1

Fast fourier Transform (FFT)

FFT is an algorithm that efficiently computes the DFT.

The DFT of a seq. $x(n)$ of length N is gn. by a Complex valued seqn. $\{X(k)\}$

$$\boxed{X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N}, \quad 0 \le K \le N-1}$$

Let $W_N \rightarrow$ be the complex valued phase factor, which is an N^{th} root of unity expressed by

$$\boxed{W_N = e^{-j2\pi/N}}$$

Hence $X(k)$ becomes

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{nk}, \quad 0 \le K \le N-1 \quad \text{--- (2)}$$

IDFT becomes,

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(k)\, W_N^{-nK}, \quad 0 \le n \le N-1$$

→ From the above eqns, it is evident that for each value K, the direct computation of $X(k)$ involves → N complex multiplications ($4N$ real multiplications), $N-1$ complex additions ($4N-2$ real additions), so we need to compute all N values of DFT, N^2 complex mul N^2 complex additions are required.

Direct Computation of DFT

No of complex additions = $N(N-1) = 512 \times (512-1)$
$= 2,61,632$.

No. of complex MUX = $N^2 = 512^2 = 262144$.

$\log_2 2^m = m$.

Radix-2 FFT

No. of complex addns : $N \log_2 N = 512 \times \log_2 512$
$= 512 \times \log_2 2^9 = 512 \times 9$
$= 4608$

No. of complex MUX = $\frac{N}{2} \log_2 N = \frac{512}{2} \times \log_2 512$

$= \frac{512}{2} \times \log_2 2^9 = \frac{512}{2} \times 9 = 2304$

% Saving

in addn = $100 - \frac{\text{No. of addn. in radix-2 FFT}}{\text{No. of addn. in Direct DFT}} \times 100$

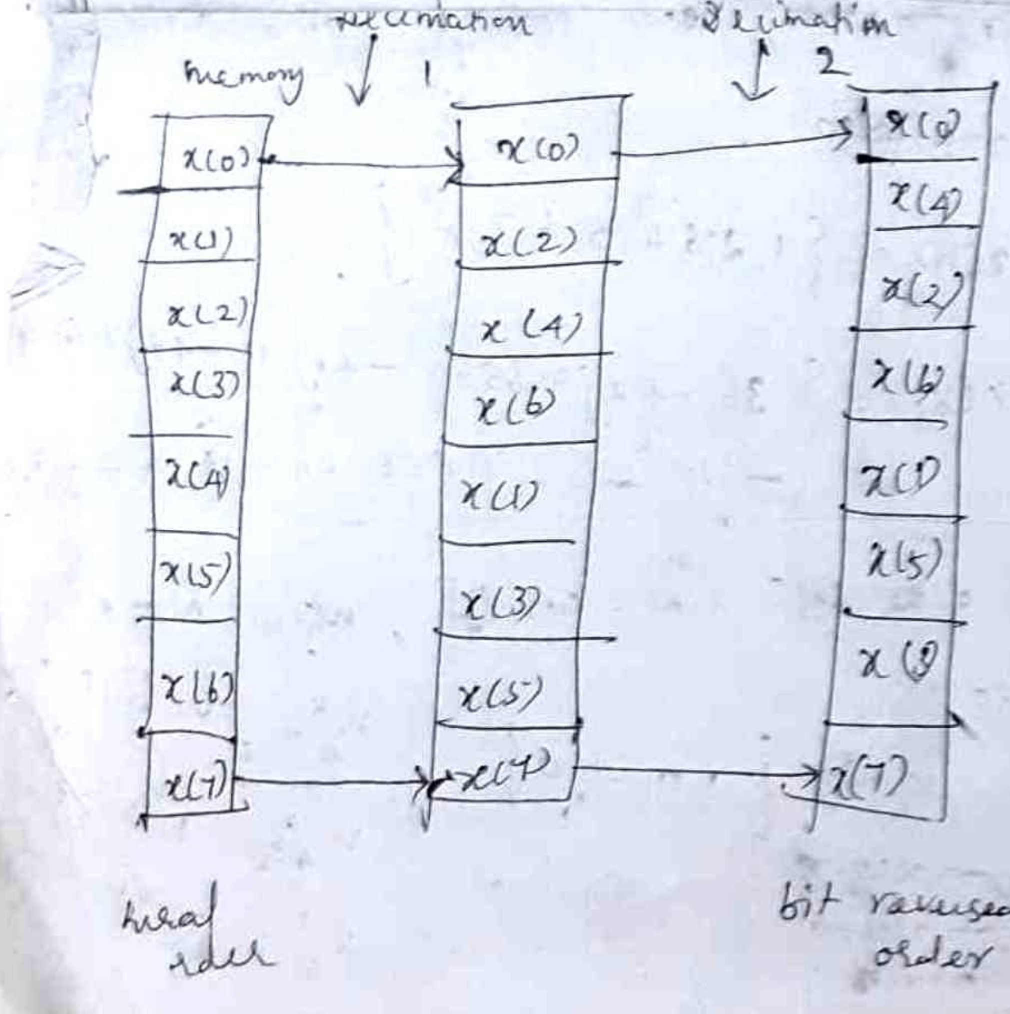
$= 100 - \frac{4608}{261632} \times 100 = 98.2\%$.

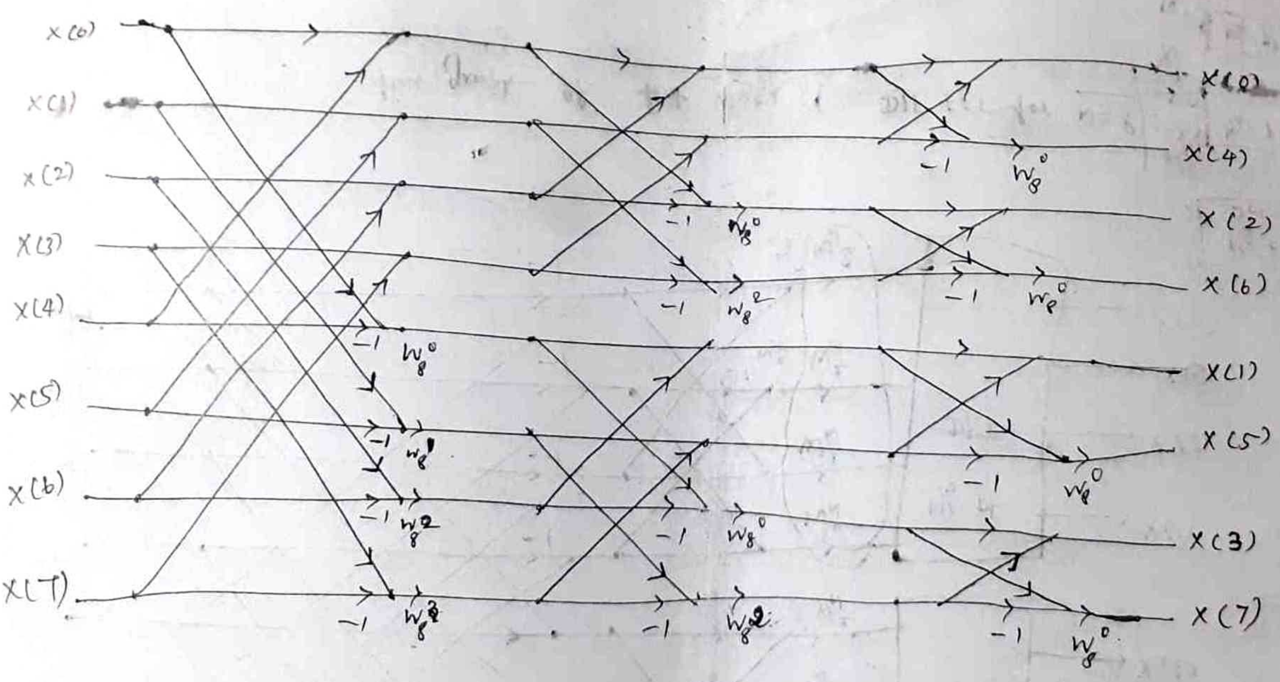
in MUX = $100 - \frac{\text{No. of MUX. in radix-2 FFT}}{\text{No. of MUX in direct DFT}} \times 100$.

$= 100 - \frac{2304}{262144} \times 100 = 99.1\%$.

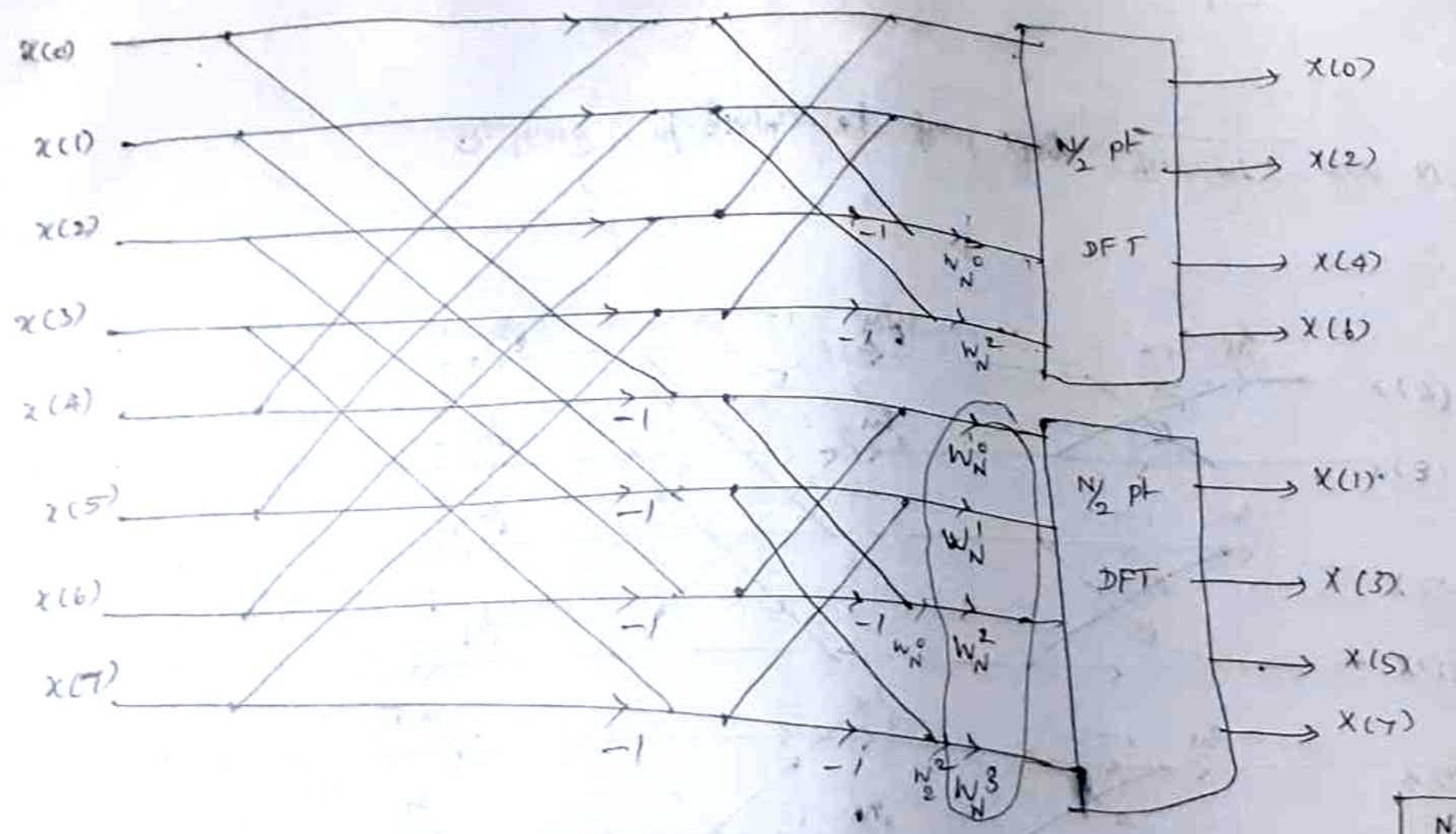
Shuffling of the data and bit reversal

	$n_2 n_1 n_0$		$\rightarrow n_0 n_2 n_1$	$\rightarrow n_0 n_1 n_2$	
0	0 0 0	0	0 0 0	0 0 0	0
1	0 0 1	4	1 0 0	1 0 0	4
2	0 1 0	1	0 0 1	0 1 0	2
3	0 1 1	5	1 0 1	1 1 0	6
4	1 0 0	2	0 1 0	0 0 1	1
5	1 0 1	6	1 1 0	1 0 1	5
6	1 1 0	3	0 1 1	0 1 1	3
7	1 1 1	7	1 1 1	1 1 1	7

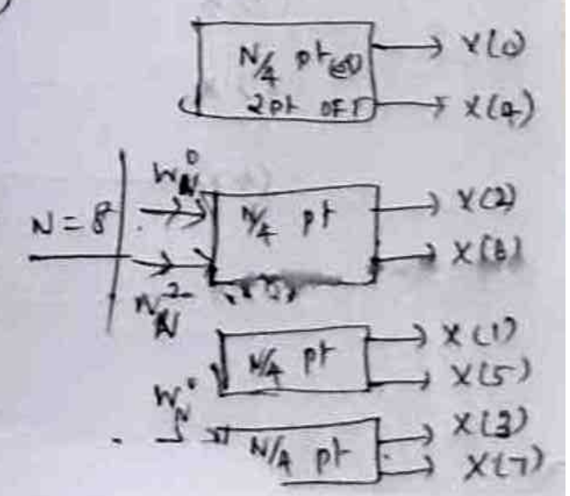




Reduced f. graph of final stage <u>DIT FFT</u> for N = 8.



flow graph of first stage of DIF FFT for N=8



Fast FT → is a method or algorithm for computing the DFT with reduced no. of calculations.

- the computational eff. is achieved if we adopt a divide & conquer approach.
- This approach is based on the decomposition of an N-point DFT into successively smaller DFTs.

In an N-point sequence, if N can be expressed as $N = r^m$, then the seqn. can be decimated into r-point sequences.

- for each r-point sequence, r-point DFT can be computed.
- from the results of r-point DFT, the r^2-point DFTs are computed.
- from the results of r^2 point DFTs, the r^3 point DFTs are computed and so on, until we get r^m point DFT.

⇒ In computing N-point DFT by this method the no. of stages of computation will be m-times

→ The no. r is called the radix of the FFT algorithm

In radix-2 FFT, the N-point sequence is decimated into 2-pt sequences and

- the 2-pt DFT for each decimated sequ. is computed.
- from the results of 2-pt. DFTs, the 4-pt, DFTs can be computed.
- from the results of 4-pt DFT, the 8-pt DFT can be computed and so on, until we get N-point DFT.

DIT Radix - 2 FFT

In DIT algorithm, the time domain sequence $x(n)$ is decimated and smaller pt. DFTs are performed. the results of smaller pt. DFTs are combined to get the result of N - pt. DFT.

In general, In DIT algo, the N-pt DFT can be realized from two numbers of $N/2$ pt. DFTs, the $N/2$ pt DFT can be realized from two numbers of $N/4$ pt DFTs and so on.

Let $x(n) \rightarrow$ be the N - sample seq.

- we can decimate $x(n)$ into two sequences of $N/2$ samp

→ Let the two sequences be $f_1(n)$ and $f_2(n)$

→ Let $f_1(n)$ consists of even numbered samples of $x(n)$

$f_2(n) \rightarrow$ odd numbered samples.

$f_1(n) = x(2n) \qquad n = 0, 1, \cdots \left(\frac{N}{2} - 1\right).$

$f_2(n) = x(2n+1) \qquad n = 0, 1, 2, \cdots (N/2 - 1)$

eqn ② can be written as.

$$\rightarrow X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$$

$$X(k) = \sum_{n=even} x(n)\, W_N^{kn} + \sum_{n=odd} x(n)\, W_N^{kn} \qquad (k=0,1,\ldots N-1) \quad \text{—③}.$$

→ when $n \rightarrow$ is replaced by $2n \rightarrow$ even no. samples selected

$n \rightarrow (2n+1) \rightarrow$ odd nok. selected.

$$\rightarrow X(k) = \sum_{n=0}^{\frac{N}{2}-1} x(2n)\, \underline{W_N^{k(2n)}} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1)\, \underline{W_N^{k(2n+1)}} \quad \text{—④}.$$

Now the phase factors can be modified.

$$W_N^{k(2n)} = \left(e^{-j2\pi}\right)^{\frac{k(2n)}{N}} = \left(e^{-j2\pi}\right)^{\frac{kn}{N/2}} = W_{N/2}^{kn}$$

$$W_N^{k(2n+1)} = \left(e^{-j2\pi}\right)^{\frac{k(2n+1)}{N}} = \left(e^{-j2\pi}\right)^{\frac{k2n}{N}} \cdot \left(e^{-j2\pi}\right)^{\frac{k}{N}}$$

$$= \left(e^{-j2\pi}\right)^{\frac{kn}{N/2}} \left(e^{-j2\pi}\right)^{\frac{k}{N}}.$$

$$= W_{N/2}^{kn} \cdot W_N^{k}$$

eqn (4) can be written as

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} f_1(n)\, W_{N/2}^{kn} + \sum_{n=0}^{\frac{N}{2}-1} f_2(n)\, W_{N/2}^{kn} \cdot W_N^{k}$$

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} f_1(n)\, W_{\frac{N}{2}}^{kn} + W_N^{k} \sum_{n=0}^{N/2-1} f_2(n) \cdot W_{N/2}^{kn} \quad \text{— (5)}$$

By the def. of DFT, the $N/2$ pt DFT of $f_1(n)$ & $f_2(n)$ are gn. by

$$\boxed{X(k) = F_1(k) + W_N^{k} F_2(k)}$$ where $k = 0, 1, 2, \ldots N-1$

From the above exp, it is clear that the N-pt. DFT of N sample sequence can be obtained from the results of N/2 pt. DFT.

8-pt DFT using Radix-2 DIT FFT

⇒ involves 3-stages of computation.

→ $N = 8 = 2^3$, ∴ $r = 2$ and $m = 3$.

→ the gn. 8-pt seq. is decimated to 2-pt sequences.

- for each 2-pt sequence, the 2-pt DFT is computed.
- from the result of 2-pt DFT, 4 pt DFT can be computed
- from the result of 4-pt DFT, 8 pt DFT can be computed.

59,

⇒ Let the gn. seqn. be $x(0) \to x(7)$, which consists of 8-samples,

- the 8-samples should be decimated into sequences of 2-samples.

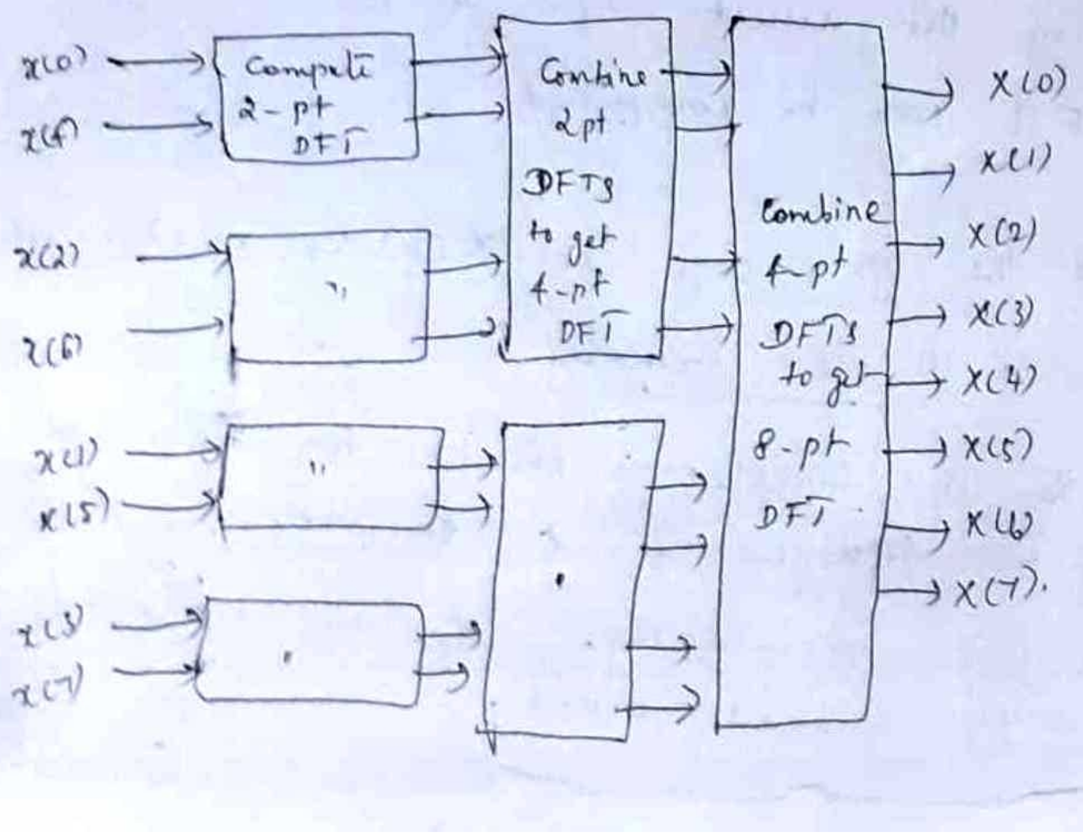
→ before decimation, they are arranged in bit reversed order.

→ $x(n)$ in bit reversed order is decimated into 4-numbers of 2-pt. sequence.

- $x(0)$ & $x(4)$
- $x(2)$ & $x(6)$
- $x(1)$ & $x(5)$
- $x(3)$ & $x(7)$

normal order		Bit rev. order	
$x(0)$	$x(000)$	$x(0)$	000
$x(1)$	(001)	$x(4)$	100
$x(2)$	(010)	$x(2)$	010
$x(3)$	011	$x(6)$	110
$x(4)$	100	$x(1)$	001
$x(5)$	101	$x(5)$	101
$x(6)$	110	$x(3)$	011
$x(7)$	111	$x(7)$	111

- Using the decimated sequences as I/p, the 8-point DFT is computed.



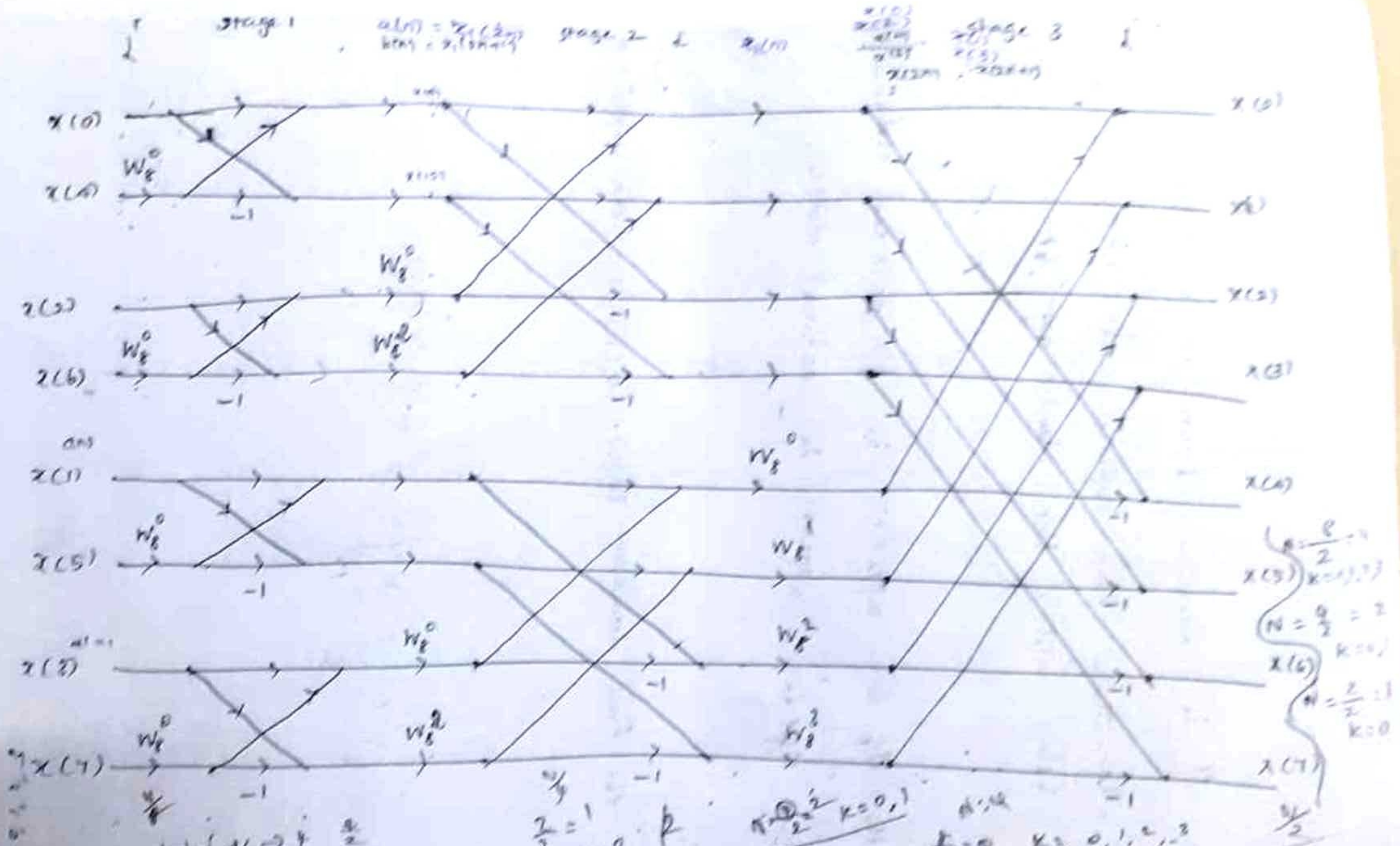


Fig: Reduced flow graph for an 8-pt DIT FFT

$W_N^k \Rightarrow W_{N/2}^k \Rightarrow W_{N/4} \Rightarrow W_N$

$a + bW$

$a - bW$

$N = 2$ $\quad F(k) + W_N^k G(k)$

$k = 0, 1$

DIT -

It is observed from basic f.graph, the algorithm has inplace calculations given below with the butterfly structure.

In DIT —

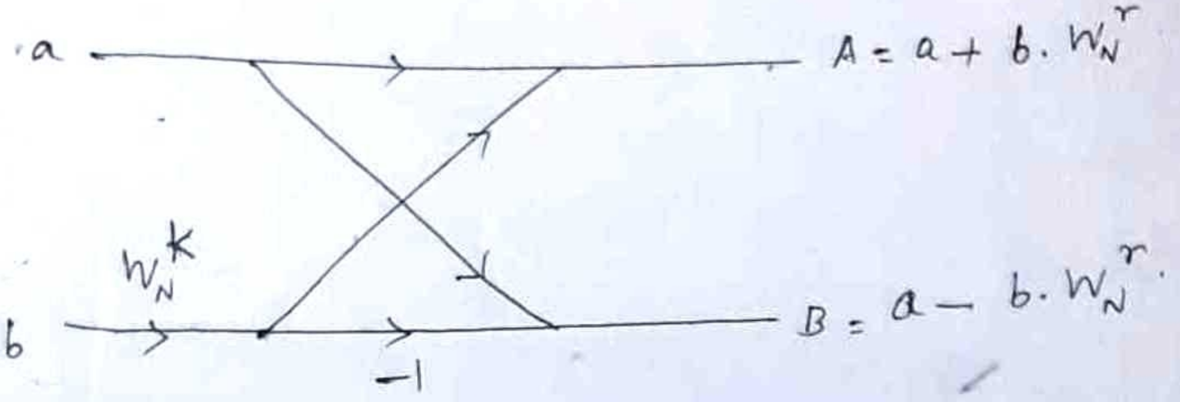
the I/P seq. appears in bit reversed order

O/P seq. appears in natural order.

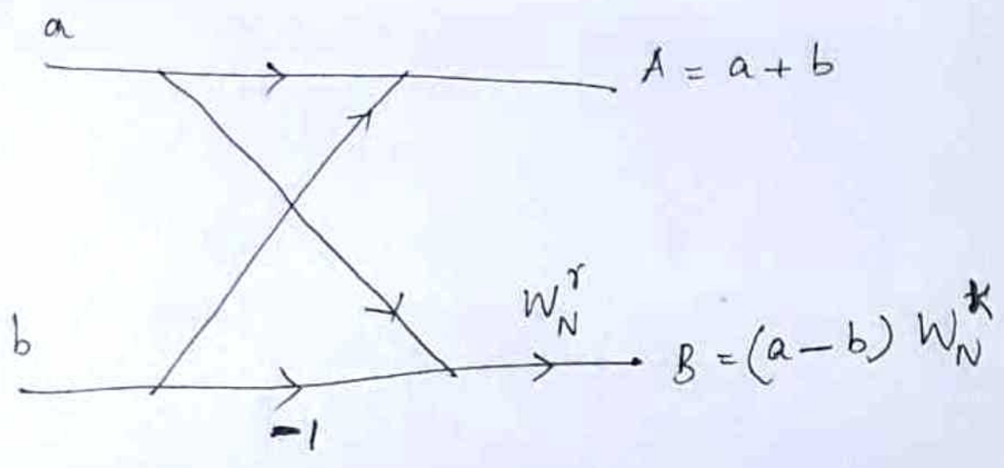
In DIT —

the time domain seqn. $x(n)$ is decimated

Basic Butterfly flow graph for the computation in the DIT FFT Algorithm.



DIF FFT



1. Given $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$.

find $X(k)$ using DIT FFT algorithm.

soln:

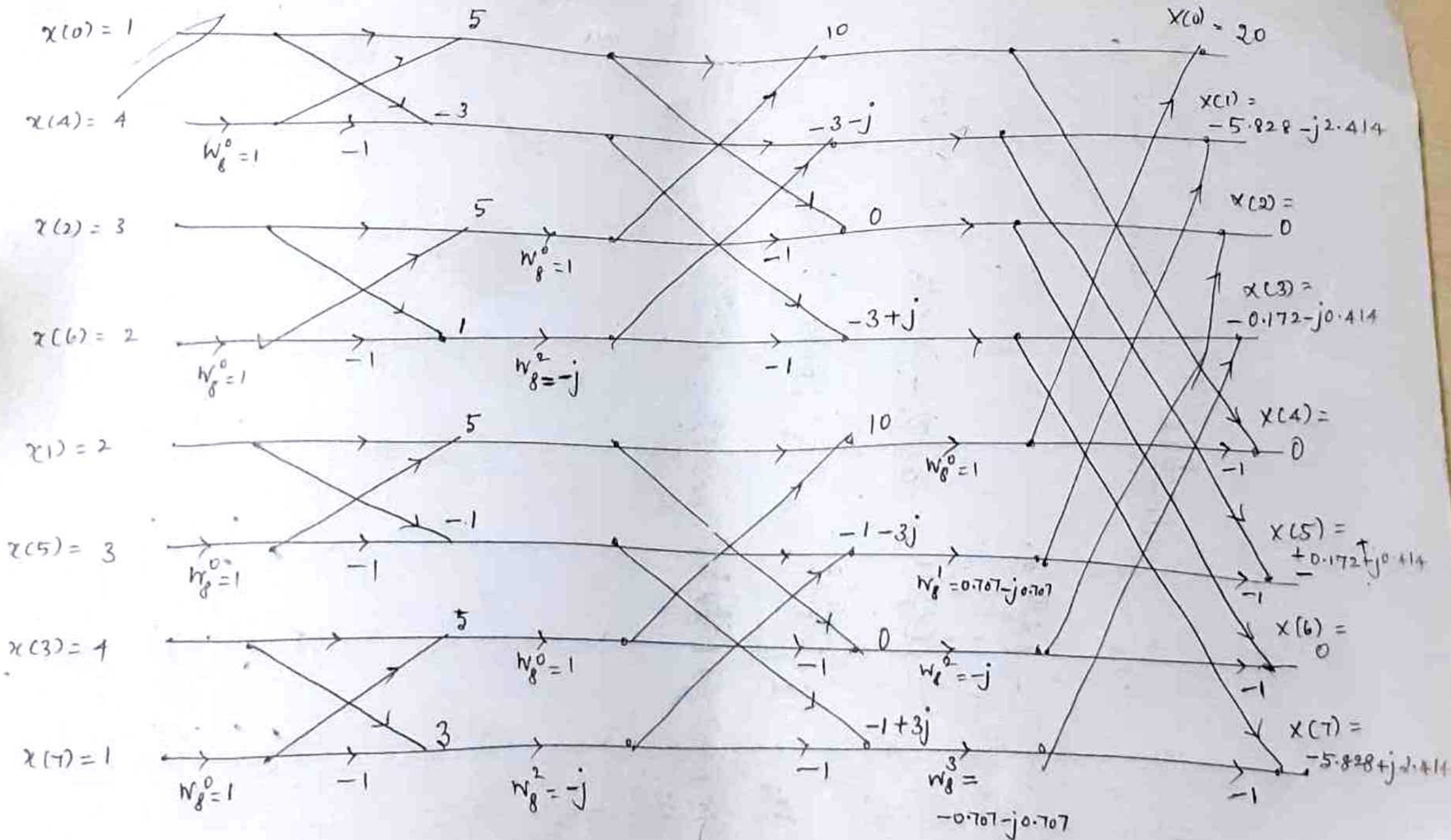
$$W_N^K = e^{-j\left(\frac{2\pi}{N}\right)k} \qquad \text{Given } N = 8$$

$$W_8^0 = e^{-j\left(\frac{2\pi}{8}\right)0} = 1.$$

$$W_8^1 = e^{-j\left(\frac{2\pi}{8}\right)1} = \cos \pi/4 - j\sin \pi/4 = 0.707 - j0.707.$$

$$W_8^2 = e^{-j\left(\frac{2\pi}{8}\right)2} = \cos \pi/2 - j\sin \pi/2 = -j$$

$$W_8^3 = e^{-j\left(\frac{2\pi}{8}\right)3} = \cos 3\pi/4 - j\sin 3\pi/4 = -0.707 - j0.707$$



$-j - j2.12 \approx 3$

$(-3-j) + (-1-3j)(0.707 - j0.707)$

$\cancel{-0.707} + j0.707 - j2.121 - \cancel{2.12}$

-2.82

2. Given $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$.

Find $X(k)$ using DIF - FFT algorithm.

Soln: Given $N = 8$,

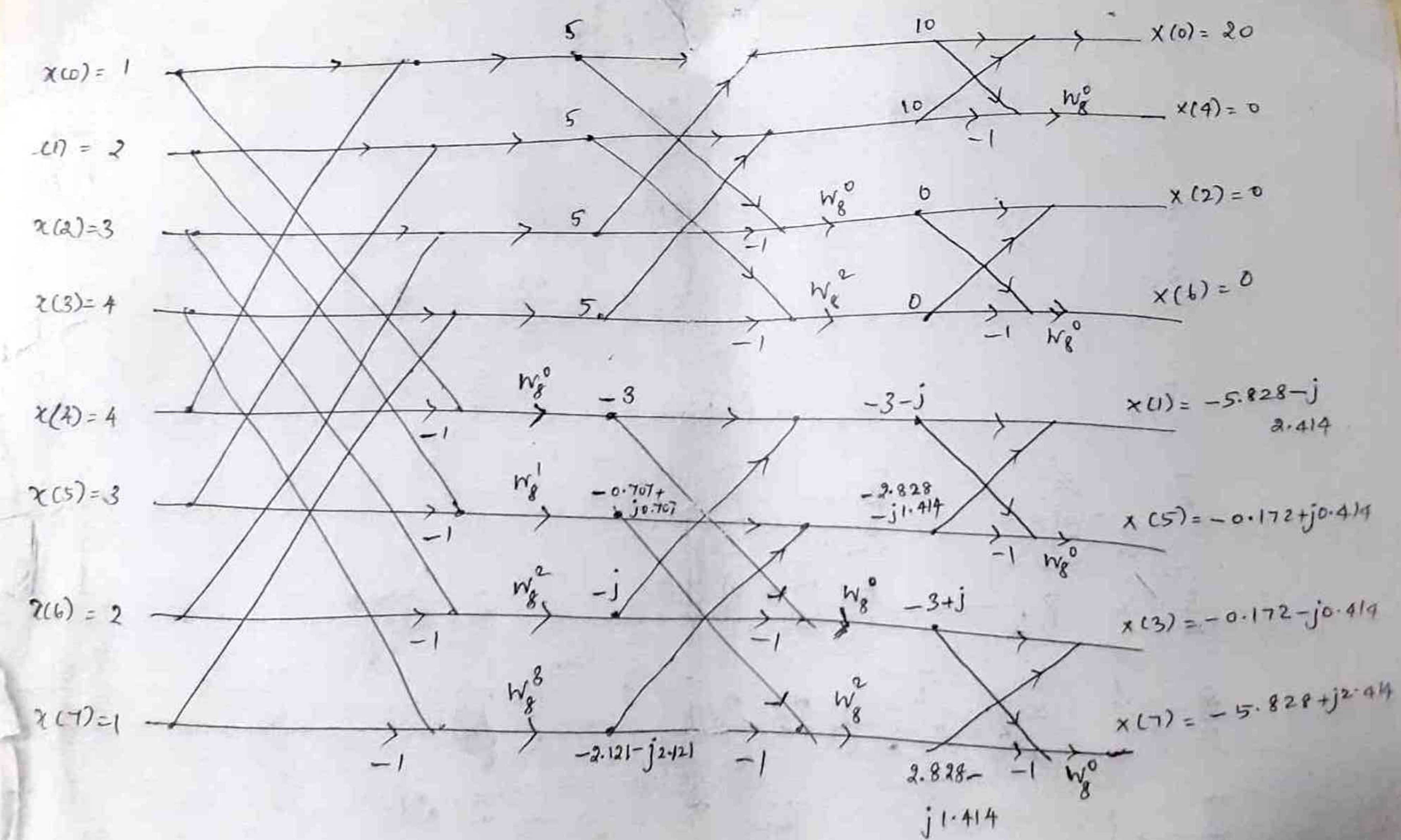
$$W_N^K = e^{-j\left(\frac{2\pi}{N}\right)K}$$

$\therefore W_8^0 = 1$

$W_8^1 = 0.707 - j0.707$

$W_8^2 = -j$

$W_8^3 = -0.707 - j0.707$



x(0) = 1
x(1) = 2
x(2) = 3
x(3) = 4
x(4) = 4
x(5) = 3
x(6) = 2
x(7) = 1
5
5
5
5
-3
-0.707+j0.707
-j
-2.121-j2.121
10
10
0
0
-3-j
-2.828-j1.414
-3+j
2.828-j1.414
X(0) = 20
X(4) = 0
X(2) = 0
X(6) = 0
X(1) = -5.828-j2.414
X(5) = -0.172+j0.414
X(3) = -0.172-j0.414
X(7) = -5.828+j2.414

$x(n) = 2^n$ and $N = 8$ find $X(k)$

using DIT FFT algo & DIF FFT algo.

$X(K) = \{255, 48.6 + j166.05, -51 + j102, -78.63 + j46.05, -85, -78.63 - j46.05, -51 - j102, 48.63 - j166.05\}$

Given $x(n) = \{0, 1, 2, 3, 4, 5, 6, 7\}$, find $X(k)$.

using DIT FFT algo & DIF FFT algo.

$X(K) = \{28, -4 + j9.656, -4 + 4j, -4 + j1.656, -4, -4 - j1.656, -4 - 4j, -4 - j9.656\}$

$x(n) = \{1, 2, 4, 8, 16, 32, 64, 128\}$

$X(K) = \{255, 48.63 + j166.05, -51 + j102, -78.63 + j46.05, -85, -78.63 - j46.05, -51 - j102, 48.6 - j166.05\}$

3. Given $x(n) = n + 1$ and ... find ...

using DIT & DIF algo.

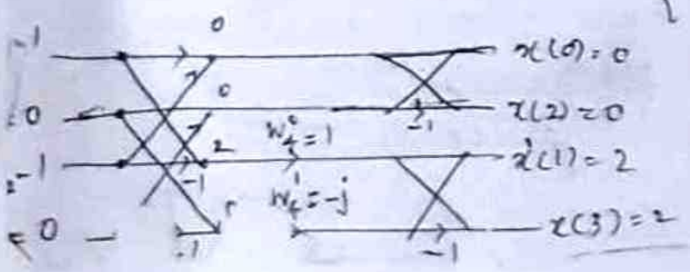
$x(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

Ans. $X(K) = \{36, -4 + j9.656, -4 + j4, -4 + j1.656, -4, -4 - j1.656, -4 - j4, -4 - j9.656\}$

Compute the DFTs of the seq. $x(n) = \cos\frac{n\pi}{2}$, where $N = 4$, using DIF FFT algo.

$N = 4$, $x(n) = \{1, 0, -1, 0\}$.

$W_N^k = e^{-j\left(\frac{2\pi}{N}\right)k}$



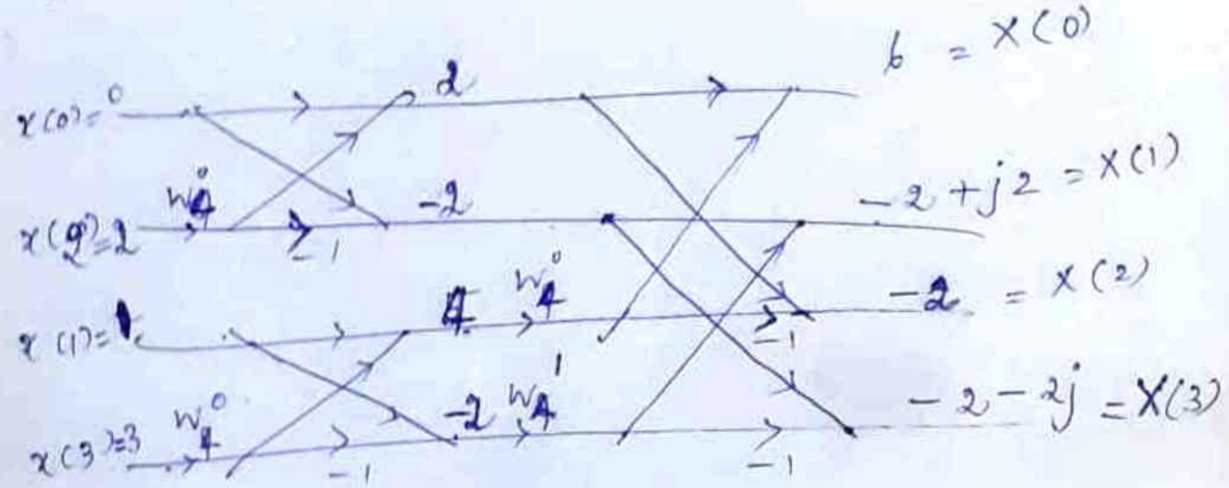
$W_4^0 = 1$

$W_4^1 = e^{-j\pi/2}$

Given $x(n) = \{0, 1, 2, 3\}$ find $X(k)$.

$N = 4$.

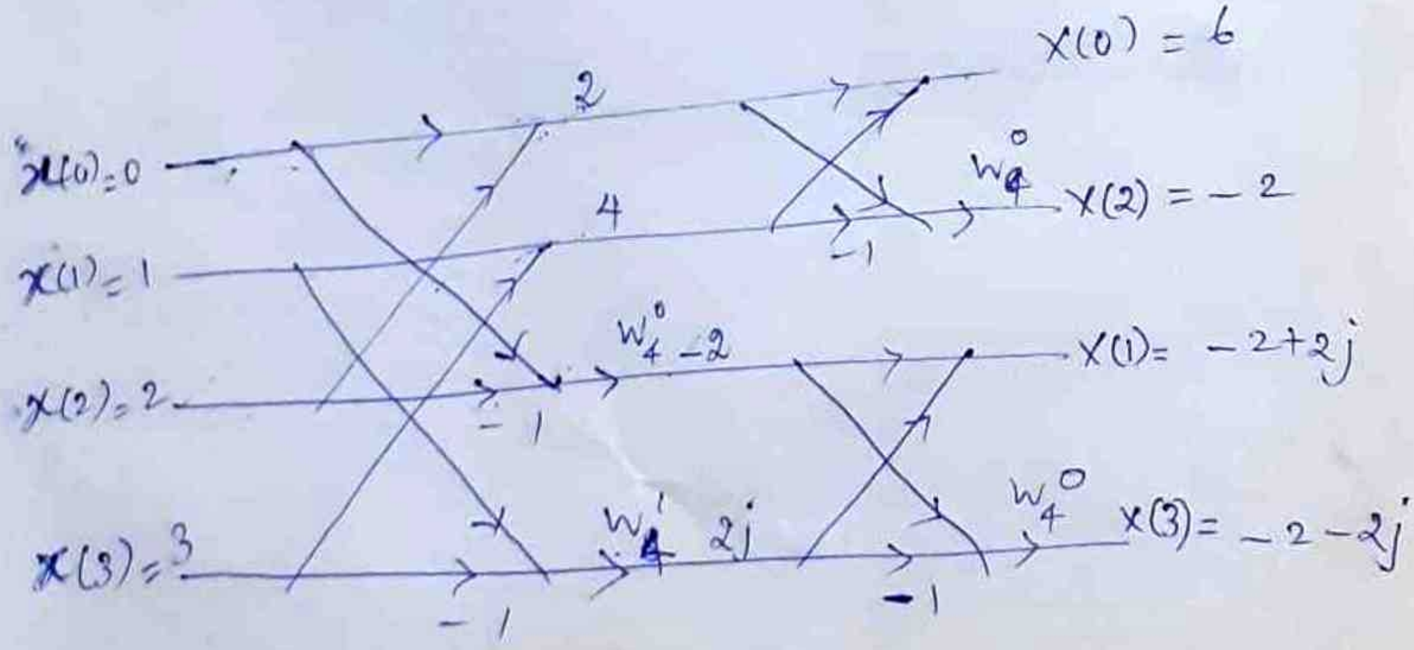
using



DIT

$W_4^0 = 1$

$W_4^1 = -j$



DIF

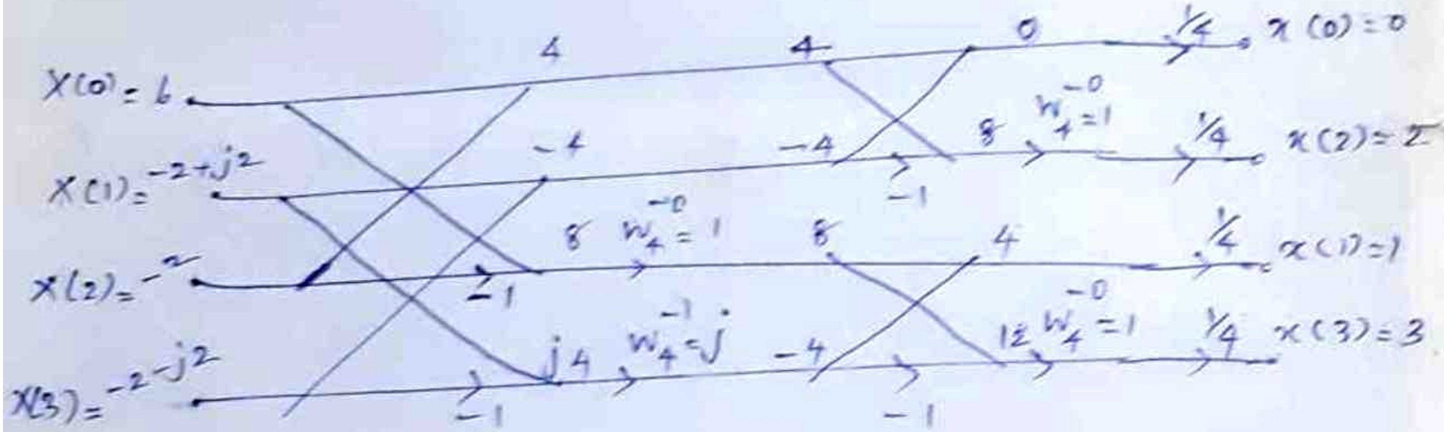
Use the 4-pt. inverse FFT and verify the DFT results {6, -2+j2, -2, -2-j2} for the given i/p seq. {0,1,2,3}.

<u>soln:</u> $W_N^{k} = e^{-j\left(\frac{2\pi}{N}\right)k}$

$W_4^{-0} = 1$

$W_4^{-1} = e^{j\pi/2} = j$

Using IFFT also, we can find the o/p seq. x(n) from the gn. i/p seq. X(K).



Given $X(k) = \{20, -5.828 - j2.414, 0, -0.172 - j0.414, 0, -0.172 + j0.414, 0, -5.828 + j2.414\}$.

find $x(n)$.

$W_N^k = e^{-j\left(\frac{2\pi}{N}\right)k}$ Given $N = 8$

$W_8^{-0} = 1$

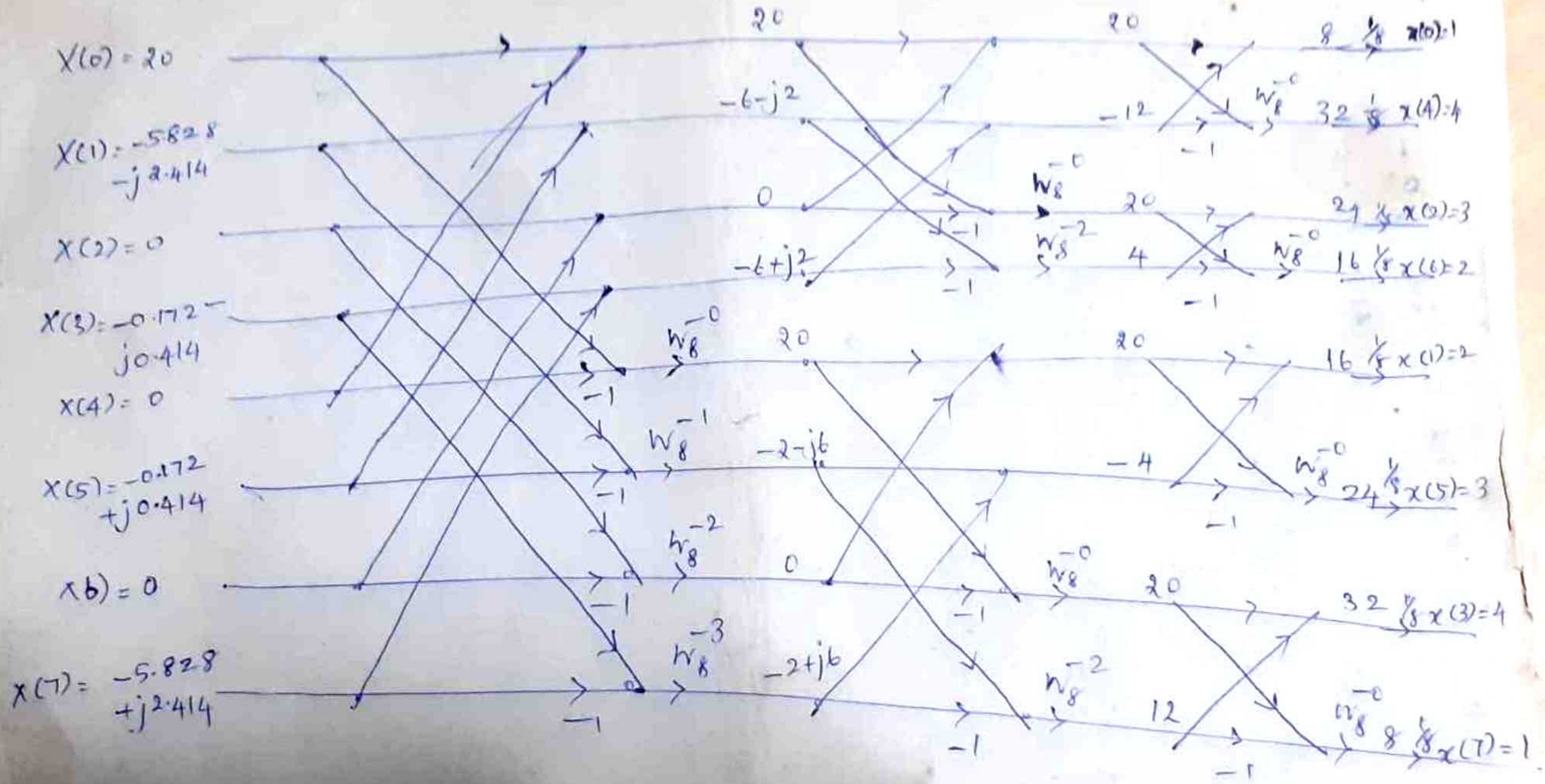
$W_8^{-1} = 0.707 + j0.707$

$W_8^{-2} = j$

$W_8^{-3} = -0.707 + j0.707$

Ans:

$x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$



X(0) = 20
X(1) = -5.828 -j2.414
X(2) = 0
X(3) = -0.172 - j0.414
X(4) = 0
X(5) = -0.172 +j0.414
X(6) = 0
X(7) = -5.828 +j2.414
20
-6-j2
0
-6+j2
20
-2-j6
0
-2+j6
20
-12
20
4
20
-4
20
12
8 1/8 x(0)=1
32 1/8 x(4)=4
24 1/8 x(2)=3
16 1/8 x(6)=2
16 1/8 x(1)=2
24 1/8 x(5)=3
32 1/8 x(3)=4
8 1/8 x(7)=1

10) Given $X(K) = \{ 255, 48.63 + j166.05, -51+j102, -78.63 + j46.05, -85, -78.63 - j46.05, -51 - j102, 48.63 - j166.05 \}$.

Find $x(n)$.

$W_N^K = e^{-j\left(\frac{2\pi}{N}\right)k}$.

$(W_N^K)^* = e^{j\frac{2\pi}{N}k}$.

$W_8^0 = 1$	$W_8^{-0} = 1$
$W_8^1 = 0.707 - j0.707$	$W_8^{-1} = 0.707 + j0.707$
$W_8^2 = -j$	$W_8^{-2} = j$
$W_8^3 = -0.707 - j0.707$	$W_8^{-3} = -0.707 + j0.707$

$x(n) = \{1, 2, 4, 8, 16, 32, 64, 128\}$.

Given, $X(k) = \{ 36, -4 + j9.656, -4 + j4, -4 + j1.656, -4, -4 - j1.656, -4 - j4, -4 - j9.656 \}$

find $x(n)$.

$x(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}$

DIF Radix - 2 FFT

In DIF, the freq. domain sequence $X(k)$ is decimated.

- In this algo, the N point time domain seqn is converted to 2 nos/. of $N/2$ point sequences
- then each $N/2$ pt. seqn. is converted to 2 nos/ of $N/4$ pt sequences. — thus we get 4 nos/. of $N/4$ pt. seqns

This process is continued until we get $N/2$ nos. of 2-pt sequences.

Finally the 2-pt DFT of each 2-pt sequence is computed.

- the 2-pt DFTs of $N/2$ nos of 2-pt. sequences will give N samples. — which is the N-pt. DFT of the time domain seqn.

→ N-pt. DFT of $x(n)$ ← 2 nos/. of $N/2$ pt DFTs

$N/2$ pt DFT ← " $N/4$ pt. DFTS

⋮

The decimation continues upto 2-pt. DFTs.

Consider a N-pt. seqn. $x(n)$.

Let the N-pt DFT of $x(n)$ is gn. by

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

$$= \sum_{n=0}^{N/2-1} x(n) W_N^{kn} + \sum_{n=N/2}^{N-1} x(n) W_N^{kn}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(n) W_N^{kn} + \sum_{n=0}^{\frac{N}{2}-1} x\left(n+\frac{N}{2}\right) W_N^{k(n+\frac{N}{2})}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(n) W_N^{kn} + \sum_{n=0}^{\frac{N}{2}-1} x\left(n+\frac{N}{2}\right) W_N^{kn} W_N^{\frac{kN}{2}}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) W_N^{kn} + (-1)^k x\left(n+\frac{N}{2}\right) W_N^{kn} \right]$$

$$\boxed{X(k) = \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) + (-1)^k x\left(n+\frac{N}{2}\right) \right] W_N^{kn}} \quad \text{——(1)}$$

$$\left[W_N^{\frac{kN}{2}} = e^{-j\frac{2\pi}{N}\cdot\frac{kN}{2}} = (e^{-j\pi})^k = (-1)^k \right]$$

Let us split $X(k)$ into even and odd numbered samples.

For even values of K, the $X(K)$ can be written as

$$X(2K) = \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) + (-1)^{2K} x\left[n+\frac{N}{2}\right)\right] W_N^{2Kn}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) + (-1)^{2K} \cdot x\left[n+\frac{N}{2}\right)\right] W_N^{2Kn}$$

$$X(2K) = \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) + x\left(n+\frac{N}{2}\right)\right] W_{N/2}^{Kn} \quad \text{for } K=0,1,\ldots \frac{N}{2}-1 \quad — (2).$$

For odd values of K, $X(K)$ can be written as

$$X(2K+1) = \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) + (-1)^{2K+1} x\left(n+\frac{N}{2}\right)\right] W_N^{(2K+1)n}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) - x\left(n+\frac{N}{2}\right)\right] W_N^{2Kn} W_N^{n}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) - x\left(n+\frac{N}{2}\right)\right] W_N^{n} W_{N/2}^{Kn} \quad K=0,1,\ldots \frac{N}{2}-1 \quad — (3)$$

Let us define new time domain sequences $g_1(n)$ & $g_2(n)$ consisting of $N/2$ samples. — (4)

Let $g_1(n) = x(n) + x\left(n+\frac{N}{2}\right)$ for $n=0,1,2,\ldots \frac{N}{2}-1$

$$g_2(n) = \left[x(n) - x\left(n+\frac{N}{2}\right)\right] W_N^{n} \quad n=0,1,\ldots \frac{N}{2}-1 \quad — (5).$$

Using eqn 5, eqn 3 can be written as

$$X(2K) = \sum_{n=0}^{\frac{N}{2}-1} g_1(n)\, W_{N/2}^{kn} \quad \text{for } K = 0, 1, 2 \ldots \frac{N}{2}-1 \quad \text{— (6)}$$

Using eqn 5, eqn 4 can be written as

$$X(2K+1) = \sum_{n=0}^{\frac{N}{2}-1} g_2(n)\, W_{N/2}^{kn} \quad \text{for } k = 0, 1, 2, \ldots \frac{N}{2}-1 \quad \text{— (7)}$$

Now $X(2K)$ is $\frac{N}{2}$ pt. DFT of $g_1(n)$ &

$X(2k+1)$ is $\frac{N}{2}$ pt. DFT of $g_2(n)$

Let $X(2K) = G_1(K)$

$X(2k+1) = G_2(K)$ —— (8)

Using eqn (8), the eqns (6) & (7) can be written as

$$G_1(K) = \sum_{n=0}^{\frac{N}{2}-1} g_1(n)\, W_{\frac{N}{2}}^{kn} \quad k = 0, 1, 2 \ldots \frac{N}{2}-1 \quad \text{— 9}$$

$$G_2(K) = \sum_{n=0}^{\frac{N}{2}-1} g_2(n)\, W_{\frac{N}{2}}^{kn} \quad k = 0, 1, 2 \ldots \frac{N}{2}-1 \quad \text{— 10}$$

$G_1(k)$ is $\frac{N}{2}$ pt sequence and it can be decimated into two numbers of $\frac{N}{4}$ pt sequences Hence the eqn. (9) can be modified as shown below

$$G_1(K) = \sum_{n=0}^{\frac{N}{2}-1} g_1(n) W_{\frac{N}{2}}^{kn} = \sum_{n=0}^{\frac{N}{4}-1} g_1(n) W_{\frac{N}{2}}^{kn} + \sum_{n=N/4}^{\frac{N}{2}-1} g_1(n) W_{N/2}^{Kn}$$

$$= \sum_{n=0}^{N/4-1} g_1(n) W_{\frac{N}{2}}^{kn} + \sum_{n=0}^{\frac{N}{4}-1} g_1\left(n+\frac{N}{4}\right) W_{\frac{N}{2}}^{K(n+N/4)}$$

$$= \sum_{n=0}^{\frac{N}{4}-1} \left[g_1(n) W_{\frac{N}{2}}^{\frac{kn}{4}} + g_1\left(n+\frac{N}{4}\right)\right] W_{\frac{N}{2}}^{kn} W_{N/2}^{\frac{kN}{4}} \quad \Bigg| \quad W_{\frac{N}{2}}^{\frac{kN}{4}} = e^{-\frac{j2\pi}{N/2}\cdot\frac{kN}{4}} = \left(e^{-j\pi}\right)^k = (-1)^k$$

$$G_1(K) = \sum_{n=0}^{\frac{N}{4}-1} \left[g_1(n) + (-1)^k g_1\left(n+\frac{N}{4}\right)\right] W_{N/2}^{kn} \quad \text{—— } 11.$$

Let us decimate $G_1(K)$ into 2 nos. of $N/4$ pt sequence

For even values of K, the eqn. (11) can be written as

$$G_1(2K) = \sum_{n=0}^{\frac{N}{4}-1} \left[g_1(n) + (-1)^{2k} g_1\left(n+\frac{N}{4}\right)\right] W_{N/2}^{2Kn}$$

$$G_1(2k) = \sum_{n=0}^{N/4-1} \left[g_1(n) + g_1\left(n+\frac{N}{4}\right)\right] W_{N/4}^{kn} \quad \text{for } k=0,1,\ldots N/4-1$$

—— (12)

For odd values of k, eqn (11) can be written as

$$G_1(2k+1) = \sum_{n=0}^{N/4-1} \left[g_1(n) + (-1)^{2k+1} g_1(n+N/4) \right] W_{N/2}^{(2k+1)n}$$

$$G_1(2k+1) = \sum_{n=0}^{\frac{N}{4}-1} \left[g_1(n) - g_1\left(n+\frac{N}{4}\right) \right] W_{N/2}^{n} W_{N/4}^{kn} \quad \text{—— (13)}$$

for $k = 0, 1, \ldots N/4 - 1$

Similarly the $N/2$ pt. sequence $G_2(k)$ can be decimated into two new of $N/4$ pt. sequences as

$$G_2(2k) = \sum_{n=0}^{\frac{N}{4}-1} \left[g_2(n) + g_2\left(n+\frac{N}{4}\right) \right] W_{N/2}^{kn} \quad \text{—— 14}$$

$$G_2(2k+1) = \sum_{n=0}^{\frac{N}{4}-1} \left[g_2(n) - g_2(n+N/4) \right] W_{N/4}^{n} W_{N/4}^{kn} \quad \text{—— 15}$$

Let us define new time domain sequences $d_{11}(n)$, $d_{12}(n)$, $d_{21}(n)$ and $d_{22}(n)$ consisting of $N/4$ samples.

Let $d_{11}(n) = g_1(n) + g_1\left(n+\frac{N}{4}\right)$

$$d_{12}(n) = \left[g_1(n) - g_1\left(n+\frac{N}{4}\right) \right] W_{N/2}^{n} \quad \text{—— 16}$$

$$d_{21}(n) = g_2(n) + g_2(n+N/4)$$

$$d_{22}(n) = \left[g_2(n) - g_2\left(n+\frac{N}{4}\right) \right] W_{N/2}^{n} \quad \text{—— 17}$$

Using eqn (16), the eqns (12) & (13) can be written as

$$G_1(2k) = \sum_{n=0}^{\frac{N}{4}-1} d_{11}(n)\, W_{N/4}^{kn} = D_{11}(k) \quad —\ 18$$

$$— \quad 19.$$

$$G_1(2k+1) = \sum_{n=0}^{\frac{N}{4}-1} d_{12}(n)\, W_{N/4}^{kn} = D_{12}(k)$$

Using eqn (17), eqns 14, 15 can be written as

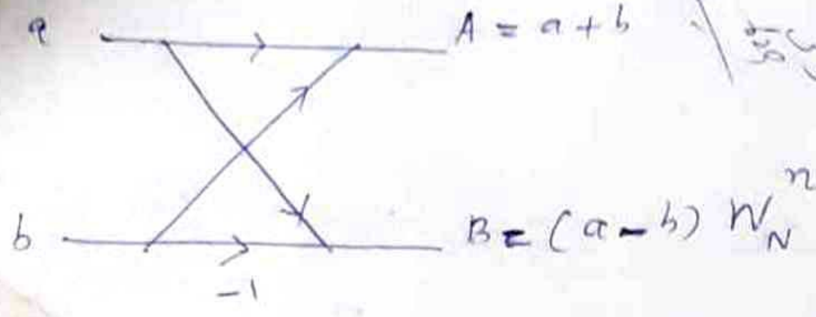
$$G_2(2k) = \sum_{n=0}^{\frac{N}{4}-1} d_{21}(n)\, W_{N/4}^{kn} = D_{21}(k) \quad —\ 20.$$

$$G_2(2k+1) = \sum_{n=0}^{N/4-1} d_{22}(n)\, W_{N/4}^{kn} = D_{22}(k) \quad —\ 21.$$

where $D_{11}(k)$, $D_{12}(k)$, $D_{21}(k)$ & $D_{22}(k)$ → $N/4$ pt. DFTs of $d_{11}(n)$, $d_{12}(n)$, $d_{21}(n)$ & $d_{22}(n)$ respe[ctively]

The Decimation of the freq. domain seqn. can be continued until the resulting sequence are reduce to 2-pt. sequences.

On observing the basic calculations, each stage involves $N/2$ butterflies of the of the type.



From above relations we get

$X(0) = G_1(0) = D_{11}(0)$

$X(4) = G_1(2) = D_{11}(1)$

$X(2) = G_1(1) = D_{12}(0)$

$X(6) = G_1(3) = D_{12}(1)$

$X(1) = G_2(0) = D_{21}(0)$

$X(5) = G_2(2) = D_{21}(1)$

$X(3) = G_2(1) = D_{22}(0)$

$X(7) = G_2(3) = D_{22}(1)$

From the above, we observe that the o/p is in bit reversed order.

Combining 3. stages of computation:

The final o/p. $D(k)$ gives $X(k)$.

$X(2k) = G_1(k)$; $k = 0, 1, 2, 3$

$X(0) = G_1(0)$

$X(2) = G_1(1)$

$X(4) = G_1(2)$

$X(6) = G_1(3)$

$X(2k+1) = G_2(k)$; $k = 0, 1, 2, 3$

$X(1) = G_2(0)$

$X(3) = G_2(1)$

$X(5) = G_2(2)$

$X(7) = G_2(3)$

$G_1(2k) = D_{11}(k)$: $k = 0, 1$

$G_1(0) = D_{11}(0)$

$G_1(2) = D_{11}(1)$

$G_1(2k+1) = D_{12}(k)$ $k = 0, 1$

$G_1(1) = D_{12}(0)$

$G_1(3) = D_{12}(1)$

$G_2(2k+1) = D_{22}(k)$ $k = 0, 1$

$G_2(1) = D_{22}(0)$

$G_2(3) = D_{22}(1)$

$G_2(2k+1) = D_{22}(k)$; $k = 0, 1$

$G_2(1) = D_{22}(0)$

$G_2(3) = D_{22}(1)$