12/8/22

1. SET THEORY

DEFINITION

A set is a well defined collection of objects

ex, Set of all positive integer, set of all states in India.

Collection of 5 best Indian actors is not a set.

Objects of a set are called members or elements of a set

enclosed within { }

NOTE: Sets are denoted by A, B, C..... Elements

are denoted by a, b, c

If A is any set contains a it is denoted by $a \in A$

If b is not in A it is denoted by $b \notin A$

REPRESENTATION OF SET

Sets are represented in two ways

1. Roster or tabular form
2. Set builder notation

ROSTER FORM

All of the elements of the set are listed if possible seperated by commas within the braces.

ex, {a, e, i, o, u}

SET

We define the element of a set by specifying a property that they have in common.

ex, $2\mathbb{Z} = \{x \mid x = 2n, n \in \mathbb{Z}\}$. $\mid \rightarrow$ Such that

CARDINALITY OF A SET. Or SIZE OF A SET.

Let A be any set, the number of distinct elements in A are called cardinality $|A|$ or $\#A$

ex, $A = \{1, 2, 3\}$

$n(A) = 3$.

NULL SET

A set which contains no elements is called null set or empty set denoted by ϕ or $\{\}$.

$|\phi| = 0$

SINGLETON SET

A set which has only one element

$A = \{a\}$ is a singleton set

A set which has cardinality n $\forall n \in \mathbb{Z}^+$ is called a finite set.

Any set that is not finite is called infinite set.

ex, $A = \{2, 4, 6, 8, \dots\}$.

A set A is said to be subset of set B if each element of A is an element of B. $A \subseteq B$

Set of all even +ve integer between 1 to 100 is a subset of all +ve integer btwn 1 to 100

PROPER SUBSET

$A \subset B$, atleast one of element of B should not be in A. $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 3, 4, 5\}$.

IMPROPER SUBSET. $A \leq B$

$A \subseteq B$, every element of A is also an element of B.

NOTE: every set is a subset of itself $A \subseteq A$

empty set is a subset of each set.

$A \subseteq B$, A is a subset then B is a superset.

EQUALITY OF

If two sets A and B are equal if and only if $A \subseteq B$, $B \subseteq A$, then $A = B$.

POWER SET

Given any set A, Collection of all subset of A is called power set of A. $P(A)$

The cardinality of power set of a set A of Cardinality n is 2^n.

$$n(P(A)) = 2^{n(A)}$$

$A = \{1, 2, 3\}$.

$P(A) = \{\{\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\},$
$\{1, 3\}, \{1, 2, 3\}\}$

$8 = n(P(A)) = 2^{n(A)} = 2^3 = 8$.

The power set of empty set is 1.

EQUIVALENT SET.

If the cardinality of 2 sets are same are called equivalent set.

OVERLAPPING.

Two sets that have atleast one common element

SET OPERATION.

Set U is called universal set. If U is the superset of all the sets A, B, C and hence U is the Universal set.

$U = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 2, 3\}$

$B = \{3, 4, 5\}$, $C = \{1, 5, 2, 3\}$.

Union of two sets denoted by $A \cup B$, set of all elements that belongs to A, B or both.

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

$A = \{1, 2, 3\}$ $B = \{1, 2, 3, 4\}$

$A \cup B = \{1, 2, 3, 4\}$.

INTERSECTION

$A \cap B$, is the common elements in A and B.

$A \cap B = \{1, 2, 3\}$. $A \cup B = \{x \mid x \in A$ and $x \in B\}$.

DISJOINT.

If two sets are disjoint then they do not contains any no common element. $A = \{1, 2\}, B = \{3, 4\}$.
$A \cap B = \phi$.

COMPLEMENT

Set of elements belongs to U but not belongs to A. U is the universal set, A is the set in U.

$\bar{A}$ or A' or $A^c = \{x \mid x \in U$ and $x \notin A\}$.

$U = \{1, 2, 3, 4, 5\}$ $A = \{1, 3, 5\}$.

$A' = U - A = \{2, 4\}$.

DIFFERENCE OF A and B or Relative complement of B with respect to A

If A and B are any two set; difference of A and B is set of element $\in A$ not belongs to B.

$A - B$ or $A \mid B = \{x \mid x \in A$ and $x \notin B\}$.

example, $A = \{1, 2, 3, 4\}$, $B = \{1, 3, 5, 7\}$.

$A - B = \{2, 4\}$.

ORDERED PAIR.

Consist of two object in a given fixed order

An ordered pair is not a set consisting of two elements

CARTESIAN PRODUCT.

If A and B are sets, the set of all ordered pair whose first component belongs to A and second component belongs to B. Denoted by $A \times B$

$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$.

$B \times A = \{(b, a) \mid a \in A \text{ and } b \in B\}$.

example, $A = \{a, b, c\}$, $B = \{1, 2\}$

$A \times B = \{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\}$

$B \times A = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\}$

$A \times B \neq B \times A$.

$A \times B = B \times A$ only when $A = B$.

SYMMETRIC DIFFERENCE.

If A and B are any two set the set of elements that belongs to A or B not to both. is called Symmetric difference of A and B.

$A \oplus B$ or $A \Delta B = (A - B) \cup (B - A)$.

$A \oplus B$ or $A \Delta B = (A \cup B) - (A \cap B)$

1. Using set builder prove ~~A∪~~ $A \cap B = B \cap A$.

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

$= \{x \mid x \in A \cap B\}$.

$= \{x \mid x \in A \text{ and } x \in B\}$.

$= \{x \mid x \in B \text{ and } x \in A\}$.

$= \{x \mid x \in B \cap A\} = B \cap B$.

2. Prove $\overline{A \cap B} = \overline{A} \cup \overline{B}$ using set builder form.

$\overline{A \cap B} = \{x \mid x \in \overline{A \cap B}\}$

$= \{x \mid x \notin A \cap B\}$

$= \{x \mid x \notin A \text{ and } x \notin B\}$.

$= \{x \mid x \in \overline{A} \text{ or } x \in \overline{B}\}$.

$= \{x \mid x \in \overline{A} \cup \overline{B}\} = \overline{A} \cup \overline{B}$.

$\overline{A \cup B} = \overline{A} \cap \overline{B}$.

$= \{x \mid x \in \overline{A \cup B}\}$

$= \{x \mid x \notin A \cup B\}$

$= \{x \mid x \notin A \text{ or } x \notin B\}$

$= \{x \mid x \in \overline{A} \text{ and } x \in \overline{B}\}$

$= \{x \mid x \in \overline{A} \cap \overline{B}\}$

$= \overline{A} \cap \overline{B}$.

Dual of any statement obtained by replacing

$\cup$ by $\cap$, $\cap$ by $\cup$, ϕ by U, U by ϕ }

PARTESIAN OF A SET.

S_1	S_2	S_3	S_4

$S = \{S_1, S_2, S_3, S_4\}$

If S is a non empty set, a collection of disjoint non empty subset of S, whose union is S is called partesian of a set.

In other words, collection of subset of S_i is a partesian if and only if set of all maxset doesn't constitute partition

Note: If n subset of a set are given the no of minterm or maxterm is 2^n.

i) $S_i \neq \phi$ for each i.

ii) $S_i \cap S_j = \phi$ for $i \neq j$

iii) $\cup S_i = S$ for all i

for example,

$A = \{1, 2, 3, 4, \ldots, 10\}$.

$A_1 = \{1, 3, 5\}$. $A_3 = \{8, 9\}$

$A_2 = \{2, 4\}$ $A_4 = \{6, 7, 10\}$

A_1, A_2, A_3, A_4 form partision of A.

MINSET.

Let $\{B_1, B_2, \ldots, B_n\}$ be a collection of subset of Set "A", The minset (or) min term generated by $B_1, B_2, \ldots B_n$ is of the form $D_1 \cap D_2 \cap D_3 \ldots \cap D_n$ where each $D_i = B_i$ (or) B_i^C.

Note: Dual of minset is maxset

let: $U = \{1, 2, 3, \ldots, 10\}$

$A = \{2, 4, 6\}$, $B = \{3, 5, 7, 9\}$.

Find the minterm & maxterm.

$D_1 = A \cap B = \{\ \}$ $A^C = \{1, 3, 5, 7, 8, 9, 10\}$

$D_2 = A \cap B^C = \{2, 4, 6\}$ $B^C = \{1, 2, 4, 6, 8, 10\}$

$D_3 = A^C \cap B = \{3, 5, 7, 9\}$

$D_4 = A^C \cap B^C = \{1, 8, 10\}$

Union of minterms is "U"

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

A set of all nonempty minterms of 'A' is the cartesian of 'A'

Maxsets

$D_1 = A \cup B$

$D_2 = A \cup B^c$

$D_3 = A^c \cup B$

$D_4 = A^c \cup B^c$

1. Prove that $(A-C) \cap (C-B) = \{\}$ (or) ϕ, where A, B, C are sets

$(A-C) \cap (C-B) = \{(x \mid x \in A \text{ and } x \notin C\} \text{ and } (x \mid x \in C \text{ and } x \notin B)\}$

$= \{x \mid x \in A \text{ and } (x \notin C \text{ and } x \in C) \text{ and } x \notin B\}$

$= \{x \mid x \in A \text{ and } x \in \phi) \text{ and } x \notin B\}$

$= \{x \mid x \in \phi \text{ and } x \notin B\}$

$= \{x \mid x \in \phi\}$

$= \phi$.

$(A-C) \cap (C-B) = (A \cap \overline{C}) \cap (C \cap \overline{B})$

$= A \cap (\overline{C} \cap C) \cap \overline{B}$

$= A \cap (\phi \cap \overline{B})$

$= A \cap \phi$

$= \phi$.

2. Prove that $A-(B\cap C)=(A-B)\cup(A-C)$, where A, B, C are sets

$A-(B\cap C)=\{x \mid x\in A \text{ and } x\notin (B\cap C)\}$

$=\{x \mid x\in A \text{ and } x\in (B\cap C)'\}$

$=\{x \mid x\in A \text{ and } x\in B'\cup C'\}$

$=\{x \mid x\in A \text{ and } x\in B' \text{ or } x\in C'\}$

$=\cancel{A\cap(B\cup}\ \{x \mid (x\in A \text{ and } x\in B') \text{ or } (x\in A \text{ and } x\notin C')\}$

$=\{x \mid (x\in A \text{ and } x\notin B) \text{ or } (x\in A \text{ and } x\notin C)\}$

$=(A-B)\cup(A-C)$

$A-(B\cap C)=A\cap(B\cap C)'$

$=A\cap(B'\cup C')$

$=(A\cap B')\cup(A\cap C')$

$=(A-B)\cup(A-C)$

3. Prove $A\cap(B-C)=(A\cap B)-(A\cap C)$

$A\cap(B-C)=\{x \mid x\in A \text{ and } x\in (B-C)\}$

$=\{x \mid x\in A \text{ and } (x\in B \text{ and } x\notin C)\}$

$=\{x \mid (x\in A \text{ and } x\in B) \text{ and } (x\in A \text{ and } x\notin C)\}$

$=(A\cap B)-(A\cap C)$.

RELATION :
A relation is a structure . relationship of (represents)
elements of a set to the elements of another set .
If A & B are set . Subset R of cartesian product
$A \times B$ is called a binary relation from A to B .
If R is a binary relation from A to B , R is set
of ordered pairs of (a, b) where $a \in A$, $b \in B$.
when $(a, b) \in R$ we use aRb. It is read as "a is
related to b".
example,
$A = \{1, 2, 3, 4\}$, $B = \{1, 7, 8\}$.
Relation is A less than B .
$R = \{(1,7), (1,8), (2,7), (2,8), (3,7), (3,8), (4,7), (4,8)\}$
$A \times B = \{(1,1), (1,7), (1,8), (2,1), (2,7), (2,8) \ldots\}$.

if R is a relation from a set A to itself R is
called relation on the set A . $R \subseteq A \times A$.

DOMAIN & RANGE
Let R be a relation from A to B $R \subseteq A \times B$
The domain D of Relation R is set of all first
element of the ordered pairs which belongs to R

domain of R.

$D(R) = \{a \mid a \in A \text{ of } (a,b) \in R \text{ for some } b \in B\}$.

RANGE

Range of the relation is set of all second elements of ordered pairs in R.

$R(R) = \{b \mid b \in B, (a,b) \in R \text{ for } a \in A\}$.

clearly the domain of a relation from A to B is a subset of A and range is subset of B.

example, $A = \{1, 2, 3, 4\}$, $B = \{r, s, t\}$.

$R = \{(1, r), (2, s), (3, r)\}$.

$D(R) = \{1, 2, 3\}$. $R(R) = \{r, s\}$.

1. Let R be a relation $A = \{2, 3, 4, 5\}$.

$B = \{3, 6, 7, 10\}$ which is defined by the expressi x/y write R as a set of ordered pairs. find domain & range. 'x divides y'

$x/y \Rightarrow y = mx$.

$R = \{(2,6), (2,10), (3,6), (3,3), (5,10)\}$.

$D(R) = \{2, 3, 5\}$

$R(R) = \{6, 3, 10\}$

TYPES OF RELATION

UNIVERSAL RELATION

A relation R on set A is said to be universal relation if $R = A \times A$.

$A = \{1, 2, 3\}$.

$R = A \times A = \{(1,1), (1,2), (2,1), (1,3), (2,2), (2,3), (3,1), (3,2), (3,3)\}$.

VOID or NULL RELATION

Relation R on set A is called void relation if R is null set

ex, $A = \{3, 4, 5\}$ then aRb, if $a + b > 0$

$R = \{ \}$.

INVERSE

When R is any relation from set A to set B

The inverse of R is denoted by $R^{-1} : B \to A$.

$R : A \to B$. whic consist of those ordered pairs got by interchanging the element of ordered pair in R. $R^{-1} = \{(b, a) \mid (a, b) \in R\}$.

if aRb then $bR^{-1}a$

ex, $A = \{2, 3, 5\}$ $B = \{6, 8, 10\}$.

$R \to a < b$.

$R = \{(2,6), (2,8), (2,10), (3,6), (3,8), (3,10), (5,10), (5,6), (5,8), (5,10)\}$.

$R^{-1} = \{(6,2),(8,2),(10,2),(6,3),(8,3),(10,3),$
$(6,5),(8,5),(10,5)\}$

OPERATIONS ON RELATION:

→ Union and Intersection of relation:

Let R and S are two relation of set A and Set B the $R \cup S$ and $R \cap S$ are defined as follows.

$R \cup S = \{(a,b) \mid (a,b) \in R$ or $(a,b) \in S\}$

(or)

$= \{(a,b) \mid a(R \cup S)b \Leftrightarrow aRb \vee aSb\}$.

$R \cap S = \{(a,b) \mid (a,b) \in R$ and $(a,b) \in S\}$.

$= \{(a,b) \mid a(R \cap S)b \Leftrightarrow aRb \wedge aSb\}$.

Let $A = \{1,2,3\}$ $B = \{2,6\}$. Define the relation R

$R = \{(1,2),(1,6),(2,6)(3,6)\}$ $S = \{(1,2),(1,6),(2,2),$
$(2,6),(3,2).(2,6)(3,6)\}$

$R \cup S = \{(1,2),(1,6),(2,2),(2,6),(3,2),(3,6)\}$

$R \cap S = \{(1,2),(1,6),(2,6),(3,6)\}$.

Difference of R & S.

$R - S = aRb$ & $a \notin b$.

$A = \{x, y, z\}$ $B = \{1, 2, 3\}$

$C = \{x, y\}$ $D = \{2, 3\}$

$R = A \to B = \{(x, 1), (x, 2), (y, 3)\}$

$S = C \to D = \{(x, 2), (y, 3)\}$.

$R - S = \{(x, 1)\}$.

R' or $\sim R$ is defined by $aR'b = aRb$

$R' = A \times B - R$.

IDENTITY RELATION

Relation R on a set A is called an identity relation on A if

$R = \{(a, a) \mid a \in A$ is denoted by $I_A\}$.

$A = \{1, 2, 3\}$.

the $R = \{(1,1), (2,2), (3,3)$ is identity relation.

Let $A = \{1, 2, 3\}$, $B = \{1, 4\}$.

$R \to a < b$.

find R, R', R^{-1}

$A \times B = \{(1,1), (1,4), (2,4), (2,1), (3,1), (3,4)\}$.

$R = \{(1, 4), (2,4), (3, 4)\}$.

$R' = A \times B - R = \{(1,1), (2,1), (3,1)\}$

$R^{-1} = \{(4,1), (4,2), (4,3)\}$

2. Let R is Relation btwn A & B, $S \rightarrow B$ & C.

if A, B, C are sets. The Composition of $R : A \rightarrow B$

$S : B \rightarrow C$

$R \circ S$ is relation from $A \rightarrow C$. $R \circ S : A \rightarrow C$

$R \circ S = \{(a,c) \mid (a,b) \in R \text{ \& } (b,c) \in S\}$.

Let, $A = \{1, 2, 3\}$, $B = \{2, 3, 6, 8, 12\}$,

$C = \{13, 17, 22\}$.

$R = \{(1,2), (1,3), (1,12), (2,3), (2,6), (2,8), (2,12)\}$

$S = \{(2,13), (2,17), (3,13), (3,22), (8,22)\}$.

$R \circ S = \{(1,17), (1,13), (1,22), (2,13), (2,22)\}$.

Let A be any non empty set and R be a relation on A.

1. R is called a reflexive relation if aRa for every $a \in A$

ie; $(a,a) \in R$ for every $a \in A$.

3. R is Symmetric if whenever aRb, then bRa

if $(a,b) \in R$ then $(b,a) \in R$.

4. R is said to be transitive if $(a,b), (b,c) \in R$ then $(a,c) \in R$.

4. R is irreflexive if $(a,a) \in R$ for all $a \in A$

5. R is antisymmetric if a R b & b R a then $a = b$.

(or) a R b then b R a.

6. R is asymmetric if it is both antisymmetric and irreflexive

NOTE: Symmetric & AntiSymmetric Relation are not opposite because the Relation R can contain both the Properties or may not.

EQUIVALENCE RELATION

A relation R on non empty set A is called an equivalence relation if reflexive, Symmetric and transitive $(a,b),(b,c)$ (a,c)

1. If R is a Relation is a set of +ve integers such that $(a,b) \in R$ if $a^2 + b$ is even. Then prove R is an equivalence relation.

Soln

$a^2 + a \in R$ is even.

$a^2 + a = a(a+1)$ is even as either a or $(a+1)$ is even.

$(a,a) \in R$, Hence R is reflexive.

When $a^2 + b$ is even a & b must be even or both odd.

The $b^2 + a$ is also even.

$\therefore$ R is Symmetric.

when a, b, c are even a^2+b, b^2+c are even

also a^2+c is also even.

when a, b, c are odd a^2+b, b^2+c are even

also a^2+c is also even.

then $(a,b) \in R$, $(b,c) \in R$, $(a,c) \in R$.

$\therefore$ It is transitive. Hence R is an equivalence Relation

2. Let $A = \{1, 2, 3, 4\}$,

$R = \{(1,1), (1,2), (2,1), (1,3), (3,1), (2,2), (3,3)\}$

$S = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

Prove R & S are reflexive, symmetry,

1. R is not reflexive because $(4,4) \notin R$.

R is symmetric $(a,b) \in R$ & $(b,a) \in R$.

R is not transitive because $(2,3) \notin R$

2. S is not reflexive because $(4,4) \notin R$

S is symmetric as $(a,b) \in R$ & $(b,a) \in R$.

S is transitive

3. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $R = \{(x,y) \mid |x-y| = 2\}$ is R an equivalence Relation

$R = \{(1,3), (2,4), (3,5), (4,6), (5,7), (7,5),$

$(6,4), (5,3), (4,2), (3,1)\}$

R is not reflexive $(a,a) \notin R \ \forall a \in A$.

R is symmetric

R is not transitive

$\therefore$ R is non equivalence relation.

4. R is a relation on a set of integers such that $\{(a,a) \in R \mid 3a+4b = 7n$ for some integer $n\}$ prove R is an equivalence relation.

1. Reflexive.

$(a,a) \in R$. if & only if $3a + 4a = 7a$

$a \in R$ the $n \in R$.

$\therefore aRa$ R is reflexive.

2. Symmetric.

$(a,b) \in R$ then $(b,a) \in R$.

$(a,b) \in R \Rightarrow 3a+4b = 7n$

$(b,a) \in R \Rightarrow 3b+4a = 7n$. $7a+7b-(3a+4b) = 7(a+b) - 7n = 3a$

$3b+4a = (7a+7b) - (3a+4b)$

$7(a+b) - 7n = 7(a+b-n)$

$3a+4b = 7(a+b-n)$

R is Symmetric.

3. Transitive

$3a+4b = 7m - 4b + 7n - 3b$

$= 7(m+n) - 7b$

$= 7(m+n-b)$

$(a, c) \in R$.

R is transitive.

PARTIAL ORDERING OR PARTIAL ORDER RELATION

A relation R on a set A is called partial ordering if and only R is reflexive, antisymmetric, transitive.

1. A set A together with partial ordering ~~are~~ R is called partially ordered set or poset.

NOTE: It is convient to denote partial ordering is $\leq$ this symbol does not necessarily mean less than or equal to as it used for real num

REPRESENTATION OF ~~GRAPHS~~ RELATION BY GRAPHS.

Let R be a relation on set A to represent R graphically

1. Each element of A is represented by a point called node or vertex.
2. Whenever element A is related to element B an arc or straight line is drawn from point A to B. It starts from A to B are called arc or edges.

The direction is indicated by an arrow the resulting diagram is called directed graph or digraph.

3. The edge of the form a, a is represented by using an arc from the vertex a back to itself is called loop.

1. In a digraph of R, the in degree of vertex is no of edges terminating at the vertex but out degree of the vertex is the no of edges leaving the vertex.
2. A relation R is reflexive if & only if there is a loop of every vertex of the digraph
3. A relation R is symmetry if & only if for every edge betwn distinct vertices in its digraph there is an edge in the opposite direction.

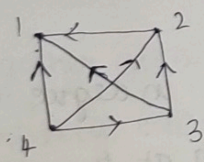
4. A relation R is transitive if & only if whenever there is an edge from vertex a to b, b to c, and there will be edge from a to c.

5. A relation R is antisymmetry if & only if there are never two edges in opposite direction between distinct vertices in other words a relation R on a set is antisymmetry if there is atmost one directed edge btwn every pair of vertices.

Example,

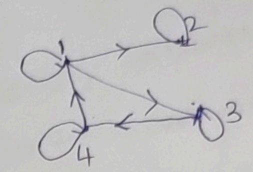
$A = \{1, 2, 3, 4\}$, $Q = \{(x, y) \mid x > y\}$.

$R = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\}$



1. It is not reflexive since no loop
2. R is not symmetric since no opposite direction edges.
3. It is transitive
4. It is anti symmetric.

2. $A = \{1, 2, 3, 4\}$, $R = \{(1,1), (1,2), (1,3), (2,2), (3,3), (3,4), (4,1), (4,4)\}$.



1. R is reflexive
2. R is not symmetry
3. R is not transitive
4. R is anti symmetry

MATRIX

If R is a relation from set A to set B where, the elements of A & B are assumed to be in a specific order.

The relation R can be represented

$M_R = [M_{ij}]$

Example,

If $A = \{a_1, a_2, a_3\}$ $B = \{b_1, b_2, b_3, b_4\}$

$R = \{(a_1, b_2), (a_2, b_1), (a_2, b_3), (a_2 b_4); (a_3 b_2), (a_3, b_4)\}$.

$$M_R = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{cccc} b_1 & b_2 & b_3 & b_4 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{array}$$

NOTE :

If R & S be two relation on A. when

$M_{R \cup S} = M_R \vee M_S$ → joint → union

$M_{R \cap S} = M_R \wedge M_S$ → intersection

$M_{R^{-1}} = (M_R)^T$ → transpose.

Example,

1. $M_R = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ $M_S = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$.

$M_{R \cup S} = M_R \vee M_S$.

$$= \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$M_{R \cap S} = M_R \wedge M_S$

$$= \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Boolean Product.

$M_{R \circ S} = M_R \cdot M_S$.

$$= \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0+0 & 0+0+1 & 1+0+0 \\ 0+1+0 & 0+0+1 & 0+0+0 \\ 1+0+0 & 0+0+0 & 1+0+0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

3. Examine if the relation R represented by $M_R = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ is an equivalence relation. Using properties of M_R.

→ Since all the elements in the main diagonal of M_R ∴ M_R is reflexive.

→ M_R is Symmetry because $M_{31} = M_{13}$, $M_{12} = M_{21}$

→ Hence $M_R^2 = M_R$ ∴ M_R is transitive.

HASSE DIAGRAM

The simplified form of a digraph of a partial ordering on a finite set that contains sufficient information about partial ordering called Hasse Diagram.

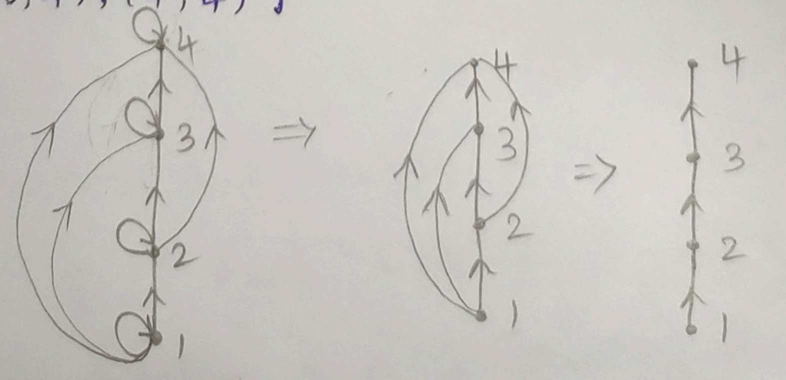
STEPS TO BE FOLLOWED TO DRAW HASSE D//

→ Create a vertex for elements of given domain → If aRb draw an edge from a to b $\overset{b}{\uparrow}_{a}$

→ Remove self loops and transitive edges → aRb, $bRc \Rightarrow aRc$.

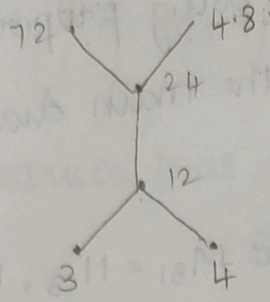
1. Let us draw HASSE diagram for partial ordering $\{(a,b) \mid a \leq b\}$ $\{1, 2, 3, 4\}$ starting from its diagraph.

$R = \{(1,1), (1,2), (1,3), (1,4), (2,3), (2,2), (2,4), (3,3), (3,4), (4,4)\}$



divisib

2. Draw the Hasse diagram for $A=\{3,4,12,24,48,72\}$

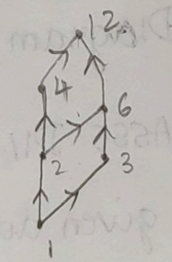


a divides b . b/a

3. Draw the Hasse diagram for $(D_{12}, /)$.

D_{12} means divisor of 12. $\frac{2}{1}$ $\frac{2}{1}$ $1/2$

$=\{1, 2, 3, 4, 6, 12\}$. subordinates



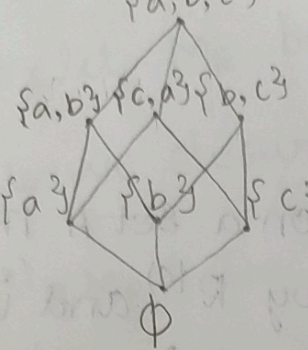
4. i) Consider the divisibility $A=\{1,2,3,4,5,6,7,8,9,10\}$

ii) Draw the Hasse diagraph for $(D_{30}, /)$

iii) where $A=\{a,b,c\}$. $P(A)$

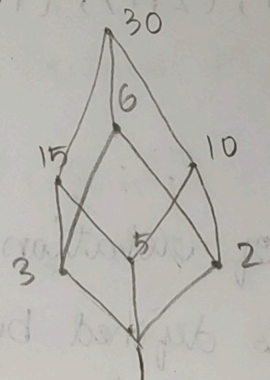
1. Draw the Hasse diagram representing subset relation on the set power $P(A)$ $A = \{a, b, c\}$

$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}\}$



2. Let D_{30} be the divisor of 30 draw the hasse diagram for $(D_{30}, |)$ the slash denotes divisibility relation

$A = \{1, 2, 3, 5, 6, 10, 15, 30\}$.



CLOSURE OF RELATION.

1. REFLEXIVE CLOSURE

Let R be a relation on set A the reflexive ~~per~~ closure of A is denoted by $R^{(r)}$ and defined as $R^{(r)} = R \cup \{(a, a) \mid a \in A\}$ it is the smallest ~~reflex~~ relation of A containing R

Let $A = \{1, 2, 3, 4\}$.

$R = \{(1,1), (1,2), (2,1), (2,3), (2,4)\}$.

$R^{(r)} = \{(1,1), (1,2), (2,1), (2,2), (3,3), (2,3), (2,4), (4,4)\}$

2. SYMMETRY CLOSURE

Let R be a relation on a set A the symmetric closure of R is denoted by $R^{(s)}$ and its defined by $R^{(s)} = R \cup \{(b,a) \mid (a,b) \in R\} = R \cup R^{-1}$

Let, $A = \{1, 2, 3\}$.

$R = \{(1,1), (1,2), (2,1), (1,3), (2,3)\}$

$R^{(s)} = \{(1,1), (1,2), (2,1), (1,3), (3,1), (2,3), (3,2)\}$

3. TRANSITIVE CLOSURE

Transitive closure of relation R. On a set A denoted by $R^{(t)}$ and its defined by !

$R^{(t)} = R \cup \{(a,c) \mid (a,b) \in R \;\&\; (b,c) \in R\}$.

if R is a relation on a set A with $|A| = n$

then $R^{(t)} = R \cup R^2 \cup R^3 \cup \cdots R^n$

Let $A = \{1, 2, 3\}$

$R = \{(1,1), (2,3), (3,1)\}$

$R^{(t)} = \{(1,1), (2,3), (3,1), (2,1)\}$.

Closure of relation in matrix form, matrix of the closure of a relation.

Let R be a relation on a set A with n elements let M_R be the matrix of R

1. Reflexive Closure $R^{(r)}$ is given by $M_{R^{(r)}}$

$M_{R^{(r)}} = M_R + I_n$ $\quad I_n \to n \times n$ unit matrix

2. Symmetric closure $R^{(s)}$ is $M_{R^{(s)}}$.

$M_{R^{(s)}} = M_R + (M_R)^T$

3. Transitive closure $R^{(t)}$ is $M_{R^{(t)}}$.

$M_{R^{(t)}} = M_R + M_{R^2} + M_{R^3} + \cdots + M_{R^n}$

where $M_{R^2} = M_R \odot M_R$.

1. Let $A = \{1, 2, 3\}$, $R = \{(1,2), (2,3), (3,1)\}$.

$$M_R = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \\ 3 & 1 & 0 & 0 \end{array}$$

1. Reflexive closure.

$M_{R^{(r)}} = M_R + I_3$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

2. Symmetric closure.

$$M_R(s) = M_R + (M_R)^T$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

3. Transitive Closure

$$M_R(t) = M_R + M_R{}^2 + M_R{}^3$$

$$M_R{}^2 = M_R \odot M_R$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0+0+0 & 0+0+0 & 0+1+0 \\ 0+0+1 & 0+0+0 & 0+0+0 \\ 0+0+0 & 1+0+0 & 0+0+0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$M_R{}^3 = M_R{}^2 \odot M_R$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_R(t) = M_R + M_R{}^2 + M_R{}^3$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$M_R = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \\ 3 & 1 & 0 & 0 \end{array}$$

one^th position

Column	Row	C/R
{3}	{2}	(3, 2)
{1, 3}	{3}	{(1,3), (3,3)}
{2, 3}	{1, 2, 3}	{(2, 1), (2, 2), (2, 3), (3,1), (3,2), (3,3)}

$$W_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$W_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$W_3 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

2. Using Warshall's algorithm find the transitive closure of the relation R on set A = {1, 2, 3, 4}.

$$M_R = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

find the transitive

Column	Row	C|R	Matrix
$\{1,4\}$	$\{1,3,4\}$	$\{(1,1),(1,3),(1,4),(4,1),(4,3),(4,4)\}$	$W_1=\begin{pmatrix}1&0&1&1\\0&1&0&0\\0&0&0&1\\1&0&1&1\end{pmatrix}$
$\{2\}$	$\{2\}$	$\{2,2\}$	$W_2=\begin{pmatrix}1&0&1&1\\0&1&0&0\\0&0&0&1\\1&0&1&1\end{pmatrix}$
$\{1,4\}$	$\{4\}$	$\{(1,4),(4,4)\}$	$W_3=\begin{pmatrix}1&0&1&1\\0&1&0&0\\0&0&0&1\\1&0&1&1\end{pmatrix}$
$\{1,3,4\}$	$\{1,3,4\}$	$\{(1,3),(1,1),(1,4),(3,1),(3,3),(3,4),(4,1),(4,3),(4,4)\}$	$W_4=\begin{pmatrix}1&0&1&1\\0&1&0&0\\1&0&1&1\\1&0&1&1\end{pmatrix}$

3. Let $A=\{1,2,3,4\}$, $R=\{(1,2),(2,1),(2,3),(3,4)$ using warshall's algorithm find transitive Closure of R.

4. Using warshall's algorithm find transitive Closure of R if $R=\{(1,1),(1,2),(2,3),(3,1),(3,$

If R is an equivalence relation on a set A then set of all elements of A R related to an element of A is called equivalence class

$[a]=\{x : x\in A, xRa\}$.

Let R be an equivalance relation on a set A then equivalance class of any element a on A as set of all elements that R related to A through R.

$[a]=\{x : x\in A, xRa\}$.

The subset of 'A' containing all the elements related to 'A' through 'R'.

$R=\{1,2,3,4,5\}$

$R=\{(x,y) \mid x-y$ is even $x,y\in A\}$.

$R=\{(1,1),(2,2),(3,3),(4,4),(4,5),(1,3),(1,5),(2,4),(3,1),(5,1),(4,2),(5,3),(3,5)\}$.

$[1]=\{1,3,5\}$
$[2]=\{2,4\}$
$[3]=\{1,3,5\}$
$[4]=\{2,4\}$
$[5]=\{1,3,5\}$.

NOTE :

1. $a\in[a]$.
2. Either $[a]\wedge[b]=\phi$ (either disjoint or identical) or $[a]=[b]$.

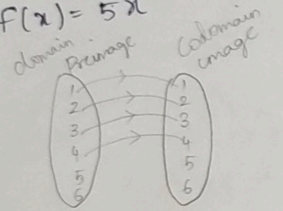
One to one or injective function

Let f maps $A \rightarrow B$ be a function F is called one to one if every distinct element of A is assigned to distinct element of B

$a \neq b \Rightarrow f(a) \neq f(b)$ $a, b \in A$. If $f(a) = f(b)$ then $a = b$. F maps $F: N \rightarrow N$

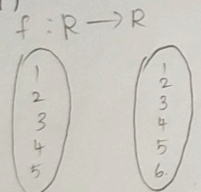
Let $F: N \rightarrow N$.

$f(x) = 5x$



example : 2 $f: R \rightarrow R$

$f(x) = x^2$



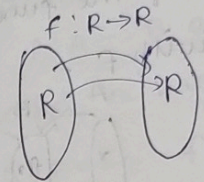
Onto function

x. Let $F: A \rightarrow B$ F is onto function if for every if $b \in B$ there is an $a \in A$ such that $f(a) = b$.

One to One Correspondence or Byection

A function $F: A \rightarrow B$ is a bijection if F is both one to one and onto.

Let $f: R \rightarrow R$ $f(x) = 2x + 1$.



One to one

$f(x) = 2x + 1$

Let $x_1, x_2 \in R$.

$f(x_1) = f(x_2)$

$2x_1 + 1 = 2x_2 + 1$

$x_1 = x_2$

Onto

$f(x) = y = 2x + 1$

$x = \frac{y-1}{2}$

$f\left(\frac{y-1}{2}\right) = y$

$\therefore$ f is a bijective function

Example - 2

Let $F: \cancel{A \rightarrow B}$ $F: Z \rightarrow Z$. Check whether the func is bijective funct $f(x) = x^2$

Let $x_1, x_2 \in Z$.

One to One

$f(x_1) = f(x_2)$

$x_1^2 = x_2^2$

$x_1 = x_2$

$\therefore$ One to one

Onto

$f(x) = y = x^2$

$y = x^2$

$x = \pm\sqrt{y}$

$\therefore$ not onto

$\therefore$ f is not a bijective function.

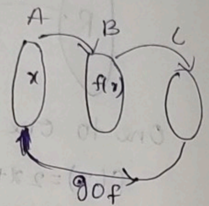
COMPOSITION

$F: A \to B$, $g: B \to C$ then the corresponding of fog is denoted by $g \circ f : A \to C$ & defined by.

$(g \circ f)(x) = g(f(x))$, $x \in A$.

$g \circ f$ is called as relative product of the function f & g or left composition of g with f.

Note: $(g \circ f)(x) = g[f(x)]$



The range of f is the domain of g

$R_f = D_g$

COMPOSITION OF FUNCTION IS NOT COMMUTATIVE

$g \circ f \neq f \circ g$

$g \circ f$ is defined only when the range f is subset of domain g.

EQUIVALITY OF FUNCTION

The func f and g are said to be equal if

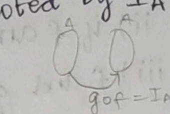
1. domain of f = domain of g.

Codomain of f = Codomain of g.

2. $f(x) = g(x)$ for all f belongs to domain of f

IDENTITY FUNCTION.

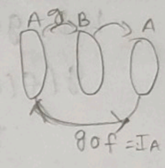
A function $f : A \to A$ defined by $f(x) = x \ \forall x \in A$ is called the identity function on A is denoted by I_A



INVERSE FUNCTION.

Let $f : A \to B$ and $g : B \to A$ then the function g is called the inverse of the function f $[g = f^{-1}]$.

$g \circ f = I_A$ $\quad f \circ g = I_B$.



Theorems : 1

1. Composition of function is associative.

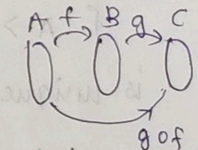
Proof :

Let $f : A \to B$, $g : B \to C$; $h : C \to D$ be three functions are defined on A, B, C.

$h \circ (g \circ f) = (h \circ g) \circ f$. $\quad$ $h \circ (f \circ g) = (h \circ f) \circ g$.

1. $g \circ f : A \to C$: $h : C \to D$, $\therefore h \circ (g \circ f) : A \to D$

$h \circ g : B \to D$. then $(h \circ g) \circ f : A \to D$.



2. Let $x \in A$, $y \in B$, $z \in C$ that $f(y) = x$ & $z = g(y)$

$(g \circ f)(x) = g[f(x)]$
$= g(y)$
$= z$.

$h \circ (g \circ f)(x) = h[g \circ f(x)]$.
$= h[z]$. — ①

$(h \circ g) \circ f(x) = h \circ g(f(x))$
$= h \circ g(y) = h(g(y)) = h(z)$ — ②

Theorem-2.

1. Let $F: A \to B$, $g: B \to C$ be a function if

i) $g \circ f$ is one to one.

ii) if f and g are onto their $g \circ f$ is onto.

iii) f and g are bijection then $g \circ f$ also bijection.

Since f and g are one to one $g \circ f: A \to C$

Let $g \circ f(a_1) = g \circ f(a_2)$ $a_1, a_2 \in A$.

$g(f(a_1)) = g(f(a_2))$.

$g(a_1) = g(a_2)$ [$\because$ f is one to one]

$a_1 = a_2$ [$\because$ g is one to one].

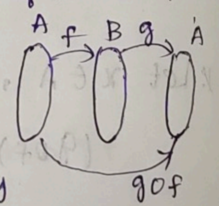
Composition of a function is onto:

Let f and g are onto, $c \in C$ as g is onto there is $b \in B$ such that $g(b) = c$, f is onto there is $a \in A$ such that $f(a) = b$.

$g \circ f(a) = g[f(a)] = g[b] = c$.

Theorem 3.

$f: A \to B$ be a function if f^{-1} exist then it is unique. In other words uniqueness of inverse function



Let g & h be inverse of f

$g: B \to A$ & $h: B \to A$

then by the definition of identity

$$\left.\begin{array}{ll} g \circ f = I_A, & f \circ g = I_B \\ h \circ f = I_A & h \circ g = I_B \end{array}\right\} - (1).$$

$h = h \circ I_B$

$= h \circ (f \circ g)$

$= (h \circ f) \circ g = I_A \circ g = g$

By the uniqueness property we have

$(f^{-1})^{-1} = (g^{-1}) = f$

Theorem 4:

Let $f: A \to B$ be any function then f^{-1} exist if and only if f is one to one and onto. In other words necessary and sufficient condition for the function $f: A \to B$ to be invertible is that f is one to one & onto.

Proof

Assume f is invertible prove that f is one to one and onto.

Let f inverse exist then there exist a unique function $f^{-1} = g: B \to A$ such that $g \circ f = I_A$, $f \circ g = I_B$.

Let $a_1, a_2 \in A$ such that $f(a_1) = f(a_2)$ where $f(a_1), f(a_2) \in B$.

Since $g: B \to A$

$g(f(a_1)) = g(f(a_2))$

$g \circ f(a_1) = (g \circ f)(a_2)$.

$I_A(a_1) = I_A(a_2)$

$a_1 = a_2$. $\therefore$ f is one to one.

Let $b \in B$ the $g(b) \in A$ $\because$ 1

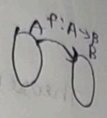
$b = I_B(b) = (f \circ g)(b)$

$= f[g(b)]$.

for every b in B there exist $g(b) \in A$ such that $f[g(b)] = b$. $\therefore$ f is onto.

Conversely let $f : A \to B$ be a bijection then we ve to prove that f is invertible.

$\because f$ is onto for every $b \in B$ an element exist $a \in A$ such that $f(a) = b$.



By $g(b) = a$ where $f(a) = b$ then g is well defined

if possible $g(b) = a_1$ & $g(b) = a_2$

for every $b \in B$ there exist an element $a \in A$

Sine f is bijective

$g : B \to A$ $g(b) = a$ where $f(a) = b$.

$g(b) = a_1$ & $g(b) = a_2$ where $a_1 \neq a_2$

$f(a_1) = b$, $f(a_2) = b$. $f(a_1) = f(a_2)$

$(g o f) a = g(f(a))$

$= g(b) = a = I_A(a)$

there $g o f = I_A$

for any $b \in B$

$(f o g) b = f(g(b))$

$= f(a) = b = I_B(b)$

$\therefore f$ is the inverse of g $\therefore f = g^{-1}$.

Theorem 5

Let $f : A \to B$, $g : B \to C$ are invertible function then $g o f : A \to C$ is also invertible $(g o f)^{-1} = f^{-1} o g^{-1}$

$f : A \to B$, $g : B \to C$.

f and g are bijective then

$g o f : A \to C$ is also bijective.

$g o f$ is bijective then it is invertible

$(g o f)^{-1}$ exist

$(g o f)^{-1} : C \to A$. — (1)

$g^{-1} : C \to B$

$f^{-1} : B \to A$

$f^{-1} o g^{-1} : C \to A$. — (2).

(1) = (2).

$(g o f)^{-1} = f^{-1} o g^{-1}$.

To prove $c \in C$ $(g o f)^{-1}(c) = (f^{-1} o g^{-1})(c)$.

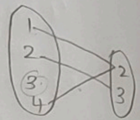
Problems

1. Determine whether each of the relation is a function with domain $\{1, 2, 3, 4\}$ if any relation is not a function explain why.

i) $R_1 = \{(1,1), (2,1), (3,1), (4,1), (3,3)\}$.

R_1 is not a function because $(3,1)$ $(3,3)$ the image of 3 is not unique.

ii) $R_2 = \{(1,2), (2,3), (4,2)\}$.



R_2 is not a function because 3 is not mapped to and element in codomain.

iii) $R_3 = \{(1,1), (2,1), (3,1), (4,1)\}$

R_3 is a function because every element in domain is mapped to element in codomain.

2. $f : R \to R$, $g : R \to R$ $f(x) = x^2 - 2$ $g(x) = x + 4$ find $g \circ f$ & $f \circ g$. Prove f,g are bijective

$$(g \circ f)(x) = g[f(x)]$$
$$= g[x^2 - 2]$$
$$= x^2 - 2 + 4$$
$$(g \circ f)(x) = x^2 + 2.$$

$$(f \circ g)(x) = f[g(x)]$$
$$= f[x + 4]$$
$$= (x + 4)^2 - 2$$
$$= x^2 + 8x + 16 - 2$$
$$= x^2 + 8x + 14.$$

composition is not Commutative.

To prove f is one to one.

$f : R \to R$.

$f(x) = f(y)$

$x^2 - 2 = y^2 - 2$.

$x^2 = y^2$

$x = \pm y$.

f is not injective

To prove f is onto.

for every $y \in Y$ there exist an element $f(x) = y$

$x^2 - 2 = y$

$x^2 = y + 2$

$x = \pm\sqrt{y + 2}$. $\notin R$.

f is not onto (surjective)

$g : R \to R$.

$g(x) = g(y)$

$x + 4 = y + 4$

$x = y$

g is one to one

$g : R \to R$.

$g(x) = y$

$x + 4 = y$

$x = y - 4$

g is on to

$\therefore$ g is bijective

2. Show that the function $f: R \to R$. $f(x) = \frac{x}{x+4}$ is one to one & onto : Find the inverse.

one to one

$f(x_1) = f(x_2)$

$\frac{x_1}{x_1+4} = \frac{x_2}{x_2+4}$

$x_1x_2 + x_1 4 = x_1x_2 + 4x_2$.

$4x_1 = x_2(4)$

$x_1 = x_2$.

f is one to one.

on to.

$f(x) = y$

$\frac{x}{x+4} = y$.

$x = y(x+4)$.

$x = xy + y(4)$

$x - xy = y(4)$

$x(1-y) = 4(y)$

$x = \frac{4(y)}{(1-y)}$.

$f\left(\frac{4y}{1-y}\right) = y$

$f\left(\frac{4y}{1-y}\right) = y$

$\frac{\frac{4y}{1-y}}{\frac{4y}{1-y}+4} = y$

$y = y$.

$\therefore$ f is onto.

$f^{-1}(y) = \frac{4y}{1-y}$.

3. Show the $f: R - \{3\} \to R - \{1\}$. $f(x) = \frac{x-2}{x-3}$ is bijective find inverse.

Let $f(x_1) = f(x_2)$

$\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$

$x_1 = x_2$

It is one to one.

Onto :

$f(x) = y$

$\frac{x-2}{x-3} = y$

$x - 2 = y(x-3)$

$x = yx - 3y + 2$

$-xy + x = 2 - 3y$

$x(1-y) = 2 - 3y$

$x = \frac{2-3y}{1-y}$.

$x = f^{-1}(y) = \frac{2-3y}{1-y}$.

$\therefore$ f inverse exist.

4. Show that $f: R \to R$ defined as $f(x) = 3x^3 - 4$ is one to one.

$f(x_1) = f(x_2)$

$3x_1^3 - 4 = 3x_2^3 - 4$

$3x_1^3 = 3x_2^3$

$x_1 = x_2$

$\therefore$ f is one to one.

fog ≠ gof

5. If $S = \{1, 2, 3, 4, 5\}$. $f = \{(1,2), (2,1), (3,4), (4,5), (5,3)\}$, $g = \{(1,3), (2,5), (3,1), (4,2), (5,4)\}$.
$h = \{(1,2), (2,2), (3,4), (4,3), (5,1)\}$. $f: S \to S$.

i) $fog = gof$.

$(fog)(1) = f(g(1)) = f(3) = 4$.

$(fog)(2) = f(g(2)) = f(5) = 3$.

$(fog)(3) = f(g(3)) = f(1) = 2$

$(fog)(4) = f(g(4)) = f(2) = 1$

$(fog)(5) = f(g(5)) = f(4) = 5$

$$f \circ g = \{(1,4),(2,3),(3,2),(4,1),(5,5)\}$$

$$(g \circ f)(1) = g[f(1)] = g(2) = 5$$

$f \circ g = g^{-1} \circ f^{-1}$

$$(g \circ f)(2) = g[f(2)] = g(1) = 3$$

$$(g \circ f)(3) = g[f(3)] = g(4) = 2$$

$$(g \circ f)(4) = g[f(4)] = g(5) = 4$$

$$(g \circ f)(5) = g[f(5)] = g(3) = 1$$

$$g \circ f = \{(2,3),(3,2),(4,4),(5,1)\}.$$

$,5)\}$

$A=\{1,2,3,4,5\}$ $B=\{1,2,3,8,9\}$ $f: A \to B$
$g: A \to A$ $f=\{(1,8),(3,9)$ $g=\{(1,2),(3,1),$
$(2,2),(4,3),(5,2)\}$ find $f \circ g$ & $g \circ f$ & $f \circ f$ & $g \circ g$
if they exist.

1. $f: Z \to N \cup 0$ defined by $f(x) = \begin{cases} 2x-1 & x>0, \\ -2x & x \leq 0. \end{cases}$
prove f is one to one & onto
find f^{-1}.