

18ECE340T- MACHINE PERCEPTION WITH COGNITION

UNIT-2

*Topics Covered: Boundary descriptors and Region
descriptors*

Reference Book
Computer and Machine Vision
by E.R.Davies

Prepared by
Dr. S.UMA MAHESWARI,
Assistant Professor/ ECE

BOUNDARY DESCRIPTORS AND REGION DESCRIPTORS

BOUNDARY DESCRIPTORS :

- Simple Descriptors. q Length of a Contour.
- By counting the number of pixels along the contour.
- For a chain coded curve with unit spacing in both directions, the number of vertical and horizontal components plus $2\frac{1}{2}$ times the number of components give the exact length of curve.

REGION DESCRIPTORS

- **REGION DESCRIPTORS:**
- In computer vision, visual descriptors or image descriptors are descriptions of the visual features of the contents in images, videos, or algorithms or applications that produce such descriptions.
- They describe elementary characteristics such as the shape, the color, the texture or the motion, among others.

Shape Analysis & Measurement

- The extraction of quantitative feature information from images is the objective of image analysis.
- The objective may be:
 - shape quantification
 - count the number of structures
 - characterize the shape of structures

Shape Measures

- The most common object measurements made are those that describe shape.
 - Shape measurements are physical dimensional measures that characterize the appearance of an object.
 - The goal is to use the fewest necessary measures to characterize an object adequately so that it may be unambiguously classified.

Shape Measures

- The performance of any shape measurements depends on the quality of the original image and how well objects are pre-processed.
 - Object degradations such as small gaps, spurs, and noise can lead to poor measurement results, and ultimately to misclassifications.
 - Shape information is what remains once location, orientation, and size features of an object have been extracted.
 - The term **pose** is often used to refer to location, orientation, and size.

Shape Descriptors

- What are shape descriptors?
 - Shape descriptors describe specific characteristics regarding the geometry of a particular feature.
 - In general, **shape descriptors** or **shape features** are some set of numbers that are produced to describe a given shape.

Shape Descriptors

- The shape may not be entirely reconstructable from the descriptors, but the descriptors for different shapes should be different enough that the shapes can be discriminated.
- Shape features can be grouped into two classes: *boundary features* and *region features*.

Distances

- The simplest of all distance measurements is that between two specified pixels (x_1, y_1) and (x_2, y_2) .
- There are several ways in which distances can be defined:

- *Euclidean*

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

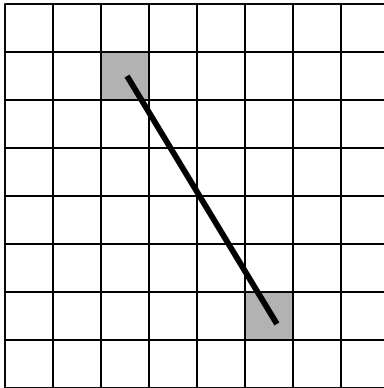
- *Chessboard*

$$d = \max (|x_1 - x_2|, |y_1 - y_2|)$$

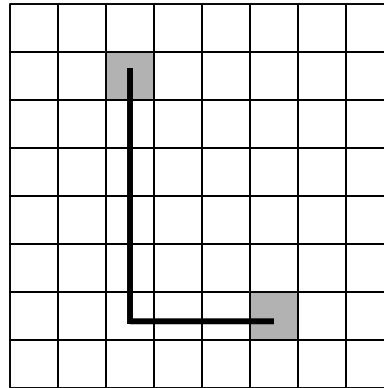
Distances

– *City-block*

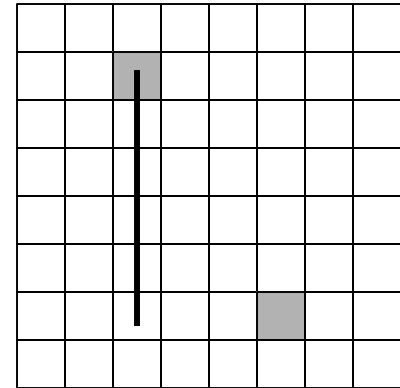
$$d = |x_1 - x_2| + |y_1 - y_2|$$



Euclidean
Chessboard

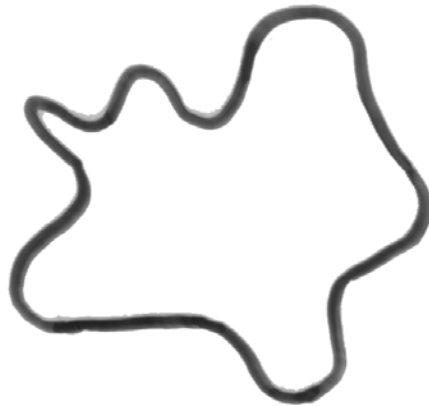


City-block

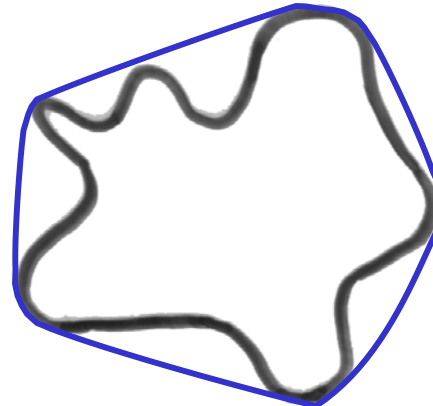


Area

- The **area** is the number of pixels in a shape.



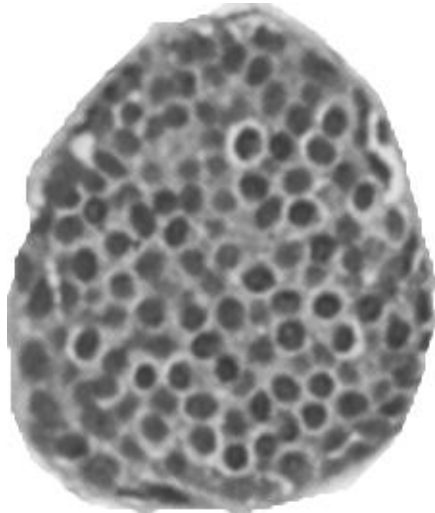
Net Area



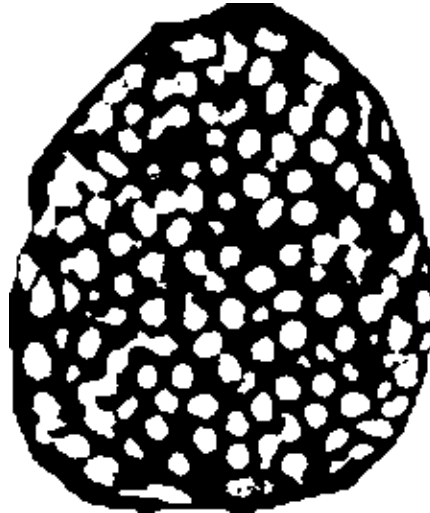
Convex Area

- The **convex area** of an object is the area of the convex hull that encloses the object.

Area



Original Image



Net Area



Filled Area

Perimeter

- The **perimeter** [length] is the number of pixels in the boundary of the object.
 - If x_1, \dots, x_N is a boundary list, the perimeter is given by:

$$\text{perimeter} = \sum_{i=1}^{N-1} d_i = \sum_{i=1}^{N-1} |x_i - x_{i+1}|$$

- The distances d_i are equal to 1 for 4-connected boundaries and 1 or $\sqrt{2}$ for 8-connected boundaries.

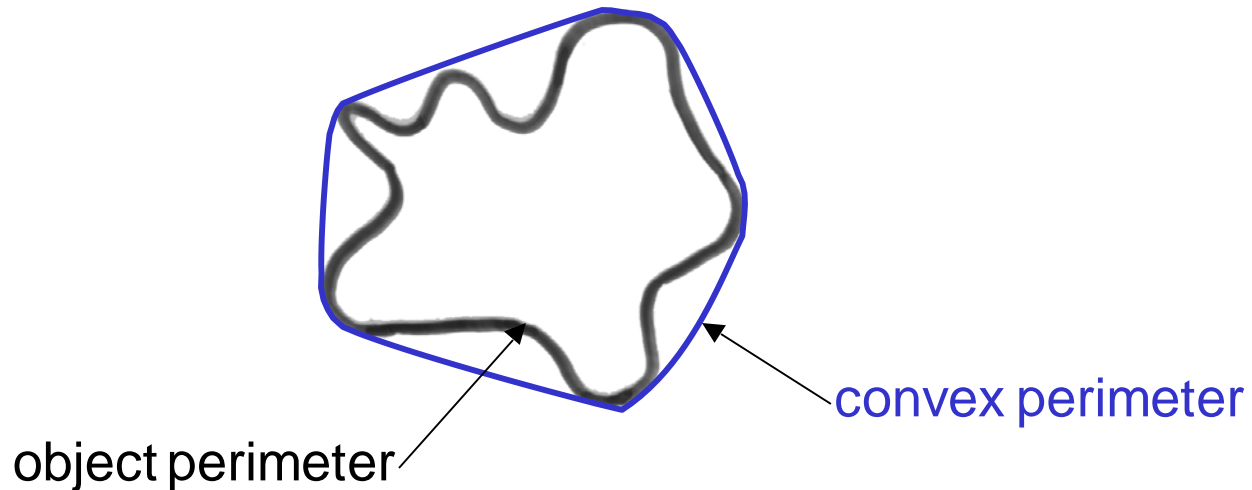
Perimeter

- For instance in an 8-connected boundary, the distance between diagonally adjacent pixels is the Euclidean measure
– The number of diagonal links in $N_4 - N_8$, and the remaining $N_8 - (N_4 - N_8)$ links in the 8-connected boundary are of one pixel unit in length. Therefore the total perimeter is:

$$\text{perimeter} = (\sqrt{2} - 1)N_4 + (2 - \sqrt{2})N_8$$

Perimeter

- The **convex perimeter** of an object is the perimeter of the convex hull that encloses the object.



Major Axis

- The **major axis** is the (x,y) endpoints of the longest line that can be drawn through the object.
 - The major axis endpoints (x_1, y_1) and (x_2, y_2) are found by computing the pixel distance between every combination of border pixels in the object boundary and finding the pair with the maximum length.

Major Axis Length

- The **major-axis length** of an object is the pixel distance between the major-axis endpoints and is given by the relation:

$$\text{major-axis length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- The result is measure of object length.

Major Axis Angle

- The **major-axis angle** is the angle between the major-axis and the x-axis of the image:

$$\text{major-axis angle} = \tan^{-1} \left(\frac{y_2 - y_1}{x_2 - x_1} \right)$$

- The angle can range from 0° to 360°.
- The result is a measure of object orientation.

Minor Axis

- The **minor axis** is the (x,y) endpoints of the longest line that can be drawn through the object whilst remaining perpendicular with the major-axis.
 - The minor axis endpoints (x_1, y_1) and (x_2, y_2) are found by computing the pixel distance between the two border pixel endpoints.

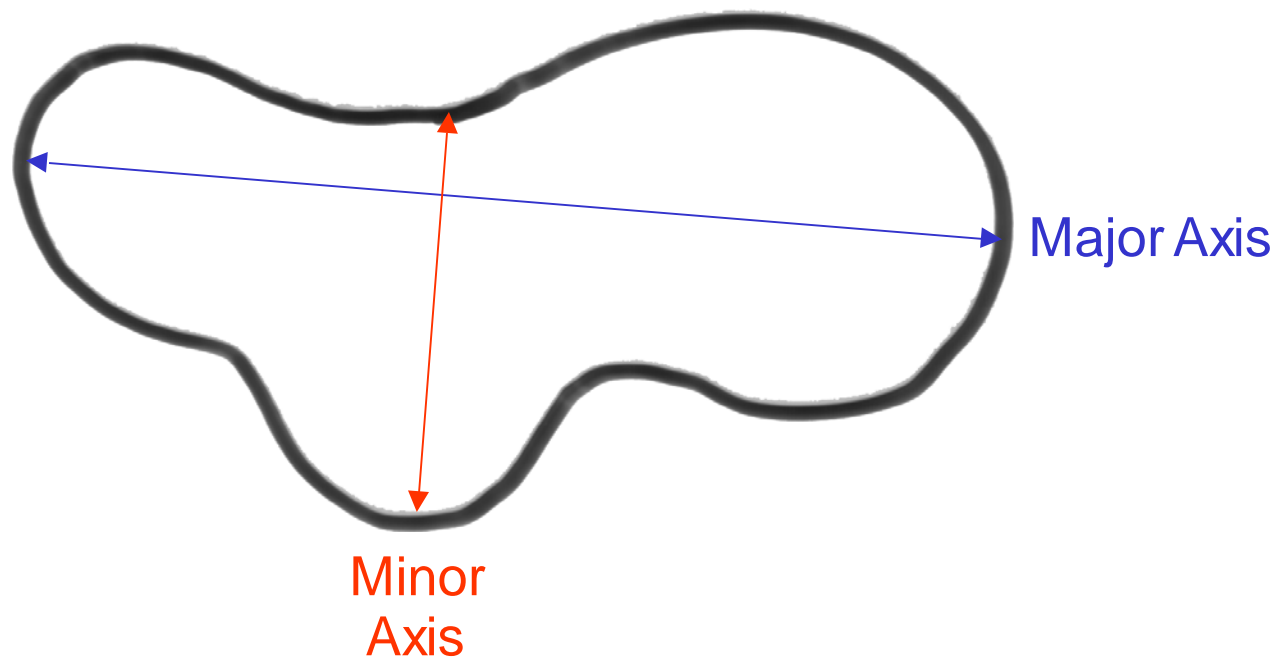
Minor Axis Length

- The **minor-axis length** of an object is the pixel distance between the minor-axis endpoints and is given by the relation:

$$\text{minor-axis length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- The result is measure of object width.

Major and Minor Axes



Compactness

- **Compactness** is defined as the ratio of the area of an object to the area of a circle with the same perimeter.

$$\text{compactness} = \frac{4\pi \cdot \text{area}}{(\text{perimeter})^2}$$

- A circle is used as it is the object with the most compact shape.
- The measure takes a maximum value of 1 for a circle
 $\pi/4$
- A square has compactness =

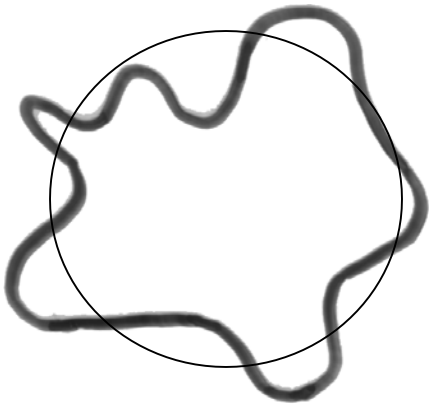
Compactness

- Objects which have an elliptical shape, or a boundary that is irregular rather than smooth, will decrease the measure.
- An alternate formulation:

$$\text{compactness} = \frac{(\textit{perimeter})^2}{4\pi \cdot \textit{area}}$$

- The measure takes a minimum value of 1 for a circle
- Objects that have complicated, irregular boundaries have larger compactness.

Compactness



low compactness



compactness=0.764



compactness=0.668

Elongation

- In its simplest form **elongation** is the ratio between the length and width of the object bounding box:

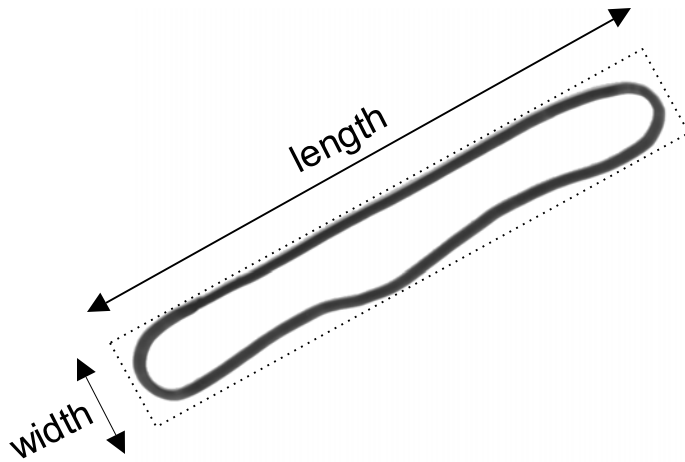
$$\text{elongation} = \frac{\text{width}_{\text{bounding-box}}}{\text{length}_{\text{bounding-box}}}$$

- The result is a measure of object elongation, given as a value between 0 and 1.
- If the ratio is equal to 1, the object is roughly square or circularly shaped. As the ratio decreases from 1, the object becomes more elongated.

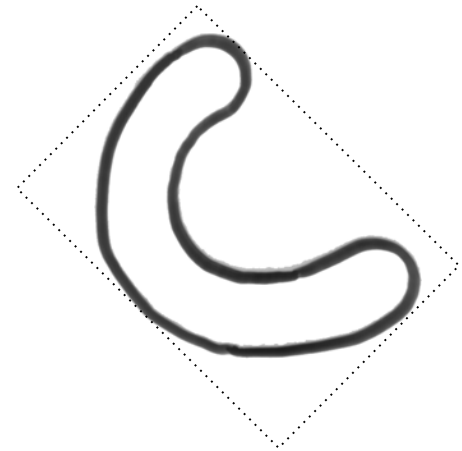
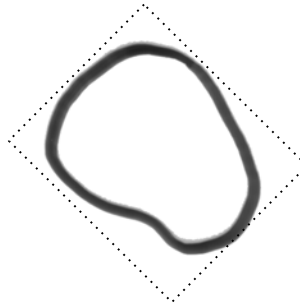
Elongation

- This criterion cannot succeed in curved regions, for which the evaluation of elongatedness must be based on maximum region thickness.
 - Elongatedness can be evaluated as a ratio of the region area and the square of its thickness.
 - The maximum region thickness (holes must be filled if present) can be determined as the number **d** of erosion steps that may be applied before the region totally disappears.
$$\text{elongation} = \frac{A}{2d^2}$$

Elongation



high elongation



low elongation

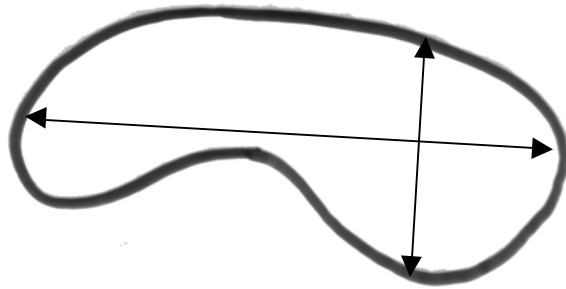
Eccentricity

- **Eccentricity** is the ratio of the length of the short (minor) axis to the length of the long (major) axis of an object:

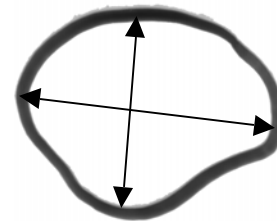
$$\text{eccentricity} = \frac{\text{axislength}_{\text{short}}}{\text{axislength}_{\text{long}}}$$

- The result is a measure of object eccentricity, given as a value between 0 and 1.
- Sometimes known as **ellipticity**.

Eccentricity



high eccentricity



low eccentricity

Eccentricity

- Eccentricity can also be calculated using central moments:

$$\text{eccentricity} = \frac{(\mu_{02} - \mu_{20})^2 + 4\mu_{11}}{\text{area}}$$

Measures of “Circularity”

- Sometimes it is useful to have measures that are sensitive only to departures of a certain type of circularity:
 - e.g. convexity (measures irregularities)
 - roundness (excludes local irregularities)

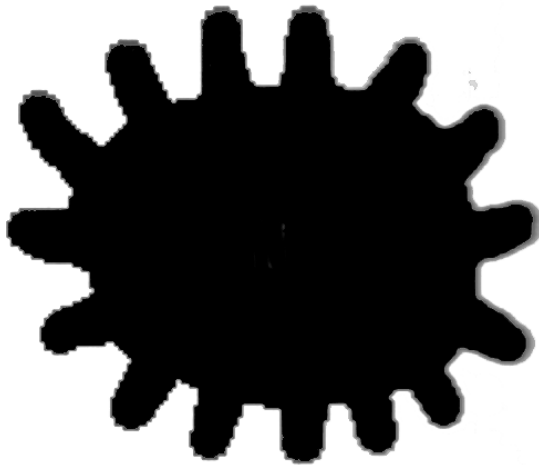
Circularity or Roundness

- A measure of **roundness** or **circularity** (area-to-perimeter ratio) which excludes local irregularities can be obtained as the ratio of the area of an object to the area of a circle with the same convex perimeter:

$$\text{roundness} = \frac{4\pi \cdot \text{area}}{(\text{convex perimeter})^2}$$

- This statistic equals 1 for a circular object and less than 1 for an object that departs from circularity, except that it is relatively insensitive to irregular boundaries.

Circularity



roundness=0.584

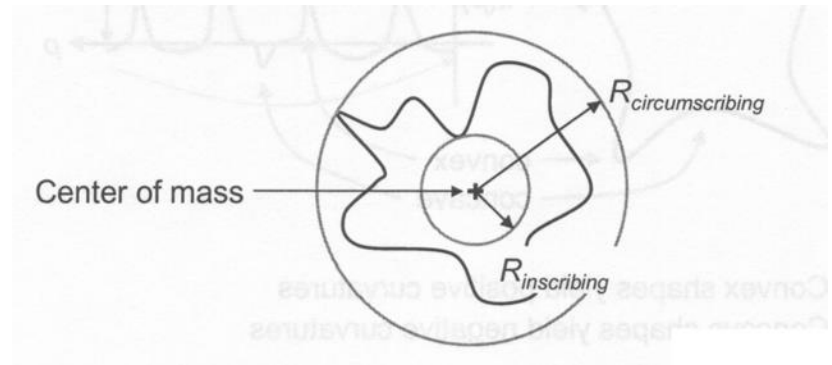


roundness=0.447

Sphericity

- **Sphericity** measures the degree to which an object approaches the shape of a “sphere”.

$$\text{sphericity} = \frac{R_{\text{inscribing}}}{R_{\text{circumscribing}}}$$



- For a circle, the value is the maximum of 1.0

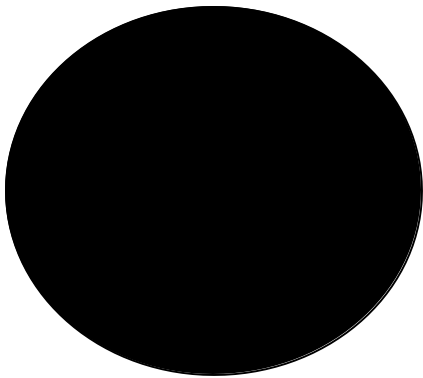
Convexity

- **Convexity** is the relative amount that an object differs from a convex object.
 - A measure of convexity can be obtained by forming the ratio of the perimeter of an object's convex hull to the perimeter of the object itself:

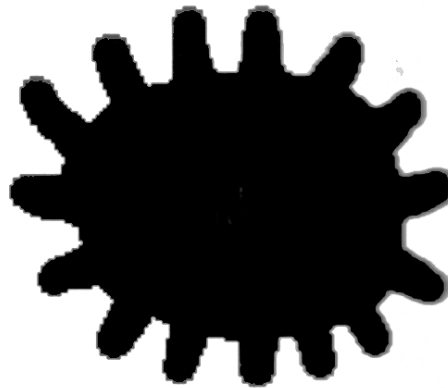
$$\text{convexity} = \frac{\text{convex perimeter}}{\text{perimeter}}$$

Convexity

- This will take the value of 1 for a convex object, and will be less than 1 if the object is not convex, such as one having an irregular boundary.



convexity=1.0



convexity=0.483



convexity=0.349

Aspect Ratio

- The aspect ratio measures the ratio of the objects height to its width.

$$\text{aspect ratio} = \frac{\text{height}}{\text{width}}$$

Caliper Dimensions

- **Caliper** or **feret diameters** are the distances between parallel tangents touching opposite sides of an object.

– At orientation θ , the caliper diameter is:

$$\max_{(x,y) \in A} (x \sin \theta + y \cos \theta) - \min_{(x,y) \in A} (x \sin \theta + y \cos \theta)$$

– Certain caliper diameters are of special interest:

- The **width** of an object ($\theta = 0^\circ$)

$$\max_{(x,y) \in A} (y) - \min_{(x,y) \in A} (y)$$

Caliper Dimensions

- The **height** of an object ($\theta = 90^\circ$)

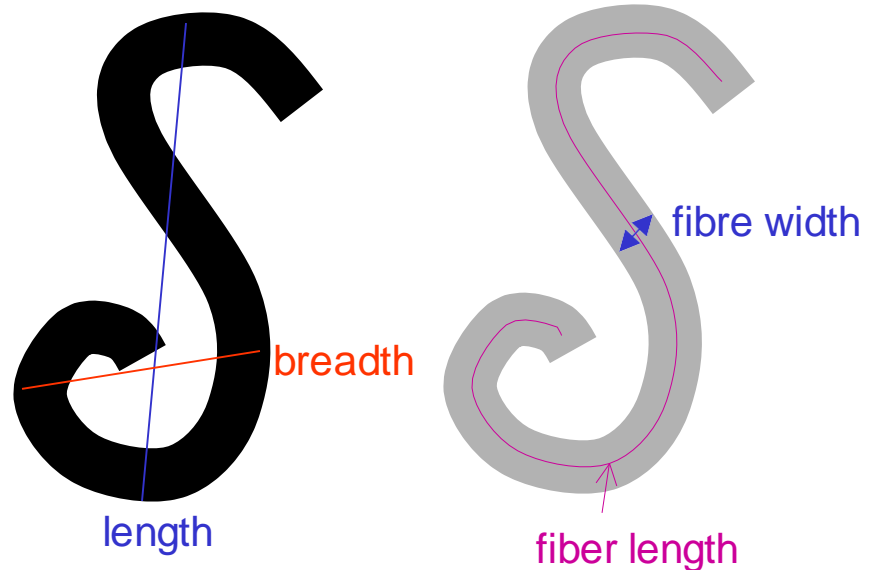
$$\max_{(x,y) \in A} (x) - \min_{(x,y) \in A} (x)$$

- The maximum caliper diameter is one definition of an objects **length**.

Curl

- The **curl** of an object measures the degree to which an object is “curled up”.

$$\text{curl} = \frac{\text{length}}{\text{fibre length}}$$



Curl

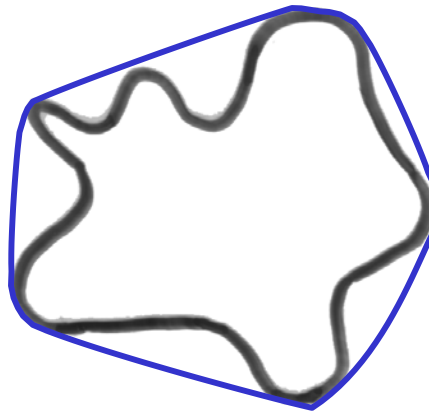
- As the measure of **curl** decreases, the degree to which they are “curled up” increases.

$$\text{fibre length} = \frac{\text{perimeter} - \sqrt{(\text{perimeter})^2 - 16 \cdot \text{area}}}{4}$$

$$\text{fibre width} = \frac{\text{area}}{\text{fibre length}}$$

Convex Hull

- The **convex hull** of an object is defined to be the smallest convex shape that contains the object.



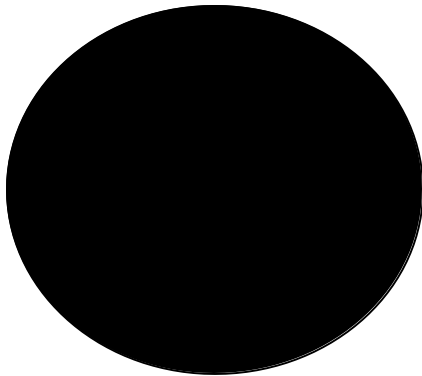
Solidity

- **Solidity** measures the density of an object.
- A measure of solidity can be obtained as the ratio of the area of an object to the area of a convex hull of the object:

$$\text{solidity} = \frac{\text{area}}{\text{convex area}}$$

Solidity

- A value of 1 signifies a solid object, and a value less than 1 will signify an object having an irregular boundary, or containing holes.



solidity=1.0



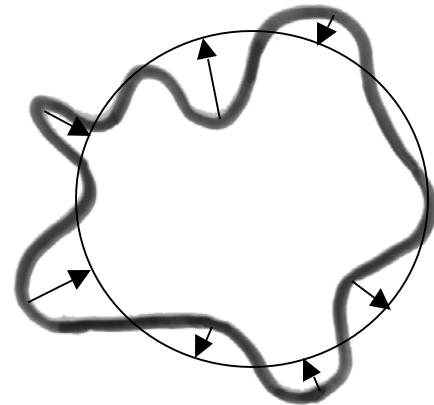
solidity=0.782



solidity=0.592

Shape Variances

- Sometimes a shape should be compared against a template.
 - A circle is an obvious and general template choice. The **circular variance** is the proportional mean-squared error with respect to solid circle.
 - It gives zero for a perfect circle and increases along shape complexity and elongation.



Shape Variances

- **Elliptic variance** is defined similarly to the circular variance. An ellipse is fitted to the shape (instead of a circle) and the mean-squared error is measured.

Rectangularity

- **Rectangularity** is the ratio of the object to the area of the minimum bounding rectangle.
 - Let F_k be the ratio of region area and the area of a bounding rectangle, the rectangle having the direction k . The rectangle direction is turned in discrete steps as before, and rectangularity measured as a maximum of this ratio F_k

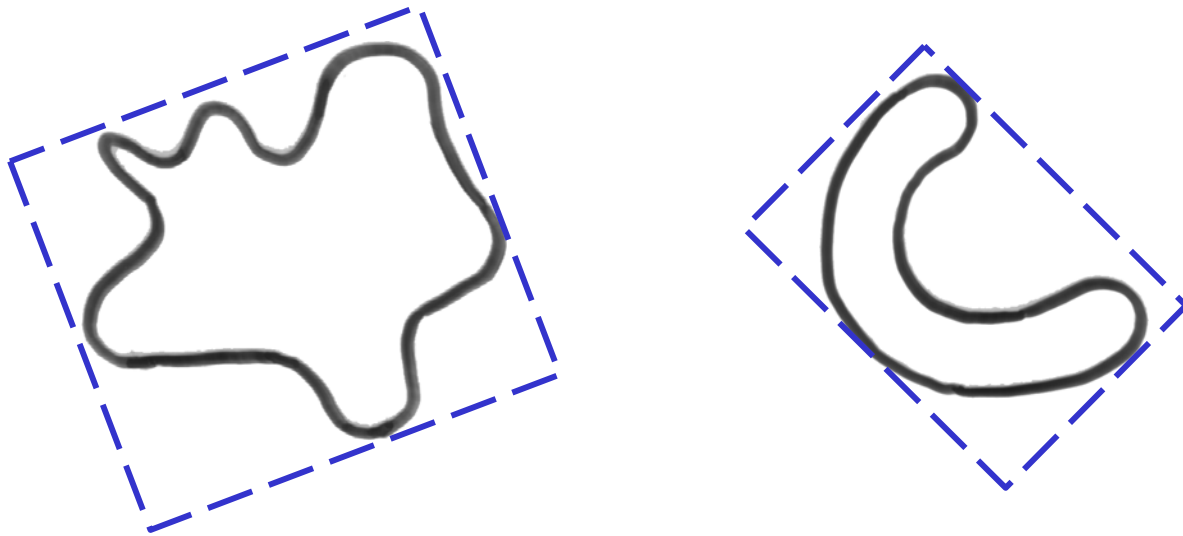
$$\text{rectangularity} = \max_k (F_k)$$

- Rectangularity has a value of 1 for perfectly rectangular object

Bounding Box

- The **bounding box** or bounding rectangle of an object is a rectangle which circumscribes the object. The dimensions of the bounding box are those of the major and minor axes.
 - The **area** of the bounding box is:
$$\text{area} = (\text{major axis length}) * (\text{minor axis length})$$
 - The **minimum bounding box** is the minimum area that bounds the shape.

Bounding Box



bounding boxes

Direction

- **Direction** is a property which makes sense in elongated regions only. If the region is elongated, direction is the direction of the longer side of a minimum bounding rectangle.
 - If the shape moments are known, the direction θ can be computed as:

$$\theta = \frac{1}{2} \tan^{-1} \left(\frac{2\mu_{11}}{\mu_{20} - \mu_{02}} \right)$$

Direction

- Elongatedness and rectangularity are independent of linear transformations translation, rotation, and scaling.
- Direction is independent on all linear transformations which do not include rotation.
- Mutual direction of two rotating objects is rotation invariant.

Orientation

- The overall direction of the shape.

Topological Descriptors

- Topological properties are useful for global descriptions of objects in an image.
 - Features that do not change with elastic deformation of the object.
 - For binary regions, topological features include the number of holes in a region, and the number of indentations, or protrusions.
 - One topological property is the number of **connected components**.

Topological Descriptors

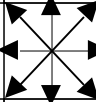
- The number of holes **H** and connected components **C** in an image can be used to define the **Euler** number.
 - The Euler number is defined as the number of components minus the number of holes:
$$\text{Euler number} = \mathbf{C} - \mathbf{H}$$
 - This simple topological feature is invariant to translation, rotation and scaling.

Boundary Descriptors

- The shape of a region can be represented by quantifying the relative position of consecutive points on its boundary.
- A **chain code** consists of a starting location and a list of directions d_1, d_2, \dots, d_N provides a compact representation of all the information in a boundary.
 - The directions d_i are estimates of the slope of the boundary.

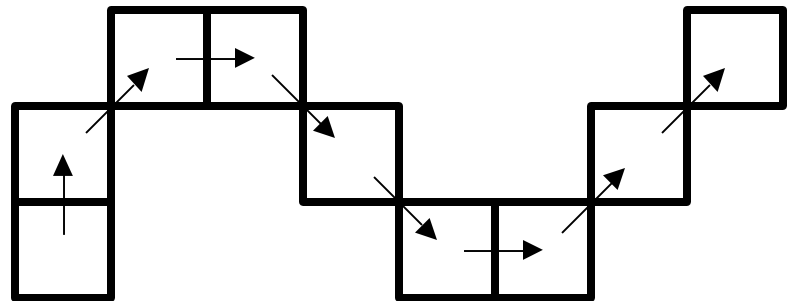
Boundary Descriptors

- Chain codes are based on 4- or 8-connectivity:

3	2	1
4		0
5	6	7

e.g.

2,1,0,7,7,0,1,1



Boundary Descriptors

- The *k-slope* of the boundary at (x_i, y_i) can be estimated from the slope of the line joining $(x_{i-k/2}, y_{i-k/2})$ and $(x_{i+k/2}, y_{i+k/2})$ for some small, even value of k . Calculated as an angle of:

$$\tan^{-1} \left(\frac{y_{i+k/2} - y_{i-k/2}}{x_{i+k/2} - x_{i-k/2}} \right)$$

measured in a clockwise direction, with a horizontal slope taken to be zero.

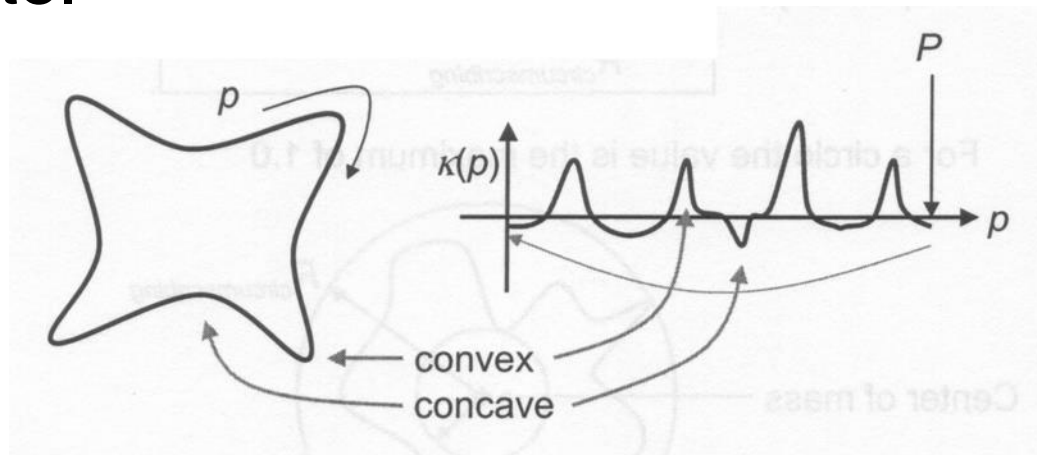
Boundary Descriptors: Curvature

- The *k-curvature* of the boundary at (x_i, y_i) can be estimated from the change in the *k-slope*:

$$\left\{ \tan^{-1} \left(\frac{y_{i+k} - y_i}{x_{i+k} - x_i} \right) - \tan^{-1} \left(\frac{y_i - y_{i-k}}{x_i - x_{i-k}} \right) \right\} \pmod{2\pi}$$

Boundary Descriptors: Curvature

- The curvature (κ) of an object is a local shape attribute.



- Convex shapes yield positive curvatures
- Concave shapes yield negative curvatures

Boundary Descriptors: Bending Energy

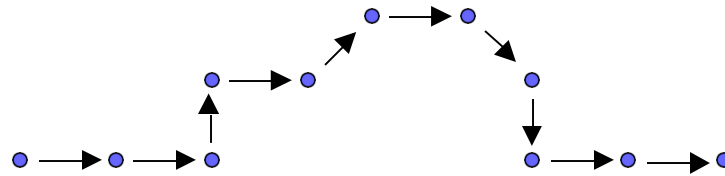
- The total bending energy E_c is a robust global shape descriptor.
 - The bending energy of a boundary may be understood as the energy necessary to bend a rod to the desired shape and can be calculated as a sum-of-squares of the boundary curvature $\kappa(p)$ over the boundary length L .

$$E_c = \frac{1}{L} \sum_{p=1}^L \kappa(p)^2 \quad \frac{2\pi}{R} \leq E_c \leq \infty$$

- The minimum value $2\pi/R$ is obtained for a circle of radius R .

Boundary Descriptors: Bending Energy

- For example:



chain-code: 0 0 2 0 1 0 7 6 0 0

curvature: 0 2 -2 1 -1 -1 -1 2 0

sum of squares: 0 4 4 1 1 1 1 4 0

Bending Energy = 16

Boundary Descriptors:

Total Absolute Curvature

- This descriptor is a measure of the **total absolute curvature** in an object:

$$\kappa_{total} = \frac{1}{L} \sum_{p=1}^L |\kappa(p)| \quad 2\pi \leq \kappa_{total} \leq \infty$$

- The minimum value is found for all convex objects.

Moment Analysis

- The evaluation of moments represents a systematic method of shape analysis.
 - The most commonly used region attributes are calculated from the three low-order moments.
 - Knowledge of the low-order moments allows the calculation of the central moments, normalised central moments, and moment invariants.

Spatial Moments

- To define the $(p,q)^{\text{th}}$ -order moment:

$$m_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} x^p y^q \quad \text{for } p, q = 0, 1, 2, \dots$$

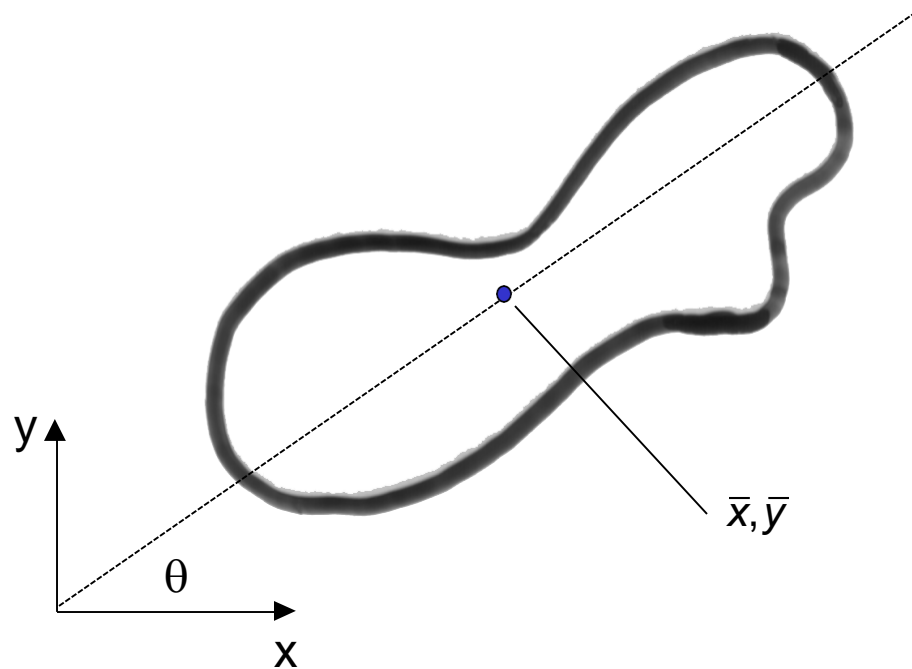
- The zeroth-order moment m_{00} simply represents the sum of the pixels contained in an object and gives a measure of the area (because $x^0=y^0=1$)

Spatial Moments

- The first-order moments in x (m_{10}) and y (m_{01}) normalised by the area can be used to specify the location of an object:
 - The *centre of gravity*, or *centroid* of an object is a measure of the object's location in the image.
 - It has two components, denoting the row and column positions of the point of balance of the object.

$$\text{centroid} = (\bar{x}, \bar{y}) = \left(\frac{m_{10}}{m_{00}}, \frac{m_{01}}{m_{00}} \right)$$

Spatial Moments



Central Moments

- The **central moments** μ_{pq} (i.e. $p+q > 1$) represent descriptors of a region that are normalised with respect to location.

$$\mu_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (x - \bar{x})^p (y - \bar{y})^q \quad \text{for } p + q > 1$$

Normalised Central Moments

- The central moments can be normalised with respect to the zeroth moment to yield the **normalised central moment**:

$$\eta_{pq} = \mu_{pq} / \mu_{00}^{\gamma}$$

$$\gamma = (p + q)/2 + 1$$

- The most commonly normalised central moment is η_{11} , the first central moment in x and y.
 - This provides a measure of the deviation from a circular region shape. A value close to zero describes a region that is close to circular.

Central Moments

- Central moments are translation invariant:
 - i.e. two objects that are identical except for having different centroids, will have identical values of
 - Central moments are not rotationally invariant μ_{pq}^t
- Central moments are not rotationally invariant
 - they will change if an object is rotated.

Central Moments

- The second-order central moments:

$$\mu_{20} = m_{20} - \frac{m_{10}^2}{m_{00}} \quad \mu_{02} = m_{02} - \frac{m_{01}^2}{m_{00}} \quad \mu_{11} = m_{11} - \frac{m_{10} m_{01}}{m_{00}}$$

- The second-order moments measure how dispersed the pixels in an object are from their centroid:
 - μ_{20} measures the object's spread over rows
 - μ_{02} measures the object's spread over columns
 - μ_{11} is a cross-product term representing spread in the direction in which both row and column indices increase.

Principal Axes

- Principal axes of an object can be uniquely defined as segments of lines crossing each other orthogonally in the centroid of the object and representing the directions with zero cross-correlation. This way, a contour is seen as an realization of a statistical distribution.

Principal Axes

- **Principal** (major and minor) **axes** are defined to be those axes that pass through the centroid, about which the moment of inertia of the region is, respectively maximal or minimal.

- The orientation of the major axis:

$$\theta = \frac{1}{2} \tan^{-1} \left[\frac{\mu_{11}}{\mu_{20} - \mu_{02}} \right]$$

which is measured clockwise, with the horizontal direction taken as zero.

- This can be used to find the minimum bounding box.

Moment Invariants

- Normalisation with respect to orientation results in rotationally invariant moments.
 - The first two are the following functions of the second-order central moments:

$$\phi_1 = \eta_{20} + \eta_{02}$$

$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$$

- The first of these statistics is the moment of inertia, a measure of how dispersed, in any direction, the pixels in an object are from their centroid.
- The second statistic measures whether this dispersion is isotropic or directional.

Moment Invariants

$$\phi_1 = \eta_{20} + \eta_{02}$$

$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$$

$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2$$

$$\phi_4 = (\eta_{03} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2$$

Central Moments

$$\mu_{10} = \mu_{01} = 0$$

$$\mu_{11} = m_{11} - m_{10}m_{01}/m_{00}$$

$$\mu_{20} = m_{20} - m_{10}^2/m_{00}$$

$$\mu_{02} = m_{02} - m_{01}^2/m_{00}$$

$$\mu_{30} = m_{30} - 3\bar{x}m_{20} + 2m_{10}\bar{x}^2$$

$$\mu_{03} = m_{03} - 3\bar{y}m_{02} + 2m_{01}\bar{y}^2$$

$$\mu_{12} = m_{12} - 2\bar{y}m_{11} - \bar{x}m_{02} + 2m_{10}\bar{y}^2$$

$$\mu_{21} = m_{21} - 2\bar{x}m_{11} - \bar{y}m_{20} + 2m_{01}\bar{x}^2$$

Radial Distance Measures

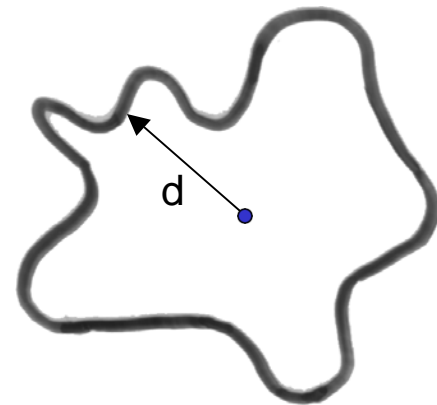
- The shape of a structure of interest can be determined by analysing its boundary, the variations and curvature of which constitute the information to be quantified.
 - Transform the boundary into a 1D signal and analysing its structure.
 - The **radial distance** is measured from the central point (centroid) in the object to each pixel $x(n)$, $y(n)$ on the boundary.

Radial Distance Measures

- Generally the centroid is used as the central point, and the radial distance:

$$d(n) = \sqrt{[x(n) - \bar{x}]^2 + [y(n) - \bar{y}]^2} \quad n = 0, 1, \dots, N - 1$$

is obtained by tracing all N pixels of the boundary.



Radial Distance Measures

- To achieve scale invariance, the **normalised radial distance** $r(n)$ is obtained by normalising $d(n)$ with the maximal distance
- The number of times the signal $r(n)$ crosses its mean and other similar metrics can be used as a measure of boundary roughness.
 - Kilday, J., Palmieri, F., and Fox, M.D., “Classifying mammographic lesions using computerized image analysis”, *IEEE Transactions on Medical Imaging*, 1993, **12**: pp.664-669

Radial Distance Measures

- The sequence $r(n)$ is further analysed to extract shape metrics such as the entropy:

$$E = \sum_{k=1}^K h_k \log h_k$$

where h_k is the k-bin probability histogram that represents the distribution of $r(n)$ as well as the statistical moments:

$$m_p = \frac{1}{N} \sum_{n=0}^{N-1} [r(n)]^p$$

Radial Distance Measures

- The central moment:

$$\mu_p = \frac{1}{N} \sum_{n=0}^{N-1} [r(n) - m_1]^p$$

- Normalised moments invariant to translation, rotation and scaling:

$$\bar{m}_p = \frac{m_p}{\mu_2^{p/2}} \quad \bar{\mu}_p = \frac{\mu_p}{\mu_2^{p/2}} \quad p \neq 2$$

Fourier Descriptor

- The information in the $r(n)$ signal can be further analysed in the spectral domain using the discrete Fourier transform (DFT):

$$a(u) = \frac{1}{N} \sum_{n=0}^{N-1} r(n) e^{-j 2\pi n u / N} \quad u = 0, 1, \dots, N - 1$$

- The “low-frequency” terms, correspond to the smooth behavior.
- The “high-frequency” terms correspond to the jagged, bumpy behavior ① roughness

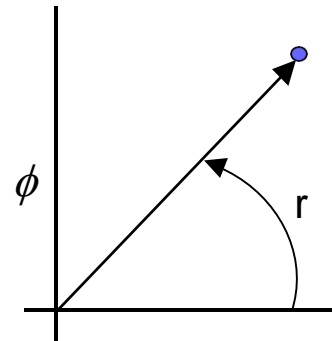
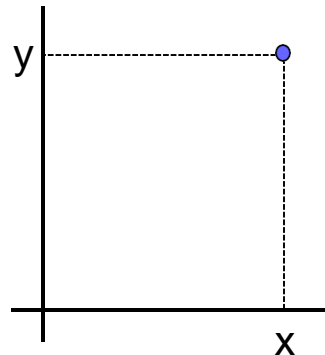
References:

Radial Distance Measures

1. Pohlman, K.A., Powell, K.A., Obuchowski, N.A., Chilcote, W.A., Grunfest-Bronlatowski, S., “Quantitative classification of breast tumours in digitized mammograms”, *Medical Physics*, 1996, **23**: pp.1337-1345 (**masses**)
2. Bruce, L.M., Kallergi, M., “Effects of image resolution and segmentation method on automated mammographic mass shape classification”, *Proceedings of the SPIE*, 1999, **3661**: pp.940-947 (**masses**)
3. Kilday, J., Palmieri, F., and Fox, M.D., “Classifying mammographic lesions using computerized image analysis”, *IEEE Transactions on Medical Imaging*, 1993, **12**: pp.664-669 (**masses**)
4. Shen, L., Rangayyan, R.M., and Desautels, J.E.L., “Application of shape analysis to mammographic calcifications”, *IEEE Transactions on Medical Imaging*, 1994, **13**: pp.263-274 (**calcifications**)

Contour-based Shape Representation & Description

- Object boundaries must be expressed in some mathematical form:
 - Rectangular (Cartesian) coordinates
 - Polar coordinates (in which boundary elements are represented as pairs of angle ϕ and distance r).



Fractal Dimension

- The **fractal dimension** is the rate at which the perimeter of an object increases as the measurement scale is reduced.
 - The fractal dimension produces a single numeric value that summarizes the irregularity of “roughness” of the feature boundary

Fractal Dimension

- Richardson dimension
 - Counting the number of strides needed to “walk” along the boundary, as a function of stride length.
number of steps \times stride-length = perimeter measurement
 - As the stride length is reduced, the path follows more of the local irregularities of the boundary and the measured parameter increases.
 - The result, plotted on log-log axes is a straight line whose slope gives the fractal dimension.

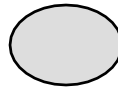
Mammogram Features

- Shape descriptors can be used to characterise features extracted from mammograms.

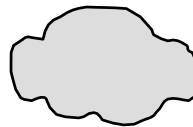
Round



Oval



Lobulated

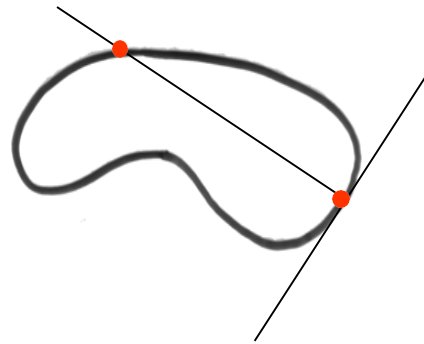


Cell Image Features

- When dealing with images containing numerous objects, such as histological or cytological cell images, shape descriptors are calculated for all the individual cells.
 - Global shape measures can be calculated from the individual image descriptors:
 - Standard deviation ① area, short-axis, long axis, perimeter, circularity,
 - Mean ① compactness, elongation, perimeter, area

Profile

- The **profile** of a binary image analyses the binary image in terms of its projections.
 - Projections can be vertical, horizontal, diagonal, circular, radial, spiral.
e.g. used in determining a region of highest density
 - A profile is the sum of the pixel values in a specified direction



Signature Analysis

- A **signature** is a one-dimensional representation of the boundary.
 - Computing the distance from the centroid of an object to the boundary as a function of angles, from 0° @ 360° in any chosen increment.
 - Harmonic analysis, or shape unrolling
 - The plot repeats every 2π g

