# Discrete Mathematics For Engineers

## Assignment - I

S. Kunal Keshan
RA2011004010051
ECE - A

## PART - B

1. Prove that $\overline{(A-B)} = \bar{A} \cup B$ analytically

Soln.

$\bar{A} \cup B$ — Let $x$ be an element.

for $\bar{A} \cup B$, — $x \in \bar{A}$ and $x \in B$

$= x \notin A$ and $x \in B$

$= x \in \bar{A}$ and $x \in \bar{B}$

$= x \in \bar{A} - \bar{B}$

$(A-B)' \Rightarrow$ $x \in A$ and $x \notin B$

$x \notin \bar{A}$ and $x \in \bar{B}$

$x \in (\bar{B} - \bar{A})$

$(A-B)' \Rightarrow x \in (\bar{A} - \bar{B})$

Hence $\bar{A} \cup B = \overline{(A-B)}$

2. If $R$ is a rel. on set $A = \{1, 2, 3, 4, 5\}$ defined by $(a, b) \in R$ if $a + b \leq 6$, then list elements $R$, $R^{-1}$ and $\bar{R}$. Find the relational matrix $M_R$, $M_{R^{-1}}$ and $M_{\bar{R}}$

soln

Given $A = \{1, 2, 3, 4, 5\}$

(i) $R = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1)\}$

(ii) $R^{-1} = \{(1,1), (2,1), (3,1), (4,1), (5,1), (1,2), (2,2), (3,2), (4,2), (1,3), (2,3), (3,3), (1,4), (2,4), (1,5)\}$

iii) $R^c = \{ (2,5), (3,4), (3,5), (4,3), (4,4), (4,5), (5,2), (5,3), (5,4), (5,5) \}$

iv) $M_R = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 & 0 \\ 3 & 1 & 1 & 1 & 0 & 0 \\ 4 & 1 & 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 & 0 & 0 \end{array}$ $\quad M_{R^{-1}} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

$M_{R^c} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$

3. If $f(x) = x+2$, $g(x) = x-2$, $\forall x \in R$, then P.T $fog = gof$.

Soln.

$(fog)(x) = f[g(x)]$

$= f[x-2]$

$= x+2+2$

$= x$ — (1)

$(gof)(x) = g[f(x)]$

$= g[x+2]$

$= x+2-2$

$= x$ — (2)

From (1) and (2), it is clear that $fog = gof$.

Hence Proved.

4. If $f, g: R \to R$, where $f(x) = ax+b$; $g(x) = 1-x+x^2$ and $(gof)(x) = 9x^2-9x+3$. find the Value of $a, b$.

Soln. $(gof)(x) = g[f(x)]$

$= g[ax+b]$

$= 1-(ax+b) + (ax+b)^2$

$9x^2-9x+3 = a^2x^2 + (2ab-a)x + b^2-b+1$

Comparing LHS and RHS, we get.

$a^2 = 9$

$a = \pm 3$

$2ab-a = -9$

$a=3,$

$b=-1$

$a=-3$

$b=2$

$b^2-b+1=3$

$b(b-2)+1(b-2)=0$

$b=2,-1$

$a = \pm 3$, $b = +2, -1$, inputing the values in $(gof)(x)$ also.

$g[3x-1] = 1-(3x-1) + (3x-1)^2$

$= 9x^2-9x+3$

$g[-3x+2] = 1-(-3x+2) + (-3x+2)^2$

$= 9x^2-9x+3$

$\therefore \boxed{a = \pm 3, \quad b = -1, 2}$

5. Verify whether the given relation R on $A = \{a, b, c, d\}$ is an equivalence relation or not.

Soln.

$R = \{(a,a), (a,c), (a,d), (b,b), (c,a), (c,c), (d,a), (d,d)\}$

(i) Reflexive: R is said to be reflexive if $R = \{(a,a) \mid a \in A\}$.

$R = \{(a,a), (b,b), (c,c), (d,d)\}$ $\therefore$ R is reflexive.

(ii) Symmetric. R is said to be symmetric if there exist $a, b \in A$ such that if $aRb$, then $bRa$ also exist.

we see that $(a,c)$ and $(c,a)$ ; $(a,d)$ and $(d,a) \in R$.

$\therefore$ R is symmetric.

(iii) Transitive: R is said to be transitive if whenever $aRb$ and $bRc$, then $aRc$.

$(a,c) \in R$ and $(c,a) \in R$ , $(a,a) \in R$ (true)

$(a,d) \in R$ and $(d,a) \in R$ , $(a,a) \in R$ (true)

$(a,a) \in R$ and $(a,c) \in R \Rightarrow (a,c) \in R$ (true)

$(d,a) \in R$ and $(a,c) \in R$ $(d,c) \notin R$ (false)

$\therefore$ R is not transitive

$\therefore$ R is not an equivalence relation.

6. Find the matrix representation of $R \cup S$ and $R \cap S$ when $R = \{(1,1), (1,3), (2,2)\}$ and $S = \{(1,2), (1,3) (2,1), (2,2), (3,3)\}$ are relations defined on $A = \{1, 2, 3\}$.

Soln.

$$M_R = \begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \qquad M_S = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_{R\cup S} = \begin{bmatrix} 1 \vee 0 & 0 \vee 1 & 1 \vee 1 \\ 0 \vee 1 & 1 \vee 1 & 0 \vee 0 \\ 0 \vee 0 & 0 \vee 0 & 0 \vee 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_{R\cap S} = \begin{bmatrix} 1 \wedge 0 & 0 \wedge 1 & 1 \wedge 1 \\ 0 \wedge 1 & 1 \wedge 1 & 0 \wedge 1 \\ 0 \wedge 0 & 0 \wedge 0 & 0 \wedge 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

## PART-C.

1. State and Prove Demorgans law of Set Theory.

Soln. (i) Law of union. $(A \cup B)^c = A^c \cap B^c$

The Complement of the union of 2 sets A & B is equal to the intersection of individual complements of the 2 sets.

Proof.

$(A \cup B)^c = \{ x \mid x \notin (A \cup B) \}$

$\Rightarrow \{ x \mid \sim(x \in (A \cup B)) \}$

$= \{ x \mid \sim(x \in A \cup x \in B) \}$

$= \{ x \mid \sim(x \in A) \cap \sim(x \in B) \}$

$= \{ x \mid x \in A^c \cap x \in B^c \}$

$\boxed{(A \cup B)^c = A^c \cap B^c}$

5

(ii) Law of intersection : $(A \cap B)^c = A^c \cup B^c$

The complement of intersection of 2 sets is equal to the union of individual complements of 2 sets

Proof.

$(A \cap B)^c = \{x \mid x \notin (A \cap B)\}$

$= \{x \mid \sim (x \in (A \cap B)\}$

$= \{x \mid \sim (x \in A) \cup \sim (x \in B)\}$

$= \{x \mid x \in \bar{A} \cup x \in \bar{B}\}$

$\boxed{(A \cap B)^c = \bar{A} \cup \bar{B}}$

2\. If A, B & C are sets, then prove the statement $(A-B)-C = A-(B \cup C)$ analytically.

Soln.

$A-(B \cup C) = \{x \mid x \in A \text{ and } x \notin B \cup C\}$

$= \{x \mid x \in A \text{ and } (x \notin B \text{ and } x \notin C)\}$

$= \{x \mid (x \in A \text{ and } x \in B) \text{ and } x \notin C\}$

$= \{x \mid x \in (A-B)-C\}$

$\boxed{A-(B \cup C) = (A-B)-C}$

Hence Proved.

3. If $R$ is a relation on set of integers | $(a, b) \in R$, iff $3a + 4b = 7n$ for some n, P.T R is an equivalence relation.

Soln. (i) Reflexive.

$$3a + 4b = 7n$$

$a = b$.

$3a + 4a = 7n$

$7a = 7n$

$a = n$, $n$ = integer, $a$ = integer.

$\therefore$ R is reflexive.

(ii) Symmetric, if $(a, b) \in R$, then $(b, a) \in R$

$3a + 4b = 7n$.

$3b + 4a = 7a + 7b - (3a - 4b)$

$= 7a + 7b - 7n$

$= 7(a + b - n) = 7m$, m is also an integer.

$3b + 4a$ is also an integer

$\therefore$ R is Symmetric.

(iii) Transitive, If $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.

$3a + 4b = 7n$

$3b + 4c = 7p$.

$3a + 7b + 4c = 7(n + p)$

$3a + 4c = 7(n + p - b) = 7q$, where q is also an integer

$\therefore$ R is transitive

$\therefore$ Since, R is Symmetric, reflexive, and transitive, R is an equivalence relation.

Hence Proved.

9

4. If R is a relation on set $A = \{1, 2, 4, 6, 8\}$, defined by $a R b$ iff $b/a$ is an integer. Show that R is partially ordering on A.

Soln. Partially ordering iff R is reflexive, anti-symmetric and transitive.

$R = \{(1,1), (2,2), (1,4), (1,6)\ (1,8)\ (2,2)\ (2,4)\ (2,6)\ (2,8)$
$(4,4)\ (4,8)\ (6,6)\ (8,8)\}$.

(i) Reflexive. If $R = \{(a,a) \mid a \in A\}$.
R contains $\{(1,1)\ (2,2)\ (4,4)\ (6,6)\ (8,8)\}$.
$\therefore$ R is reflexive.

(ii) Anti-Symmetric, R is said to be antisymmetric, if $a R b$ and $b R a$.

$\frac{a}{b}$ is an integer and $\frac{b}{a}$ is also an integer
$a = b$, here R is anti-symmetric.

(iii) Transitive, if $a R b$, $b R c \Rightarrow a R c$.

$(1,2) \in R$ and $(2,2) \in R$, $(2,4) \in R$, $(4,6) \in R$, $(2,8) \in R$.
$\Rightarrow (1,2)\ (1,4)\ (1,6)\ (1,8) \in R$ –
$(2,4) \in R$ and $(4,4)\ (4,8) \in R \Rightarrow (2,4)\ (2,8) \in R$.
$\therefore$ R is transitive.

5. Let $R = \{(1,1)\ (1,3)\ (1,5)\ (2,3)\ (2,4)\ (3,3)\ (3,5)$
$(4,2)\ (4,4)\ (5,4)\}$ be relation on
$A = \{1, 2, 3, 4, 5\}$. Find the transitive closure using Warshall's algorithm.

$R$: $W_0$ $\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

| | Position of 1s in column | Positions of 1s in row | | |
|---|---|---|---|---|
| 1 | 1 | 1, 3, 5 | (1,1) (1,3) (1,5) | $\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$ |
| 2 | 4 | (3, 4) | (4,3) (4,4) | $\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$ |
| 3 | (1, 2, 3) | (1, 3, 5) | (1,1) (1,3) (1,5) (2,1) (2,3) (2,5) (3,1) (3,3) (3,5) | $\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$ |
| 4 | (2, 4, 5) | (2, 3, 4) | (2,2) (2,3) (2,4) (4,2) (4,3) (4,4) (5,2) (5,3) (5,4) | $\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$ |
| 5 | (1, 2, 3) | (1, 2, 3, 4) | (1,1) (1,2) (1,3) (1,4) (2,1) (2,2) (2,3) (2,4) (3,1) (3,2) (3,3) (3,4) | $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$ |

$\therefore$ Transitive Closure $R^{\infty}$ :

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

6. Show that the composition of invertible function is invertible.

Soln. Consider $f: A \to B$, $g: B \to C$ are 2 invertible function.

They are bijective. To show that $g \circ f$ is invertible

(i) $g \circ f$ is 1-1.

WKT $f \to$ 1-1 and $g$ 1-1.

Let $(g \circ f)(a_1) = (g \circ f)(a_2)$

$g[f(a_1)] = g[f(a_2)]$

$f(a_1) = f(a_2)$

$a_1 = a_2$

$g[a_1] = g[a_2]$.

$a_1 = a_2$ $\quad g \circ f$ is 1-1.

(ii) $g \circ f$ is onto.

Let $c \in C$, since $g$ is onto, there exist $b \in B \mid c = g(b)$.
Since $f$ is onto, there will be an element $a \in A$,

$b = f(a)$

$c = g(b) = g(f(a)) = g \circ f(a)$.

$\forall c \in A$, $\exists$ $a \in A \mid (g \circ f)(a) = c$.

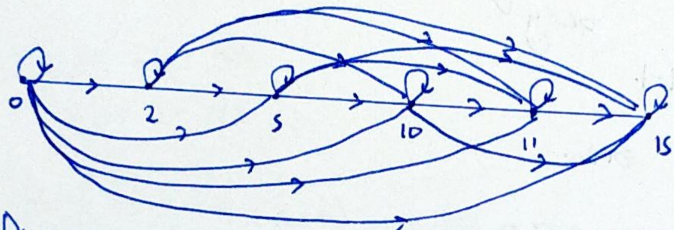
Then $g \circ f : A \to C$ is onto.

$\therefore$ $g \circ f$ is bijective and thus invertible.

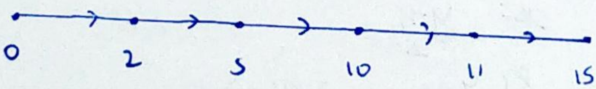
7. Draw the hasse-diagram for the "less than (or) equal to" relation on {0, 2, 5, 10, 11, 15} starting from digraph.

Soln.

R: { (0,0) (0,2) (0,5) (0,10) (0,11) (0,15)
(2,2) (2,5) (2,10) (2,11) (2,15) (5,5)
(5,10) (5,11) (5,15) (10,10), (10,11) (10,15)
(11,11) (11,15) (15,15) }.



Hasse Diagram.



8. If $f: Z \times Z \to Z$ defined by $f(m,n) = 2m + 3n$, then determine whether it is 1-1 or onto.

Soln.

1-1:

$f(m_1, n_1) = f(m_2, n_2)$

$2m_1 + 3n_1 = 2m_2 + 3n_2$.

$2(m_1 - m_2) \neq 3(n_2 - n_1)$

$f(m,n)$ is not 1-1.

Onto

$f(m, n) = 2m + 3n$

let $x \in Z \times Z$ $\quad y \in Z$ $\quad f(-1, 1) = 2 \cdot (-1) + 3(1)$
$= 1 \in Z$

$f(m,n)$ is onto.

9. If $f: Z \to N$ is defined by $f(x) = \begin{cases} 2x-1, & x > 0 \\ -2x, & x \leq 0. \end{cases}$

P.T $f$ is bijective.

Sln.

(i) $f$ is 1-1.

Case 1: $x > 0$

$2x-1 = 2y-1$

$x = y$

Case 2 $x \leq 0$

$-2x = -2y$

$x = y$

$\therefore$ $f$ is 1-1.

(ii) $f$ is onto.

Case 1 $x > 0$

$y = 2x-1$

$x = \frac{y+1}{2}$

$f\left(\frac{y+1}{2}\right) = 2\frac{(y+1)}{2} - 1 = y.$

Case 2 $x \leq 0$

$y = -2x$

$x = -y/2.$

for $\forall$ $y$ in $N \cup \{0\}$, there exists $x$ in $Z$ such that $x = -y/2$.

$\therefore$ $f$ is onto

$f$ is bijective.

10 PT R is an equivalence relation where a R b iff
$3a+b$ is a multiple of 4.

Soln i) Reflexive. $R: \{(a,a) \mid a \in A\}$

$3a+b=4n$

$3a+a=4n$ $a=b$

$4a=4n$ , ∴ R is reflexive.

ii) Symmetric $(a,b) \in R \Rightarrow (b,a) \in R$.

$3a+b=4n$

$3b+a=4k$.

$4(a+b)=4(n+k)$ divisible by 4.

R is Symmetric

(iii) Transitive : $(a,b) \in R$, $(b,c) \in R \Rightarrow (a,c) \in R$.

$3a+b=4n$

$3b+c=4k$.

+

$3a+4b+c=4(n+k)$

$3a+c=4(n+k-b) \Rightarrow$ divisible by 4.

R is Transitive.

∴ R is an equivalence relation.