Problem 1

a) Jaccard $(A, B) = \frac{4}{8} = 0.5$

Jaccard $(B, C) = \frac{4}{8} = 0.5$

Jaccard $(A, C) = \frac{4}{8} = 0.5$

$A = [4, 5, 0, 5, 1, 0, 3, 2]$

$B = [0, 3, 4, 3, 1, 2, 1, 0]$

$C = [2, 0, 1, 3, 0, 4, 5, 3]$

$$\cos(A,B) = \frac{0+15+0+15+1+0+3+0}{\sqrt{16+25+0+25+1+0+9+4}\sqrt{0+9+16+9+1+4+1+0}} = \frac{34}{\sqrt{80}\sqrt{40}}$$

$$= 0.601$$

$$\cos(B,C) = \frac{0+0+4+9+0+8+5+0}{\sqrt{0+9+16+9+1+4+1+0}\sqrt{4+0+1+9+0+16+25+9}} = \frac{26}{\sqrt{40}\sqrt{64}} = 0.514$$

$$\cos(A,C) = \frac{8+0+0+15+0+0+15+6}{\sqrt{16+25+0+25+1+0+9+4}\sqrt{4+0+1+9+0+16+25+9}} = \frac{44}{\sqrt{80}\sqrt{64}} = 0.615$$

b)

	a	b	c	d	e	f	g	h
A	1	1	0	1	0	0	1	0
B	0	1	1	1	0	0	0	0
C	0	0	0	1	0	1	1	1

$Jaccard(A,B) = \frac{2}{5} = 0.4$ (< part-A)

$Jaccard(B,C) = \frac{1}{6} = 0.1667$ (< part-A)

$Jaccard(A,C) = \frac{2}{6} = 0.3333$ (< part-A)

$A = [1, 1, 0, 1, 0, 0, 1, 0]$

$B = [0, 1, 1, 1, 0, 0, 0, 0]$

$C = [0, 0, 0, 1, 0, 1, 1, 1]$

$$\cos(A,B) = \frac{1+1}{\sqrt{4}\sqrt{3}} = \frac{2}{2\sqrt{3}} = 0.577$$

$$\cos(B,C) = \frac{1}{\sqrt{3}\sqrt{4}} = \frac{1}{2\sqrt{3}} = 0.289$$

$$\cos(A,C) = \frac{2}{\sqrt{4}\sqrt{4}} = \frac{2}{4} = 0.5$$

c) Normalized:

	a	b	c	d	e	f	g	h
A	0.67	1.67	0	1.67	-2.34	0	-0.34	-1.34
B	0	0.67	1.67	0.67	-1.34	-0.34	-1.34	0
C	-1	0	-2	0	0	1	2	0

$A = [0.67, 1.67, 0, 1.67, -2.34, 0, -0.34, -1.34]$

$B = [0, 0.67, 1.67, 0.67, -1.34, -0.34, -1.34, 0]$

$C = [-1, 0, -2, 0, 0, 1, 2, 0]$

$$\cos(A,B) = \frac{(1.67 \times 0.67) + (1.67 \times 0.67) + (-2.34 \times -1.34) + (-0.34 \times -1.34)}{\sqrt{13.41} \times \sqrt{7.39}}$$

$$= \frac{5.829}{9.95} = 0.5858$$

$$\cos(B,C) = \frac{(1.67 \times -2) + (0.67 \times 0) + (-0.34) + (-1.34 \times 2)}{\sqrt{7.39}\ \sqrt{10}} = \frac{-6.36}{8.6}$$

$$= -0.74$$

$$\cos(A,C) = \frac{(-0.67) + 0 + (-0.34 \times 2) + (0)}{\sqrt{13.41}\ \sqrt{10}} = \frac{-1.35}{11.58}$$

$$= -0.12$$