Points of diminishing returns.

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Given: A data set with (x, y) values corresponding to a specific key. There are in total 173 keys. The *aim* of the project is to analyze the data set and then predict, for each key, the point of diminishing returns.

1 Assumptions and some observations:

- As the objective is finding point of diminishing returns, we assume all the data sets, corresponding to each key, is decreasing, non-monotonically, in y for all x larger than some value.
- We assume that the data is non-cyclical and we further assume that the data does't have seasonality.
- We assume that the noise in y is not highly correlated.

2 Analysis

- The strategy of finding the points is as follows:

  Point of diminishing returns is the points after which the *marginal* increase in y starts decreasing. Therefore, we need to computer the first and second derivatives of the function and then look for the x value where the second derivative is changing sign. (This point is also called inflection point.)
- However, due to noise in the data and lot of variance, we first have to smooth the data in y. For this we tried "moving average" or "rolling average" which is a low-pass filter.
- To get better results, we used 'Savitzky-Golay' filter after trying others. This works by convolution with low-degree polynomials by least squared method.
- After applying the filter three times, we get a reasonably smooth curve.

- Then to look for point of diminishing returns we write a program, podr, that starts from the right of x axis and looks for the first x values where the second derivative is positive and larger than

$$0.5 * \sigma / \sqrt{samplesize},$$

where σ is the standard deviation of the second derivative. This is just the half of *unbiased sample deviation.*

3 Remarks and Practical context

- The choice of filter here can certainly be different. Moreover, to compute the derivative, we used Numpy's diff, where other finite-difference methods and Fast Fourier transforms could have been used as well.
- In another direction, the data could be modeled, after smoothing, by a curve using non-linear regression. Then the second-derivative could be computed easily. Predictions of different models can then be tested using statistical tool and criteria like AIC, BIC.
- (Bonus): In online keyword auction setting, besides several economic scenarios, the present data can be thought of representing incremental ROAS (return on ad spend) overt time. At a given point in time, ROAS becomes negative impacting the profits.