(1) The orbital period of a satellite is 650 min. Determine the semi-major axis of the elliptical orbit.

Sol

$T = 650\ min = 650 \times 60\ sec = 39000\ sec$

$\mu = GM$

$= 6.67 \times 10^{-11} \times 5.98 \times 10^{24}$

$\mu = 39.8 \times 10^{13}\ Nm^2/kg$

The orbital time period (T) is given by,

$$T = 2\pi \sqrt{\frac{a^3}{\mu}} = 2\pi \sqrt{\frac{a^3}{GM}}$$

$$39000 = 3.14 \times 2 \sqrt{\frac{a^3}{39.8 \times 10^{13}}}$$

$$\sqrt{\frac{a^3}{39.8 \times 10^{13}}} = 6210.191$$

$$a^3 = 1534945636.7398 \times 10^{13}$$

$$a = 535.424 \times 10^4$$

$$\boxed{a = 5350\ Km}$$

(2) The apogee and perigee of an elliptical satellite orbits are 3000 km and 200 km. Determine the eccentricity, semi-major axis and semi-minor axis.

Sol.

$r_a = 3000 \text{ Km}$

$r_p = 200 \text{ Km}$

Eccentricity, $e = \frac{r_a - r_p}{r_a + r_p} = \frac{3000-200}{3000+200}$

$\boxed{e = 0.875}$

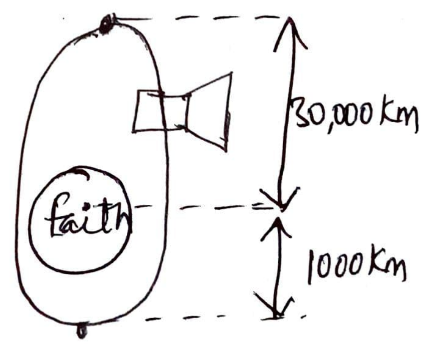
Semi-major axis, $a = \frac{r_a + r_p}{2} = \frac{3000+200}{2}$

$\boxed{a = 1600 \text{ Km}}$

Semi-minor axis, $b = \frac{r_a - r_p}{2} = \frac{3000-200}{2}$

$\boxed{b = 1400 \text{ Km}}$

3 A satellite is moving in an elliptical orbit as shown in fig. Determine its semi-major axis



Sol

Apogee = 30,000 Km

Perigee = 1000 Km.

Semi-major axis $= \frac{\text{Apogee} + \text{Perigee}}{2}$

$= \frac{30,000 + 1,000}{2} = 15500 \text{ Km}$

4. Determine the apogee, perigee and orbit eccentricity in a satellite's elliptical eccentric orbit, if the farthest and closest points from earth's surface are 20,000 km and 500 km respectively. The radius of earth is 6370 Km.

Sol

$$\text{Apogee} = 20,000 + 6370$$
$$= 26370 \text{ Km}$$

$$\text{Perigee} = 500 + 6370$$
$$= 6870 \text{ Km}.$$

$$\text{orbit eccentricity } e = \frac{\text{Apogee} - \text{Perigee}}{\text{Apogee} + \text{Perigee}}$$

$$e = \frac{26370 - 6870}{26370 + 6870} = 0.587$$

5. A satellite is moving in a circular orbit for the period of 1 day. Calculate the radius of orbit (given that $\mu = 39.8 \times 10^{13}$ Nm^2/Kg).

Sol The mean motion is given by

$$n = \frac{2\pi}{T}$$

Where T = 1 day

$$n = \frac{2 \times 3.14}{24 \times 60 \times 60}$$

$$n = 7.27 \times 10^{-5} \text{ rad/sec}$$

Kepler's third law gives,

$$a = \left(\frac{\mu}{n^2}\right)^{1/3} = \left[\frac{39.8 \times 10^{13}}{(7.27 \times 10^{-5})^2}\right]^{1/3}$$

$$\boxed{a = 42241 \text{ km}}$$

6) Calculate the apogee and perigee heights for the orbital parameters given as $e = 0.00115$, $a = 7192.3$ km, ~~Assume~~ where ~~consider a mean~~ earth radius of 6371 km.

Sol

$$R = 6371 \text{ km}$$

Apogee and perigee distance can be calculated as.

$$r_a = a(1+e)$$

$$= 7192.3(1+0.00115) = 7200.6 \text{ km}$$

$$r_p = a(1-e)$$

$$= 7192.3(1-0.00115) = 7184.1 \text{ km}$$

(5)

Thus the apogee and perigee heights will be

h_a (or) $H_a = r_a - R = 7200.6 - 6371$

$= 829.6$ Km

h_p or $H_p = r_p - R = 7184.1 - 6371$

$= 813.1$ Km

Formulas:-

The value of eccentricity 'e' determine the type of orbit.

~~7) Perigee distance~~

1) $e = \begin{cases} 0 \rightarrow \text{circle} \\ <1 \rightarrow \text{Ellipse} \\ 1 \rightarrow \text{Parabola} \\ >1 \rightarrow \text{Hyperbola} \end{cases}$

2) orbital period, $T = 2\pi\sqrt{\frac{a^3}{\mu}}$

$\mu = GM = 3.98 \times 10^{14}\ m^3/s^2$

3) Satellite velocity for a circular orbit $= \sqrt{\frac{\mu}{a}}$

4) Eccentricity $e = \frac{c}{a} = \frac{r_a - r_p}{r_a + r_p}$

5) Semi-major axis, $a = \frac{r_a + r_p}{2} = \frac{P}{1-e^2}$

6) Apogee distance, $r_a = a + c = a(1+e)$

7) Perigee distance, $r_p = a - c = a(1-e)$

8) Locus parameter, $P = a(1-e^2) = \dfrac{2 r_a r_p}{(r_a + r_p)}$

9) Semi-minor axis,

$$b = a(1-e^2)^{1/2} = (r_a r_p)^{1/2}$$