① Consider the interface between a glass slab with $n_1 = 1.48$ and air for which $n_2 = 1.00$. What is the critical angle for light traveling in the glass?

$$\phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}(0.676) = 42.5^\circ.$$

② A light ray traveling in air ($n_1 = 1.00$) is incident on a smooth flat slab of crown glass, which has a R.I. $n_2 = 1.52$. If the incoming ray makes an angle of $\phi_1 = 30.0^\circ$ with respect to the normal, what is the angle of refraction ϕ_2 in the glass?

$$\sin\phi_2 = \frac{n_1}{n_2}\sin\phi_1$$

Degree to radian
$1^\circ = 0.0174533$ radian

$$= \frac{1.00}{1.522}\sin(30.0^\circ).$$

$$\sin\phi_2 = 0.329.$$

$$\phi_2 = \sin^{-1}(0.329) = 19.2^\circ.$$

③ Consider a Multimode silica fiber that has a core refractive index $n_1 = 1.480$ and cladding index $n_2 = 1.460$. Find ⓐ Critical angle ⓑ Numerical aperture, and ⓒ the acceptance angle.

ⓐ $\phi_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1}\frac{1.460}{1.480} = $ ~~0.9865~~ ~~Radian~~ 1.4062 radian → degree

$$= 1.4062 / 0.0174533$$

$$= 80.58^\circ.$$

ⓑ $NA = \sqrt{n_1^2 - n_2^2} = 0.242.$

ⓒ Acceptance angle $\theta_A = \sin^{-1}(NA) = 0.2449$

$$= 0.2449 / 0.01745 = 14.03^\circ.$$

(4) Consider a multimode fiber that has a core refractive index of 1.480 and a core-cladding index difference 2.0% ($\Delta = 0.020$). Find the (a) NA (b) Acceptance angle (c) Critical angle.

(a) For multimode fiber $NA = n_1\sqrt{2\Delta}$

But $\Delta = 0.020$.

$n_1 = 1.480$.

$NA = 1.480\sqrt{2 \times 0.02}$

$NA = 0.296$

(b) $\theta_A = \sin^{-1}(NA) = \sin^{-1}(0.296)$

$= 17.22^\circ$.

(c) Critical angle: $\phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$.

$n_2 = n_1(1-\Delta)$.

$= 1.48(1-0.02) = 1.450$

$\phi_c = \sin^{-1}(n_2/n_1) = \sin^{-1}(1.450/1.480)$.

$= 1.369$. Radians.

$= \frac{1.369}{0.174533} = ~~78.5~~^\circ\ 7.85^\circ$

(5) A Step Index fiber has a normalized frequency $V = 26.6$ at a 1300 nm wavelength. If the core radius is 25 μm, what is the Numerical aperture?

$V = \frac{2\pi a}{\lambda} \times NA$.

$NA = V \times \frac{\lambda}{2\pi a}$

$= \frac{26.6 \times 1300 \times 10^{-9}}{2 \times \pi \times 25 \times 10^{-6}} = 0.22$.

(6) Consider a fiber that has a core refractive index of 1.480, a cladding index of 1.476 and a core radius of 4.4 μm. Show that the wavelength at which this fiber becomes single mode is λ_c = 1250nm.

$$V = \frac{2\pi a}{\lambda} \times (n_1^2 - n_2^2)^{1/2}$$

$$= \frac{2 \times \pi \times 4.4 \times 10^{-6}}{1250 \times 10^{-9}} (1.48 - 1.476)^{1/2}$$

$$= 1.398$$

$$V < 2.405$$

Hence this fiber becomes a single mode.

(7) Consider a multimode step index fiber with a 62.5 μm core diameter and a core-cladding index difference of 1.5 percent. If the core refractive index is 1.48, estimate the normalized frequency of the fiber and the total number of modes supported in the fiber at a wavelength of 850nm.

$n_1 = 1.48$

$\Delta = 1.5\% = 0.015$

$dia = 62.5 \times 10^{-6}$

$\lambda = 850\,nm = 850 \times 10^{-9}$

$$V = \frac{2\pi a}{\lambda} \times NA$$

If multimode Step Index fiber $NA = n_1\sqrt{2\Delta}$.

$$V = \frac{2 \times \pi \times \frac{62.5}{2} \times 10^{-6}}{850 \times 10^{-9}} \times 1.48\sqrt{2 \times 0.015}$$

In problem given diameter → $radius = \frac{dia}{2} = 31.25 \times 10^{-6}$

$$V = 59.2$$

$M \simeq \frac{V^2}{2} = 1753.$

(8) Suppose we have a multimode step-index fiber that has a core radius of 25 μm, a core index of 1.48 and an index difference $\Delta = 0.01$, what are the number of modes in the fiber at wavelengths 860, 1310 and 1550 nm?

$n_1 = 1.48$

$a = 25 \times 10^{-6}$

$\Delta = 0.01$

$\lambda = 860 \times 10^{-9}$.

(a) $V = \frac{2\pi a}{\lambda} n_1 \sqrt{2\Delta}$

$= \frac{2 \times \pi \times 25 \times 10^{-6} \times 1.48 \sqrt{2 \times 0.01}}{860 \times 10^{-9}}$

$= 38.2.$

$M = \frac{V^2}{2} = 729$

(b) @ 1310 nm $\quad V = 25.1$ and $M = 315$.

(c) @ 1550 nm $\quad V = 21.2$ and $M = 224$.

(9) Suppose we have three multimode step index fibers, each of which has a core index of 1.48 and an index difference $\Delta = 0.01$. Assume the three fibers have core diameters of 50, 62.5 and 100 μm. What are the number of modes in these fibers at a wavelength of 1550 nm?

$\lambda = 1550 \times 10^{-9}$

$n_1 = 1.48$

$\Delta = 0.01$.

dia = 50 μm

$a = \frac{50}{2} = 25$ μm

(a) $V = \frac{2\pi a}{\lambda} NA \quad$ where $\quad NA = n_1 \sqrt{2\Delta}.$

$= \frac{2 \times \pi \times 25 \times 10^{-6}}{1550 \times 10^{-9}} 1.48 \sqrt{2 \times 0.01} = 21.2.$

$M \simeq \frac{V^2}{2} = 224$.

(b) @ dia of 62.5 μm $\rightarrow$ $a = 31.25$ μm.

$V = 26.5$ and $M = 351$.

(c) @ ~~di~~ dia of 100 μm $\rightarrow$ $a = 50$ μm.

$V = 42.4$ and $M = 898$.

$\frac{2 \times \pi \times 50 \times 10^{-6}}{850 \times 10^{-9}} \times$

(10) Consider a multimode step-index optical fiber that has a core radius of 25 μm, a core index of 1.48 and an index diff. $\Delta = 0.01$. Find the percentage of optical power that propagates in the cladding @ 840 nm.

$a = 25$ μm

$n_1 = 1.48$.

$\Delta = 0.01$.

$\lambda = 840$ nm.

$$V = \frac{2\pi a}{\lambda} NA = \frac{2\pi a}{\lambda} n_1 \sqrt{2\Delta}$$

$$= \frac{2 \times \pi \times 25 \times 10^{-6}}{840 \times 10^{-9}} 1.48 \sqrt{2 \times 0.01}$$

$$V = 39$$

$$M = \frac{V^2}{2} = 760$$

$$\frac{P_{cladding}}{P} = \frac{4}{3\sqrt{M}}$$

$$= 0.05$$

$\therefore$ 5% of the optical power propagates in the cladding.

If $\Delta = 0.03$

then there are 242 modes in the fiber and about 9% of the power propagates in the cladding.

A manufacturing engineer wants to make an OF that has core index of 1·48 & cladding index of 1·478. What should the core size be for single mode operation @ 1550nm?

Soln:-

$V \leq 2.405$ must be satisfied for single mode operation

$$a = \frac{V\lambda}{2\pi} \frac{1}{\sqrt{n_1^2 - n_2^2}} = 7.7\,\mu m.$$

If this fiber also should be single mode at 1310nm, then the core radius must be less than 6.5 µm.

An appln engineer has an optical fiber that has a 3.0 µm core radius and a NA of 0·1. Will this fiber exhibit single mode operation at 800nm.

Soln:

$$V = \frac{2\pi a}{\lambda} NA = 2.356.$$

$\therefore V < 2.405$, this fiber will exhibit single mode operation at 800nm.