Multi-carrier Modulation and OFDM

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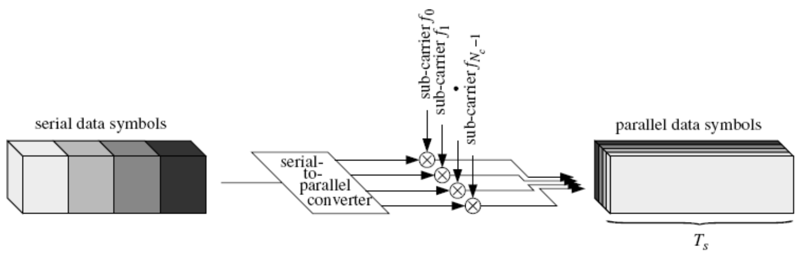
Multi-carrier systems: basic idea

- Typical mobile radio channel is a fading channel that is *flat* or *frequency selective*
- For high bandwidth applications channel is frequency selective and delay spread dictates throughput
- Multicarrier modulation is a technique where multiple low data rate carriers are combined by a transmitter to form a composite high data rate transmission
- In a classic multi-carrier system, the available spectrum is split into several non-overlapping frequency sub channels. The individual data elements are modulated into these sub channels and are thus frequency multiplexed

Multi-carrier transmission

- Converts a high-data rate bit stream into multiple lower-data rate substreams
- Each substream is modulated onto a different carrier



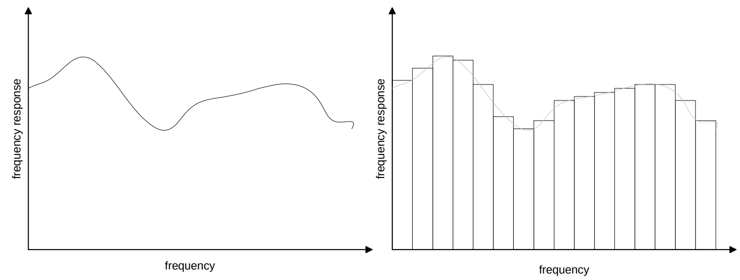
(Fazel and Kaiser, 2003)

Advantage

- Since symbols are transmitted at a lower rate, the effects of delay spread are reduced
  - Reduced inter-symbol interference
- This in turn reduces the complexity of the equalizer

Robustness to delay spread

- MC-modulation increases the symbol time by modulating into narrow sub-channels



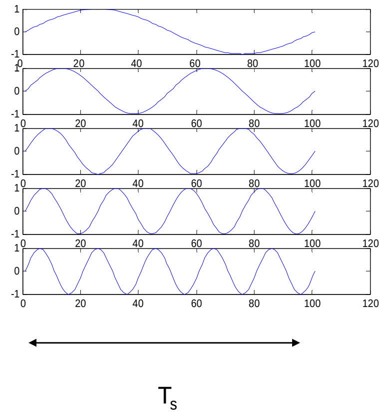
Channel frequency responses for a single carrier and multicarrier system. In the multicarrier system each sub channel only undergoes slight distortion

Orthogonal Frequency Division Multiplexing

- In classic multicarrier system guard bands have to be inserted, resulting in poor spectral efficiency
- A more efficient approach is to allow the spectra of individual subcarriers to overlap
- Problem: If individual subcarriers are overlapping isn’t there interference between carriers?
- Answer: No! If subcarrier tones are separated by the inverse of the signaling symbol duration, independent separation of frequency multiplexed tones is possible
  - This ensures that the spectra of individual sub channels are zeros at other subcarrier frequencies

OFDM carrier

- Orthogonal waveforms are generated by using signals that have integer number of cycles in the duration T_s



Subcarriers in OFDM

OFDM symbols

- Consider N_c complex-valued source symbols:

$$S_n,\ n = 0, 1, \ldots, N_c - 1$$

- These symbols are transmitted in parallel using N_c sub-carriers
  - All of these symbols combined are referred to as an OFDM symbol
- If the source symbol duration is T_d, then the OFDM symbol duration is $T_s = N_c T_d$
- The N_c sub-carriers have a spacing of

$$f = \frac{1}{T_s}$$

OFDM carriers

- The baseband information of the kth carrier can be expressed as

$$\underbrace{(x_k + jy_k)}_{\text{data symbol}}\underbrace{(\cos 2\pi kft + j\sin 2\pi kf)}_{k^{th}\text{ carrier}}$$

- The OFDM signal is the sum of all the signals in each of its subcarriers which can be written as (usually implemented using IFFT)

$$s(t) = \sum_{k=0}^{N_c-1} (x_k + jy_k)(\cos 2\pi kft + j\sin 2\pi kf)$$

Recovering the individual symbols

- The individual modulated symbols at the receiver are recovered using the FFT
- The *k*th output from the FFT is:

$$z_k = \int_0^{T_s} s(t)(\cos 2\pi kft - j\sin 2\pi kf)dt =$$

$$\sum_{n=0}^{N_c-1}\left\{\int_0^{T_s}(x_n + jy_n)\cos 2\pi nf(\cos 2\pi kft - j\sin 2\pi kf)dt + j\int_0^{T_s}(x_n + y_n)\sin 2\pi nf(\cos 2\pi kft - j\sin 2\pi kf)dt\right\}$$

Solving these integrals…

- Trigonometry reminder:

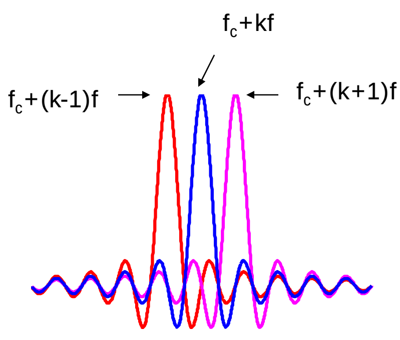
$$2\cos A\cos B = \cos(A-B) + \cos(A+B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

$$2\sin A\cos B = \sin(A-B) + \sin(A+B)$$

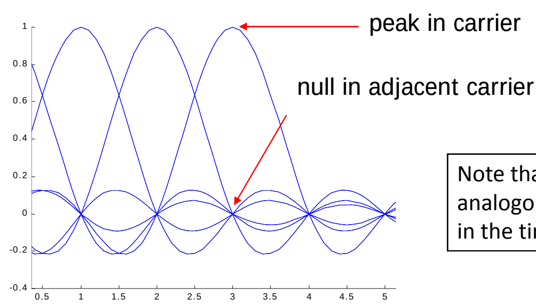
OFDM spectrum

- The individual spectra of the subcarriers are sinc functions
- Zero crossings occur at every integer multiple of *f* and hence no Inter-Carrier Interference occurs in the frequency domain
- Note the analogy with time-domain sinc pulses



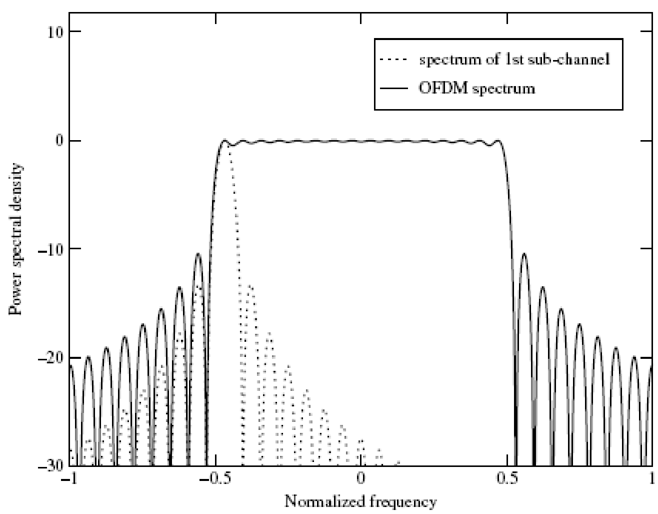
Spectral efficiency

- For *N* sub-carriers, the bandwidth of conventional FDM is 2*N*/*T* while that of OFDM is (*N*+1)/*T*. By allowing the sub-carrier spectra to overlap, OFDM improves the spectral efficiency.



Note that this is analogous to sinc pulses in the time-domain

Power spectral density example (N_c = 16)



Guard interval

- As N_c increases, the OFDM symbol duration T_s becomes large as compared to the duration of the impulse response τ_{max} of the channel
- To completely eliminate ISI, must add a guard interval $T_g \geq \tau_{max}$
- The new duration of the OFDM symbol is then

$$T_s' = T_s + T_g$$

Sampled sequence with cyclic guard extension

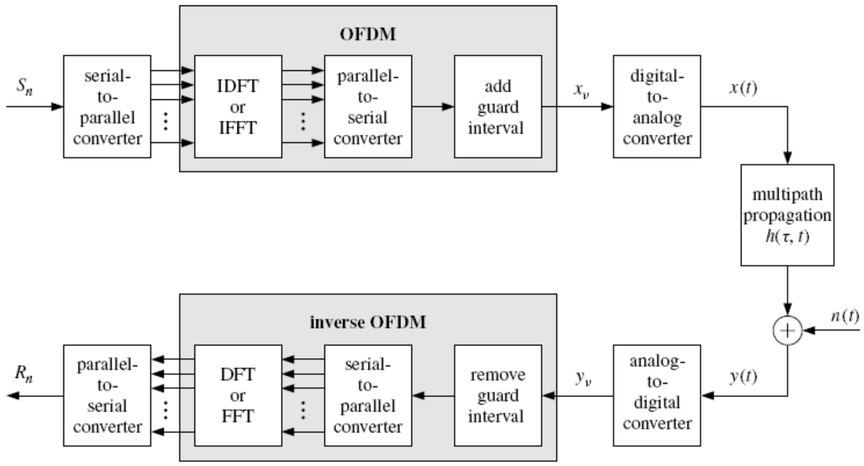
- The length of the guard interval L_g must be

$$L_g \geq \left\lceil \frac{\tau_{\max} N_c}{T_s} \right\rceil$$

- The sampled signal with the guard extension becomes

$$x_v = \frac{1}{N_c} \sum_{n=0}^{N_c - 1} S_n e^{j2\pi n v / N_c}, \; v = -L_g, \ldots, N_c - 1$$

OFDM transmitter and receiver



Matrix notation

- Complex-valued source symbols, transmitted in parallel as an OFDM symbol

$$\mathbf{s} = \begin{pmatrix} S_0 & S_1 & \dots & S_{N_c - 1} \end{pmatrix}^T$$

- N_c x N_c channel matrix

$$\mathbf{H} = \begin{pmatrix} H_{0,0} & 0 & \dots & 0 \\ 0 & H_{1,1} & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & H_{N_c-1, N_c-1} \end{pmatrix}$$

(Why is this a diagonal matrix?)

Matrix notation (cont'd)

- Additive noise

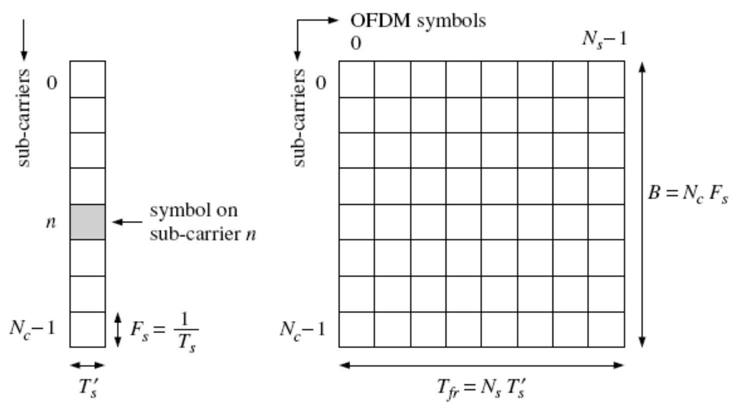
$$\mathbf{n} = \left(N_0 \quad N_1 \quad \ldots \quad N_{N_c-1}\right)^T$$

- Received signals

$$\mathbf{r} = \left(R_0 \quad R_1 \quad \ldots \quad R_{N_c-1}\right)^T$$

$$\mathbf{r} = \mathbf{Hs} + \mathbf{n}$$

An OFDM frame



(a) OFDM symbol (b) OFDM frame

OFDM advantages

- High spectral efficiency for large number of subcarriers: nearly rectangular frequency-domain representation of signal
- Low-complexity receivers: due to low ICI and ISI if guard interval is long enough
- Flexible spectrum adaptation: good for DSA
- Different modulation can be applied to different subcarriers to suit the transmission conditions on each subcarrier

OFDM disadvantages

- High peak-to-average power ratio (PAPR): requires highly linear power amplifiers
- Some loss of spectral efficiency due to guard interval
- Average frequency and time synchronization is required
- More sensitive to Doppler spread than single-carrier systems

OFDM example: DVB-T

Bandwidth	8 MHz	
# of carriers	1705 (2k FFT)	6817 (8k FFT)
Symbol duration T_s	224 μs	896 μs
Carrier spacing F_s	4.464 kHz	1.116 kHz
Guard time T_g	$T_s/32$, $T_s/16$, $T_s/8$, $T_s/4$	
Modulation	QPSK, 16-QAM, 64-QAM	
FEC coding	Reed Solomon + convolutional with code rate ½ up to 7/8	
Max. data rate	31.7 Mbps	

OFDM example: IEEE 802.11a

Bandwidth	20 MHz
# of carriers	52 (64 FFT)
Symbol duration T_s	4 μs
Carrier spacing F_s	312.5 kHz
Guard time T_g	0.8 μs
Modulation	BPSK, QPSK, 16-QAM, 64-QAM
FEC coding	Convolutional code with rate ½ up to 3/4
Max. data rate	54 Mbps

OFDM example: IEEE 802.16a

Bandwidth	From 1.5 to 28 MHz
# of carriers	256
Symbol duration T_s	from 8 to 125 μs (depending on the bandwidth)
Guard time T_g	from 1/32 up to ¼ of T_s
Modulation	QPSK, 16-QAM, 64-QAM
FEC coding	Reed Solomon + convolutional code with rate ½ up to 5/6

OFDM applications

- Wireline
  - Asymmetric Digital Subscriber Loop (ADSL)
- Wireless
  - Digital Audio Broadcasting (DAB)
  - Digital Video Broadcasting-Terrestrial (DVB-T)
  - Integrated Services Digital Broadcasting-Terrestrial (ISDB-T)
  - Wireless LAN (IEEE 802.11(a), HiperLAN/2)
  - Wireless MAN (IEEE 802.16 a/b)