Lecture 7: Entry

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ECON 416-1

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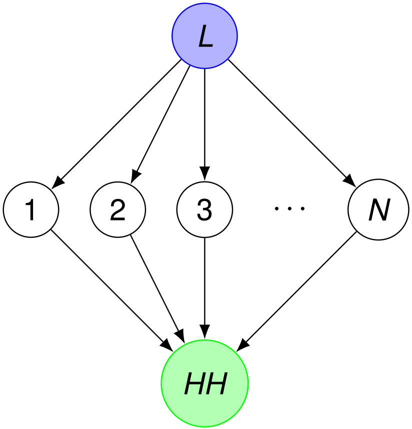
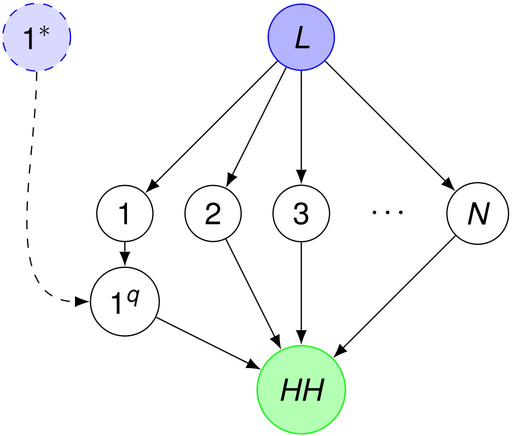
Recap from last time

- Last time, we discussed an isomorphism between distortions and technology.

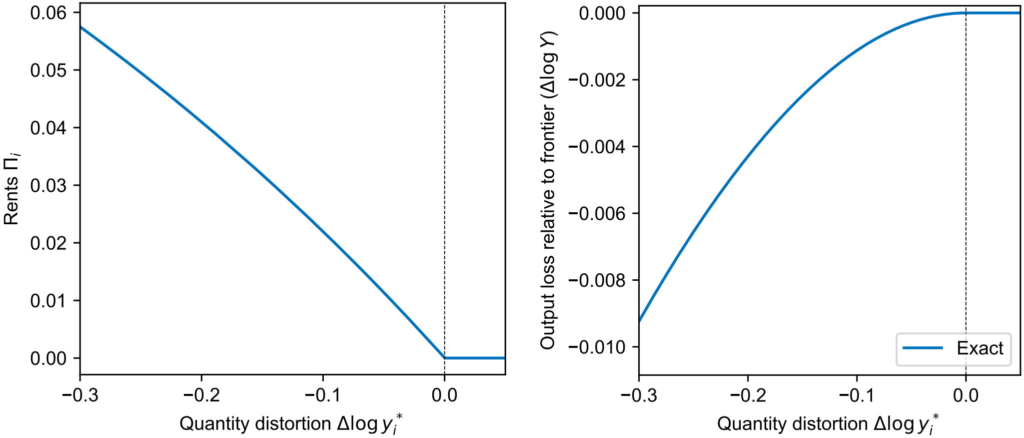


(a) Observed economy.



(b) Implementation with quota.

Distance to Frontier Example



- Local gains from relaxing distortion given by rents Π_i.
- Rents go to zero when distortion no longer binds.

Nonlinearities: Multiple Quotas

- Method scales up to multiple interacting quotas

$$\Delta \log Y \approx \mathbf{\Pi}' d \log \mathbf{y}^* + \frac{1}{2} (d \log \mathbf{y}^*)' \frac{d\mathbf{\Pi}}{d \log \mathbf{y}^*} (d \log \mathbf{y}^*),$$

- Quota demand system $\frac{d\mathbf{\Pi}}{d \log \mathbf{y}^*}$ summarizes responses of rents to quotas.

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Illustrative Example: Multiple Quotas

- Consider horizontal economy. Response to changes in quotas on firms 1 and 2:

$$\Delta \log Y \approx \underbrace{\Pi_1 d\log y_{1^*} + \Pi_2 d\log y_{2^*}}_{\text{First order}}$$
$$\underbrace{+(1/2)\left(\frac{\partial \Pi_1}{\partial \log y_1^*}(d\log y_{1^*})^2 + \frac{\partial \Pi_2}{\partial \log y_2^*}(d\log y_{2^*})^2 + 2\frac{\partial \Pi_1}{\partial \log y_2^*}(d\log y_{1^*})(d\log y_{2^*})\right)}_{\text{Second order}}.$$

- $\frac{\partial \Pi_1}{\partial \log y_2^*}$ determines whether relaxing y_{1^*} amplifies/reduces gains from relaxing y_{2^*}.
- In horizontal economy, $\frac{\partial \Pi_1}{\partial \log y_2^*} > 0$ if

$$\theta < 1 - \frac{(\lambda_1 - \Pi_1)(\lambda_2 - \Pi_2)}{(1 - \lambda_1 - \lambda_2)\Pi_1 \Pi_2}.$$

Necessary condition is that $\theta < 1$, i.e., outputs of firms 1 and 2 are complements.

Nonlinearities: Multiple Quotas

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- Quota demand system $\frac{d\mathbf{\Pi}}{d \log \mathbf{y}^*}$ summarizes responses of rents to quotas.
- Similarly, gains from eliminating quotas simultaneously given by:

$$\Delta \log Y \approx -\frac{1}{2} \mathbf{\Pi}' \left[\frac{d\mathbf{\Pi}}{d \log \boldsymbol{y}^*} \right]^{-1} \mathbf{\Pi}.$$

- If i's rents fall when j's quota relaxed, then

gains from relaxing both quotas $<$ sum of gains from relaxing each.

Empirical Example: China's Textile & Clothing Exports

- 1975–1994 Multi-Fiber Agreement capped China's textile & clothing exports to US, EU.
- Staged phase-out:
  - Jan 2002 (Phase III): Knit fabrics, gloves, dressing gowns, brassieres, etc.
  - Jan 2005 (Phase IV): Silk, wool, and cotton textiles, other apparel categories, etc.
- Obtain quota demand system using initial rents & response of exports to liberalization.
- Use quota auction prices for initial rents: $\Pi_{\text{Phase III}} = \$520\text{M}$, $\Pi_{\text{Phase IV}} = \$1583\text{M}$.

Quota Demand System: China's Textile and Clothing Exports

1. Use quota auction prices for initial quota profits.

2. Use response of export quantities to phase-out to recover quota demand system, H.

3. Use H to estimate gains from relaxing any subset of quotas.

Quota Demand System: China's Textile and Clothing Exports

1. Use quota auction prices for initial quota profits.
    - Market prices for quotas to export in each product category in 2001.
    - Quota profits in 2025 USD, $\Pi_{\text{Phase III}} = \$520\text{M}$, $\Pi_{\text{Phase IV}} = \$1583\text{M}$.
2. Use response of export quantities to phase-out to recover quota demand system, H.
3. Use H to estimate gains from relaxing any subset of quotas.

Quota Demand System: China's Textile and Clothing Exports

1. Use quota auction prices for initial quota profits.

2. Use response of export quantities to phase-out to recover quota demand system, H.

   - Because profits go to zero when quotas are removed, $H = \partial \Pi / \partial \log y$ solves:

   $$\begin{aligned} \Pi_{\text{Phase III}} &= (d \log y^{\text{III}}_{\text{Phase III}})\, H_{11}\,, \\ \Pi_{\text{Phase III}} &= (d \log y^{\text{IV}}_{\text{Phase III}})\, H_{11} + (d \log y^{\text{IV}}_{\text{Phase IV}})\, H_{12}\,, \\ \Pi_{\text{Phase IV}} &= (d \log y^{\text{III}}_{\text{Phase IV}})\, H_{21} + (d \log y^{\text{IV}}_{\text{Phase IV}})\, H_{22}\,, \end{aligned}$$

   where $d \log y^{\text{III}}_x$ ($d \log y^{\text{IV}}_x$) is the change in exports for x goods following Phase III (IV).

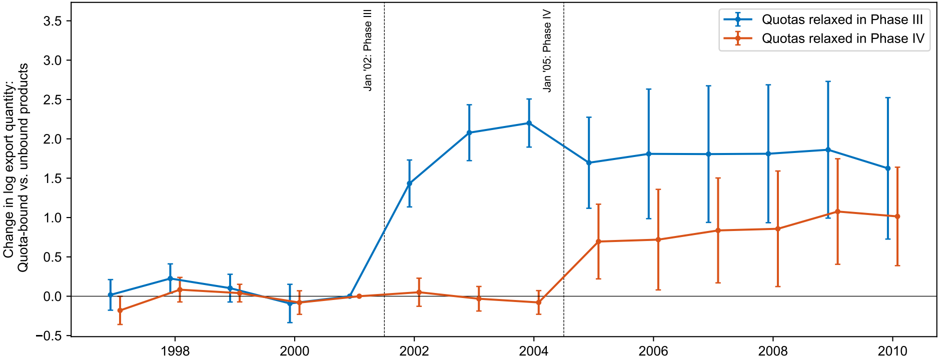
   - Symmetry implies $H_{12} = H_{21}$.

3. Use H to estimate gains from relaxing any subset of quotas.

Quota Demand System: China's Textile and Clothing Exports

$$\log y_{ict} = \beta_t^{\text{Phase III}} \left(\text{Binding}_c \times 1\{c \text{ quota relaxed in Phase III}\} \times 1\{\text{year} = t\}\right)$$
$$+ \beta_t^{\text{Phase IV}} \left(\text{Binding}_c \times 1\{c \text{ quota relaxed in Phase IV}\} \times 1\{\text{year} = t\}\right) + \alpha_t + \delta_i + \varepsilon_{ict},$$

- E.g., $\beta_t^{\text{Phase III}}$ is change in Phase III good exports in t relative to unconstrained goods.



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Quota Demand System: China's Textile and Clothing Exports

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3. Use *H* to estimate gains from relaxing any subset of quotas.

   - Estimated inverse quota demand system:

$$\Pi = \begin{bmatrix} \Pi_{\text{Phase III}} \\ \Pi_{\text{Phase IV}} \end{bmatrix} = \begin{bmatrix} \$520\text{M} \\ \$1583\text{M} \end{bmatrix}, \qquad \frac{d \log \boldsymbol{\Pi}}{d \log \mathbf{y}^*} = \begin{bmatrix} -0.472 & -0.200 \\ -0.066 & -1.149 \end{bmatrix}.$$

Quota Demand System: China's Textile and Clothing Exports

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Intervention	Efficiency gains (2025 USD)
(A) Relaxing Phase III quotas only	\$565M
(B) Relaxing Phase IV quotas only	\$706M
(C) Relaxing both Phase III and IV quotas	\$1075M
Difference: $C-(A+B)$	\$196M

Distance to Frontier: Argentina's Capital Controls

- In September 2019, Argentina reimposed capital controls, restricting amount that households and firms could access foreign exchange markets.
- Restrictions led to decline in capital outflows from \$7.2B to $<$\$1.5B per month.
- What are the efficiency losses from these restrictions?
  - Permitted transactions take place at official exchange rate $\bar{e}$.
  - Unconstrained transactions take place at black-market exchange rate e.
  - Gap between $\bar{e}$ and e are profits earned by permit to exchange under controls.

Distance to Frontier: Argentina's Capital Controls

- Option 1: $\Delta \log Y \approx -\frac{1}{2}\Pi_i d \log y_{i^*} \approx -\frac{1}{2}\left(\log e/\bar{e}\right) dy_{i^*}$.
  Measure distortion dy_{i^*} as gap relative to 2016–2019 outflows.

- Option 2: Elasticity of exchange rate to currency purchases, $\log e/\bar{e} = \theta\left(dy_i^*/\text{GDP}\right)$.
  Then, $\frac{\Delta Y}{\text{GDP}} \approx -\frac{1}{2}\frac{1}{\theta}\left(\log e/\bar{e}\right)^2$. (Blanchard et al. 2015, Adler et al. 2019.)

Distance to Frontier: Argentina's Capital Controls

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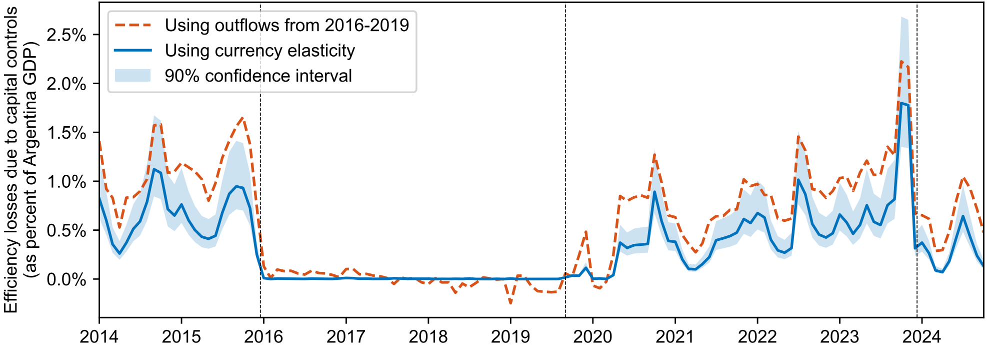

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Increasing Returns to Scale and Trade

- One reason for trade: differences in technology and factor endowments.
- However, in 1960s–1970s, economists noted that most trade occurs between similar countries (incomes, endowments).
- Moreover, intra-industry trade (e.g., Germany both imports and exports cars).
- “New trade theory” justifies trade with increasing returns and love-of-variety.

Increasing Returns to Scale and Trade

- One reason for trade: differences in technology and factor endowments.
- However, in 1960s–1970s, economists noted that most trade occurs between similar countries (incomes, endowments).
- Moreover, intra-industry trade (e.g., Germany both imports and exports cars).
- “New trade theory” justifies trade with increasing returns and love-of-variety.
- Today, we will start with an adapted version of Krugman (1979).
- Will also help us think about real GDP vs. welfare with entry.

Setup: Production

- Continuum of varieties indexed by θ. Mass M of varieties produced (endogenous).

- Firms pay fixed entry cost f_e in units of labor plus variable labor costs.
  Cost of producing $y_\theta L$ units of output is

$$C_\theta(y_\theta L) = w\frac{y_\theta L}{A_\theta} + wf_e,$$

  where w is the wage and A_θ is the firm's productivity.

- We will have free entry, so in eq., $w = I = 1$ (take per-capita income as numeraire).

- As in Krugman (1979), we will start by assuming all varieties are symmetric, so that

$$C(yL) = \frac{yL}{A} + f_e.$$

  Fixed costs $\Rightarrow$ "micro increasing returns to scale."

Setup: Preferences

- Krugman uses non-homothetic variable elasticity of substitution (VES) preferences.
- We will use HSA preferences (Matsuyama and Ushchev, 2017).
- Mass L of households with identical preferences.
- For each household, expenditure share on variety θ is defined by $s(\cdot)$ function,

$$\frac{p_\theta y_\theta}{I} = s_\theta(\frac{p_\theta}{P}).$$

  where y_θ is per-capita consumption and I is per-capita income.
- Aggregator P defined implicitly by

$$\int_0^M s_\theta(\frac{p_\theta}{P})d\theta = 1.$$

Setup: Preferences

- What are real output Y and the ideal price index P^Y?
- Locally, we know from Shephard's Lemma that the elasticity of the ideal price index w.r.t. a price change is given by the expenditure share:

$$\frac{\partial \log P^Y}{\partial \log p_\theta} = \frac{p_\theta y_\theta}{I} = s_\theta(\frac{p_\theta}{P}).$$

- Integrating across varieties,

$$\begin{aligned} d\log P^Y &= \int_0^M s_\theta(\frac{p_\theta}{P}) \left[d\log \frac{p_\theta}{P} + d\log P \right] d\theta \\ &= d\log P + \int_0^M s_\theta(\frac{p_\theta}{P}) \left[d\log \frac{p_\theta}{P} \right] d\theta. \end{aligned}$$

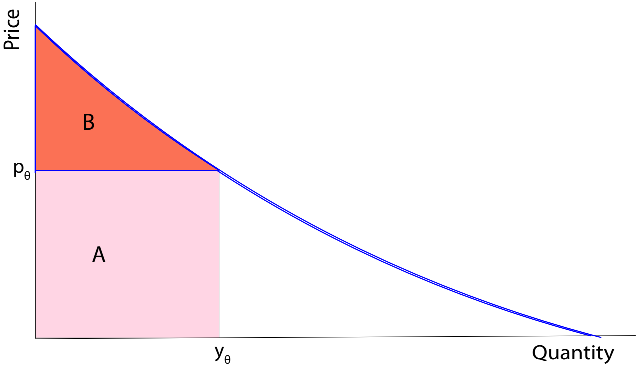
- Integrating across price changes from ∞ to p_θ / P,

$$\log P^Y = \log P - \int_0^M \left[\int_{p_\theta/P}^{\infty} \frac{s_\theta(\xi)}{\xi} d\xi \right] d\theta + \text{const}.$$

Consumer Surplus Ratio

- Define consumer surplus ratio δ_θ. Captures ratio of inframarginal surplus to sales,

$$\delta_\theta = \frac{\int_0^{y_\theta} p_\theta(y)dy}{p_\theta y_\theta} = \frac{A+B}{A} = 1 + \frac{\int_{p_\theta/P}^{\infty} \frac{s_\theta(\xi)}{\xi} d\xi}{s_\theta\left(\frac{p_\theta}{P}\right)}.$$



Setup: Preferences

- What is the change in output (welfare)?

$$\log Y = -\log P^Y = -\log P + \int_0^M \left[\int_{p_\theta/P}^{\infty} \frac{s_\theta(\xi)}{\xi} d\xi \right] d\theta - \text{const.}$$

Differentiating (and using symmetry),

$$d\log Y = -d\log P + M\left[\int_{p/P}^{\infty} \frac{s(\xi)}{\xi} d\xi \right] d\log M - Ms(\frac{p}{P}) d\log \frac{p}{P}$$

$$= \underbrace{(\delta - 1)\, d\log M}_{\substack{\text{Gains from} \\ \text{new varieties}}} \underbrace{-d\log p}_{\substack{\text{Gains from} \\ \text{price changes}}} .$$

- For new varieties, consumer gets entire inframarginal utility, $\delta d\log M$, but crowds out $d\log M$ sales of existing varieties.

- Change in real GDP (as measured by statistical agencies) is

$$d\log Q = -\int_0^M s_\theta(\frac{p_\theta}{P}) d\log p_\theta d\theta = -d\log p.$$

Other Demand Statistics

- We have already defined consumer surplus ratio δ_θ.
- Demand elasticity σ_θ is

$$\sigma_\theta(\frac{p_\theta}{P}) = -\frac{\partial \log y_\theta}{\partial \log p_\theta} = 1 - \frac{\frac{p_\theta}{P} s'_\theta(\frac{p_\theta}{P})}{s(\frac{p_\theta}{P})}.$$

- Markup $\mu_\theta(\frac{p_\theta}{P}) = \sigma_\theta/(\sigma_\theta - 1)$ given by Lerner formula.
- Pass-through of idiosyncratic cost shocks ρ_θ given by

$$\rho_\theta(\frac{p_\theta}{P}) = \frac{1}{1 - \frac{\frac{p_\theta}{P}\mu'_\theta(\frac{p_\theta}{P})}{\mu_\theta(\frac{p_\theta}{P})}}.$$

Krugman (1979): *"This assumption [...] that the elasticity of demand rises when the price of a good is increased, seems plausible. In any case, it seems to be necessary if this model is to yield reasonable results, and I make the assumption without apology."*

Free Entry

- Free entry sets profits net of fixed costs to zero:

$$p_\theta y_\theta L - C(y_\theta L) = \left(1 - \frac{1}{\mu(\frac{p_\theta}{P})}\right) p_\theta y_\theta L - f_e = 0.$$

- Since $p_\theta y_\theta / I = s_\theta(\frac{p_\theta}{P}) = 1/M$, we have

$$M = \left(1 - \frac{1}{\mu}\right) \frac{L}{f_e}.$$

M increasing in μ, L; decreasing in f_e.

Macro Increasing Returns to Scale

- Suppose we increase the mass of households by $d\log L$.
- What happens to the price aggregator and to prices? Recall:

$$Ms\left(\frac{p}{P}\right) = 1.$$

$$\Rightarrow \qquad d\log P = d\log p - \frac{1}{\sigma - 1} d\log M = -\frac{1}{\rho}\frac{1}{\sigma - 1} d\log M.$$

- For firm prices,

$$d\log p = \frac{d\log \mu\left(\frac{p}{P}\right)}{d\log \frac{p}{P}} d\log \frac{p}{P} = \left(1 - \frac{1}{\rho}\right) d\log \frac{p}{P} = (1-\rho)\, d\log P.$$

Place ρ weight on own marginal cost (unchanged) and $1-\rho$ on market aggregator.

Macro Increasing Returns to Scale

- What happens to entry? Recall:

$$M = \left(1 - \frac{1}{\mu}\right)\frac{L}{f_e}.$$

- So adjustment in entry is:

$$d\log M = (1-\rho)(\sigma-1)\,d\log P + d\log L = \rho\, d\log L.$$

Why does fall in P decrease entry?

Macro Increasing Returns to Scale

- What happens to entry? Recall:

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Why does fall in P decrease entry?

Macro Increasing Returns to Scale

$$
\begin{aligned}
d\log Y &= (\delta-1)\, d\log M - d\log p \\
&= \underbrace{(\delta-1)\,\rho\, d\log L}_{\substack{\text{Gains from}\\ \text{new varieties}}} + \underbrace{(1-\rho)(\mu-1) d\log L}_{\substack{\text{Gains from}\\ \text{price changes}}} \\
&= (\delta-1) d\log L + (1-\rho)(\mu-\delta) d\log L.
\end{aligned}
$$

- First term is what we would get if we scaled up economy without changing allocation of labor between entry vs. variable production.
- Second term reflects how allocation is changing due to changes in markups.
- If $\rho < 1$, is allocation to entry increasing or decreasing?
- For $\rho < 1$, why does this term depend on $\mu > \delta$?

Macro Increasing Returns to Scale

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- Second term often referred to as "pro-competitive effect" of trade.
- Notice that reducing markups is not per se beneficial (reducing distortion).
- Here, the allocation of labor to entry is potentially inefficient.
- If markups are too high, too many firms incentivized to enter.

Macro Increasing Returns to Scale

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- Mankiw and Whinston (1986) "business stealing" and non-appropriability externalities.
- Entrant generates consumer surplus in excess of revenues it captures, $\delta - 1$.
- Entrant causes all other firms to contract output, leading to loss in profits of $\mu - 1$.
- If $\delta > \mu$, increasing entry is beneficial, because additional consumer surplus dominates business stealing.

Macro Increasing Returns to Scale

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&= (\delta - 1)d\log L + (1-\rho)(\mu - \delta)d\log L.
\end{aligned}
$$

- Notice that the change in (measured) real GDP is

$$
d\log Q = (1-\rho)(\mu - 1)d\log L.
$$

Real GDP increases only if $\rho < 1$. Moreover, it increases even if $\mu < \delta$.

Macro Increasing Returns to Scale

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- How do macro returns to scale compare to micro returns to scale?
- Micro returns to scale is ratio of average cost to marginal cost $= (\mu-1)$.

Macro Increasing Returns to Scale

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&= (\delta - 1) d\log L + (1-\rho)(\mu - \delta) d\log L \\
&= \underbrace{(\mu - 1) d\log L}_{\text{Micro returns to scale}} - \rho(\mu - \delta) d\log L.
\end{aligned}$$

- How do macro returns to scale compare to micro returns to scale?
- Micro returns to scale is ratio of average cost to marginal cost $= (\mu - 1)$.

Trade Between Two Countries

- We considered a size in the labor force in a closed economy.
- Isomorphic to an economy of size L opening to trade with an infinitesimal country dL with identical preferences and technologies.
- Trade is balanced and varieties of similar goods.

Trade Between Two Countries

- We considered a size in the labor force in a closed economy.
- Isomorphic to an economy of size L opening to trade with an infinitesimal country dL with identical preferences and technologies.
- Trade is balanced and varieties of similar goods.
- Mundell (1957): trade and factor mobility are substitutes.
- If there were frictions to trade but not to migration, there would be an incentive for workers to move to the region which starts with a larger labor force.
- More populous regions offers greater real wage and variety of goods.
- In fact, with large starting population, can even dominate inferior technology.

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Aggregate Increasing Returns to Scale: Heterogeneous Firms

- What happens if we allow for realistic firm heterogeneity?
- Same model, now allow for different productivities A_θ.
- Will generate heterogeneous σ_θ, μ_θ, δ_θ.

Aggregate Increasing Returns to Scale: Heterogeneous Firms

- What happens if we allow for realistic firm heterogeneity?
- Same model, now allow for different productivities A_θ.
- Will generate heterogeneous σ_θ, μ_θ, δ_θ.
- Allow for a "selection margin" by having both entry costs and overhead costs.

$$\pi_\theta = \left(1 - \frac{1}{\mu_\theta}\right) p_\theta y_\theta L - f_o.$$

- Types θ ordered by profitability. Only types $\theta \geq \theta^*$ enter, where $\pi_{\theta^*} = 0$.
- Free entry determines mass of entrants (who learn productivity after entering),

$$\int_{\theta^*}^{1} \pi_\theta d\theta \geq f_e.$$

Social Inefficiency

- Three margins of inefficiency: relative size, entry, selection.
- Excessive relative size θ' vs. θ iff:

$$\mu_{\theta'} < \mu_{\theta}.$$

- Excessive entry iff:

$$\mathbb{E}_{\lambda}[\delta_{\theta}] < \mathbb{E}_{\lambda}[\mu_{\theta}{}^{-1}]^{-1}.$$

- Excessive selection iff:

$$\delta_{\theta^*} > \mathbb{E}_{\lambda}[\delta_{\theta}].$$

  where $\mathbb{E}_{\lambda}[\cdot]$ is sales-weighted expectation.

Technical and Allocative Efficiency

- Welfare function:
$$Y = \mathcal{Y}(L, \mathcal{X}).$$
where $\mathcal{X}$ is share of labor allocated to each type and use.

- Technical and allocative efficiency:
$$d\log Y = \underbrace{\frac{\partial \log \mathcal{Y}}{\partial \log L} d\log L}_{\text{technical efficiency}} + \underbrace{\frac{\partial \log \mathcal{Y}}{\partial \mathcal{X}} d\mathcal{X}}_{\text{allocative efficiency}} .$$

Welfare

- Change in welfare per capita:

$$d \log Y = \underbrace{\left(\mathbb{E}_\lambda[\delta_\theta] - 1\right) d \log L}_{\text{technical efficiency}} + \underbrace{\left(\xi^{\varepsilon} + \xi^{\theta^*} + \xi^{\mu}\right) \bar{\mu}\, d \log L}_{\text{allocative efficiency}},$$

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where

$$\textit{Darwinian}: \xi^{\varepsilon} = \left(\mathbb{E}_\lambda[\delta_\theta] - 1\right) Cov_\lambda[\sigma_\theta, \mu_\theta^{-1}] > 0,$$

Welfare

- Change in welfare per capita:

$$d\log Y = \underbrace{\left(\mathbb{E}_{\lambda}[\delta_{\theta}] - 1\right) d\log L}_{\text{technical efficiency}} + \underbrace{\left(\xi^{\varepsilon} + \xi^{\theta^*} + \xi^{\mu}\right) \bar{\mu}\, d\log L}_{\text{allocative efficiency}},$$

where

$$\text{Darwinian}: \xi^{\varepsilon} = \left(\mathbb{E}_{\lambda}[\delta_{\theta}] - 1\right) Cov_{\lambda}[\sigma_{\theta}, \mu_{\theta}^{-1}] > 0,$$

$$+\ \text{Selection}: \xi^{\theta^*} = (\mathbb{E}_{\lambda}[\delta_{\theta}] - \delta_{\theta^*})\lambda_{\theta^*}\gamma_{\theta^*}\left(\mathbb{E}_{\lambda}\left[\frac{\sigma_{\theta^*}}{\sigma_{\theta}}\right] - 1\right) \lesseqgtr 0,$$

Welfare

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$$+\,\textit{Pro-competitive}: \xi^{\mu} = \mathbb{E}_\lambda\left[(1-\rho_\theta)\sigma_\theta\left(1 - \frac{\mathbb{E}_\lambda[\delta_\theta]}{\mu_\theta}\right)\right]\mathbb{E}_\lambda\left[\sigma_\theta^{-1}\right] \lesseqgtr 0.$$

Example I: Technical Efficiency Only

Suppose preferences are CES.

$$s_\theta\left(\frac{p_\theta}{P}\right) = \left(\frac{p_\theta}{P}\right)^{1-\sigma}.$$

Change in welfare per capita:

$$d\log Y = \underbrace{\left(\mathbb{E}_\lambda[\delta_\theta] - 1\right) d\log L}_{\text{technical efficiency}} + \underbrace{0}_{\text{allocative efficiency}},$$

$$= \underbrace{(\mu - 1)\, d\log L}_{\text{Micro returns to scale}}.$$

Example II: Darwinian Effect Only

Suppose zero overhead costs and preferences given by

$$s_\theta\left(\frac{p_\theta}{P}\right) = \left(\frac{p_\theta}{P}\right)^{1-\sigma_\theta}.$$

Change in welfare per capita:

$$d\log Y = \underbrace{\left(\mathbb{E}_\lambda[\delta_\theta] - 1\right) d\log L}_{\text{technical efficiency}} + \underbrace{(\xi^\varepsilon)\,\bar{\mu}\, d\log L}_{\text{allocative efficiency}},$$

where

$$\textit{Darwinian}: \xi^\varepsilon = \left(\mathbb{E}_\lambda[\delta_\theta] - 1\right) Cov_\lambda[\sigma_\theta, \mu_\theta^{-1}] > 0.$$

High-μ firms insulated from $\downarrow$ price index due to low elasticity

$\Rightarrow$ market expansion alleviates cross-sectional misallocation.

Example III: Darwinian Effect + Selection Only

Same setup, but reintroduce overhead costs.

Change in welfare per capita:

$$d \log Y = \underbrace{\left(\mathbb{E}_\lambda[\delta_\theta] - 1\right) d \log L}_{\text{technical efficiency}} + \underbrace{\left(\xi^{\varepsilon} + \xi^{\theta^*}\right) \bar{\mu}\, d \log L}_{\text{allocative efficiency}},$$

where

$$\textit{Selection}: \xi^{\theta^*} = \left(\mathbb{E}_\lambda[\delta_\theta] - \delta_{\theta^*}\right)\lambda_{\theta^*}\gamma_{\theta^*}\left(\mathbb{E}_\lambda\left[\frac{\sigma_{\theta^*}}{\sigma_\theta}\right] - 1\right) \lesseqgtr 0.$$

Positive if market expansion increases selection cutoff ($\sigma_{\theta^*} > \mathbb{E}_\lambda[\sigma_\theta]$) and selection is too weak ($\delta_{\theta^*} < \mathbb{E}_\lambda[\delta_\theta]$).

Example IV: Pro-Competitive Effect Only

Suppose homogeneous firms (Krugman 1979 example).

Change in welfare per capita:

$$d \log Y = \underbrace{(\delta - 1) d \log L}_{\text{technical efficiency}} + \underbrace{\xi^{\mu} \mu \, d \log L}_{\text{allocative efficiency}},$$

where

$$\textit{Pro-competitive}: \xi^{\mu} = (1 - \rho)\left(1 - \frac{\delta}{\mu}\right) \lesseqgtr 0.$$

Positive if markups fall ($\rho < 1$) and entry is initially excessive ($\mu > \delta$).

Aggregate Markup

- Change in aggregate markup:

$$d\log\bar{\mu} = \left(\zeta^{\varepsilon} + \zeta^{\theta^*} + \zeta^{\mu}\right)\bar{\mu}\, d\log L,$$

where

$$\begin{aligned}
&\textit{Darwinian}: \zeta^{\varepsilon} = (\bar{\mu}-1)Cov_{\lambda}\left[\sigma_{\theta}\mu_{\theta}^{-1}\right] \geq 0,\\
&+\textit{Selection}: \zeta^{\theta^*} = \lambda_{\theta^*}\gamma_{\theta^*}\left(\frac{\bar{\mu}}{\mu_{\theta^*}} - 1\right)\left(\mathbb{E}_{\lambda}\left[\frac{\sigma_{\theta^*}}{\sigma_{\theta}}\right] - 1\right),\\
&+\textit{Pro-competitive}: \zeta^{\mu} = -\mathbb{E}_{\lambda}\left[\frac{\bar{\mu}-1}{\sigma_{\theta}}\right]\mathbb{E}_{\lambda}[(\sigma_{\theta}-1)(1-\rho_{\theta})] \leq 0.
\end{aligned}$$

Measured Real GDP

- Change in real GDP,

$$d \log Q = -\mathbb{E}_\lambda [d \log p_\theta] = \mathbb{E}_\lambda [1 - \rho_\theta] \mathbb{E}_\lambda \left[\frac{1}{\sigma_\theta} \right] \bar{\mu} \, d \log L.$$

- Real GDP only increases if $\rho_\theta < 1$.
- If $\rho_\theta = 1$, real GDP per capita is invariant to market size, even though welfare is increasing.

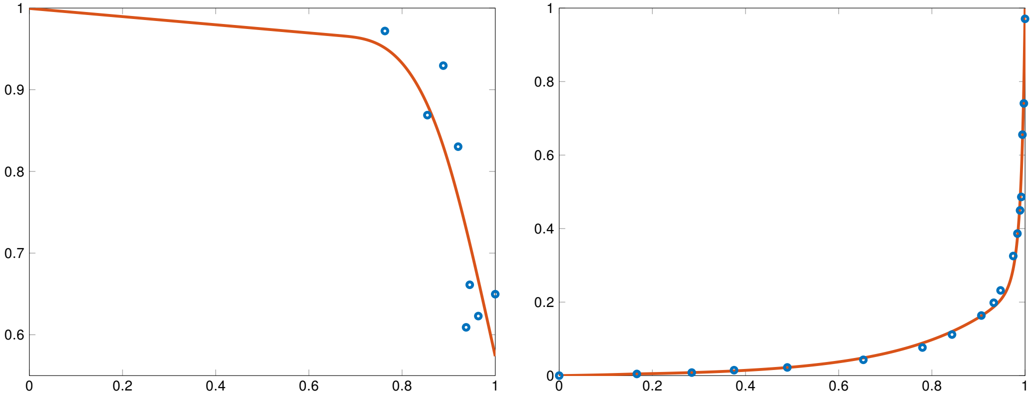
Non-Parametric Calibration

- Will not use off-the-shelf functional form for s_θ.
- Belgian data for manufacturing firms.
- Sales and pass-throughs by firm size for ProdCom sub-sample (price and quantity data) from Amiti et al. (19).
- Can back out primitives as solution to ODEs
(up to $\bar{\delta}$ and $\bar{\mu}$).

Non-Parametric Calibration

- Inputs:
  - λ_θ, ρ_θ (data);
  - $\bar{\mu} = 1/[\mathbb{E}_\lambda[1/\mu_\theta]]$ and $\bar{\delta} = \mathbb{E}_\lambda[\delta_\theta]$ (postulates).
- Outputs:
  - μ_θ, σ_θ, $A_\theta B_\theta$, δ_θ, γ_θ ;
  - f_e, f_o, $\Upsilon(\cdot)$.

Fitting to Data



(a) Cumulated average pass-through

(b) Cumulative sales share distribution.

Non-Parametric Calibration (Key Equations)

- Changes in λ_θ:

$$\frac{d \log \lambda_\theta}{d\theta} = \frac{\rho_\theta}{\mu_\theta - 1} \frac{d \log(A_\theta B_\theta)}{d\theta}.$$

- Changes in μ_θ:

$$\frac{d \log \mu_\theta}{d\theta} = (1 - \rho_\theta) \frac{d \log(A_\theta B_\theta)}{d\theta}.$$

Non-Parametric Calibration

- Recover μ_θ by solving:

$$\frac{d\log\mu_\theta}{d\theta} = \frac{(\mu_\theta - 1)(1-\rho_\theta)}{\rho_\theta}\frac{d\log\lambda_\theta}{d\theta} \quad \text{s.t.} \quad \mathbb{E}_\lambda[\mu_\theta^{-1}]^{-1} = \bar{\mu}.$$

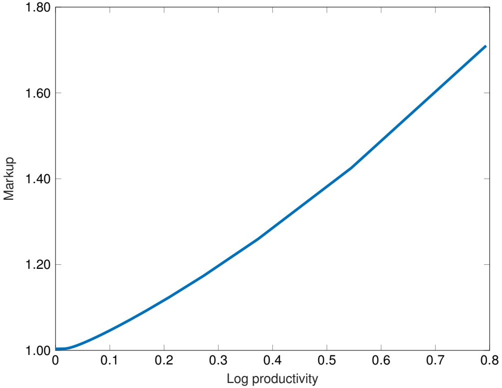
- Recover δ_θ by solving:

$$\frac{d\log\delta_\theta}{d\theta} = \frac{\mu_\theta - \delta_\theta}{\delta_\theta}\frac{d\log\lambda_\theta}{d\theta} \quad \text{s.t.} \quad \mathbb{E}_\lambda[\delta_\theta] = \bar{\delta}.$$

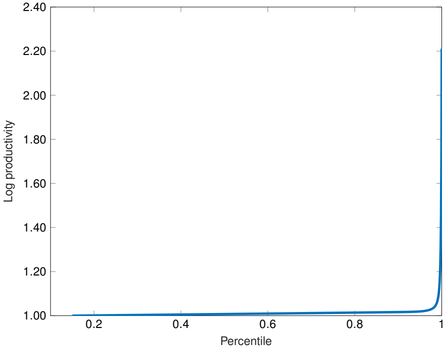
Postulates for Boundary Conditions

- Calibrate $\bar{\mu} = 1.09$.
- Take one of two values for $\bar{\delta}$:
  - $\bar{\delta} = \bar{\mu}$ (efficient entry);
  - $\bar{\delta} = \delta_{\theta^*}$ (efficient selection).

Estimates

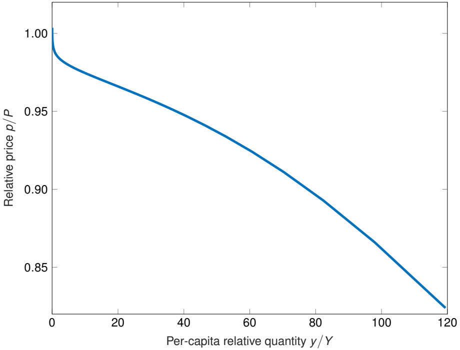


(a) Markup μ_θ ($\bar{\mu} = 1.09$)

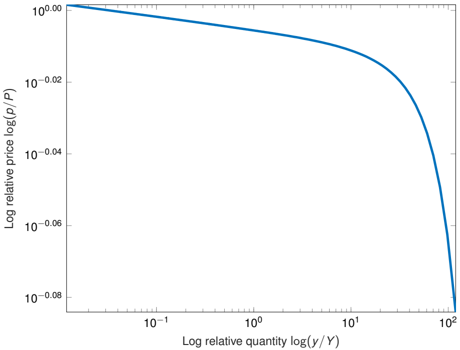


(b) Productivity $\log A_\theta$ ($\bar{\mu} = 1.09$)

Residual Demand Curve

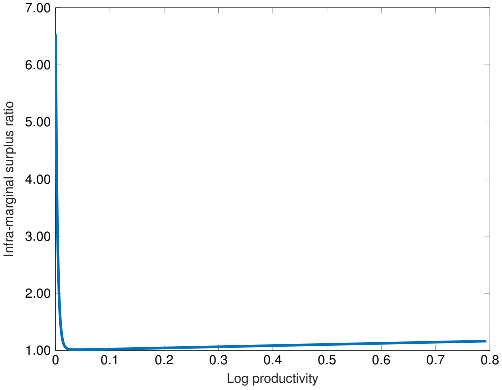


(a) Residual demand curve (efficient entry, $\bar{\mu} = 1.09$).

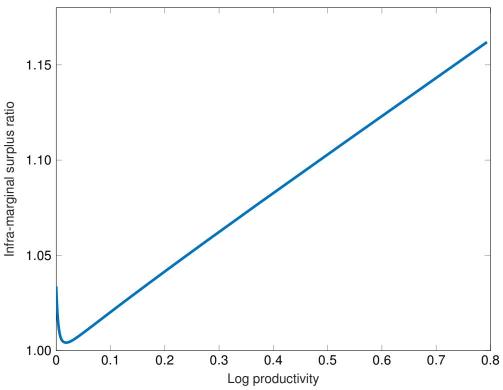


(b) Log-log residual demand curve (efficient entry, $\bar{\mu} = 1.09$).

Estimates (Efficient Selection vs. Efficient Entry)



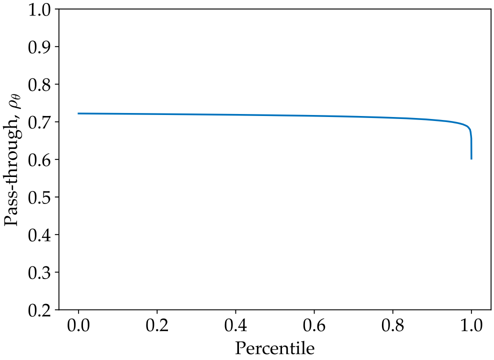
(a) Infra-marginal surplus ratio δ_θ (efficient selection, $\bar{\mu} = 1.045$).



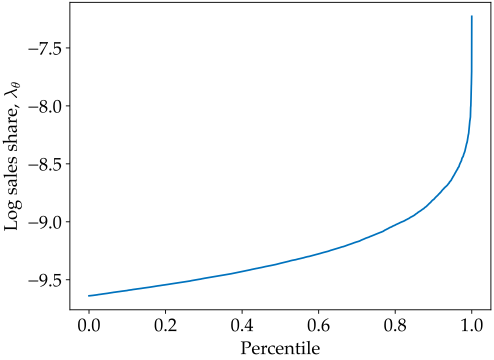
(b) Infra-marginal surplus ratio δ_θ (efficient entry, $\bar{\mu} = 1.045$).

Comparison to Klenow–Willis

- Other approach is to use off-the-shelf functional form, e.g., Klenow & Willis (2016).
- Standard calibration: Elasticity ($\sigma = 5$), "super-elasticity" ($\varepsilon = 1.6$), $A_\theta \sim$ Pareto(8).



(a) Pass-through ρ_θ.



(b) Log sales share λ_θ.

Counterfactual: 1% Population Shock

	$\bar{\mu} = 1.090$		
	$\bar{\delta} = \delta_{\theta^*}$	$\bar{\delta} = \bar{\mu}$	KW
Welfare	0.259	0.278	
Tech. effic.	0.033	0.090	
Alloc. effic.	0.225	0.188	

Counterfactual: 1% Population Shock

	$\bar{\mu} = 1.090$		
	$\bar{\delta} = \delta_{\theta^*}$	$\bar{\delta} = \bar{\mu}$	KW
Welfare	0.259	0.278	
Tech. effic.	0.033	0.090	
Alloc. effic.	0.225	0.188	
Darwinian effect	0.235	0.631	
Selection effect	0.000	-0.344	
Markups effect	-0.010	-0.099	
GDP per capita	0.043	0.043	
Aggregate markup	0.494	0.494	

Table: The elasticity of welfare and real GDP per capita to population with heterogeneous firms.

Counterfactual: 1% Population Shock

	$\bar{\mu} = 1.090$		
	$\bar{\delta} = \delta_{\theta^*}$	$\bar{\delta} = \bar{\mu}$	KW
Welfare	0.259	0.278	0.268
Tech. effic.	0.033	0.090	0.260
Alloc. effic.	0.225	0.188	0.008
Darwinian effect	0.235	0.631	0.009
Selection effect	0.000	-0.344	-0.000
Markups effect	-0.010	-0.099	-0.001
GDP per capita	0.043	0.043	
Aggregate markup	0.494	0.494	

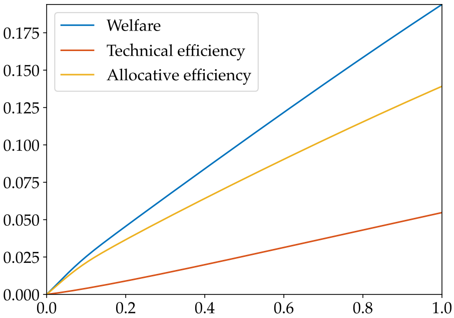
Table: The elasticity of welfare and real GDP per capita to population with heterogeneous firms.

Counterfactual: 1% Population Shock (Homogenous Firms)

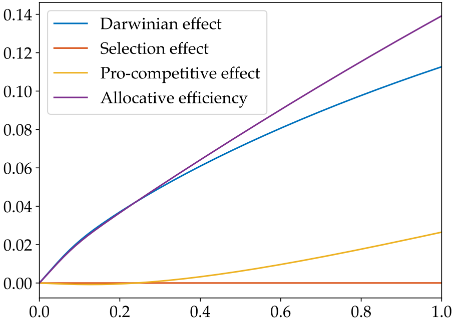
	$\bar{\mu} = 1.090$	
	$\bar{\delta} = \delta_{\theta^*}$	$\bar{\delta} = \bar{\mu}$
Welfare	0.061	0.090
Technical efficiency	0.033	0.090
Allocative efficiency	0.027	0.000
Real GDP per capita	0.043	0.043
Average markup	-0.043	-0.043

Table: The elasticity of welfare and real GDP per capita to population with homogenous firms.

Counterfactual: Nonlinearities

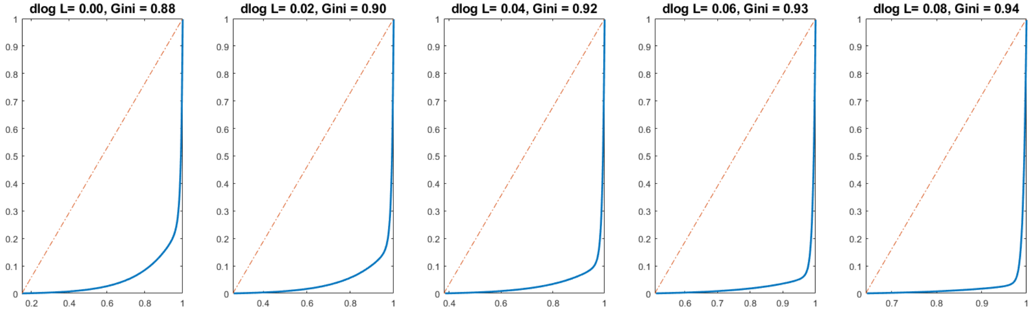


(a) Welfare: change in technical & allocative efficiency with $\Delta \log L$ (efficient selection, $\bar{\mu} = 1.09$).



(b) Darwinian, selection, and pro-competitive effects with $\Delta \log L$ (efficient selection, $\bar{\mu} = 1.09$).

Changes in Industrial Concentration



As market becomes larger, market is becoming more concentrated.

Counterfactual: Entry tax

	$\bar{\mu} = 1.090$	
$\bar{\delta} = \delta_{\theta^*}$	$\bar{\delta} = \bar{\mu}$	
Welfare	-0.155	-0.161
Darwinian effect	-0.215	-0.579
Other effects	0.060	0.418

Counterfactual: Entry tax

	$\bar{\mu} = 1.090$	
$\bar{\delta} = \delta_{\theta^*}$	$\bar{\delta} = \bar{\mu}$	
Welfare	-0.155	-0.161
Darwinian effect	-0.215	-0.579
Other effects	0.060	0.418
Welfare w/ homog. firms	0.027	0.000

Table: Change in welfare due to an entry tax.

- Entry subsidy spurs beneficial Darwinian reallocations.

Taking Stock

- With entry/exit of goods, changes in real GDP (measured using price changes of continuing varieties) need not coincide with changes in welfare.
- Non-appropriability: new products create consumer surplus $>$ sales.
- Business stealing: new products reduce output of other firms.
- Efficiency of entry depends on strength of these two margins.
- Due to composition effects, changes in aggregate markup do not necessarily align with changes in market competitiveness or welfare.