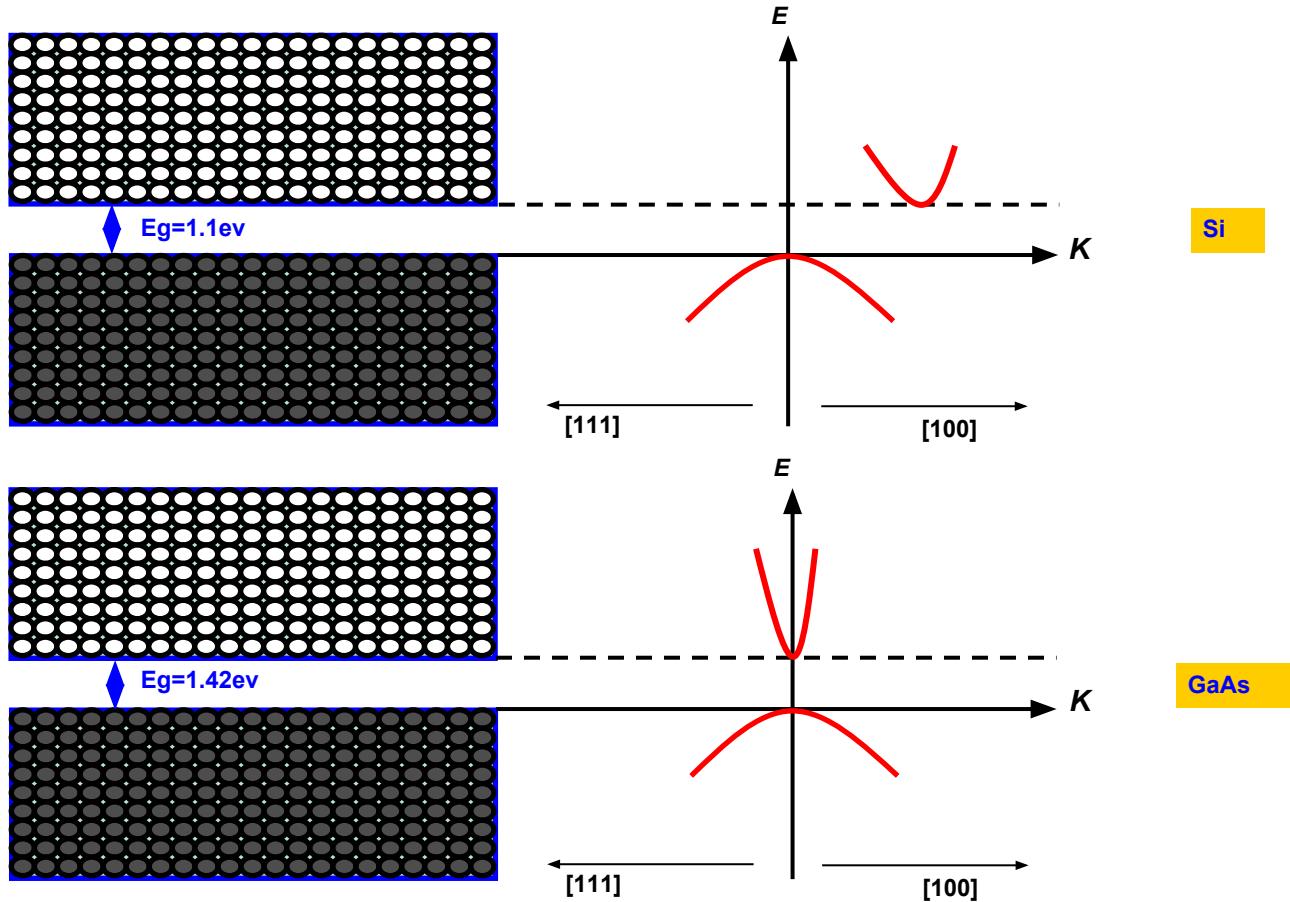


Figure 15.1-4 Approximating the  $E$ - $K$  diagram at the bottom of the conduction band and at the top of the valence band of Si and GaAs by parabols.

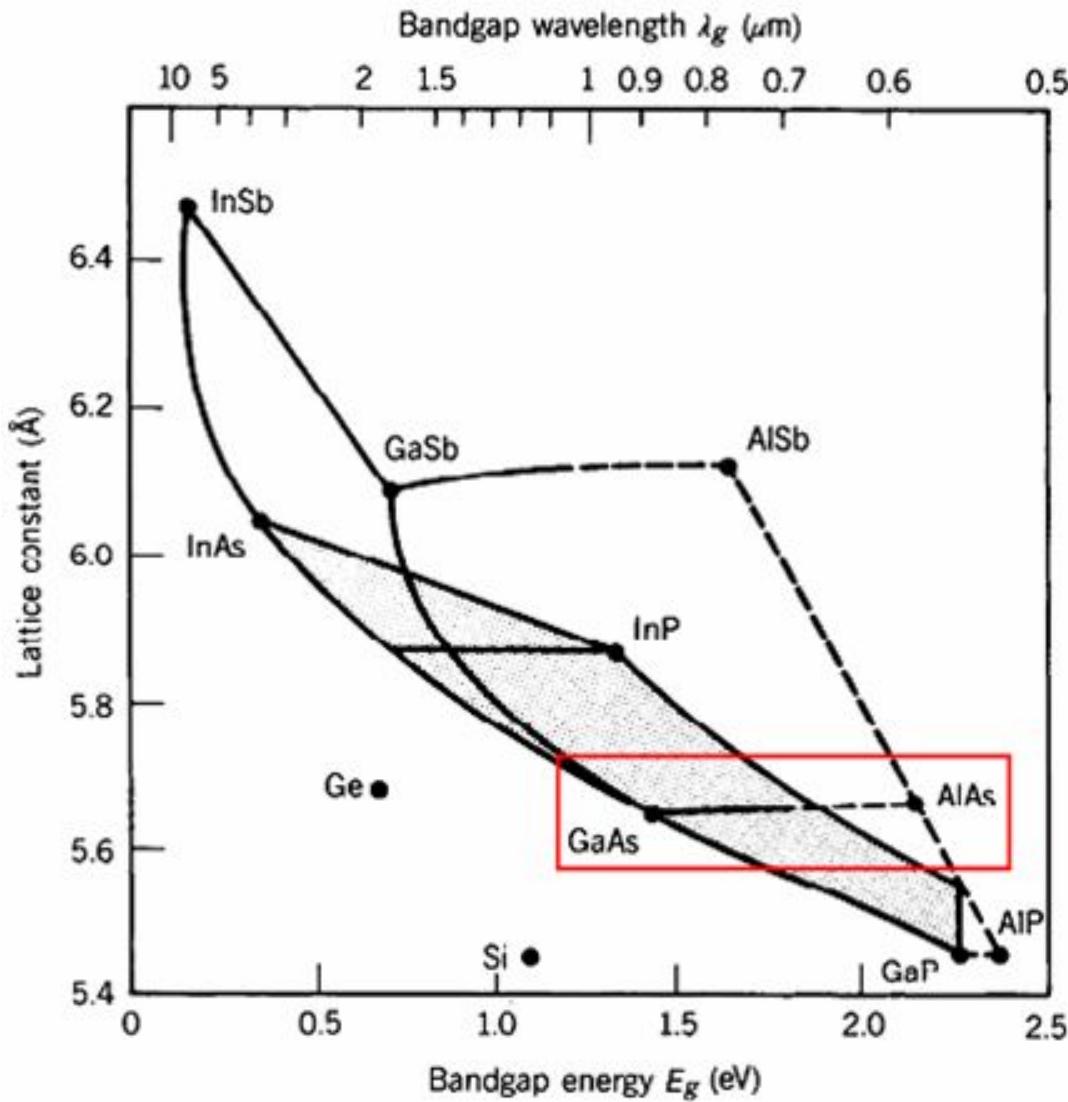
The direct-gap semiconductors such as GaAs are efficient photon emitters, while the indirect-gap counterparts are not.

# Semiconducting materials

# Semiconducting materials

- Si: widely used for making photon detectors but not useful for fabricating photon emitters due to its indirect bandgap.
- **GaAs, InP GaN etc.: used for making photon detectors and sources.**
- Ternary and quaternary semiconductors:  
 $\text{Al}_x\text{Ga}_{1-x}\text{As}$ ,  $\text{In}_x\text{Ga}_{1-x}\text{As}_y\text{P}_{1-y}$  etc.  
tunable bandgap energy with variation of  $x$  and  $y$ .

# Semiconducting materials



$\text{Al}_x\text{Ga}_{1-x}\text{As}$  is **lattice matched** to GaAs, means it can be grown on the GaAs without introducing strains.

Solid and dashed curves represent **direct-gap** and **indirect-gap** compositions respectively.

We can see that a material may have direct bandgap for one mixing ratio  $x$  and an indirect bandgap for a different  $x$ .

# Doped semiconductors

- Dopants: alter the concentration of mobile charge carriers by many orders of magnitude.
- $n$ -type: predominance of mobile electrons

$$n \gg p$$

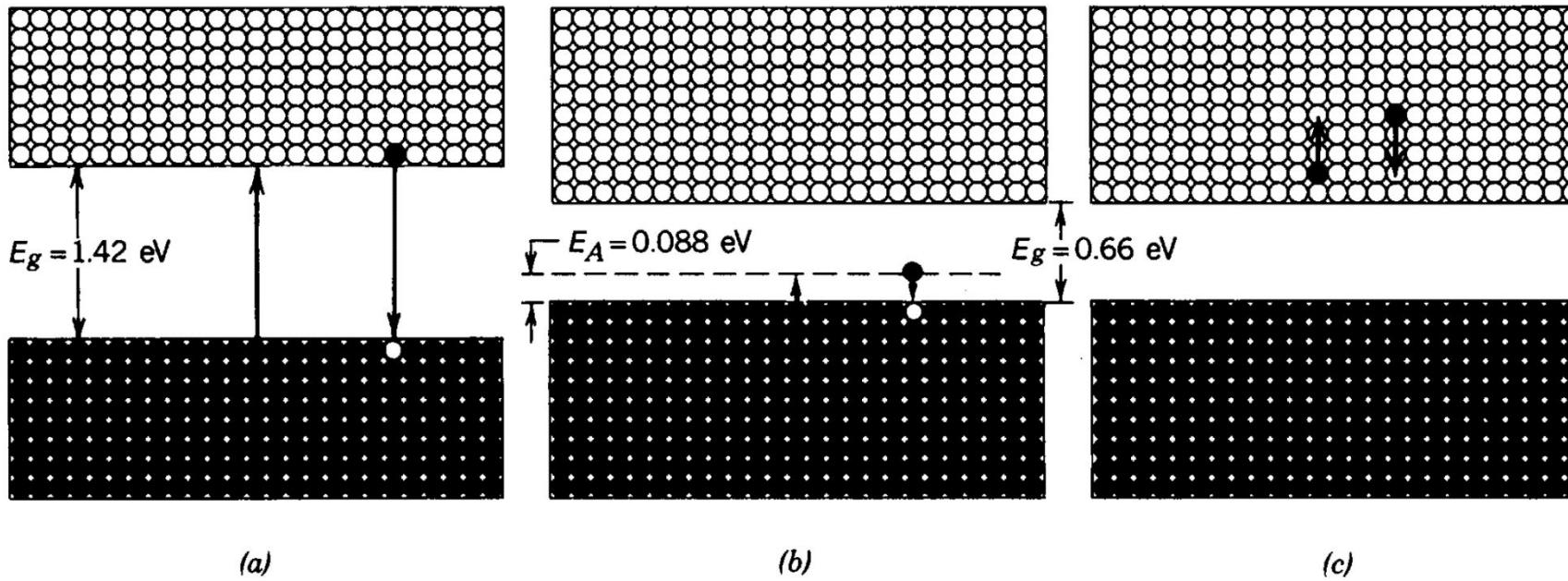
- $p$ -type: predominance of holes

$$p \gg n$$

# **INTERACTIONS OF PHOTONS WITH CHARGE CARRIERS (ELECTRONS AND HOLES)**

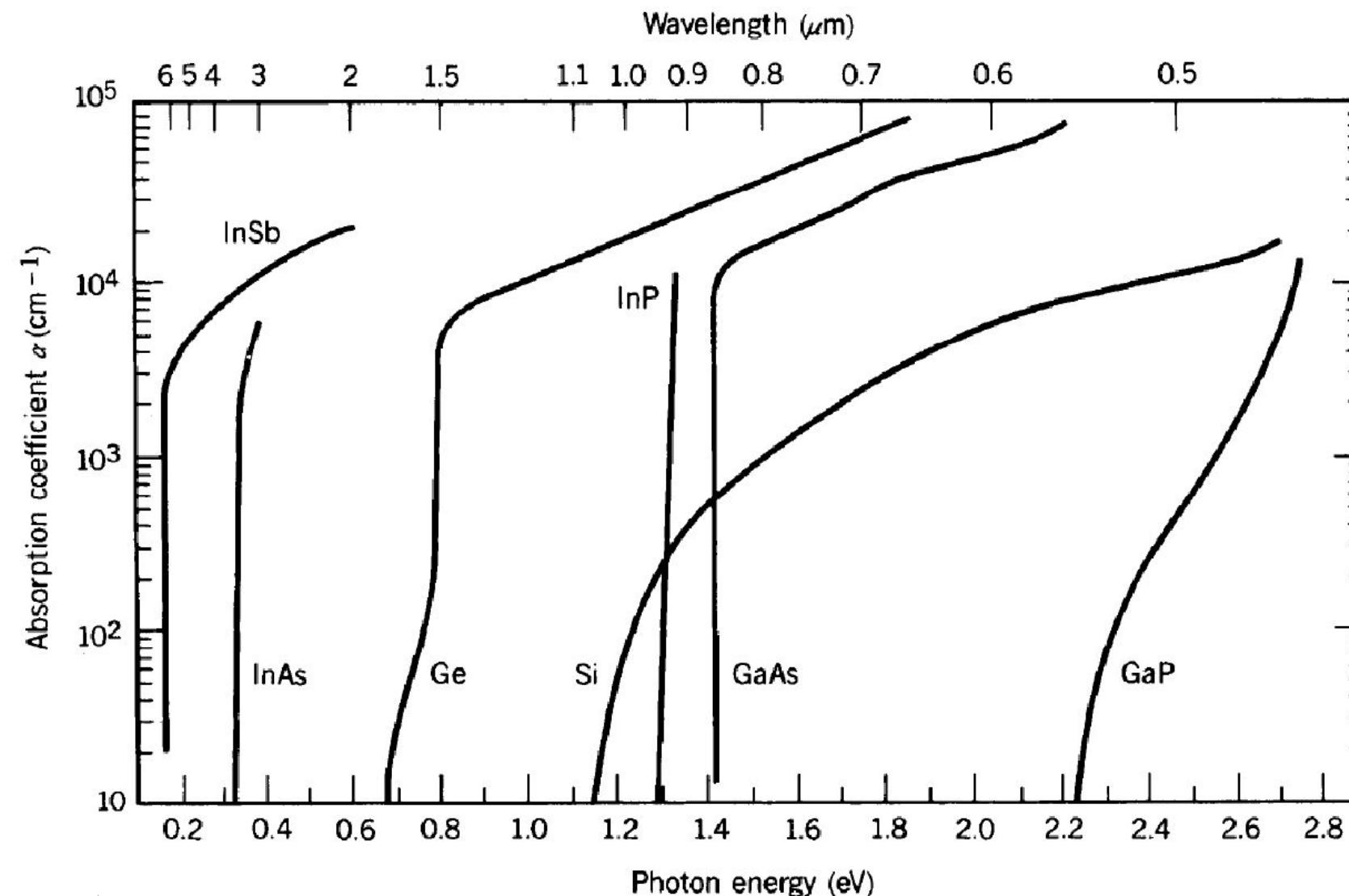
**Mechanisms leading to absorption and emission of photons in a semiconductor:**

- **Band-to-Band (Inter-band) Transitions.**
- **Impurity-to-Band Transitions.**
- **Free-Carrier (Intraband) Transitions**
- **Phonon Transitions**
- **Excitonic Transitions.**

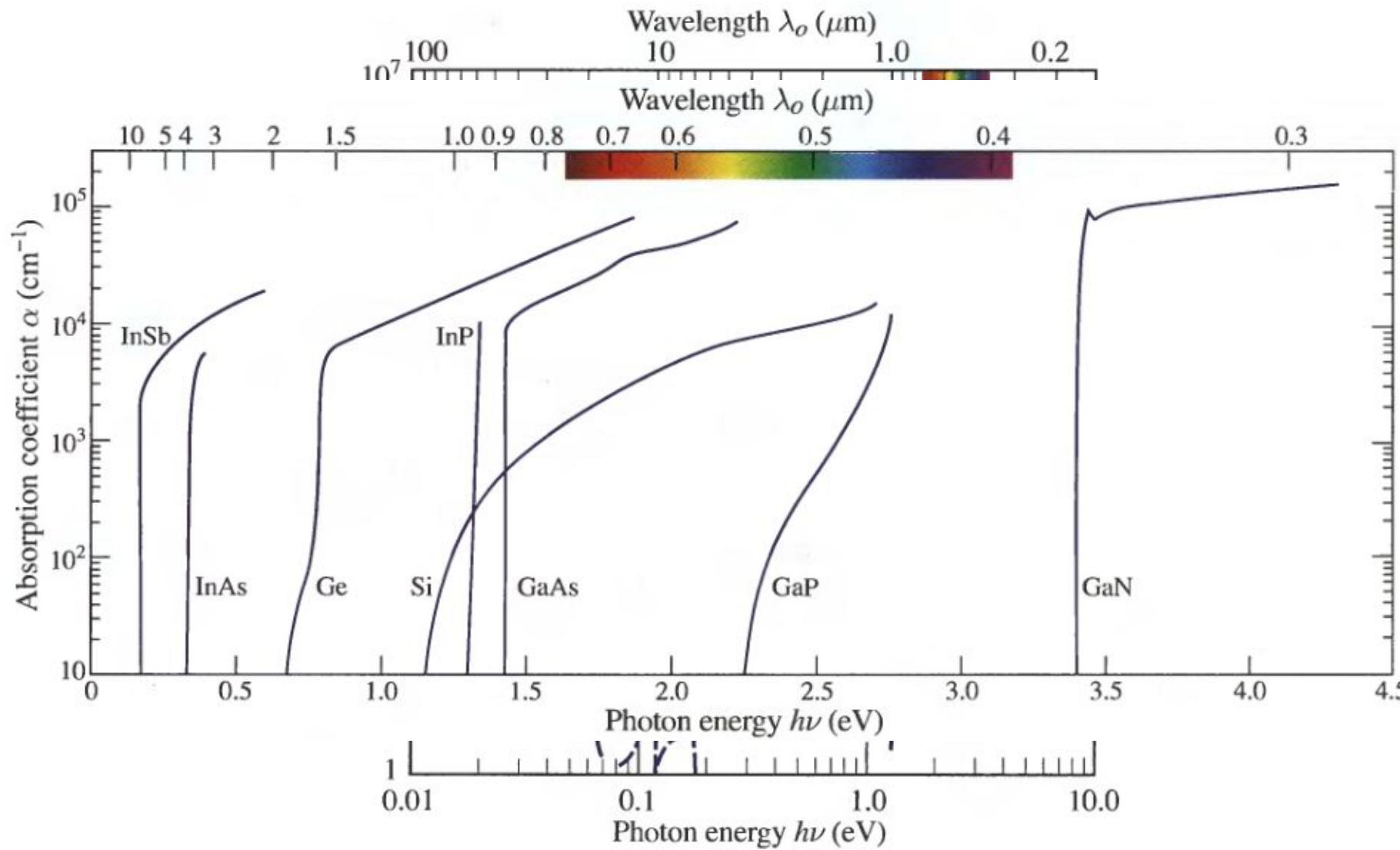


Examples of absorption and emission of photons in a semiconductor. (a) Band-to-band transitions in GaAs can result in the absorption or emission of photons of wavelength  $\lambda_g = hc_0 / E_g = 0.87 \mu\text{m}$ . (b) The absorption of a photon of wavelength  $\lambda_A = hc_0 / E_A = 14 \mu\text{m}$  results in a valence-band to acceptor-level transition in Hg-doped Ge (Ge:Hg). (c) A free-carrier transition within the conduction band.

# Absorption coefficient for some semiconductor materials



# Absorption coefficient for some semiconductor materials



- For photon energies greater than the bandgap energy  $E_g$ , the absorption is dominated by band-to-band transitions

- **Absorption edge:**

The spectral region where the material changes from being relatively transparent to strongly absorbing

- Direct-gap semiconductors have a more abrupt absorption edge than indirect-gap materials

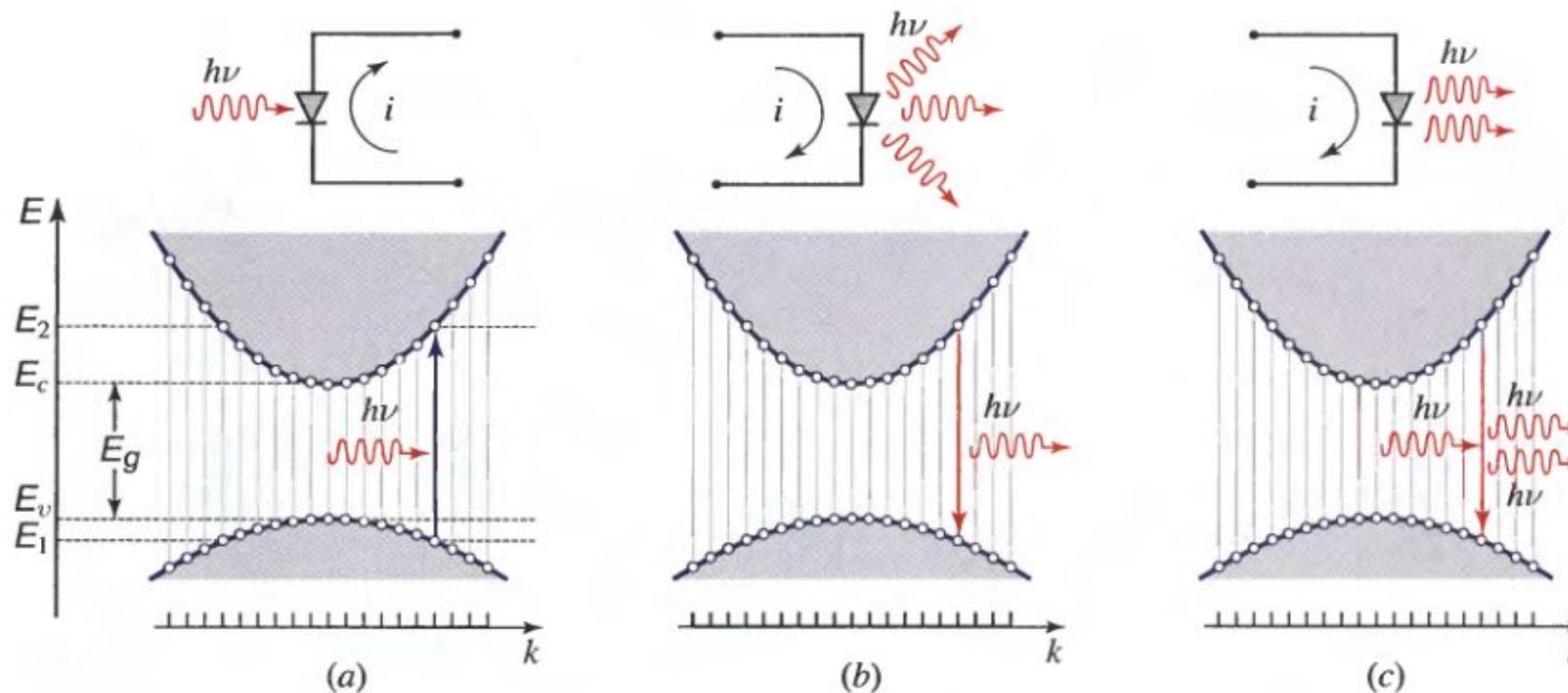
# Band-to-Band Absorption and Emission

**Bandgap Wavelength:**

$$\lambda_g (\mu m) = \frac{1.24}{E_g (eV)}$$

The quantity  $\lambda_g$  is also called the cutoff wavelength.

# Absorption and Emission



- (a) The absorption of a photon results in the generation of an electron-hole pair. This process is used in the photodetection of light.
- (b) The recombination of an electron-hole pair results in the spontaneous emission of a photon. Light-emitting diodes (LEDs) operate on this basis.
- (c) Electron-hole recombination can be stimulated by a photon. The result is the induced emission of an identical photon. This is the underlying process responsible for the operation of semiconductor injection lasers.

# Conditions for Absorption and Emission

- Conservation of Energy

$$E_2 - E_1 = h\nu$$

- Conservation of Momentum

$$p_2 - p_1 = h\nu / c = h / \lambda \dots \text{or}$$
$$\dots k_2 - k_1 = 2\pi / \lambda$$

# Conditions for Absorption and Emission

- Energies and Momenta of the Electron and Hole with Which a Photon Interacts

$$E_2 = E_c + \frac{m_r}{m_c} (h\nu - E_g)$$

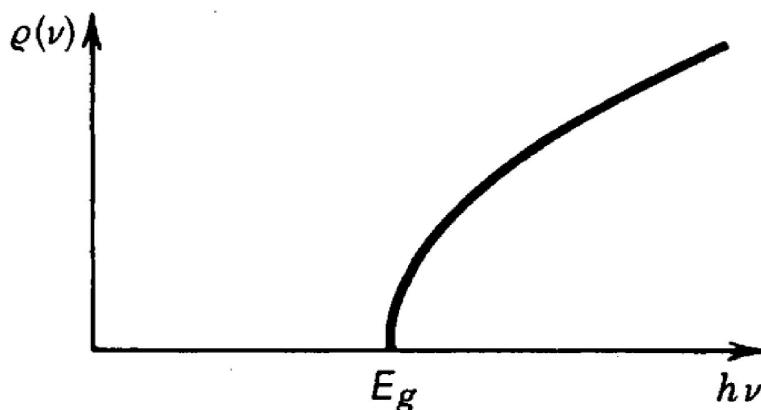
$$E_1 = E_v - \frac{m_r}{m_v} (h\nu - E_g) = E_2 - h\nu$$

# Optical Joint Density of States

The density of states  $\rho(\nu)$  with which a photon of energy  $h\nu$  interacts under conditions of energy and momentum conservation in a direct-gap semiconductor is determined by:

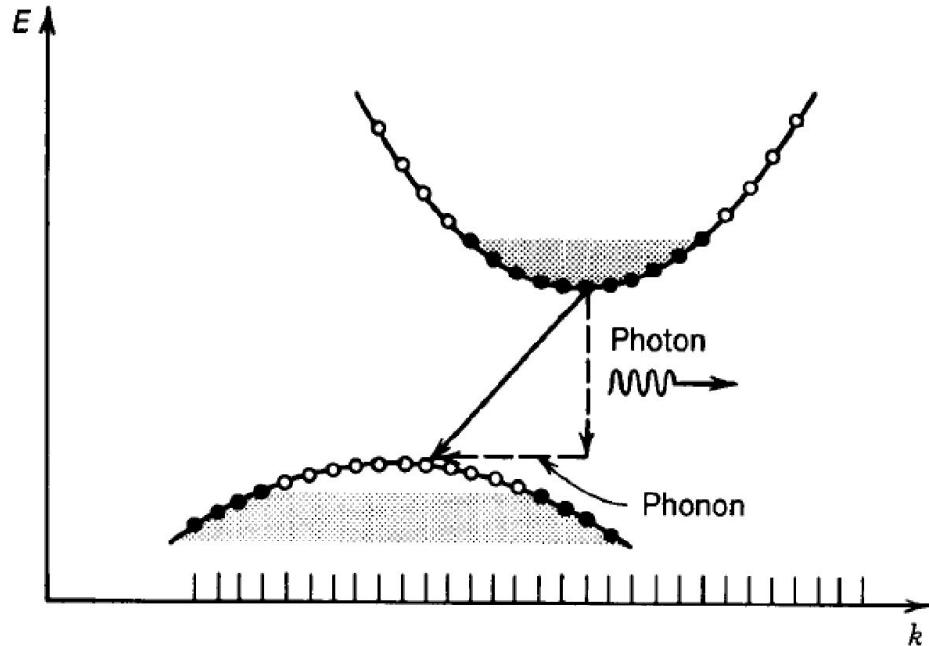
$$\rho(\nu) = \frac{(2m_r)^{3/2}}{\pi\hbar^2} (h\nu - E_g)^{1/2}, h\nu \geq E_g$$

And illustrated as follow:



The density of states with which a photon of energy  $h\nu$  interacts increases with  $h\nu - E_g$  in accordance with a square-root law

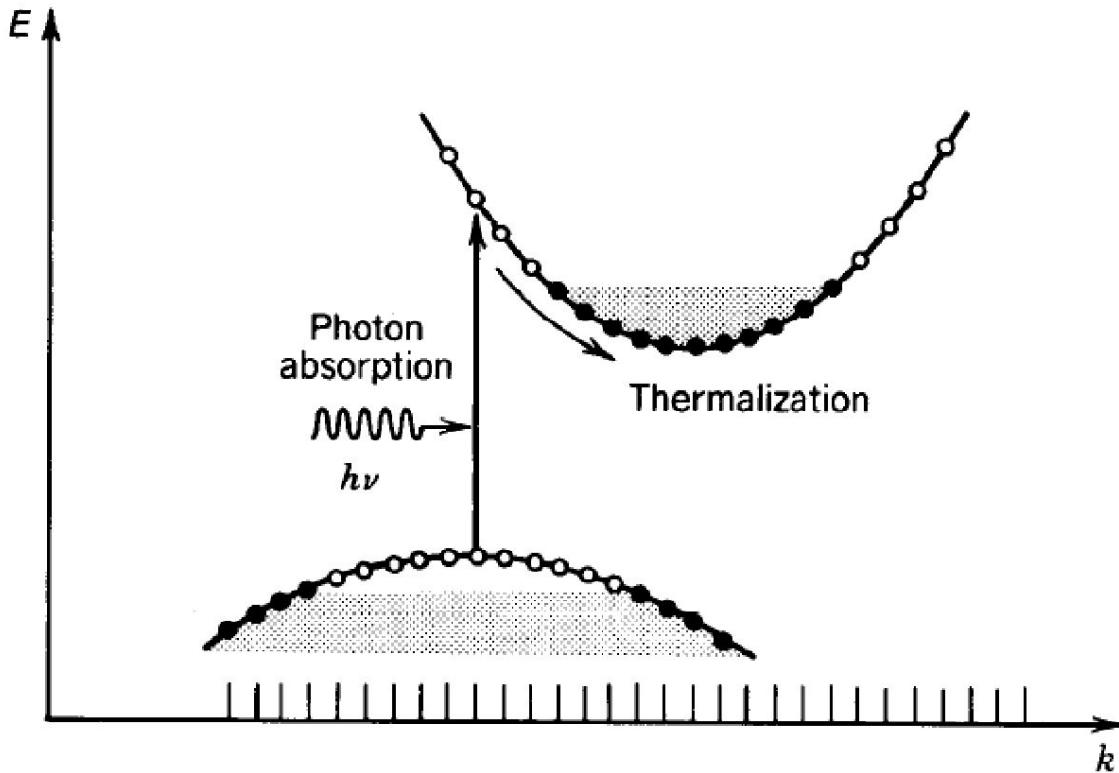
# Photon Emission Is **Unlikely** in an Indirect-Gap Semiconductor !



**Photon emission  
in an indirect-gap  
semiconductor**

The recombination of an electron near the bottom of the conduction band with a hole near the top of the valence band requires the exchange of energy and momentum. The energy may be carried off by a photon, but one or more phonons are required to conserve momentum. This type of multiparticle interaction is unlikely.

# Photon Absorption is **Not Unlikely** in an Indirect-Gap Semiconductor !



**Photon absorption in an indirect-gap semiconductor**

The photon generates an excited electron and a hole by a vertical transition; the carriers then undergo fast transitions to the bottom of the conduction band and top of the valence band, respectively, releasing their energy in the form of phonons. Since the process is sequential it is not unlikely.

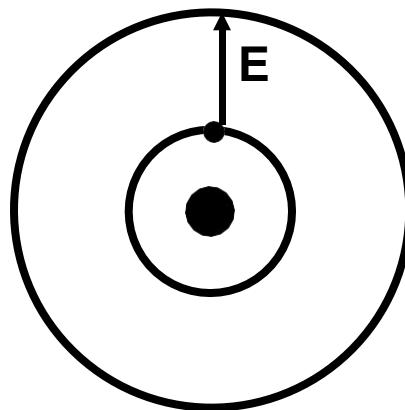
# **Rates of Absorption and Emission**

The probability densities of a photon of energy  $h\nu$  being emitted or absorbed by a semiconductor material in a direct band-to-band transition are mainly determined by three factors:

- Occupancy probabilities
- Transition probabilities
- Density of states

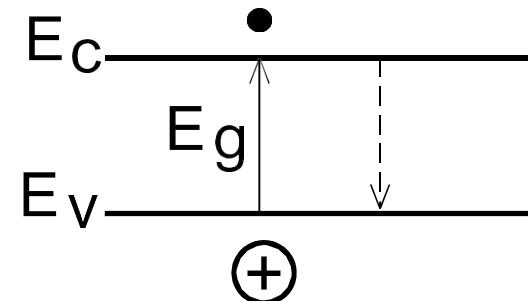
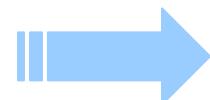
# diagrams

- In the atoms, the larger the radius, the higher the electron potential energy
- Hence, electron position can be described either by radius or by its potential energy
- In the semiconductor crystal: the atom orbits OVERLAP; radius-based description becomes impractical.  
Energy-based description works well:
- The highest orbit filled with electrons becomes the VALENCE BAND
- The higher orbit (nearly empty) becomes the CONDUCTION BAND



Single atom

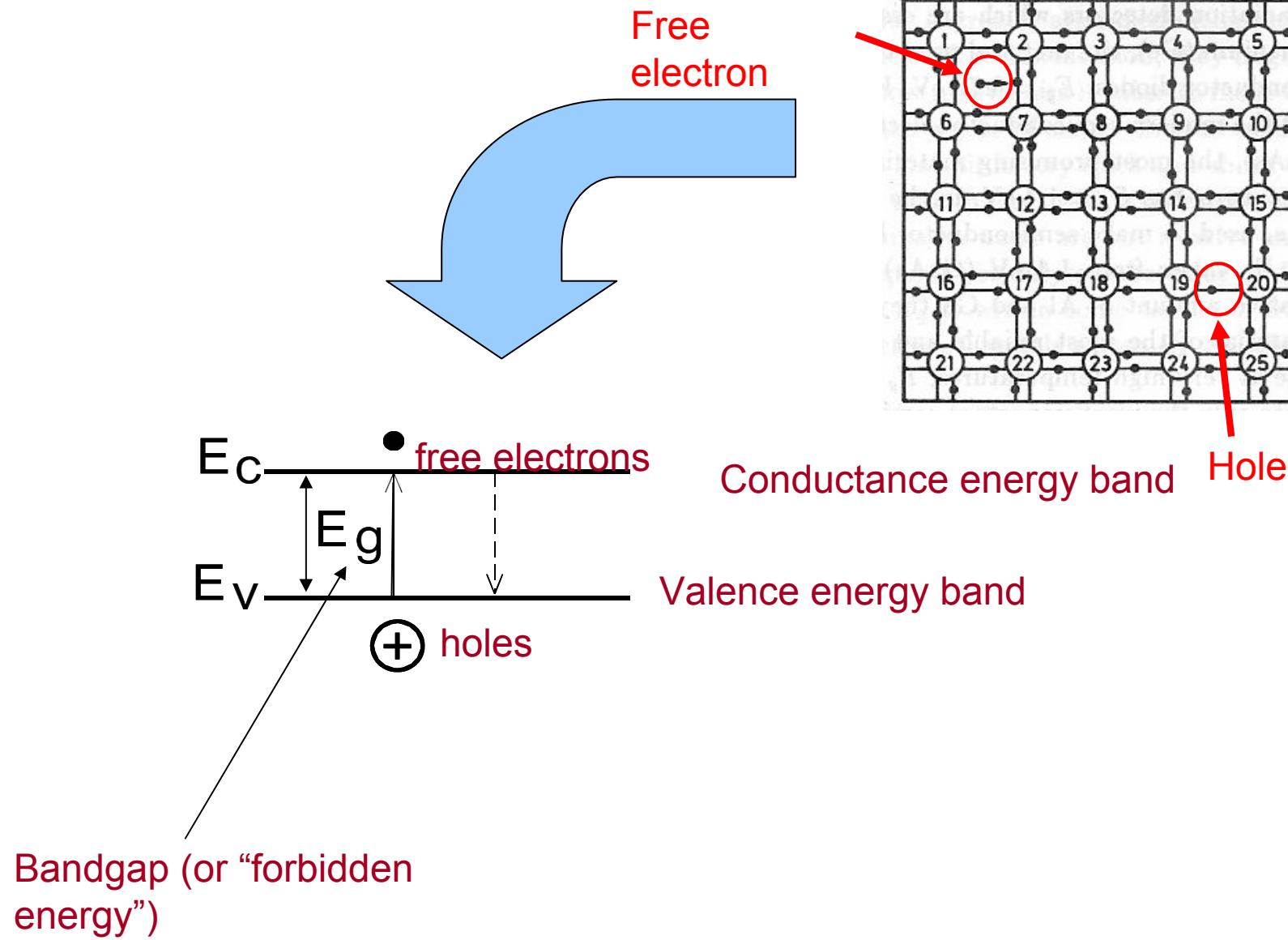
Excited electrons cannot move



Crystal

Excited electrons can move (free electrons)

# Energy band diagrams



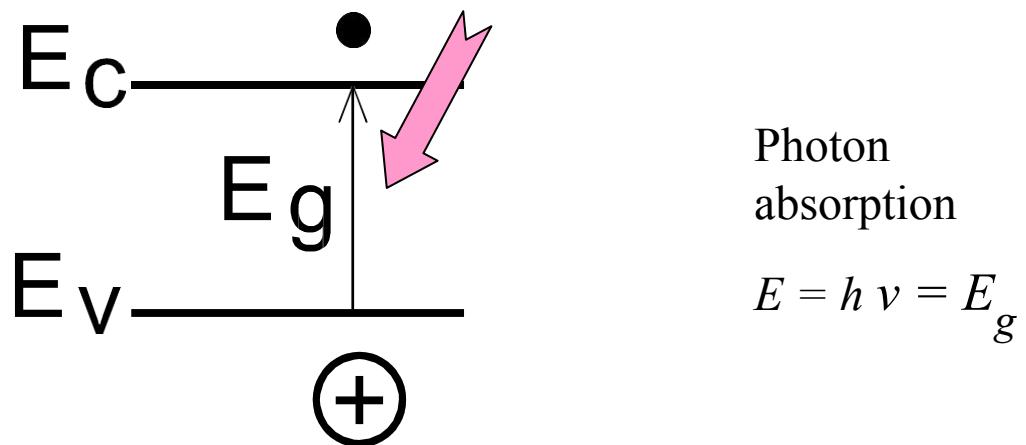
# Optical processes in semiconductors: radiation and absorption

Related electrical processes:  
electron - hole pair generation and recombination

## Absorption Related electrical process:

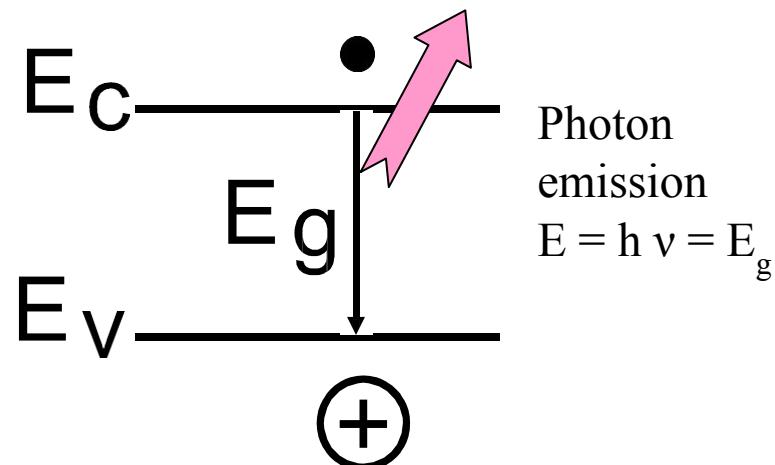
### Electron - Hole pair generation

- The photon with the energy exceeding the bandgap energy of semiconductor can be absorbed.
- The photon disappears; the photon energy excites the electron from the valence band into the conduction band.
- As a result, one e-h pair is being created



# Radiation related electrical process: Electron - Hole Pair Recombination

- When the excited electron meets the hole in the valence band, it may occupy that place. As a result the e-h pair disappear; this process is called *recombination*.
- During recombination, the electron energy is released as a photon with the energy closed to the bandgap energy of the semiconductor.



## Electron and hole concentrations under illumination

- We define  $n_0$  and  $p_0$  as the electron and hole concentrations in the absence of illumination (“dark” concentrations).
- $\Delta n$  and  $\Delta p$  are the additional concentrations generated by light.

$$n = \Delta n + n_0$$

$$p = \Delta n + n_i^2/n_0$$

Note that in the equilibrium for ANY semiconductor,

$$n \times p = n_i^2$$

Under illumination:

$$n \times p \approx (\Delta n + n_0) \times (\Delta n + p_0) >$$

## Generation rate

When the semiconductor is under CONSTANT illumination, the photons are being absorbed at a constant rate; absorbed photons *GENERATE* electron- hole pairs

Therefore the concentration of *e-h* pairs MUST linearly increase with the time.

The GENERATION RATE,  
G, is the number of electron-hole pair generated per unit time:

$$\Delta n = \Delta p = G \times t;$$

How does the semiconductor sample  
come to a steady-state condition under illumination?

## Recombination rate

The probability of electron and hole "annihilation", or the RECOMBINATION rate, is proportional to both electron and hole concentrations:

$$R \sim n \times p = B_r \times n \times p$$

Therefore, when  $n$  and  $p$  increase due to illumination, the RECOMBINATION rate,  $R$ , also increases.

The  $e-h$  concentration increases until the increasing recombination rate would compensate it.

Under the steady state condition we have:

$$G = R;$$

# Steady-state e-h pair concentration

$$\mathbf{G} = \mathbf{R};$$

$$\mathbf{R} = \mathbf{B}_r \times \mathbf{n} \times \mathbf{p}$$

The steady-state concentration of photo-electrons (and photo-holes):

$$\mathbf{G} = \mathbf{B}_r \times \mathbf{n} \times \mathbf{p} \quad \square \quad \mathbf{n} \times \mathbf{p} = \mathbf{G} / \mathbf{B}_r$$

$$n \times p \approx (\Delta n + n_0) \times (\Delta n + p_0)$$

Under strong illumination,  $\Delta n \gg n_0$  and  $\Delta n \gg p_0$

$$n \times p \approx \Delta n^2 \quad \square \quad G = B_r \times \Delta n^2:$$

$$\Delta n = (G / B_r)^{1/2}$$

# Spontaneous and Excessive Recombination Rate

In case of direct electron - hole recombination

$$R_{sp} = B_r(n_o + \Delta n)(p_o + \Delta p)$$

$$R_{sp} = B_r[n_o p_o + \Delta n(n_o + p_o) + (\Delta n^2)]$$

The spontaneous recombination rate in the equilibrium as

$$R_{sp}^o = B_r n_o p_o$$

$$R_{sp}^{ex} = B_r \Delta n [n_o + p_o + \Delta n]$$

$$R_{sp} = R_{sp}^o + R_{sp}^{ex}$$

## Electron and hole LIFETIME

The recombination rate  $R$  is often expressed as

$$R = \frac{\Delta n}{\tau}$$

The lifetime,  $\tau$ , determines the mean time an electron spends before recombining with hole.

$$R_{sp}^{ex} = B_r \Delta n [n_o + p_o + \Delta n]$$

As follows from

at very high excitation level,  $\Delta n \gg n_0, p_0$

$$\tau_r \cong \frac{1}{B_r \Delta n}$$

At low excitation level,  $\Delta n \ll n_0, p_0$

$$\tau_r \cong \frac{1}{B_r(n_o + p_o)}$$

## Example

Optical beam irradiating an intrinsic semiconductor (GaAs) produces  $0.5 \times 10^{23} \text{ cm}^{-3}/\text{s}$  electron-hole pairs.

The steady state concentration of photoelectrons is  $\Delta n = 10^{14} \text{ cm}^{-3}$ .

- 1) Find the electron /hole recombination lifetime  $\tau$ .
- 2) Find the radiative recombination coefficient  $B_r$

# Solution

In steady state,

$$G = R$$

The recombination rate,

$$R = \frac{\Delta n}{\tau}$$

Therefore,

$$G = \frac{\Delta n}{\tau}$$

The lifetime

$$\tau = \frac{\Delta n}{G}$$

$$\tau = \frac{\Delta n}{G} = \frac{10^{14} \text{ cm}^{-3}}{0.5 \cdot 10^{23} \text{ cm}^{-3} \text{ s}^{-1}} = 2 \cdot 10^9 \text{ s} = 2 \text{ ns}$$

**The electron /hole recombination lifetime  $\tau$  is 2 ns**

# Solution

In GaAs,  $n_i \sim 10^5 \text{ cm}^{-3}$ , therefore,  $\Delta n \gg n_0$ .

In this case,

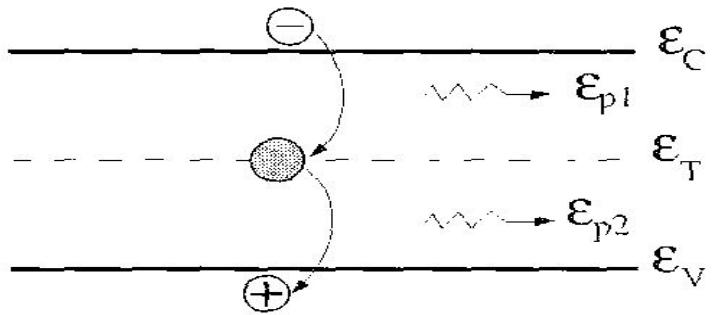
$$\tau_r \approx \frac{1}{B_r \Delta n}$$

$$B_r = \frac{1}{\Delta n \tau_r} = \frac{1}{10^{14} \text{ cm}^{-3} \cdot 2 \cdot 10^{-9} \text{ s}} = 5 \cdot 10^{-6} \text{ cm}^3 \text{s}^{-1}$$

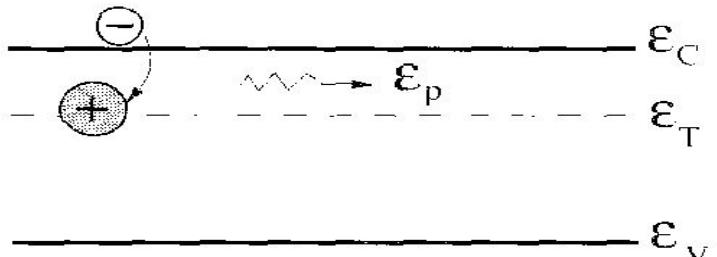
The radiative recombination coefficient  $B_r$  is  $5 \times 10^{-6} \text{ cm}^3 \text{s}^{-1}$

# Radiative and nonradiative recombination

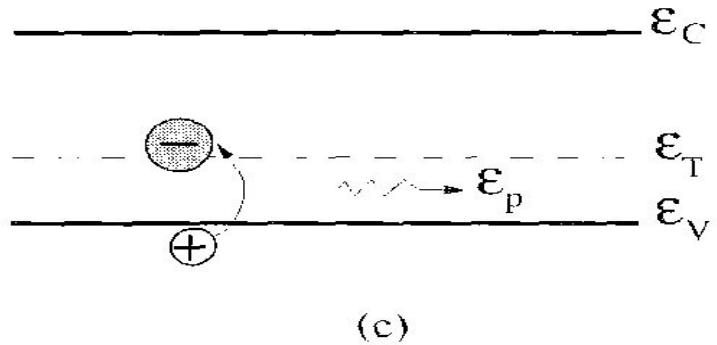
Nonradiative recombination typically does not produce photons;  
The electron energy is being transferred to phonons, i.e. into the heat.



(a)



(b)



(c)

# Radiative and nonradiative recombination

When both radiative and nonradiative recombination processes take place in semiconductor,

$$R_{\text{total}} = R_r + R_{nr} = R_{sp}$$

$$\frac{1}{\tau} = \frac{1}{\tau_r} + \frac{1}{\tau_{nr}}$$

## **Effect of Electric Field on Absorption - Franz—Keldysh effect**

- A strong electric field applied to a semiconductor tilts the band edges, valence band maximum, and conduction band minimum, and the wave functions for electrons and holes penetrate into the band-gap.
- The photo- assisted tunnelling therefore occurs at a wavelength longer than the wavelength corresponding to the band-gap energy.
- As a result, the fundamental absorption edge shifts to a longer wavelength.
- This phenomenon is called the Franz - Keldysh effect and it is remarkable when the field strength is more than  $10^4$  V/cm.

# Effect of Electric Field on Absorption - Franz—Keldysh effect

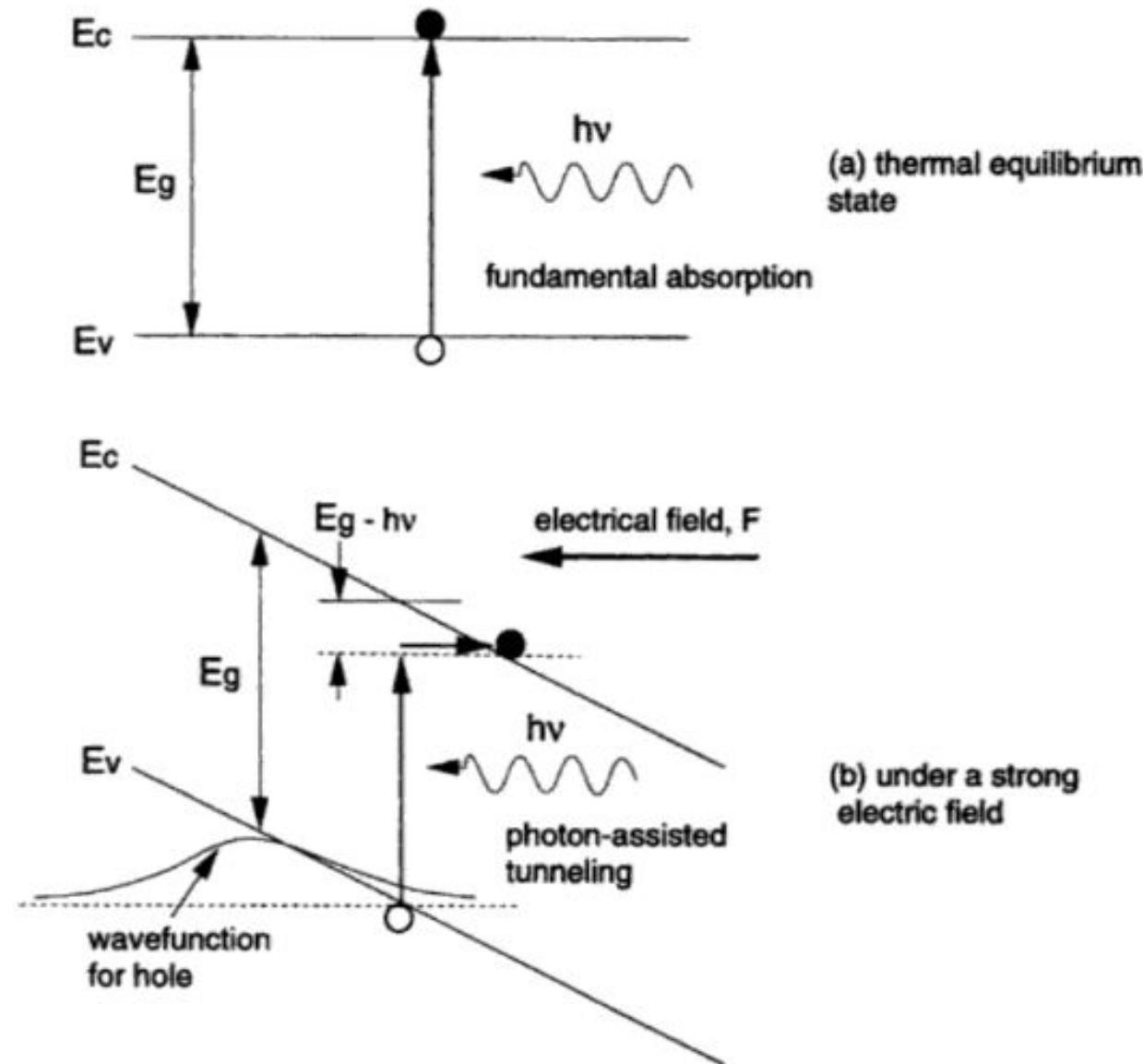


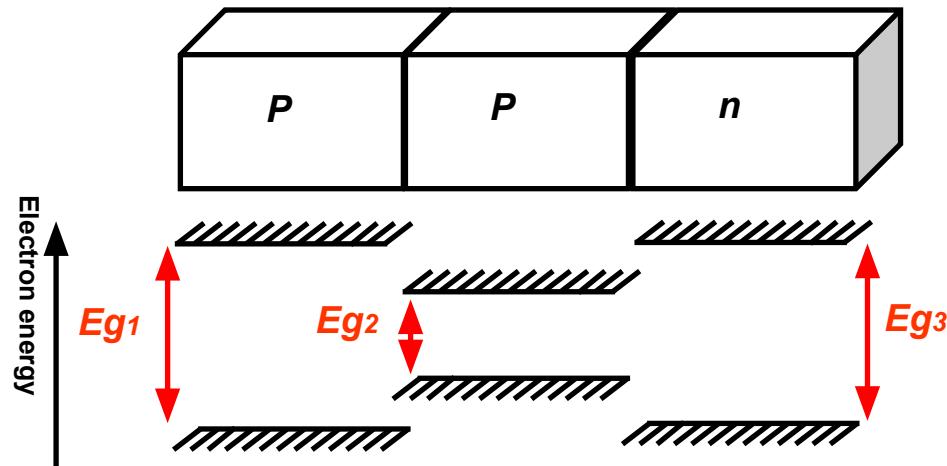
Fig: Schematic diagram of the tilting of energy bands under a strong electric field and of the Franz-Keldysh effect

## **Effect of Electric Field on Absorption - Franz—Keldysh effect**

- The electric-field-dependent absorption coefficient becomes small as photon energy,  $h\nu$ , decreases from band-gap energy,  $E_g$ , and it changes exponentially with  $(E_g - h\nu)$ .
- Consequently, light can be modulated by changing the absorption coefficient (intensity modulation) if the wavelength of the incident light is near the fundamental absorption edge.
- For light having a wavelength longer than that corresponding to the band-gap energy, however, the change in refractive index is dominant and a phase modulator is formed.
- These changes are proportional to the square of the intensity of the electric field.

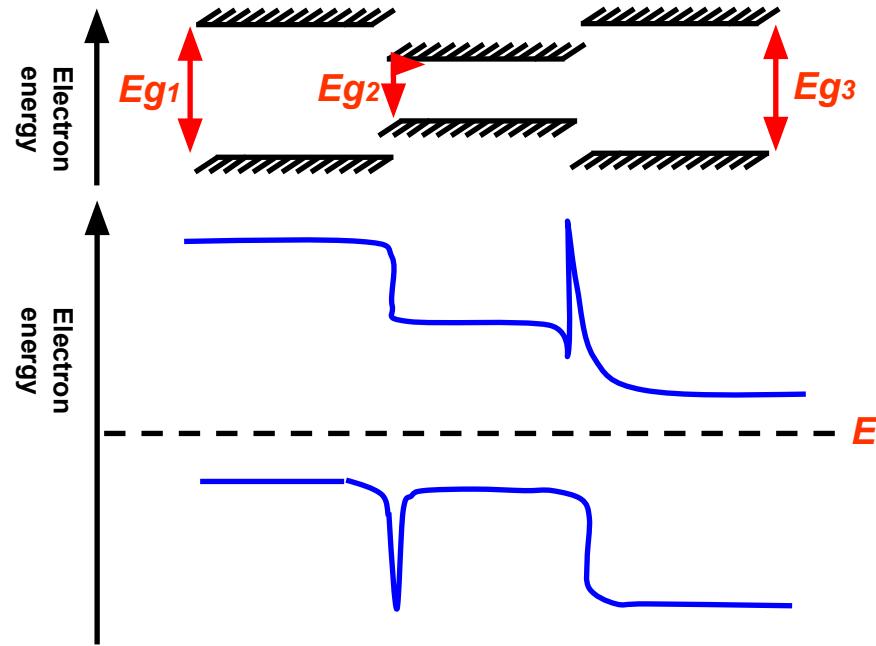
# Heterojunctions

Junctions between different semiconductor materials are called **heterojunctions**.



They can provide substantial improvement in the performance of electronic and optoelectronic devices

Junctions between materials of different bandgap create localized jumps in the energy-band diagram



A potential energy discontinuity provides a barrier that can be useful in preventing selected charge carriers from entering regions where they are undesired.

This property used in a p-n junction can reduce the proportion of current carried by minority carriers, and thus to increase injection efficiency

- Discontinuities in the energy-band diagram created by two heterojunctions can be useful for confining charge carriers to a desired region of space
- Heterojunctions are useful for creating energy-band discontinuities that accelerate carriers at specific locations
- Semiconductors of different bandgap type can be used in the same device to select regions of the structure where light is emitted and where light is absorbed
- Heterojunctions of materials with different refractive indices create optical waveguides that confine and direct photons.

# References

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