Circular Arrangements

Circular arrangements are permutations in which objects are arranged in a circle.

To calculate the number of ways in which n objects can be arranged in a $\odot^{le}$, we arbitrarily fix the position of one object, so the remaining (n−1) objects can be arranged as if they were on a straight line in (n−1)! ways.

ie the number of arrangements in a circle $\Big\} = (n-1)!$

Problems :-

(1) Find the number of ways can 6 people be seated at a round table?

The number of ways will be $(6-1)!$

$= 120$.

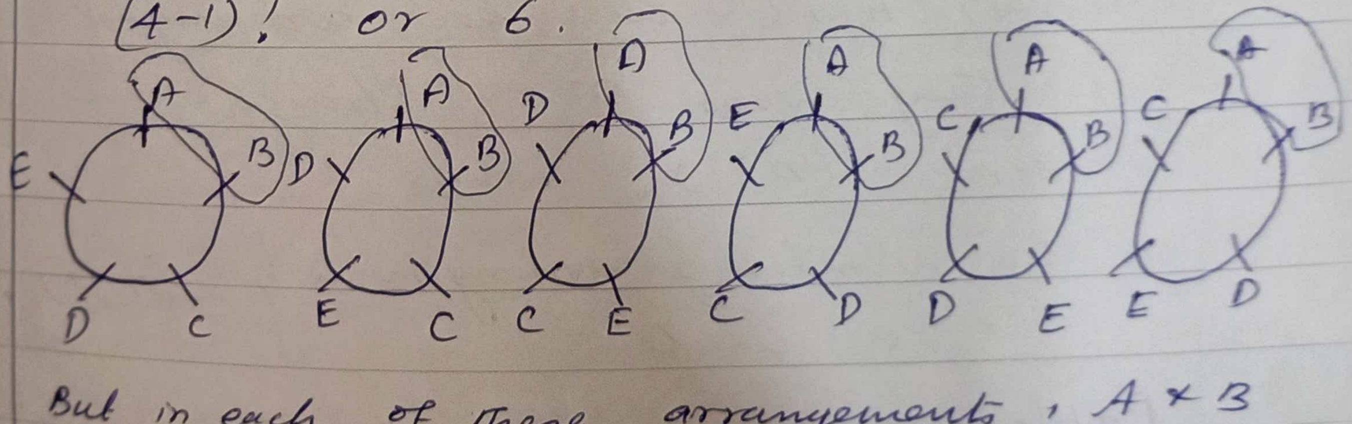
2. Find the number of ways in which 5 people A, B, C, D & E can be seated at a round table, such that

(i) A & B always sit together

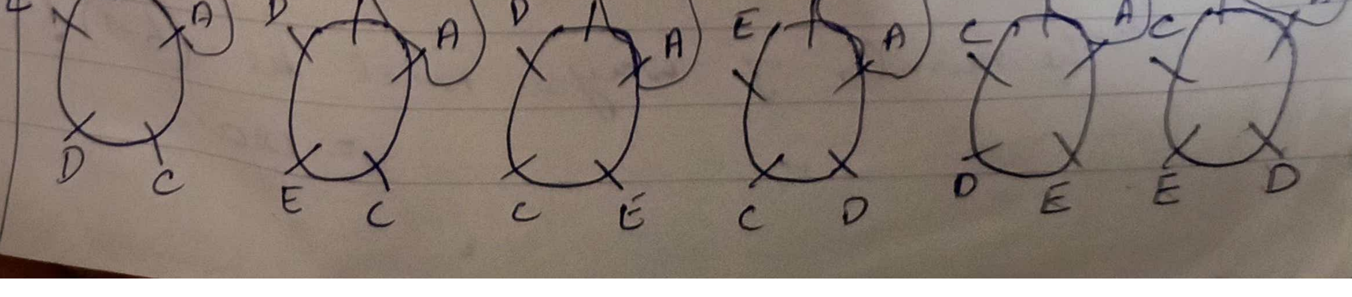
(ii) C & D never sit together.

<u>Soln</u> If we wish to seat A & B together in all arrangements, we can consider these two as one unit, along with 3 others. So, effectively we've to arrange 4 people in a circle, the number of ways being

$(4-1)!$ or 6.



But in each of these arrangements, A & B can themselves interchange places in 2 ways.



∴ The total number of ways will be

6×2 or 12.

(ii)

The number of ways in which 5 people arranged in a table is $(5-1)!$

$= 4! = 24$.

The number of cases in which C & D are seated together is $= 3! \times 2!$

$= 12$ ways.

∴ The required number of ways in which C & D never sit together is

$= 24 - 12 = 12$ ways.

3. In how many ways can 3 men & 3 women be seated at a round table such that no two men sit together?

Since we don't want the men to be seated together, the only way to do this is to make the men & women sit alternately. We will first seat the 3 women on alternate seats

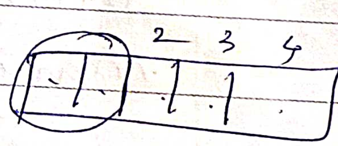
After fixing the position of the women the arrangement on the remaining seats is equivalent to a linear arrangement.
$\therefore$ the total number of ways in this case will be $2! \times 3!$

or 12 ways.

4) In how many ways can 5 people be seated around a circular table if 2 people insist on sitting beside each other?

<u>Soln</u>

$n = 4$



$P = (n-1)!$

$= 3! \times 2!$

$= 6 \times 2 = 12$ ways.

(ii) if 2 people refuse to sit beside each other.

To solve this you have to subtract the number of people who insist on sitting beside each other from the number of permutations with no restrictions.

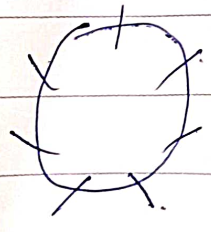
| 1 | 2 | 3 | 4 5 | − | (1) | 3 | 4 | 5 | = | ~~X~~ | 3 |

$4!$ − $(4-1)!\,2!$ $= 24 - 12$

$= 12$ ways.

(5) In how many ways can 7 peop[le] be arranged about a circular ta[ble]

(ii) If two of the people insist o[n] sitting next to each other, how many arrangements are possible



$= (7-1)! = 6!$

$= 720$

$1!\ 5!\ 2!$

$= 240$ ways.

| (1 | 2) | 3 | 4 | 5 | 6 | 7 |

Note :-

The number of circular permutations of n different objects taken r at a time is $\frac{{}^nP_r}{r}$ when clockwise & anticlockwise order are treated as different.

$= \frac{{}^nP_r}{2r}$ when clockwise & anticlockwise order are treated as same.

The number of circular permutations of n different objects taken all n at a time (or taken altogether)

$= \frac{{}^nP_n}{n} = (n-1)!$ when clockwise & anticlockwise order are treated as different.

ie The number of circular permutations of n things taken all at a time in one direction is

$= \frac{{}^nP_n}{2n} = \frac{1}{2}(n-1)!$

$= \frac{1}{2}$ [All possible circular arrangements] when the above two orders are treated as same.

Important Note :-

(i) The following situations leads identical nature of clockwise, anticlockwise arrangements.

(i) Formation of garland with flowers.

(ii) Formation of necklace with beads

(iii) Persons sitting around a table with distinct neighbours.

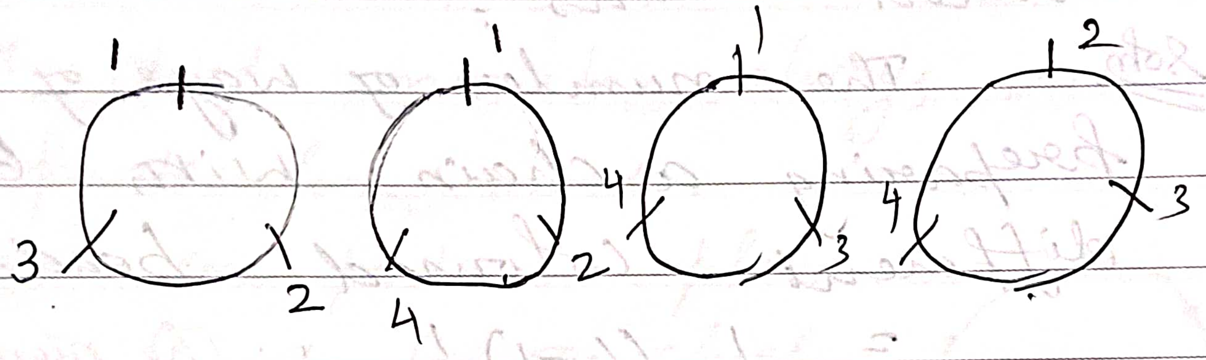
b. Write all possible circular permutations using two digits 1, 2, 3, 4 by taken three at a time

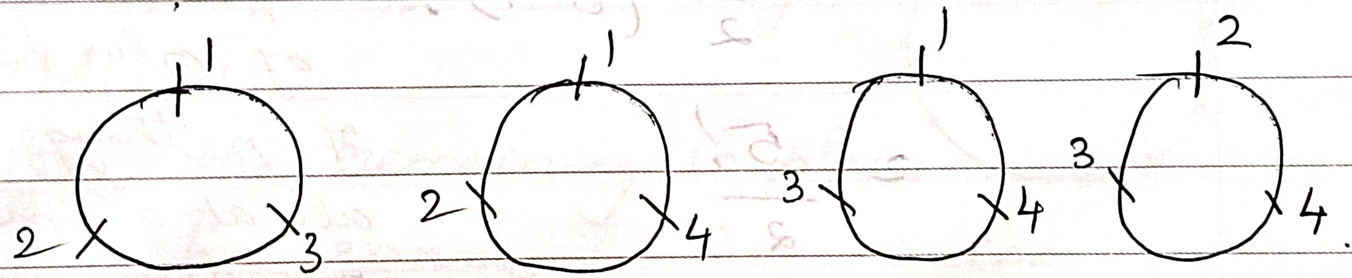
Soln by the formula, we have number of circular permutations of n different things taken r at a time $\frac{{}^{n}P_{r}}{r}$

$$= \frac{{}^{4}P_{3}}{3} = \frac{\lfloor 4}{3 \cdot \lfloor 4-3} = \frac{\lfloor 4}{3 \cdot 1}$$

$$= \frac{1 \times 2 \times 3 \times 4}{3}$$

$$= 8.$$





Thus the number of required circular permutation is 8.

suppose when clockwise & anticlockwise arrangements are treated as identical then the arrangement is

$$= \frac{{}^{n}P_{r}}{2r} = \frac{{}^{4}P_{3}}{2 \times 3} = \frac{1 \times 2 \times 3 \times 4}{2 \times 3}$$

$$= 4 \text{ ways}.$$

Imp

7\. Find the number of ways of preparing a chain with 6 different coloured beads?

Soln The number of ways of preparing a chain with 6 different coloured beads

$= \frac{1}{2}(6-1)!$

$= \frac{5!}{2}$

$= 3 \times 4 \times 5$

$= 60.$

∵ The number of circular permutations of n things taken all at a time in one directions is $\frac{1}{2}(n-1)!$

8\. Find the number of ways of arranging 7 persons around a circle.

Soln The number of ways of arranging 7 persons around a circle

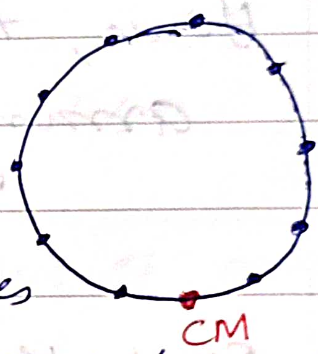
$= (7-1)!$

$= 6!$

∵ The number of circular permutations of n different things taken all at a time is $(n-1)!$

9. Find the number of ways of arranging the chief minister & 10 cabinet ministers at a circular table so that the chief minister always sits in a particular seat.

Soln One particular place is fixed for the chief Minister.

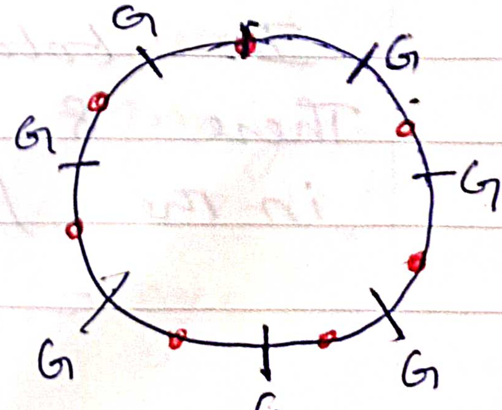


Now, the remaining 10 places can be arranged for 10 cabinet ministers in $10!$ ways.

∵ In a circular arrangement if we fix one place, then it becomes linear permutations.

10. Find the number of ways of arranging 7 gents & 4 ladies around a circular table if no two ladies wish to sit together.

Soln 7 gents around a circular table can be arranged in $(7-1)!$



$= 6!$ ways.

In between these gents, there are 7 places.

∴ arrangements of 4 ladies in these 7 places is $^7P_4$ ways

Hence, the required number of arrangements is

$$= 6!\ ^7P_4.$$

11 Find the number of ways of preparing a garland with 3 yellow, 4 pink & 2 red roses of different sizes such that the two red roses come together.

soln Let us treat the two red roses as one unit & the remaining 7 roses as 7 units.

In total there are 8 units.

These 8 units can be arranged in the form of a garland in

$$= \frac{(8-1)!}{2} = \frac{7!}{2} \text{ ways.}$$

& the 2 red roses can be arranged among themselves in $2!$ ways.

$\therefore$ The required number of arrangement is $= \frac{7!}{2} \times 2!$

$= 7! = 5040$

Find the number of ways of arranging 6 boys & 6 girls around a circular table so that

(i) all the girls sit together.

(ii) No two girls sit together.

(iii) boys & girls sit alternately.

Soln

Let us treat 6 girls as 1 unit & remaining 6 boys as 6 units

In total there are 7 units

These 7 units can be arranged in a circular table in $(7-1)!$

$= 6!$ ways.

& the 6 girls can be arranged among themselves in 6! ways.

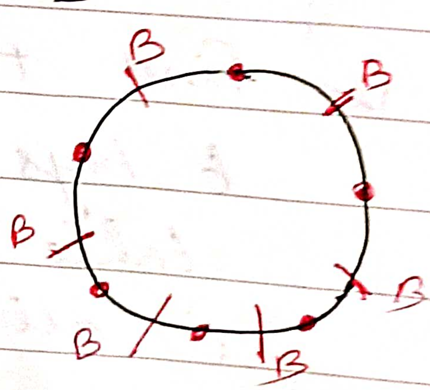
∴ The required number of arrangements is $6! \times 6!$

(ii) no two girls sit together:

Firstly, we arrange the 6 boys in a circular table in $(6-1)! =$

$= 5!$ ways

& in between the 6 boys there are 6 places.



These 6 places can be arranged by 6 girls in 6! ways

∴ The required number of arrangements

$= 5! \times 6!$

(iii) boys & girls sit alternately

$= 5! \times 6!$

13. Find the number of ways of sitting 5 Indians, 4 Americans & 3 Russians at a round table so that (i) all the Indians sit together
(ii) no two Russians sit
(iii) Persons of the same nationality sit together.

Soln. Let us treat 5 Indians as 1 unit & the remaining 4 Americans 3 Russians as 7 units

In total there are 8 units.

These 8 units can be arranged around a circle in $(8-1)!$ $= 7!$ ways.

& the 5 Indians can be arranged among themselves in $5!$ ways

$\therefore$ The required number of arrangements $= 7! \times 5!$

(ii) no two Russians sit together

Firstly, arrange 5 Indians &
4 Americans in a round table
in $(5+4-1)! = 8!$ ways.
In between these 9 persons
there are 9 gaps.
These 9 gaps can be filled by
3 Russians in ${}^9P_3$ ways.
∴ The required number of arrangements
$= 8! \cdot {}^9P_3$.

(iii) Persons of the same nationality
sit together
Let us treat 5 Indians as
1 unit, 4 Americans as 1 unit
& 3 Russians as 1 unit.
In total there are 3 units.
Now, we can arrange these 3
units in a round table in
$(3-1)! = 2!$ ways.
Now the 5 Indians, 4 Americans
& 3 Russians can be arranged

among themselves in
5! 4! & 3! ways resp.
∴ The required number of arrangements
is $2! \times 5! \times 4! \times 3!$

14. Find the number of ways of arranging 6 red roses & 3 yellow roses of different sizes into a garland. In how many of them
(i) all the yellow roses are together
(ii) no two yellow roses are together.

Soln The number of arrangement of 6 red roses & 3 yellow roses in a garland is

$$\frac{(6+3-1)!}{2} = \frac{8!}{2}$$

$$= 20160 \text{ ways}$$

(ii) all the yellow roses are together
Let us treat 3 yellow roses as 1 unit & 6 red roses as 6 units. In total there are 7 units.
These 7 units can be arranged in the form of a garland in

$$\frac{(7-1)!}{2} = \frac{6!}{2} = 360 \text{ ways.}$$

& the 3 yellow roses can be arranged among themselves in 3! ways
∴ The required number of arrangement is

$$= 360 \times 3!$$
$$= 2160.$$

(iii) No two yellow roses are together
Firstly, we arrange the 6 red roses in the form of a garland in

$$\frac{(6-1)!}{2} = \frac{5!}{2} \text{ ways.}$$

Among these 6 red roses there are 6 places.

[illegible]

$=$ [illegible]

$=$ [illegible]

∴ The [illegible]

$=$ [illegible]

$=$ [illegible]

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