INVESTMENT ALLOCATION WITH FACTOR MODELS AND CONE PROGRAMMING

CHRISTOPHER NIEVES (CDN25), KUNAL RAJANI (KR386), ANTON TONEV (ADT59), RONI YOSOFOV (RY96)

Abstract. The goal of our project is to utilize factor models to explain returns and optimize the Sharpe ratio to create a portfolio that outperforms the S&P 500. After refining our data we have a universe of 335 stock in which we can invest. We rebalance our portfolio quarterly and incorporate factor models and Sharpe ratio optimization through cone programming to form the portfolio. The rest of the paper is organized as follows: Section 1 is a short introduction ot our paper, Section 2 gives a brief idea of the data available and what kind of choices we made to reach the final universe of stocks, Section 3 gives an idea of the general methodoloy used in the paper, Section 4 describes thre results that we have reached, section 5 presents the significance test we performed, Section 6 presents the resulsts of different sensitivity analysis and Section 7 summarizes the project and gives suggestions for further research.

1. Introduction

In this project we will build a quarterly rebalancing portfolio that will try to outperform the S&P 500 index. Since it is not easy to come with prices and factors of 500 stocks in the past 10 years, we are not able to make a 1:1 replication of the S&P 500 index. Another reason we couldnt make such a replication is because some stocks in the index change throughout time, therefore, it can be overwhelming to track the evolution of the index closely. To overcome or at least mitigate the problems, we decided to set a base year and pick about 335 stocks (Appendix 1) that were in the S&P 500 index at that time. We kept track of those stocks throughout our analysis even if the stocks were removed from the S&P 500 index. The reason we could do this is because the general idea and performance is maintained and can be judged against any existing S&P 500 index. In the first project we used past returns and the Markowitz Mean-Variance Framework to come up with best estimates of optimal portfolios. In this project we will be using Multi-Factor Models to estimate future returns of each stock that we picked. After completing this step (and after calculating the covariance matrix), we decided to use Cone Programming to locate the maximal Sharpe Ratio. Sharpe Ratio is the excess return per unit of risk we earn on our investment:

$$S(R) = E(X)/\sqrt{(VAR(X)}$$

We carefully selected a number of firm specific factors to make our predictions realistic. Since firm specific factors are released every quarter, we used quarterly analysis and rebalancing. The following states the factors and specific explanations why we chose them:

2. Data

Our initial goal was to use all the stocks that comprise the S&P 500 index. The biggest problem with this idea is that the stocks in the index change often due to mergers, acquisitions or stocks being dropped and added to the index. Therefore, we decided to start our project with the stocks part of the S&P 500 in 2008. The next challenged we encountered with the data was that some stocks did not have realiable data or did not have data at all for some of the factors we used in our factor model. We had to drop some of the stocks again and we reached our final number of stocks of 360. Because of the regressions we use in our paper we decided to get rid of the outliers in each quarter because they would unnecessary alter the regression results. Therefore we refined the data in such a way that each observation of a factor model or return above the 95th percentile and below the 5the precentile was given a value equal to the mentioned percentiles. The code for this part can be seen in the appendix.

3. Methodology

3.1. **Factor Models.** Based on the big universe of stocks we are working with we decided that factor models would be an approach that can produce desirable results. The advantage of the factor models is that they shift the emphasis from the stocks themselves to a smaller set of "factors" driving the market. After screening several factors we decided to use nine of them for our model. Therefore, the data driving our model is a much smaller set compared to the 360 stocks we use and this of course leads to computational efficiency. Each of factors we use will be discussed below and a reason would be given why it was chosen. In order to find each of the factor premia we used the following regression:

$$r_i = \sum_{j=1}^{m} \beta_{ij} f_j + u_i$$

where $i = 1, \dots, 360$ and $j = 1, \dots 6$. The return for each asset r_i is regressed on each factor loading β_{ij} multiplied by the return on each factor f_i. As usual, the residual term u_i is the unexplained by the model return which can also be considered as the "stock-specific" return.

3.1.1. *Deciding the relevant factors.*

Value Factors: The first value factor that we decided to use is the "Price-Earnings Ratio - **P/E Ratio**". It is a factor that measures the ratio of the company's current share price to its per-share earnings. It is calculated as:

Market Value per Share / Earnings per Share (EPS)

For example, if a company trades at \$50 a share and its earnigs over the last fiscal year were \$2 per share, then the P/E ratio for the company would be simply : 25 (\$50/\$2). In general, a high P/E suggests that investors are expecting higher earnings growth in the future compared to companies with lower P/E. Stocks with negative earnings were given the maximum P/E value since they are least effective when measured using P/E. Therefore, we classified this factor as a value factor.

The next value factor that we focused on is the "Book-To-Market Ratio - **B/M Ratio**". It is a ratio that describes the value of the company by comapring its book value to its market value. It is calculated as follows:

Book Value of the Company / Market Value of the Company

It is best used as a measure that identifies under/overvalued companies. Simply put, one can charactarize a company as an undervalued if its B/M Ratio is above 1 and as overvalued if the B/M is below 1.

The third and last value factor that we used is the "Price-To-Sales Ratio - **PSR Ratio**". It measures the stock to its own past performance or some other benchmark. It is just the ratio of the price per share of the company and its revenue per share:

Share Price / Revenue Per Share

All the three value factors that we mentioned above have limitations and should not be used on their own. They either work for comparing stocks only in their own industries or do not tell us the whole story of the comapny. Therefore, we combined the three value factors together and added profit factors.

Profit Factors: The first profit factor that we use is the "Return on Equity - **ROE**". It is a factor that measures the company's profitability by computing how much value a company produced with the shareholder's money. It can be computed as follows:

Net Income / Shareholders' Equity

It is generally used to compare the profitability of one company to its industry peers.

The other profit factor that we included in our factor model is the "Return on Assets - **ROA**". It is similar to the ROE discussed above but it measures the profitability of the company not relative to its equity but to its total assets. One can think of the ROA as measuring the efficiancy with which the company uses it assets. Although ROA also includes interest expense in the numerator we didn't have reliable data to include it in our analysis. Though Net Income also provides a good indicative measure of ROA. It is calculuated as follows:

Net Income / Total Assets

The more efficiant a company is in converting its assets to profits the higher one's expectation for stock of the company is. For both ROA and ROE we decided to include the changes in their quarterly values to see if they would be better predictors of future returns.

Health Factor. The health factor that we used measures as one would expect the health of the company. The "Debt-To-Equity Ratio - **D/E Ratio**" measures the financial leverage of the company. It is found by dividing the company's Total Liabilities by its Shareholders' Equity:

Total Liabilities / Shareholders' Equity

If the D/E ratio is high this means that the company is highly leveraged and this might cause volatile future earnings.

Momentum Factor: The Fear & Greed indicator. For our previous project, we analyzed a technical indicator called Trender, which returns a positive or negative signal based on the recent price history. This signal was well fit to act as a constraint (e.g. only buy stocks with a positive signal), but the fact that it was binary limits the indicators usefulness in the context of a cross-sectional factor model.

However, Bloomberg has another closely related indicator called the Fear & Greed indicator which seemed to be a good way to incorporate momentum into our model. The Fear & Greed index can be seen as a measure of buying or selling pressure. At a value of zero, the indicator suggests that there is equal buying and selling pressure. Positive values indicate buying pressure on the stock and negative values indicate selling pressure. A screenshot of the S&P 500s price alongside the corresponding Fear and Greed values is shown below. For a more detailed description of how we calculate this indicator, please refer to Appendix A: Fear & Greed Indicator Calculations.

3.1.2. *Computing the factor returns and covariances.* The next important step in implementing the factor model is to compute the factor returns and covariances. We gathered the data for each of our factors from the third quarter of 2001 until the first quarter of 2011. We obtained the factor returns $f_{j,t}$ by running the following regression:

$$r_{i,t} = \sum_{j=1}^{m} \beta_{i,j,t} \times f_{j,t} + \epsilon_{i,t}$$

where $r_{i,t}$ is the return on the i-th stock at time t, $\beta_{i,j,t}$ is the factor loading for stock i, factor j and time t and $\epsilon_{i,t}$ is the usual residual we obtain by running a regression. After obtaining all the $f_{j,t}$ from the previous periods we decided to define the an exponentially weighted moving average $\mu_j = EWMA(f_{j,t})$. We also defined the covariance matrix of the factor returns to be

$$\sum = Cov(F)$$

where

$$F = \begin{matrix} f_{1,t} & f_{2,t} & \dots & \dots & f_{m,t} \\ \dots & & & & \dots \\ \dots & & & & \dots \\ \\ f_{1,k} & f_{2,k} & \dots & \dots & f_{m,k} \end{matrix}$$

3.1.3. *Computing the expected value and the covariance of the asset returns.* To obtain the covariance matrix of the asset returns we start with the covariance matrix of the factor returns and use the following formula

$$V = B \times F \times B^T + \Delta$$

where V is the covariance matrix for the asset returns, F is as defined above the covariance matrix of the factor returns, B is a factor loading matrix for the next period and Δ is the diagonal matrix whose (i, i) entry is the vraiance of the residula u_i and can also be described as the matrix of the stock-specific risk.

3.2. **Cone Programming.** In the project we found optimal portfolios using Second - Order Cone Programming techniques. Our objective function was to minimize

$$\frac{\sqrt{(x^T \times V \times x)}}{(SharpeRatio)}$$

where x is the vector of weights and V is the covariance matrix, which is also a positive semi-definite matrix. Sharpe ratio is a performance measure that shows the reward-to-risk (SD) ratio. The higher the ratio, the more rewording the investment is.

The reason we dont maximize the Sharpe ratio but minimize the value above is just because of working with more comfortable numbers and calculations. One of the pluses of second-order cone programming is that it allows us to take care of convex functions and in MATLAB we can use the cvx plugin to implement this optimization technique.

A SOCP optimization problem is of the form:

$$min \qquad \frac{x \times V \times x}{(\mu^T \times x - \mu_0)}$$

OR

$$min \qquad t$$

Such That:

$$||U \times x||^2 \leq t \times (\mu^T \times x - \mu_{0)}$$

$$t \times (\mu^T \times x - \mu_0) \geq 0$$

$$\sum_{i=1}^{335} x_i = 1$$

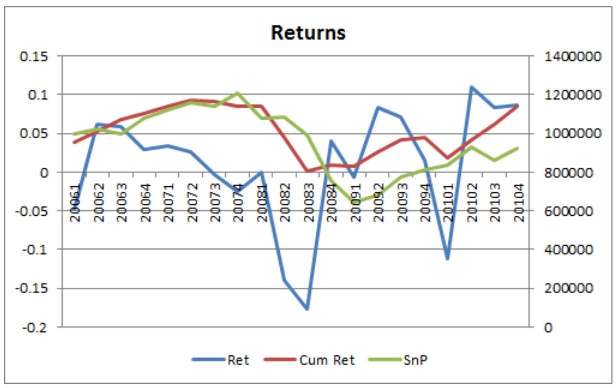
$$x_i \leq k$$

$$x_i \geq l$$

where k is the upper bound of the amount of the portfolio that can be invested in one stock and the last inequality puts a restriction on the amount of shortselling. In our analysis we take as default values of k and l equal to 0.1 and -0.1 respectively. We decided to take the risk free rate to be 0 since we had taken S&P to be the benchmark portfolio and hence were comparing our results to the absolute returns by S&P. Furthermore, we optimize the Sharpe ratio to compute weights and since risk free rate is a constant, it would not alter our results in any significant way.

4. Results

After running our dynamic factor models and the optimization program for 2006Q1 to 2010Q4 we obtain the following results.



We see that at the end of 2010Q4 we are able to achieve a cumulative return of 13.8% as against almost negligible for S&P. We achieve this with a Sharpe Ratio of 0.12 as against S&Ps 0.05. This seems to be quite competitive performance and an indication that factor models are able to predict the relationships between stocks and break it into just a few dimensions.

When we look at our performance as we emerged from the crisis we see that we were able to obtain a 36.6% cumulative return from 2009Q1 with a Sharpe ratio of 0.56 which is quite impressive.

Of course we need to study whether these outcomes are just a stroke of luck or they have a strong foundation underneath. We look at the average of the coefficients for each factor from the 20 estimates that we obtained for each quarter.

	FnG	B/M	P/Sales	P/E	ROA	ROE	ROE_change	ROA_chng	D/E
Mean	0.0003	0.0412	0.0359	0.0011	-0.0764	0.2520	0.0059	-0.0003	-0.0023
Stdev	0.0065	0.0807	0.4201	0.0317	1.0276	0.6129	0.0377	0.0415	0.0110

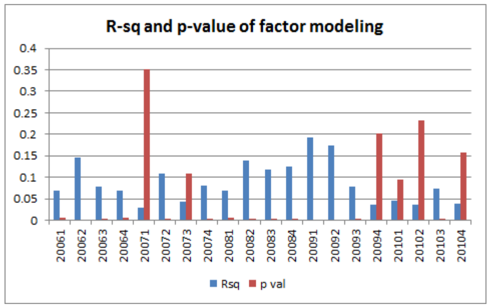
We see that as expected, momentum is positively correlated with the returns indicating that previous price increases signal a forthcoming price increase.

Among Value Factors, B/M has a positive premia which shows that a high B/M stock that is undervalued will tend to increase in value in the future. On the contrary we see that P/E and P/Sales also have a positive premia that indicates that a high priced stock as compared to its fundamentals will continue to increase in value. This seems counterintuitive and is prevalent in the high errors of these quantities.

As for Profit Factors, a negative premia associated with ROA and its change is again counterintuitive and this is upheld by the high error in its estimation. Apart from that it is also surprising that the premia for ROE and its change is positive given that both these factors are highly correlated. This behavior can be attributed to randomness since the std errors of ROA estimates are quite large.

D/E, as expected, has a negative premia with the prices of stocks having a high leverage dropping in the future as compared to firms that seem to have a balanced capital structure. When analyzing models with each of the factors seperately we reached the conclusion that the D/E factor actually performs the best. More details and results can be seen in the Sensitivity Analysis part.

Although the premia associated with each factor seems explicable lets also look at the significance of these estimates.



The above figure shows that the R-square is quite stable and seems to average between 5%-10%. Although this seems quite a low value, in the dynamic world of finance this explanation of variance can be quite helpful in forming portfolios as we saw from our performance above. Moreover, the estimates thus obtained are always within 90% confidence limits except 4 periods when they were quite insignificant.

Note that even though the R-sq is relatively low and pvalue is relatively high during 2010, we are able to obtain a 30% return. Although we had a bull run in 2010, our methodology was able to identify outperforming stocks as can be seen by the 10% that the S&P performed during the period.

In the next few sections we shall look at the significance of these results in more detail by studying the correlation between the factors and applying bootstrapping to study any bias in the estimation of factor premia. We will then look into sensitivity analysis by considering different investment horizons and looking at the impact of each factor individually to our strategy. We will then try to identify any industry or size specific bias by running our analysis for different categories broken by industry and size.

Our aim is to investigate all such dimensions and then make an informed conclusion to the application of factor models and optimization using Sharpe ratio for portfolio formation.

5. Significance Tests

5.1. Correlations.

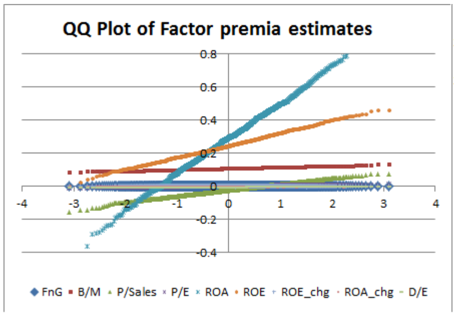
	FnG	B/M	P/Sales	P/E	ROA	ROE	ROE_chng	ROA_chng	D/E
FnG	1	0.0225	-0.0227	-0.0347	0.0525	0.0522	0.0413	0.043	-0.0011
B/M	0.0225	1	-0.187	-0.1	-0.4698	-0.4171	-0.0702	-0.0697	0.0867
P/Sales	-0.0227	-0.187	1	0.5268	0.1301	-0.0304	0.0177	0.0163	-0.2773
P/E	-0.0347	-0.1	0.5268	1	0.0417	-0.0188	0.0064	-0.0001	-0.1505
ROA	0.0525	-0.4698	0.1301	0.0417	1	0.7561	0.3267	0.3198	-0.3081
ROE	0.0522	-0.4171	-0.0304	-0.0188	0.7561	1	0.3258	0.3224	0.0811
ROE_chng	0.0413	-0.0702	0.0177	0.0064	0.3267	0.3258	1	0.9664	-0.0631
ROA_chng	0.043	-0.0697	0.0163	-0.0001	0.3198	0.3224	0.9664	1	-0.0563
D/E	-0.0011	0.0867	-0.2773	-0.1505	-0.3081	0.0811	-0.0631	-0.0563	1

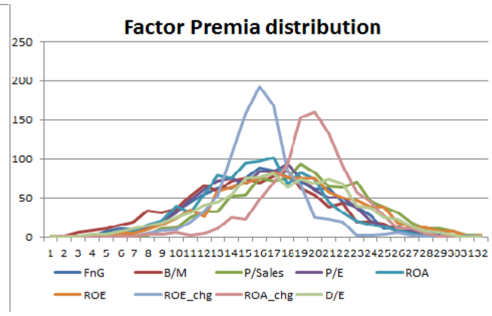
We see that ROA and ROE as well as the changes in ROA and ROE are highly correlated. This is quite intuitive since Equity forms a part of Assets and hence these ratios should move in the same direction until the debt structure of the firm changes. Hence it is ideal to include just one of the factors from the two groups.

Although to not such an extent we see that B/M is also somewhat negatively correlated with ROA and ROE which is not very intuitive.

We see that the P/Sales and the P/E are also quite correlated and it makes sense since Earnings are a part of the sales. Hence it would probably make sense to just include one of them as a factor. When we perform analysis on the models which use each of the factors seperately we see that most of the models perform simiarly in terms of wealth generation. However, the D/E performs the best. All the factors are significant at different times. To access the power of each of the factors we also report the Sharpe Ratio for the models that use each of the factors separately.

5.2. **Bootstrapping.**





The above bootstrapping was done taking the entire 10 year period (2000Q3 to 2010Q4) into account. The procedure is as follows:

1. Since we have 14405 records for the ~10 years for 335 stocks we sampled the entire data to obtain 14405 records with replacement.

2. Next we ran regression for a factor model including all the factors for the above sample to obtain the regression coefficients (factor premia).

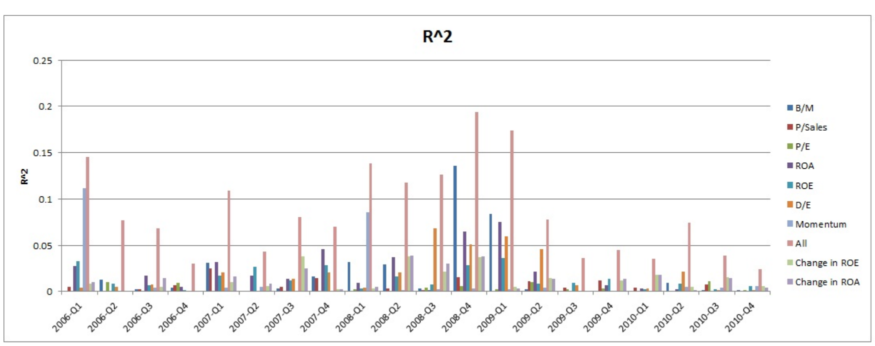
3. The above process was repeated for 1000 samples and the distribution of the factor premia for each factor is plotted above about its mean.

We see that the distributions appear normal and their qq plots also indicate a strong resemblance to normal distribution. This leads us to believe that there is no bias in the population and in fact it also goes on to reject any possibility of a potential bias due to different time periods or different companies. Hence we can say that the estimates are a true representation of the population factor premia subject to the p-values that determine the confidence which would be different for each regression.

From the above distribution we can see that the change in ROE and ROA have a very centered distribution indicating that their factor premia is estimated with a higher accuracy than others and hence its p-value tends to be generally lower than for the other factors.

6. Sensitivity Analysis

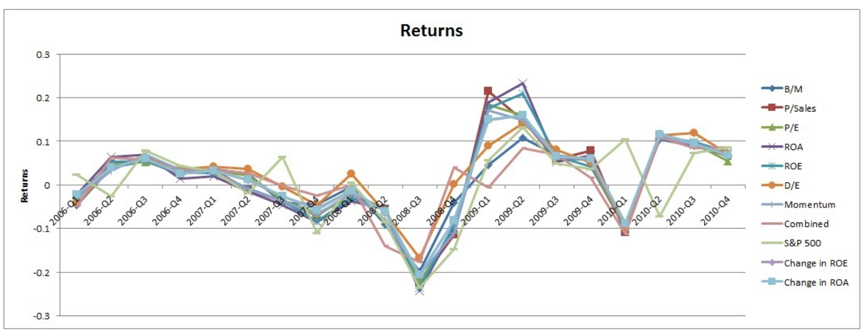
6.1. **Factor Model Sensitiviy.** In this section we would explore what is the performance of our model depending on the factors used. We run our regression eight times: seven for each of the factors separately and one more for all the factors combined. Below are plots of the R^2, p-value, Return and Wealth performance of each of the portfolios. We also report the Sharpe ratio for each of the portfolios. Below is a graph of the R^2 of each of the models:



The R^2 analysis is helpful in explaining which of the factors contribute the most to our final combined model. From what we see the P/Sales and P/E ratios contribute the least to the value of the R^2 of the combined model. One can argue that the reason for this is that both ratios are very firm specific and might be better used to compare companies that have similar structures and products or are at least in the same industry. However, this is common for almost all such ratios and cannot be used as the single reason explaining this result. Another reason for the insignificance of the P/E ratio is that its denominator (EPS) is based on an accounting measure of earnings that is susceptible to forms of manipulation, making the quality of the P/E only as good as the quality of the underlying earnings number. Therefore, comparing just the P/E of companies to make an investment decision would be a bad idea. The reason for the insignificance of the P/Sales ratio might be that it doesn't take any expenses or debt into account, the ratio is somewhat limited in the story it tells.

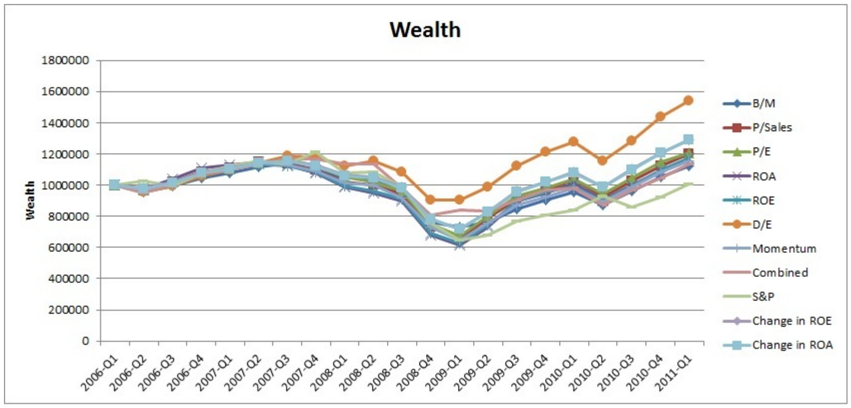
We see that the momentum factor (the so called fear and greed) contributes the most to our combined model in the beginning of 2008. However, for the other periods it does not improve our model a lot. As explained above the momentum factor predicts the direction of stock price evolution based on the pressure of buying or selling in the market. The so-called profit ratios: ROE and ROA (and the change in each of them) and the D/E ratio explain a significant portion of the returns in the end of 2008. During the other periods they all vary in an unsystematic way.

We see that the R^2 of the combined model reaches values up to 0.19. Although, this is not a value one would want for his model it still shows that when combined the factors we chose help explain up to 19% of the returns in some periods. Unfortunately, in the last quarter of 2010 our model explained only around 2.5% of the variability in the returns. As one can see the R^2 of the combined model is not as much as the sum of the R^2 of each of the models with one factor only. The reason for this is the correlation between the different factors. This might not be intuitive to someone not familiar with regressions. Now, let us analyzie the returns of each of the models.



The returns the different models produce vary significantly through time but in any given time period they are very close with few exceptions mainly in the middle of 2009. From the end of 2006 to the most intense part of the financial crisis at the end of 2008 the model that uses D/E as a factor gives higher returns than the rest of the models. Unfortunately, there are many time periods that those returns are negative. However, this is to be expected because as mentioned above all those ratios cannot be used to make a sound investment decisions because they are very company or industry specific and they rarely tell all the story of the company.

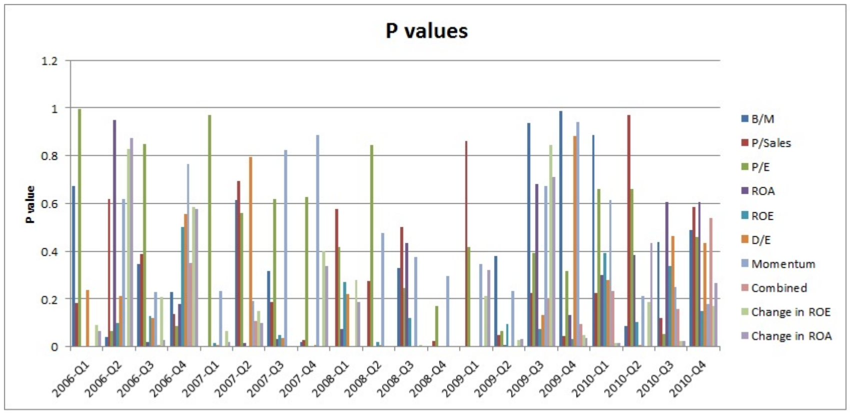
All the models significantly outperformed the index. The first two quarters 2009 all our models outperform the S&P 500. The models have positive returns while the index is still under pressure from the financial crisis and is in the negative returns area. The other period when our models outperformed the index is the third quarter of 2010. However, once the FED introduces its bond buying program widely known as Quantitative Easing (QE) the index returns quickly jumped in positive returns area. Following is a graph of the wealth evolution of portfolios formed by each of the models:



Looking at the evolution of the wealth of a person who invests \$1mln in each of the portfolios and in the S&P 500 at the beginning of 2006 we see that his portfolio that uses only the D/E as a factor would have given him a final wealth of \$ 1 539 331. This portfolio performs the best. A possible explanation is that the D/E is the most consistent factor across all firms irrespective of their industry or size because it looks at their capital structure. All portfolios using factor models outperform the S&P 500 no matter what factors they use. Unfortunately, we see that the portfolio that uses the model with all factors combined performs second worst after the index. The rest of the portfolios perform somehow similarly and produce a final wealth value between \$1.15mln and \$1.20mln. An interesting period is the beginning of 2009 when the model with all factors together outperformed all but one of the portfolios. This of course might be due to chance. We know that the final wealth is not the best estimator of the performance of a trading strategy because it does not account for the risk exposure. Because of this we take a look at the Sharpe Ratios of each of the portfolios and of the standard deviations of their returns. Following is a table of comparing the average returns, the standard deviation of returns and the Sharpe Ratio for each of the models we have and the S&P 500 index:

Models	Average Returns	SD of Returns	Sharpe Ratio
B/M	0.00869418	0.077380468	0.112356255
P/Sales	0.013906119	0.100422242	0.138476487
P/E	0.013953724	0.09526609	0.146471051
ROA	0.013480298	0.10823357	0.124548215
ROE	0.013271812	0.102715667	0.129209228
D/E	0.024674936	0.077009931	0.320412393
Change in ROA	0.016518198	0.088757475	0.18610487
Change in RoE	0.016717728	0.08871724	0.18843833
Momentum	0.011479149	0.093447228	0.122840984
All	0.007078529	0.077001488	0.091927167
S&P 500	0.004642414	0.093099434	0.049865119

We see that the highest average return expectedly is the one for the model using only D/E as a factor. This model does not have the smallest standard deviation of returns but it is very close to the smallest one the one for the model using all the factors combined. The model with D/E as a factor also has the highest Sharpe Ratio proving that even when adjusting for risk its portfolio performs the best. A good result we reach is that all the portfolios have considerably higher Sharpe Ratios than the S&P 500. Therefore after adjusting for risk the portfolios again perform better than the index. S&P 500 also seems to have one of the highest standard deviations of returns. This of course together with its having the smallest average returns explains its smallest Sharpe Ratio.



From the graph above we see that most of the times the p-values of our models are high which makes them insignificant. However, there are several cases when they are zero or very close to zero and they are significant. The models using all the factors combined or the D/E only as a factor or ROE has more small values than the rest. This shows that those models are more significant than the rest. As already discussed above the model using D/E only as a factor performs the best compared on several measures so it was expected that it will be more significant than the rest. The high p values might be explained by the fact that we are considering a mix of all sectors and companies and this reduces the significance of each of the factors and models. To make a more clear distinction we explore the models for each of the industries in the following parts of our paper. The beta coefficient that our models find can very a lot and this explains the high p values that we observe. Therefore, the results might not be significant with enough level of confidence but they still explain enough of the variance. This means that the true factor loadings lie somewhere in the range of that confidence interval.

6.2. **Boundary Sensitivity.** In this section we are trying to see the sensitivity of the 'CVX' (Cone Programming) by changing the upper and lower bounds of the weights of the portfolio. Cone programming, unlike Mean-Variance Markowitz framework, doesn't seem to be sensitive to bound changes. The upper and lower bounds were changed from 40% and down to -20%, but the returns in each quarter didn't change by significance numbers (unless we allowed long only). We can clearly conclude that the Cone Programming is a more sophisticated technique that doesnt centralize the investment in very few stocks, as it would have happened in our first project had we allowed more relaxed weight boundaries.

For the scenarios that we allowed shorting (please refer to scenarios #1-#5 in the appendix), the returns didn't change by much; therefore, we should expect the risk to be constant as well. By a quick calculation of two risk measurements (Standard Deviation and the Sharpe Ratio), we can see that they are pretty constant as we predicted.

On the other hand, in the scenario that we didnt allow shorting and increased the upper bound of the weights to 20%, we did drastically better, especially in 2009. We can clearly see in this scenario (scenario #6 in the appendix) that we took more risk but we got rewarded for this risk and our Sharpe Ratio doubled from the previous scenarios. The way to reason this outcome is to look at the year 2008 and the constraints of our Cone Program. In cone programming we look at how the companies did in the past, so since 2008 was a bad year, the optimization programming may be biased toward this year and may want to decide to short some stocks to reduce the risk of having a bad year again. However, here we dont allow shorting (our personal view may be that 2009 is going to be a strong year) and our constraints force the investment in some of the 335 stocks. Therefore, we are forced to go long with all of our wealth and the great year works in our favor (79% yearly return).

To sum up, we conclude that it more depends on whether we short or dont, rather than the number of the bounds (once we decide if we allow shorting or not).

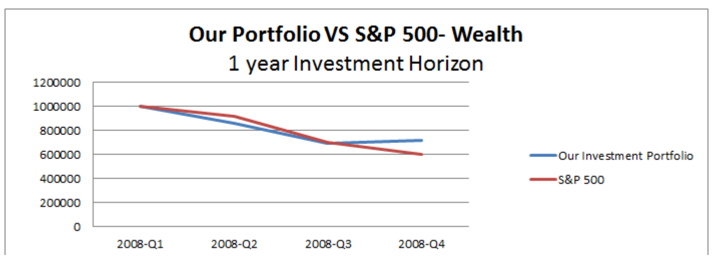
6.3. **Investment Horizon.** (Note: All numbers are in the Appendix)

In this analysis our goal is to test three different investment horizons and see whether the length of the investment can influence the performance of our portfolio overall. We chose to concentrate on three investment horizons that are used by hedge funds, 1 year- 3 years-5 years. In addition, to assure that our results are consistent, we did the investment horizon analysis on two different periods, 2006-2010 and 2004-2008. During these periods, we paid attention at the final wealth and two risk measures (Standard Deviation and the Sharpe Ratio). So, we are certain that these calculations will give us a pretty good sense on how good our investment strategy is and whether it works better in the short or long run. Moreover, we should analyze closely the volatility during the periods of the regression (for calculating the mu, the last three periods are most important) and the volatility during our optimization analysis.

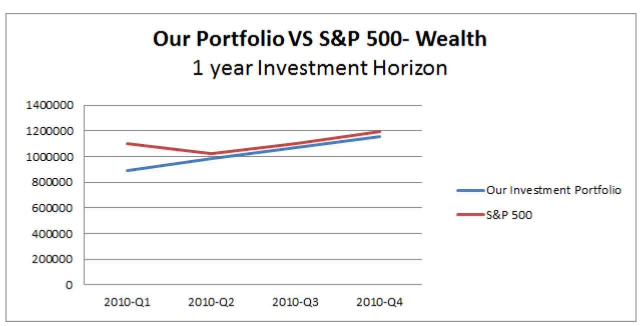
Note: our initial wealth is $1,000,000. The upper bound of the weights is 10% and the lower bound is -10%. This holds for all of the analyses in this part.

6.3.1. *1-Year Investment Horizon :*

2004-2008. The 1 year horizon held from 2008 Q1 to 2008 Q4. We underperformed the S&P 500 until 2008 Q4. One main reason we can think of is because of the volatility (we will touch upon the VIX in the next analysis section) or perhaps because the CVX had a better picture after underperforming in three periods. If we look at the risk measures for this investment, we had a better Sharpe Ratio (-0.620991063 VS -1.141104376) overall but our investment was more volatile. In general, we could have achieved better results by investing in risk free assets.

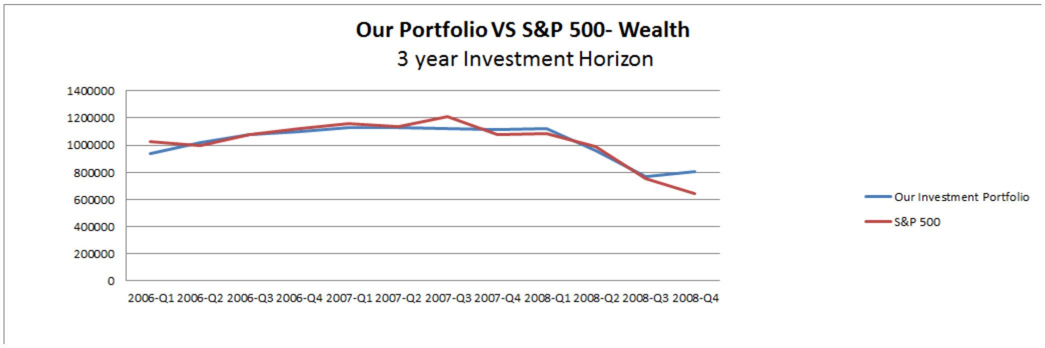


2006-2010: The 1 year horizon held from 2010 Q1 to 2010 Q4. We underperformed the S&P 500 throughout the whole investment but ended up to be a few thousand dollars below the S&P 500 by the end of 2010 Q4. If we look the risk measures, we accepted higher investment volatility and achieved a lower Sharpe Ratio. Clearly this investment wasnt successful and would like to look at the volatility to see if we can reason such poor results. Perhaps also, our investment strategy is more successful for longer investment horizons.

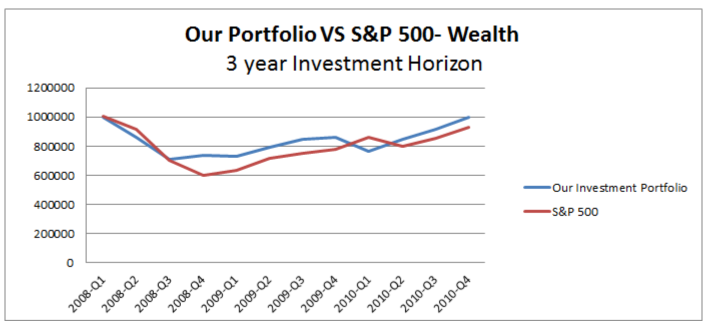


6.3.2. *3-Year Investment Horizon.*

2004-2008: The 3-year investment horizon held from 2006 Q1 to 2008 Q4. In 2006 Q1, at first we did poorly but then we were able to reach the S&P 500 wealth and outperform it in the last quarter. Overall, we had a less volatile investment than the S&P 500 and we achieved a better Sharpe Ratio. Here again, we could have achieved better results with investing in risk free assets.

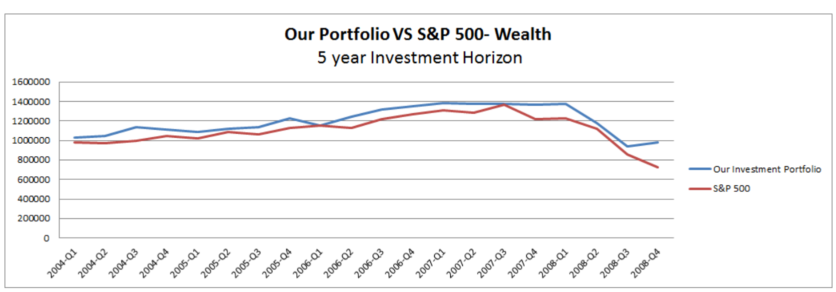


2006-20010: The 3-year investment horizon held from 2008 Q1 to 2010 Q4. This was a very volatile period and we seem to be able to recover all of the losses from 2008 in the later 2 years. We ended up to breakeven and the S&P lost around $70,000 during this investment horizon. In this analysis we also achieved a better Sharpe Ratio with less volatile returns.

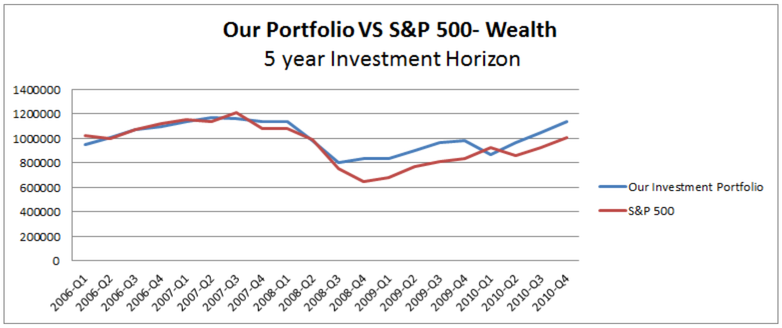


6.3.3. *5-Year Investment Horizon.*

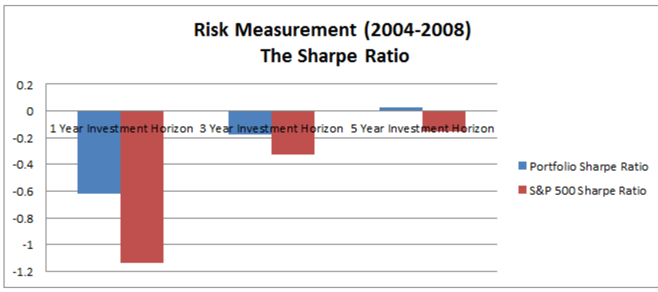
2004-2008: The 5 year horizon held from 2004 Q1 to 2008 Q4. We outperformed the S&P 500 and we almost reached our initial investment after 2008 Q4, while the S&P 500 was down about $300,000. We were above our benchmark throughout the investment (with better Sharpe Ratio and lower return volatility). 2008 Q4 was the quarter that put us apart from the benchmark as we were able to achieve the return of 4.6% while the S&P 500 had -14.7%.

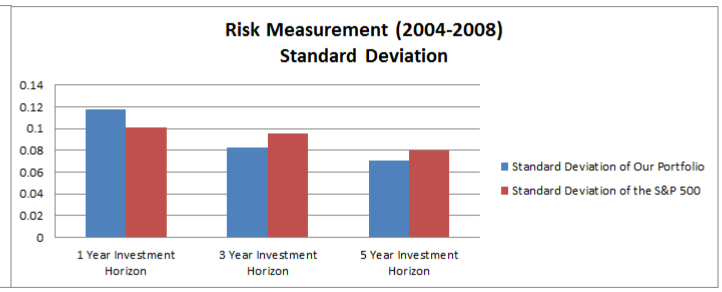


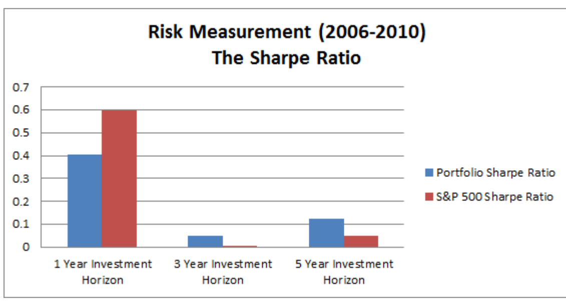
2006-20010: The 5 year horizon held from 2006 Q1 to 2010 Q4. We are pretty close to the S&P 500 investment value, until 2008 Q3, where the volatility is high and we seem to do better. As in the previous results, we have less volatile results and a better Sharpe Ratio (even though its very low). So, overall we had a very successful investment here and we ended up with about $114,000 profit while the S&P 500 had a profit of only $5,000.

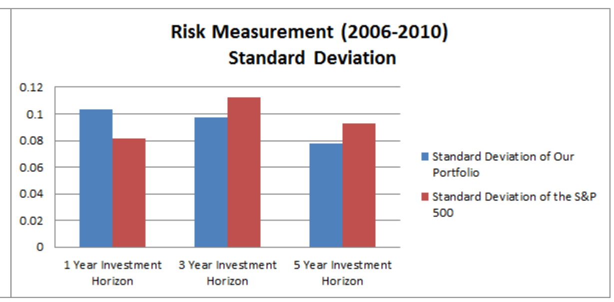


Risk Measures For The Analysis Above:









In conclusion, we had better success in the 3-year and the 5-year investment horizons. From our analysis they were successful in both of the periods, 2004-2008 and 2006-2010. So, we have a strong belief that our investment strategy is powerful. To have a better idea about our conclusion, we should look at the volatility during these periods and see if high or low volatility had an influential impact on our results.

6.4. **Volatility Index of the S&P 500 (VIX).** The VIX measures the implied volatility of the S&P 500 index options and is a good measure of the volatility in the market. Usually, during uncertain times the volatility is higher. And, if the volatility is higher, the risk increases. The reason we would like to analyze the volatility in the market during 2004-2010 is because we would like to understand how the volatility in the market impacts our results.

In the previous section we had a number of very successful quarters with respect to the S&P 500. For instance,

2006-Q2 (2006-2010 investment) we gained 6.1% and the benchmark lost 2.5%.
2007-Q4 (2006-2010 investment) we lost 2.4% and the benchmark lost 11%
2008-Q4 (2006-2010 investment) we gained 4$ and the benchmark lost 14.7%
2004-Q1 (2004-2008 investment) we gained 2.8% while the benchmark lost 2.1%

There were also periods that we did worse than the S&P 500. For instance,

2006-Q1 (2004-2008 investment) we lost 6% while the benchmark gained 2.4%
2008-Q2 (2004-2008 investment) we lost 14.2% while the benchmark lost 8.5%

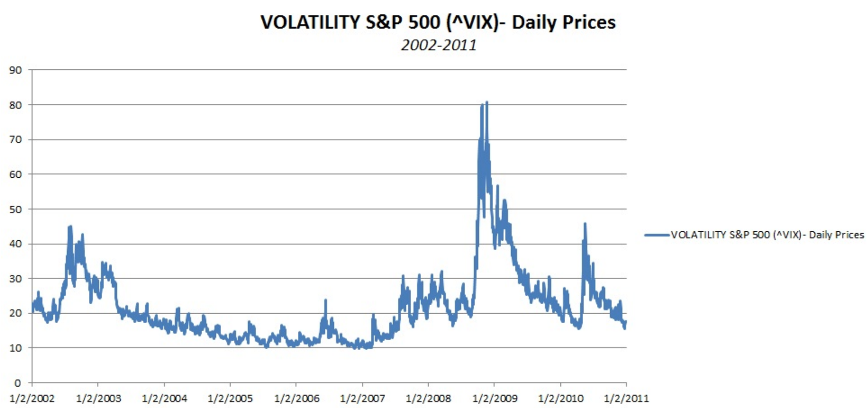
If we look at the 2006-2010 investment we did well in 2006Q2, we can see that the volatility is low during this time and a few periods before 2006Q2 (those few periods are important to determine the properties of the mu matrix for the CVX optimization). In the previous section we thought that we should do better during high volatility period, but seems that we can do well during low volatility period (did it happen by chance? Lets see).

In 2007Q4 we bad times and concerns about the economy are increasing, as a result we have high VIX throughout 2007, so the mu matrix incorporates the high volatility and so is the optimization. As a result, the high volatility during the regression period and the optimization period helped us achieve good results.

Now, lets look at 2004Q1, beginning of 2003 and 2002 are very volatile periods. 2003 Q4 has low volatility but the mu still had some impact from the beginning of 2003; as a result, we seemed to be doing well during such periods.

Lastly, we underperformed during 2006Q1. This period is pretty stable with no surprises. It seems as if the regression and the optimization had a hard time performing during such a period.

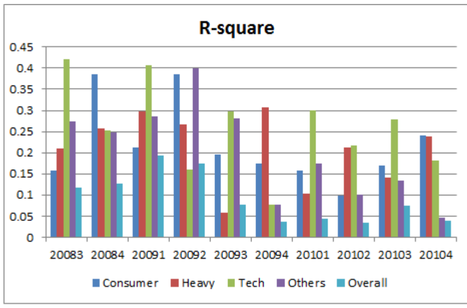
To sum up, we cant really state strict conclusion whether we perform better during high or low volatility. We seem to be performing good and bad during both periods, however, we can see that we performed better more often during high volatility than during low volatility.

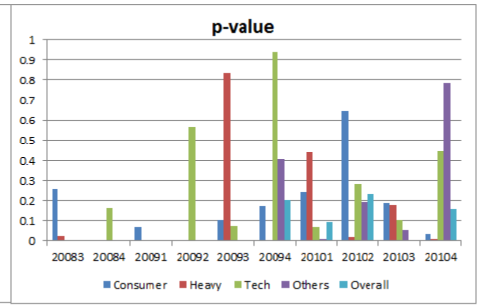


6.5. **Industry wide analysis.** For the purpose of this analysis we categorized our entire stock database into 4 categories based on the industry codes. The idea is to see if our factors work for an industry better than other. Our classification is as follows:

Category	SIC Codes	No. of cos.	Names of some companies
Heavy Industries	1010, 1510, 2010	89	3M, Boeing, Chevron, Dupont, Exxon, GE, Lockheed Martin, Schlumberger
Consumer Products	2520/30/40/50, 3010/20/30	73	Amazon, Coca Cola, Comcast, Fossil, Gap, Home Depot, Lowe's, Macy's
Software and Tech	4510/20/30, 5010	51	AMD, Analog Devices, Apple, AT&T, Broadcom, Cisco, Citrix, Dell, Intel, MS, Qualcomm, TI
Others		122	Abbott, Aetna, Amex, Citi, Ford, Pepco, Pfizer, Southwest, Ventas

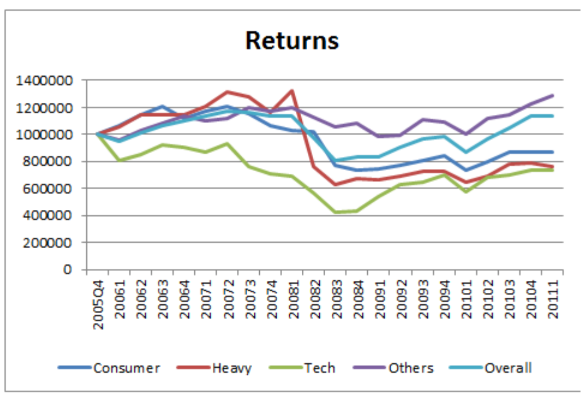
We ran the factor models and the optimization program on these industries separately and here are the results. Note that for the sake of clarity we have shown the statistics from 2008Q3 onwards and the complete series can be found in the appendix.





We can clearly see that the factors are able to explain the variance in the tech sector much more than any other sector on a consistent basis. However, the p-value of the tech sector is also quite high which means that the actual coefficients are not estimated with a high level of confidence. For eg. In 2009Q2 even though the factors are able to explain 15% variance, they are able to estimate the coefficients (i.e. the factor premia) with only 40% confidence. This means that the return vector that the factor loadings would create might not be a true estimate of the actual returns and hence it might not lead to an accurate optimization and the final wealth might turn out to be lower. On the other hand we see that the Other industry (and in fact overall as well) is able to obtain an R-square of sometimes ~25% and other times >10% and all the while with a very low p-value (except for 2009Q4 and 2010Q2 and Q4). This seems to suggest that its factor premia estimates are accurate to explain as much variance and hence should also be good predictors.

This can be tested by looking at the returns over the entire period:



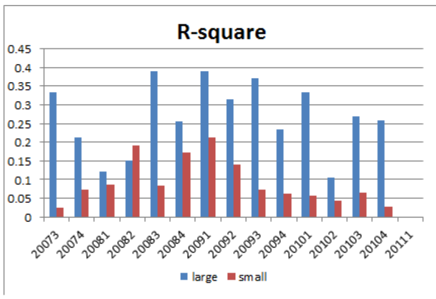
We see that even though Tech had the highest R-square, the low confidence with which its estimates are computed causes it to have a lower absolute return. From the above graph we can see that of the $1MM invested in Tech in 2006, we are left with $736K at the end of 2010 as against $1.28MM in Others.

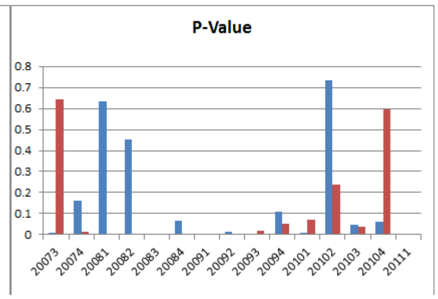
Comparing the Sharpe Ratio we see that both Tech and Consumer have a Sharpe Ratio of -0.04 while Others is able to obtain a Sharpe Ratio of 0.2 as against 0.12 Overall.

This seems to suggest that p-values play a stronger role in determining the success of factor models in portfolio optimization.

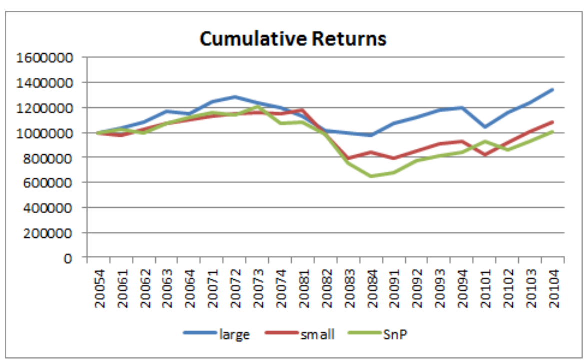
6.6. **Size of Companies.** Now we tried to understand if our methodology is able to produce any different results based on the size of the company. Traditionally, factor analysis works better on large stocks than small ones. This is because large stocks are popular and hence their prices have an emotional and behavioral bias in them while small stocks are more fundamentally driven. Since our universe of stocks is limited we ran all the factors on these two groups to measure the effectiveness. We define small cap to be the companies with their market value < the average of all companies ($24Bn) and large cap as the rest. From 335 stocks we obtained about 60 large cap (like Amex, AT&T, Boeing, Cisco, Exxon, Intel, IBM, Pepsico, PnG, Target, etc.) and about 270 small cap (like Airgas, Altera, Apache, Avon, BMC, ConAgra, Dollar Tree, Gannett, Goodyear, etc.). We created this list using 2000Q3 as the base. However there can be transfers between these lists as time progresses. We checked to see that at the end of 2010 there were about 75 large caps of which 45 were from our former list. Hence we acknowledge that there is diffusion between the two lists and hence this part can be used to get a directional read rather than actual figures.

We obtain the following results when running the factor models on these two datasets separately. We are showing the results from 2007Q3 for the sake of clarity and the entire graphs can be found in the appendix.





We can clearly see that the factor models are able to consistently predict the variation in the returns of large stocks much more than small stocks. For large stocks the factor models are able to predict about 25% of the variation while for small stocks its about 10%. On the other hand, the estimates are not significant enough a few times for both large and small stocks. Based on p-values it seems that both the stock categories stand together but large stocks outperforms tremendously when R-square is concerned. Hence, as expected, this helps take the cumulative returns from large stocks to a much higher level than the small stocks which can be seen in the below graph.

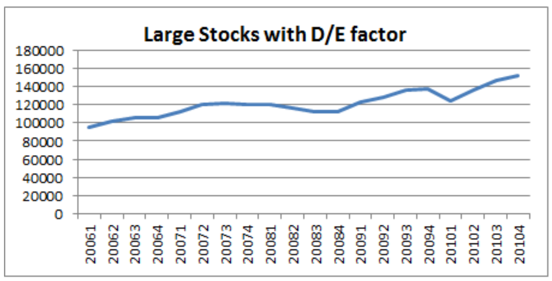


We see that there was a big drop in the small stocks from 2008Q1 to 2008Q3 which is essentially when large stocks took the lead. When compared with SnP 500 we see that it also took a plunge which seems to indicate that the stability of the large stocks portfolio can be attributed to our factor analysis and optimization program. Lets also look at the Sharpe Ratio for all these categories. We obtain a Sharpe ratio of 0.26 for the Large stocks, 0.09 for the small stocks and 0.05 for the SNP 500. We see that with our methodology we are able to obtain a much higher Sharpe ratio than SNP 500 and also more than all 335 stocks combined.

The above analysis clearly suggests that our methodology is able to provide superior returns on large stocks especially during bad times.

Conclusion

We realized that in the dynamic world of finance it is not easy to explain the variability in the returns that can be used to predict future returns although factor models do help to some extent in optimizing the portfolio. We studied that though our results made financial sense and were not biased, they were not always significant based on traditional confidence limits of 95%. When we looked at different time horizons and volatility conditions we saw that our investment horizon did better during longer time horizons in the periods 2004-2008 and 2006-2010. Additionally, we noticed that our investment strategy outperformed the benchmark during highly volatile periods in many instances. Though, we couldnt make a strict conclusion that this always holds. From the individual analysis of all factors we saw that for our world of stocks D/E seems to add the most value. Since industry didnt seem to be a differentiating classification, we ran our methodology only on large caps with D/E as a factor and we were able to achieve a return of 52.4% from 2006 to 2010 with a Sharpe Ratio of 0.442.



Although this seems to be a very impressive performance there is a whole lot of further analysis that we should look at that can help strengthen this result even further.

Further Analysis

During our project we came across a host of dimensions along which if we progress we would be able to strengthen our results and our understanding. If we have the data it would be interesting to assess a larger stock universe so we have a good range of the values pertaining to all factors and the behavioral biases of investors in less popular companies can also be understood. We would like to look at a longer time horizon since such factors tend to have a bigger impact on the prices in the long run. It will also be interesting to study any non linear relationships between the stock prices and factors. For eg., log(size) is a popular factor than size since an increase in the log tends to show a significant impact on the value of the company. We can also further our analysis by studying what stocks our optimization program assigns weights to based on the different factor premia and if we are able to foresee any changes in the company based on the factor that the program loads on. It may also improve our models if we are able to include macroeconomic factors but that is a challenge in cross sectional factor models that we are working on.

Appendix

Appendix A: Fear & Greed Indicator Calculations

To calculate the Fear & Greed indicator, we perform computations over 3 different time intervals. At least 70 days of price history is needed in order to calculate the Fear & Greed. While Bloomberg has released some general information on how this indicator is calculated, many of the specifics were left to our own discretion.

Period 1 - Lookback Period (10 days)

True Range: This period is the interval we are calculating the True Range over. We find the difference between the highest ask price and the lowest bid price that occurred over the past ten trading days.

TR_{inc}: Equal to the current True Range value if the securitys closing price has increased in the past 10 days and equals 0 otherwise.

TR_{dec}: Equal to the current True Range value if the securitys closing price has decreased in the past 10 days and equals 0 otherwise.

Period 2 - Short-Term Averaging (20 days)

$EMAvg_{20}(TR_{inc})$: An exponential moving average of the past 20 values of TR_inc, including any zeros. An identical computation is carried out on the TR_dec.

$Diff_{20}$: Subtract the current value of EMAvg20(TR_inc) by EMAvg20(TR_dec). A positive value indicates buying pressure while a negative value indicates selling pressure.

Period 3 - Long-Term Averaging (50 days)

Equivalent calculations as performed in Period 2 except that a 50-day exponential moving average is used.

Once these calculations are carried out, we can calculate the value for the Fear and Greed:

$$rawFG(t) = diff_{20}(t) - diff_{50}(t-9)$$

for $t > 69$.

This equation attempts to detect recent buying pressure that is in excess of its historical levels. Note that $diff_{50}$ is only able to be computed after 60 days of data (10 days until True Range can be calculated + 50 days needed for the moving average).

The ultimate value for the Fear & Greed is undefined for t $<$ 70. Otherwise, we can find the value recursively:

$$FG(t) = \frac{rawFG(t) + rawFG(t-1)}{2} \qquad for\ t = 70$$

$$FG(t) = \frac{2}{3} \times rawFG(t) + \frac{1}{3} \times FG(t-1) \qquad for\ t > 70$$

Appendix B: Sensitivity Analysis-Investment Weights Boundaries

#1	*Upper*	*10%*			**#2**	*Upper*	*20%*		
	Lower	*-10%*				*Lower*	*-10%*		
	Quaterly returns		**Returns**	**Wealth**		**Quaterly returns**		**Returns**	**Wealth**
2006-Q1	-0.0486				**2006-Q1**	-0.0486			
2006-Q2	0.0614				**2006-Q2**	0.0614			
2006-Q3	0.0582	**2006**	10.022%	1100217.431	**2006-Q3**	0.0582	**2006**	10.022%	1100217.431
2006-Q4	0.0296				**2006-Q4**	0.0296			
2007-Q1	0.0346				**2007-Q1**	0.0346			
2007-Q2	0.0265				**2007-Q2**	0.0265			
2007-Q3	-0.0025	**2007**	3.362%	1137206.042	**2007-Q3**	-0.0025	**2007**	3.362%	1137206.042
2007-Q4	-0.0243				**2007-Q4**	-0.0243			
2008-Q1	0.0002				**2008-Q1**	0			
2008-Q2	-0.1395				**2008-Q2**	-0.1395			
2008-Q3	-0.1773	**2008**	-26.311%	837999.8393	**2008-Q3**	-0.1773	**2008**	-26.325%	837832.2728
2008-Q4	0.0407				**2008-Q4**	0.0407			
2009-Q1	-0.0059				**2009-Q1**	-0.0059			
2009-Q2	0.0836				**2009-Q2**	0.0836			
2009-Q3	0.0704	**2009**	17.126%	981515.8437	**2009-Q3**	0.0704	**2009**	17.126%	981319.5798
2009-Q4	0.0158				**2009-Q4**	0.0158			
2010-Q1	-0.1121				**2010-Q1**	-0.1121			
2010-Q2	0.1097				**2010-Q2**	0.1097			
2010-Q3	0.0836	**2010**	15.9814%	*1138376.003*	**2010-Q3**	0.0836	**2010**	15.9814%	***1138148.374***
2010-Q4	0.0863				**2010-Q4**	0.0863			
Initial Wealth	*$1,000,000*				*Initial Wealth*	*$1,000,000*			
Risk Analysis					**Risk Analysis**				
Expected Quaterly Returns	0.00952				*Expected Quaterly Returns*	0.00951			
SD	0.077764033				*SD*	0.07776531			
Sharpe ratio	0.122421634				*Sharpe ratio*	0.12229104			

#3	*Upper*	*5%*			**#4**	*Upper*	*5%*		
	Lower	*-20%*				*Lower*	*-5%*		
	Quaterly returns		**Returns**	**Wealth**		**Quaterly returns**		**Returns**	**Wealth**
2006-Q1	-0.0483				**2006-Q1**	-0.049			
2006-Q2	0.0614				**2006-Q2**	0.0616			
2006-Q3	0.0582	**2006**	10.056%	1100564.357	**2006-Q3**	0.0582	**2006**	9.996%	1099962.091
2006-Q4	0.0296				**2006-Q4**	0.0296			
2007-Q1	0.0346				**2007-Q1**	0.0346			
2007-Q2	0.0265				**2007-Q2**	0.0265			
2007-Q3	-0.0023	**2007**	3.351%	1137084.325	**2007-Q3**	0.0008	**2007**	3.831%	1142371.489
2007-Q4	-0.0246				**2007-Q4**	-0.0231			
2008-Q1	0.0098				**2008-Q1**	0.0092			
2008-Q2	-0.1424				**2008-Q2**	-0.1414			
2008-Q3	-0.1799	**2008**	-26.032%	841083.0824	**2008-Q3**	-0.1797	**2008**	-26.057%	844708.3584
2008-Q4	0.0415				**2008-Q4**	0.0403			
2009-Q1	-0.0047				**2009-Q1**	-0.0028			
2009-Q2	0.084				**2009-Q2**	0.0817			
2009-Q3	0.069	**2009**	17.134%	985195.8671	**2009-Q3**	0.0701	**2009**	17.206%	990050.6523
2009-Q4	0.0156				**2009-Q4**	0.0154			
2010-Q1	-0.1128				**2010-Q1**	-0.113			
2010-Q2	0.1137				**2010-Q2**	0.1137			
2010-Q3	0.0849	**2010**	16.4580%	***1147339.116***	**2010-Q3**	0.0859	**2010**	16.1314%	***1149759.773***
2010-Q4	0.0864				**2010-Q4**	0.0826			
Initial Wealth	*$1,000,000*				*Initial Wealth*	*$1,000,000*			
Risk Analysis					**Risk Analysis**				
Expected Quaterly Returns	0.01001				*Expected Quaterly Returns*	0.01006			
SD	0.078719729				*SD*	0.07833648			
Sharpe ratio	0.127159991				*Sharpe ratio*	0.12842037			

#5	*Upper*	*2%*		
	Lower	*-2%*		
	Quaterly returns		**Returns**	**Wealth**
2006-Q1	-0.0564			
2006-Q2	0.063			
2006-Q3	0.0566	**2006**	9.119%	1091189.899
2006-Q4	0.0296			
2007-Q1	0.0344			
2007-Q2	0.0276			
2007-Q3	-0.0096	**2007**	2.485%	1127554.952
2007-Q4	-0.0265			
2008-Q1	0.0174			
2008-Q2	-0.164			
2008-Q3	-0.1853	**2008**	-27.421%	818363.0515
2008-Q4	0.0474			
2009-Q1	0.0029			
2009-Q2	0.0621			
2009-Q3	0.0541	**2009**	15.402%	944407.6143
2009-Q4	0.0278			
2010-Q1	-0.1072			
2010-Q2	0.1137			
2010-Q3	0.0934	**2010**	16.9914%	***1104876.107***
2010-Q4	0.0761			
Initial Wealth	*$1,000,000*			
Risk Analysis				
Expected Quaterly Returns	0.007855			
SD	0.080464301			
Sharpe ratio	0.097620932			

#6	*Upper*	*20%*		
	Lower	*0%*		
	Quaterly returns		**Returns**	**Wealth**
2006-Q1	-0.0623			
2006-Q2	0.0535			
2006-Q3	0.0874	**2006**	13.307%	1133073.039
2006-Q4	0.0548			
2007-Q1	0.0352			
2007-Q2	0.002			
2007-Q3	-0.0577	**2007**	-7.888%	1013432.844
2007-Q4	-0.0576			
2008-Q1	-0.0234			
2008-Q2	-0.1954			
2008-Q3	-0.2414	**2008**	-39.449%	613638.7406
2008-Q4	0.0158			
2009-Q1	0.2934			
2009-Q2	0.2122			
2009-Q3	0.0451	**2009**	79.210%	1099704.409
2009-Q4	0.0937			
2010-Q1	-0.1119			
2010-Q2	0.1147			
2010-Q3	0.1088	**2010**	19.2512%	***1311410.968***
2010-Q4	0.0864			
Initial Wealth	*$1,000,000*			
Risk Analysis				
Expected Quaterly Returns	0.022665			
SD	0.125480111			
Sharpe ratio	0.180626235			

Appendix C: Investment Horizon Analysis (1-year VS 3-years VS 5-years)- 2006-2010

Upper Bound=10%, Lower Bound= -10%, Initial Wealth= $1,000,000.00

1 Year Investment Horizon				
	Return	Wealth	S&P Quaterly Returns	Wealth
2010-Q1	-0.1121	887900	0.105059272	1105059.272
2010-Q2	0.1097	985302.63	-0.071703646	1025822.492
2010-Q3	0.0836	1067673.93	0.07412854	1101865.216
2010-Q4	0.0863	1159814.19	0.086929331	1197649.622
Expected Quaterly Return	0.041875		0.048603374	
Standard Deviation	0.103317		0.081202341	
Sharpe Ratio	0.405307		0.598546464	

3 Year Investment Horizon				
	Return	Wealth	S&P Quaterly Returns	Wealth
2008-Q1	0.0002	1000200	0.005106815	1005106.815
2008-Q2	-0.1395	860672.1	-0.085313837	919357.2957
2008-Q3	-0.1773	708074.9367	-0.235627831	702731.1305
2008-Q4	0.0407	736893.5866	-0.14747871	599093.2502
2009-Q1	-0.0059	732545.9144	0.056824236	633136.2664
2009-Q2	0.0836	793786.7529	0.131380255	716317.8702
2009-Q3	0.0704	849669.3403	0.049327581	751652.0982
2009-Q4	0.0158	863094.1159	0.036363987	778985.1656
2010-Q1	-0.1121	766341.2655	0.105059272	860824.7797
2010-Q2	0.1097	850408.9023	-0.071703646	799100.5042
2010-Q3	0.0836	921503.0865	0.07412854	858336.6581
2010-Q4	0.0863	1001028.803	0.086929331	932951.2894
Expected Quaterly Return	0.004625		0.000416333	
Standard Deviation	0.09703		0.112026305	
Sharpe Ratio	0.047666		0.003716384	

5 Year Investment Horizon				
	Return	Wealth	S&P Quaterly Returns	Wealth
2006-Q1	-0.0486	951400	0.023850072	1023850.072
2006-Q2	0.0614	1009815.96	-0.025903968	997328.292
2006-Q3	0.0582	1068587.249	0.079332007	1076448.347
2006-Q4	0.0296	1100217.431	0.043760977	1123554.778
2007-Q1	0.0346	1138284.955	0.030683335	1158029.186
2007-Q2	0.0265	1168449.506	-0.018281536	1136858.634
2007-Q3	-0.0025	1165528.382	0.064668412	1210377.476
2007-Q4	-0.0243	1137206.042	-0.110257006	1076924.88
2008-Q1	0.0002	1137433.484	0.005106815	1082424.536
2008-Q2	-0.1395	978761.5127	-0.085313837	990078.7451
2008-Q3	-0.1773	805227.0965	-0.235627831	756788.6382
2008-Q4	0.0407	837999.8393	-0.14747871	645178.4263
2009-Q1	-0.0059	833055.6402	0.056824236	681840.1975
2009-Q2	0.0836	902699.0918	0.131380255	771420.5362
2009-Q3	0.0704	966249.1078	0.049327581	809472.8454
2009-Q4	0.0158	981515.8437	0.036363987	838908.5057
2010-Q1	-0.1121	871487.9176	0.105059272	927043.6223
2010-Q2	0.1097	967090.1422	-0.071703646	860571.2143
2010-Q3	0.0836	1047938.878	0.07412854	924364.1022
2010-Q4	0.0863	1138376.003	0.086929331	1004718.455
Expected Quaterly Return	0.00952		0.004642414	
Standard Deviation	0.077764		0.093099434	
Sharpe Ratio	0.122422		0.049865118	

Appendix D: Investment Horizon Analysis (1-year VS 3-years VS 5-years)- 2004-2008

Upper Bound=10%, Lower Bound= -10%, Initial Wealth= $1,000,000.00

1 Year Investment Horizon				
	Return	Wealth	S&P Quaterly Returns	Wealth
2008-Q1	0.0051	1005100	0.005106815	1005106.815
2008-Q2	-0.1426	861772.74	-0.085313837	919357.2957
2008-Q3	-0.2008	688728.7738	-0.235627831	702731.1305
2008-Q4	0.0462	720548.0432	-0.14747871	599093.2502
Expected Quaterly Return	-0.073025		-0.115828391	
Standard Deviation	0.117594285		0.101505518	
Sharpe Ratio	-0.62099106		-1.141104376	

3 Year Investment Horizon				
	Return	Wealth	S&P Quaterly Returns	Wealth
2006-Q1	-0.0598	940200	0.023850072	1023850.072
2006-Q2	0.0799	1015321.98	-0.025903968	997328.292
2006-Q3	0.0603	1076545.895	0.079332007	1076448.347
2006-Q4	0.0245	1102921.27	0.043760977	1123554.778
2007-Q1	0.0267	1132369.268	0.030683335	1158029.186
2007-Q2	-0.0055	1126141.237	-0.018281536	1136858.634
2007-Q3	-0.0028	1122988.041	0.064668412	1210377.476
2007-Q4	-0.0077	1114341.033	-0.110257006	1076924.88
2008-Q1	0.0051	1120024.173	0.005106815	1082424.536
2008-Q2	-0.1426	960308.7256	-0.085313837	990078.7451
2008-Q3	-0.2008	767478.7335	-0.235627831	756788.6382
2008-Q4	0.0462	802936.251	-0.14747871	645178.4263
Expected Quaterly Return	-0.01470833		-0.031288439	
Standard Deviation	0.082696867		0.095293853	
Sharpe Ratio	-0.17785841		-0.328336386	

5 Year Investment Horizon				
	Return	Wealth	S&P Quaterly Returns	Wealth
2004-Q1	0.0285	1028500	-0.021067428	978932.5719
2004-Q2	0.015	1043927.5	-0.005039285	973999.4519
2004-Q3	0.087	1134749.193	0.025850488	999177.8133
2004-Q4	-0.0216	1110238.61	0.045186693	1044327.354
2005-Q1	-0.0178	1090476.363	-0.020672666	1022738.324
2005-Q2	0.0287	1121773.034	0.066845313	1091103.587
2005-Q3	0.0171	1140955.353	-0.022014617	1067083.359
2005-Q4	0.0743	1225728.336	0.060538024	1131682.477
2006-Q1	-0.0598	1152429.781	0.023850072	1158673.185
2006-Q2	0.0799	1244508.921	-0.025903968	1128658.952
2006-Q3	0.0603	1319552.809	0.079332007	1218197.731
2006-Q4	0.0245	1351881.853	0.043760977	1271507.254
2007-Q1	0.0267	1387977.098	0.030683335	1310521.337
2007-Q2	-0.0055	1380343.224	-0.018281536	1286562.995
2007-Q3	-0.0028	1376478.263	0.064668412	1369762.98
2007-Q4	-0.0077	1365879.38	-0.110257006	1218737.015
2008-Q1	0.0051	1372845.365	0.005106815	1224960.88
2008-Q2	-0.1426	1177077.616	-0.085313837	1120454.766
2008-Q3	-0.2008	940720.4309	-0.235627831	856444.4405
2008-Q4	0.0462	984181.7148	-0.14747871	730137.1195
Expected Quaterly Return	0.001735		-0.012291737	
Standard Deviation	0.070258375		0.07974975	
Sharpe Ratio	0.024694565		-0.154128852	

Appendix E: The R^2's and the P-Values for Industry Wise and Size Wise Analysis