Reg.No. | | | | | | | | | | | | | | | |

FACULTY OF ENGINEERING AND TECHNOLOGY, SRM IST
DEPARTMENT OF MATHEMATICS
Cycle Test – II
Academic Year 2018 – 2019 [ODD]

Program offered: B.Tech (CSE/SWE)
Year/Sem : III/V
Max. Marks: 50

Duration : 1Hr 40 minutes
Date of Exam: 19.9.2018

Course code & title: 15MA201 / Transforms & Boundary Value Problems

SET - A

12/10 5

Part – A (5 × 4 = 20 marks)
Answer all the questions.

Sl.No.	Question	Course outcome	Bloom's Taxonomy	Marks
1	State Dirichlet's conditions for the existence of the Fourier series $f(x)$ in $(C, C+2l)$	a	Knowledge	4
2	Find the half range Fourier sine series for the function $f(x) = x$ in $0 < x < \pi$. $\frac{2}{n}(-1)^{n+1}$	e	Application	4
3	Find the Root Mean square value for the function $f(x) = x - x^2$ in $-1 < x < 1$. $\sqrt{8/15}$	e	Analysis	4
4	State one dimensional wave equation and write its all possible solutions.	e	Application	4
5	Classify the following Partial Differential equations (i) $f_{xx} + 2f_{xy} + 4f_{yy} = 0$ (ii) $4u_{xx} = u_t$ elliptic parabolic	a	Knowledge	4

Part B(3 × 10 = 30 marks)
Answer ANY THREE questions.

Sl.No.	Question	Course outcome	Bloom's Taxonomy	Marks
6.	Find the Fourier series expansion of the period $2l$ for the function $f(x) = (l-x)^2$ in the interval $(0, 2l)$. Deduce the sum series $\sum_{n=1}^{\infty} \frac{1}{n^2}$. $a_0 = \frac{2l^2}{3}$ $a_n = \frac{4l^2}{n^2\pi^2}$	a, e $b_n = 0$	Application $x = 0$ deduction - 3m	10
7.	Find the Fourier series expansion of period 2π for the function $y = f(x)$ which is defined in $(0, 2\pi)$ by means of the table of values given below. Find the series upto the second harmonic. x: 0, $\frac{\pi}{3}$, $\frac{2\pi}{3}$, π, $\frac{4\pi}{3}$, $\frac{5\pi}{3}$, 2π y: 1.0, 1.4, 1.9, 1.7, 1.5, 1.2, 1.0 $a_0 = 2.9$ $a_1 = -0.37$ $a_2 = -0.1$ $b_1 = 0.17$	e	Application $b_2 = -0.06$	10
8.	A tightly stretched string with fixed end points $x = 0$ and $x = l$ initially at rest in its equilibrium position. If its set vibrating giving each point a velocity $\frac{\partial y}{\partial t} = 3x(l-x)$. Find the displacement.	a, e $A=0$ $d = n\pi/l$ $C=0$	Application $B_n = \frac{12 l^3}{n^4\pi^4 a}$ $[1-(-1$	10
9.	A rod of length 20 cm has its end A and B kept at $0°c$ and $100°c$ respectively until steady state condition prevail. If the temperature at each end is then suddenly reduced to $0°c$ and maintained so , find the temperature u(x , t) at a distance x from A at time t.	a, e $u(x) = 5x$	Application $=0$	10

Set A Answer key

15MA201 - Transforms & Boundary Value Problems

Slot C_1

Part A

1. Dirichlet's conditions : (4 marks)

If a function $f(x)$ is defined in $c \le x \le c+2l$, it can be expanded as a Fourier series of the form $\frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}\right)$, provided the following Dirichlet's conditions are satisfied.

(i) $f(x)$ is single valued and finite in $(c, c+2l)$

(ii) $f(x)$ is continuous or piecewise continuous with finite number of finite discontinuities in $(c, c+2l)$

(iii) $f(x)$ has a finite number of maxima or minima in $(c, c+2l)$

2.
$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx \quad \text{— (1 mark)}$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} x \sin nx \, dx = \frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - 1\left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[-\frac{\pi}{n}(-1)^n\right] = \frac{2}{n}(-1)^{n+1} \quad \text{— (2 marks)}$$

$$f(x) = 2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin nx \quad \text{— (1 mark)}$$

3. Root mean square value of $f(x) = \sqrt{\dfrac{\int_{-1}^{1}(x-x^2)^2\,dx}{2}}$

$$= \sqrt{\frac{1}{2}\times 2\int_0^1 (x^2+x^4)\,dx} = \sqrt{\frac{8}{15}} \quad \text{— (4 marks)}$$

4. $$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \quad \text{—— (1m)}$$

The various possible solutions of the wave equation are

(i) $y(x,t) = (A_1 e^{\lambda x} + B_1 e^{-\lambda x})(C_1 e^{\lambda at} + D_1 e^{-\lambda at})$

(ii) $y(x,t) = (A_2 \cos \lambda x + B_2 \sin \lambda x)(C_2 \cos \lambda at + D_2 \sin \lambda at)$

(iii) $y(x,t) = (A_3 x + B_3)(C_3 t + D_3)$ —— (3 marks)

5. (i) $B^2 - 4AC = 4 - 4(1)(4) = 4 - 16 = -12 < 0$ Elliptic —— (2m)

(ii) $B^2 - 4AC = 0 - 4(0) = 0$ parabolic —— (2m)

Part B

6. $$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} \quad \text{—— (1m)}$$

$$a_0 = \frac{1}{l}\int_0^{2l} (l-x)^2 dx = \frac{2l^2}{3} \quad \text{—— (2m)}$$

$$a_n = \frac{1}{l}\int_0^{2l} (l-x)^2 \cos\frac{n\pi x}{l} dx$$

$$= \frac{1}{l}\left[(l-x)^2\left(\frac{\sin\frac{n\pi x}{l}}{n\pi/l}\right) - 2(l-x)(-1)\left(\frac{-\cos\frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right) + 2\left(\frac{-\sin\frac{n\pi x}{l}}{\frac{n^3\pi^3}{l^3}}\right)\right]_0^{2l}$$

$$= \frac{4l^2}{n^2\pi^2} \quad \text{—— (2m)}$$

$$b_n = \frac{1}{l}\int_0^{2l} (l-x)^2 \sin\frac{n\pi x}{l} dx$$

$$= \frac{1}{l}\left[(l-x)^2\left(\frac{-\cos\frac{n\pi x}{l}}{n\pi/l}\right) - 2(l-x)(-1)\left(\frac{-\sin\frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right) + 2\left(\frac{\cos\frac{n\pi x}{l}}{\cdots}\right)\right]^{2l}$$

Scanned with CamScanner

$$b_n = 0 \quad \text{—— (2m)}$$

$$\therefore f(x) = \frac{l^2}{3} + \sum_{n=1}^{\infty} \frac{4l^2}{n^2\pi^2} \cos\frac{n\pi x}{l} \quad \text{—— ①}$$

Here 0 is a point of discontinuity

$\therefore$ Put $x = 0$ in ①

$$\frac{f(0) + f(2l)}{2} = \frac{l^2 + l^2}{2} = l^2$$

$$l^2 = \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2}, \quad \therefore \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \quad \text{—— (3m)}$$

7. $f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + b_1 \sin x + b_2 \sin 2x$ —(1m)

x	y	$y\cos x$	$y\sin x$	$y\cos 2x$	$y\sin 2x$
0	1.0	1	0	1	0
$\pi/3$	1.4	0.7	1.212	−0.7	1.212
$2\pi/3$	1.9	−0.95	1.65	−0.95	−1.645
π	1.7	−1.7	0	1.7	0
$\frac{4\pi}{3}$	1.5	−0.75	−1.299	−0.75	1.299
$\frac{5\pi}{3}$	1.2	0.6	−1.039	−0.6	−1.039
					−0.1732
	$\Sigma y = 8.7$	$\Sigma y\cos x = -1.1$	$\Sigma y \sin x = 0.5196$	$\Sigma y\cos 2x = -0.3$	$\Sigma y \sin 2x = -0.1732$

—(4marks)

$$a_0 = 2\left[\frac{\Sigma y}{6}\right] = 2.9 \qquad b_1 = 2\left[\frac{\Sigma y \sin x}{6}\right] = 0.17$$

$$a_1 = 2\left[\frac{\Sigma y \cos x}{6}\right] = -0.37 \qquad b_2 = 2\left[\frac{\Sigma y \sin 2x}{6}\right] = -0.06$$

—(5marks)

$$a_2 = 2\left[\frac{\Sigma y \cos 2x}{6}\right] = -0.1$$

$$\therefore y = 1.45 + (-0.37\cos x + 0.17 \sin x) + (-0.1\cos 2x)$$

8. Wave equation is $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ —— (1m)

The b.c's are (i) $y(0,t)=0$ $t \geq 0$; (ii) $y(l,t)=0$

(iii) $y(x,0)=0$ (iv) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 3x(l-x)$ $0 \leq x \leq l$ —— (2m)

—— (1m)

The correct soln is $y(x,t)=(A\cos\lambda x + B\sin\lambda x)(C\cos\lambda at + D\sin\lambda at)$ —— (1)

Apply (i) b.c in (1) $\boxed{A=0}$, $y(x,t)=B\sin\lambda x(C\cos\lambda at + D\sin\lambda at)$ —— (2)

Apply (ii) b.c in (2), $\boxed{\lambda = \frac{n\pi}{l}}$ in 2)

—— (2m)

$y(x,t)= B\sin\frac{n\pi x}{l}\left(C\cos\frac{n\pi at}{l} + D\sin\frac{n\pi at}{l}\right) \longrightarrow (3)$

Apply (iii) b.c in (3)

$y(x,0)= B\sin\frac{n\pi x}{l}(C)=0 \Rightarrow \boxed{C=0}$ in (3)

The most general soln is $y(x,t)=\sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}\sin\frac{n\pi at}{l}$ —— (4)

Diff (4) partially w.r.to 't' —— (2m)

$\frac{\partial y}{\partial t} = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}\cos\frac{n\pi at}{l}\cdot\frac{n\pi a}{l}$

$\frac{\partial y(x,0)}{\partial t} = \sum_{n=1}^{\infty}\left(B_n\frac{n\pi a}{l}\right)\sin\frac{n\pi x}{l} = 3x(l-x)$ —— (5)

$\therefore B_n\cdot\frac{n\pi a}{l} = \frac{2}{l}\int_0^l 3x(l-x)\sin\frac{n\pi x}{l}\,dx$

$B_n = \frac{12l^3}{n^4\pi^4 a}[1-(-1)^n]$, $B_n = \begin{cases}\frac{24l^3}{n^4\pi^4 a} & n \text{ is odd}\\ 0 & n \text{ is even}\end{cases}$

$\therefore y(x,t)=\sum_{n=1,3,5}^{\infty}\frac{24l^3}{n^4\pi^4 a}\sin\frac{n\pi at}{l}\sin\frac{n\pi x}{l}$ —— (2m)

9. Heat equation is $\frac{\partial u}{\partial t} = \alpha^2\frac{\partial^2 u}{\partial x^2}$

When steady state conditions exists, the b.c's are

(i) $u(0)=0$ Apply (i), (ii) b.c's, $u(x)=\frac{100x}{l}$ 5r

(ii) $u(l)=100$ —— (2m). Steady state is

The end B is reduced to zero. The changed in to unsteady state

The b.c's are (i) $u(0,t)=0$, (ii) $u(l,t)=0$

(iii) $u(x,0)=\frac{100x}{l}$ 5r —— (2m)

The correct solution is

$$u(x,t) = (A\cos\lambda x + B\sin\lambda x)\, e^{-\alpha^2\lambda^2 t} \quad \text{—— (1)} \quad \text{—— 1m}$$

Apply (i) b.c in (1) $A=0$ in (1) —— 1m

$$u(x,t) = B\sin\lambda x \; e^{-\alpha^2\lambda^2 t} \quad \text{—— (2)}$$

Apply (ii) b.c in (2), $\lambda = \frac{n\pi}{l}$ —— 1m

$$u(x,t) = B\sin\frac{n\pi x}{l}\; e^{\frac{-\alpha^2 n^2\pi^2 t}{l^2}} \quad \text{—— (3)}$$

The most general soln is

$$u(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l}\; e^{\frac{-\alpha^2 n^2\pi^2 t}{l^2}} \quad \text{—— (4)}$$

Apply (iii) b.c in (4) —— 1m

$$u(x,0) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l} = \frac{100x}{l} \quad \text{—— (5)}$$

The half range F.S. series is $f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$ —— (6)

From (5) & (6) $\boxed{b_n = B_n}$

$$B_n = \frac{2}{l}\int_0^l \frac{100x}{l}\sin\frac{n\pi x}{l}\,dx = \frac{200}{n\pi}(-1)^{n+1}$$

Subst. in (4),

$$u(x,t) = \sum_{n=1}^{\infty} \frac{200}{n\pi}(-1)^{n+1} \sin\frac{n\pi x}{l}\; e^{\frac{-\alpha^2 n^2\pi^2 t}{l^2}}$$

—— 2m

Reg.No. | | | | | | | | | | | | | | |

FACULTY OF ENGINEERING AND TECHNOLOGY, SRM IST
DEPARTMENT OF MATHEMATICS
Cycle Test – II
Academic Year 2018 – 2019 [ODD]

Program offered: B.Tech (CSE/SWE/EIE) — **Duration : 1Hr 40 minutes**
Year/Sem : III/V — **Date of Exam: 19.9.2018**
Max. Marks: 50

Course code & title: 15MA201 / Transforms & Boundary Value Problems
SET - B

Part – A (5 × 4 = 20 marks)
Answer all the questions.

Sl.No.	Question	Course outcome	Bloom's Taxonomy	Marks
1	Find the half range Fourier cosine series for the function $f(x) = x$ in $0 < x < \pi$. $a_0 = \pi$	a	Knowledge	4
2	Find the RMS (Root Mean Square) value for the function $f(x) = x + 2$ in $-2 < x < 2$. $4/\sqrt{3}$	e	Application	4
3	Find constant term in the Fourier series expansion of $f(x) = x^2$ in $-\pi < x < \pi$. $2\pi/3$	e	Analysis	4
4	State one dimensional heat equation and write its all possible solutions.	e	Application	4
5	Classify the following PDE: (a). $y^2 u_{xx} - 2xyu_{xy} + x^2 u_{yy} + 2u_x - 3u = 0$. parabolic (b). $y^2 u_{xx} + u_{yy} + u_x^2 + u_y^2 + 5 = 0$. elliptic	a	Knowledge	4

Part B(3 × 10 = 30 marks)
Answer ANY THREE questions.

Sl.No.	Question	Course outcome	Bloom's Taxonomy	Marks
6.	Find the half-range cosine series of $f(x) = (\pi - x)^2$ in $(0, \pi)$. Hence find the sum of the series $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\infty$. $a_0 = \frac{2}{3}\pi^2$, $a_n = 4/n^2$	a, e	Application	10
7.	Find the Fourier series as far as the second harmonic to represent the function $f(x)$ with period 6, given in the following table: x: 0, 1, 2, 3, 4, 5 $f(x)$: 9, 18, 24, 28, 26, 20 $l = 3$, $a_0 = 41.66$, $b_2 = 0$	e	Application	10
8.	A tightly stretched string with fixed end points x=0 and $x = l$ is initially in a position given by $y(x,0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement y at any time and at any distance from the end x=0. $A = 0$, $\alpha = n\pi/l$, $D = 0$, $B_1 = 3y_0/4$, $B_3 = -y_0/4$	a, e	Application	10
9.	Find the solution of the equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ that satisfies the condition $u(0,t) = 0$ and $u(l,t) = 0$ for $t \geq 0$ and $u(x,0) = \begin{cases} x, & for\ 0 < x < l/2 \\ l - x, & for\ l/2 < x < l \end{cases}$. $A = 0$, $\alpha = n\pi/l$, $B_n = \frac{4l}{\pi^2 n^2} \sin\frac{n\pi}{2}$	a, e	Application	10

15MA201 - SET-B (C1-Slot Answerkey.) (C1)

① $f(x) = \frac{a_0}{2} + \sum_{1}^{\infty} a_n \cos nx$ ⟶ (1 marks)

$a_0 = \pi$ —— (1 marks)

$a_n = \begin{cases} -4/\pi n^2 & \text{if } n \text{ odd} \\ 0 & \text{if } n \text{ even} \end{cases} = \frac{2}{\pi}\left[\frac{1}{n^2}\left((-1)^n - 1\right)\right]$ —— (1 marks)

$x = \frac{\pi}{2} - \frac{4}{\pi}\sum_{odd}\frac{1}{n^2}\cos nx$ —— (1 - marks)

② $\bar{y} = \sqrt{\frac{\int_a^b y^2 dx}{b-a}} = 4/\sqrt{3}$ —— (4 marks)

③ $a_0 = \frac{2}{\pi}\int_0^{\pi} x^2 dx = \frac{2\pi^2}{3}$ —— (4 marks)

④ $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$; —— (1 marks)

(i) $u(x,t) = (A_1 e^{\lambda x} + B_1 e^{-\lambda x})\, C_1 e^{\alpha^2 \lambda^2 t}$ ⟶ (1 marks) (2

(ii) $u(x,t) = (A_2 \cos\lambda x + B_2 \sin\lambda x)\, C_2 e^{-\alpha^2\lambda^2 t}$; (iii) $u(x,t) = (A_3 x + B_3)\, C_3$

⑤ (a) $B^2 - 4AC = 4x^2y^2 - 4x^2y^2 = 0$ —— (2 marks) (parabolic)

(b) $B^2 - 4AC = -4y^2 < 0$ elliptic (2 marks)

⑥ $f(x) = \frac{a_0}{2} + \sum a_n \cos nx$ ⟶ (1 marks)

$a_0 = 2/3\, \pi^2$ (2 marks).

$a_n = \frac{4}{n^2}$, if $n \neq 0$ (2 marks)

$f(x) = \frac{\pi^2}{3} + 16\sum \frac{1}{n^2}\cos nx$ (1 marks)

Parseval's $\frac{a_0^2}{4} + 1/2 \sum a_n^2 = \frac{1}{\pi}\int_0^{\pi}(\pi - x)^4 dx$ (2 marks)

$\sum \frac{1}{n^4} = \frac{\pi^4}{90}$. (2 marks)

⑧. $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$; $\underset{(i)}{y(0,t) = 0}$; $\underset{(ii)}{y(l,t) =}$

$\underset{(iii)}{\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0}$; $\underset{(iv)}{y(x,0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)}$;

CS $y(x,t) = (A\cos\lambda x + B\sin\lambda x)(C\cos\lambda at + D\sin$

(i) ⟹ $A = 0$; (ii) ⟹ $\lambda = \frac{n\pi}{l}$; (iii) ⟹ $D = 0$

MSG $y(x,t) = \sum B_n \sin\frac{n\pi x}{l}\cos\frac{n\pi at}{l}$

(iv) $y(x,0) = \frac{y_0}{4}\left(3\sin\frac{\pi x}{l} - \sin\frac{3\pi x}{l}\right)$

$B_1 = \frac{3y_0}{4}$; $B_3 = \frac{-y_0}{4}$; $B_n = 0$ for $n \neq$

(1) $y(x,t) = \frac{3y_0}{4}\sin\frac{\pi x}{l}\cos\frac{\pi at}{l} - \frac{y_0}{4}\sin\frac{3\pi x}{l}\cos\frac{3\pi t}{l}$

⑦ $2l = 6 \Rightarrow l = 3$ —— (1 marks)

$f(x) = \frac{a_0}{2} + a_1\cos\frac{\pi x}{3} + a_2\cos\frac{2\pi x}{3} + b_1\sin\frac{\pi x}{3} + b_2\sin\frac{2\pi x}{3}$ (2 marks)

(4 marks)
$\sum y = 125$; $a_0 = 41.66$
$\sum y\cos\pi x/3 = -25$; $a_1 = -8.33$
$\sum y\sin\pi x/3 = -3.4$; $b_1 = -1.13$
$\sum y\cos\frac{2\pi x}{3} = -19$; $a_2 = -6.33$
(? marks)

⑨ $u(x,t) = (A\cos\lambda x + B\sin\lambda x)\, e^{-\alpha^2}$

$u(0,t) \Rightarrow A = 0$; $u(l,t) = 0 \Rightarrow \lambda = \frac{n\pi}{l}$

MGS $u(x,t) = \sum B_n \sin\frac{n\pi x}{l} e^{-\frac{\alpha^2 n^2\pi^2 t}{l^2}}$ ⟶ ②

$B_n = \frac{2}{l}\left[\int_0^{l/2} x\sin\frac{n\pi x}{l}dx + \int_{l/2}^{l}(l-x)\sin\frac{n\pi x}{l}dx\right]$

$= \frac{4l}{n^2\pi^2}\sin\frac{n\pi}{2}$

$u(x,t) = \frac{4l}{\pi^2}\sum\frac{1}{n^2}\sin\frac{n\pi}{2}\sin\frac{n\pi x}{l} e^{-\alpha^2 n^2 \ldots}$

SET C

Reg.No

SRM INSTITUTE OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF MATHEMATICS
CYCLE TEST 2
15MA201- TRANSFORMS AND BOUNDARY VALUE PROBLEMS

Date: 19-09-2018

Time: 100 Minutes
Max. marks : 50

Part –A (5 × 4 = 20 marks)
Answer **ALL** the Questions.

1. Find the constant term a_0 of Fourier series for $f(x) = \begin{cases} x, & -1 < x < 0 \\ x+2, & 0 < x < 1 \end{cases}$ 2
2. Find the Root mean square value of $f(x) = (3-x)^4, 0 < x < 3$. $27 = 3^3$
3. Classify the following Partial Differential equation $u_{xx} - y^4 u_{yy} = 2y^3 u_y$. hypo. per $y \neq 0$ $y = 0$
4. State any four assumptions while deriving one dimensional wave equation.
5. Find the steady state temperature of a rod of length 10 cm which has the ends A and B kept at temperature 30°C and 100°C respectively. $7x+30$

Part – B (3 × 10 = 30 marks)
Answer any **THREE** Questions.

$a_0 = \frac{8}{3}$, $a_n = \frac{4}{n^2\pi^2}$ parseval

6. Express $f(x) = (x-1)^2, 0 < x < 1$ as Fourier cosine series and hence deduce $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots$
7. Find the Fourier cosine series for y=f(x) in $(0, \pi)$ upto first three harmonics for the following data.

x	0	π/6	π/3	π/2	2π/3	5π/6
y	10	12	15	20	17	11

$a_0 = 28.33$
$a_1 = 3.2886$
$a_2 = -4.833$
$a_3 = -4$

8. The ends of a uniform string of length $2l$ are fixed. The initial diaplacement is $y(x,0) = kx(2l-x), (0,2l)$, while the initial velocity is zero. Find the displacement at any distance x from the end $x = 0$ at any time t.
9. Obtain the solution of the equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ satisfying the following conditions. (i) $u \neq \infty, t \to \infty$, (ii) $u = 0$ for $x = 0$ & $x = \pi$ for any value of t. $(iii) u = \pi x - x^2$ when $t = 0$ in the range $(0, \pi)$.

$B_n = \frac{16kl^2}{n^3\pi^3}(1-(-1)^n)$ — B1m

$A = 0$, $\lambda = n$ } - 5m

$B_n = \frac{2}{\pi n^3}[1-(-1)^n]$ - 3m

total - 10

Scanned with CamScanner

15MA201 - TRANSFORMS AND BOUNDARY VALUE PROBLEM

ANSWER KEY - CYCLE TEST - II

SLOT C1, DATE - 19.9.2018, SET - C

Part - A

3. $B^2 - 4AC = 4y^4 \geq 0$ pde is of $\begin{cases} \text{hyperbolic if } y \neq 0 \\ \text{parabolic if } y = 0 \end{cases}$ — (4m)

1. $a_0 = \frac{1}{1}\left[\int_{-1}^{0} x\,dx + \int_0^1 (x+2)\,dx\right] = 2$ — (4m)

2. RMS $= \sqrt{\dfrac{\int_0^3 (3-x)^6\,dx}{3}} = \sqrt{\dfrac{3^6}{7}}$ — (3m) formula — (1m)

4. String is perfectly elastic, ... (any four assumptions) — (4m)

5. $u(0,t) = 30$, $u(10,t) = 100$ — (2m), $u(x,t) = Ax + B = 7x + 30$ — (2m)

Part - B

6. $a_0 = \frac{2}{1}\int_0^1 (x-1)^2\,dx = \frac{2}{3}$ — (2m); $a_n = \frac{4}{n^2\pi^2}$ — (3m) $f(x) = \frac{1}{3} + \sum \frac{4}{n^2\pi^2}\cos n\pi x$ — (1m)

Parseval's id: $\frac{2}{1}\int_0^1 (x-1)^4\,dx = \frac{2}{9} + \sum \frac{16}{n^4\pi^4} \Rightarrow \sum \frac{1}{n^4} = \frac{\pi^4}{90}$ — (4m)

7.

x	$f(x)$	$f(x)\cos x$	$f(x)\cos 2x$	$f(x)\cos 3x$
0	10	10	10	10
$\pi/6$	12	10.392	6	0
$\pi/3$	15	7.5	-7.5	-15
$\pi/2$	20	0	-20	0
$2\pi/3$	17	-8.5	-8.5	17
$5\pi/6$	11	-9.526	5.5	0
	85	9.866	-14.5	12

→ (5m)

$a_0 = \frac{2}{n}\sum f(x) = \frac{85}{3} = 28.33$

$a_1 = \frac{2}{6} \times 9.866 = 3.2886$

$a_2 = \frac{2}{6} \times (-14.5) = -4.833$

$a_3 = \frac{2}{6} \times 12 = 4$

(4m)

$f(x) = 14.165 + 3.2886\cos x - 4.833\cos 2x + 4\cos 3x + \cdots$ — (1m)

8. (i) $y(0,t) = 0$, (ii) $y(2l,t) = 0$ (iii) $y(x,0) = kx(2l-x)$ (iv) $\left.\frac{\partial y}{\partial t}\right|_{t=0} = 0$ — (1m)

$y(x,t) = (A\cos\lambda x + B\sin\lambda x)(C\cos\lambda at + D\sin\lambda at)$ — (1m)

(i) $\Rightarrow A = 0$ (ii) $\Rightarrow \lambda = \frac{n\pi}{2l}$ (iv) $\Rightarrow D = 0$. → (3m)

MGS is $y(x,t) = \sum B_n \sin\frac{n\pi x}{2l}\cos\frac{n\pi a t}{2l}$ — (1m)

(iii) $\Rightarrow$ $B_n = \frac{2}{2l}\int_0^{2l} kx(2l-x)\sin\frac{n\pi x}{2l}\,dx = \frac{16kl^2}{n^3\pi^3}(1-(-1)^n)$ — (3m)

$y(x,t) = \sum \frac{16kl^2}{n^3\pi^3}[1-(-1)^n]\sin\frac{n\pi x}{2l}\cos\frac{n\pi at}{2l}$ — (1m)

9. By (i) suitable solution is

$u(x,t) = (A\cos\lambda x + B\sin\lambda x)\, e^{-\lambda^2 \alpha^2 t}$ — (1m)

When $x=0$, $u=0 \Rightarrow A=0$

$x=\pi$, $u=0 \Rightarrow \lambda = n$ — (4m)

The MGS is $u(x,t) = \sum B_n \sin nx\, e^{-n^2\alpha^2 t}$ — (1m)

(iv) $\Rightarrow$ $B_n = \frac{2}{\pi}\int_0^{\pi} (\pi x - x^2)\sin nx\, dx = \frac{2}{\pi n^3}\left[1-(-1)^n\right]$ — (3m)

$u(x,t) = \sum \frac{2}{\pi n^3}\left(1-(-1)^n\right) \sin nx\, e^{-n^2\alpha^2 t}$ — (1m)

Scanned with CamScanner