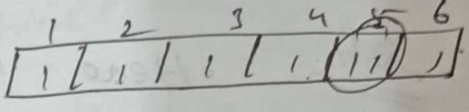
Pigeonhole principle :-

If $(n+1)$ pigeon occupies 'n' pigeon holes then atleast one hole has more than one pigeon.



7 P

Pigeon → object
Pigeonholes → boxes.

The Extended Pigeonhole principle :-

(or) generalisation of the pigeonhole principle :-

If m pigeon occupies 'n' holes $(m>n)$ then atleast one hole has more than

$\left\lfloor \frac{m-1}{n} \right\rfloor + 1$ pigeons. (or floor function)

Here $\lfloor x \rfloor$ denotes greatest integer less than or equal to x, which is a real number.

Note :- $\lfloor 5.3 \rfloor = 5$
$\lfloor 2.9 \rfloor = 2$
↳ integer value only consider

If n pigeonholes are occupied by $n+1$ or more pigeon then atleast one pigeonhole occupied by more than one Pigeon

* If n pigeons are accomodated in m pigeonholes $(n>m)$ then atleast one pigeonhole will contains two or more pigeons.
more than one pigeon

problems :-

1. Show that, among 100 people, at least 9 of them were born in the same month.

$\frac{100}{12}$

Soln Here No. of pigeon $= m$

$= $ no. of people $= 100$

No. of Holes $= n =$ No. of month $= 12$

Then by generalized pigeon hole principle, we have

$$\left\lfloor \frac{m-1}{n} \right\rfloor + 1 = \left\lfloor \frac{100-1}{12} \right\rfloor + 1$$

$\lfloor 8.25 \rfloor + 1$

$= 9$, were born in the same month.

$\left\lceil \frac{100}{12} \right\rceil$

8.33, $12 \overline{)100}$, 96, 40, 36, 4

(2). Show that if seven colours are used to paint 50 bicycles, at least 8 bicycles will be the same colour.

Soln Here, No. of pigeon $= m$

$=$ No. of bicycle $= 50$. ①

No. of Holes $= n =$ No. of colours $= 7$

By generalized pigeon hole principle,

we have

$$\left\lfloor \frac{m-1}{n} \right\rfloor + 1 = \left\lfloor \frac{50-1}{7} \right\rfloor + 1$$

$$= 7 + 1 = 8$$

colour denote - m pigeon hole

bicycle - n pigeon

& $m < n$

$\frac{50}{7} = 7$

$\therefore$ 7 bicycles each of 7 colour.

$\therefore$ atleast 8 bicycles will have the same colour. [remainder 1. will be a colour from the 7.]

$\therefore$ 8 bicycles may have a same colour.

(3) prove that in any group of six people there must be atleast three mutual friends or three mutual enemies.

Soln Let the six people be A, B, C, D, E & F.

Fix A. The remaining five people can accommodate into two groups namely

[5 people with 2 groups]

(1) Friends of A &

(2) Enemies of A.

Now by generalized pigeon hole principle atleast one of the group must contain $\left\lfloor \frac{5-1}{2} \right\rfloor + 1 = 3$ people.

the same grade if there are five possible grade A, B, C, D & F ?

Soln Number of grads = Number of pigeonholes

$= 5 = n$

Let m be the number of Students (pigeon) in discrete mathematics class

Now, by generalized pigeonhole principle, we have

$\left\lfloor \frac{m-1}{n} \right\rfloor + 1 = 6$

$\left\lfloor \frac{m-1}{5} \right\rfloor + 1 = 6$

$\left\lfloor \frac{m-1}{5} \right\rfloor = 5$ ~~539~~ 561 66

$m-1 = 25$

$m = 25+1$

$= 26$

$\therefore$ Total number of Students = 26 students

(5) Show that in a group of 50 students atleast 5 are born in same month.

Here, No. of pigeon $= m$

$= $ no. of students

$= 50.$

No. of Holes $= n =$ No. of months

$= 12.$

Then by generalized pigeonhole principle we have

$$\left\lfloor \frac{m-1}{n} \right\rfloor + 1 = \left\lfloor \frac{50-1}{12} \right\rfloor + 1$$

$$= \left\lfloor \frac{49}{12} \right\rfloor + 1$$

$$= 4 + 1 = 5.$$

$\therefore$ atleast 5 students are born in same month.

How many students do you need in a school to guarantee that there are atleast 2 students who have the same first two initials?

Soln. objects → student
└→ N+1

boxes
└→ N initials

How many initials are possible.

AA	AB	----	AZ
BA	BB	----	BZ
⋮			
ZA	ZB	----	ZZ.

↓ 26 ↓ 26.

$26^2 = 676$

There are 676 boxes of first two initials. It should be 677

(7) How many friends must you have to guarantee that atleast five of them will have birthday in the same month.

m = no. of friends

M = 12

∴ by generalized Pigeonhole principle.

$\left\lfloor \frac{m-1}{M} \right\rfloor + 1 = 5$.

$$\left\lfloor \frac{n-1}{12} \right\rfloor + 1 = 5$$

$\frac{49}{12}$ 4.

$$\frac{n-1}{12} = 4.$$

$$n-1 = 48$$

$$n = 49.$$

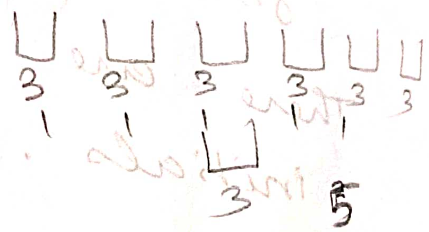
Among 49 friends atleast five of them will have birthday in same month

(8) Suppose there are 26 students & 7 cars to transport them. Show that atleast one car must have 4 or more passengers.

Soln

m = no. of pigeon = no. of students

$= 26.$



n = no. of pigeonhole

= no. of cars $= 7.$

$\therefore$ by Generalized pigeonhole principle, we have. ie $\left\lfloor \frac{m-1}{n} \right\rfloor + 1.$

$$\left\lfloor \frac{26-1}{7} \right\rfloor + 1 = \left\lfloor \frac{25}{7} \right\rfloor + 1$$

$$= \lfloor 3.5 \rfloor + 1 = 4.$$

$\therefore$ one car must have 4 or more students (passengers).

9) Prove that if any 30 peoples are selected, then we may choose a subset of 5 so that all 5 were born on the same day of the week.

Soln $m = 30$.

$n = 7$ days of the week.

$\therefore$ By generalized pigeon hole principle we have $\left\lfloor \frac{m-1}{n} \right\rfloor + 1$.

$\left\lfloor \frac{30-1}{7} \right\rfloor + 1 = \left\lfloor \frac{29}{7} \right\rfloor + 1 = \lfloor 4.1 \rfloor + 1$

$= 5$.

ie 5 people were born on the same day of the week.

10. Prove that if 6 integers are selected from $\{3,4,5,6,7,8,9,10,11,12\}$ there must be 2 integers whose sum is 15.

Soln Label 5 boxes with pairs of numbers that sum to 15 as shown.

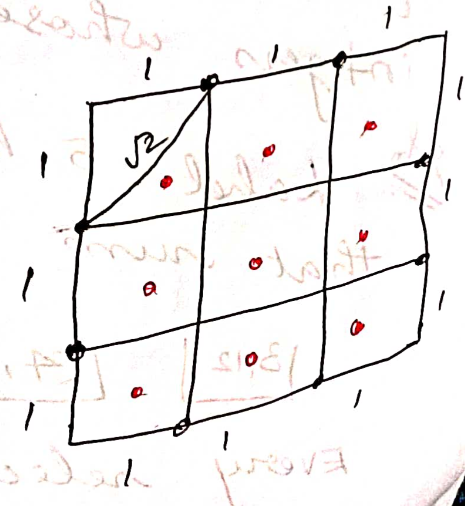
$\boxed{3,12}$ $\boxed{4,11}$ $\boxed{5,10}$ $\boxed{6,9}$ $\boxed{7,8}$

Every selected integer is placed into

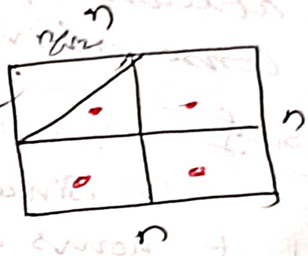
the box with the matching label
we need to place 6 selected integers
into 5 boxes. So by the PHP, a
box must have atleast 2 integers.
so there must be 2 integers
selected that sum to 15.

11. Prove that if 10 points are placed in a
3cm by 3cm square then two points must
be less than or equal to $\sqrt{2}$ cm apart.

Sol: Divide the square into 9 squares that
are 1 cm by 1 cm.
We have 10 points to place in the 9 squares
so, by the PHP, there must be a square
that has more than 1 pt on or
within its boundary. So this square
has two pts less than or equal to
$\sqrt{2}$ cm apart.



6. Given a $n \times n$ square prove that if 5 points are placed randomly inside the square then two of them are atmost $(n/\sqrt{2})$ unit apart.



Soln Divide the square into 4 squares that are $n/\sqrt{2}$ by $\frac{n}{\sqrt{2}}$ unit.

We have 5 points to place with 4 squares, so by the PHP, there must be a square that has more than 1 pt on or within its boundary. So this square has two pts less than or equal to $\frac{n}{\sqrt{2}}$

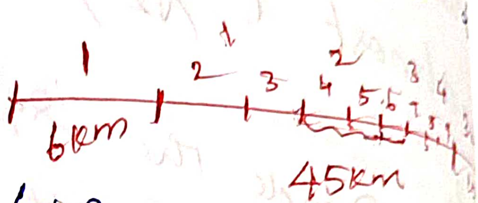
$$\left(\sqrt{(n/2)^2 + (n/\sqrt{2})^2}\right) = \sqrt{\frac{n^2}{4} + \frac{n^2}{4}} = \sqrt{\frac{2n^2}{4}}$$

$$= \frac{n}{\sqrt{2}}$$

* If n objects are put in m boxes & $n > m$ Then atleast one box will contains two or more objects.

A man hiked For 10 hours & covered total distance of 45km. It is known that hiked 6km in the First hour & only in the last hour. Show that he must have atleast 9 km within a certain period of consecutive hours.

proof!-



Since the man hiked $6+3 = 9$km in the First & last hours. he must have hiked $45-9 = 36$km in the period from second to ninth hours.

If we combine the second & third hours together, the fourth & fifth hours together etc. & eighth & Ninth hours together, we have 4 time periods.

Let us now treat 4 time periods as pigeon holes & 36 km as 36 pigeons.

using the generalized Pigeonhole principal, the least no. of pigeons accommodated in one pigeon holes

$$= \left\lfloor \frac{36-1}{4} \right\rfloor + 1 = \lfloor 8.75 \rfloor + 1$$

$$= 8+1 = 9$$

the man must have hiked at-least 9km in one time period of 2 consecutive hours.

FACULTY OF ENGINEERING AND TECHNOLOGY

SRM UNIVERSITY
(Under section 3 of UGC Act 1956)

ADDITIONAL SHEET

REG. No.

DATE

BRANCH

NAME

SEMESTER

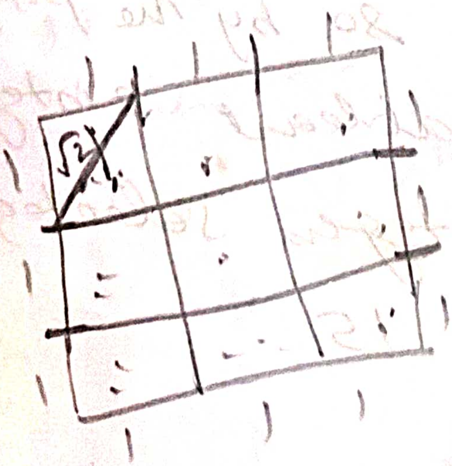
SUBJECT

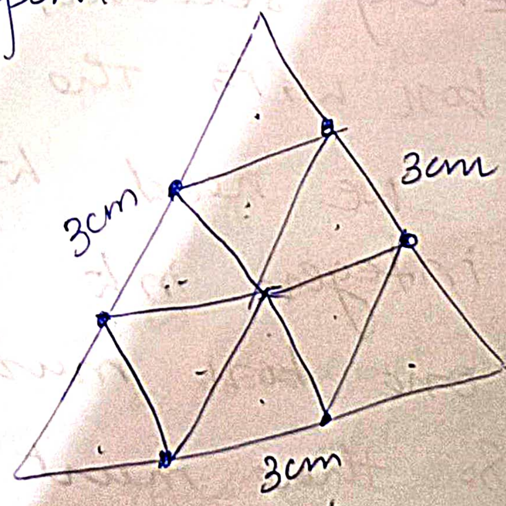
Prove that if 10 points are placed in a 3cm by 3cm square then two points must be less than or equal to $\sqrt{2}$cm apart.

Soln Divide the square into 9 squares that are 1cm by 1cm

we have 10 points to place & 9 squares. So, by the PHP, there must be a square that has more than 1 pt on or within its boundary

So this square has two pts less than or equal to $\sqrt{2}$cm apart





1

If you have more pigeons than [holes]
then atleast one hole must have
atleast two pigeons.

4 → pigeons
3 → pigeons holes

11 1 1.

Prove that if 6 integers are
Selected from {3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
there must be 2 integers whose sum
is 15.

3, 4, 5, 6, 7, 8

3 → 12
4 → 11
5 → 10
6 → 9
7 → 8.

$\boxed{3,12}$ $\boxed{4,11}$ $\boxed{5,10}$ $\boxed{6,9}$ $\boxed{7,8}$

__Soln__ Label 5 boxes with pairs
of numbers that sum to 15 as shown
Every selected integer is placed into the
box with the matching label.
We need to place 6 selected
integers into 5 boxes. So by the PHP
one box must have atleast 2 integers.
So there must be 2 integers selected
that sum to 15.

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ADDITIONAL SHEET

DATE ____________ REG. No. ____

NAME ____________ BRANCH ____

SUBJECT ____________ SEMESTER ____

1. How many students do you need in a school to guarantee that there are atleast 2 students who have the same first two initials?

objects → students   boxes
↳ N+1   ↳ N
      initials.

Students

How many initials are possible.

AA AB --- AZ
BA BB --- BZ
⋮
ZA ZB --- ZZ.

↓ ↓ 26 26. $26^2 = 676$

There are 676 boxes of first two initials.

It should be 677.

D → 13
S 13
A 13
CL 13
52

$\frac{1040}{12}$) =

9 cards are drawn from a standard deck show that atleast one suit occurs three times

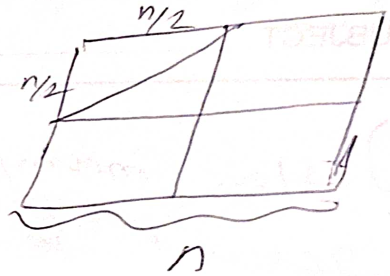
9 cards (P) $\frac{9}{4} = 2.25$
4 suit (PH)

Hence atleast one suit must occur 3 times.

(2) Given a $n \times n$ square prove that if 5 points are placed randomly inside the square then two of them are atmost $(n/\sqrt{2})$ unit apart

n = 5 points

m = boxes → square.



$$= \sqrt{(n/2)^2 + (n/2)^2}$$

$$= \sqrt{\frac{n^2}{4} + \frac{n^2}{4}} = \sqrt{\frac{2n^2}{4}} = \frac{n}{\sqrt{2}}.$$

5 point 4 boxes. There will be atleast one by PHP boxes, by PSP there will be atleast one ~~point~~ boxes which will have atleast two points, by PSP. the max distance is of two points be $n/\sqrt{2}$.

$$\begin{array}{c|cc} R & 4 & 5 \\ \hline 1 & 1 & 1 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{array}$$

FACULTY OF ENGINEERING AND TECHNOLOGY

ADDITIONAL SHEET

DATE Set theory

NAME Relation

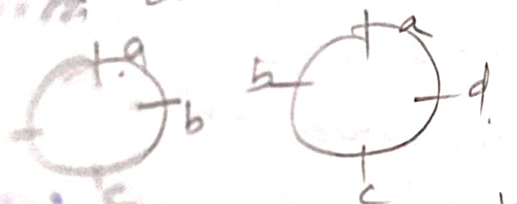
SUBJECT & Functions.

REG. No.

BRANCH

SEMESTER

Let ABC be an equilateral triangle with AB = 1. Show that by selecting 10 points there are at least two with distance $\leq 1/3$ apart.

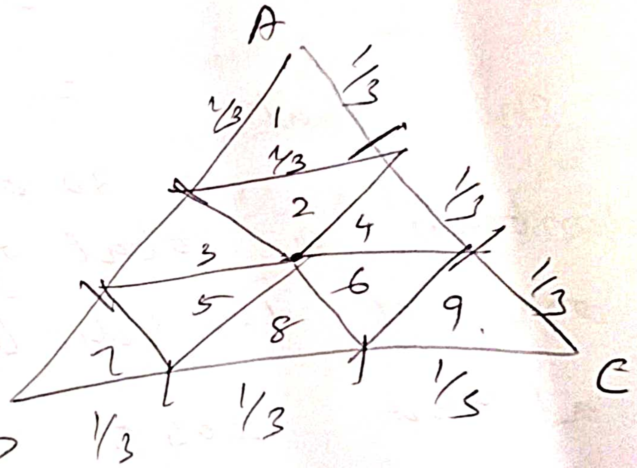


The neigh of a b+d

" " b a+c

c b+d

d → a+c



ceil [ ] = The greatest integer $\geq x$ whole no : above.

floor ⌊ ⌋ = The greatest integer $\leq x$ whole no : below.

$\lfloor 9/5 \rfloor = \lfloor 1.8 \rfloor = 1$

↳ nearest integer 2

D 1.8 2

(1) How many different rooms are needed to assign 500 classes if there are 45 different time periods in the university time table are available.

Sol) $n = 500$ $m = 45$

Acco, to extended pigeonhole

Required no. of rooms $= \lfloor \frac{n-1}{m} \rfloor + 1$

$$= \left\lfloor \frac{499}{45} \right\rfloor + 1$$

$$= \lfloor 11.08 \rfloor + 1$$

$$= 12.08$$

(2) Show that if 30 dictionaries in a library contain a total of 61,327 pages then one of the dictionaries must have atleast 2045 pages.

pages - pigeon $= n$
diction - pigeonhole $= m$

$$\left\lfloor \frac{n-1}{m} \right\rfloor + 1 = 2045$$

$$\left\lfloor \frac{61327-1}{30} \right\rfloor + 1 = 2045$$

$$\left\lfloor \frac{61326}{30} \right\rfloor = 2045.$$

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ADDITIONAL SHEET

DATE ____________ REG. No. ____________

NAME ____________ BRANCH ____________

SUBJECT ____________ SEMESTER ____________

(3) How many friends must you have to guarantee that atleast five of them will have birthday in the same month.

n = no. of friends.

$m = 12$

$\left\lfloor \frac{n-1}{m} \right\rfloor + 1 = 5$

$\left\lfloor \frac{n-1}{12} \right\rfloor + 1 = 5.$

$\frac{n-1}{12} = 4$

$n-1 = 48$

$n = 49.$

Among 49 friends, atleast five of them will have birthday in same month.

$$\begin{array}{r} 4 \\ 12 \overline{)\,49} \\ 48 \\ \hline 1 \end{array}$$

(a) suppose there are 26 students & 7 [cars] to transport them. show that at least one car must have 4 or more passengers.

$$7\overline{)25}^{3.5}$$
$$\frac{21}{40}$$

$n = 26 \rightarrow$

$m = 7 \rightarrow$

$\left\lfloor \frac{26-1}{7} \right\rfloor + 1$

$\left\lfloor \frac{25}{7} \right\rfloor + 1 = \lfloor 3.5 \rfloor + 1$

$= 4$

one car must have 4 or more.

(b) prove that if any 30 peoples are selected, then we may choose a subset of 5 so that all 5 were born on the same day of the week.

$$7\overline{)29}^{4}$$
$$\frac{28}{1}$$

$n = 30$

$m = 7$ days of the week

$\left\lfloor \frac{n-1}{m} \right\rfloor + 1 = \left\lfloor \frac{30-1}{7} \right\rfloor + 1 = \left\lfloor \frac{29}{7} \right\rfloor + 1 = \lfloor 4.1 \rfloor + 1$

$= 5$

5 people were born on the same day of the week

$1 \to 366$

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SRM UNIVERSITY (Under section 3 of UGC Act 1956)

ADDITIONAL SHEET

DATE ______ REG. No. 1.3 = 2

NAME ______ BRANCH ______

SUBJECT ______ SEMESTER ______

$366 \overline{)500}$ 366 1340 1098 242

366×3 1098 264

How many students would be required to place soda order, one soda per student in order to ensure that atleast one of the 6 listed sodas would be ordered by atleast two students.

Let m be the number of students (pigeons)

n be the number of soda (P-H)

$n = 6$ (given)

$\therefore$ minimum number of students $= 7$

$\lceil \frac{m}{n} \rceil = \lceil \frac{7}{6} \rceil = 2$

or. $\lfloor \frac{n-1}{m} \rfloor + 1 = 2$

$\lfloor \frac{n-1}{6} \rfloor + 1 = 2$

$\lfloor \frac{n-1}{6} \rfloor = 1$

$n = 7$

$\lceil \frac{30}{7} \rceil \xrightarrow{\text{ceiling}} = \lceil 4.\rceil = 5$ (or) $\lfloor \frac{n-1}{m} \rfloor + 1$

$= \lfloor \frac{29}{7} \rfloor + 1$

$= \lfloor 4. \rfloor + 1 = 4+1$

$= 5.$

$\lfloor \frac{n-1}{12} \rfloor + 1 = 2$

$n-1 = 1$

$\frac{n-1}{12} = 1$

$n = 12 \times 3 + 1$

$= \cancel{36+1} = \cancel{37}$

$n = 13$

Find the minimum number of students in a class so that two students were born in same month.

Let m be the number of students (Pigeon)

n be the number of months in a year (pi Hole)

As $n = 12$

$m > n$ $\lceil \frac{m}{n} \rceil = \lceil \frac{13}{12} \rceil = \lceil 1.08 \rceil$

$= 2.$

$\therefore$ minimum number of students in class $= 13$

$\lfloor \frac{n-1}{m} \rfloor + 1 = 3.$ $\lfloor \frac{n-1}{12} \rfloor + 1 = 2$ $\lfloor \frac{n-1}{4} \rfloor = 2.$ 2 2 2 2

$\lfloor \frac{n-1}{4} \rfloor + 1 = 3.$

$\lfloor \frac{n-1}{12} \rfloor + 1 = 3$ $\lfloor \frac{n-1}{12} \rfloor = 2$

$n-1 = 8$

$n = 9$

$\lfloor \frac{n-1}{12} \rfloor = 1$

$n = 12+1$

$= 13.$