National Institute of technology Calicut

A Deemed University

NIT Campus PO, Kerala – 673601

Department Of Mechanical Engineering

ME3112D: METROLOGY AND INSTRUMENTATION

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Assignment Report on

MONTE CARLO SIMULATION

Problem no. : 4

Submitted By:

NAME: KUNAL PATHAK

ROLL NO: B211095ME

BATCH: ME04

GROUP: D

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Problem statement

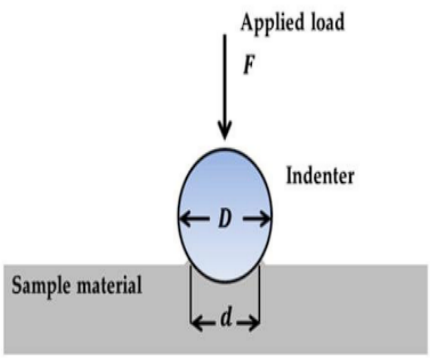
Group D

Problem 4

Figure shows a simple model for the measurement of Brinell hardness. This test is executed by applying a load on a sphere made of a hard material over the surface of the test sample The model used here for the Brinell hardness (HB) is represented in Equation:

$$HB = \frac{0.204\,F}{\pi D\,(D - \sqrt{D^2 - d^2}\,)}$$

where F is the applied load (N), D is the indenter diameter (mm) and d is the diameter of the indentation mark (mm).



Compute the mean, standard uncertainty and 95% coverage interval for Brinell hardness using MCS using $2\text{x}10^5$ trials. Plot histogram representing the resulting PDF for Brinell hardness estimated by Monte Carlo simulation

Input Source	Type	PDF	PDF Parameters
Load (F)	B	Uniform	Mean: 34500 N; Range: 500 N
Intender diameter (D)	B	Gaussian	Mean: 8 mm; Uncertainty: 0.003 mm with 95% level of confidence
Diameter of the mark (d)	A	Triangular	Mean: 2.25 mm, Standard uncertainty: 0.005 mm

Introduction

Uncertainty:

A non-negative parameter, associated with the result of a measurement that characteristics the dispersion of the values that could reasonably be attributed to the measurand.

Type A evaluation of measurement uncertainty:

Evaluation of a component of measurement uncertainty by a statistical analysis of measured quantity values obtained under defined measurement conditions.

Type B evaluation of measurement uncertainty:

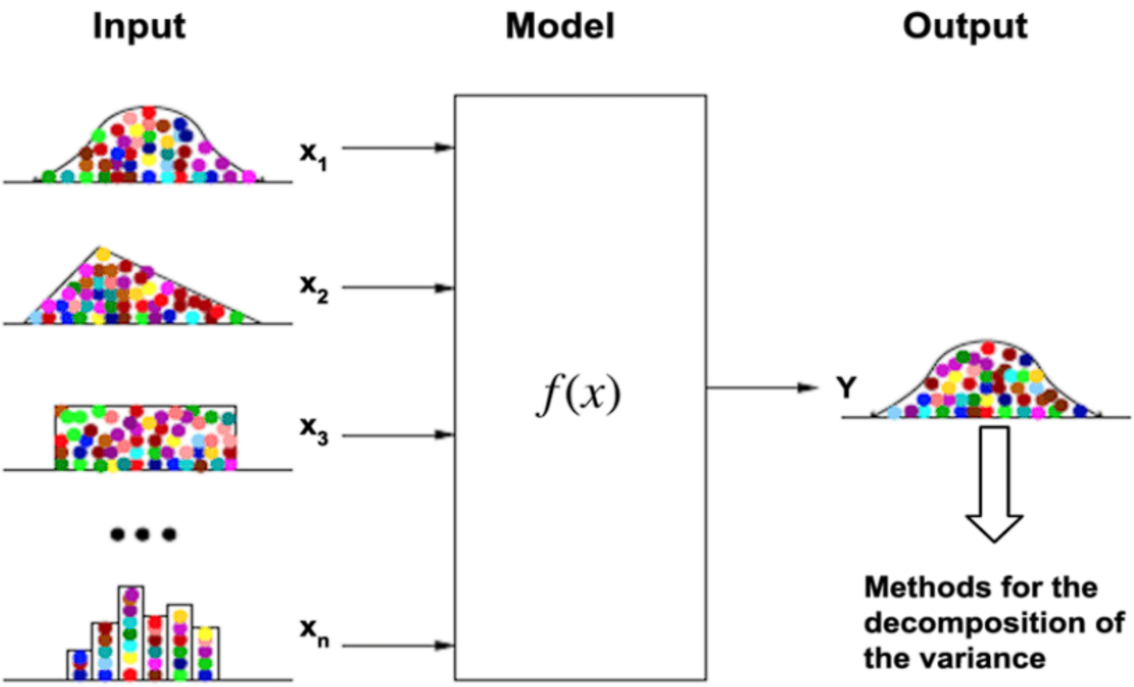
Evaluation of a component of measurement uncertainty determined by means other than a Type A evaluation of measurement uncertainty.

Monte Carlo simulation:

Monte Carlo simulation is a technique that uses random sampling to estimate the possible outcomes of an uncertain event or problem. It involves generating multiple scenarios based on probability distributions and running a simulation for each scenario. It can be applied in various fields of knowledge where there is interference from random variables.

Monte Carlo simulation is a method for risk analysis that involves generating models with various input values to forecast outcomes. Managing a Monte Carlo simulation is typically done using software or programs, which calculate the probabilities associated with each variable. The process begins by creating random datasets for the input variables and inputting them into the program. The simulation then computes the probability for each input.

Overview of Monte Carlo Simulation approach



To employ Monte Carlo simulation for resolving uncertainties, the following steps are usually followed:

1. Clearly define the problem and identify the input variables.
2. Establish the probability distribution for each input variable.
3. Generate random values for each input.
4. Execute the model using these random values.
5. Record the outcomes.
6. Repeat steps 3-5 numerous times, often thousands of iterations.
7. Analyze the results to determine the probability distribution of the output variable(s).

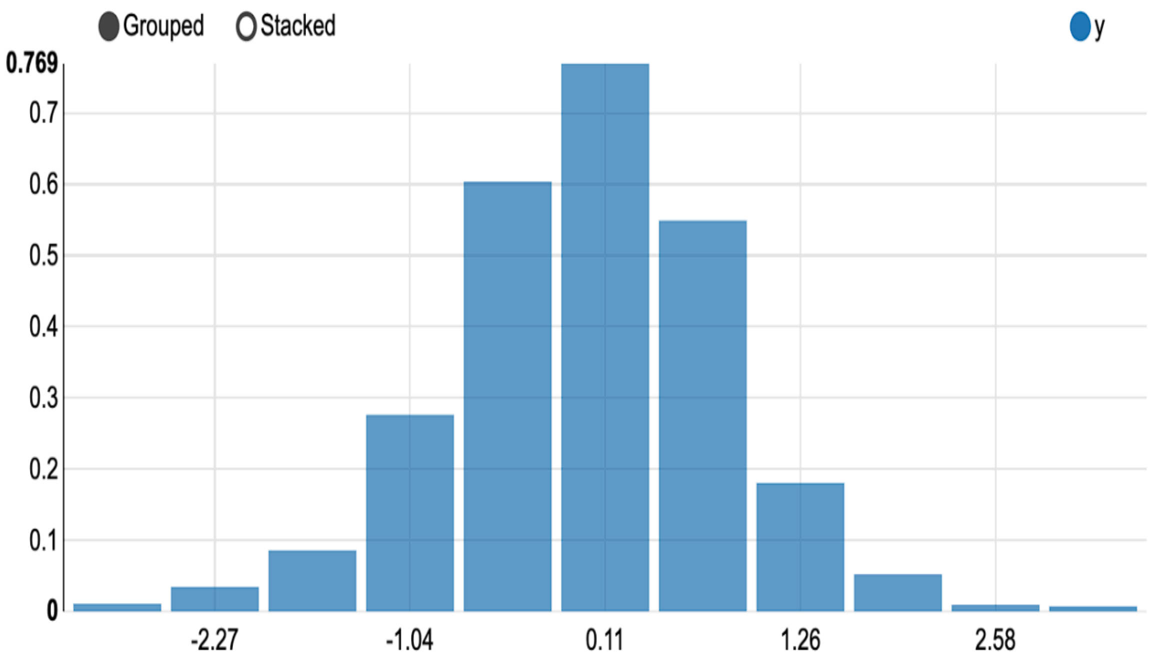
You can conduct multiple Monte Carlo simulations by adjusting the underlying parameters used for data simulation. Additionally, it's important to compute the variation within a sample by calculating the variance and standard deviation, which are common measures of spread. Variance represents the expected value of the squared difference between the variable and its expected value, while the standard deviation is the square root of the variance. Smaller variances are typically considered preferable.

Theory

Monte Carlo simulation involves repeatedly sampling random inputs of a variable and aggregating the results. The variable with probabilistic characteristics is assigned random values, and the model is then computed based on these values. The outcome is recorded, and the process is iterated numerous times, often hundreds or thousands. Once the simulation concludes, the results are averaged to estimate the final value.

This technique is employed to examine how a model reacts to randomly generated inputs and generally follows a three-step process:

1. Randomly generate "N" inputs or scenarios.
2. Conduct simulations for each of the "N" inputs using a computerized model of the system under analysis.
3. Analyze the outcomes to ascertain the probability distributions associated with the results.
4.



Example output of Monte Carlo simulation

PROBLEM CODE (USING PYTHON)

Program Code Link:

[Metrology Monte Carlo Simulation code]()

```python
import numpy as np
import matplotlib.pyplot as plt

def brinell_hardness(F, D, d):
    numerator = 0.204 * F
    denominator = np.pi * D * (D - np.sqrt(D**2 - d**2))
    return numerator / denominator

def generate_load_samples(size):
    return np.random.uniform(34000, 35000, size)

def generate_D_samples(size):
    return np.random.normal(8, 0.003, size)

def generate_d_samples(size):
    return np.random.triangular(2.2, 2.25, 2.3, size)

trials = 2 * 10**5

loads = generate_load_samples(trials)
D = generate_D_samples(trials)
d = generate_d_samples(trials)

hardness_values = brinell_hardness(loads, D, d)

mean_hardness = np.mean(hardness_values)
std_dev_hardness = np.std(hardness_values)
std_error_hardness = std_dev_hardness / np.sqrt(trials)

lower_bound = np.percentile(hardness_values, 2.5)
upper_bound = np.percentile(hardness_values, 97.5)

print("Mean Brinell Hardness:", mean_hardness)
print("Standard Deviation:", std_dev_hardness)
print("Standard Uncertainty:", std_error_hardness)
print("95% Coverage Interval:", (lower_bound, upper_bound))
```

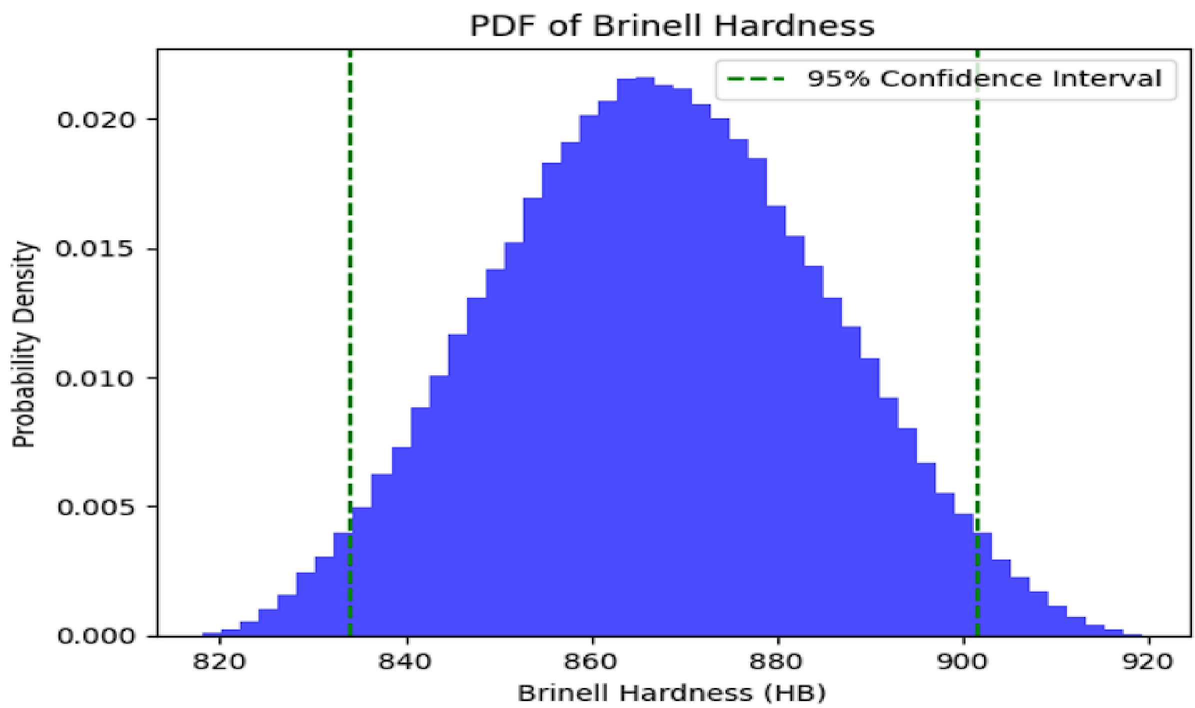
```
plt.hist(hardness_values, bins=50, density=True, alpha=0.7, color='b')
plt.title('PDF of Brinell Hardness')
plt.xlabel('Brinell Hardness (HB)')
plt.ylabel('Probability Density')

plt.axvline(x=lower_bound, color='green', linestyle='--', label='95% Confidence Interval')
plt.axvline(x=upper_bound, color='green', linestyle='--')

plt.legend()
plt.show()
```

Output:

```
Mean Brinell Hardness: 867.3871134522923
Standard Deviation: 17.574928714883765
Standard Uncertainty: 0.03929873530619312
95% Coverage Interval: (834.1395277558785, 901.7457488315194)
```



RESULTS AND DISCUSSIONS

1. Utilized Monte Carlo simulation to estimate Brinell hardness (HB) of a material.

2. Generated random samples for input parameters (load, indenter diameter, and indentation mark diameter) based on respective probability distribution functions.

3. Computed Brinell hardness for each trial using provided equation.

4. Ran simulation for 200,000 trials.

5. Calculated mean, standard deviation, and 95% coverage interval of obtained hardness values.

Mean	= 867.3871134522923HB
Standard uncertainty	=0.03929873530619312
95% coverage interval	=[834.1395277558785, 901.7457488315194]