MODE THEORY FOR CIRCULAR WAVEGUIDES

Maxwell's Equation:

To consider Maxwell's equation that gives relationship between electric and magnetic fields.

* Assuming a dielectric material having no current and free charges

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad — (1)$$

$$\nabla \times H = \frac{\partial D}{\partial t} \quad — (2)$$

$$\nabla \cdot D = 0 \quad — (3)$$

$$\nabla \cdot B = 0 \quad — (4)$$

$D = \varepsilon E$; $B = \mu H$

ε - permitivity
μ - permeability

Take curl on Eqn (1)

$$\nabla \times (\nabla \times E) = -\mu \frac{\partial (\nabla \times H)}{\partial t}$$

$$= -\mu \frac{\partial}{\partial t}\left(\frac{\partial D}{\partial t}\right)$$

$$= -\mu \varepsilon \frac{\partial^2 E}{\partial t^2} \quad — (5)$$

Using vector Identity,

$$\nabla \times (\nabla \times E) = \nabla \cdot (\nabla \cdot E) - \nabla^2 E$$

We know that, $\nabla \cdot E = 0$ & $\nabla \cdot H = 0$

$$\Rightarrow \quad -\nabla^2 E = \nabla \times (\nabla \times E)$$

$$= -\mu\varepsilon \frac{\partial^2 E}{\partial t^2}.$$

$$\boxed{\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \quad — (6)}$$

$$\boxed{\nabla^2 H = \varepsilon\mu \frac{\partial^2 H}{\partial t^2} \quad — (7)}$$

Equation (6) & (7) are the standard Wave Equations.

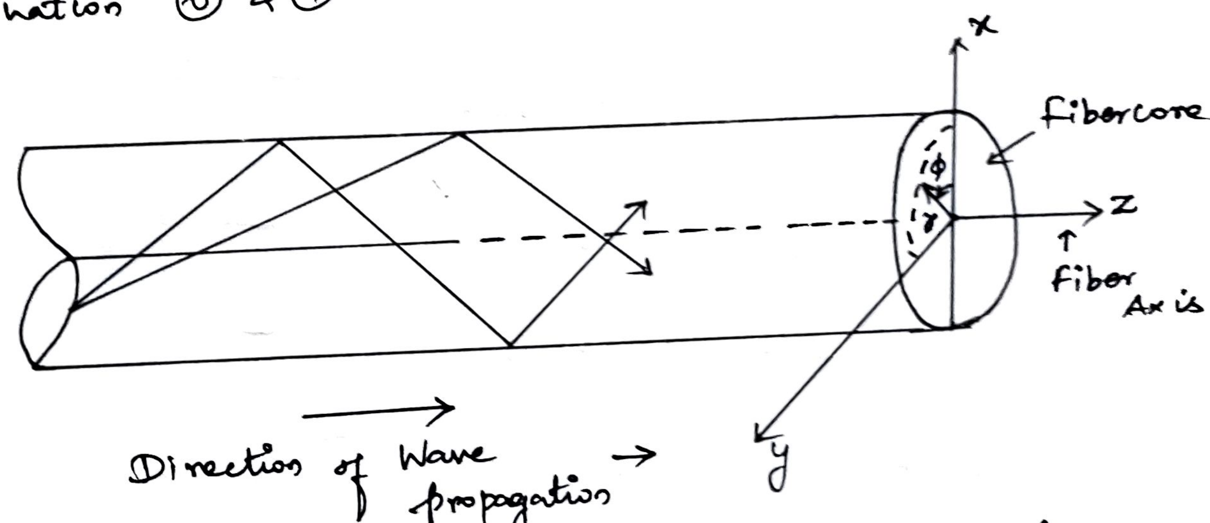


Fig: Cylindrical co-ordinate system used for analyzing electromagnetic wave propagation in an optical fiber.

If the electromagnetic waves are to propagate along the z-axis

$$E = E_0 (r, \phi)\, e^{j(\omega t - \beta z)} \quad — (8)$$

$$H = H_0 (r, \phi)\, e^{j(\omega t - \beta z)} \quad — (9)$$

which are harmonic in time 't' & coordinate 'z'.

* Parameter 'β' is the z-component to the propagation vector and determined by boundary conditions.

$$\nabla \times E = \begin{bmatrix} a_r & a_\phi & a_z \\ \partial/\partial r & \frac{1}{r}\partial/\partial\phi & \partial/\partial z \\ E_r & E_\phi & E_z \end{bmatrix}$$

$$\nabla \times E = a_r\left[\frac{1}{r}\frac{\partial E_z}{\partial\phi} - \frac{\partial E_\phi}{\partial z}\right] - a_\phi\left[\frac{\partial E_z}{\partial r} - \frac{\partial E_r}{\partial z}\right]$$

$$+ a_z\left[\frac{\partial E_\phi}{\partial r} - \frac{1}{r}\frac{\partial E_r}{\partial\phi}\right]$$

$$= \frac{a_r}{r}\left[\frac{\partial E_z}{\partial\phi} - r\frac{\partial E_\phi}{\partial z}\right] - a_\phi\left[\frac{\partial E_z}{\partial r} - \frac{\partial E_r}{\partial z}\right]$$

$$+ \frac{a_z}{r}\left[r\frac{\partial E_\phi}{\partial r} - \frac{\partial E_r}{\partial\phi}\right] \quad - (10)$$

From Eqn. (8) $\Rightarrow E_\phi = E_0(r,\phi)\, e^{j(\omega t - \beta z)}$

Diff. eqn. (8) w.r.t 'z', we get

$$\frac{\partial E_\phi}{\partial z} = E_0(r,\phi)\, e^{j(\omega t-\beta z)} \cdot -j\beta$$

$$= -j\beta E_\phi$$

$$\frac{\partial E_\phi}{\partial z} = -j\beta E_\phi.$$ Similarly $\frac{\partial E_r}{\partial z} = -j\beta E_r.$

Substitute the above values in eqn. (10)

$$\nabla \times E = \frac{a_r}{r}\left[\frac{\partial E_z}{\partial\phi} + jr\beta E_\phi\right] - a_\phi\left[\frac{\partial E_z}{\partial r} + j\beta E_r\right]$$

$$+ \frac{a_z}{r}\left[r\frac{\partial E_\phi}{\partial r} - \frac{\partial E_r}{\partial\phi}\right]$$

From Maxwell's equation, W.K.T

$$\nabla \times E = -j\omega\mu H \quad ; \quad \nabla \times H = j\omega\varepsilon E.$$

$$\therefore \nabla \times E = -j\omega\mu \left[H_r \vec{a_r} + H_\phi \vec{a_\phi} + H_z \vec{a_z} \right]$$

Equate like coefficients

$$\frac{1}{r}\left[\frac{\partial E_z}{\partial \phi} + j r \beta E_\phi\right] = -j\omega\mu H_r \quad \text{— (A)}$$

$$\frac{\partial E_z}{\partial r} + j\beta E_r = j\omega\mu H_\phi \quad \text{— (B)}$$

$$\frac{1}{r}\left[r\frac{\partial E_\phi}{\partial r} - \frac{\partial E_r}{\partial \phi}\right] = -j\omega\mu H_z \quad \text{— (C)}$$

Similarly,

$$\nabla \times H = j\omega\varepsilon \left[E_r \vec{a_r} + E_\phi \vec{a_\phi} + E_z \vec{a_z} \right]$$

Equate like coefficients

$$\frac{1}{r}\left[\frac{\partial H_z}{\partial \phi} + j r \beta H_\phi\right] = j\omega\varepsilon E_r \quad \text{— (D)}$$

$$\frac{\partial H_z}{\partial r} + j\beta H_r = -j\omega\varepsilon E_\phi \quad \text{— (E)}$$

$$\frac{1}{r}\left[r\frac{\partial H_\phi}{\partial r} - \frac{\partial H_r}{\partial \phi}\right] = j\omega\varepsilon E_z \quad \text{— (F)}$$

Take (A) & (E) to get E_ϕ Value.

From (A) $\Rightarrow H_r = -\frac{1}{j\omega\mu r}\left[\frac{\partial E_z}{\partial \phi} + jr\beta E_\phi\right]$ — (11)

Substitute eqn. (11) in (E) to get E_ϕ value.

$$\Rightarrow -\left[\frac{\partial H_z}{\partial r} + j\beta\left(-\frac{1}{j\omega\mu r}\left[\frac{\partial E_z}{\partial \phi} + jr\beta E_\phi\right]\right)\right] = j\omega\varepsilon E_\phi$$

$$\Rightarrow -\left[\frac{\partial H_z}{\partial r} - \frac{j\beta}{j\omega\mu r}\left(\frac{\partial E_z}{\partial \phi} + jr\beta E_\phi\right)\right] = j\omega\varepsilon E_\phi$$

$$\Rightarrow -\frac{\partial H_z}{\partial r} + \frac{\beta}{\omega\mu r}\frac{\partial E_z}{\partial \phi} = j\omega\varepsilon E_\phi - \frac{j\beta^2 E_\phi}{\omega\mu}$$

$$= jE_\phi\left[\omega\varepsilon - \frac{\beta^2}{\omega\mu}\right]$$

$$\omega\varepsilon - \frac{\beta^2}{\omega\mu} = \frac{\omega^2\mu\varepsilon - \beta^2}{\omega\mu} = \frac{q^2}{\omega\mu} \quad \text{where, } q^2 = \omega^2\mu\varepsilon - \beta^2.$$

$$= jE_\phi\frac{q^2}{\omega\mu}.$$

$$\Rightarrow -\frac{\partial H_z}{\partial r} + \frac{\beta}{\omega\mu r}\frac{\partial E_z}{\partial \phi} = jE_\phi\frac{q^2}{\omega\mu}$$

$$E_\phi = \frac{\omega\mu}{jq^2}\left[-\frac{\partial H_z}{\partial r} + \frac{\beta}{\omega\mu r}\frac{\partial E_z}{\partial \phi}\right]$$

$$= \frac{-j}{q^2}\left[-\omega\mu\frac{\partial H_z}{\partial r} + \frac{\beta}{r}\frac{\partial E_z}{\partial \phi}\right]$$

$$E_\phi = \frac{j}{q^2}\left[\omega\mu \frac{\partial H_z}{\partial r} - \frac{\beta}{r}\frac{\partial E_z}{\partial \phi}\right] \quad \text{— (12)}$$

To get value of E_r:

Solve (B) & (D) to get value of E_r.

From (B) $\Rightarrow H_\phi = \frac{1}{j\omega\mu}\left[\frac{\partial E_z}{\partial r} + j\beta E_r\right]$

Substitute the above H_ϕ value in (D)

$$\Rightarrow \frac{1}{r}\left[\frac{\partial H_z}{\partial \phi} + jr\beta\left\{\frac{1}{j\omega\mu}\left(\frac{\partial E_z}{\partial r} + j\beta E_r\right)\right\}\right] = j\omega\varepsilon E_r$$

$$\Rightarrow \frac{1}{r}\left[\frac{\partial H_z}{\partial \phi} + \frac{jr\beta}{j\omega\mu}\frac{\partial E_z}{\partial r} + \frac{j^2 r \beta^2 E_r}{j\omega\mu}\right] = j\omega\varepsilon E_r.$$

$$\Rightarrow \frac{1}{r}\frac{\partial H_z}{\partial \phi} + \frac{jr\beta}{j\omega\mu r}\frac{\partial E_z}{\partial r} + \frac{j^2 r\beta^2 E_r}{rj\omega\mu} = j\omega\varepsilon E_r.$$

$$\Rightarrow \frac{1}{r}\frac{\partial H_z}{\partial \phi} + \frac{\beta}{\omega\mu}\frac{\partial E_z}{\partial r} + \frac{j\beta^2 E_r}{\omega\mu} = j\omega\varepsilon E_r.$$

$$\Rightarrow \frac{1}{r}\frac{\partial H_z}{\partial \phi} + \frac{\beta}{\omega\mu}\frac{\partial E_z}{\partial r} = j\omega\varepsilon E_r - \frac{j\beta^2 E_r}{\omega\mu}$$

$$= jE_r\left[\omega\varepsilon - \frac{\beta^2}{\omega\mu}\right]$$

$$= jE_r\left[\frac{\omega^2\mu\varepsilon - \beta^2}{\omega\mu}\right]$$

$$= jE_r\frac{q^2}{\omega\mu} \quad \left[\because q^2 = \omega^2\mu\varepsilon - \beta^2\right]$$

$$\Rightarrow \frac{1}{r}\frac{\partial H_z}{\partial \phi} + \frac{\beta}{\omega\mu}\frac{\partial E_z}{\partial r} = jE_r \frac{q^2}{\omega\mu}.$$

$$E_r = \frac{\omega\mu}{jq^2}\left[\frac{1}{r}\frac{\partial H_z}{\partial \phi} + \frac{\beta}{\omega\mu}\frac{\partial E_z}{\partial r}\right]$$

$$\boxed{E_r = -\frac{j}{q^2}\left[\frac{\omega\mu}{r}\frac{\partial H_z}{\partial \phi} + \beta\frac{\partial E_z}{\partial r}\right]} \quad \text{— (13)}$$

Similarly,

$$H_r = \frac{-j}{q^2}\left(\beta\frac{\partial H_z}{\partial r} - \frac{\omega\varepsilon}{r}\frac{\partial E_z}{\partial \phi}\right) \quad \text{— (14)}$$

$$H_\phi = \frac{-j}{q^2}\left(\frac{\beta}{r}\frac{\partial H_z}{\partial \phi} + \omega\varepsilon\frac{\partial E_z}{\partial r}\right) \quad \text{— (15)}$$

Substitute,

E_r & E_ϕ in (C) i.e. (12) & (13) in (C)

H_r & H_ϕ in (F) i.e. (14) & (15) in (F)

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E_z}{\partial \phi^2} + q^2 E_z = 0 \quad \text{— (16)}$$

$$\frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r}\frac{\partial H_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 H_z}{\partial \phi^2} + q^2 H_z = 0 \quad \text{— (17)}$$

Eqns (16) & (17) contain either only E_z or H_z.

→ This appears to imply that the E and H are uncoupled.

* In general, coupling of E_z and H_z is required by the boundary conditions of the Electromagnetic field.

* If the boundary conditions, do not lead to coupling between the field components, the mode solutions can be obtained in which $E_z = 0$ or $H_z = 0$.

* When $E_z = 0 \rightarrow$ The modes are called Transverse Electric modes (TE Modes).

* When $H_z = 0 \rightarrow$ The modes are called Transverse Magnetic modes (TM modes).

* Hybrid modes exist if both E_z and H_z are non-zero.

WAVE EQUATIONS FOR STEP INDEX FIBERS

Use the results obtained to find the guided modes in Step-index fibers.

* By seperation of variables method

$$E_z = A\, F_1(r)\, F_2(\phi)\, F_3(z)\, F_4(t) \quad — (18)$$

The time and z dependent factors are given by

$$F_3(z)\, F_4(t) = e^{j(\omega t - \beta z)} \quad — (19)$$

Since the wave is sinusoidal in time and propagate in the z-direction.

$$F_2(\phi) = e^{j\nu\phi} \quad — (20)$$

The 'ν' can be positive or negative

Substitute (20) in (18), the wave equation for E_z (16) becomes

$$\boxed{\frac{\partial^2 F_1}{\partial r^2} + \frac{1}{r}\frac{\partial F_1}{\partial r} + \left(q^2 - \frac{\nu^2}{r^2}\right) F_1 = 0} \quad — (21)$$

which is a well-known differential Equation for Bessel function.

Eqn (21) must be solved for the regions inside the core and outside the core.

Inside Core:

Guided modes must remain finite as $r \to 0$.

$r<a$:

Solutions are the bessel function of the first kind of order ν.

For these function, use common fn

$J_\nu(ur)$, $u^2 = k_1^2 - \beta^2$ with $k_1 = \frac{2\pi n_1}{\lambda}$.

$$E_z(r<a) = A\, J_\nu(ur)\, e^{j\nu\phi}\, e^{j(\omega t - \beta z)} \quad \text{--- (22)}$$

$$H_z(r<a) = B\, J_\nu(ur)\, e^{j\nu\phi}\, e^{j(\omega t - \beta z)} \quad \text{--- (23)}$$

A & B - arbitary constants.

Outside Core: Solutions must decay to zero as $r \to \infty$.

$r>a$:

Modified bessel fn $K_\nu(wr)$

$w^2 = \beta^2 - k_2^2$; $k_2 = \frac{2\pi n_2}{\lambda}$.

$$E_z(r>a) = C\, K_\nu(wr)\, e^{j\nu\phi}\, e^{j(\omega t - \beta z)} \quad \text{--- (24)}$$

$$H_z(r>a) = D\, K_\nu(wr)\, e^{j\nu\phi}\, e^{j(\omega t - \beta z)} \quad \text{--- (25)}$$

C & D - arbitary constants.

From the definition it is seen that

$K_\nu(wr) \to e^{-wr}$ as $wr \to \infty$

$K_\nu(wr) = 0$. it follows $w>0$.

This intuen implies that $\beta \geq k_2$, which represents a cut off condition.

The cut off condition is the point at which a mode is no longer bound to the core region.

A second condition on β deduced from $J_v(ur)$. Inside the core the parameter 'u' must be real for F_1.

which gives $k_1 \geq \beta$.

$$n_2 k = k_2 \leq \beta \leq k_1 = n_1 k.$$

$k = \frac{2\pi}{\lambda}$ free space propagation constant.

MODAL EQUATION:

→ The solution for β can be determined from the boundary conditions.

→ The boundary conditions require the tangential components E_ϕ and E_z of E inside and outside of the dielectric interface $(r=a)$.

→ Similarly for H_ϕ and H_z.

→ Consider the first ~~tanteng~~ tangential component of E.

For Inner core-cladding, consider $E_z = E_{z1}$

Outer core-cladding, consider $E_z = E_{z2}$.

Subtract (22) and (24) to find E_z.

$$E_{z1} - E_{z2} = A J_v(ur) - C K_v(wr) = 0$$

$(\because r = a)$

$$= A J_v(ua) - C K_v(wa) = 0 \quad \text{—} (26)$$

To find $E\phi$

~~Subtract~~ Sub. (22) and (23) in (13) to find $E\phi_1$

Sub. (24) and (25) in (15) to find $E\phi_2$.

$$E\phi_1 - E\phi_2 = \frac{-j}{u^2}\left[A \frac{jv\beta}{a} J_v(ua) - B\omega\mu u J_v'(ua)\right]$$

$$- \frac{j}{w^2}\left[C \frac{jv\beta}{a} K_v(wa) - D\omega\mu w K_v'(wa)\right] = 0 \quad \text{—} (27)$$

Similarly for H

$$H_{z1} - H_{z2} = B J_v(ua) - D K_v(wa) = 0 \quad \text{—} (28)$$

$$H\phi_1 - H\phi_2 = \frac{-j}{u^2}\left[B \frac{jv\beta}{a} J_v(ua) + A\omega\varepsilon_1 u J_v'(ua)\right]$$

$$- \frac{j}{w^2}\left[D \frac{jv\beta}{a} K_v(wa) + C\omega\varepsilon_2 w K_v'(wa)\right]$$

$$= 0 \quad \text{—} (29)$$

Set of four Equations with four unknown coefficients A, B, C, D.

$$\begin{array}{cccc} A & B & C & D \\ \end{array}$$

$$\begin{bmatrix} J_\nu(ua) & 0 & -K_\nu(wa) & 0 \\ \frac{\beta\nu}{au^2} J_\nu(ua) & \frac{j\omega\mu}{u} J_\nu'(ua) & \frac{\beta\nu}{aw^2} K_\nu(wa) & \frac{j\omega\mu}{w} K_\nu'(wa) \\ 0 & J_\nu(ua) & 0 & -K_\nu(wa) \\ -\frac{j\omega\varepsilon_1}{u} J_\nu'(ua) & \frac{\beta\nu}{au^2} J_\nu(ua) & -\frac{j\omega\varepsilon_2}{w} K_\nu'(wa) & \frac{\beta\nu}{aw^2} K_\nu(wa) \end{bmatrix} = 0 \quad -(30)$$

By determinant, find the Eigen value Equation

$$(J_\nu + K_\nu)(k_1^2 J_\nu + k_2^2 K_\nu) = \left(\frac{\beta\nu}{a}\right)^2 \left(\frac{1}{u^2} + \frac{1}{w^2}\right)^2$$

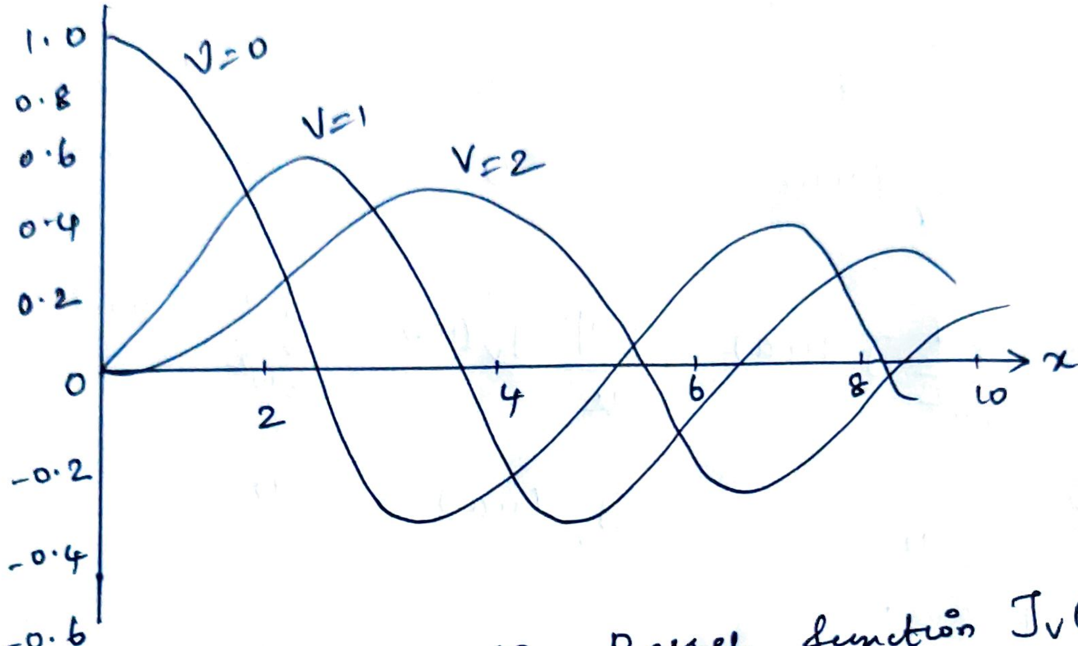
where,

$$J_\nu = \frac{J_\nu'(ua)}{u J_\nu(ua)} \qquad K_\nu = \frac{K_\nu'(wa)}{w K_\nu(wa)} \quad -(31)$$

Solving eqn (31) for β, it will be found only discrete values restricted for β.

→ For any particular mode, characteristics can be found by Eqn. (31).

MODES IN STEP INDEX FIBER.

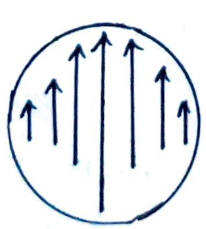


Variation of the Bessel function $J_v(x)$ for the first three orders ($v=0,1,2$) plotted as a function of x.

Because of the oscillatory behaviour of J_v, there will be 'm' roots for a given 'v' value.

→ The roots will be designated by β_{vm} & corresponding modes are TE_{vm}, TM_{vm}, EH_{vm}, HE_{vm}

Lowest order mode.

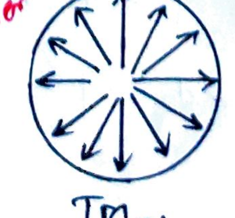


HE_{11}

HE_{21}

First set of higher order modes.

TE_{01}



TM_{01}

For the dielectric fiber waveguide, all modes are hybrid modes, except $V=0$.

when $V=0$,

$$J_0 + K_0 = 0 \qquad (\because J_0'(z) = -J_1(z))$$

or using the relation for J_v' & K_v'.

$$\frac{J_1(ua)}{u J_0(ua)} + \frac{K_1(wa)}{w K_0(wa)} = 0$$

which corresponds to TE_{om} modes $(E_z = 0)$

$$k_1^2 J_0 + k_2^2 K_0 = 0$$

(or)

$$\frac{k_1^2 J_1(ua)}{u J_0(ua)} + \frac{k_2^2 K_1(wa)}{w K_0(wa)} = 0$$

which corresponds to TM_{om} modes.

$V \neq 0$, more complex and numerical methods are needed to solve.

By simplified, accurate approximatation, core & cladding refractive index are nearly the same.

$$n_1 - n_2 << 1 \rightarrow \text{Weakly guided modes.}$$

V	Mode	Cut-off
0	TE_{0m}, TM_{0m}	$J_0(ua) = 0$
1	HE_{1m}, EH_{1m}	$J_1(ua) = 0$
≥ 2	$EH_{\nu m}$	$J_\nu(ua) = 0$
	$HE_{\nu m}$	$\left(\frac{n_1^2}{n_2^2}+1\right) J_{\nu-1}(ua) = \frac{ua}{\nu-1} J_\nu(ua)$.

Normalized propagation constant,

$$b = \frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2}.$$

The modes are cut-off when $(\beta/k) = n_2$.

HE_{11}, no cut off, core diameter $\to 0$

Approximately,

$$V = \frac{2\pi a}{\lambda}\sqrt{NA} \leq 2.405.$$

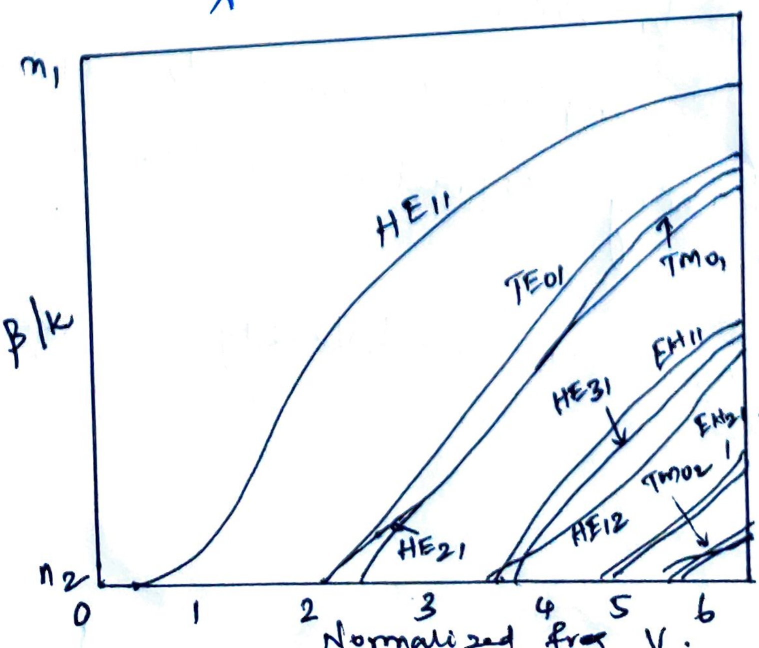


Fig: Plots for propagation Const. (β/k) vs Normalized frequency V for a fewest of lowest order modes.

LINEARLY POLARIZED MODES

→ By now, Exact Analysis of the fiber mode is complex.

→ However, a simple but high accurate approximation based on the principle of step index fiber $\Delta << 1$.

→ Weakly guiding fiber Approximation,

When $\Delta << 1$, $k_1^2 \approx k_2^2 \approx \beta^2$.

Using this approximation.

$$J_v + K_v = \pm \frac{v}{a}\left(\frac{1}{u^2} + \frac{1}{w^2}\right)$$

$$TE_{om} = \frac{J_1(ua)}{u J_0(ua)} + \frac{K_1(wa)}{w K_0(wa)} = 0$$

$$TM_{om} = \frac{k_1^2 J_1(ua)}{u J_0(ua)} + \frac{k_2^2 K_1(wa)}{w K_0(wa)} = 0$$

By using +ve and −ve signs.

$$\frac{J_{v+1}(ua)}{u J_v(ua)} + \frac{K_{v+1}(wa)}{w K_v(wa)} = 0 \longrightarrow EH$$

$$\frac{J_{v-1}(ua)}{u J_v(ua)} - \frac{K_{v-1}(wa)}{w K_v(wa)} = 0 \longrightarrow HE.$$

$$j = \begin{cases} 1 & \text{for } TE, TM \\ v+1 & \text{for } EH \\ v-1 & \text{for } HE \end{cases}$$

$$\frac{u\, J_{j-1}(ua)}{J_j(ua)} = -\frac{w\, K_{j-1}(wa)}{K_j(wa)} .$$

Last two equations show that, within the weakly guiding approximation.

This means that these modes are degenerate

LP_{jm} → 'j' means one half the number of maxima / minima intensity that occurs while the angular coordinate changes from 0 to 2π radians.

→ 'm' means the number of maxima intensity that occurs while the radial co-ordinate changes from 0 to ∞.

LP_{0m} derived from an HE_{1m} mode

LP_{1m} " " " TE_{0m}, TM_{0m}, HE_{2m}

LP_{vm} $(v \geq 2)$ " " " $HE_{v+1\,m}$ and $EH_{v-1\,m}$.

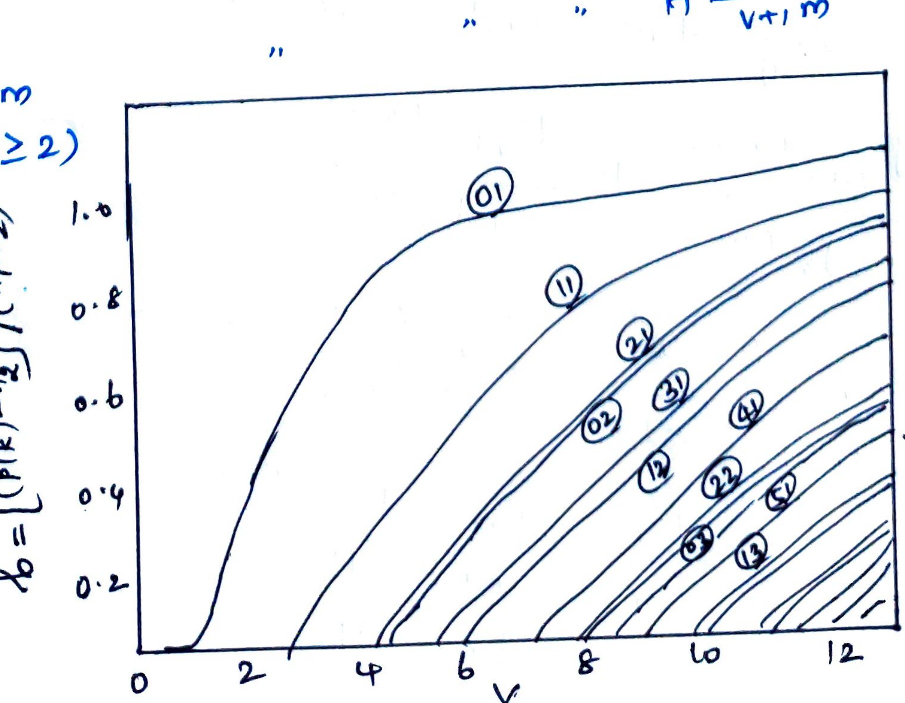


Fig: plots for the propagation constant 'b' as a fn. of V for various LP_{jm} modes

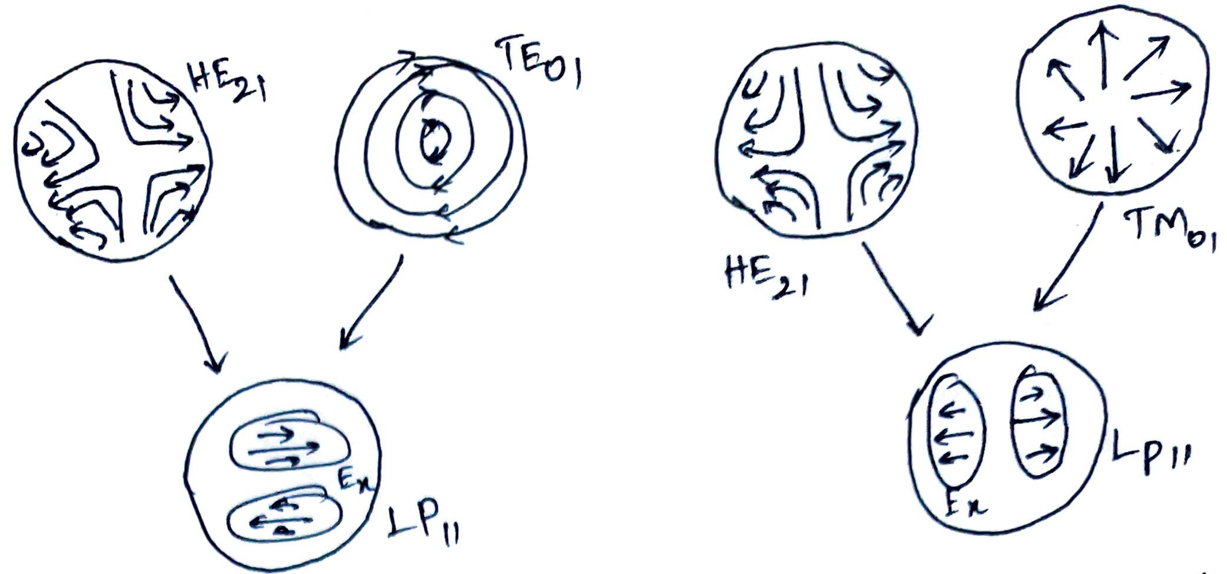


Fig: Composition of two LP_{11} modes from exact modes and their transverse electric field and intensity distributions.

→ These imperfections break the circular symmetry of the Ideal fiber.

→ The modes propagate with different phase velocities and the difference between their effective refractive indices is called **Fiber Birefringence**.

$$B_f = n_y - n_x$$

$$\beta = k_0 (n_y - n_x) \qquad \left(\because k_0 = \frac{2\pi}{\lambda}\right).$$

* If light is injected into the fiber so that both modes are excited.

* The one will be delayed in phase relative to the other as they propagate.

* When the phase difference is integral multiple of 2π, the two modes will beat at this point, and the input polarization will be produced. The length over which this beating occurs is the "Fiber beat length"

$$L_p = 2\pi/\beta \;;\; B_f = \frac{\lambda}{L_p}.$$

LIST OF FORMULAS

1) Refractive Index $= \dfrac{\text{Speed of light in Vaccum (c)}}{\text{Speed of light in material (v)}}$

2) Snell's law $\Rightarrow$ $n_1 \sin\theta_1 = n_2 \sin\theta_2$.

3) Critical Angle $\Rightarrow$ $\theta_c = \sin^{-1}\left(\dfrac{n_2}{n_1}\right)$.

4) Acceptance Angle $\Rightarrow$ $\theta_{a(max)} = \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right)$
$= \sin^{-1}(NA)$.

5) Numerical Aperture $\Rightarrow$ i) $NA = \sin\theta_a$.
ii) $NA = n_1\sqrt{2\Delta}$. $\Delta = \dfrac{n_1^2 - n_2^2}{2n_1^2}$.

6) Normalized frequency $\Rightarrow$ $V = \dfrac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$
$= \dfrac{2\pi a}{\lambda}(NA)$.

i) Number of guided modes for Step Index fiber $M = \dfrac{V^2}{2}$
ii) " " " " Graded " " $M = \dfrac{V^2}{4}$.

7) Cut-off Wavelength $\Rightarrow$ $\lambda_c = \dfrac{2\pi a}{V_c}\sqrt{n_1^2 - n_2^2}$.
$= \dfrac{2\pi a n_1 \sqrt{2\Delta}}{V_c}$.

i) cut off normalized frequency $V_c = 2.405$ for Step Index fiber

8) Mode-field diameter $\Rightarrow$ $E(r) = E_0 \exp\left(-\frac{r^2}{W_0^2}\right)$.
(Gaussian Appr.)

W_0 - width of E.F. distribution

$$2W_0 = 2\left[\frac{2\int_0^\infty r^3 E^2(r)\,dr}{\int_0^\infty r E^2(r)\,dr}\right]^{1/2}$$

9) Fiber Birefringence $\Rightarrow$ $B_f = n_y - n_x$

or $B_f = k_0 (n_y - n_x)$.

10) Fiber Beat Length $\Rightarrow$ $L_p = \frac{2\pi}{\beta}$