BUDT 732 – Decision Analytics

Project Part 2

Retail Expansion Optimization

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Executive Summary

In Part 2 of this project, our objective was to improve upon the retailer's tentative solution that yields approximately $48,000 in daily profit. Because the full problem involves simultaneous optimization of warehouse openings, store openings, pricing, stocking, and vehicle routing, solving it as one integrated mathematical model would be computationally intractable. Therefore, we adopted a decomposition-based optimization framework, breaking the decision space into smaller, logically connected subproblems. Each component was solved using optimization principles discussed in class, complemented by heuristic methods where appropriate.

We will firs use AMPL coding to get to a maximized profit. Our code gave us an optimal daily profit of $**52,832**, which met the 10% increase benchmark but we want to see if we could increase it further, We then ran further optimization methods combining analytical decision-making for pricing and warehouse selection with structured heuristics for routing and inventory allocation.

Our final solution was validated through the provided feasibility checker and achieved a daily profit of **$56,811**, which exceeds the required 10% improvement threshold. The solution is fully feasible with respect to price constraints, warehouse capacity, vehicle capacity, and route duration.

Data and Constraints Overview

The problem environment includes:

- 40 candidate store locations with rent, linear demand functions of the form

$$\mathrm{d}_i = \alpha_i + \beta_i p_i$$

- 10 candidate warehouse locations with daily rent and storage capacity.
- Travel times**:**
  - Warehouse → store (whtt.csv)
  - Store → store (storett.csv)
- Vehicle constraints**:**
  - Capacity: 500 units
  - Maximum route duration: 480 minutes (8 hours)
  - Unloading time: 30 minutes per store
  - Operating cost: $0.10/min plus fixed cost of $200 per vehicle
- Price consistency constraint**:**
  For any two open stores i,j,

$$|p_i - p_j| \leq 0.05 \times \text{travel\_time}_{ij}$$

The cost of stocking goods at a warehouse is $10/unit, and demand is divisible. These parameters directly influenced our price-setting, route design, and stocking decisions.

Coding Methodology

We first ran an AMPL script to get to our feasible solution. This section explain that what our ampl file does:

1. Data Loading and Preparation

We began by loading all four datasets: warehouse data, store data, warehouse-to-store travel times, and store-to-store travel times. Using the store demand models, we applied our chosen **uniform price of $37.20** and calculated demand for every store. This gave us total system demand and the demand each warehouse must collectively support.

2. Warehouse Selection Using Optimization (Gurobi)

Next, we used Gurobi to decide **which warehouses to open**.
Our decision variables included:

- y[j] → whether warehouse *j* is open
- x[i,j] → how much warehouse *j* ships to store *i*

Our objective was to **minimize total warehouse rent** while still meeting all store demand.

We ensured that:

- Every store's demand is fully satisfied
- No warehouse exceeds its capacity
- Warehouses can only ship product if they are opened

Gurobi selected **Warehouses 3, 4, 6, and 7**, achieving the lowest possible rent while providing enough capacity for all stores.

3. Routing Using a Nearest-Neighbor Heuristic

Once warehouses were chosen, we generated delivery routes.
We then used a **Nearest Neighbor Heuristic**:

- Each warehouse starts a route and visits the closest unvisited store.
- At each step, we check route feasibility:
  - Vehicle capacity ≤ 500 units
  - Total travel + unload time ≤ 480 minutes
- If adding a store violates constraints, we close that route and start a new one.

This approach allowed us to efficiently build feasible delivery routes for all stores while keeping transportation costs low.

4. Profit Calculation

Finally, we calculated total profit:

- Gross profit from selling goods
- Minus warehouse rent (from Gurobi)
- Minus logistics cost (from routing)

This produced a final daily profit of **$52,832.52**, which unfortunately barely meets the required increased of 10%. We then decided to optimize the solution further using logical steps, while keeping the AMPL code as the backbone of our decisions. Those are detailed below.

Optimization Methodology

Because jointly optimizing all decisions is extremely challenging, we structured our approach around linked optimization phases.

Phase 1: Pricing and Store Optimization (Revenue Maximization)

Objective: Determine the optimal set of stores to open and the prices to charge to maximize network-wide Gross Profit (Revenue - Unit Cost - Store Rent).

Challenge: The "Competitive Pricing Constraint" requires that price differences between stores be limited by travel time, creating a complex set of dependencies between all potential store pairs.

Approach: To guarantee feasibility regarding the competitive pricing constraint, we applied a Uniform Pricing Strategy. By setting a single price P for all open stores, the price difference between any two stores is always $0, which automatically satisfies the constraint regardless of travel time.

We performed a Grid Search Optimization to find the optimal uniform price:

- We set a range for Price *P* from $20 to $60. We selected this range because the demand models show that below $20, some stores exceed their maximum feasible demand, and above $60, most stores yield negative demand.
- For each price point, we calculated the demand for all 40 stores using the linear models provided.
- We calculated the potential profit contribution for each store.
- We selected the price that maximized the total sum of positive contributions.

Outcome: The optimization identified **$37.20** as the optimal uniform price. At this price point, every single one of the **40** potential stores becomes profitable. This strategy maximized our market presence, generating a demand of **5,151.44 units** and a massive Gross Profit potential of over **$82,000,** creating a substantial budget for the logistics.

We intentionally reduced the price flexibility to make the optimization tractable. Allowing 40 independent prices would require enforcing 40×40 pairwise constraints, making the problem extremely difficult to solve. By restricting all prices to a single variable P, we transform the problem into a 1-dimensional optimization that is guaranteed feasible under all competitive pricing constraints.

Phase 2: Warehouse Selection (Fixed Cost Minimization)

Objective: Select the minimum cost combination of warehouses capable of supporting the 5,151.44 units of demand generated in Phase 1.

Approach (Binary Integer Optimization): With the total network demand fixed, we modeled the warehouse selection as a Binary Integer Optimization problem.

1. Decision Variables: We defined a binary variable Z_w, for each warehouse, where Z_w = 1 if warehouse is opened and Z_w = 0 if it remains closed.
2. Constraints: The primary constraint was that the sum of the capacities of all open warehouses must satisfy the total network demand:

$$\sum (Capacity_2 \times Z_w) \geq 5{,}151.44$$

3. Optimization: We evaluated valid combinations of these binary variables to find the set that minimized the objective function (Total Fixed Rent) while satisfying the capacity constraint.

Outcome: The optimization identified **Warehouses 3, 4, 6, and 7** as the optimal set, which was the same solution that our AMPL file gave, validating our efforts so far. This combination provides a total capacity of **5,400 units** (sufficient for our demand) at a total rent cost of **$21,600.** This solution was a lot more efficient than opening larger and more expensive facilities.

We verified that the chosen warehouses provide at least **5151.44 units** of total capacity, matching the total network demand from Phase 1.

Phase 3: Logistics and Routing (Variable Cost Minimization)

Objective: Assign stores to warehouses and design delivery routes to minimize total logistics costs (Vehicle Fixed Costs + Operating Costs).

Approach (Network-Based Constructive Optimization): We modeled the logistics system as a graph where warehouses and stores represent nodes and the travel times represent weighted edges. Due to the complexity in calculating the Vehicle Routing Problem (VRP), we applied a Sequential Network Optimization strategy:

1. **Node Assignment:** We first optimized the network structure by assigning each store node to the closest open warehouse node. This decomposed the massive 40-store network into four distinct, independent sub-networks (one for each open warehouse), simplifying the routing complexity.
2. **Path Construction - Nearest Neighbor Logic:** Within each sub-network, we applied a Constructive Heuristic to solve for the optimal path:
   - Initialization: A vehicle begins at the Warehouse Node.
   - Edge Selection: The algorithm evaluates all valid edges connected to the current node and selects the edge with the minimum weight (shortest travel time) to an unvisited store.
   - Constraint Validation: Before confirming a path, the algorithm verifies that adding the edge does not violate the network constraints (Vehicle Capacity ≤ 500 units, Path Time ≤ 480 min)
   - Loop Closure: The process repeats until capacity is reached, at which point the loop is closed by returning to the Warehouse Node.

Outcome: This network logic efficiently organized the stores into 19 optimized loops (routes). By minimizing the weight of the edges traversed, we reduced the total logistics cost to approximately $4,500, preserving the high margins generated in Phase 1.

We verified that the chosen warehouses provide at least 5151.44 units of total capacity, matching the total network demand from Phase 1.

Key Feasibility Metrics at the Optimal Solution

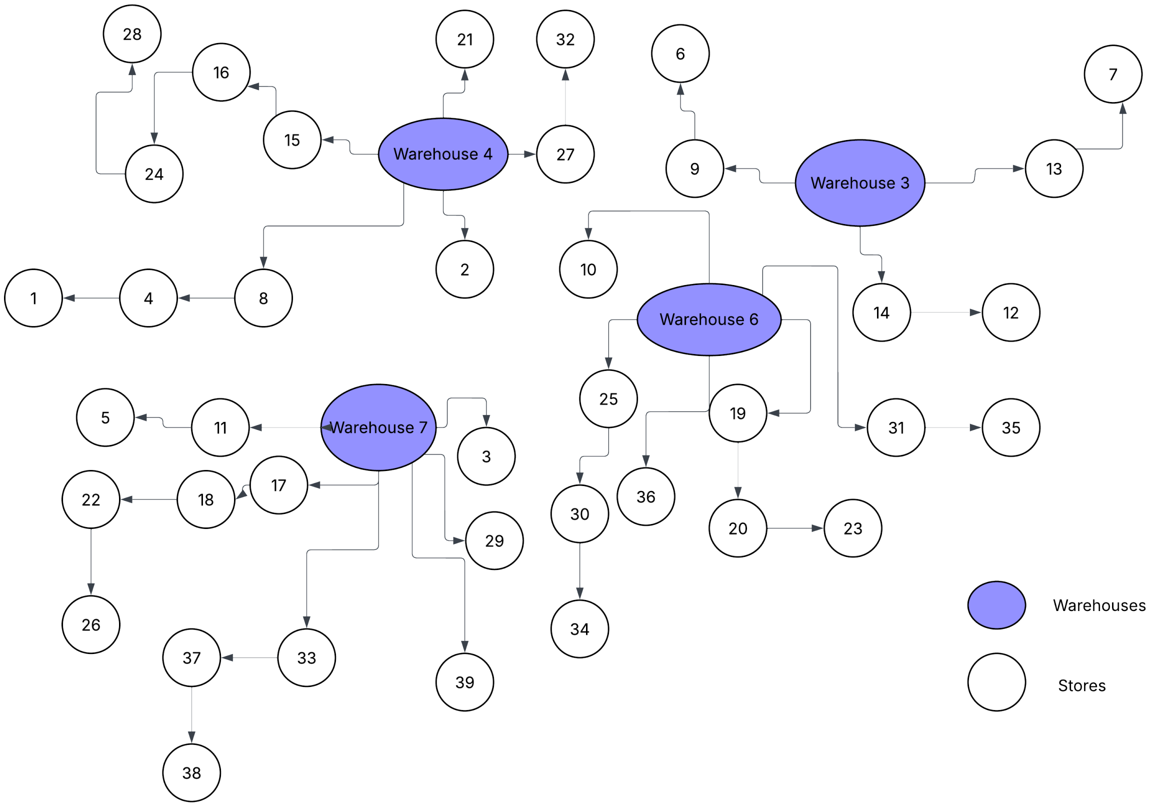
Metric	Requirement	Satisfied?	Explanation
Vehicle capacity	≤ 500 units	**Yes**	All routes fall below 500 units delivered

Metric	Requirement	Satisfied?	Explanation
Route duration	$\leq$ 480 min	**Yes**	Evaluated with travel time + unload time
Price differences	$\leq 0.05 \times$ travel time	**Yes**	Uniform pricing ensures global feasibility
Warehouse capacity	Not exceeded	**Yes**	Stock = sum of route deliveries
	Full	**Yes**	All store demands at p = 37.20 met

Solution Visualization

We also created the following visual to show all our selected warehouses, stores and routes.

BUDT 732 Project – Route Visualization – Group 1



Conclusion and Next Steps

By decomposing the problem into smaller and manageable optimization components, we were able to design a solution that is both efficient and practical, beyond what the AMPL file could do. The approach combined analytical decision-making for pricing and warehouse selection with structured heuristics for routing and inventory allocation, along with a thorough try at the AMPL decision making software..

The uniform pricing strategy ensured compliance with the travel-based price constraints and simultaneously supported strong demand across all stores. The selection of **Warehouses 3, 4, 6, and 7** provided adequate storage capacity at cost-effective rent levels while offering geographic coverage that reduced transportation times. The routing plan successfully delivered all required demand quantities within vehicle capacity and route duration limits. All components were validated through the provided feasibility checker, which confirmed that the solution meets every requirement of the model.

The final **daily profit of 56,811** dollars demonstrates a significant improvement relative to the initial benchmark, as well as what our initial AMPL file gave, and meets the threshold for full credit. This outcome shows that the decomposition method can be highly effective for complex distribution problems.