Signals and System (EC-205) Lec-1 24-08-2022

Weightage: CWS → 15%
PRS → 25%
MTE → 20%
ETE → 40%

Text-books
1. Signals & Systems by Alan V. Oppenheim, Pearson education.
2. → Numerical Purpose → Schaum's outline of S&S, McGraw Hills.

* Signals

Signals are physical qty. which carries some information. "OR"
Any physical qty. which varies with one or more independent variables (time, space, temp)

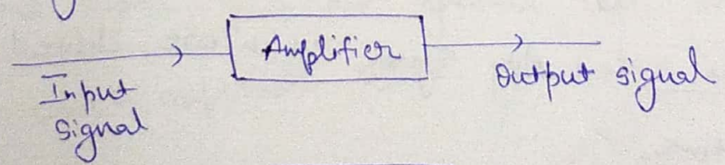
time → Audio signal, Music ", FM

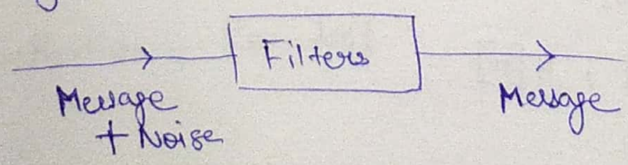
space → picture

Video signals, EM Wave → Time & space Varying

Applications area of S&S.
1. Communication systems: AM, FM
2. Bio medical system: ECG: Electrocardiogram, Echo
3. Satellite communication.
4. Aircraft system.
5. RADAR
6. Instrumentation & Control System.

System: Any physical set of components or piece of Code that process the signal & extract the desired behaviour from the signal.





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* Classification of signals :

(1) Continous time signals & Discrete time signals

CT signals : Those signals which ~~are~~ is defined for all set of time from (a, b). These signals are also known as Analog signals.

→ All naturally occuring signals are Analog signals.

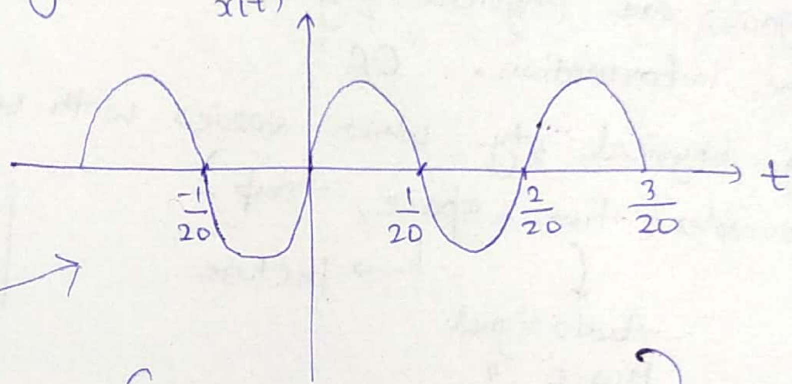
→ Represented by $x(t)$, $y(t)$.

(1) $x(t) = \sin(2\pi 10t)$

$n\pi = 2\pi 10t$

$$\boxed{t = \frac{n}{20}}$$

$n = 0, \pm 1, \pm 2, \ldots$

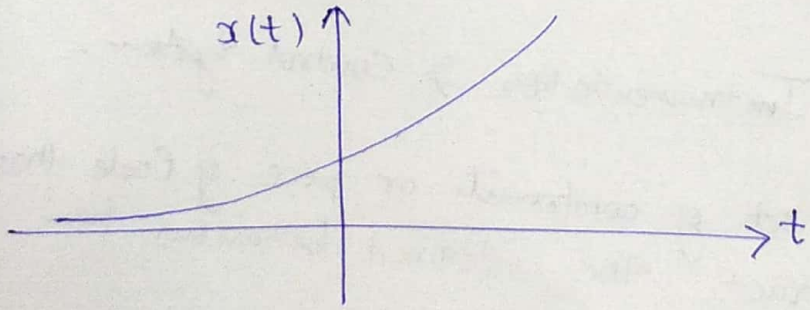


Freq = 10 Hz

Time period $= \frac{2}{20} = \frac{1}{10}$

freq $= \frac{1}{T} = 10$ Hz

(2) $x(t) = e^t$ for $-\infty \leq t \leq \infty$



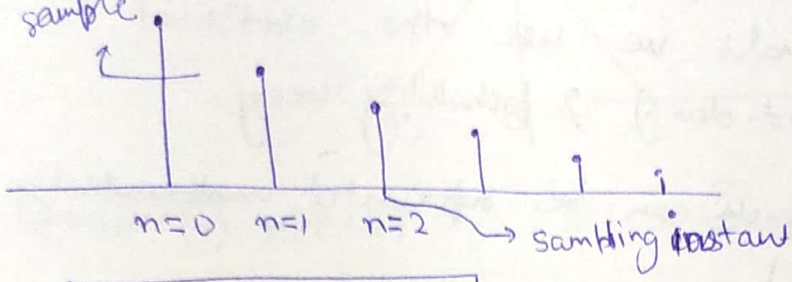
\# Analog signals can take infinite values.

\## Discrete Time signals → Any signal which are defined for discrete set of time.

Represented by $x[n]$, $y[n]$

$$x[n] = \begin{cases} \left(\frac{1}{2}\right)^n & n \geq 0 \\ 0 & \text{else} \end{cases}$$

Analog signal
↓ sampling
Discrete signal



* Sampling Theorem

sampling rate → $f_s \geq 2 f_m$ f_m = maxi. freq.

sampling interval ← $T_s \leq \frac{1}{2 f_m}$

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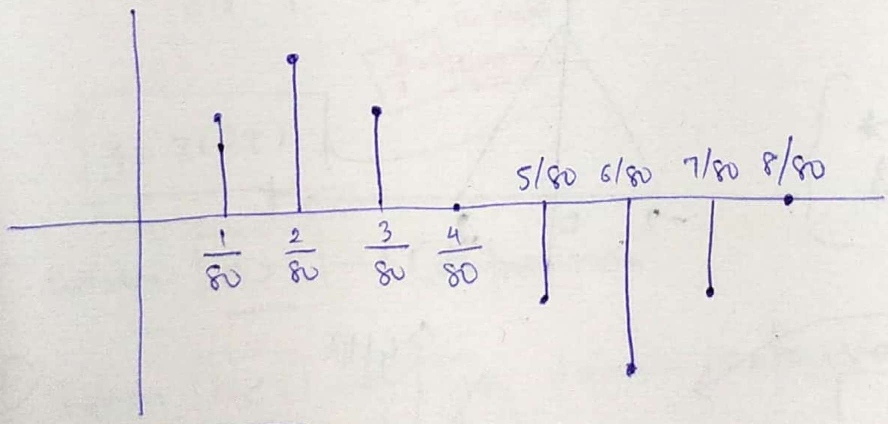
Q. Choose $\frac{1}{80}$ → draw discrete time signal

sampling interval.



Digital signal → are special case of discrete time signals in which amplitude levels are also quantized (discrete)

* Quantization - Roundoff.

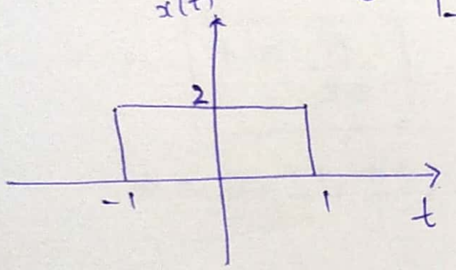
* Random & ~~disc~~ deterministic signals.

Random – Any signal which cannot be represented mathematically.
All naturally occuring & noise signals are Random signals.
To Analyse the random signals we use the statistical method (Mean, Variance, Std. devi) & probability Theory.

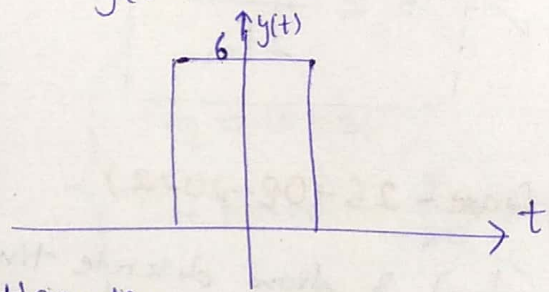
Deterministic signal – Which can be represented mathematically.
Example : $E = E_0 \sin(Kx - \omega t)$

* Various Operations on signals.

① Amplitude ~~Shifting~~ Scaling $\boxed{y(t) = K\,x(t)}$



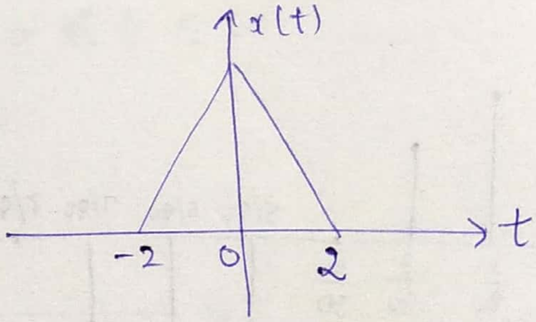
$y(t) = 3x(t)$



Reverse of Amplification = Attenuation
e.g. $y(t) = 0.3x(t)$

② Time Shifting
$y(t) = x(t + t_0)$

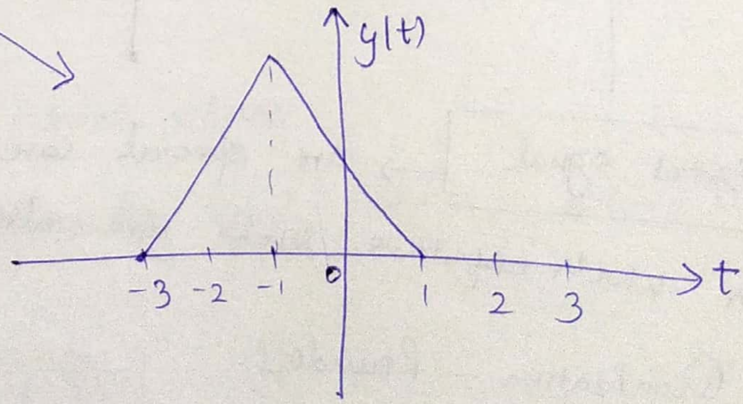
① $t_0 > 0$ $x(t)$



$y(t) = x(t+1)$

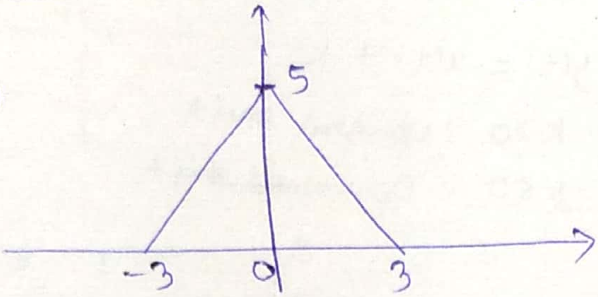
left shift

when t_0 = +ve

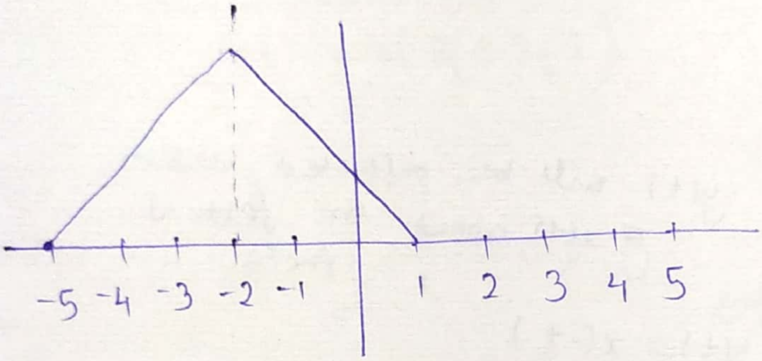


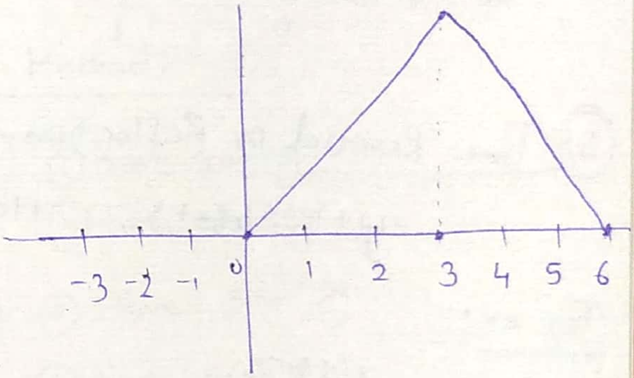
(ii) $\boxed{t_0 < 0}$ → Right Shift (Delay operation)

Q. $x(t)$



Draw ① $x(t+2)$ ② $x(t-3)$

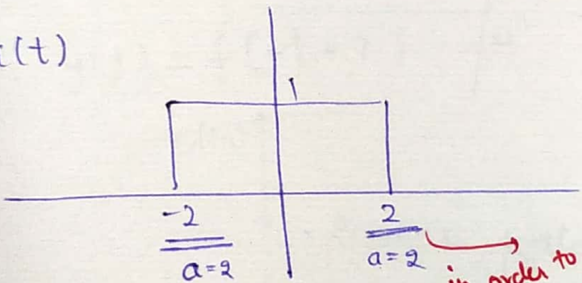




③ Time scaling $y(t) = x(at)$

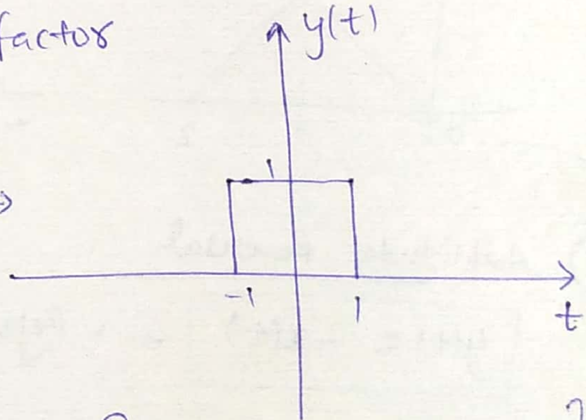
a: Scaling factor

$x(t)$



in order to convert $\boxed{\frac{2}{2}=1}$

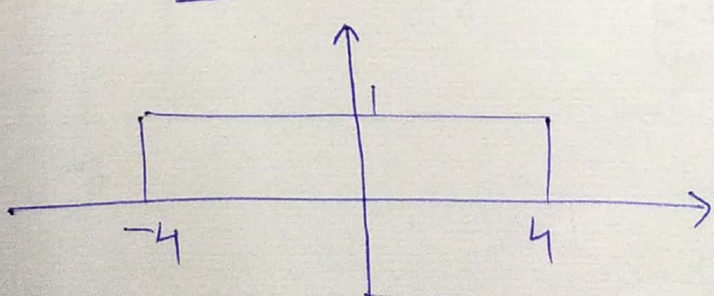
$\boxed{y = x(2t)}$



* No change in Amplitude
* Compression in time.

① When $\boxed{a>1}$ → Compression.

② When $\boxed{a<1}$ → Time domain expansion.



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4) ~~Time Reversal~~

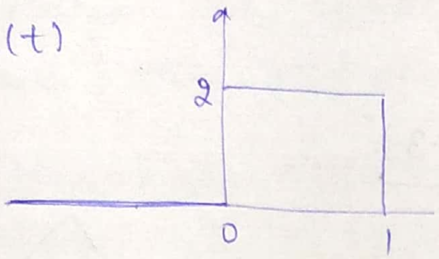
↳ Amplitude shifting

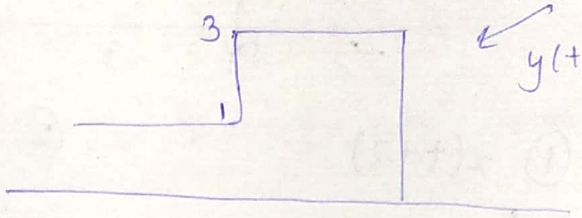
$y(t) = x(t) + K$

$K > 0$: upward shift

$K < 0$: Downward shift

x(t)





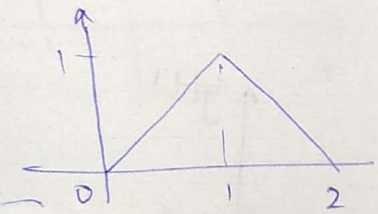
$y(t) = x(t) + 1$

5) Time Reversal or Reflection.

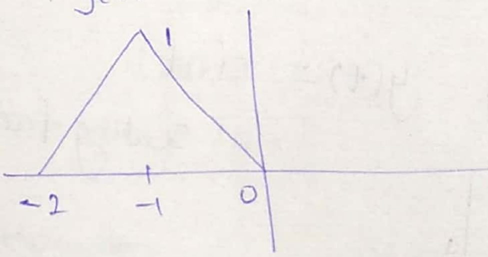
$y(t) = x(-t)$ Here $y(t)$ will be reflected version of $x(t)$ about the Vertical Axis.

For ex:

x(t)



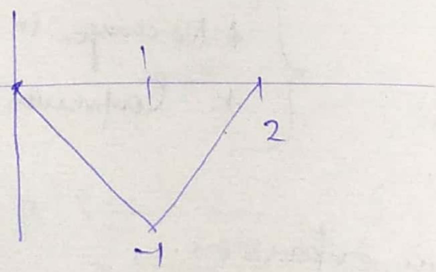
$y(t) = x(-t)$



6) Amplitude Reversal

$\boxed{y(t) = -x(t)}$ → Rejection along x-axis.

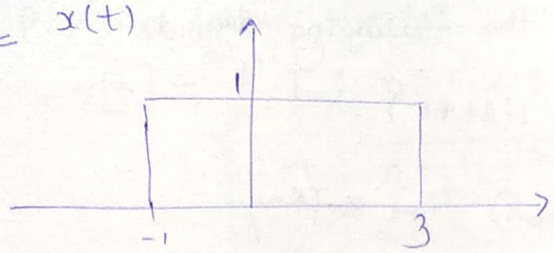
$y(t) = -x(t)$



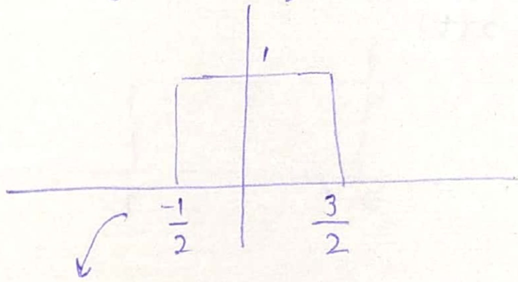
7) Time shifting & time scaling

$y(t) = x(at+b)$

Q. $x(t)$



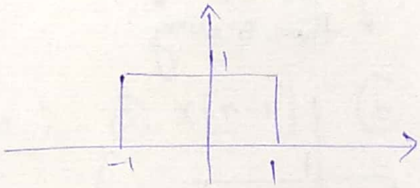
Plot $y(t) = x(2t+1)$



$v(t) = x(2t)$

left shift

$y(t) = x\left(2\left(t+\frac{1}{2}\right)\right)$



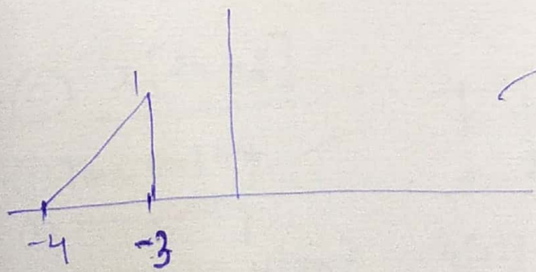
Methods

1. (a) time scaling
   (b) time shifting
2. (a) time shifting
   (b) time scaling.

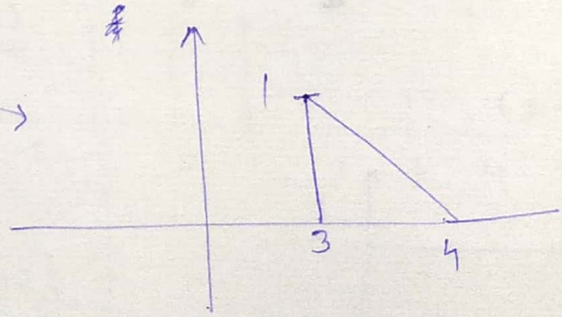
8) Time shifting & time Reversal.

$y(t) = f(-t+7)$

Step 1 time shifting

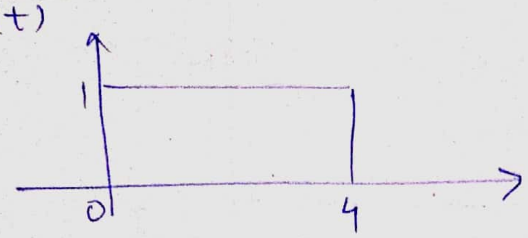


Step. 2 time ~~scaling~~ Reversal

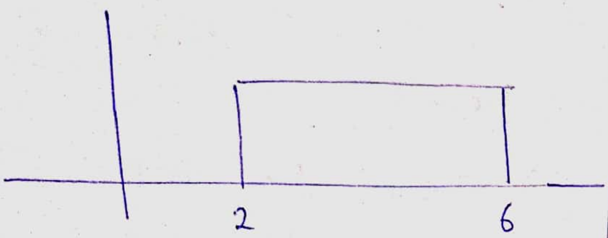


Q. For a given signal $x(t)$, plot the following signals.

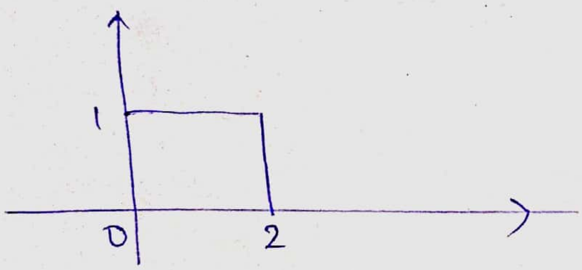
$x(t)$



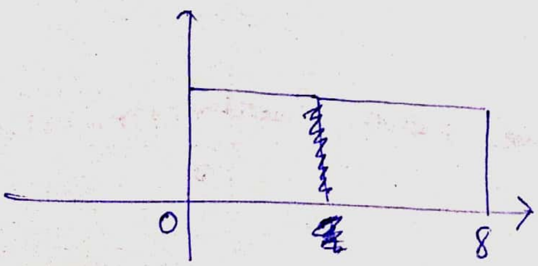
(a) $x(t-2)$



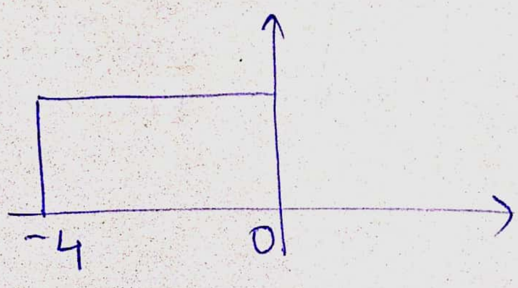
(b) $x(2t)$



(c) $x\left(\left(\frac{t}{2}\right)\right)$ $\boxed{a = \frac{1}{2}}$

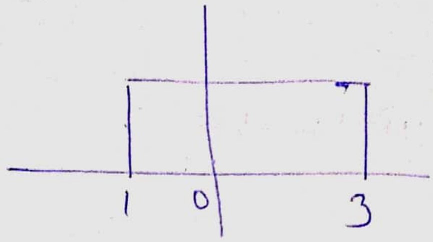


(d) $x(-t)$

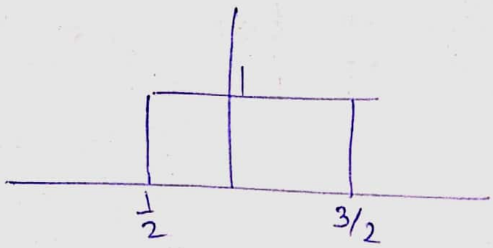


(e) $x(2t+1)$

Step (1) Time shifting



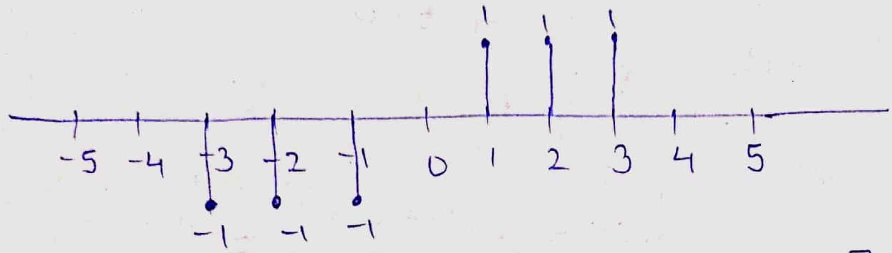
time scaling



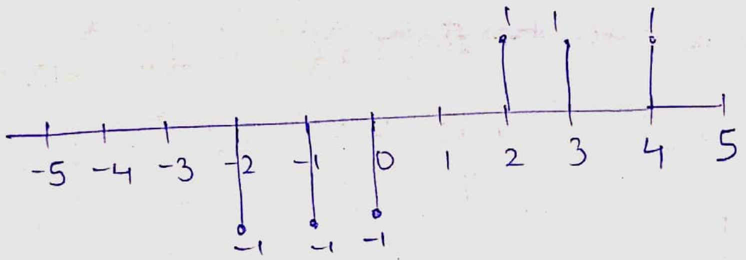
(f) $x(-t+3)$

Q. Plot the follow. signals.

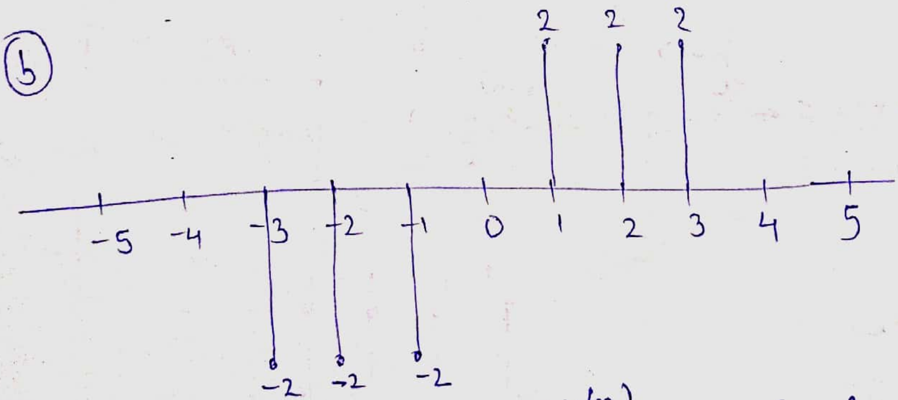
$$x[n] = \begin{cases} 1 \; ; & n = 1, 2, 3 \\ -1 \; ; & n = -1, -2, -3 \\ 0 \; ; & n = 0, \; |n| > 3 \end{cases}$$



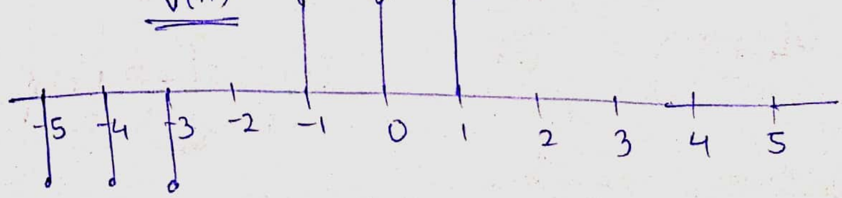
Plot : (a) $x[n-1]$ (b) $2x[n]$ (c) $x[2n+2]$



(b)



v(n)



(c) $x[2n+2]$

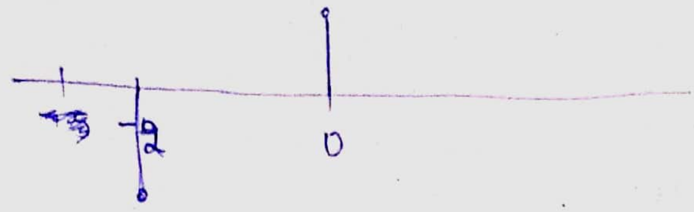
① Time shifting

② — scaling

$y(n) = v(2n)$ $y[+1] = v[+2]$

$y[0] = v[0]$ $y[-2] = v[-4]$

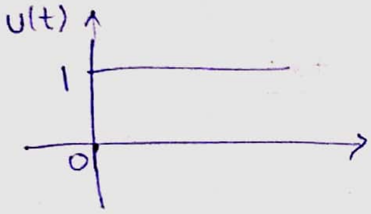
$y[-1] = v[-2]$ $y[+2] = v[+4]$



Basic or Elementary Signals.

① Unit step function $\rightarrow u(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases}$

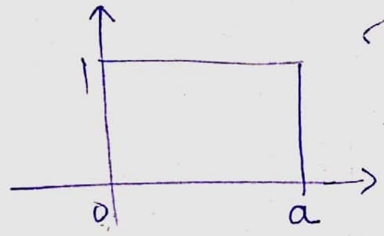
This fxn. is discontinuous at $\boxed{t=0}$



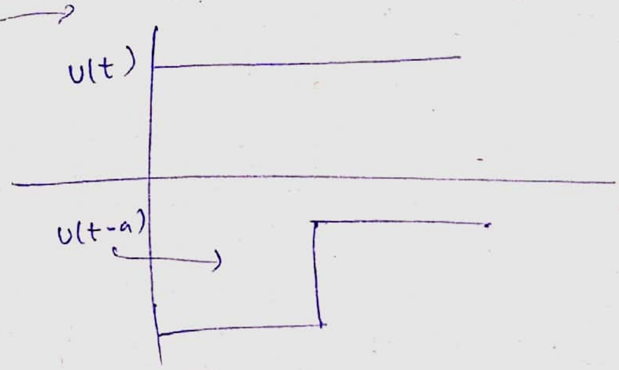
$$\boxed{U(0) = \frac{U(0^+) + U(0^-)}{2}} = \frac{1}{2}$$

The unit step fxn. is used as a basic ~~fxn to ge~~ signal to generate several discontinuous wave form

e.g $x(t) =$



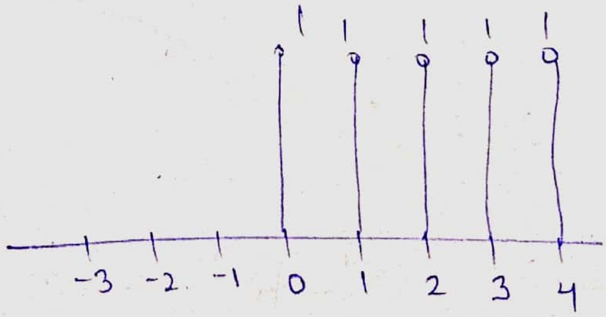
$x(t) = u(t) - u(t-a)$



② Unit Ramp fxn.

→ unit step sequence

$$u[n] = \begin{cases} 1 & ; n \geq 0 \\ 0 & \text{else} \end{cases}$$



Unit Ramp function

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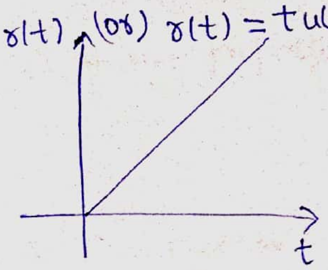
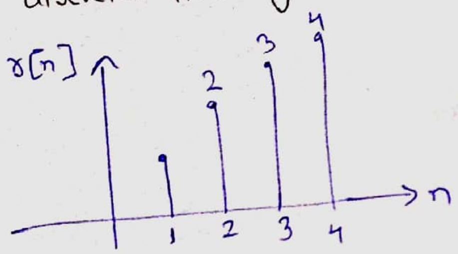
$$r(t) = \begin{cases} t \,; & \text{for } t \geq 0 \\ 0 & \text{else} \end{cases}$$

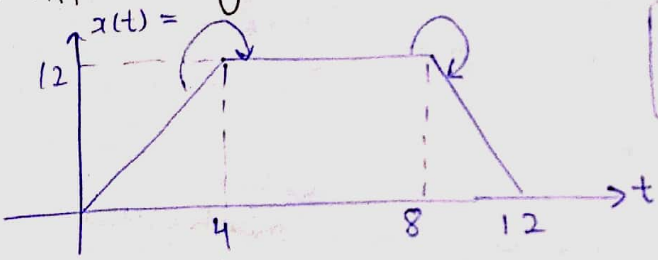
(or) $r(t) = t\,u(t)$



For discrete time signal $\rightarrow$ $r[n] = \begin{cases} n \,; & \text{for } n \geq 0 \\ 0 & \text{else} \end{cases}$

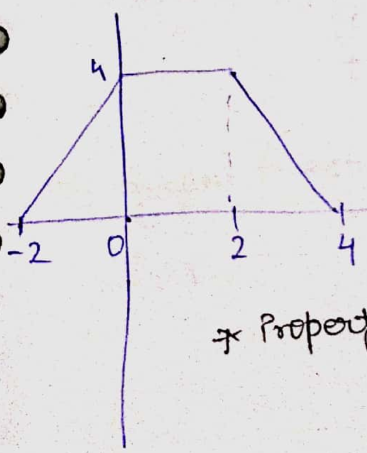


* Application of Ramp signal.



$$\boxed{x(t) = 3r(t) - 3r(t-4) - 3r(t-8) + 3r(t-12)}$$

Q. Plot the signal as $2r(t+2) - 2r(t) - 2r(t-2)$ -ve



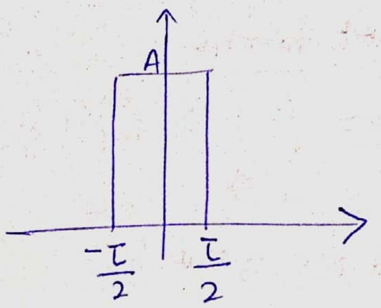
* Property : $r(t) = \int_{-\infty}^{t} u(\tau)\,d\tau$

$$u(t) = \frac{d\,r(t)}{dt}$$

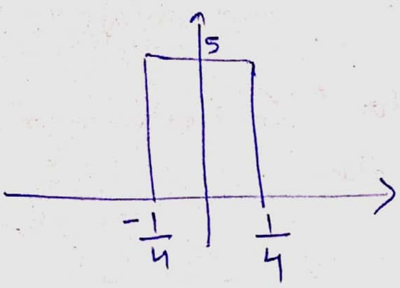
Rectangular fxn. or GATE Function.

$A\,\text{rect}\left(\frac{t}{\tau}\right)$ τ: width of pulse

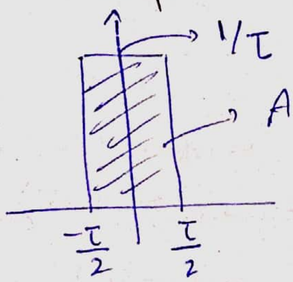
A: height/ of pulse Amplitude

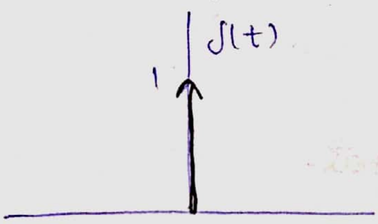


Plot $5\,\text{rec}(2t) = 5\,\text{rec}\left(\frac{t}{1/2}\right)$



* Unit-Impulse function OR Dirac Delta fxn.





$\delta(t) = 0$ for $t \neq 0$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

Normally at the arrow of impulse fxn we write a number, this number represents the area under the pulse.

* Scaled Impulse function: $A\delta(t)$, A = Weight of an impulse.

* Properties of Impulse fxn.

① Product property – $x(t)\delta(t) = x(0)\delta(t)$

Q. $$\int_{-\infty}^{\infty} \delta(t) \cos\left(\frac{3t}{2}\right) dt = \int_{-\infty}^{\infty} \cos(0)\,\delta(t)\,dt$$

$$= 1\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

(2) Shifting property –

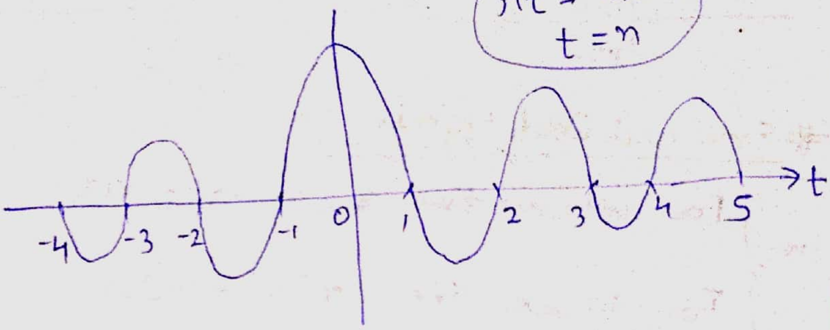
$$\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$$

(3) Scaling property – $\boxed{\delta(at) = \frac{1}{|a|}\,\delta(t)}$

Sinc function

$\pi t = n\pi$
$t = n$

$$x(t) = \text{sinc}(t)$$
$$= \frac{\sin \pi t}{\pi t}$$



Q. $x(t) = 5\,\text{sinc}(5t)$

$= \cancel{5\,\text{sinc}}$

$= \frac{\sin \pi 5t}{\pi 5t}$

$\cancel{\pi t = n\pi}$

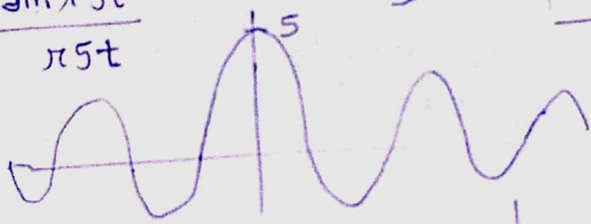
$\cancel{5t = n\pi}$

$\cancel{t = \frac{n\pi}{5}}$

$\pi 5t = n\pi$

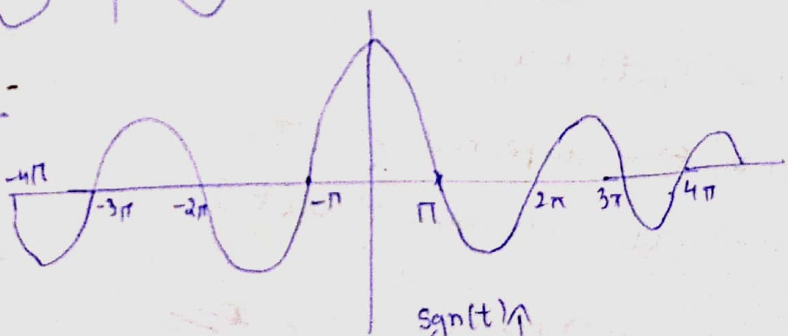
$\boxed{t = \pm \frac{n}{5}}$ $n = 1, 2, 3, \ldots$

$=$



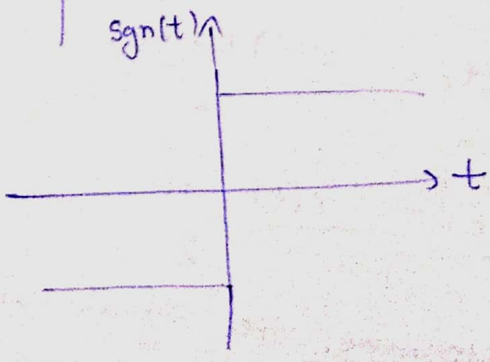
Sampling function –

$$Sa(t) = \frac{\sin t}{t}$$



Signum function

$$\text{Sgn}(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t = 0 \\ -1 & \text{for } t < 0 \end{cases}$$



$$\boxed{\text{sgn}(t) = \frac{1+u(t)}{2}}$$

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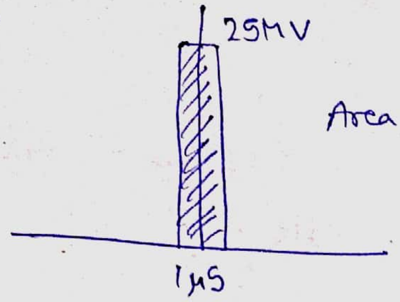
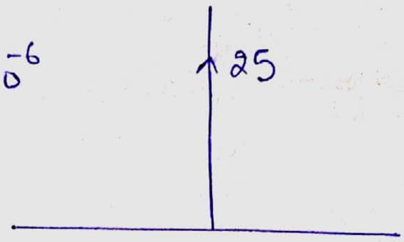
Q. For a 25 MV Voltage applied for 1 μ sec, Model it as impulse function.

→



$$\text{Area} = 25\times10^{6} \times 1\times10^{-6}$$
$$= 25$$



$25\,\delta(t)$

Even and Odd Signals.

Even:
- For continous time $x(-t) = x(t)$
- For discrete time $x[-n] = x[n]$

Odd:
- For C.T $x(-t) = -x(t)$
- $x[-n] = -x[n]$

$x(0) = 0$

* Any signal can be broken into sum of 2 signals one of which is even & other is odd.

$$x(t) = x_E(t) + x_o(t)$$

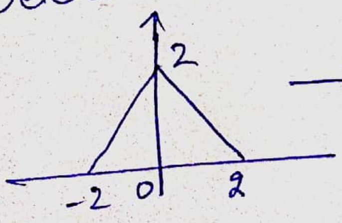
$x_E(t)$: Even component of $x(t)$

$x_o(t)$: Odd component of $x(t)$

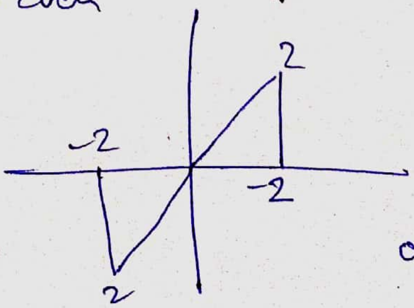
$$x_E(t) = \frac{x(t) + x(\overset{-ve}{-t})}{2}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

Q. Determine the signal is odd/Even

① 

→ even

② 

odd

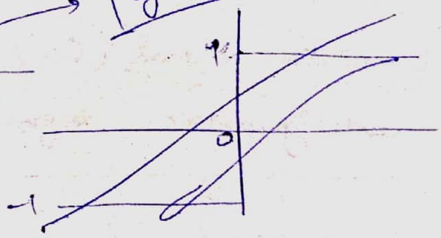
Q. Draw the Even component of given signal.

(a) $x(t) = u(t)$

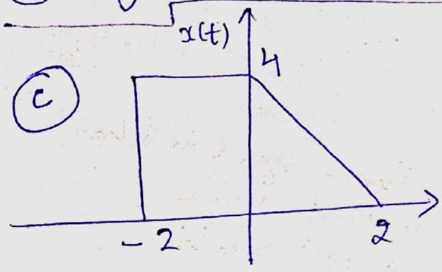
$= x_E(t) + x_o(t)$

$= \boxed{\underbrace{\tfrac{1}{2}}_{\text{even}} + \underbrace{\tfrac{1}{2}\,\mathrm{sgn}(t)}_{\text{odd}}}$

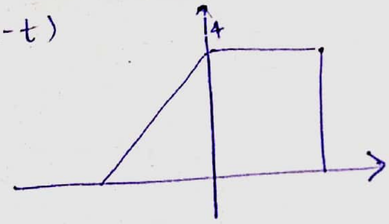
(b) $\mathrm{sgn}(t) \longrightarrow 0 + \mathrm{sgn}(t) \longrightarrow \boxed{\mathrm{sgn}(t)}$ → odd fxn.



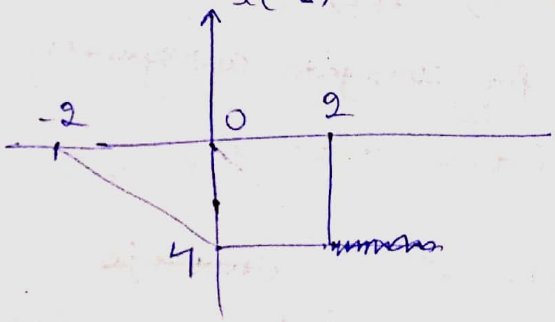
(c)



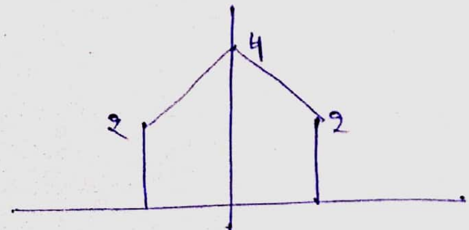
→ $x(-t)$

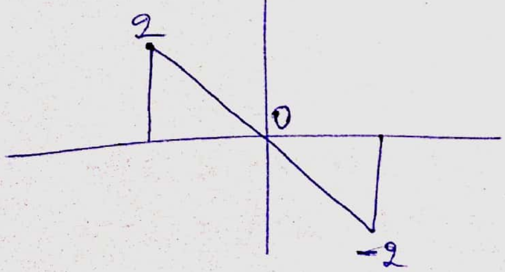


$x(-t)$



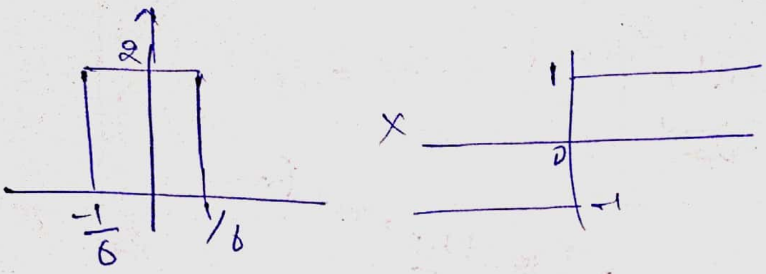
~~x(t)+x(-t)~~ $\dfrac{x(t)+x(-t)}{2}$





Q. Determine whether the given signal is odd or Even.

$x(t) = \underbrace{2\,\text{rect}(3t)}_{\text{even}} \times \underbrace{\text{sgn}(t)}_{\text{odd}} = \text{odd}$



conjugate symmetric signal (CS) & Conjugate anti symmetric signal (CAS)

For any complex signal,

condition for conjugate symmetry → Those signal in which Real part → even, Img. part ⇒ odd.

$x^*(-t) = x(t)$

Condition for conjugate anti symmetry

$x^*(-t) = -x(t)$

Q. $x(t) = e^{jt} = \cancel{e\cos t + je}$

→ $x(t) = \underbrace{\cos t}_{\text{even}} + j\underbrace{\sin t}_{\text{odd}}$ → Conjugate Symm

$x(t) = e^{jt}$

$x(-t) = e^{-jt}$

$x^*(-t) = e^{jt}$

$x(t) = x^*(-t)$

This is conjugate symmetry

Q. $x(t) = j e^{jt}$

$\rightarrow$ $\boxed{x^*(-t) = -x(t)}$ $\rightarrow$ CAS

(OR) odd conjugate signal

$\Downarrow$

Real part $\rightarrow$ odd

Img part $\rightarrow$ even

$= j[\cos t + j\sin t]$

$= -\sin t + j\cos t$

Q. $x(t) = e^{jt^2}$ $\rightarrow$ Neither.

Periodic & A-periodic signals.

For any cont. signal $x(t)$

$x(t) = x(t+T)$

T: period of given signal

Fundamental period $\rightarrow$ smallest T.

$x[n] = x[n+N]$

N_0: FP

Q. Calculate funda. time perio. for $x(t) = A e^{j\omega_0 t}$ —(i)

$\rightarrow$ $x(t) = x(t+T)$

$x(t+T) = A e^{j\omega_0 (t+T)}$

$= A e^{j\omega_0 t} \cdot e^{j\omega_0 T}$ —(ii)

$\Rightarrow e^{j\omega_0 T_0} = e^{j2\pi}$

$$\boxed{T_0 = \frac{2\pi}{\omega_0}}$$

Q. $x(t) = A\sin(\omega_0 t + \theta)$

$x(t) = A\cos(\omega_0 t + \theta)$

$T_0 = \frac{2\pi}{\omega_0}$

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$A_0 e^{j\omega_0 t} \longrightarrow FTP = \frac{2\pi}{\omega_0}$

$A \sin(\omega_0 t + \theta) \longrightarrow FTP = \frac{2\pi}{\omega_0}$

Determine FTP of given signal.

1) $5 \sin(4\pi t + 30^\circ) = \frac{2\pi}{4\pi} = \frac{1}{2}$ sec

2) $10 + 5 \sin(4\pi t + 30^\circ) = \frac{1}{2}$ sec.

3) $5 \sin(4\pi(t \pm 2)) + 30^\circ) = \frac{1}{2}$ sec.

4) $5 \sin(-4\pi t + 30^\circ) = \frac{1}{2}$ sec

6) $\sin^2(4\pi t) = \frac{1 - \cos(8\pi t)}{2}$

$T = \frac{2\pi}{8\pi} = 1/4$

7) $\sin^2 2t + \cos 3\pi t$

$T_1 = \frac{2\pi}{2} = \pi$

$\hookrightarrow T_2 = \frac{2\pi}{3\pi} = 2/3$

The sum of 2 or more than 2 periodic signal will be periodic if Ratio of their F.T.P is a rational No. × Not periodic.

$\frac{T_1}{T_2} = \frac{3\pi}{2} =$ not a rational no.

8) $\sin 2t + \cos \sqrt{2} t \longrightarrow \times$

9) $\sin 7\pi t + \cos 3\pi t$

F.T.P = LCM of (T_1, T_2)

$T_1 = \frac{2\pi}{7\pi}$, $T_2 = \frac{2\pi}{3\pi}$

$T_0 = LCM \text{ of } (T_1 \,\&\, T_2)$

$= LCM \text{ of } \left(\frac{2}{7}, \frac{2}{3}\right)$

$= \frac{LCM \text{ of } (2, 2)}{HCF \text{ of } (7, 3)} = \frac{2}{1}$

* Discrete signal.

Q. Calculate the F.T.P of

$\cancel{x[n] = e^{j\omega_0 n}}$.

$x[n] = e^{j\omega_0 n}$

$x[n] = x[n+N]$

N: Time period

$x[n] = x[n + mN_0]$

$m = \pm 1, \pm 2, ---$

N_0 = fundamental T.P

$x[n] = e^{j\omega_0 n}$

$x[n+N] = e^{j\omega_0 (n+N)}$

$e^{j\omega_0 n} = e^{j\omega_0 N} \cdot e^{j\omega_0 n}$

$\therefore e^{j\omega_0 N} = 1$

$= e^{j2\pi m}$

$\therefore \boxed{N = \frac{2\pi}{\omega_0} m}$

For any integer value of m if N becomes an integer, then $x[n]$ is periodic.

Q.1 $x[n] = e^{j3\pi n}$

$\therefore N = \frac{2\pi}{3\pi} m$

$N = 2/3^m$

For $m = \pm 3, \pm 6, \pm 9$

$N_0 = \frac{2}{3} \times 3$

FTP $\Rightarrow N_0 = 2$

Q2 $x[n] = \sin(\pi^2 n)$

$N = \frac{2\pi}{\omega_0} \times m$

$= \frac{2\pi}{\pi^2} \times m$

$= \frac{2}{4} \times m$

Q. Calculate F.T.P of

$e^{j\left(\frac{2\pi}{3}n\right)} + e^{j\left(\frac{3\pi}{4}n\right)}$

$N_1 = \frac{2\pi}{\frac{2\pi}{3}} \times m_1$

$= 3m_1$

$m_1 = 1$

$N_{0_1} = 3$

$N_2 = \frac{2\pi \times m_2}{\frac{3\pi}{4}}$

$= \boxed{\frac{8 m_2}{3}}$

$m_2 = 3$,

N_2 becomes integer

$N_{0_2} = 8$

F.T.P = LCM of (3, 8)

= 24

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Energy & Power signal

Energy signal are those signals for which energy is finite
Power signal ——————————————— power is finite

For continous time $x(t)$

$$\underset{(\text{Energy})}{E} = \int_{-\infty}^{\infty} x^2(t)\,dt$$

Time period -

$$\left.\begin{matrix}\text{Mean} \\ \text{square} \\ \text{value}\end{matrix}\right] = P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x^2(t)\,dt$$

Q. $x(t) = e^{-at}u(t)$ $\boxed{a<1}$

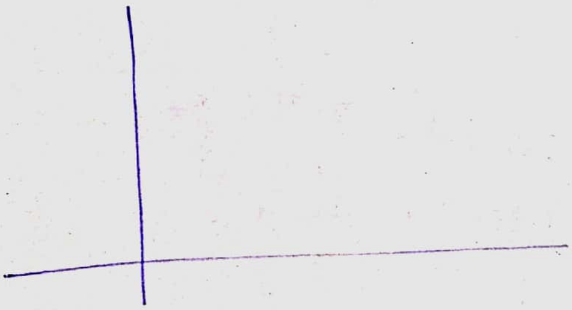
Energy : $E < \infty$

power : $P < \infty$

$$E = \int_0^{\infty} (e^{-at})^2\,dt$$

$$E = \int_0^{\infty} e^{-2at}\,dt$$

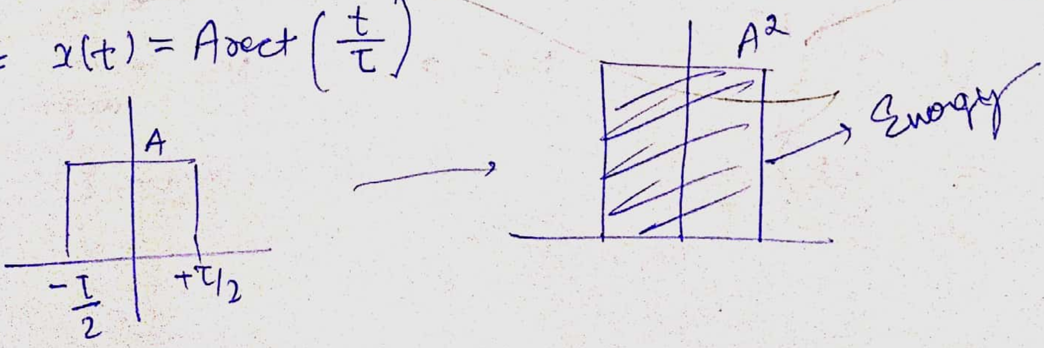
$$E = \frac{1}{2a}$$



$$P = \lim_{T\to\infty} \frac{1}{2T}\int_0^{T} e^{-2at}\,dt$$

$$= 0$$

Q. $x(t) = A\,\text{rect}\left(\frac{t}{\tau}\right)$



All the bounded (Amplitude) signal as well as finite duration signal are energy signal.

$$E = \int_{-\tau/2}^{\tau/2} A^2 dt$$

$\boxed{E = A^2 \tau}$ Joules $\boxed{P = 0}$

Q. $x(t) = u(t) \longrightarrow$ unit step fxn./signal.

$E = \infty$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_0^T 1 dt = \frac{1}{2T} \times T = \frac{1}{2}$$

* Power signal $\longrightarrow$ $\boxed{P \text{ is finite } \& \ E = \infty}$

Q. $x(t) = r(t) \rightarrow$ Ramp.

$\rightarrow$ $\boxed{\begin{matrix} E = \infty \\ P = \infty \end{matrix}}$ $\rightarrow$ Neither Energy nor Power signal. $\rightarrow$ unbounded signals.

~~Q. $x(t) = \frac{A}{t}$.~~

Q. $x[n] = a^n u[n]$

(i) $a < 1$

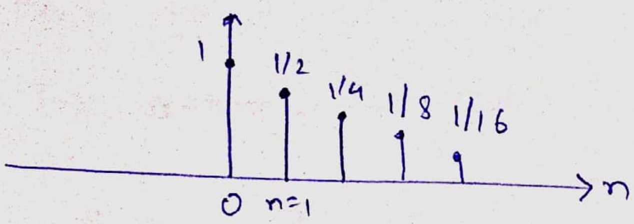
(ii) $a > 1$

$$E = \sum_{n=-\infty}^{\infty} x^2(n), \qquad P = \lim_{N \to \infty} \frac{1}{(2N+1)} \sum_{n=-\infty}^{\infty} x^2(n)$$

07-Sept-2022

① $x[n] = a^n u[n]$ for $a < 1$

Let $a = 1/2$

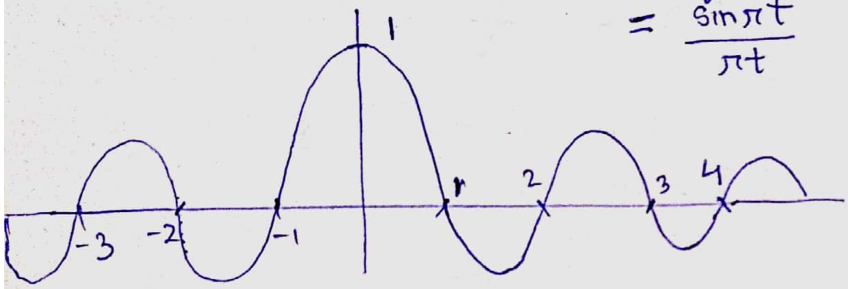


$$E = \sum_{-\infty}^{\infty} x^2[n] = \sum_{n=0}^{\infty} (a^n)^2 = \sum_{n=0}^{\infty} a^{2n} = 1 + a^2 + a^4 + \cdots \infty$$

$$= \boxed{\frac{1}{1-a^2}}$$

Q. Calculate energy of signal $\boxed{\operatorname{sinc} t}$

$$= \frac{\sin \pi t}{\pi t}$$



→ finite duration & Amplitude in decaying

↓

Energy

$$E = \int_{-\infty}^{\infty} \operatorname{sinc}^2 t \, dt = 2 \int_0^{\infty} \frac{\sin^2 \pi t}{\pi^2 t^2} \, dt$$

Let $\pi t = x$

$dt = \frac{dx}{\pi}$

$$E = \frac{2}{\pi} \int_0^{\infty} \frac{\sin^2 x}{x^2} \, dx = \frac{2}{\pi} \times \left(\frac{\pi}{2}\right) = \underline{1 \text{ Joule}}$$

Power $= 0$

$$\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \operatorname{sinc}^2 t \, dt = 0$$

* $x(t) \Longrightarrow E$ for $\boxed{a>1}$

$x(at) \longrightarrow \frac{E}{|a|}$

Q. $E[\text{sinc}(5t)] = \frac{1}{5}$

Q. Calculate energy of $\boxed{Sa(t)}$ → Sampling fxn.

$$Sa(t) = \frac{\sin t}{t}$$

$$= \text{sinc}\left(\frac{t}{\pi}\right)$$

$$= E\left[\text{sinc}\left(\frac{t}{\pi}\right)\right] = \pi$$

Q. $I = \int_{-\infty}^{\infty} \text{sinc}^2(4t)\,dt$ = energy of sinc (4t)

$= \frac{1}{4}$ Joule

Q. For Complex signals.

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

Calculate energy of $x(t) = A e^{j\omega_0 t}$

→ $x(t) = A e^{j\omega_0 t}$

$$E = \int_{-\infty}^{\infty} |A e^{j\omega_0 t}|^2\,dt = 0$$

Power $= A^2$ Watt

Periodic signals. $\longrightarrow$ All periodic signal are Power signal

Q. $A\sin(\omega_0 t + \theta)$

$$\boxed{E = \infty}$$

$$P = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt \;\Big\|\; \frac{1}{2T}\int_{-T}^{T} x^2(t)\,dt$$

$$P = \frac{1}{2T}\int_{-T}^{T} A^2 \sin^2(\omega_0 t + \theta)\,dt \qquad T = \frac{2\pi}{\omega_0}$$

$$P = \frac{A^2}{2T}\int_{-T}^{T} \sin^2(\omega_0 t)\,dt$$

$$\cancel{P = \frac{A^2}{2T}\int_{-T}^{T} \frac{1 - \cos(2\omega_0 t)}{2}\,dt}$$

$$\cancel{P = \frac{A^2}{2T}}$$

$$\cancel{P = \frac{A^2}{(2T)}\left[\int_{-T}^{T} 1\right]}$$

$$P = \frac{1}{2T}\int_{-T}^{T} A^2\left(\frac{1 - \cos 2(\omega_0 t + \theta)}{2}\right)dt$$

$$P = \frac{1}{2T}\left[\int_{-T}^{T}\frac{A^2}{2}\,dt - \cancel{\int_{-T}^{T}\frac{\cos 2(\omega_0 t + \theta)}{2}\,dt}^{\,\to 0}\right]$$

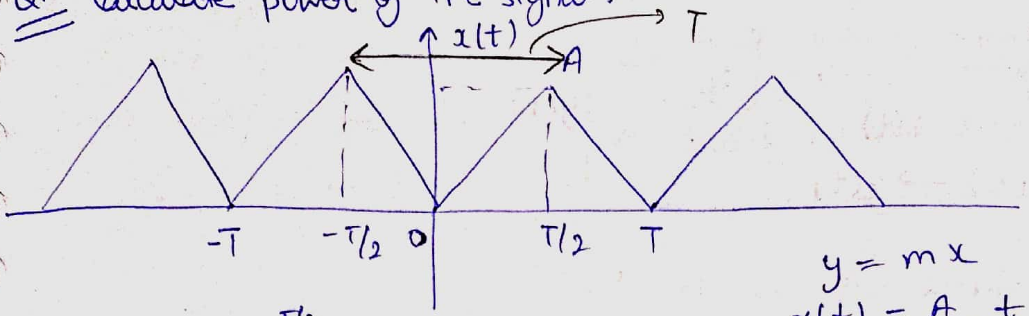
$$P = \frac{A^2}{4T}\int_{-T}^{T} dt$$

$$\boxed{P = \frac{A^2}{2}\ \text{Watt}}$$

$$\boxed{\text{Power} = (\text{RMS value})^2}$$

$$= \left(\frac{A}{\sqrt{2}}\right)^2 = \frac{A^2}{2}$$

Q. Calculate power of the signal.



$y = mx$

$x(t) = \frac{A}{T/2} t$

$x(t) = \frac{2A}{T} t$

$$P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

$$P = \frac{2}{T} \int_{0}^{T/2} x^2(t)\,dt$$

$$P = \frac{2}{T} \int_{0}^{T/2} \left(\frac{2A}{T} t\right)^2 dt$$

$$\cancel{\frac{4A}{T^2}\left(\frac{t^2}{2}\right)_0^{T/2}}$$

$$\cancel{\frac{4A}{T^2}\left[\frac{T^2}{2}\right]}$$

$$\cancel{\frac{4A}{T^2(2)}\left(\frac{T^2}{4} - 0\right)}$$

$$\frac{2}{T} \cdot \frac{4A^2}{T^2} \cdot \left(\frac{t^3}{3}\right)_0^{T/2}$$

$$\frac{8A^2}{T^3(3)} \cdot \frac{T^3}{8} = A^2/3$$

$$\boxed{P = \frac{A^2}{3}}$$

① $x[n] = u[n]$

② $x[n] = 2^n u[n]$

neither E or P

$$E = \sum_{n=0}^{\infty} 2^{2n} < \infty$$

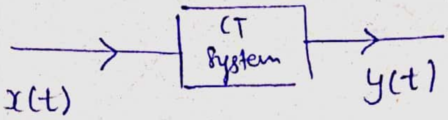
$$P = \lim_{N \to \infty} \left(\frac{1}{2N+1} \sum_{n=0}^{N} x^2[n] \right)$$

$$= \lim_{N \to \infty} \left(\frac{1}{2N+1} \sum_{n=0}^{N} 1 \right) \Rightarrow \lim_{N \to \infty} \frac{N+1}{2N+1}$$

$$= \frac{1}{2}$$

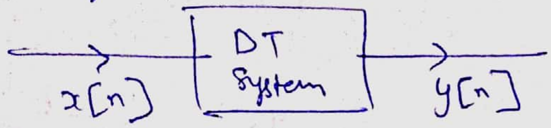
* Systems —

⊛ Continous time system



Symbolically $x(t) \rightarrow y(t)$

⊛ Discrete time system



$$x[n] \rightarrow y[n]$$

(I) Static & dynamic system

Static system or Memoryless signal : Those system in which present output depends only on the present input. For e.g $y(t) = 3x(t)$

$y[n] = (x[n]+1)^2$

Dynamic system → with memory : Those system in which present output depends also on past & future input. For e.g $y(t) = x(t+1)$

$y(0) = x(1)$

Date 09-09-2022

I. Static & Dynamic
II. Causal and non-Causal system.
III. Time variant & Time invariant systems.
IV. Invertible & Non-invertible system
V. Linear & Non-linear system
VI. Stable & Unstable system.

(I) Static & Dynamic ↳ with memory

↓

Memoryless

Ex: 1. $y(n) = x(n-1) \rightarrow$ Dynamic

2. $y(n) = \sum_{K=-\infty}^{n} x(K)$: Accumulator.

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II. Causal & Non-Causal System

→ The system in which present output depends only on present input and/or past input. ~~inputs~~

$y[n] = x[n-1]$

$n = 0$

$n = -1$

$n = +1$

$y[0] = x[-1]$

$y[1] = x[0]$

$y[-1] = x[-2]$

* Causal system are also called Non-anticipative system

* All the physicals realizable system are causal systems.

Non-Causal → Present output ~~also~~ depends also on future input

* Anti causal system → Present output depends only on future input.

Examples. ① $y(t) = x(t-1)$ → Causal → $y(0) = x(-1)$

$y(-1) = x(-2)$

$y(1) = x(0)$

② $y(t) = x(t^2)$ → Non-causal

③ $y(t) = x^2(t)$ → causal

④ $y(t) = x(2t)$ → Non-causal

⑤ $y(t) = x(t^2+1)$ → Anticausal.

III. Time variant & Time invariant.

Time invariant: A system is time invariant if a time shift in the input signal result in the identical time shift in the output signal.

Step 1. $y(t-t_0)$ → Response to a system due to delayed input

Step 2 $y(t, t_0)$

→ output is delayed by t_0

$\boxed{\text{if } y(t-t_0) = y(t, t_0)}$ → Time invariant

Ex. (1) $y(t) = x(t-1)$

$y(t-t_0) = x(t-t_0-1)$

$y(t,t_0) = x(t-t_0-1)$

$y(t-t_0) = y(t,t_0) \longrightarrow$ Time invariant

(2) $y(t) = \sin(x(t))$

$y(t-t_0) = \sin(x(t-t_0))$

$y(t,t_0) = \sin(x(t-t_0))$

$\Rightarrow y(t-t_0) = y(t,t_0) \rightarrow$ Time invariant

(3) $y(t) = x(t^2)$

$y(t-t_0) =$ Replace $t \rightarrow (t-t_0)$

$= x((t-t_0)^2)$

$y(t,t_0) = x(t^2-t_0)$

$\Rightarrow y(t-t_0) \neq y(t,t_0)$ Time variant.

(4) $y(t) = t\,x(t)$

$\rightarrow y(t-t_0) = (t-t_0)\,x(t-t_0)$

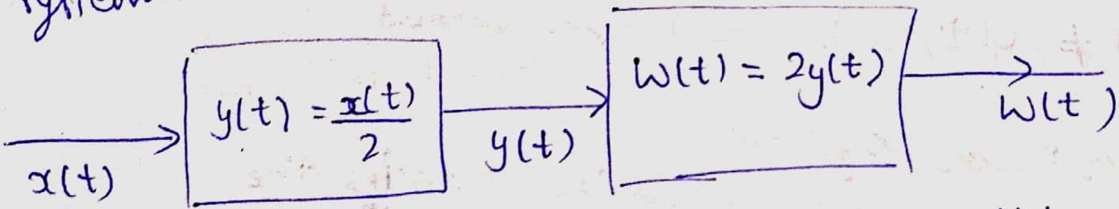
~~$y(t,t_0) =$~~

$y(t,t_0) = t\,x(t-t_0)$

T- variant.

IV Invertible & Non-invertible system

For invertible system, an inverse system will exist which when cascaded with original signal result in input to the first system.

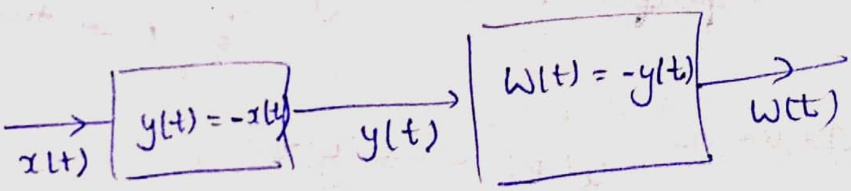


$$= 2 \times \frac{x(t)}{2}$$

$$= \boxed{x(t)}$$

$x(t)$	$y(t)$
0	0
-1	-1/2
1	1/2

Ex. ① $y(t) = -x(t)$ → invertible



② $y(t) = x(-t)$ → invertible

③ $y(t) = x(t^2)$ → Non-invertible

④ $y(t) = x(2t)$ → invertible

⑤ $y(t) = x(t+2)$ → invertible

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Q. Determine whether the give system is time vari or time invariant

$y(t) = x(2t)$

① $y(t-t_0) = x(2(t-t_0))$

② $y(t, t_0) = x(2t-t_0)$

$y(t-t_0) \neq y(t, t_0) \Rightarrow$ time variant.

(V) Linear & Non-linear System.

Linear system are those systems which satisfies the property of Superposition.

Superposition property (a) Homogeneity (Scaling)
(b) Additive.

Additive:

$x(t) \longrightarrow y(t)$

$x_1(t) \longrightarrow y_1(t)$

$x_2(t) \longrightarrow y_2(t)$

$x_1(t) + x_2(t) \longrightarrow y_1(t) + y_2(t)$

Homogeneity:

$x(t) \longrightarrow y(t)$

$x_1(t) \longrightarrow y_1(t)$

$ax_1(t) \longrightarrow ay_1(t)$.

* Superposition Property:

$x(t) \longrightarrow y(t)$, $\quad x_1(t) \longrightarrow y_1(t)$

$x_2(t) \longrightarrow y_2(t)$

$$\boxed{ax_1(t) + bx_2(t) \longrightarrow ay_1(t) + by_2(t)}$$

For any linear system

$x(t) \longrightarrow y(t)$

$0 \cdot x(t) \longrightarrow 0 \cdot y(t)$

$0 \longrightarrow 0$

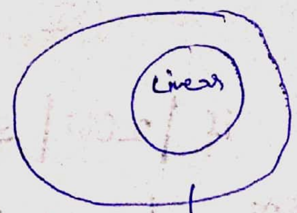
Zero in | zero out property.

E.g. $y(t) = x(t) + 3$

$x(t) = 0$

$y(t) = 3$

Non-linear.



Ques. Comment of Linearity.

① $y(t) = x^2(t)$

Sol. $a x_1(t) \longrightarrow a y_1(t) = a x_1^2(t)$

$b x_2(t) \longrightarrow a y_2(t) = b x_2^2(t)$

$x_3(t) = a x_1(t) + b x_2(t) \longrightarrow y_3(t)$

$= x_3^2(t)$

$= a \cdot a x_1^2(t) + b^2 x_2^2(t) + 2ab\, x_1(t)\, x_2(t)$

$= a^2 y_1(t) + b^2 y_2(t) + \_\_\_\_$

$\neq a y_1(t) + b y_2(t)$

② $\boxed{y(t) = K x(t)}$

↳ Linear

③ $y(t) = t\, x(t) \longrightarrow$ Linear

④ $y[n] = x[n] + 4$

$a x_1(n) + b x_2(n) \xrightarrow{\times} a y_1(n) + b y_2(n)$

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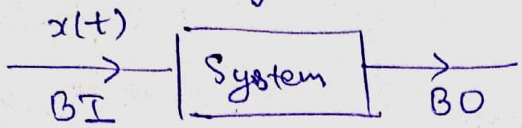
VI) Stable & Unstable System

Any system which satisfies BIBO (Bounded i/p Bounded o/p) stability criteria are called Stable system.



$$\text{if } |x(t)| \le M_x < \infty \text{ (bounded i/p)}$$

and also $$|y(t)| \le M_y < \infty \text{ (bounded o/p)}$$

Q. $y(t) = \dfrac{x(t)}{\sin t}$

→ if $|x(t)| < \infty$

$t = 0, \pi, 2\pi$

$y(t) = \infty$ unstable.

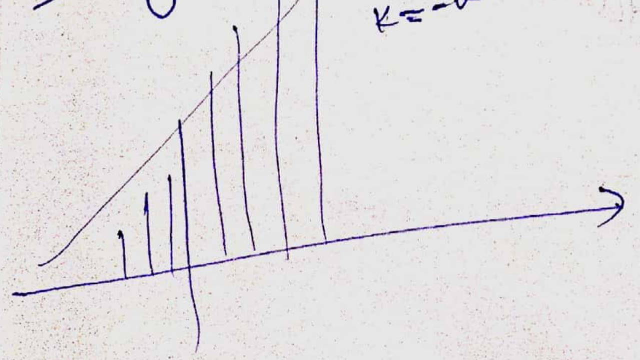
Q. $y(t) = x(t) + 3$

if $|x(t)| \le M_x < \infty$

$|y(t)| \le M_x + 3 \le M_y < \infty$

→ stable.

Q. $y[n] = \displaystyle\sum_{K=-\infty}^{n} x(K)$ → $\boxed{\text{unstable}}$



LTI System:

(Linear Time invariant System)

Those systems which are linear as well as time invariant.

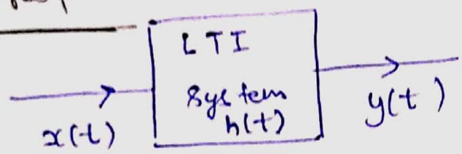
Any LTI system will follow

(a) Superpositⁿ property (b) Time invariance property.

→ For any LTI system, it exhibits two important characteristics

↳ (1) Impulse response
(2) Transfer function } only for LTI

* **Impulse response** –



if $x(t) = \delta(t)$

$y(t) =$ impulse response $(h(t))$

$\delta(t) \longrightarrow h(t)$

* ~~Transfer function~~

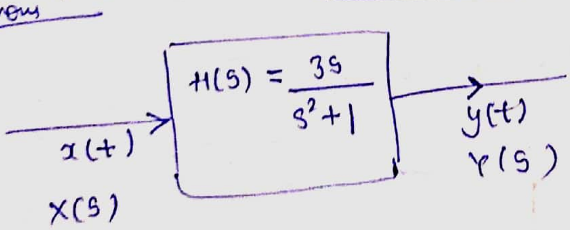
CT LTI system

DT LTI system : if $x[n] = \delta[n]$, $y[n] = h[n]$: 2 impulse response of DT LTI system

* **Transfer function.**

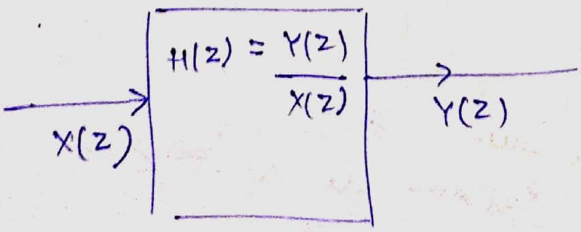
CT LTI : $H(S) = \dfrac{Y(S)}{X(S)}$ → Analyzing the LTI system in frequency domain

C ↓ continuous, T ↳ time



DT LTI

D ↓ Discrete



* h(t) of any LTI system is known, y(t) for any input can be determined. ~~for discrete times~~

For Discrete Time

$x[n] =$

Any x(n) can be represented as linear combination of delayed impulse.

$$\boxed{x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]}$$ → Shifting property of impulse function.

Convolution Theorem -

$$\delta[n] \longrightarrow h[n]$$

TI property → $\delta[n-k] \longrightarrow h[n-k]$

Scaling property $x[k]\,\delta[n-k] \longrightarrow x[k]\,h[n-k]$

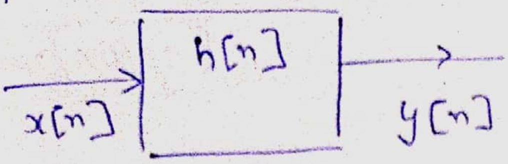
$$\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k] \longrightarrow \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

$$x[n] \longrightarrow y[n]$$

$$\boxed{y[n] = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)}$$

↓ Convolution Sum

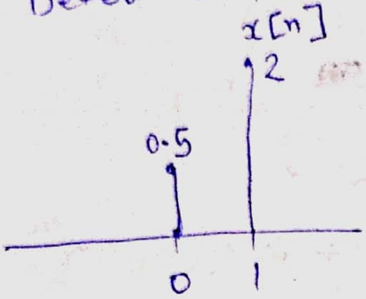
The output of DTLTI System is convolution sum of x[n] & h[n].

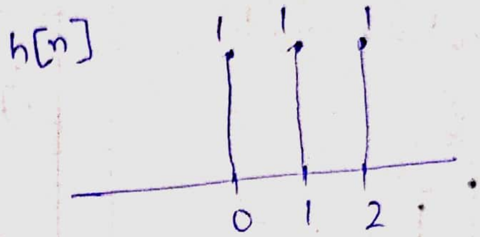


$y[n] = \underbrace{x[n] * h[n]}_{\text{convolution sum}}$

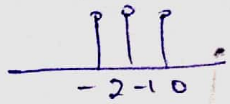
$$= \sum_{K=-\infty}^{\infty} x(K)\, h(n-K) = \sum_{K=-\infty}^{\infty} h(K)\, x(n-K)$$

Q. Determine the output of LTI system of

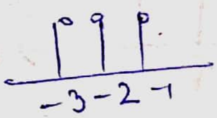




$\rightarrow$ $y(0) = \sum_{K=-\infty}^{\infty} x(K)\, h(-K)$

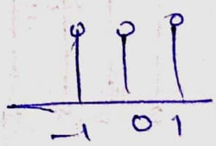


$= 0.5$



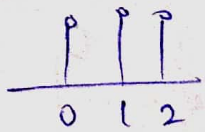
$y(-1) = \sum_{K=-\infty}^{\infty} x(K)\, h(-K-1) = 0$

$y[n] = 0$ for $n < 0$

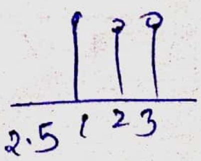


$y(1) = \sum_{K=-\infty}^{\infty} x(K)\, h(-K+1)$

$= 0.5 + 2 = 2.5$

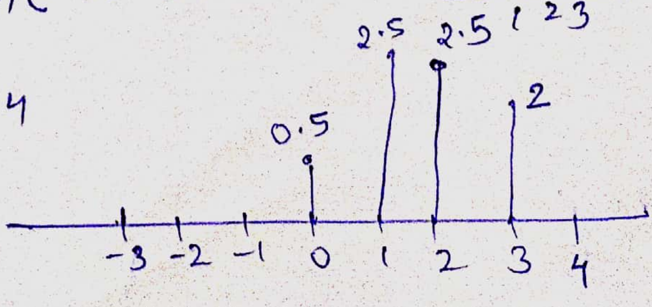


$y(2) = \sum_{K=-\infty}^{\infty} x(K)\, h(-K+2) = 2.5$



$y(3) = \sum_{K=-\infty}^{\infty} x(K)\, h(-K+3) = 2$

$y[n] = 0$ for $n \geq 4$



Q. $x[n] = \alpha^n u[n]$ Calculate the response of the system.

$h[n] = u[n]$

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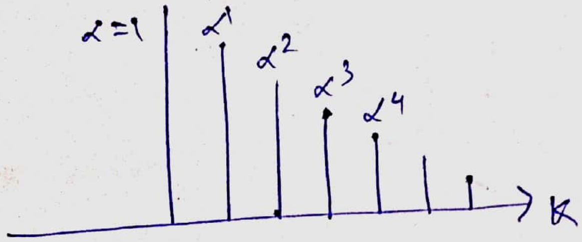
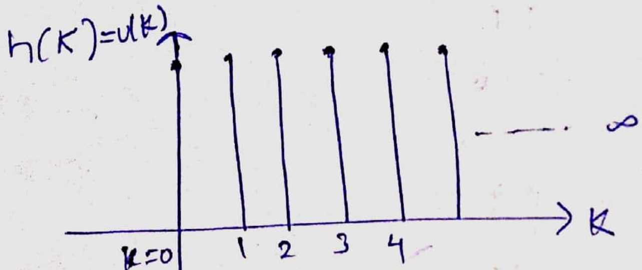
$\rightarrow y(n) = x(n) * h(n)$

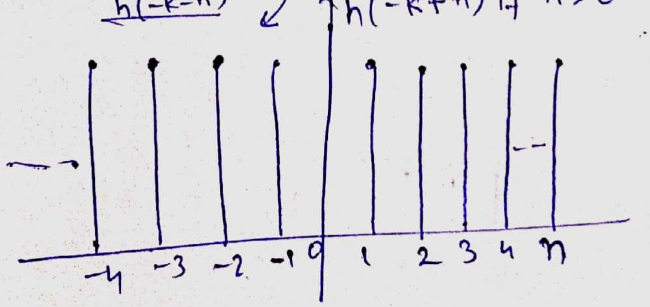
$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$x(k) = \alpha^k u[k], \ \alpha < 1$





$h(-k-n)$? $h(-k+n)$ if $n>0$



For $n<0$: $y(n) = 0$

for $n>0$

$$y(n) = \sum_{k=0}^{n} \alpha^k \cdot 1$$

$$= \alpha^0 + \alpha^1 + \alpha^2 + \cdots \alpha^n$$

$$= 1 + \alpha^1 + \alpha^2 + \cdots + \alpha^n$$

$$= \frac{a(1-r^n)}{(1-r)}, \quad |r|<1$$

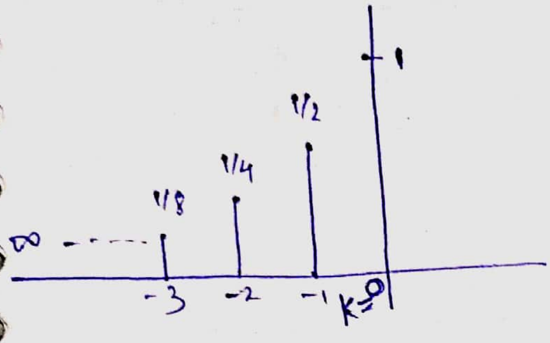
$$= \frac{1(1-\alpha^{n+1})}{1-\alpha}$$

$$y(n) = \begin{cases} 0 & \text{if } n<0 \\ \dfrac{1-\alpha^{n+1}}{1-\alpha}, & \text{if } n \geq 0 \end{cases}$$

Q. $x[n] = 2^n u[-n]$ Calculate $y[n]$.

$h[n] = u[n]$

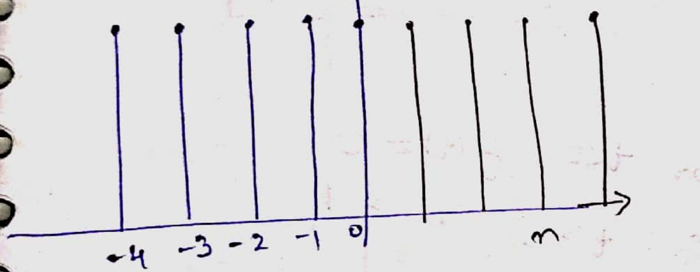
$\rightarrow$ $x(k) = 2^k u[-k]$



~~h(k) = u(~~

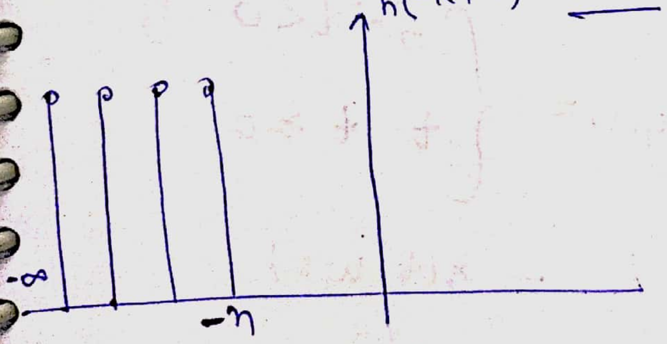
$h(-k) = u(-k)$

$h(-k+n)$ $n \geq 0$



* $h(-k+n)$ when $n<0$

$h(-k+n)$ $n<0$



$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{0} 2^k$$

$\Rightarrow$ Let $k = -r$

$$= \sum_{0}^{\infty} \left(\frac{1}{2}\right)^r$$

$$= \frac{a}{1-r}$$

$$= \frac{1}{1-1/2}$$

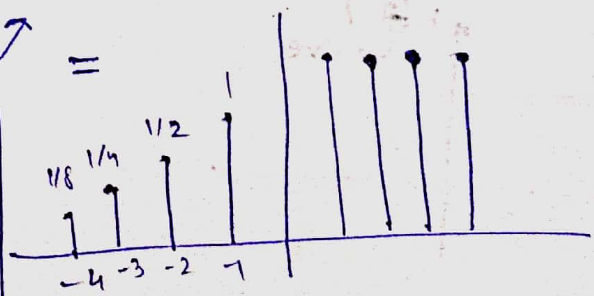
$$= 2$$

$$y(n) = \sum_{k=-\infty}^{n} 2^k \cdot 1 \qquad \cancel{k=-r}$$

$k = -r$

$$\sum_{r=-n}^{\infty} 2^{-r} = 2^{n+1}$$

=



Continous time LTI System

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$h(t)$: impulse response of CTLTI system.

Convolution theorem

$$\delta(t) \longrightarrow h(t)$$

Time invariant $\delta(t-\tau) \longrightarrow h(t-\tau)$

$$x(\tau)\,\delta(t-\tau) \longrightarrow x(\tau)\,h(t-\tau)$$

scaling

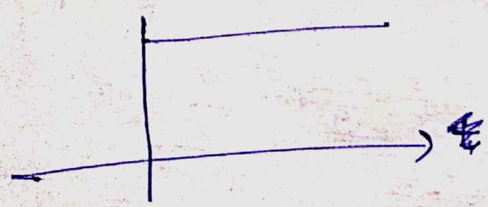
Additive :

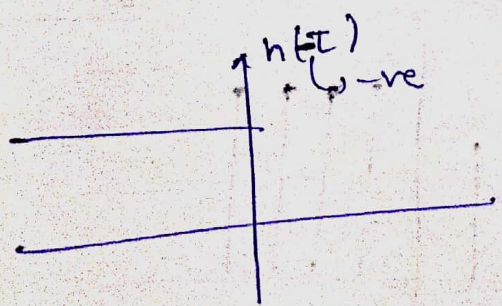
$$\boxed{\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau \longrightarrow \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau}$$

Q. $x(t) = u(t)$

$h(t) = u(t)$

$\rightarrow$ $x(\tau)$





For $t<0$: $y(t) = 0$

for $t \geq 0$:

$$y(t) = \int_{\tau=0}^{t} 1 \cdot 1 \cdot d\tau = t$$

$$y(t) = \begin{cases} 0 & t<0 \\ t & t \geq 0 \end{cases}$$

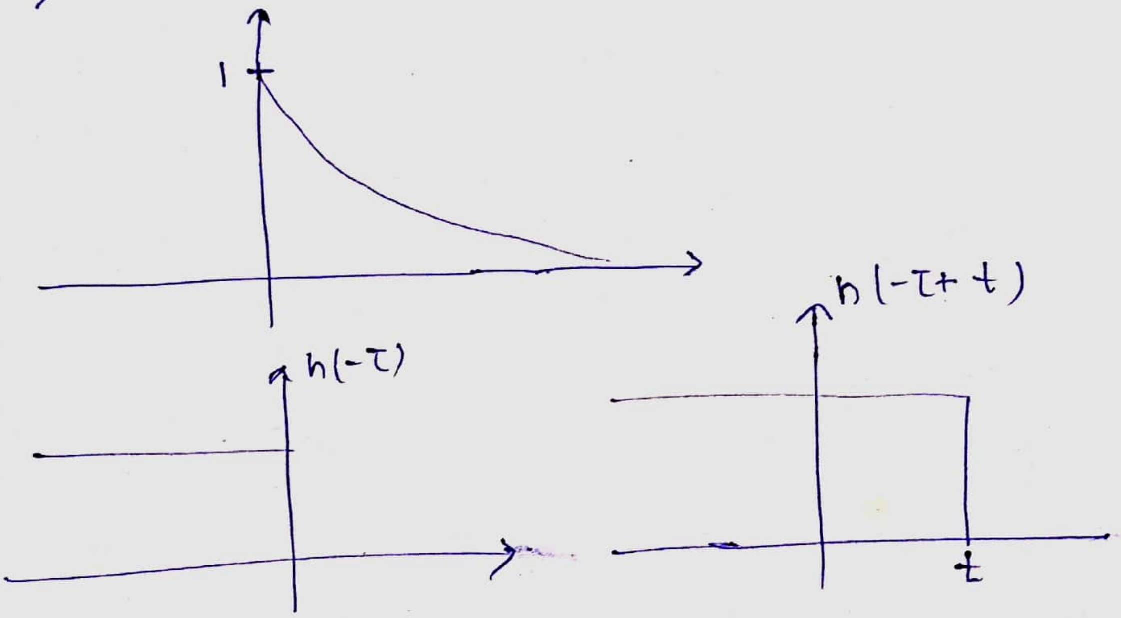
$$= r(t)\,u(t)$$

Q. $x(t) = e^{-at}u(t)$; $a>0$

$h(t) = u(t)$

$y(t) = ?$

$\Rightarrow \quad x(\tau) = e^{-a\tau}u(\tau)$



$t \geq 0$

$$y(t) = \int_{\tau=0}^{t} e^{-a\tau}\, d\tau$$

$$y(\tau) = \frac{1}{a}\left(1 - e^{-at}\right)u(t)$$