Permutation :-

For any set A, a function $\phi : A \to A$ is called a permutation of A if ϕ is 1-1 & onto. (bijective function)

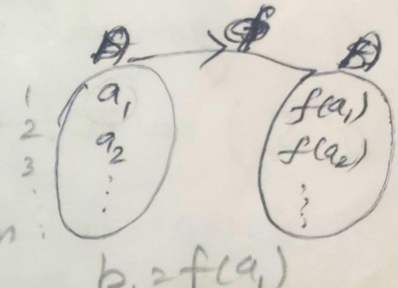
If A is any nonempty set and S_A is the set of permutations of A, then the set S_A forms a group under permutation multiplication.

If A is the finite set $\{1, 2, 3 \ldots n\}$ then the group S_n consisting of all of the permutations of A under permutation multiplication is called permutation group of degree n. the symmetric group or permutation group. $(S_n, *)$ form a group.

Representation of permutation :-

We denote a permutation ϕ on the set in a ~~two~~ rowed notation.

$$\phi = \begin{pmatrix} a_1 & a_2 & \ldots & a_n \\ b_1 & b_2 & \ldots & b_n \end{pmatrix} \begin{matrix} \to \text{domain} \\ \to \text{codomain} \end{matrix}$$



$b_1 = f(a_1)$

$n \quad n-1 \quad \ldots \quad 1$

We n

$n!$

Suppose the set S has two elements then the possible permutation of S

$S = \{1, 2\}$

are

$$S_A = \left\{ \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \right\}$$

2 ways

$S_3 = \{1, 2, 3\}$ $\qquad O(S_3) = 3!$

Then S_3 consists of

$$P_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \qquad P_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \qquad P_4 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

$$P_5 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \qquad P_6 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

To prove $(S_3, *)$ is a group

In general $O(S_n) = n!$ as there are $n!$ permutations of n elements.

$*$	P_1	P_2	P_3	P_4	P_5	P_6
P_1	P_1	P_2	P_3	P_4	P_5	P_6
P_2	P_2	P_3	P_1	P_6	P_4	P_5
P_3	P_3	P_1	P_2	P_5	P_6	P_4
P_4	P_4	P_5	P_6	P_1	P_2	P_3
P_5	P_5	P_6	P_4	P_3	P_1	P_2
P_6	P_6	P_4	P_5	P_2	P_3	P_1

$$P_1 * P_1 = P_1 \circ P_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} = P_1$$

1 2 3
3 1 2
2 3 1

$$P_2 * P_2 = P_2 \circ P_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} = P_3$$

1 2 3
1 2 3

$$P_2 * P_3 = P_3 \circ P_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} = P_1.$$

$$P_2 * P_4 = P_4 \circ P_2 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} = P_6.$$

$$P_2 * P_5 = P_5 \circ P_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} = P_4.$$

$$P_2 * P_6 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} = P_5.$$

$P_3 * P_2 = P_2 \circ P_3 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 8 \end{pmatrix} = P_1$

$P_3 * P_3 = P_3 \circ P_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} = P_2$

$P_3 * P_4 = P_4 \circ P_3 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} = P_5$

$P_3 * P_5 = P_5 \circ P_3 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} = P_6$

$P_3 * P_6 = P_6 \circ P_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} = P_4$

$P_4 * P_5 = P_5 \circ P_4 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} = P_2$

$P_5 * P_3 = P_3 \circ P_5 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} = P_4$

$P_4 * P_2 = P_2 \circ P_4$

$= \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$

$\begin{matrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{matrix}$

$= P_5$

$P_4 * P_3 = P_3 \circ P_4$

$= \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$

$= P_6$

$P_4 * P_4$

$= \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$

$P_5 * P_2$

$= \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$

like this we can fill the table.

It is clear that entries in the table are members of S_3. $\therefore$ closure property holds

$$P_2 * (P_4 * P_6) = (P_2 * P_4) * P_6$$

$$P_2 * P_3 = P_6 * P_6$$

$$P_1 \qquad P_1$$

Thus associativity is satisfied

Now $P_1 * P_i = P_i * P_1 = P_i$ for $i = 1, 2 \ldots 6$

Thus the existence of the identity element is verified ($\because e = P_1$)

Also $P_1^{-1} = P_1$, $P_2^{-1} = P_3$, $P_3^{-1} = P_2$

$P_4^{-1} = P_4$. $P_5^{-1} = P_5$, $P_6^{-1} = P_6$

Thus $(S_3, *)$ is a group.

$P_4 * P_2 = P_5$; $P_2 * P_4 = P_6 \neq P_5$

$\therefore$ $*$ is not commutative

$\therefore$ S_3 is not abelian group under $*$.

Types of permutation :-

Equality of permutation :-

If $f = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ & $g = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \end{pmatrix}$

then $f = g$.

identity permutation :-

$I = \begin{pmatrix} 1 & 2 & \cdots & n \\ 1 & 2 & \cdots & n \end{pmatrix}$ is the identity permutation of degree n.

Inverse permutation :-

$f = \begin{pmatrix} a_1 & a_2 & \cdots & a_n \\ b_1 & b_2 & \cdots & b_n \end{pmatrix}$ & $f^{-1} = \begin{pmatrix} b_1 & b_2 & \cdots & b_n \\ a_1 & a_2 & \cdots & a_n \end{pmatrix}$

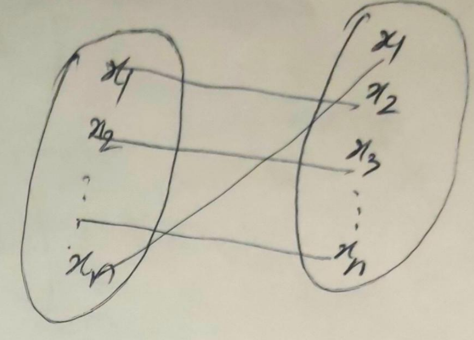
cyclic permutation :-

A permutation f on a set S is called a cyclic permutation of length l if there exists $x_1, x_2, \ldots, x_l \in S$ s.t $f(x_1) = x_2$, $f(x_2) = x_3$ ---- $f(x_{l-1}) = x_l$

$f(x_l) = x_1$

Eg :- $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ is a cyclic permutation of length 3

This permutation is also expressed as $(1\ 2\ 3)$



order of permutation :-

Let S_n be a permutation group defined over n symbols & $f \in S_n$

If m is the least +ve integer such that

$f^m = I$.

where I is the identity permutation in S_n. Then m is called order of f & is denoted by $o(f) = m$.

In other words :-

The order is found by finding the lcm of length of the cycles.

Example :- If the permutations of the elements $\{1, 2, 3, 4, 5\}$ are given by

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 4 & 5 \end{pmatrix}, \quad \delta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 5 & 4 \end{pmatrix}$$

$$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & 4 \end{pmatrix}$$

Find $\alpha\beta$, $\beta\alpha$, α^2, $\gamma\beta$, δ^{-1} & $\alpha\beta\gamma$

$$\gamma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 1 & 2 \end{pmatrix}$$

Soln

$$\alpha\beta = \alpha * \beta = \beta \circ \alpha.$$

$$\begin{array}{llllll} \alpha : & 1 & 2 & 3 & 4 & 5 \\ & 2 & 3 & 1 & 4 & 5 \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \beta & 2 & 3 & 1 & 5 & 4 \end{array}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$$

$$\beta\alpha = \beta * \alpha = \alpha \circ \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$$

β : 1 2 3
1 2 3 5
2 3 1 5 4

α 1 2 3 4 5
2 3 1 4 5
3 1 2 4 5

$$\alpha^2 = \alpha * \alpha = \alpha \circ \alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 4 & 5 \end{pmatrix}$$

$$\gamma\beta = \beta \circ \gamma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 3 & 1 & 2 \end{pmatrix}$$

1 2 3 4 5
5 4 3 1 2
4 5 3 1 2

$$\delta^{-1} = \begin{pmatrix} 3 & 2 & 1 & 5 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 5 & 4 \end{pmatrix}$$

order of the column in this representation of σ is immaterial. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$ & $\begin{pmatrix} 2 & 4 & 3 & 1 \\ 4 & 3 & 1 & 2 \end{pmatrix}$ represents the same

$\alpha\beta$: 1 2 3 4 5
2 3 1 5 4
4 3 5 2 1

$$\alpha\beta\gamma = \alpha\beta * \gamma = \gamma \circ (\alpha\beta) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 5 & 2 & 1 \end{pmatrix}$$

solving the equation $\alpha x = \beta$ means finding the value of x that satisfies the equation. premultiplying by α^{-1}, the given equation becomes

$$\alpha^{-1}\alpha x = \alpha^{-1}\beta$$

$$\text{i.e. } ex = \alpha^{-1}\beta$$

ie

$x = \alpha^{-1}\beta$

$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 5 & 4 \end{pmatrix}$

α^{-1}	1	2	3	4	5
	3	1	2	4	5
β	3	1	2	5	4

(2) If $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 5 & 4 & 6 & 2 \end{pmatrix}$

$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 1 & 6 & 2 & 4 \end{pmatrix}$ are two elements of the symmetric group S_6

Find $\alpha\beta$, $\beta\alpha$, α^2, β^2, α^{-1}, β^{-1}.

Soln

$\alpha\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 5 & 4 & 6 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 1 & 6 & 2 & 4 \end{pmatrix}$

$\alpha\beta = \beta \circ \alpha.$

$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 1 & 6 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 5 & 4 & 6 & 2 \end{pmatrix}$

	1	2	3	4	5	6
α	3	1	5	4	6	2
β	1	5	2	6	4	3

$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 5 & 2 & 6 & 4 & 3 \end{pmatrix}$

$\beta\alpha = \alpha \circ \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 3 & 2 & 1 & 4 \end{pmatrix}$

$$\alpha^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 5 & 4 & 6 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 5 & 4 & 6 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 6 & 4 & 2 & 1 \end{pmatrix}$$

$$\beta^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 1 & 6 & 2 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 1 & 6 & 2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 5 & 4 & 3 & 6 \end{pmatrix}$$

$$\alpha^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 6 & 1 & 4 & 3 & 5 \end{pmatrix}$$

$$\beta^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 2 & 6 & 1 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 1 & 5 & 4 & 6 & 2 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}$$

$\downarrow$

column of the permutations are rearranged so that the image of the elements is not changed

(3) If α, β are elements of the symmetric group S_4 given by

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \ \& \ \beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$$

Find $\alpha\beta$, $\beta\alpha$, α^2 & α^{-1}. Find also the order of α, β, & $\alpha\beta$.

$$\alpha\beta = \beta\circ\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$$

$$\beta\alpha = \alpha\circ\beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$$

$$\alpha^2 = \alpha\cdot\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$$

$$\alpha^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$$

$O(\alpha) = (1\ 2\ 3\ 4)$

$O(\alpha) = 4.$

$(1\ 2\ 3\ 4)$

$O(\alpha\beta) = 4.$

$\beta = (1\ 2\ 4)(3)$

length of the cycle $= 3 \times 1$

Lcm of $(3, \times 1)$
$= 3$

$O(\beta) = 3.$

II method

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$$

$$\alpha^2 = \alpha \cdot \alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$$

$$\alpha^3 = \alpha^2 \cdot \alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$$

$$\alpha^4 = \alpha^3 \cdot \alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = I.$$

$\therefore O(\alpha) = 4.$

$$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$$

$$\beta^2 = \beta \cdot \beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \end{pmatrix}$$

$$\beta^3 = \beta^2 \cdot \beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

$$= I$$

$O(\beta) = 3.$

$$\alpha\beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$$

$$(\alpha\beta)^2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$$

$$(\alpha\beta)^3 = (\alpha\beta)^2 \cdot (\alpha\beta)$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$$

$$(\alpha\beta)^4 = (\alpha\beta)^3 \cdot (\alpha\beta) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

$o(\alpha\beta) = 4.$

(4) Determine the order of each of the permutations on the set $\{1, 2, 3, 4, 5\}$

where $P_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 2 & 5 & 1 \end{pmatrix}$, $P_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 3 & 5 \end{pmatrix}$

$P_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 2 & 4 & 3 \end{pmatrix}$ & $P_4 = P_1 P_3$.

Soln

$P_1 = (1\ 4\ 5)(2\ 3) \quad = \quad \text{Lcm}(2,3)$

= length of the 1st cycle 3 $\quad = 6.$

" " " 2 cy 2

$o(P_1) = 6.$

$P_2 = (1\ 2)\ (3\ 4)\ 5$

$= 2 \times 2 \times 1$

$\text{Lcm}(2,2,1) = 2$

$o(P_2) = 2.$

$P_3 = 1\ (2\ \cancel{3}\ 3)\ 4$

$= 1 \times 3 \times 1$

$\text{Lcm}(1,3) = 3.$

$o(P_3) = 3.$

$P_4 = P_1 P_3 = P_3 \circ P_1$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 2 & 4 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 2 & 5 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 3 & 1 \end{pmatrix}$$

$= (1\ 4\ 3\ 5)\ (2)$

$\text{lcm}(4,1) = 4$

$o(P_4) = 4$