Unit: 3 ⇒ Finite Impulse Response Filters.

Adv:

→ FIR always stable.

→ FIR exactly linear phase & easily designed.

→ can be realized both recursive & non-recursive.

→ Free from limit cycle oscillations.

→ Excellent methods available for FIR design.

Disadv:-

→ the implementation of narrow transition band FIR filters are very costly, as it requires considerably more arithmetic operations and hardware components such as multipliers, adders and delay elements.

→ More memory required and execution time are very high.

Linear phase FIR Filters:-

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}, \quad h(n) \Rightarrow \text{Impulse response.}$$

The Fourier transform of $h(n)$ is

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n) e^{-j\omega n}$$ which is periodic in frequency with period "2π".

$$H(e^{j\omega}) = \pm \underbrace{|H(e^{j\omega})|}_{\text{Magnitude response}} \, e^{j\theta(\omega)}$$ ↳ phase response.

$$\text{Group delay} = \tau_g = \frac{-d\theta(\omega)}{d\omega} \qquad \text{phase delay} = \tau_p = \frac{-\theta(\omega)}{\omega}$$

→ For FIR Filter with linear phase we can define

$$\theta(\omega) = -\alpha\omega, \quad -\pi \le \omega \le \pi.$$

α = constant phase delay in samples.

$$\sum_{n=0}^{N-1} h(n) e^{-j\omega n} = \pm |H(e^{j\omega})| e^{j\theta(\omega)}.$$

which gives.

$$\sum_{n=0}^{N-1} h(n) \cos\omega n = \pm |H(e^{j\omega})| \cos\theta(\omega) \rightarrow (1)$$

and

$$-\sum_{n=0}^{N-1} h(n) \sin\omega n = \pm |H(e^{j\omega})| \sin\theta(\omega). \rightarrow (2)$$

By taking ratio of (2) & (1).

$$\frac{\sum_{n=0}^{N-1} h(n) \sin\omega n}{\sum_{n=0}^{N-1} h(n) \cos\omega n} = \frac{\sin\alpha\omega}{\cos\alpha\omega} \qquad \boxed{\because \theta(\omega) = -\alpha\omega.}$$

After simplifying

$$\sum_{n=0}^{N-1} h(n) \sin(\alpha - n)\omega = 0 \Rightarrow \text{which will be "zero"}$$

when

$$h(n) = h(N-1-n) \quad \text{and } \alpha = \frac{N-1}{2}.$$

∴ FIR filter will have constant phase and group delay when the impulse response is $\boxed{\text{symmetrical}}$ about $\alpha = \frac{N-1}{2}$.

$$h(n) = -h(N-1-n), \quad \alpha = \frac{N-1}{2} \rightarrow \tau_g \text{ constant}$$

for $\boxed{\text{antisymmetrical}}$ $\quad \tau_p$ = not constant.

Case I:- symmetrical, N=odd.

$$H'(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n$$

$$\theta(\omega) = -\alpha\omega = -\left(\frac{N-1}{2}\right)\omega.$$

$$a(0) = h\left(\frac{N-1}{2}\right), \quad a(n) = 2h\left(\frac{N-1}{2} - n\right)$$

case II:- symmetrical, N= Even

$$H'(e^{j\omega}) = \sum_{n=1}^{N/2} b(n) \cos(n - 1/2)\omega$$

$$\theta(\omega) = -\alpha\omega = -\left(\frac{N-1}{2}\right)\omega$$

$$b(n) = 2h\left(\frac{N}{2} - n\right)$$

case III:- Antisymmetry, N=ODD.

$$H'(e^{j\omega}) = H'(e^{j\omega})\, e^{-j\omega(N-1)/2}\, e^{j\pi/2} = \sum_{n=1}^{\frac{N-1}{2}} c(n) \sin \omega n$$

$$\theta(\omega) = \pi/2 - \alpha\omega = \pi/2 - \left(\frac{N-1}{2}\right)\omega.$$

case IV:- Antisymmetry, N= Even

$$H'(e^{j\omega}) = \sum_{n=1}^{N/2} d(n) \sin \omega(n - 1/2)$$

$$\theta(\omega) = \pi/2 - \alpha\omega, \quad d(n) = 2h\left(\frac{N}{2} - n\right)$$

Design an ideal LPF with a freq respn

$$H_d(e^{j\omega}) = 1, \text{ for } -\pi/2 \le \omega \le \pi/2$$
$$= 0, \text{ for } \pi/2 \le |\omega| \le \pi. \text{ Find the}$$

values of h(n) for N=11, Find H(z). plot the magnitude response.

$H_d(e^{j\omega}) \Rightarrow$ freq response of FIR filter.

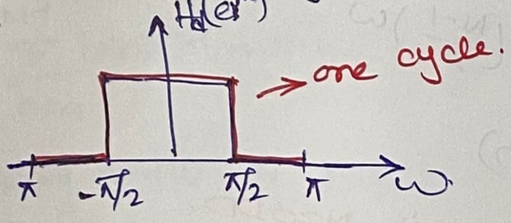
$h_d(n) \Rightarrow$ Fourier coefficients.

For a symmetrical impulse response having symmetry at $n=0$.

$$h(n) = h(-n).$$

soln:

→ The freq response of LPF with $\omega_c = \pi/2$ is shown in fig.



$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n) e^{-j\omega n}$$

Impulse response $h_d(n)$ can be found by.

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega.$$

$$= \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} e^{j\omega n} d\omega.$$

$$= \frac{1}{2\pi j n} e^{j\omega n} \Big|_{-\pi/2}^{\pi/2} = \frac{1}{n\pi(2j)} \left[e^{j\pi n/2} - e^{-j\pi n/2} \right]$$

$$h_d(n) = \frac{\sin \frac{\pi}{2} n}{n\pi}. \quad -\infty \le n \le \infty. \quad \rightarrow (1)$$

Truncating (1) in to '11' samples, we have.

$$h(n) = \frac{\sin \frac{\pi}{2} n}{\pi n}, \text{ for } |n| \le \left(\frac{N-1}{2}\right) 5 \quad \rightarrow (2).$$

$$= 0, \text{ otherwise}$$

Need to find '11' values of h(n).

for $n=0$. h(n) becomes indeterminate

$$h(0) = \lim_{n\to 0} \frac{\sin \frac{\pi}{2} n}{n\pi} = \frac{1}{2} \lim_{n\to 0} \frac{\sin \frac{\pi}{2} n}{\frac{\pi n}{2}} = \frac{1}{2}$$

$$\boxed{\because \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1}$$

(or)

$$h(0) = h_d(0) = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} d\omega = \frac{1}{2\pi} \omega \Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{\pi}{2\pi} = \frac{1}{2} = 0.5.$$

$h(n) = h(-n)$ for symmetry

for $n=1$

$$h(1) = h(-1) = \frac{\sin \pi/2}{\pi} = \frac{1}{\underset{(3.14)}{\pi}} = 0.3183.$$

$h(2) = h(-2) = 0$. $h(3) = h(-3) = \frac{\sin 3\pi/2}{3\pi} = -\frac{1}{3\pi} = -0.106$.

$h(4) = h(-4) = 0$, $h(5) = h(-5) = \frac{1}{5\pi} = 0.063$

The transfer function of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} [h(n)(z^n + z^{-n})]$$

$$= 0.5 + \sum_{n=1}^{5} [h(n)(z^n + z^{-n})]$$

$$= 0.5 + 0.3183(z^1 + z^{-1}) - 0.106(z^3 + z^{-3})$$
$$+ 0.063(z^5 + z^{-5})$$

It not a realizable filter physically.

convert to realizable filter by multiplying it by $z^{-\alpha}$, $\alpha = \frac{N-1}{2}$. (i.e) $z^{-(\frac{N-1}{2})}$ Here z^{-5}.

$H'(z) = z^{-5} H(z)$

$= z^{-5}[0.5 + 0.3183(z+z^{-1}) - 0.106(z^{3}+z^{-3}) + 0.063(z^{5}+z^{-5})]$

$= 0.063 - 0.106z^{-2} + 0.318z^{-4} + 0.5z^{-5} + 0.318z^{-6} - 0.106z^{-8}$
$+ 0.063z^{-10}.$

$\rightarrow$ eq. (1)

From eqn (1).

$h(0) = h(10) = 0.06366,$ $h(2) = h(8) = -0.106$

$h(1) = h(9) = 0,$ $h(3) = h(7) = 0$

$h(4) = h(6) = 0.3183,$ $h(5) = 0.5.$

The freq response of this realizable filter is given by.

$$H'(e^{j\omega}) = \sum_{n=0}^{5} a(n) \cos \omega n.$$

$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.5$

$a(n) = 2h\left(\frac{N-1}{2} - n\right) = 2h(5-n) = \cancel{2}.$

$a(1) = 2h(5-1) = 2h(4) = 0.6366.$

$a(2) = 0,$ $a(3) = 2h(2) = -0.212,$ $a(4) = 0,$

$a(5) = 2h(0) = 0.127.$

$H'(e^{j\omega}) = 0.5 + 0.6366 \cos\omega - 0.212 \cos 3\omega + 0.127 \cos 5\omega$

The magnitude in dB can be calculated by varying 'ω' from 0 to π and tabulate below.

$$|H(e^{j\omega})|_{dB} = 20 \log |H'(e^{j\omega})|$$

A

w(in deg)	0	10	20	30	40	50	60
$\|H\|_{dB}$	0.4	0.21	-0.26	-0.517	-0.21	0.42	0.77
ω	70	80	90	100	110	120	130
$\|H\|_{dB}$	0.21	-1.79	-6	-14.56	-31.89	-20.6	-26
ω	140	150	160	170	180		
$\|H\|_{dB}$	-32	-24.7	-30.55	-32	-26.		

Magnitude response.