



C H A P T E R

The pn Junction

P R E V I E W

Up to this point in the text, we have been considering the properties of the semiconductor material. We calculated electron and hole concentrations in thermal equilibrium and determined the position of the Fermi level. We then considered the nonequilibrium condition in which excess electrons and holes are present in the semiconductor. We now wish to consider the situation in which a p-type and an n-type semiconductor are brought into contact with one another to form a pn junction.

Most semiconductor devices contain at least one junction between p-type and n-type semiconductor regions. Semiconductor device characteristics and operation are intimately connected to these pn junctions, so considerable attention is devoted initially to this basic device. The pn junction diode itself provides characteristics that are used in rectifiers and switching circuits. In addition, the analysis of the pn junction device establishes some basic terminology and concepts that are used in the discussion of other semiconductor devices. The fundamental analysis techniques used for the pn junction will also be applied to other devices. Understanding the physics of the pn junction is, therefore, an important step in the study of semiconductor devices.

The electrostatics of the pn junction is considered in this chapter and the current-voltage characteristics of the pn junction diode are developed in the next chapter. ■

7.1 | BASIC STRUCTURE OF THE pn JUNCTION

Figure 7.1a schematically shows the pn junction. It is important to realize that the entire semiconductor is a single-crystal material in which one region is doped with acceptor impurity atoms to form the p region and the adjacent region is doped with donor atoms to form the n region. The interface separating the n and p regions is referred to as the *metallurgical junction*.

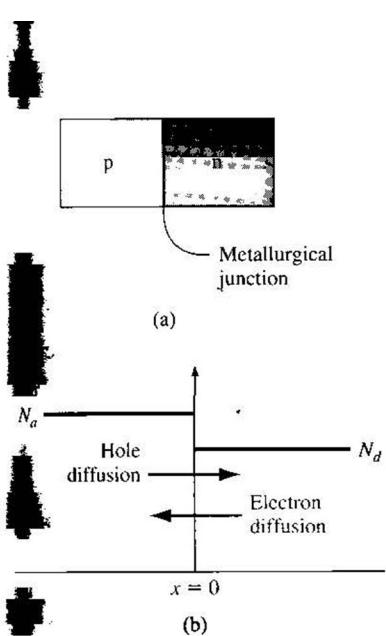


Figure 7.1 | (a) Simplified geometry of a pn junction; **(b)** doping profile of an ideal uniformly doped pn junction.

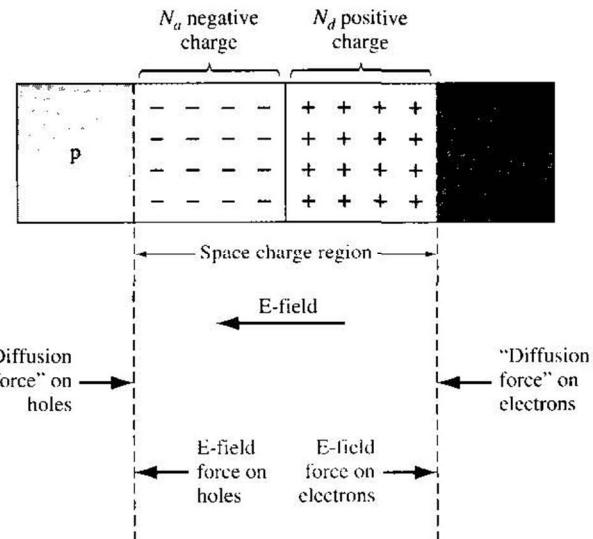


Figure 7.2 | The space charge region, the electric field, and the forces acting on the charged carriers.

The impurity doping concentrations in the p and n regions are shown in Figure 7.1b. For simplicity, we will consider a *step junction* in which the doping concentration is uniform in each region and there is an abrupt change in doping at the junction. Initially, at the metallurgical junction, there is a very large density gradient in both the electron and hole concentrations. Majority carrier electrons in the n region will begin diffusing into the p region and majority carrier holes in the p region will begin diffusing into the n region. If we assume there are no external connections to the semiconductor, then this diffusion process cannot continue indefinitely. As electrons diffuse from the n region, positively charged donor atoms are left behind. Similarly, as holes diffuse from the p region, they uncover negatively charged acceptor atoms. The net positive and negative charges in the n and p regions induce an electric field in the region near the metallurgical junction, in the direction from the positive to the negative charge, or from the n to the p region.

The net positively and negatively charged regions are shown in Figure 7.2. These two regions are referred to as the *space charge region*. Essentially all electrons and holes are swept out of the space charge region by the electric field. Since the space charge region is depleted of any mobile charge, this region is also referred to as the *depletion region*; these two terms will be used interchangeably. Density gradients still exist in the majority carrier concentrations at each edge of the space charge region. We can think of a density gradient as producing a "diffusion force" that acts on the majority carriers. These diffusion forces, acting on the electrons and holes at the edges of the space charge region, are shown in the figure. The electric field in the space charge region produces another force on the electrons and holes which is in the

opposite direction to the diffusion force for each type of particle. In thermal equilibrium, the diffusion force and the E-field force exactly balance each other.

7.2 | ZERO APPLIED BIAS

We have considered the basic pn junction structure and discussed briefly how the space charge region is formed. In this section we will examine the properties of the step junction in thermal equilibrium, where no currents exist and no external excitation is applied. We will determine the space charge region width, electric field, and potential through the depletion region.

7.2.1 Built-in Potential Barrier

If we assume that no voltage is applied across the pn junction, then the junction is in thermal equilibrium—the Fermi energy level is constant throughout the entire system. Figure 7.3 shows the energy-band diagram for the pn junction in thermal equilibrium. The conduction and valance band energies must bend as we go through the space charge region, since the relative position of the conduction and valence bands with respect to the Fermi energy changes between p and n regions.

Electrons in the conduction band of the n region see a potential barrier in trying to move into the conduction band of the p region. This potential barrier is referred to as the *built-in potential barrier* and is denoted by V_{bi} . The built-in potential barrier maintains equilibrium between majority carrier electrons in the n region and minority carrier electrons in the p region, and also between majority carrier holes in the p region and minority carrier holes in the n region. This potential difference across the junction cannot be measured with a voltmeter because new potential barriers will be formed between the probes and the semiconductor that will cancel V_{bi} . The potential V_{bi} maintains equilibrium, so no current is produced by this voltage.

The intrinsic Fermi level is equidistant from the conduction band edge through the junction, thus the built-in potential barrier can be determined as the difference

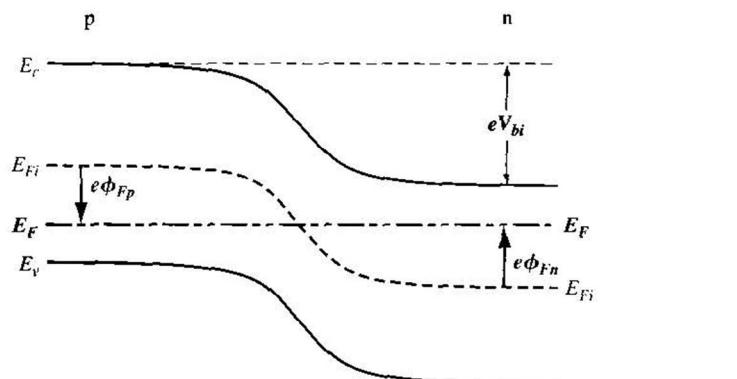


Figure 7.3 | Energy-band diagram of a pn junction in thermal equilibrium.

between the intrinsic Fermi levels in the p and n regions. We can define the potentials ϕ_{Fn} and ϕ_{Fp} as shown in Figure 7.3, so we have

$$V_{bi} = |\phi_{Fn}| + |\phi_{Fp}| \quad (7.1)$$

In the n region, the electron concentration in the conduction band is given by

$$n_0 = N_c \exp \left[\frac{-(E_c - E_F)}{kT} \right] \quad (7.2)$$

which can also be written in the form

$$n_0 = n_i \exp \left[\frac{E_F - E_{Fi}}{kT} \right] \quad (7.3)$$

where n_i and E_{Fi} are the intrinsic carrier concentration and the intrinsic Fermi energy, respectively. We may define the potential ϕ_{Fn} in the n region as

$$e\phi_{Fn} = E_{Fi} - E_F \quad (7.4)$$

Equation (7.3) may then be written as

$$n_0 = n_i \exp \left[\frac{-(e\phi_{Fn})}{kT} \right] \quad (7.5)$$

Taking the natural log of both sides of Equation (7.5), setting $n_0 = N_d$, and solving for the potential, we obtain

$$\phi_{Fn} = \frac{-kT}{e} \ln \left(\frac{N_d}{n_i} \right) \quad (7.6)$$

Similarly, in the p region, the hole concentration is given by

$$p_0 = N_a = n_i \exp \left[\frac{E_{Fi} - E_F}{kT} \right] \quad (7.7)$$

where N_a is the acceptor concentration. We can define the potential ϕ_{Fp} in the p region as

$$e\phi_{Fp} = E_{Fi} - E_F \quad (7.8)$$

Combining Equations (7.7) and (7.8), we find that

$$\phi_{Fp} = +\frac{kT}{e} \ln \left(\frac{N_a}{n_i} \right) \quad (7.9)$$

Finally, the built-in potential barrier for the step junction is found by substituting Equations (7.6) and (7.9) into Equation (7.1), which yields

$$V_{bi} = \frac{kT}{e} \ln \left(\frac{N_a N_d}{n_i^2} \right) = V_t \ln \left(\frac{N_a N_d}{n_i^2} \right) \quad (7.10)$$

where $V_t = kT/e$ and is defined as the thermal voltage.

At this time, we should note a subtle but important point concerning notation. Previously in the discussion of a semiconductor material, N_d and N_a denoted donor and acceptor impurity concentrations in the same region, thereby forming a compensated semiconductor. From this point on in the text, N_d and N_a will denote the net donor and acceptor concentrations in the individual n and p regions, respectively. If the p region, for example, is a compensated material, then N_a will represent the difference between the actual acceptor and donor impurity concentrations. The parameter N_d is defined in a similar manner for the n region.

EXAMPLE 7.1**Objective**

To calculate the built-in potential barrier in a pn junction.

Consider a silicon pn junction at $T = 300\text{ K}$ with doping densities $N_a = 1 \times 10^{18}\text{ cm}^{-3}$ and $N_d = 1 \times 10^{15}\text{ cm}^{-3}$. Assume that $n_i = 1.5 \times 10^{10}\text{ cm}^{-3}$.

Solution

The built-in potential barrier is determined from Equation (7.10) as

$$V_{bi} = (0.0259) \ln \left[\frac{(10^{18})(10^{15})}{(1.5 \times 10^{10})^2} \right] = 0.754\text{ V}$$

If we change the acceptor doping from $N_a = 1 \times 10^{18}\text{ cm}^{-3}$ to $N_a = 1 \times 10^{16}\text{ cm}^{-3}$, but keep all other parameter values constant, then the built-in potential barrier becomes $V_{bi} = 0.635\text{ V}$.

Comment

The built-in potential barrier changes only slightly as the doping concentrations change by orders of magnitude because of the logarithmic dependence.

TEST YOUR UNDERSTANDING

E7.1 Calculate the built-in potential barrier in a silicon pn junction at $T = 300\text{ K}$ for

(a) $N_a = 5 \times 10^{17}\text{ cm}^{-3}$, $N_d = 10^{16}\text{ cm}^{-3}$ and (b) $N_a = 10^{15}\text{ cm}^{-3}$,

$N_d = 2 \times 10^{16}\text{ cm}^{-3}$. [Ans. (a) 0.796 V, (b) 0.653 V]

E7.2 Repeat E7.1 for a GaAs pn junction. [Ans. (a) 1.26 V, (b) 1.12 V]

7.2.2 Electric Field

An electric field is created in the depletion region by the separation of positive and negative space charge densities. Figure 7.4 shows the volume charge density distribution in the pn junction assuming uniform doping and assuming an abrupt junction approximation. We will assume that the space charge region abruptly ends in the n region at $x = +x_n$ and abruptly ends in the p region at $x = -x_p$ (x_p is a positive quantity).

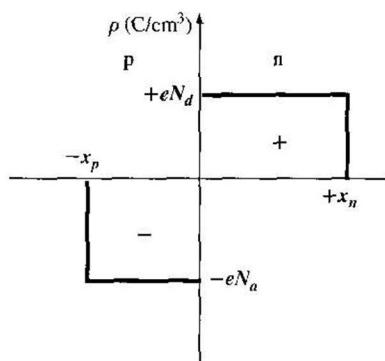


Figure 7.4 The space charge density in a uniformly doped pn junction assuming the abrupt junction approximation.

The electric field is determined from Poisson's equation which, for a one-dimensional analysis, is

$$\frac{d^2\phi(x)}{dx^2} = \frac{-\rho(x)}{\epsilon_s} = -\frac{dE(x)}{dx} \quad (7.11)$$

where $\phi(x)$ is the electric potential, $E(x)$ is the electric field, $\rho(x)$ is the volume charge density, and ϵ_s is the permittivity of the semiconductor. From Figure 7.4, the charge densities are

$$\rho(x) = -eN_a \quad -x_p < x < 0 \quad (7.12a)$$

and

$$\rho(x) = eN_d \quad 0 < x < x_n \quad (7.12b)$$

The electric field in the p region is found by integrating Equation (7.11). We have that

$$E = \int \frac{\rho(x)}{\epsilon_s} dx = - \int \frac{eN_a}{\epsilon_s} dx = \frac{-eN_a}{\epsilon_s} x + C_1 \quad (7.13)$$

where C_1 is a constant of integration. The electric field is assumed to be zero in the neutral p region for $x < -x_p$ since the currents are zero in thermal equilibrium. As there are no surface charge densities within the pn junction structure, the electric field is a continuous function. The constant of integration is determined by setting $E = 0$ at $x = -x_p$. The electric field in the p region is then given by

$$E = \frac{-eN_a}{\epsilon_s} (x + x_p) \quad -x_p \leq x \leq 0 \quad (7.14)$$

In the n region, the electric field is determined from

$$E = \int \frac{(eN_d)}{\epsilon_s} dx = \frac{eN_d}{\epsilon_s} x + C_2 \quad (7.15)$$

where C_2 is again a constant of integration. The constant C_2 is determined by setting $E = 0$ at $x = x_n$, since the E-field is assumed to be zero in the n region and is a continuous function. Then

$$E = \frac{-eN_d}{\epsilon_s}(x_n - x) \quad 0 \leq x \leq x_n \quad (7.16)$$

The electric field is also continuous at the metallurgical junction, or at $x = 0$. Setting Equations (7.14) and (7.16) equal to each other at $x = 0$ gives

$$N_a x_p = N_d x_n \quad (7.17)$$

Equation (7.17) states that the number of negative charges per unit area in the p region is equal to the number of positive charges per unit area in the n region.

Figure 7.5 is a plot of the electric field in the depletion region. The electric field direction is from the n to the p region, or in the negative x direction for this geometry. For the uniformly doped pn junction, the E-field is a linear function of distance through the junction, and the maximum (magnitude) electric field occurs at the metallurgical junction. An electric field exists in the depletion region even when no voltage is applied between the p and n regions.

The potential in the junction is found by integrating the electric field. In the p region then, we have

$$\phi(x) = - \int E(x) dx = \int \frac{eN_a}{\epsilon_s} (x + x_p) dx \quad (7.18)$$

or

$$\phi(x) = \frac{eN_a}{\epsilon_s} \left(\frac{x^2}{2} + x_p \cdot x \right) + C'_1 \quad (7.19)$$

where C'_1 is again a constant of integration. The potential difference through the pn junction is the important parameter, rather than the absolute potential, so we may arbitrarily set the potential equal to zero at $x = -x_p$. The constant of integration is

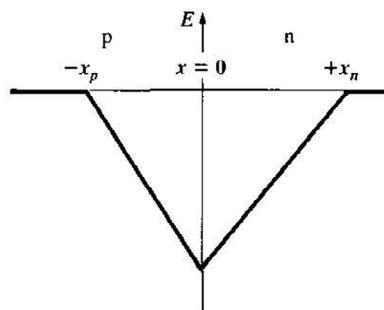


Figure 7.5 | Electric field in the space charge region of a uniformly doped pn junction.

then found as

$$C'_1 = \frac{eN_a}{2\epsilon_s} x_p^2 \quad (7.20)$$

so that the potential in the p region can now be written as

$$\phi(x) = \frac{eN_a}{2\epsilon_s} (x + x_p)^2 \quad (-x_p \leq x \leq 0) \quad (7.21)$$

The potential in the n region is determined by integrating the electric field in the n region, or

$$\phi(x) = \int \frac{eN_d}{\epsilon_s} (x_n - x) dx \quad (7.22)$$

Then

$$\phi(x) = \frac{eN_d}{\epsilon_s} \left(x_n \cdot x - \frac{x^2}{2} \right) + C'_2 \quad (7.23)$$

where C'_2 is another constant of integration. The potential is a continuous function, so setting Equation (7.21) equal to Equation (7.23) at the metallurgical junction, or at $x = 0$, gives

$$C'_2 = \frac{eN_a}{2\epsilon_s} x_p^2 \quad (7.24)$$

The potential in the n region can thus be written as

$$\phi(x) = \frac{eN_d}{\epsilon_s} \left(x_n \cdot x - \frac{x^2}{2} \right) + \frac{eN_a}{2\epsilon_s} x_p^2 \quad (0 \leq x \leq x_n) \quad (7.25)$$

Figure 7.6 is a plot of the potential through the junction and shows the quadratic dependence on distance. The magnitude of the potential at $x = x_n$ is equal to the

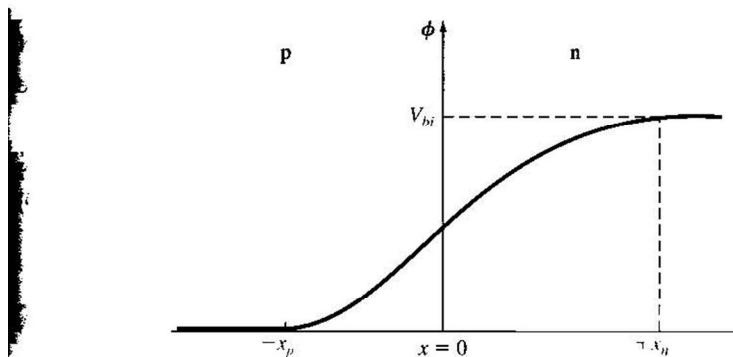


Figure 7.6 | Electric potential through the space charge region of a uniformly doped pn junction.

built-in potential barrier. Then from Equation (7.25), we have

$$V_{bi} = |\phi(x = x_n)| = \frac{e}{2\epsilon_s} (N_d x_n^2 + N_a x_p^2) \quad (7.26)$$

The potential energy of an electron is given by $E = -e\phi$, which means that the electron potential energy also varies as a quadratic function of distance through the space charge region. The quadratic dependence on distance was shown in the energy-band diagram of Figure 7.3, although we did not explicitly know the shape of the curve at that time.

7.2.3 Space Charge Width

We can determine the distance that the space charge region extends into the p and n regions from the metallurgical junction. This distance is known as the space charge width. From Equation (7.17), we may write, for example,

$$x_p = \frac{N_d x_n}{N_a} \quad (7.27)$$

Then, substituting Equation (7.27) into Equation (7.26) and solving for x_n , we obtain

$$x_n = \left\{ \frac{2\epsilon_s V_{bi}}{e} \left[\frac{N_a}{N_d} \right] \left[\frac{1}{N_a + N_d} \right] \right\}^{1/2} \quad (7.28)$$

Equation (7.28) gives the space charge width, or the width of the depletion region, x_n extending into the n-type region for the case of zero applied voltage.

Similarly, if we solve for x_p from Equation (7.17) and substitute into Equation (7.26), we find

$$x_p = \left\{ \frac{2\epsilon_s V_{bi}}{e} \left[\frac{N_d}{N_a} \right] \left[\frac{1}{N_a + N_d} \right] \right\}^{1/2} \quad (7.29)$$

where x_p is the width of the depletion region extending into the p region for the case of zero applied voltage.

The total depletion or space charge width W is the sum of the two components, or

$$W = x_n + x_p \quad (7.30)$$

Using Equations (7.28) and (7.29), we obtain

$$W = \left\{ \frac{2\epsilon_s V_{bi}}{e} \left[\frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2} \quad (7.31)$$

The built-in potential barrier can be determined from Equation (7.10), and then the total space charge region width is obtained using Equation (7.31).

Objective

EXAMPLE 7.2

To calculate the space charge width and electric field in a pn junction.

Consider a silicon pn junction at $T = 300$ K with doping concentrations of $N_a = 10^{16} \text{ cm}^{-3}$ and $N_d = 10^{15} \text{ cm}^{-3}$.

Solution

In Example 7.1, we determined the built-in potential barrier as $V_{bi} = 0.635$ V. From Equation (7.31), the space charge width is

$$\begin{aligned} W &= \left\{ \frac{2\epsilon_s V_{bi}}{e} \left[\frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2} \\ &= \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.635)}{1.6 \times 10^{-19}} \left[\frac{10^{16} + 10^{15}}{(10^{16})(10^{15})} \right] \right\}^{1/2} \\ &= 0.951 \times 10^{-4} \text{ cm} = 0.951 \mu\text{m} \end{aligned}$$

Using Equations (7.28) and (7.29), we can find $x_n = 0.864 \mu\text{m}$, and $x_p = 0.086 \mu\text{m}$.

The peak electric field at the metallurgical junction, using Equation (7.16) for example, is

$$E_{\max} = \frac{-eN_d x_n}{\epsilon_s} = \frac{-(1.6 \times 10^{-19})(10^{15})(0.864 \times 10^{-4})}{(11.7)(8.85 \times 10^{-14})} = -1.34 \times 10^4 \text{ V/cm}$$

Comment

The peak electric field in the space charge region of a pn junction is quite large. We must keep in mind, however, that there is no mobile charge in this region; hence there will be no drift current. We may also note, from this example, that the width of each space charge region is a reciprocal function of the doping concentration: The depletion region will extend further into the lower-doped region.

TEST YOUR UNDERSTANDING

- E7.3 A silicon pn junction at $T = 300$ K with zero applied bias has doping concentrations of $N_d = 5 \times 10^{16} \text{ cm}^{-3}$ and $N_a = 5 \times 10^{15} \text{ cm}^{-3}$. Determine x_n , x_p , W , and $|E_{\max}|$.

$$x_p = 4.11 \times 10^{-5} \text{ cm}, W = 4.52 \times 10^{-5} \text{ cm}, |E_{\max}| = 3.18 \times 10^4 \text{ V/cm}$$

(Ans. $x_n = 4.11 \times 10^{-6} \text{ cm}$,

- E7.4 Repeat E7.3 for a GaAs pn junction.

$$|E_{\max}| = 3.86 \times 10^4 \text{ V/cm}$$

(Ans. $x_n = 5.60 \times 10^{-6} \text{ cm}$, $x_p = 5.60 \times 10^{-5} \text{ cm}$, $W = 6.19 \times 10^{-5} \text{ cm}$,

7.3 | REVERSE APPLIED BIAS

If we apply a potential between the p and n regions, we will no longer be in an equilibrium condition—the Fermi energy level will no longer be constant through the system. Figure 7.7 shows the energy-band diagram of the pn junction for the case

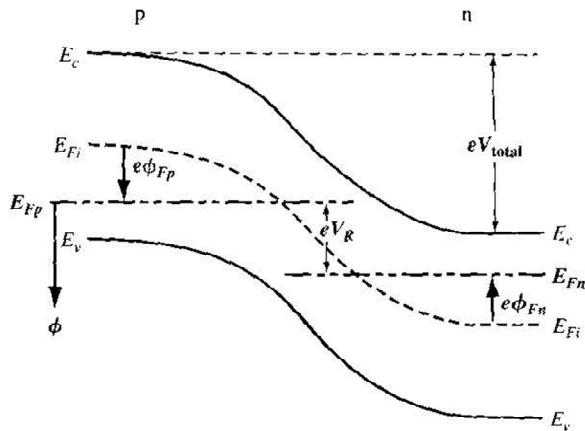


Figure 7.7 Energy-band diagram of a pn junction under reverse bias.

when a positive voltage is applied to the n region with respect to the p region. As the positive potential is downward, the Fermi level on the n side is below the Fermi level on the p side. The difference between the two is equal to the applied voltage in units of energy.

The total potential barrier, indicated by V_{total} , has increased. The applied potential is the reverse-bias condition. The total potential barrier is now given by

$$V_{\text{total}} = |\phi_{F_n}| + |\phi_{F_p}| + V_R \quad (7.32)$$

where V_R is the magnitude of the applied reverse-bias voltage. Equation (7.32) can be rewritten as

$$V_{\text{total}} = V_{bi} + V_R \quad (7.33)$$

where V_{bi} is the same built-in potential barrier we had defined in thermal equilibrium.

7.3.1 Space Charge Width and Electric Field

Figure 7.8 shows a pn junction with an applied reverse-bias voltage V_R . Also indicated in the figure are the electric field in the space charge region and the electric field E_{app} , induced by the applied voltage. The electric fields in the neutral p and n regions are essentially zero, or at least very small, which means that the magnitude of the electric field in the space charge region must increase above the thermal-equilibrium value due to the applied voltage. The electric field originates on positive charge and terminates on negative charge; this means that the number of positive and negative charges must increase if the electric field increases. For given impurity doping concentrations, the number of positive and negative charges in the depletion region can be increased only if the space charge width W increases. The space charge width W increases,

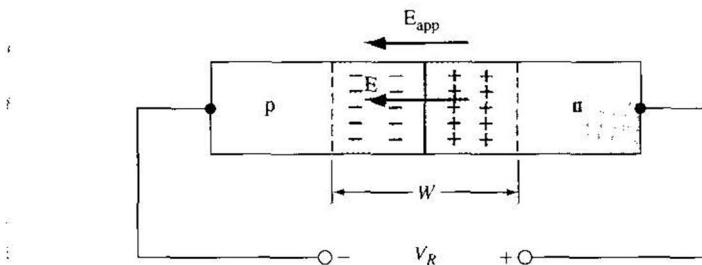


Figure 7.8 | A pn junction, with an applied reverse-bias voltage, showing the directions of the electric field induced by V_R and the space charge electric field.

therefore, with an increasing reverse-bias voltage V_R . We are assuming that the electric field in the bulk n and p regions is zero. This assumption will become clearer in the next chapter when we discuss the current-voltage characteristics.

In all of the previous equations, the built-in potential barrier can be replaced by the total potential barrier. The total space charge width can be written from Equation (7.31) as

$$W = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left[\frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2} \quad (7.34)$$

showing that the total space charge width increases as we apply a reverse-bias voltage. By substituting the total potential barrier V_{total} into Equations (7.28) and (7.29), the space charge widths in the n and p regions, respectively, can be found as a function of applied reverse-bias voltage.

Objective

EXAMPLE 7.3

To calculate the width of the space charge region in a pn junction when a reverse-bias voltage is applied.

Again consider a silicon pn junction at $T = 300$ K with doping concentrations of $N_a = 10^{16}$ cm $^{-3}$ and $N_d = 10^{15}$ cm $^{-3}$. Assume that $n_i = 1.5 \times 10^{10}$ cm $^{-3}$ and let $V_R = 5$ V.

Solution

The built-in potential barrier was calculated in Example 7.1 for this case and is $V_{bi} = 0.635$ V. The space charge width is determined from Equation (7.34). We have

$$W = \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.635 + 5)}{1.6 \times 10^{-19}} \left[\frac{10^{16} + 10^{15}}{(10^{16})(10^{15})} \right] \right\}^{1/2}$$

so that

$$W = 2.83 \times 10^{-4} \text{ cm} = 2.83 \mu\text{m}$$

Comment

The space charge width has increased from $0.951 \mu\text{m}$ at zero bias to $2.83 \mu\text{m}$ at a reverse bias of 5 V.

The magnitude of the electric field in the depletion region increases with an applied reverse-bias voltage. The electric field is still given by Equations (7.14) and (7.16) and is still a linear function of distance through the space charge region. Since x_n and x_p increase with reverse-bias voltage, the magnitude of the electric field also increases. The maximum electric field still occurs at the metallurgical junction.

The maximum electric field at the metallurgical junction, from Equations (7.14) and (7.16), is

$$E_{\max} = \frac{-eN_d x_n}{\epsilon_s} = \frac{-eN_a x_p}{\epsilon_s} \quad (7.35)$$

If we use either Equation (7.28) or (7.29) in conjunction with the total potential barrier, $V_{bi} + V_R$, then

$$E_{\max} = -\left\{ \frac{2e(V_{bi} + V_R)}{\epsilon_s} \left(\frac{N_a N_d}{N_a + N_d} \right) \right\}^{1/2} \quad (7.36)$$

We can show that the maximum electric field in the pn junction can also be written as

$$E_{\max} = \frac{-2(V_{bi} + V_R)}{W} \quad (7.37)$$

where W is the total space charge width.

DESIGN EXAMPLE 7.4
**Objective**

To design a pn junction to meet maximum electric field and voltage specifications.

Consider a silicon pn junction at $T = 300 \text{ K}$ with a p-type doping concentration of $N_a = 10^{18} \text{ cm}^{-3}$. Determine the n-type doping concentration such that the maximum electric field is $|E_{\max}| = 3 \times 10^5 \text{ V/cm}$ at a reverse-bias voltage of $V_R = 25 \text{ V}$.

Solution

The maximum electric field is given by Equation (7.36). Neglecting V_{bi} compared to V_R , we can write

$$|E_{\max}| \approx \left\{ \frac{2eV_R}{\epsilon_s} \left(\frac{N_a N_d}{N_a + N_d} \right) \right\}^{1/2}$$

or

$$3 \times 10^5 = \left\{ \frac{2(1.6 \times 10^{-19})(25)}{(11.7)(8.85 \times 10^{-14})} \left(\frac{10^{18} \cdot N_d}{10^{18} + N_d} \right) \right\}^{1/2}$$

which yields

$$N_d = 1.18 \times 10^{16} \text{ cm}^{-3}$$

■ Conclusion

A smaller value of N_d results in a smaller value of $|E_{\max}|$ for a given reverse-bias voltage. The value of N_d determined in this example, then, is the maximum value that will meet the specifications.

TEST YOUR UNDERSTANDING

- E7.5** (a) A silicon pn junction at $T = 300 \text{ K}$ is reverse-biased at $V_R = 8 \text{ V}$. The doping concentrations are $N_a = 5 \times 10^{16} \text{ cm}^{-3}$ and $N_d = 5 \times 10^{15} \text{ cm}^{-3}$. Determine x_n , x_p , W , and $|E_{\max}|$. (b) Repeat part (a) for a reverse bias voltage of $V_R = 12 \text{ V}$.

$$\begin{aligned} x_n &= 1.73 \times 10^{-5} \text{ cm}, W = 1.90 \times 10^{-4} \text{ cm}, |E_{\max}| = 1.34 \times 10^5 \text{ V/cm} \\ W &= 1.57 \times 10^{-4} \text{ cm}, |E_{\max}| = 1.11 \times 10^5 \text{ V/cm}; (b) x_p = 1.73 \times 10^{-4} \text{ cm}, \\ &\text{[Ans. (a) } x_n = 1.43 \times 10^{-5} \text{ cm, } x_p = 1.43 \times 10^{-5} \text{ cm,} \text{ (b) } W = 1.57 \times 10^{-4} \text{ cm, } |E_{\max}| = 1.11 \times 10^5 \text{ V/cm}]\end{aligned}$$

- E7.6** The maximum electric field in a reverse-biased GaAs pn junction at $T = 300 \text{ K}$ is to be $|E_{\max}| = 2.5 \times 10^5 \text{ V/cm}$. The doping concentrations are $N_d = 5 \times 10^{15} \text{ cm}^{-3}$ and $N_a = 8 \times 10^{15} \text{ cm}^{-3}$. Determine the reverse-bias voltage that will produce this maximum electric field. (Ans. 72.5 V)

7.3.2 Junction Capacitance

Since we have a separation of positive and negative charges in the depletion region, a capacitance is associated with the pn junction. Figure 7.9 shows the charge densities in the depletion region for applied reverse-bias voltages of V_R and $V_R + dV_R$. An increase in the reverse-bias voltage dV_R will uncover additional positive charges in the n region and additional negative charges in the p region. The junction capacitance is defined as

$$C' = \frac{dQ'}{dV_R} \quad (7.38)$$

where

$$dQ' = eN_d dx_n = eN_a dx_p \quad (7.39)$$

The differential charge dQ' is in units of C/cm^2 so that the capacitance C' is in units of farads per square centimeter (F/cm^2), or capacitance per unit area.

For the total potential barrier, Equation (7.28) may be written as

$$x_n = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left[\frac{N_a}{N_d} \right] \left[\frac{1}{N_a + N_d} \right] \right\}^{1/2} \quad (7.40)$$

The junction capacitance can be written as

$$C' = \frac{dQ'}{dV_R} = eN_d \frac{dx_n}{dV_R} \quad (7.41)$$

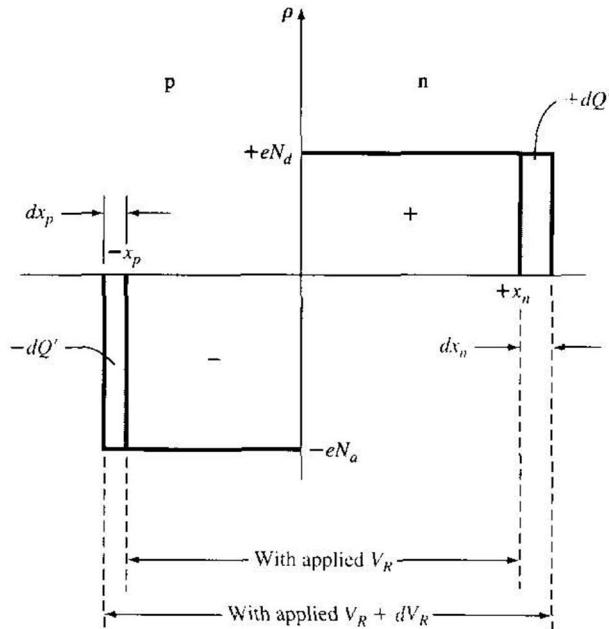


Figure 7.9 | Differential change in the space charge width with a differential change in reverse-bias voltage for a uniformly doped pn junction.

so that

$$C' = \left\{ \frac{e\epsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right\}^{1/2} \quad (7.42)$$

Exactly the same capacitance expression is obtained by considering the space charge region extending into the p region x_p . The junction capacitance is also referred to as the *depletion layer capacitance*.

EXAMPLE 7.5

Objective

To calculate the junction capacitance of a pn junction.

Consider the same pn junction as that in Example 7.3. Again assume that $V_R = 5$ V.

■ Solution

The junction capacitance is found from Equation (7.42) as

$$C' = \left\{ \frac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(10^{16})(10^{15})}{2(0.635 + 5)(10^{16} + 10^{15})} \right\}^{1/2}$$

or

$$C' = 3.66 \times 10^{-9} \text{ F/cm}^2$$

If the cross-sectional area of the pn junction is, for example, $A = 10^{-4} \text{ cm}^2$, then the total junction capacitance is

$$C = C' \cdot A = 0.366 \times 10^{-12} \text{ F} = 0.366 \text{ pF}$$

■ Comment

The value of junction capacitance is usually in the pF, or smaller, range.

If we compare Equation (7.34) for the total depletion width W of the space charge region under reverse bias and Equation (7.42) for the junction capacitance C' , we find that we can write

$$C' = \frac{\epsilon_s}{W} \quad (7.43)$$

Equation (7.43) is the same as the capacitance per unit area of a parallel plate capacitor. Considering Figure 7.9, we may have come to this same conclusion earlier. Keep in mind that the space charge width is a function of the reverse bias voltage so that the junction capacitance is also a function of the reverse bias voltage applied to the pn junction.

7.3.3 One-Sided Junctions

Consider a special pn junction called the one-sided junction. If, for example, $N_a \gg N_d$, this junction is referred to as a p⁺n junction. The total space charge width, from Equation (7.34), reduces to

$$W \approx \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{eN_d} \right\}^{1/2} \quad (7.44)$$

Considering the expressions for x_n and x_p , we have for the p⁺n junction

$$x_p \ll x_n \quad (7.45)$$

and

$$W \approx x_n \quad (7.46)$$

Almost the entire space charge layer extends into the low-doped region of the junction. This effect can be seen in Figure 7.10.

The junction capacitance of the p⁺n junction reduces to

$$C' \approx \left\{ \frac{e\epsilon_s N_d}{2(V_{bi} + V_R)} \right\}^{1/2} \quad (7.47)$$

The depletion layer capacitance of a one-sided junction is a function of the doping concentration in the low-doped region. Equation (7.47) may be manipulated to give

$$\left(\frac{1}{C'} \right)^2 = \frac{2(V_{bi} + V_R)}{e\epsilon_s N_d} \quad (7.48)$$

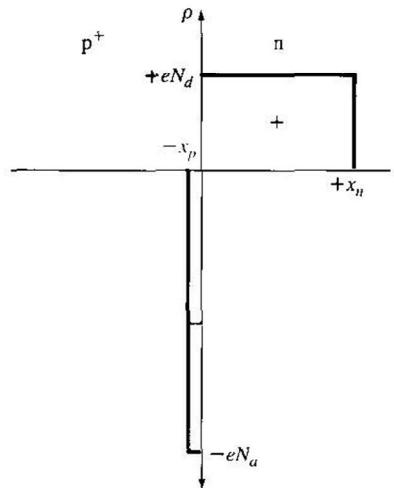


Figure 7.10 | Space charge density of a one-sided p^+ n junction.

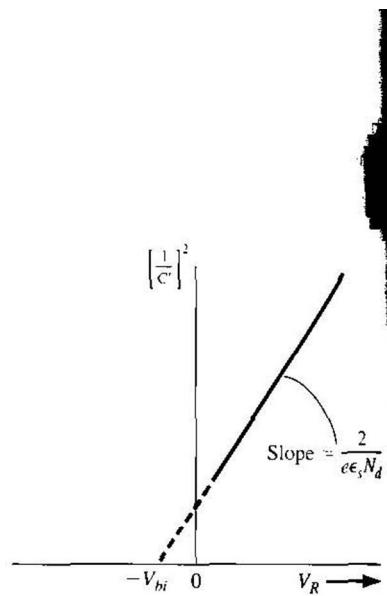


Figure 7.11 | $(1/C')^2$ versus V_R of a uniformly doped pn junction.

which shows that the inverse capacitance squared is a linear function of applied reverse-bias voltage.

Figure 7.11 shows a plot of Equation (7.48). The built-in potential of the junction can be determined by extrapolating the curve to the point where $(1/C')^2 = 0$. The slope of the curve is inversely proportional to the doping concentration of the low-doped region in the junction; thus, this doping concentration can be experimentally determined. The assumptions used in the derivation of this capacitance include uniform doping in both semiconductor regions, the abrupt junction approximation, and a planar junction.

EXAMPLE 7.6

Objective

To determine the impurity doping concentrations in a p^+ n junction given the parameters from Figure 7.11.

Assume a silicon p^+ n junction at $T = 300$ K with $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$. Assume that the intercept of the curve in Figure 7.11 gives $V_{bi} = 0.855$ V and that the slope is $1.32 \times 10^{15} (\text{F/cm}^2)^{-2} (\text{V})^{-1}$.

■ Solution

The slope of the curve in Figure 7.11 is given by $2/e\epsilon_s N_d$, so we may write

$$N_d = \frac{2}{e\epsilon_s (\text{slope})} = \frac{2}{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(1.32 \times 10^{15})}$$

or

$$N_d = 9.15 \times 10^{15} \text{ cm}^{-3}$$

from the expression for V_{bi} , which is

$$V_{bi} = V_t \ln \left(\frac{N_a N_d}{n_i^2} \right) = \frac{kT}{e} \ln \left(\frac{N_a N_d}{n_i^2} \right)$$

we can solve for N_a as

$$N_a = \frac{n_i^2}{N_d} \exp \left(\frac{eV_{bi}}{kT} \right) = \frac{(1.5 \times 10^{10})^2}{9.15 \times 10^{15}} \exp \left(\frac{0.855}{0.0259} \right)$$

which yields

$$N_a = 5.34 \times 10^{18} \text{ cm}^{-3}$$

Comment

The results of this example show that $N_a \gg N_d$; therefore the assumption of a one-sided junction was valid.

A one-sided pn junction is useful for experimentally determining the doping concentrations and built-in potential.

TEST YOUR UNDERSTANDING

- E7.7 A silicon pn junction at $T = 300$ K has doping concentrations of $N_d = 3 \times 10^{16} \text{ cm}^{-3}$ and $N_a = 8 \times 10^{15} \text{ cm}^{-3}$, and has a cross-sectional area of $A = 5 \times 10^{-5} \text{ cm}^2$. Determine the junction capacitance at (a) $V_R = 2$ V and (b) $V_R = 5$ V.
[Ans. (a) 0.694 pF, (b) 0.478 pF]
- E7.8 The experimentally measured junction capacitance of a one-sided silicon n⁺p junction biased at $V_R = 4$ V at $T = 300$ K is $C = 1.10 \text{ pF}$. The built-in potential barrier is found to be $V_{bi} = 0.782$ V. The cross-sectional area is $A = 10^{-4} \text{ cm}^2$. Find the doping concentrations.
[Ans. $N_a = 7 \times 10^{15} \text{ cm}^{-3}$, $N_p = 4.17 \times 10^{17} \text{ cm}^{-3}$]

*7.4 | NONUNIFORMLY DOPED JUNCTIONS

In the pn junctions considered so far, we have assumed that each semiconductor region has been uniformly doped. In actual pn junction structures, this is not always true. In some electronic applications, specific nonuniform doping profiles are used to obtain special pn junction capacitance characteristics.

7.4.1 Linearly Graded Junctions

If we start with a uniformly doped n-type semiconductor, for example, and diffuse acceptor atoms through the surface, the impurity concentrations will tend to be like those shown in Figure 7.12. The point $x = x'$ on the figure corresponds to the metallurgical junction. The depletion region extends into the p and n regions from the metallurgical junction as we have discussed previously. The net p-type doping

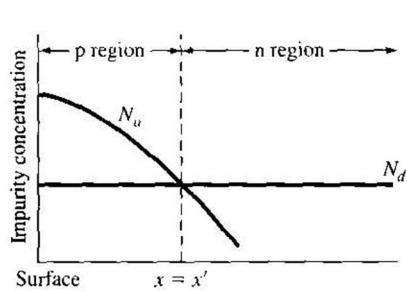


Figure 7.12 | Impurity concentrations of a pn junction with a nonuniformly doped p region.

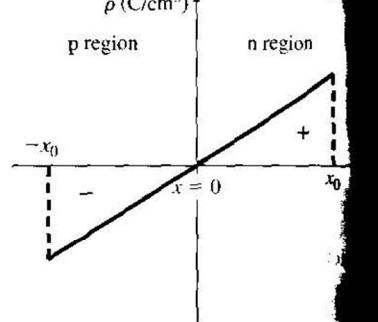


Figure 7.13 | Space charge density in the depletion region of a linearly graded pn junction.

concentration near the metallurgical junction may be approximated as a linear function of distance from the metallurgical junction. Likewise, as a first approximation, the net n-type doping concentration is also a linear function of distance extending into the n region from the metallurgical junction. This effective doping profile is referred to as a linearly graded junction.

Figure 7.13 shows the space charge density in the depletion region of the linearly graded junction. For convenience, the metallurgical junction is placed at $x = 0$. The space charge density can be written as

$$\rho(x) = eax \quad (7.49)$$

where a is the gradient of the net impurity concentration.

The electric field and potential in the space charge region can be determined from Poisson's equation. We can write

$$\frac{dE}{dx} = \frac{\rho(x)}{\epsilon_s} = \frac{eax}{\epsilon_s} \quad (7.50)$$

so that the electric field can be found by integration as

$$E = \int \frac{eax}{\epsilon_s} dx = \frac{ea}{2\epsilon_s} (x^2 - x_0^2) \quad (7.51)$$

The electric field in the linearly graded junction is a quadratic function of distance rather than the linear function found in the uniformly doped junction. The maximum electric field again occurs at the metallurgical junction. We may note that the electric field is zero at both $x = +x_0$ and at $x = -x_0$. The electric field in a nonuniformly doped semiconductor is not exactly zero, but the magnitude of this field is small, so setting $E = 0$ in the bulk regions is still a good approximation.

The potential is again found by integrating the electric field as

$$\phi(x) = - \int E dx \quad (7.52)$$

If we arbitrarily set $\phi = 0$ at $x = -x_0$, then the potential through the junction is

$$\phi(x) = \frac{-ea}{2\epsilon_s} \left(\frac{x^3}{3} - x_0^2 x \right) + \frac{ea}{3\epsilon_s} x_0^3 \quad (7.53)$$

The magnitude of the potential at $x = +x_0$ will equal the built-in potential barrier for this function. We then have that

$$\phi(x_0) = \frac{2}{3} \cdot \frac{eax_0^3}{\epsilon_s} = V_{bi} \quad (7.54)$$

Another expression for the built-in potential barrier for a linearly graded junction can be approximated from the expression used for a uniformly doped junction. We can write

$$V_{bi} = V_t \ln \left[\frac{N_d(x_0)N_a(-x_0)}{n_i^2} \right] \quad (7.55)$$

where $N_d(x_0)$ and $N_a(-x_0)$ are the doping concentrations at the edges of the space charge region. We can relate these doping concentrations to the gradient, so that

$$N_d(x_0) = ax_0 \quad (7.56a)$$

and

$$N_a(-x_0) = ax_0 \quad (7.56b)$$

Then the built-in potential barrier for the linearly graded junction becomes

$$V_{bi} = V_t \ln \left(\frac{ax_0}{n_i} \right)^2 \quad (7.57)$$

There may be situations in which the doping gradient is not the same on either side of the junction, but we will not consider that condition here.

If a reverse-bias voltage is applied to the junction, the potential barrier increases. The built-in potential barrier V_{bi} in the above equations is then replaced by the total potential barrier $V_{bi} + V_R$. Solving for x_0 from Equation (7.54) and using the total potential barrier, we obtain

$$x_0 = \left\{ \frac{3}{2} \cdot \frac{\epsilon_s}{ea} (V_{bi} + V_R) \right\}^{1/3} \quad (7.58)$$

The junction capacitance per unit area can be determined by the same method as we used for the uniformly doped junction. Figure 7.14 shows the differential charge dQ' which is uncovered as a differential voltage dV_R is applied. The junction capacitance is then

$$C' = \frac{dQ'}{dV_R} = (eax_0) \frac{dx_0}{dV_R} \quad (7.59)$$

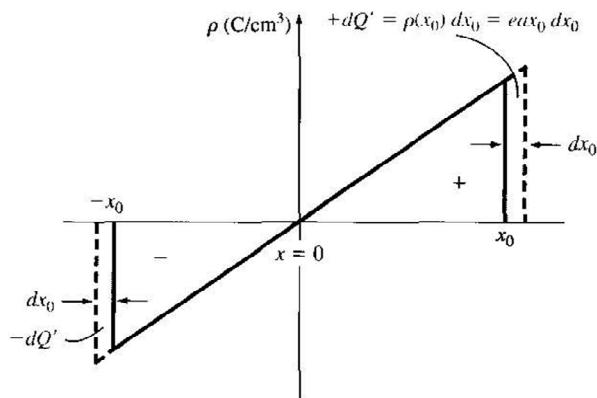


Figure 7.14 | Differential change in space charge width with a differential change in reverse-bias voltage for a linearly graded pn junction.

Using Equation (7.58), we obtain¹

$$C' = \left\{ \frac{ea\epsilon_s^2}{12(V_{bi} + V_R)} \right\}^{1/3} \quad (7.60)$$

We may note that C' is proportional to $(V_{bi} + V_R)^{-1/3}$ for the linearly graded junction as compared to $C'\alpha(V_{bi} + V_R)^{-1/2}$ for the uniformly doped junction. In the linearly graded junction, the capacitance is less dependent on reverse-bias voltage than in the uniformly doped junction.

7.4.2 Hyperabrupt Junctions

The uniformly doped junction and linearly graded junction are not the only possible doping profiles. Figure 7.15 shows a generalized one-sided p⁺n junction where the generalized n-type doping concentration for $x > 0$ is given by

$$N = Bx^m \quad (7.61)$$

The case of $m = 0$ corresponds to the uniformly doped junction and $m = +1$ corresponds to the linearly graded junction just discussed. The cases of $m = +2$ and $m = +3$ shown would approximate a fairly low-doped epitaxial n-type layer grown on a much more heavily doped n⁺ substrate layer. When the value of m is negative, we have what is referred to as a *hyperabrupt junction*. In this case, the n-type doping is larger near the metallurgical junction than in the bulk semiconductor. Equation (7.61) is used to approximate the n-type doping over a small region near $x = x_0$ and does not hold at $x = 0$ when m is negative.

¹In a more exact analysis, V_{bi} in Equation (7.60) is replaced by a gradient voltage. However, this analysis is beyond the scope of this text.

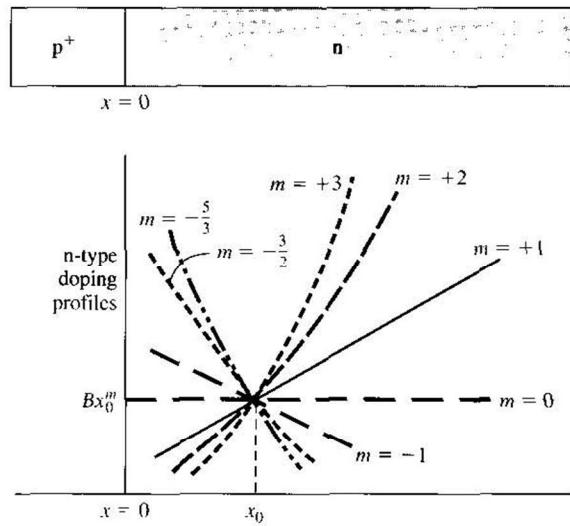


Figure 7.15 | Generalized doping profiles of a one-sided p⁺n junction.
(From Sze [14].)

The junction capacitance can be derived using the same analysis method as before and is given by

$$C' = \left\{ \frac{eB\epsilon_s^{(m+1)}}{(m+2)(V_{bi} + V_R)} \right\}^{1/(m+2)} \quad (7.62)$$

When m is negative, the capacitance becomes a very strong function of reverse-bias voltage, a desired characteristic in *varactor diodes*. The term *varactor* comes from the words *variable reactor* and means a device whose reactance can be varied in a controlled manner with bias voltage.

If a varactor diode and an inductance are in parallel, the resonant frequency of the LC circuit is

$$f_r = \frac{1}{2\pi\sqrt{LC}} \quad (7.63)$$

The capacitance of the diode, from Equation (7.62), can be written in the form

$$C = C_0(V_{bi} + V_R)^{-1/(m+2)} \quad (7.64)$$

In a circuit application, we would, in general, like to have the resonant frequency be a linear function of reverse-bias voltage V_R , so we need

$$C \propto V^{-2} \quad (7.65)$$

From Equation (7.64), the parameter m required is found from

$$\frac{1}{m+2} = 2 \quad (7.66a)$$

or

$$m = -\frac{3}{2} \quad (7.66b)$$

A specific doping profile will yield the desired capacitance characteristic.

7.5 | SUMMARY

- A uniformly doped pn junction was initially considered, in which one region of a semiconductor is uniformly doped with acceptor impurities and the adjacent region is uniformly doped with donor impurities. This type of junction is called a homojunction.
- A space charge region, or depletion region, is formed on either side of the metallurgical junction separating the n and p regions. This region is essentially depleted of any mobile electrons or holes. A net positive charge density, due to the positively charged donor impurity ions, exists in the n region and a net negative charge density, due to the negatively charged acceptor impurity ions, exists in the p region.
- An electric field exists in the depletion region due to the net space charge density. The direction of the electric field is from the n region to the p region.
- A potential difference exists across the space-charge region. Under zero applied bias, this potential difference, known as the built-in potential barrier, maintains thermal equilibrium and holds back majority carrier electrons in the n-region and majority carrier holes in the p region.
- An applied reverse bias voltage (n region positive with respect to the p region) increases the potential barrier, increases the space charge width, and increases the magnitude of the electric field.
- As the reverse bias voltage changes, the amount of charge in the depletion region changes. This change in charge with voltage defines the junction capacitance.
- The linearly graded junction represents a nonuniformly doped pn junction. Expressions for the electric field, built-in potential barrier, and junction capacitance were derived. The functional relationships differ from those of the uniformly doped junction.
- Specific doping profiles can be used to obtain specific capacitance characteristics. A hyperabrupt junction is one in which the doping decreases away from the metallurgical junction. This type of junction is advantageous in varactor diodes that are used in resonant circuits.

GLOSSARY OF IMPORTANT TERMS

abrupt junction approximation The assumption that there is an abrupt discontinuity in space charge density between the space charge region and neutral semiconductor region.

built-in potential barrier The electrostatic potential difference between the p and n regions of a pn junction in thermal equilibrium.

depletion layer capacitance Another term for junction capacitance.

depletion region Another term for space charge region.

hyperabrupt junction A pn junction in which the doping concentration on one side decreases away from the metallurgical junction to achieve a specific capacitance-voltage characteristic.

junction capacitance The capacitance of the pn junction under reverse bias.

linearly graded junction A pn junction in which the doping concentrations on either side of the metallurgical junction are approximated by a linear distribution.

metallurgical junction The interface between the p- and n-doped regions of a pn junction.

one-sided junction A pn junction in which one side of the junction is much more heavily doped than the adjacent side.

reverse bias The condition in which a positive voltage is applied to the n region with respect to the p region of a pn junction so that the potential barrier between the two regions increases above the thermal-equilibrium built-in potential barrier.

space charge region The region on either side of the metallurgical junction in which there is a net charge density due to ionized donors in the n-region and ionized acceptors in the p region.

space charge width The width of the space charge region, a function of doping concentrations and applied voltage.

varactor diode A diode whose reactance can be varied in a controlled manner with bias voltage.

CHECKPOINT

After studying this chapter, the reader should have the ability to:

- Describe why and how the space charge region is formed.
- Draw the energy band diagram of a zero-biased and reverse-biased pn junction.
- Define and derive the expression of the built-in potential barrier voltage.
- Derive the expression for the electric field in space charge region of the pn junction.
- Describe what happens to the parameters of the space charge region when a reverse bias voltage is applied.
- Define and explain the junction capacitance.
- Describe the characteristics and properties of a one-sided pn junction.
- Describe how a linearly graded junction is formed.
- Define a hyperabrupt junction.

REVIEW QUESTIONS

1. Define the built-in potential voltage and describe how it maintains thermal equilibrium.
2. Why is an electric field formed in the space charge region? Why is the electric field a linear function of distance in a uniformly doped pn junction?
3. Where does the maximum electric field occur in the space charge region?
4. Why is the space charge width larger in the lower doped side of a pn junction?
5. What is the functional dependence of the space charge width on reverse bias voltage?
6. Why does the space charge width increase with reverse bias voltage?
7. Why does a capacitance exist in a reverse-biased pn junction? Why does the capacitance decrease with increasing reverse bias voltage?

8. What is a one-sided pn junction? What parameters can be determined in a one-sided pn junction?
9. What is a linearly graded junction?
10. What is a hyperabrupt junction and what is one advantage or characteristic of such a junction?

PROBLEMS

Section 7.2 Zero Applied Bias

-  **7.1** (a) Calculate V_{bi} in a silicon pn junction at $T = 300$ K for (a) $N_d = 10^{15} \text{ cm}^{-3}$ and $N_a = (i) 10^{15}, (ii) 10^{16}, (iii) 10^{17}, (iv) 10^{18} \text{ cm}^{-3}$. (b) Repeat part (a) for $N_d = 10^{18} \text{ cm}^{-3}$.
-  **7.2** Calculate the built-in potential barrier, V_{bi} , for Si, Ge, and GaAs pn junctions if they each have the following dopant concentrations at $T = 300$ K:
- (a) $N_d = 10^{14} \text{ cm}^{-3}$ $N_a = 10^{17} \text{ cm}^{-3}$
 - (b) $N_d = 5 \times 10^{16}$ $N_a = 5 \times 10^{16}$
 - (c) $N_d = 10^{17}$ $N_a = 10^{17}$
-  **7.3** (a) Plot the built-in potential barrier for a symmetrical ($N_a = N_d$) silicon pn junction at $T = 300$ K over the range $10^{14} \leq N_a = N_d \leq 10^{19} \text{ cm}^{-3}$. (b) Repeat part (a) for a GaAs pn junction.
-  **7.4** Consider a uniformly doped GaAs pn junction with doping concentrations of $N_a = 5 \times 10^{18} \text{ cm}^{-3}$ and $N_d = 5 \times 10^{16} \text{ cm}^{-3}$. Plot the built-in potential barrier voltage, V_{bi} , versus temperature for $200 \leq T \leq 500$ K.
-  **7.5** An abrupt silicon pn junction at zero bias has dopant concentrations of $N_a = 10^{17} \text{ cm}^{-3}$ and $N_d = 5 \times 10^{15} \text{ cm}^{-3}$, $T = 300$ K. (a) Calculate the Fermi level on each side of the junction with respect to the intrinsic Fermi level. (b) Sketch the equilibrium energy-band diagram for the junction and determine V_{bi} from the diagram and the results of part (a). (c) Calculate V_{bi} using Equation (7.10), and compare the results to part (b). (d) Determine x_n , x_p , and the peak electric field for this junction.
-  **7.6** Repeat problem 7.5 for the case when the doping concentrations are $N_a = N_d = 2 \times 10^{16} \text{ cm}^{-3}$.
-  **7.7** A silicon abrupt junction in thermal equilibrium at $T = 300$ K is doped such that $E_c - E_F = 0.21 \text{ eV}$ in the n region and $E_F - E_v = 0.18 \text{ eV}$ in the p region. (a) Draw the energy band diagram of the pn junction. (b) Determine the impurity doping concentrations in each region. (c) Determine V_{bi} .
-  **7.8** Consider the uniformly doped GaAs junction at $T = 300$ K. At zero bias, only 20 percent of the total space charge region is to be in the p region. The built-in potential barrier is $V_{bi} = 1.20 \text{ V}$. For zero bias, determine (a) N_a , (b) N_d , (c) x_n , (d) x_p , and (e) E_{max} .
-  **7.9** Consider the impurity doping profile shown in Figure 7.16 in a silicon pn junction. For zero applied voltage, (a) determine V_{bi} , (b) calculate x_n and x_p , (c) sketch the thermal equilibrium energy band diagram, and (d) plot the electric field versus distance through the junction.

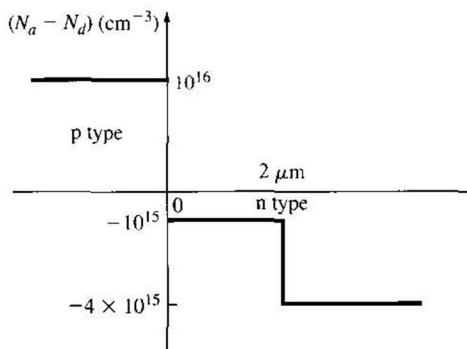


Figure 7.16 | Figure for Problem 7.9.

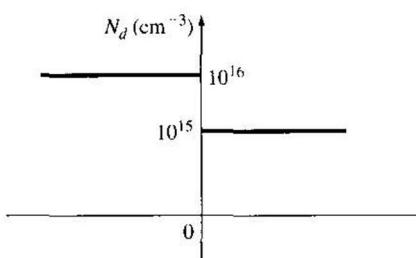


Figure 7.17 | Figure for Problem 7.12.

- *7.10** A uniformly doped silicon pn junction is doped to levels of $N_d = 5 \times 10^{15} \text{ cm}^{-3}$ and $N_a = 10^{16} \text{ cm}^{-3}$. The measured built-in potential barrier is $V_{bi} = 0.40 \text{ V}$. Determine the temperature at which this result occurs. (You may have to use trial and error to solve this problem.)
- 7.11** Consider a uniformly doped silicon pn junction with doping concentrations $N_a = 5 \times 10^{17} \text{ cm}^{-3}$ and $N_d = 10^{17} \text{ cm}^{-3}$. (a) Calculate V_{bi} at $T = 300 \text{ K}$. (b) Determine the temperature at which V_{bi} decreases by 1 percent.
- 7.12** An “isotype” step junction is one in which the same impurity type doping changes from one concentration value to another value. An n-n isotype doping profile is shown in Figure 7.17. (a) Sketch the thermal equilibrium energy band diagram of the isotype junction. (b) Using the energy band diagram, determine the built-in potential barrier. (c) Discuss the charge distribution through the junction.
- 7.13** A particular type of junction is an n region adjacent to an intrinsic region. This junction can be modeled as an n-type region to a lightly doped p-type region. Assume the doping concentrations in silicon at $T = 300 \text{ K}$ are $N_d = 10^{16} \text{ cm}^{-3}$ and $N_a = 10^{12} \text{ cm}^{-3}$. For zero applied bias, determine (a) V_{bi} , (b) x_n , (c) x_p , and (d) $|E_{max}|$. Sketch the electric field versus distance through the junction.
- 7.14** We are assuming an abrupt depletion approximation for the space charge region. That is, no free carriers exist within the depletion region and the semiconductor abruptly changes to a neutral region outside the space charge region. This approximation is adequate for most applications, but the abrupt transition does not exist. The space charge region changes over a distance of a few Debye lengths, where the Debye length in the n region is given by

$$L_D = \left[\frac{\epsilon_s k T}{e^2 N_d} \right]^{1/2}$$

Calculate L_D and find the ratio of L_D/x_n for the following conditions. The p-type doping concentration is $N_a = 8 \times 10^{17} \text{ cm}^{-3}$ and the n-type doping concentration is (a) $N_d = 8 \times 10^{14} \text{ cm}^{-3}$, (b) $N_d = 2.2 \times 10^{16} \text{ cm}^{-3}$, and (c) $N_d = 8 \times 10^{17} \text{ cm}^{-3}$.

- 7.15** Examine how the electric field versus distance through a uniformly doped pn junction varies as the doping concentrations vary. For example, consider $N_d = 10^{18} \text{ cm}^{-3}$ and let $10^{14} \leq N_a \leq 10^{18} \text{ cm}^{-3}$, then consider $N_d = 10^{14} \text{ cm}^{-3}$ and let



$10^{14} \leq N_a \leq 10^{18} \text{ cm}^{-3}$, and finally consider $N_d = 10^{16} \text{ cm}^{-3}$ and let $10^{14} \leq N_a \leq 10^{18} \text{ cm}^{-3}$. What can be said about the results for $N_a \geq 100N_d$ or $N_d \geq 100N_a$? Assume zero applied bias.

Section 7.3 Reverse Applied Bias

- 7.16** An abrupt silicon pn junction has dopant concentrations of $N_a = 2 \times 10^{16} \text{ cm}^{-3}$ and $N_d = 2 \times 10^{15} \text{ cm}^{-3}$ at $T = 300 \text{ K}$. Calculate (a) V_{bi} , (b) W at $V_R = 0$ and $V_R = 8 \text{ V}$, and (c) the maximum electric field in the space charge region at $V_R = 0$ and $V_R = 8 \text{ V}$.
- 7.17** Consider the junction described in Problem 7.11. The junction has a cross-sectional area of 10^{-4} cm^2 and has an applied reverse-bias voltage of $V_R = 5 \text{ V}$. Calculate (a) V_{bi} , (b) x_n, x_p, W , (c) E_{max} , and (d) the total junction capacitance.
- 7.18** An ideal one-sided silicon n⁺p junction has uniform doping on both sides of the abrupt junction. The doping relation is $N_d = 50N_a$. The built-in potential barrier is $V_{bi} = 0.752 \text{ V}$. The maximum electric field in the junction is $E_{max} = 1.14 \times 10^5 \text{ V/cm}$ for a reverse-bias voltage of 10 V. $T = 300 \text{ K}$. Determine (a) N_a, N_d (b) x_p for $V_R = 10$, and (c) C'_j for $V_R = 10$.
- 7.19** A silicon n⁺p junction is biased at $V_R = 10 \text{ V}$. Determine the percent change in (a) junction capacitance and (b) built-in potential if the doping in the p region increases by a factor of 2.
- 7.20** Consider two p⁺n silicon junctions at $T = 300 \text{ K}$ reverse biased at $V_R = 5 \text{ V}$. The impurity doping concentrations in junction A are $N_a = 10^{18} \text{ cm}^{-3}$ and $N_d = 10^{15} \text{ cm}^{-3}$, and those in junction B are $N_a = 10^{18} \text{ cm}^{-3}$ and $N_d = 10^{16} \text{ cm}^{-3}$. Calculate the ratio of the following parameters for junction A to junction B: (a) W , (b) $|E_{max}|$, and (c) C'_j .
- 7.21** (a) The peak electric field in a reverse-biased silicon pn junction is $|E_{max}| = 3 \times 10^5 \text{ V/cm}$. The doping concentrations are $N_d = 4 \times 10^{15} \text{ cm}^{-3}$ and $N_a = 4 \times 10^{17} \text{ cm}^{-3}$. Find the magnitude of the reverse-bias voltage. (b) Repeat part (a) for $N_d = 4 \times 10^{16} \text{ cm}^{-3}$ and $N_a = 4 \times 10^{17} \text{ cm}^{-3}$. (c) Repeat part (a) for $N_d = N_a = 4 \times 10^{17} \text{ cm}^{-3}$.
- 7.22** Consider a uniformly doped GaAs pn junction at $T = 300 \text{ K}$. The junction capacitance at zero bias is $C_j(0)$ and the junction capacitance with a 10-V reverse-bias voltage is $C_j(10)$. The ratio of the capacitances is

$$\frac{C_j(0)}{C_j(10)} = 3.13$$

Also under reverse bias, the space charge width into the p region is 0.2 of the total space charge width. Determine (a) V_{bi} and (b) N_a, N_d .

- 7.23** GaAs pn junction at $T = 300 \text{ K}$ has impurity doping concentrations of $N_a = 10^{16} \text{ cm}^{-3}$ and $N_d = 5 \times 10^{16} \text{ cm}^{-3}$. For a particular device application, the ratio of junction capacitances at two values of reverse bias voltage must be $C'_j(V_{R1})/C'_j(V_{R2}) = 3$ where the reverse bias voltage $V_{R1} = 1 \text{ V}$. Determine V_{R2} .
- 7.24** An abrupt silicon pn junction at $T = 300 \text{ K}$ is uniformly doped with $N_a = 10^{18} \text{ cm}^{-3}$ and $N_d = 10^{15} \text{ cm}^{-3}$. The pn junction area is $6 \times 10^{-4} \text{ cm}^2$. An inductance of 2.2 millihenry is placed in parallel with the pn junction. Calculate the resonant frequency of the circuit for reverse-bias voltages of (a) $V_R = 1 \text{ V}$ and (b) $V_R = 10 \text{ V}$.

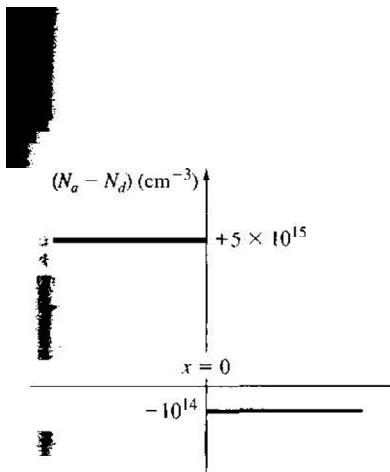


Figure 7.18 | Figure for Problem 7.27.

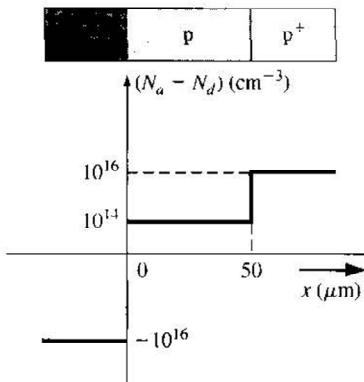


Figure 7.19 | Figure for Problem 7.28.

- 7.25** A uniformly doped silicon p⁺n junction at $T = 300 \text{ K}$ is to be designed such that at a reverse-bias voltage of $V_R = 10 \text{ V}$, the maximum electric field is limited to $E_{\max} = 10^6 \text{ V/cm}$. Determine the maximum doping concentration in the n region.
- 7.26** A silicon pn junction is to be designed which meets the following specifications at $T = 300 \text{ K}$. At a reverse-bias voltage of 1.2 V , 10 percent of the total space charge region is to be in the n region and the total junction capacitance is to be $3.5 \times 10^{-12} \text{ F}$ with a cross-sectional area of $5.5 \times 10^{-4} \text{ cm}^2$. Determine (a) N_a , (b) N_d , and (c) V_{bi} .
- 7.27** A silicon pn junction at $T = 300 \text{ K}$ has the doping profile shown in Figure 7.18. Calculate (a) V_{bi} , (b) x_n and x_p at zero bias, and (c) the applied bias required so that $x_n = 30 \mu\text{m}$.
- 7.28** Consider a silicon pn junction with the doping profile shown in Figure 7.19. $T = 300 \text{ K}$. (a) Calculate the applied reverse-bias voltage required so that the space charge region extends entirely through the p region. (b) Determine the space charge width into the n⁺-region with the reverse-bias voltage calculated in part (a). (c) Calculate the peak electric field for this applied voltage.
- 7.29** (a) A silicon p⁺n junction has doping concentrations of $N_a = 10^{18} \text{ cm}^{-3}$ and $N_d = 5 \times 10^{15} \text{ cm}^{-3}$. The cross-sectional area of the junction is $A = 5 \times 10^{-5} \text{ cm}^2$. Calculate the junction capacitance for (i) $V_R = 0$, (ii) $V_R = 3 \text{ V}$, and (iii) $V_R = 6 \text{ V}$. Plot $1/C^2$ versus V_R . Show that the slope of the curve can be used to find N_d and that the intersection with the voltage axis yields V_{bi} . (b) Repeat part (a) if the n-type doping concentration changes to $N_d = 6 \times 10^{16} \text{ cm}^{-3}$.
- 7.30** The total junction capacitance of a one-sided silicon pn junction at $T = 300 \text{ K}$ is measured at $V_R = 50 \text{ mV}$ and found to be 1.3 pF . The junction area is 10^{-5} cm^2 and $V_{bi} = 0.95 \text{ V}$. (a) Find the impurity doping concentration of the low-doped side of the junction. (b) Find the impurity doping concentration of the higher-doped region.
- 7.31** Examine how the capacitance C' and the function $(1/C')^2$ vary with reverse-bias voltage V_R as the doping concentrations change. In particular, consider these plots versus N_a for $N_a \geq 100N_d$ and versus N_d for $N_d \geq 100N_a$.
- *7.32** A pn junction has the doping profile shown in Figure 7.20. Assume that $x_n > x_0$ for all reverse-bias voltages. (a) What is the built-in potential across the junction? (b) For

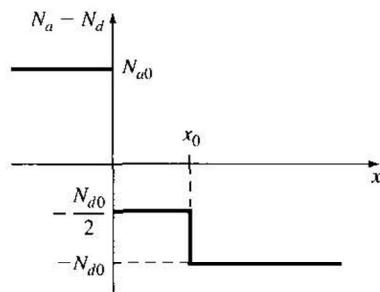


Figure 7.20 | Figure for Problem 7.32.

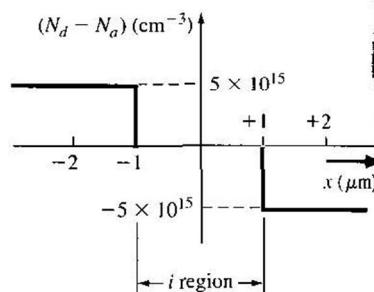


Figure 7.21 | Figure for Problem 7.33.

the abrupt junction approximation, sketch the charge density through the junction.

(c) Derive the expression for the electric field through the space charge region.

- *7.33 A silicon PIN junction has the doping profile shown in Figure 7.21. The "I" corresponds to an ideal intrinsic region in which there is no impurity doping concentration. A reverse-bias voltage is applied to the PIN junction so that the total depletion width extends from $-2 \mu\text{m}$ to $+2 \mu\text{m}$. (a) Using Poisson's equation, calculate the magnitude of the electric field at $x = 0$. (b) Sketch the electric field through the PIN junction. (c) Calculate the reverse-bias voltage that must be applied.

Section 7.4 Nonuniformly Doped Junctions

- 7.34 Consider a linearly graded junction. (a) Starting with Equation (7.49), derive the expression for the electric field given in Equation (7.51). (b) Derive the expression for the potential through the space charge region given by Equation (7.53).
- 7.35 The built-in potential barrier of a linearly graded silicon pn junction at $T = 300 \text{ K}$ is $V_{bi} = 0.70 \text{ V}$. The junction capacitance measured at $V_R = 3.5 \text{ V}$ is $C' = 7.2 \times 10^{-9} \text{ F/cm}^2$. Find the gradient, a , of the net impurity concentration.

Summary and Review



- 7.36 A one-sided p⁺n silicon diode at $T = 300 \text{ K}$ is doped at $N_o = 10^{18} \text{ cm}^{-3}$. Design the junction so that $C_j = 0.95 \text{ pF}$ at $V_R = 3.5 \text{ V}$. Calculate the junction capacitance when $V_R = 1.5 \text{ V}$.
- 7.37 A one-sided p⁺n junction with a cross-sectional area of 10^{-5} cm^2 has a measured built-in potential of $V_{bi} = 0.8 \text{ V}$ at $T = 300 \text{ K}$. A plot of $(1/C_j)^2$ versus V_R is linear for $V_R < 1 \text{ V}$ and is essentially constant for $V_R > 1 \text{ V}$. The capacitance is $C_j = 0.082 \text{ pF}$ at $V_R = 1 \text{ V}$. Determine the doping concentrations on either side of the metallurgical junction that will produce this capacitance characteristic.
- *7.38 Silicon, at $T = 300 \text{ K}$, is doped at $N_{d1} = 10^{15} \text{ cm}^{-3}$ for $x < 0$ and $N_{d2} = 5 \times 10^{16} \text{ cm}^{-3}$ for $x > 0$ to form an n-n step junction. (a) Sketch the energy-band diagram. (b) Derive an expression for V_{bi} . (c) Sketch the charge density, electric field, and potential through the junction. (d) Explain where the charge density came from and is located.
- *7.39 A diffused silicon pn junction has a linearly graded junction on the p side with $a = 2 \times 10^{19} \text{ cm}^{-4}$, and a uniform doping of 10^{15} cm^{-3} on the n side. (a) If the

depletion width on the p side is $0.7 \mu\text{m}$ at zero bias, find the total depletion width, built-in potential, and maximum electric field at zero bias. (b) Plot the potential function through the junction.

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