Product
how many no of ways
both can occur

UNIT-2 PERMUTATION AND COMBINATION.

$$^nC_r = \frac{n!}{(n-r)!\,r!} \qquad ^nP_r = \frac{n!}{(n-r)!}$$

Sum rule
no of ways it occur
not simultaneously

1. Suppose three persons enter into the car where there are 5 seats. In how many ways can they take their seats.

$$^5P_3 = \frac{5!}{(5-3)!}$$

$$= \frac{5\times 3\times 2\times 4\times 1}{\cancel{2}} = 5\times 3\times 4\times 1 =$$

— — — — —
5 ways 4 way 3 wa 2 ways 1 ways

2. In how many ways can one select ~~diff~~ two books from diff subjects among 6 distingt computer, 3 maths, 2 chemistry.

3. Suppose you have 6 new year cards sharing 4 friends

$$^6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!}$$

$$= 6\times 5\times 4\times 3\times 1$$

4. In how many no of ways can 4 girls & 5 boys are arranged in a row such that 4 girls are sitting together

$6! \times 4!$

girls B_1 B_2 B_3 B_4 B_5

4! 6!

5. A student has to answer ten qns choosing atleast 4 from each from part A&B. if there are 6 in part A 7 in part B in how many no of way a student choose

A	B
6	7
4	6
5	5
6	4

6. There are 8 can of soup on a shelf 3 of the can contain tomato soup other contains something.

3 → are identical

$$\frac{8!}{3!\,1!\,1!\,1!\,1!\,1!}$$

Circulation

circular permutations in which objects are arranged in a circle.

To calc the no of ways in which n objects can be arranged in a circle be arbitarily fix position of one Object so, the remaining n-1 objects can be arranged as if they were on a straight line (ie) the no arrangements in a circle is $(n-1)!$

NOTE: The no of circular permutation of n different objects taken are at a time is $\frac{^nP_r}{r}$ [Clockwise & anticlockwise order are treated as different]

No of circular permutation of n different objects taken all at a time $\frac{^nP_n}{n} = \frac{n!}{n} = (n-1)!$

NOTE: The following situation leads to identical nature of clockwise & anti clock wise

1. formation of garland with flowers
2. formation of necklace with beads.
3. A person sitting around table with distinct neighbours.

1. Find the no of ways 6 people can be seated in a round table

$$(n-1)! = (6-1)! = 5! = 120.$$

2. Find the no of ways in which 5 people A, B, C, D, E seated in a round table such that A,B should sit together.

$$(4-1)! \times 2! = 3!\,2! = 12.$$

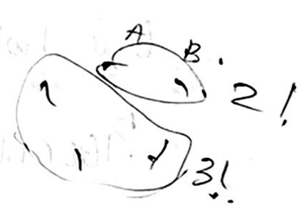
3. In how many ways can 3 men & 3 women be seated in a round table such that no two men should sit together.

men position $(3-1)! = 2!$

women position $3!$ tot $= 3! \times 2! = 12$

4. In how many ways can 5 people can be seated around circular table 1. two people insist to sit together
2. If two people refuse to sit each other.

1. $3!\,2! = 12$



2. No of ways 2 people sit together = 12 ways

No of ways 2 people refuse to sit $= (4-1)! - 12$

$= 24 - 12$

$= 12.$

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4. Write all possible circular permutation using 1, 2, 3, 4

$$\frac{{}^4P_3}{3} = 8.$$

5. Find the no of ways preparing a chain with 6 diff colored beads.

$$\frac{(n-1)!}{2} = \frac{(5-1)!}{2} =$$

6. Find the no of ways of arranging chief minister & 10 cabinet minister in a circular table so that the chief minister should sit in one position.

$10!$

7. $n+1$ pigeon occupies n pigeon holes at least one hole has

PIGEON HOLE PRINCIPLE

Generalization of pigeon hole principle.

If m pigeon occupies n holes then at least 1 hole has more than $\left\lfloor \frac{m-1}{n} \right\rfloor + 1$ pigeon

floor

$\lfloor x \rfloor \rightarrow$ floor function . $8.5 \rightarrow 8$.

only integer

1. Show that among 100 people atleast 9 of them were born in the same mothn.

no of people (pigeon) $m = 100$.

no of holes (no of month) $= 12$.

$$\left\lfloor \frac{100-1}{12} \right\rfloor + 1 = \left\lfloor \frac{99}{12} \right\rfloor + 1$$

$$= \lfloor 8.2 \rfloor + 1$$

$$= 9.$$

2. Show that if 7 color are used to paint 50 bicycles atleast 8 bicycle have same color.

no of bicycles (pigeon) $m = 50$.

no of colors (holes) $n = 7$.

$$\therefore \left\lfloor \frac{50-1}{7} \right\rfloor + 1 = \lfloor 7 \rfloor + 1 = 8$$

3. In a grp of six or more person atleast 3 are mutual frnds or atleast 3 are mutual strangers.

Consider the grp has exactly 6 person A, B, C, D, E, F.
fixed

fix A and create a pigeon hole has consist of frnds to A and consist of strangers to A.

m, no of pigeons (total people ext A) $= 5$

n, no of holes $= 2$.

$$\left\lfloor \frac{5-1}{2} \right\rfloor + 1 = \left\lfloor \frac{4}{2} \right\rfloor + 1 = 2 + 1 = 3$$

Pigeon hole-1 | hole-2
BCD | EF

Case 1.
any two of these 3 persons are frnds then the hole 1 contains mutually 3 frnds.

Case 2.
any two pigeon hole 2 has three person's namely $\Rightarrow$ D, E, F two of these 3 are strangers $\therefore$ hole-2 ~~contains~~ $\therefore$ hole-2 contains 3 mutual strangers.
$\Rightarrow$ none of them are mutually strangers

If A & B are finite subset of finite set U then

$$|A \cup B| = |A| + |B| - |A \cap B|$$

where modus of A $\rightarrow$ cardinality of set A

This principle can be extended by finite no of finite sets

$$|A_1 \cup A_2 \cup \ldots \cup A_n| = \sum_i |A_i| + \sum_{i<j} |A_i \cap A_j| + \sum_{i<j<k} |A_i \cap A_j \cap A_k| + \ldots + (-1)^{n+1} |A_1 \cap A_2 \cap \ldots \cap A_n|.$$

1. Find the no of integers betwn 1 to 250 both inclusive that are not divisible by 2, 3, 5, 7

Let A, B, C, D be the set of integers that lie btwn 1 & 250 and are divisible by 2, 3, 5 & 7 respectively

$$|A| = \left\lfloor \frac{250}{2} \right\rfloor = 125 \qquad |A \cap B \cap C \cap D| = \left\lfloor \frac{250}{210} \right\rfloor = 1$$

$$|B| = \left\lfloor \frac{250}{3} \right\rfloor = 83 \qquad |B \cap C \cap D| = \left\lfloor \frac{250}{105} \right\rfloor = 2$$

$|C| = \left\lfloor \frac{250}{5} \right\rfloor = 50$ $\qquad |B \cap C| = \left\lfloor \frac{250}{15} \right\rfloor = 16$

$|D| = \left\lfloor \frac{250}{7} \right\rfloor = 35$ $\qquad |C \cap D| = \left\lfloor \frac{250}{35} \right\rfloor = \cancel{8}7$

$|A \cap B| = \left\lfloor \frac{250}{6} \right\rfloor = 41$ $\qquad |A \cap B \cap C| = \left\lfloor \frac{250}{30} \right\rfloor = 8$

$|A \cap C| = \left\lfloor \frac{250}{10} \right\rfloor = 25$ $\qquad |A \cap B \cap D| = \left\lfloor \frac{250}{42} \right\rfloor = 5$

$|A \cap D| = \left\lfloor \frac{250}{14} \right\rfloor = 17$ $\qquad |A \cap C \cap D| = \left\lfloor \frac{250}{70} \right\rfloor = 3$

$|B \cap D| = \left\lfloor \frac{250}{21} \right\rfloor = 11$

By the principle of inclusion exclusion
The no of integers btwn 1 & 250 atleast one of the integers.

$= 125 + 83 + 50 + 35 - 41 - 25 - 17 - 16 - 7 - 11 + 8 + 5 + 3$
$+ 2 - 1$

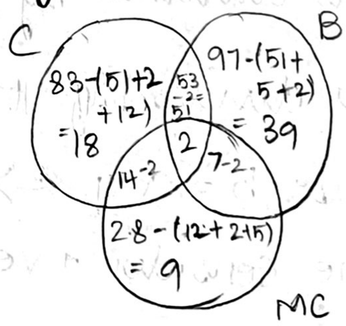
$= 193$ $\qquad = 250 - 193$
$= 57$

2. In a survey of 150 clg students reveals that 83 own car, 97 own bikes, 28 own motor cycle, 53 own a car and a bike, 14 own car & motor cycle, 7 bike & motor 2 own all all three. How many own a bike & nothing else.
How many do not own any of the three

$n|C| = 83$

$n|B| = 97$

$= 150 - |C \cup B \cup$



Number theory is the part of mathematics to investigate more specifically the properties of positive integers

Basic properties of divisibility theory and congruences Congruences

DIVISIBILITY THEORY

Let A & B are two integers with $A \neq 0$ a divides b or b is divisible by a or a is a divisor or factor of b or b is a multiple of b. $(a|b)$

$a|b$ if $b=aq$ for some $q \in Z$.

The statement a divides b is written and its negation is denoted by $a \nmid b$

ex, 66 is divisible by 11 => $11|66$

$66 = 11 \times 6$

ex, -7 divides 42 => $7|42$

$42 = (-7) \times 6$

0 is divisible by every integers

If a divides b then -a is also divides b because $b = aq$ => $b = -a \times -q$. it is enough if we consider +ve divisor of integers

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Theorem 1

Let $a, b, c \in Z$ a set of integers

i) if $a|b$ & $a|c$ then $a|b+c$.

ii) if $a|b$ & $b|c$ then $a|c$

iv) if $a|b$ & $a|c$ then $a|mb+nc$

iii) if $a|b$ then $a|mb$ for $m \in Z$.

i) Since $a|b, a|c$ it follows from the definition

$b = ma$, $c = ma$ where m & n are integers

$b+c = (m+n)a$.

$a|b+c$.

ii) $a|b$ & $b|c$

$b = ma$ & $c = mb$ $m, n \in Z$.

$c = nb$

$= n(ma)$

$= (nm)(a)$

$\therefore a|c$.

iii) Since $a|b \Rightarrow b = ma$

$mb = m(na)$

$mb = (mn)a$

$a|mb$.

$a|b \Rightarrow b = ma$

$a|c \Rightarrow c = ma$.

~~a|mb+nc~~ $a|mb$.

v) $a|b \Rightarrow a|mb$

$a|c \Rightarrow a|nc$

~~a+a~~ $a|mb+nc$.

A positive integer is ~~is~~ $b > 1$ is called prime if the only divisor of b is 1 & b.

A positive integer is $b > 1$ is called Composite if the divisor of b is

NOTE :-

i) A number that is not a prime is divisible by prime.

ii) A number that is prime

FUNDAMENTAL THEOREM OF ARITHMETIC.

Every positive integer $n > 1$ can be written uniquely as that product of prime numbers where the prime factors are written in order of increasing si

$n = 2$ Since 2 is a prime number.

if n is a prime

if n is composite

Let us assume that the theorem holds good for +ve integer $< n$

$n = ab \quad \because a, b < n$

each of a & b can be expressed as product of primes

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1. If $n > 1$ the composite integer and p is a prime factor of n then $p \leq \sqrt{n}$ or prove that a positive integer n is a prime number if no prime $p \leq \sqrt{n}$ divides n

Prove n is composite

PROOF :-

$n = ab$ for some integer a & b $1 < a, b < n$.

without loss of generality $a \leq b$

let $a \geq \sqrt{n}$.

then $b \geq a \geq \sqrt{n}$. implies $n = ab \geq \sqrt{n} \cdot \sqrt{n} \geq n$.

which is a contradiction

One of the factor, a should not be greater than or equal to $\sqrt{n}$.

$a \leq \sqrt{n}$

This divisor is either prime or by the fundamental theorem of arithmetic $\sqrt{96} = 9.76$

n has a divisor $\leq \sqrt{n}$. 2 3 5 7

NOTE: To test the given integer n is prime it is enough to see that it is not divisible by any prime $\leq \sqrt{n}$.

1. Find the prime factorization of 7007. 2, 3, 5, 7

$$\frac{7007}{7} = \frac{1001}{7} = \frac{143}{11} = \frac{13}{13}$$

Prime factors of $7007 = 7^2 \times 11 \times 13$.

DIVISION ALGORITHM

When a & b are any 2 integer $b > 0$, There exist a unique integer q & r such that $a = bq + r$ when $0 \le r < b$.

Proof :

1. What are the quotient and reminder when -11 is divided by 3.

$$-11 = (-4)(3) + 2$$

Let a & b be any two integer we say that b is a divisor of a written as $b|a$ ~~if~~ $a = qd$ for integer q.

Let a & b be any two integer then an integer d is called a common divisor of a & b by $d|a$ & $d|b$. $3|21$ $3|15$

3 is common divisor of 15, 21

GREATEST COMMON DIVISOR (GCD)

Let a, b are non zero integer then an integer $d \ge 1$ is called Greater Common divisor if the follown condition satisfies

i) $d|a$ & $d|b$.

ii) For any integer n, if $n|a$ & $n|b$ for $n|d$.

If a & b are set to be

If GCD of $a_i, b_j = 1$ then $1 \le i < j \le n$ are pairwise

EUCLID'S ALGORITHM

Euclid's algorithm

If r_1 is the remainder when a is divided by b, r_2 is the remainder when b is diveded by r_1, r_3 is remainder when r_2 is divided by r_1

$r_{a+1} = 0$ then the last non-zero remainder r_k is the GCD of a, b.

Finding GCD using prime factorization

If the prime factorization of a & b ar $a = P_1^{a_1}, P_2^{a_2}, \dots P_n^{a_n}$

$b = P_1^{b_1}, P_2^{b_2}, \dots P_n^{b_n}$. Each exponent is a non-negative integer where all the prime occurring in prime factorization either a or b included with both factorization with 0 exponent if necessary

$$\gcd(a,b) = P_1^{\min(a_1,b_1)}, P_2^{\min(a_2,b_2)} \dots P_n^{\min(a_n,b_n)}.$$

1. gcd (120, 500) using euclid's algorithm & prime factorization.

$120 = 2^{2} \times 3 \times 5$

$500 = 2^2 \times 5^3 \times 3^0$.

$$\gcd(120,500) = 2^{\min(2,2)} \times 3^{\min(0,1)} \times 5^{\min(1,3)}$$

$$= 2^2 \times 3^0 \times 5^1.$$

Euclid algorithm,

Remainder

$500 \div 120 = 20$

$120 \div 20 = 0$.

20 is the gcd of (a, b) => (500, 120)

2. Find $(1575, 231)$ by using Euclid algorithm.

$a = 1575 \div 231 =$

$1575 = (231)q + 189$

$231 = (189)q + 42$

$189 = (42)q + 21$

$42 = 21(q) + 0$.

gcd of $(1575, 231)$ is 21.

8. GCD of (a,b) can be expressed as integral of linear Combination of a, b such that $GCD(a,b) = ma + nb$

3. Express gcd of $(252, 198) = 18$

$252 = 198(1) + 54$

$198 = 3 \times 54 + 36$

$54 = 36 \times 1 + 18$

$36 = 18 \times 2 + 0$.

$\cancel{45}/4$

36

18×2

36

18 is the gcd of $(252, 198)$.

$ma + nb = GCD(a, b)$

If a, b, c are positive integer such that $\gcd(a,b)=1$ and $a|bc$ then $a|c$.

Proof:

Let a & b are relatively prime integer $\gcd(a,b)=1$, there exist an integer such that $ma+nb=1$ —①

$$mac + nbc = c$$

$$mac + nba = c$$

$$(mc+nb)a = c.$$

$$a|c.$$

$a|bc$

$bc = ka \quad k \in \mathbb{Z}.$

3. $\gcd(ka, kb) = k\gcd(a,b)$

4. If $\gcd(a,b)=d$, then $\gcd\left(\frac{a}{d}, \frac{b}{d}\right)=1$

5. $\gcd(a,b)=1$ then for any integer c,

$$\gcd(ac, b) = \gcd(c, b)$$

6. If each of $a_1, a_2, \ldots, a_n$ is a coprime of b is $\gcd(a,b)=1$ then the product $a_1, a_2, a_3 \ldots, a_n$ is also coprime to b is $\gcd(a_1, a_2 \ldots, a_n, b)=1$.

7. If p is a prime & $p|a_1, a_2, \ldots a_n$ where each a_i is an integer then $p|a_i$ for some i.

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LCM (Least Common Multiple)

If a & b are +ve integer then the smallest +ve integer that is divisible by both a & b is called least common multiple of a & b. & its denoted by $LCM(a,b)$

NOTE! even if either both a & b are -ve $LCM(a,b)$ is always positive.

Alternate definition of $LCM(a,b)$

If the prime factorization of a & b are

$a = P_1^{a_1} P_2^{a_2} P_3^{a_3} \dots P_n^{a_n}$

$b = P_1^{b_1} P_2^{b_2} P_3^{b_3} \dots P_n^{b_n}$.

Condition stated in the alternate definition:

$LCM(a,b) = P_1^{max(a_1, b_1)} P_2^{max(a_2 b_2)} P_3^{max(a_3,b_3)} \dots P_n^{max(a_n,b_n)}$

for example $LCM(24, 30)$

$24 = 2^3 \times 3 \times 5^0$

$30 = 2 \times 3 \times 5$ $\quad LCM(a,b) = 2^3 \times 3 \times 5 = 120$

If a & b are two +ve integer then GCD(a,b)

$\therefore GCD(a,b)\ LCM(a,b) = ab$.

Proof

Use prime factorization find LCM & GCD of (231, 1575) and also verify GCD (m, n) LCM(m, n) = mn.

$231 = 3 \times 11 \times 7 \times 5^0$

$1575 = 3^2 \times 5^2 \times 7 \times 11^0$

$$\begin{array}{r|l} 3 & 1575 \\ \hline 5 & 525 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 231 \\ \hline 11 & 77 \\ \hline & 7 \end{array}$$

$GCD(231, 1575) = 3 \times 11^0 \times 7 \times 5^0 = 21$

$LCM(231, 1575) = 3^2 \times 5^2 \times 11 \times 7 = 17325$

$GCD(231, 1575)\ LCM(231, 1575) = 363,825$ — (1)

$231 \times 1575 = 363,825$ — (2).

(1) = (2).

2. Find the integer m & n such that $512m + 320n = 64$

$ma + nb = GCD(a, b)$

$GCD(512, 320)$

$512 = 320 + 192$

$320 = 192 + 128$ $\qquad 64 \times 2$

$192 = 128 + 64$

$128 = 64 + 0$

$GCD(512, 320) = 64$

$64 = 192 - 128$

$= 192 - (320 - 1 \times 192)$

$= 2 \times 192 - 320$

$= 2(512 - 320) - 320$.

$= 2 \times 512 - 3 \times 320$.

$m = 2 \quad n = -3$.

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UNIT 3 - MATHEMATICAL LOGIC.

can be true or false not both.

declarative Sentence

ATOMIC PREPOSITION. (primary / Primitive / Simple)

Preposition that does not contain any of the logical operators or connectives are called atomic preposition.

MOLECULAR OR COMPOOND PREPOSITION, OR COMPOSITE STATEMENT

statement constructed by combining one or more atomic statement using connectives are called compound preposition

Truth value of preposition.

If a preposition is true its truth value is true denoted by T or 1.

If a preposition is false its truth value is false denoted by F or 0.

LOGICAL OPERATORS OR CONNECTIVES

Used to combine two or more proposition five logical operators or connectives

	Name	Symbol
NOT	Negation	$\sim$ or $\neg$
And	Conjuction	$\wedge$
or	disjunction.	$\vee$
If then	Conditional	$\rightarrow$
If and only if	Biconditional	$\leftrightarrow$

NEGATION

Let P be a preposition that simply says that

if P is true then $\neg P$ is false

if P is false then $\neg P$ is true.

$\neg P \Rightarrow$ "negation of P".

"It is not the case that P"

P	$\neg P$
T	F
F	T

CONJUCTION

Let p & q be two preposition is the preposition that is true when both p & q are true and false otherwise.

ex; Today is Sunday $\to$ p

It is raining today $-$ q.

$p \wedge q$ is Today is Sunday and it is raining today.

p	q	$\wedge$
F	F	F
F	T	F
T	F	F
T	T	T

DISJUNCTION

Let p, q be any preposition The preposition $p \vee q$ the ~~stat~~ is false only when both are false otherwise true.

P	q	$\vee$
F	F	F
F	T	T
T	F	T
T	T	T

CONDITIONAL STATEMENT

If p & q are any two statement then the statement $p \to q$ has "If p then q" is called conditional statement. It is false only when p is true and q is false otherwise true.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

BI CONDITIONAL

If p & q are any two statement then biconditional statement is $p \leftrightarrow q$ 'iff p i is true whenever both p & q has same truth value.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Conditional

Alternate way to express $p \to q$, → p implies q

→ p is sufficient for q.

→ p only if q

→ q necessary of p

→ q if p

→ q whenever p

We shall avoid the word implies

Variation in Conditional statement.

→ if p conditional q is Conditional preposition the q Conditional p is called converse of $p \to q$.

2. $\neg p \to \neg q$ is called inverse of $p \to q$,

3. $\neg q \to \neg p$ is called contrapositive of $p \to q$

for example,

If it rains today then I will stay at home.

p: It rains today q: I will stay at home.

Converse : If I will stay at home then it rains today

inverse : If I ~~does~~ not rains today then it will not stay at home.

Contrapositive : If It will not stay at home then it does not rain today.

Different ways to represent biconditional

1. P is necessary & Sufficient for q & vice versa.
2. If P then q and conversely
3. P if and only if q

Order of precedance for logical operator.

Conditional or Implication

Name	Operator	Precedance.
$\neg$	Negation	1
$\wedge$	Conjuction	2
$\vee$	disjunction	3
$\to$	Conditional	4
$\leftrightarrow$	Biconditional	5.

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$P \vee \neg P$
$T + F$ T.
$F + T$

Tautology

An expression involving logical variables that is always true for all possible truth values of variables in it.

example.

P	$\neg P$	$P \vee \neg P$
T	F	T
F	T	T

Contradiction

An expression involving logical variables that is always false for all possible truth values of variables in it.

P	$\neg P$	$P \wedge \neg P$
T	F	F
F	T	F

Contigency.

A Compound preposition that is neither tautogy and nor contradiction is called contingency.

P	q	$P \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

1. Write a Symbolic form, you can access internet from campus only if you ~~major~~ are a computer science major or a fresher.

P: You can access internet from college
q: You are Computer Science major or a fresher.

P only if q = $P \to q$.

$P \to (q \vee \neg r)$

2. It is necessary to have a valid password to login to the server.

P: to login to the server
q: valid password.

$P \to q$.

3. Home team wins whenever it is raining.

P: Raining
q: Home team. q whenever p.

$p \to q$.

4. for you to get an 'o' grade in this course, it is necessary & sufficient that you have to learn discrete maths.

P: for you to get an 'o' grade

$P \leftrightarrow q$

Construct the truth table for the following.

1. $(p \vee q) \vee \neg p$.

p	q	$\neg p$	$p \vee q$	$(p \vee q) \vee \neg p$
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

2. truth table for $p \vee q \rightarrow p \wedge q$.

p	q	$p \vee q$	$p \wedge q$	$p \vee q \rightarrow p \wedge q$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

3. Show that the following are tautology
$\neg q \wedge (p \rightarrow q) \rightarrow \neg p$.

p	q	$\neg p$	$\neg q$	$(p \rightarrow q)$	$\neg q \wedge (p \rightarrow q)$	$\neg q \wedge (p \rightarrow q) \rightarrow \neg p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	F	F	T
F	F	T	T	T	T	T

2. $\neg(q \to r) \land r \land (p \to q) \equiv F$

c

p	q	r	$q \to r$	$\neg(q \to r)$	$\neg(q \to r) \land r$	$p \to q$	$c \land (p \to q)$	F
T	T	T						
T	T	F						
T	F	T						
T	F	F						
F	T	T						
F	T	F						
F	F	T						
F	F	F						

Duality - Law

The dual of compound preposition that contains only logical operators $\lor, \land, \neg$ is the preposition obtained by replacing each joint by $\land$, each $\land$ by $\lor$, each T by F, each F by T where T & F are special variable representing compound preposition that are Tautology & contradiction respt. Dual of preposition A is denoted by A^*

ex, $A = (P \lor q) \land T$

$A^* = (P \land q) \lor F$

Logical Equivalence.

The compound preposition p & q are said to be logically equivalent if $p \leftrightarrow q$ is a tautology logical equivalence is denoted by $\equiv$ or $\Leftrightarrow$

P	T	$P \land T$
T	T	T
F	T	F

$P \land T \equiv P$.

Tautological implication

→ Let R & S be two compound preposition then R is said to be logically imply S if.

→ $R \rightarrow S$ is a tautology is denoted by $R \Rightarrow S$

NOTE.

$R \rightarrow S$ is tautology $R \Rightarrow S$

$R \leftrightarrow S$ is tautology $R \Rightarrow S$ or $R \equiv S$.

Name of The law	primal form	Dual form.
1. Identity law	$P \vee F \equiv P$	$P \wedge T \equiv P$
2. Domination law (Null law)	$P \vee T \equiv T$	$P \wedge F \equiv F$
3. Idempotent (Self repeat)	$P \vee P \equiv P$	$P \wedge P \equiv P$
4. Double Negation (Involution law)	$\neg(\neg P) \equiv P$	
5. Commutative law	$P \vee q \equiv q \vee P$	$P \wedge q \equiv q \wedge P$
6. Associative law	$(P \vee q) \vee r \equiv P \vee (q \vee r)$	$(P \wedge q) \wedge r \equiv P \wedge (q \wedge r)$
7. Distributive law	$P \vee (q \wedge r) \equiv (P \vee q) \wedge (P \vee r)$	$P \wedge (q \vee r) \equiv (P \wedge q) \vee (P \wedge r)$
8. De morgan's law	$\neg(P \vee q) \equiv \neg P \wedge \neg q$	$\neg(P \wedge q) \equiv \neg P \vee \neg q$
9. Absorption law	$P \vee (P \wedge q) \equiv P$	$P \wedge (P \vee q) \equiv P$
10. Negtion law (complement)	$P \vee \neg P \equiv T$	$P \wedge \neg P \equiv F$

1. Prove without truth table.

$(p \wedge q) \rightarrow p \vee q \Leftrightarrow T$

$(p \wedge q) \rightarrow p \vee q \equiv \neg(p \wedge q) \vee p \vee q$ (logical equivalen
$p \rightarrow q \equiv \neg p \vee q$)

$\equiv (\neg p \vee \neg q) \vee p \vee q$ (Demorgan's law)

$\equiv \neg p \vee (\neg q \vee (p \vee q))$ (Associative law)

$\equiv \neg p \vee ((\neg q \vee q) \vee p))$ (negation law)

$\equiv \neg p \vee (T \vee p)$

$\equiv \neg p \vee T$

$\equiv T.$

2. Prove $(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \equiv p \wedge q$ using laws.

LHS : $(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \equiv (\neg p \vee q) \wedge ((p \wedge p) \wedge q)$ (Associ

$\equiv (\neg p \vee q) \wedge (p \wedge q)$ (Idemp

$\equiv (\neg p \vee q) \wedge (q \wedge p)$ commuta

$\equiv (\neg p \vee q \wedge q) \wedge p$ (Associative)

$\equiv (\neg p \vee q) \wedge p.$

$\equiv (\neg p \vee q) \wedge p.$

$\equiv F \vee (q \wedge p)$

$= q \wedge p$

$= p \wedge q.$

3. Prove $p \rightarrow (q \rightarrow p) \equiv \neg p \rightarrow (p \rightarrow q)$

LHS: $p \rightarrow (q \rightarrow p) \equiv \neg p \vee (q \rightarrow p)$ (equivalence involving Conditional)

$\equiv \neg p \vee (\neg q \vee p)$ "

$\equiv \neg p \vee (p \vee \neg q)$

$\equiv (\neg p \vee p) \vee \neg q$

$\equiv T \vee \neg q$

$\equiv T$ — ① (Domination law)

RHS: $\neg p \rightarrow (p \rightarrow q) \equiv \neg(\neg p \vee p \rightarrow q)$

$\equiv \neg(\neg p) \vee (\neg p \vee q)$ (Double Negation)

$\equiv p \vee (\neg p \vee q)$

$\equiv (p \vee \neg p) \vee q)$

$\equiv T \vee q$

$\equiv T$ — ②.

from ① & ②.

① $\equiv$ ②

$\therefore p \rightarrow (q \rightarrow p) \equiv \neg p \rightarrow (p \rightarrow q)$

All tautologies are equivalent to one another.

4. Determine which of the follwn Statements are tautologies or contradiction without truth table.

PRINCIPLE OF MATHEMATICAL INDUCTION.

Let $P(n)$ be a statement involving positive integer prove that If $P(1)$ is true.

Induction: Consider $P(k)$ is true. then $P(k+1)$ is also true then we can conclude the $P(k)$ is true for all positive integer.

1. Prove that $1+3+5+\cdots+(2n-1) = n^2$ by using mathetical induction.

Let $P(n) = 1+3+5+\cdots+(2n-1) = n^2$.

Put $n=1$

$P(1) = 2(1)-(1) = 1$

$n^2 = 1^2 = 1$

$\therefore P(1)$ is true.

Assume that the above statement is true for $P(k)$

to Prove that $P(k+1)$ is true.

$P(k+1) = 1+3+5+\cdots+(2k-1)+(2(k+1)-1) = (k+1)^2$

$= 2(k+1) - 1 + k^2$

$= 2k+2-1+k^2$

$= 2k+1+k^2 = (k+1)^2$

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2. $1.2 + 2.3 + 3.4 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$.

$P(n) = 1.2 + 2.3 + 3.4 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$.

$P(1) = 1.2 + 2.3 + 3.4 + \dots + 1(1+1) = \frac{1(1+1)(1+2)}{3}$.

$2 = \frac{2(3)}{3}$

$2 = 2$.

P(1) is true.

$P(K) = 1.2 + 2.3 + 3.4 + \dots + K(K+1) = \frac{K(K+1)(K+2)}{3}$.

to prove P(K) is true.

Then P(K+1) should be true.

$P(K+1) = 1.2 + 2.3 + 3.4 + \dots + K(K+1) + (K+1)(K+1+1)$
$= \frac{(K+1)(K+2)(K+3)}{3}$

$\cancel{K}(K+1) + (K+1)(K+2)$

3. Prove $n! \geq 2^{n-1}$ for all $n \geq 1$

$P(n) = n! \geq 2^{n-1} \quad \forall n \geq 1$

$P(1) = 1! \geq 2^0$

$P(1) = 1! \geq 1$

$\therefore P(1)$ is true.

Let $P(k)$ is true.

$P(k) = k! \geq 2^{k-1}$

The $P(k+1)$ must be true.

$P(k+1) = (k+1)! \geq 2^{k+1-1}$

$\Rightarrow k!(k+1) \geq 2^k$

we know $k! \geq 2^{k-1}$.

$2^{k-1}(k+1) \geq 2^k$

$2^{k-1} \times k + 2^{k-1} \geq 2^k$

$2 \cdot 2^{k-1} \geq 2^k$

$2^k \geq 2^k$

4. Prove $8^n - 3^n$ is divisible by 5.

Let $P(n) = 8^n - 3^n$ is divisible by 5

$P(1) = 8^1 - 3^1 = 5$ is divisible by 5.

$P(1)$ is true.

Assume $P(k) = 8^k - 3^k$ is divisible by 5 is true

$P(k+1) = 8^{k+1} - 3^{k+1}$

$= 8^k \cdot 8 - 3^k \cdot 3.$

$= 8^k(5+3) - 3^k \cdot 3$

$= 3(8^k - 3^k) + 5 8^k = 3(5^k) + 5(8^k) = 5\,[(5^{k-1})3 + 8^k]$

is divisible by 5

ARGUMENT - Sequence of statement that ends in conclusion. Set of one or more premices and conclusion. The argument is true only if it is not possible to prove all premices true and a conclusion false.

THEORY OF INFERENCE

1. If I love cat then I love dog.

I love cat $\rightarrow p$

I love dog $= q$

If I love cat then I love dog : $p \rightarrow q$.

$$\frac{p \rightarrow q^{T} \rightarrow T}{\therefore q}$$

$p \rightarrow q$ is T and

2. Determine whether $H_1 : p \rightarrow q$ $H_2 : p$. $H_1 \wedge H_2$

p	q	$p \rightarrow q$	$H_1 \wedge H_2$	$\therefore q$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	F

Rule P : use all the premises

1- If it rains heavily then travelling will be difficult. If students arrive on time then travelling will not be difficult. They arrive on time H

Rule T, use the rules of inference

It rains heavily : p.

Travelling is difficult : q

Student arrive on time : r

Rule CP → Conditional Proof.

$H_1 : p \rightarrow q$.

$H_2 : r \rightarrow \neg q$

$H_3 : r$

$C \rightarrow \neg p$.

$$\begin{array}{l} p \rightarrow q \\ r \rightarrow \neg q \\ r \\ \hline \neg p \end{array}$$

Step no	Statement	Reason
1	$p \rightarrow q$	Rule P
2	$\neg q \rightarrow \neg p$	Rule T Contrapositive
3	$r \rightarrow \neg q$	Rule P
4.	$r \rightarrow \neg p$	Rule T using hypothetical syllogism
5	r	Rule P
6.	$\neg p$	Rule T modus ponens

∴ The Conclusion is Valid.

2. show that $t \wedge s$ can be derived from the premices, $p \to q$, $q \to \neg r$, r, $p \vee (t \wedge s)$

Step no	Statement	Reason
1.	$p \to q$	Rule P.
2.	$q \to \neg r$	Rule P
3.	$p \to \neg r$	Rule T, hypothetical Sylogism.
4.	r	Rule P.
5.	$\neg p$	Rule T, modeum ponen
6.	$p \vee (t \wedge s)$	Rule P
7.	$t \wedge s$	Rule T.