Lecture 14:
The Missing Intercept Problem

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ECON 416-1

Roadmap

- Aggregation and general equilibrium forces are at the heart of macroeconomics.
- E.g., Milton Friedman parable on giving one man money vs. giving every man money.
- In empirical measurement, partial vs. general equilibrium often recast in terms of a “missing intercept” problem.
- Today, we will discuss the problem and a few solutions.
- We will continue discussing more solutions next class (active area of research!).

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Potential outcomes framework

- Potential outcomes framework for regional data from Chodorow-Reich (2020).
- Let D_{it} be treatment (e.g., govt spending) in region i in period t. $\boldsymbol{D}_t = (D_{1t}, \ldots, D_{Nt})$.
- Let D_t^{agg} be aggregate treatment (e.g., nat'l monetary policy), which can depend on $\boldsymbol{D}_t$.
- Observed output in region i, Y_{it}, is given by

$$Y_{it} = \mathcal{Y}_{it}(\boldsymbol{D}_t, D_t^{\text{agg}}(\boldsymbol{D}_t)).$$

- Last class we discussed the canonical potential outcomes framework, in which D_{it} is binary and $\mathcal{Y}_{it}$ depends only on D_{it}. In that special case, we could write:

$$Y_{it} = \mathcal{Y}_{it}(D_{it}) = \mathcal{Y}_{it}(0) + D_{it}\left(\mathcal{Y}_{it}(1) - \mathcal{Y}_{it}(0)\right).$$

Potential outcomes framework: SUTVA

$$Y_{it} = \mathcal{Y}_{it}(\boldsymbol{D}_t, D_t^{\text{agg}}(\boldsymbol{D}_t)) \qquad \text{vs.} \qquad Y_{it} = \mathcal{Y}_{it}(D_{it}) = \mathcal{Y}_{it}(0) + D_{it}\left(\mathcal{Y}_{it}(1) - \mathcal{Y}_{it}(0)\right).$$

- Standard causal inference uses Stable Unit Treatment Value Assumption (SUTVA).
- Each unit's potential outcome depends only on its own treatment.
- E.g., giving medicine to Patient A does not affect health of Patient B.
- In our setting, outcomes explicitly depend on treatments to other units.
  - May depend directly on other units' treatments (e.g., effects on prices).
  - May depend on aggregate policy responses (e.g., national monetary policy).
- We will consider how violations of SUTVA affect estimates of treatment effects.

Aggregate causal effects

- Define effect on aggregate outcome of change to treatment in all regions,

$$\beta^{\text{agg}} = \sum_{j=1}^{N} \frac{\mathcal{Y}_{jt}(\boldsymbol{D}_t + \Delta 1, D_t^{\text{agg}}(\boldsymbol{D}_t + \Delta 1)) - \mathcal{Y}_{jt}(\boldsymbol{D}_t, D_t^{\text{agg}}(\boldsymbol{D}_t))}{N\Delta}.$$

  E.g., effect of increasing government spending in all regions by Δ on aggregate output.

- Compare to effect across regions of just changing treatment in region i,

$$\beta^{\text{all regions}} = \sum_{j=1}^{N} \frac{\mathcal{Y}_{jt}(\boldsymbol{D}_t + \Delta \iota_i, D_t^{\text{agg}}(\boldsymbol{D}_t + \Delta \iota_i)) - \mathcal{Y}_{jt}(\boldsymbol{D}_t, D_t^{\text{agg}}(\boldsymbol{D}_t))}{\Delta}.$$

  E.g., effect on aggregate output of just increasing government spending in i.

Aggregate causal effects

- Suppose treatment effects are linear and regions are symmetric, i.e.,

$$\mathcal{Y}_{it}(\boldsymbol{D}_t, D_t^{\text{agg}}) = \alpha + \delta D_{it} + \gamma \sum_{j \neq i}^{N} D_{jt} + \eta D_t^{\text{agg}}.$$

  If either

  - $\eta = 0$ (national treatment does not effect regional output), or
  - $D_t^{\text{agg}}(\boldsymbol{D}_t + \Delta 1) = D_t^{\text{agg}}(\boldsymbol{D}_t + \Delta \iota_i)$ (treating more regions does not affect nat'l treatment),

  then $\beta^{\text{agg}} = \beta^{\text{all regions}}$.

- Interpretation: $\beta^{\text{agg}} \neq \beta^{\text{all regions}}$ because of a "SUTVA-macro" violation.
  - Regional outcomes depend on an aggregate (macro) treatment.
  - Aggregate treatment changes when we treat all units compared to just treating one unit.

Aggregate causal effects

$$\mathcal{Y}_{it}(\boldsymbol{D}_t, D_t^{\text{agg}}) = \omega + \alpha D_{it} + \gamma \frac{1}{N-1} \sum_{j \neq i}^{N} D_{jt} + \eta D_t^{\text{agg}}, \qquad \text{and} \qquad D_t^{\text{agg}} = \psi + \phi \frac{1}{N} \sum_j D_{jt}.$$

$$\Rightarrow \mathcal{Y}_{jt}(\boldsymbol{D}_t, D_t^{\text{agg}}) = \omega + \alpha D_{jt} + \gamma \frac{1}{N-1} \sum_{k \neq j}^{N} D_{kt} + \eta \left(\psi + \phi \frac{1}{N} \sum_k D_{kt} \right).$$

- Change in outcomes due to treating all units vs. treating just unit i:

$$\begin{aligned}
\mathcal{Y}_{jt}(\boldsymbol{D}_t + \Delta 1, D_t^{\text{agg}}(\boldsymbol{D}_t + \Delta 1)) &= \mathcal{Y}_{jt}(\boldsymbol{D}_t, D_t^{\text{agg}}) + \alpha\Delta + \gamma\Delta + \eta\phi\Delta \\
\sum_j \mathcal{Y}_{jt}(\boldsymbol{D}_t + \Delta 1, D_t^{\text{agg}}(\boldsymbol{D}_t + \Delta 1)) &= \sum_j \mathcal{Y}_{jt}(\boldsymbol{D}_t, D_t^{\text{agg}}) + \alpha N\Delta + \gamma N\Delta + \eta\phi N\Delta. \\
\mathcal{Y}_{jt}(\boldsymbol{D}_t + \Delta\iota_i, D_t^{\text{agg}}(\boldsymbol{D}_t + \Delta\iota_i)) &= \mathcal{Y}_{jt}(\boldsymbol{D}_t, D_t^{\text{agg}}) + \alpha\Delta I_{ij} + \gamma\Delta(1 - I_{ij})/(N-1) + \eta\phi/N\Delta, \\
\sum_j \mathcal{Y}_{jt}(\boldsymbol{D}_t + \Delta\iota_i, D_t^{\text{agg}}(\boldsymbol{D}_t + \Delta\iota_i)) &= \sum_j \mathcal{Y}_{jt}(\boldsymbol{D}_t, D_t^{\text{agg}}) + \alpha\Delta + \gamma\Delta + \eta\phi/N\Delta.
\end{aligned}$$

$$\beta^{\text{agg}} = \alpha + \gamma + \eta\phi \qquad \text{vs.} \qquad \beta^{\text{all regions}} = \alpha + \gamma + \eta\phi/N.$$

Micro causal effects

- Define effect of changing treatment in region i on i's outcomes,

$$\beta^{\text{micro}} = \frac{\mathcal{Y}_{it}(\boldsymbol{D}_t + \Delta\iota_i, D_t^{\text{agg}}(\boldsymbol{D}_t + \Delta\iota_i)) - \mathcal{Y}_{it}(\boldsymbol{D}_t, D_t^{\text{agg}}(\boldsymbol{D}_t))}{\Delta}.$$

  E.g., effect on regional output of just increasing government spending in i.

- Same "experiment" as $\beta^{\text{all regions}}$, but now we are just counting effect on region i.

- Under prior assumptions,

$$\begin{aligned} \beta^{\text{micro}} &= \alpha + \eta\phi/N, \\ \beta^{\text{all regions}} &= \alpha + \gamma + \eta\phi/N, \\ \beta^{\text{agg}} &= \alpha + \gamma + \eta\phi. \end{aligned}$$

  $\beta^{\text{micro}} = \beta^{\text{all regions}}$ if $\gamma = 0$; otherwise, spillovers across regions lead them to differ.

Micro causal effects

- Finally, define difference-in-difference estimator

$$\beta^{\text{DiD}} = \frac{\left[Y_{it+1} - Y_{it}\right] - \frac{1}{N-1}\sum_{j\neq i}^{N}\left[Y_{jt+1} - Y_{jt}\right]}{\Delta}.$$

  Suppose potential outcomes function remains stable between t and $t+1$.

- Under prior assumptions,

$$\begin{aligned} \beta^{\text{DiD}} &= \frac{\alpha\Delta + \eta\phi\Delta - \frac{1}{N-1}\sum_{j\neq i}^{N}\left[\frac{1}{N-1}\gamma\Delta + \eta\phi\Delta\right]}{\Delta} \\ &= \alpha - \gamma/\left(N-1\right). \end{aligned}$$

- Given symmetric impact of aggregate treatment on regions, effect of change in aggregate treatment cancels.

- Control group also affected by spillovers γ.

Causal effects: Summary

$$
\begin{aligned}
\beta^{\text{DiD}} &= \alpha - \gamma/(N-1), \\
\beta^{\text{micro}} &= \alpha + \eta\phi/N, \\
\beta^{\text{all regions}} &= \alpha + \gamma + \eta\phi/N, \\
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1. As $N \to \infty$, $\beta^{\text{DiD}} \to \beta^{\text{micro}}$.
   - Spillovers γ on other regions and spillovers through aggregate policy $\eta\phi$.
   - These spillovers are violations of "SUTVA-micro": treating one unit affects others.
   - As $N \to \infty$, spillovers vanish, and diff-in-diff becomes consistent estimator of micro effect.

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   - These spillovers are violations of "SUTVA-micro": treating one unit affects others.
   - As $N \to \infty$, spillovers vanish, and diff-in-diff becomes consistent estimator of micro effect.
2. Even as $N \to \infty$, $\beta^{micro} \neq \beta^{\text{all regions}}$, because de minimis spillovers may aggregate into macroeconomically relevant magnitude.
3. $\beta^{\text{all regions}} \neq \beta^{\text{agg}}$ because of aggregate treatment response.

β^{DiD} vs. β^{micro}: Detail

- So far, we have compared β^{DiD} and β^{micro} under certain parametric assumptions.

- We can generalize (Chodorow-Reich, 2020). Suppose we instead define β^{micro} as

$$\beta^{\text{micro}} = \frac{\mathcal{Y}_{it}(\boldsymbol{D}_t + \Delta\iota_i, D_t^{\text{agg}}) - \mathcal{Y}_{it}(\boldsymbol{D}_t, D_t^{\text{agg}})}{\Delta}.$$

  i.e., hold aggregate treatment fixed.

- Assumptions: $\mathcal{Y}_i(\cdot)$ stable between t and $t+1$ and $\mathcal{Y}_i(\boldsymbol{D}_t, D_t^{\text{agg}}) = \mathcal{Y}_i(\boldsymbol{D}_t) + \nu(D_t^{\text{agg}})$.

$$\Rightarrow \beta^{\text{DiD}} - \beta^{\text{micro}} = -\frac{1}{\Delta(N-1)} \sum_{j \neq i}^{N} \left[\mathcal{Y}_j(\boldsymbol{D} + \Delta\iota_i, D_t^{\text{agg}}) - \mathcal{Y}_j(\boldsymbol{D}, D_t^{\text{agg}})\right].$$

  Only need that spillovers are *on average* equal to zero.

Other issues

- So far we have assumed treatment effects are linear and all regions are symmetric.
  - In practice, different units have different sizes.
  - Spillovers may also vary depending on e.g., distance between units.
  - Spillovers may depend on minimum / maximum treatment, not just average.
  - Aggregate treatment may also be more responsive to some units.
  - Effects of small and large local treatments may differ (concavity/convexity).
- These all make the connection between estimated DiD statistics and aggregate counterfactuals even more difficult!
- Some work has been done on each of these, but plenty more to do.

Example: Allocation of labor with production subsidies

- Suppose labor is supplied inelastically but is perfectly mobile across regions.
- Single producer in each region with revenue $p\tau_i L_i^{1-1/\gamma}$, where $\tau_i - 1$ are subsidies.
- In competitive labor market, allocation of labor is

$$L_i = \frac{\tau_i^\gamma}{\sum_{j=1}^N \tau_j^\gamma}, \qquad \Rightarrow \qquad \frac{d\log L_j}{d\log \tau_i} = \gamma I_{ij} - \frac{1}{N}\gamma.$$

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So, we find:

$$\beta^{\text{micro}} = \frac{N-1}{N}\gamma, \qquad \beta^{\text{DiD}} = \gamma, \qquad \beta^{\text{all regions}} = 0.$$

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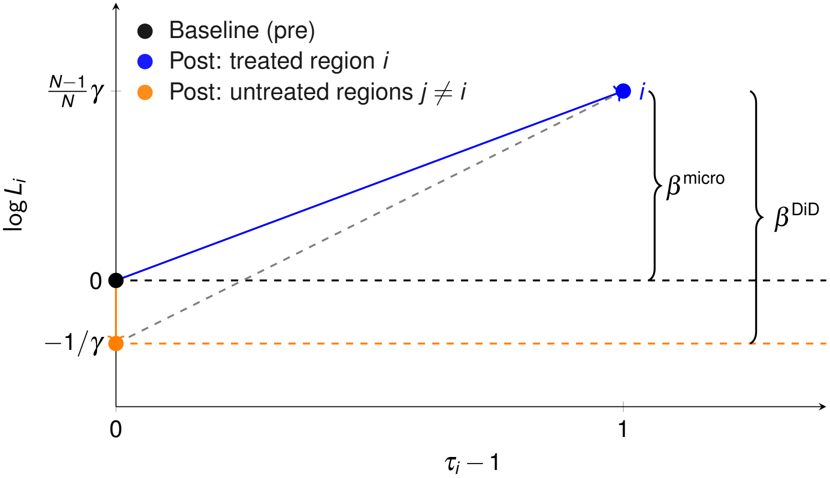
  So, we find:

$$\beta^{\text{micro}} = \frac{N-1}{N}\gamma, \qquad \beta^{\text{DiD}} = \gamma, \qquad \beta^{\text{all regions}} = 0.$$

- $\beta^{\text{DiD}} \to \beta^{\text{micro}}$ as $N \to \infty$, but both differ from $\beta^{\text{all regions}}$.
- In this case, $\beta^{\text{all regions}}$ coincides with β^{agg}, i.e., effect of raising all subsidies.

Missing intercept problem

- Visualizing the effect of production subsidy on labor across regions:



Missing intercept problem

- The "missing intercept problem" refers (somewhat confusingly) to:
  1. $\beta^{\text{DiD}} \neq \beta^{\text{micro}}$ due to violations of SUTVA-micro.
  2. $\beta^{\text{micro}} \neq \beta^{\text{all regions}}$ due to aggregation to economically relevant spillover.
  3. $\beta^{\text{all regions}} \neq \beta^{\text{agg}}$ due to "missing" response of aggregate policy (SUTVA-macro).
- In the plot on previous slide:
  1. Shift in intercept makes $\beta^{\text{DiD}} \neq \beta^{\text{micro}}$ if i is large.
  2. Even if shift in intercept is small, cumulating shift over $N-1$ units may be large.
  3. Response of aggregate policy may shift curve up / down.
- "Missing intercept" colloquially focuses most on (2) and (3).

Missing intercept problem

- Missing intercept often describes wedge between PE and GE effects.
- In macro, the missing intercept may be the most interesting part of problem!
- Keynes (1936) Paradox of Thrift:

$$S_i = s_i Y_i$$

$$Y_i = \bar{Y} + \frac{1}{N} \sum_j (1 - s_j) Y_j.$$

- In partial equilibrium (holding income fixed), $\frac{\partial \log S_i}{\partial \log s_i} = 1$.
- However, changing savings rate for all individuals changes intercept (income).

$$Y = \frac{\bar{Y}}{s} \qquad \text{so that} \qquad \frac{d \log S}{d \log s} = 0.$$

Missing intercept problem

- Keynes (1936): *"For although the amount of his own saving is unlikely to have any significant influence on his own income, the reactions of the amount of his consumption on the incomes of others makes it impossible for all individuals simultaneously to save any given sums. Every such attempt to save more by reducing consumption will so affect incomes that the attempt necessarily defeats itself."*

- Janet Yellen (2009), on Minsky: *"A process of balance sheet deleveraging has spread to nearly every corner of the economy. Consumers are pulling back on purchases, especially on durable goods, to build their savings. Businesses are cancelling planned investments and laying off workers to preserve cash. And, financial institutions are shrinking assets to bolster capital and improve their chances of weathering the current storm. Once again, Minsky understood this dynamic. He spoke of the paradox of deleveraging, in which precautions that may be smart for individuals and firms—and indeed essential to return the economy to a normal state—nevertheless magnify the distress of the economy as a whole."*

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Direct summation

- Comparing β^{micro} and $\beta^{\text{all regions}}$, the only difference is the sum across all units j:

$$\beta^{\text{all regions}} = \sum_{j=1}^{N} \frac{\mathcal{Y}_{jt}(\boldsymbol{D}_t + \Delta\iota_i, D_t^{\text{agg}}(\boldsymbol{D}_t + \Delta\iota_i)) - \mathcal{Y}_{jt}(\boldsymbol{D}_t, D_t^{\text{agg}}(\boldsymbol{D}_t))}{\Delta}.$$

$$\beta^{\text{micro}} = \frac{\mathcal{Y}_{it}(\boldsymbol{D}_t + \Delta\iota_i, D_t^{\text{agg}}(\boldsymbol{D}_t + \Delta\iota_i)) - \mathcal{Y}_{it}(\boldsymbol{D}_t, D_t^{\text{agg}}(\boldsymbol{D}_t))}{\Delta}.$$

- If you have an exogenous shock to the treatment of unit i, then why not just sum across all units' responses?

Direct summation

- Comparing β^{micro} and $\beta^{\text{all regions}}$, the only difference is the sum across all units j:

$$\beta^{\text{all regions}} = \sum_{j=1}^{N} \frac{\mathcal{Y}_{jt}(\boldsymbol{D}_t + \Delta\iota_i, D_t^{\text{agg}}(\boldsymbol{D}_t + \Delta\iota_i)) - \mathcal{Y}_{jt}(\boldsymbol{D}_t, D_t^{\text{agg}}(\boldsymbol{D}_t))}{\Delta}.$$

$$\beta^{\text{micro}} = \frac{\mathcal{Y}_{it}(\boldsymbol{D}_t + \Delta\iota_i, D_t^{\text{agg}}(\boldsymbol{D}_t + \Delta\iota_i)) - \mathcal{Y}_{it}(\boldsymbol{D}_t, D_t^{\text{agg}}(\boldsymbol{D}_t))}{\Delta}.$$

- If you have an exogenous shock to the treatment of unit i, then why not just sum across all units' responses?

- If using diff-in-diff, concern that other factors changing from t to $t+1$.

- Still, may be able to compare change from t to $t+1$ to other periods where you can argue there was no shock.

Direct summation: Example

(a) Patent 4,946,778 granted to Genex on Aug, 7 1990, "Single Polypeptide Chain Binding Molecules."

(c) Patent 6,317,722 granted to Amazon.com on Nov 13, 2001, "Use Of Electronic Shopping Carts To Generate Personal Recommendations."

- E.g., Kogan, Papanikolaou, Seru & Stoffman (2017) measure stock market reactions to patent grants.
- Aggregate effects on growth $\neq$ micro effects because of spillovers across firms.
- Positive tech spillovers, negative business stealing.
- Bloom, Schankerman & Van Reenen (2013) measure stock reactions for companies using same technology and in same product market.
- Presumably, one could also measure aggregate stock market movements around patent grants.

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Estimate effects at different levels of aggregation

- A related idea is to measure effects at different levels of aggregation.
- For example, if spillovers only operate within a geographic area, this is akin to summing over all units that have spillovers.
- Suppose we have regions $r = 1, \ldots, R$, each of which contain firms $i = 1, \ldots, N$.
- Suppose potential outcomes function given by

$$\mathcal{Y}_{irt}(\boldsymbol{D}_t, D_t^{\text{agg}}) = \alpha + \delta D_{irt} + \gamma_1 \sum_{j \neq i}^{N} D_{jrt} + \gamma_2 \sum_{s \neq r} \sum_{j=1}^{N} D_{jst}.$$

  - $\gamma_1 \gg \gamma_2$: stronger spillovers within region r compared to outside region.
  - Silence aggregate treatment for this example.

Estimate effects at different levels of aggregation

- Under these assumptions, micro / regional / aggregate causal effects:

$$\beta^{\text{micro}} = \frac{\mathcal{Y}_{irt}(\boldsymbol{D}_t + \Delta\iota_i) - \mathcal{Y}_{irt}(\boldsymbol{D}_t)}{\Delta} = \delta,$$

$$\beta^{\text{regional},i} = \sum_{j=1}^{N} \frac{\mathcal{Y}_{jrt}(\boldsymbol{D}_t + \Delta\iota_i) - \mathcal{Y}_{jrt}(\boldsymbol{D}_t)}{\Delta} = \delta + (N-1)\gamma_1,$$

$$\beta^{\text{regional, all}} = \sum_{j=1}^{N} \frac{\mathcal{Y}_{jrt}(\boldsymbol{D}_t + \Delta 1_r) - \mathcal{Y}_{jrt}(\boldsymbol{D}_t)}{N\Delta} = \delta + (N-1)\gamma_1,$$

$$\beta^{\text{agg}} = \sum_{r=1}^{R}\sum_{j=1}^{N} \frac{\mathcal{Y}_{jrt}(\boldsymbol{D}_t + \Delta 1) - \mathcal{Y}_{jrt}(\boldsymbol{D}_t)}{NR\Delta} = \delta + (N-1)\gamma_1 + (R-1)(N-1)\gamma_2.$$

- Notice that we could have looked at aggregate response of region r to one firm's shock, or taken aggregate response of region r to aggregate region shock.

- $\beta^{\text{regional}} \to \beta^{\text{agg}}$ as $\gamma_2 \to 0$.

Estimate effects at different levels of aggregation: Example

TABLE 3. Firm-level elasticity of marginal costs to changes in quantities

	Baseline		Excl. labor	
	$\Delta \log mc_{it}$	$\Delta \log \mathcal{C}_{it}$	$\Delta \log mc_{it}$	$\Delta \log \mathcal{C}_{it}$
	(1)	(2)	(3)	(4)
$\Delta \log y_{it}$	0.168**	1.082***	0.256***	1.182***
	(0.076)	(0.075)	(0.082)	(0.081)
Year × Ind. FE	✓	✓	✓	✓
Observations	267,011	267,011	267,010	267,010
F-Stat	171.91	171.91	171.91	171.91
Returns to scale	0.86	0.92	0.80	0.85

Note: This table reports the results of estimating equation (16). It report IV results with the instrument defined in (17). Columns (1) and (3) use the change in marginal costs as outcome variable; columns (2) and (4) use the change in variable costs as outcome variable. In columns (1) and (2), variable costs are materials and labor, while columns (3) and (4) exclude labor from variable costs. Regressions are weighted by firm-level lagged sales (top and bottom 1% winsorized). Standard errors are clustered at the firm level. ***, ** and * indicate significance at the 1%, 5% and 10% levels, respectively. The last row in the Table presents our estimates for the returns to scale parameter *a*.

- Herreño, Pinardon-Touati, and Thie (2025) estimate slope of Phillips curve in India.
- Key parameter is elasticity of aggregate marginal costs to output gap.
- They begin by estimating elasticity of firm-level marginal costs to output $\approx$ 0.08–0.26.

Estimate effects at different levels of aggregation: Example

TABLE 4. Aggregate elasticity of marginal costs to changes in quantities

	District		Industry	
	$\Delta \log \text{mc}$	$\Delta \log \mathcal{C}$	$\Delta \log mc$	$\Delta \log \mathcal{C}$
$\Delta \log y$	0.583***	1.487***	0.703**	1.704***
	(0.144)	(0.140)	(0.310)	(0.311)
Year FE	✓	✓	✓	✓
F Stat	103.08	103.08	26.83	26.83
N	7,707	7,707	1,211	1,211

Note: This table reports the results of estimating the aggregated version of equation 16. Columns (1) and (2) show district-level results with the instrument defined in (19). Columns (3) and (4) show industry-level results with the instrument defined in (20). Columns (1) and (3) use the change in marginal costs as outcome variable; columns (2) and (4) use the change in variable costs as outcome variable. Regressions are weighted by district(industry)-level lagged sales (top 1% winsorized). ***, ** and * indicate significance at the 1%, 5% and 10% levels, respectively.

- Aggregating at the district or industry level, elasticity of marginal costs to output is much higher, 0.49–0.70!
- Missing intercept: how input prices change with aggregate output.

Estimate effects at different levels of aggregation: Example

TABLE E.10. Coefficients for input-level prices

	Industry (1)	District (2)	Max (3)
Labor	0.29***	0.78***	0.78
Material (non-energy)	0.51***	0.42***	0.51
Energy	-0.03	-0.18	-0.03

Note: This table reports the results of estimating the aggregated version of equation 16. Column (1) shows industry-level results with the instrument defined in (20). Column (2) shows district-level results with the instrument defined in (19). Column (3) shows the maximum of columns (1) and (2). The outcome variables are the log change in the district-level wage (line 1) non-energy materials (line 2) and energy (line 3). Regressions are weighted by district(industry)-level lagged sales (top 1% winsorized). ***, ** and * indicate significance at the 1%, 5% and 10% levels, respectively.

- Can directly observe response of labor and material input prices to aggregate output at industry/district level.
- These factor price changes absorbed in Year $\times$ Industry FEs in firm specification.
- Energy prices don't respond, because close to national market.
- What does this imply for nationwide elasticity of marginal costs to output?

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Estimate spillovers directly

- What if we try to measure spillovers across units directly?
- For each unit $i = 1, ..., N$, estimate

$$Y_i = \beta_0 + \beta_1 D_i + \sum_{j \neq i} \delta_{ij} D_j + \varepsilon_{ij}.$$

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$1 + (N-1) \times N$ parameters, N observations.

- What if we assume unit j has same effect on all other units:

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- What if we assume unit j has same effect on all other units:

$$Y_i = \beta_0 + \beta_1 D_i + \sum_{j \neq i} \delta_j D_j + \varepsilon_{ij}.$$

  Still $N+1$ parameters and N observations.

- Clearly, we need some structure.

Estimate spillovers directly

- Oftentimes, researchers solve this problem using average treatment among neighbors.
- Using notation from example with R regions and N firms per region:
$$Y_{ir} = \beta_0 + \beta_1 D_{ir} + \delta \frac{1}{N} \sum_{j \neq i} D_{jr} + \varepsilon_{ij}.$$
  Now just two parameters with RN observations.
- Worth considering whether your problem indicates a different summary across units.
  - For example, perhaps firms learn from their most productive peers, not average peer.
  - Maybe teenagers are influenced by their worst-behaving friend, not the average.
- E.g., consider whether spillovers heterogeneous in treatment or firm characteristics:
$$Y_{ir} = \beta_0 + \beta_1 D_{ir} + \frac{1}{N} \sum_{j \neq i} \left(\delta_0 D_{jr} + \delta_1 D_{jr}^2 + \delta_2 \left(D_{jr} \times x_{jr} \right) \right) + \varepsilon_{ij}.$$

Estimate spillovers directly: Distance function

- Another option is to estimate spillovers using a continuous distance function:

$$Y_i = \beta_0 + \beta_1 D_i + \gamma \sum_j f\left(d(i,j)\right) D_j + \varepsilon_{ij}.$$

  where $d(i,j)$ is the distance between observations i and j and f is decreasing.

- Distance measure $d(i,j)$ should be theoretically grounded.

- Examples in spatial literature, IO literature (e.g., where $d(i,j)$ measures differences in product attributes).

Estimate spillovers directly: Distance function

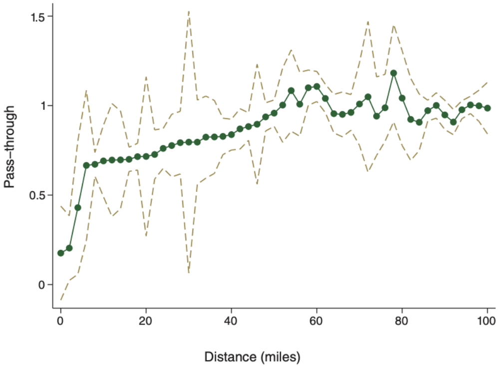

FIGURE 2. LOCAL LINEAR REGRESSION ESTIMATES OF THE EFFECT OF EXCISE TAXES ON CIGARETTE PRICES BY DISTANCE TO A LOWER-TAX BORDER

Notes: Each point represents the cigarette tax coefficient estimate from a local linear regression with a window of ten miles using a triangular kernel. The dotted lines show the bounds of the 95 percent confidence interval, which is calculated using standard errors that are clustered at the census tract level. The estimates include UPC, state and month fixed effects as well as the demographic characteristics shown in Table 2.

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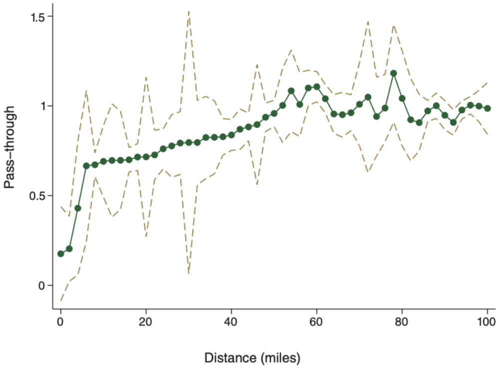
Estimate spillovers directly: Distance function



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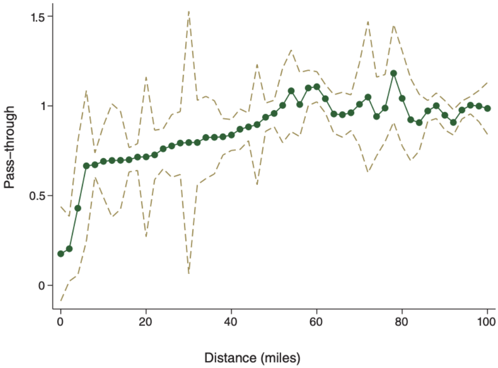
- Harding, Leibtag, and Lovenheim (2012) show that response to tax change depends on distance to untreated stores at border.
- Given specification,

$$Y_i = \beta_0 + \beta_1 D_i + \gamma \sum_j f\left(d(i,j)\right) D_j + \varepsilon_{ij},$$

assuming $f(\cdot)$ properly normalized,

  - $\beta_1 + \gamma \approx 1$,

Estimate spillovers directly: Distance function



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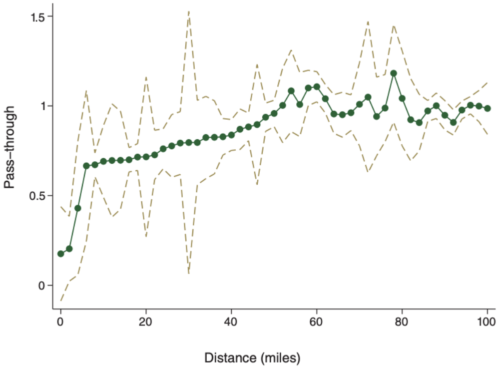

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  - Clearly this also requires some nonlinear aggregation of competitors!

Table of Contents

Huber (2018): Lender shock

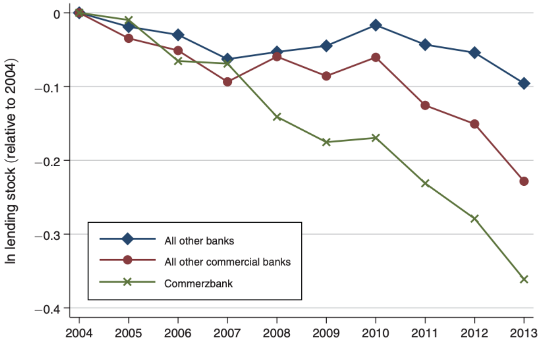

Figure 1. The Lending Stock of German Banks

Notes: This figure plots the ln lending stock to German non-financial customers, relative to the year 2004, in 2010 billions of euros. The data for Commerzbank include lending by branches of Commerzbank and Dresdner Bank. I sum their lending stock for the years before the 2009 take-over, using data from the annual reports. For all other banks, I use aggregated data from the Deutsche Bundesbank on German banks and subtract lending by Commerzbank. For all other commercial banks, I subtract lending by Commerzbank, the savings banks, the Landesbanken, and the cooperative banks.

- Huber (2018) illustrates many of these techniques.
- Studies lending cut by Commerzbank in Germany.
- Unlike other economies, Germany had no housing boom-bust, banking panic, or sovereign debt crisis.

Huber (2018): Lender shock

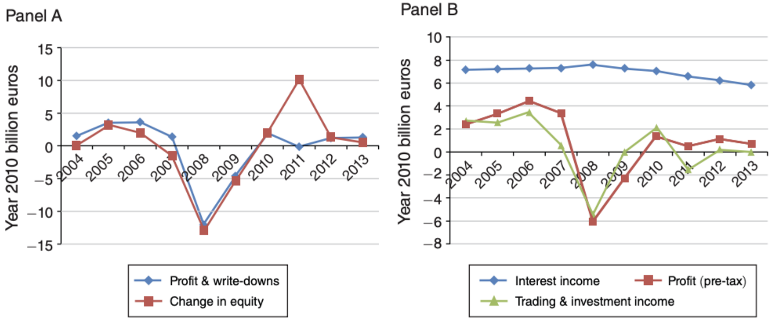

FIGURE 2. COMMERZBANK'S EQUITY CAPITAL, WRITE-DOWNS, AND PROFITS

- Trading losses from outside loan portfolio, from exposure to US asset-backed securities and large Icelandic banks.
- Basel II requirements (4% of risk-weighted assets in equity) and higher cost of external funding requires retrenching loan book.

Huber (2018): Lender shock

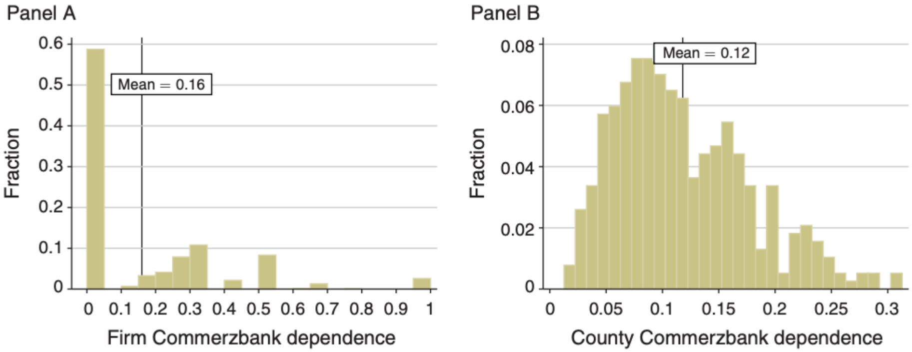

FIGURE 3. FIRM AND COUNTY COMMERZBANK DEPENDENCE

Note: The figure shows histograms of firm Commerzbank dependence for the 2,011 firms in the firm panel (panel A) and of county Commerzbank dependence for the 385 counties in the dataset (panel B).

- Firm dependence $CB\,dep_{fc}$ defined as number of relationship banks that are subsidiaries of Commerzbank / total # of relationship banks.

Huber (2018): First stage

TABLE 3—FIRM SURVEY ON BANKS' WILLINGNESS TO GRANT LOANS

	2007 (1)	2008 (2)	2009 (3)	2010 (4)	2011 (5)	2012 (6)
Firm *CB dep*	−0.111	−0.095	−0.473	−0.316	0.059	0.379
	(0.157)	(0.140)	(0.190)	(0.182)	(0.197)	(0.184)
Dep. var. from 2006	0.631	0.522	0.380	0.365	0.335	0.206
	(0.041)	(0.047)	(0.051)	(0.055)	(0.055)	(0.050)
Observations	856	988	1,032	946	898	503
R^2	0.460	0.371	0.204	0.213	0.207	0.199
Industry fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
State fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Size bin fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
ln age	Yes	Yes	Yes	Yes	Yes	Yes

Notes: This table reports estimates from cross-sectional firm regressions for different years. The outcome variable is the answer to the question: "How do you evaluate the current willingness of banks to grant loans to businesses: cooperative (coded as 1), normal (0), or restrictive (−1)?" It is standardized to have zero mean and unit variance. The coefficients are interpreted as the standard deviation increase in banks' willingness to grant loans from increasing Commerzbank dependence by one. The control variables include fixed effects for 36 industries, 16 federal states, 4 size bins (1–49, 50–249, 250–999, and over 1,000 employees in the year 2006), and the ln of firm age. Standard errors are clustered at the level of the county.

- Characteristics of firms borrowing from Commerzbank not appreciably different.
- Also did not report borrowing trouble before 2009 or after 2010.

Huber (2018): Effects on firm employment

TABLE 6—FIRM EMPLOYMENT AND COMMERZBANK DEPENDENCE

	(1)	(2)	(3)	(4)	(5)
Firm *CB dep* $\times$ *d*	−0.044	−0.047	−0.053		
	(0.021)	(0.016)	(0.015)		
Low bank debt dep. $\times$ firm *CB dep* $\times$ *d*				−0.035	
				(0.032)	
High bank debt dep. $\times$ firm *CB dep* $\times$ *d*				−0.071	
				(0.020)	
$(0 < \text{firm } CB\ dep \leq 0.25) \times d$					0.007
					(0.016)
$(0.25 < \text{firm } CB\ dep \leq 0.5) \times d$					−0.017
					(0.008)
$(0.5 < \text{firm } CB\ dep \leq 1) \times d$					−0.065
					(0.018)
Observations	12,066	12,066	12,066	12,066	12,066
R^2	0.026	0.098	0.124	0.125	0.125
Number of firms	2,011	2,011	2,011	2,011	2,011
Firm fixed effects	Yes	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes	Yes
County fixed effects $\times$ *d*	No	Yes	Yes	Yes	Yes
Size bin fixed effects $\times$ *d*	No	Yes	Yes	Yes	Yes
ln age $\times$ *d*	No	Yes	Yes	Yes	Yes
Industry fixed effects $\times$ *d*	No	No	Yes	Yes	Yes
Import and export share $\times$ *d*	No	No	Yes	Yes	Yes

Notes: This table reports estimates from firm OLS panel regressions. The outcome in all columns is firm ln employment. Firms with low (high) bank debt dependence have up to (over) 50 percent of their liabilities with banks. The control variables, the standard error calculations, the years covered by the data, and the definition of R^2 are explained in Table 4.

- Bank debt of CB-dependent firms dropped 17–21% in 2009–2012.
- About one-for-one with 17% decline in CB's aggregate lending stock.
- Year and county $\times$ *d* fixed effects mean we are comparing firms with different CB dependence within county.

Huber (2018): Effects on firm employment

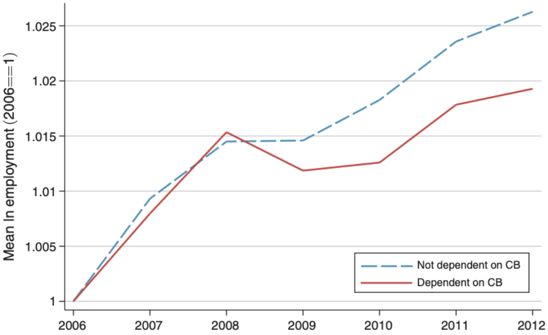

FIGURE 4. FIRM EMPLOYMENT EFFECTS

Notes: This figure plots the time series of the mean ln employment of firms with and without Commerzbank as one of their relationship banks. The time series are divided by their 2006 value. The data are from the firm panel.

- Parallel trends for firms with low and high CB dependence before shock.
- Employment also plateaued for firms without CB dependence after 2008.
- Is some of decline due to indirect effects from firms affected by lending cut?

Huber (2018): Effects on county employment

TABLE 8—COUNTY OUTCOMES AND COMMERZBANK DEPENDENCE (OLS)

Outcome:	GDP (1)	GDP (2)	GDP (3)	Empl. (4)	Net migr. (5)
County *CB dep* × *d*	−0.132	−0.165	−0.141	−0.138	0.003
	(0.063)	(0.066)	(0.077)	(0.042)	(0.006)
Observations	5,005	5,005	5,005	5,005	1,925
R^2	0.301	0.341	0.350	0.494	0.592
Number of counties	385	385	385	385	385
County fixed effects	Yes	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes	Yes
Former GDR fixed effects × *d*	No	Yes	Yes	Yes	Yes
Industry shares × *d*	No	Yes	Yes	Yes	Yes
Export and import shares × *d*	No	Yes	Yes	Yes	Yes
Landesbank in crisis × *d*	No	Yes	Yes	Yes	Yes
Population × *d*	No	No	Yes	No	No
Population density × *d*	No	No	Yes	No	No
GDP per capita × *d*	No	No	Yes	No	No
Debt index × *d*	No	No	Yes	No	No
Estimator	OLS	OLS	OLS	OLS	OLS

Notes: This table reports estimates from county OLS panel regressions of county outcomes on Commerzbank dependence (*CB dep*) interacted with *d*, a dummy for the years following the lending cut, 2009 to 2012. The outcome in columns 1 to 3 is ln GDP, in column 4 ln employment, and in column 5 net migration (immigration − out-migration) normalized by 2006 employment. The industry shares are 17 variables, giving the fraction of firms in each of the 17 industries in 2006 (agriculture, mining, manufacturing, utilities, recycling, construction, retail trade and vehicle repairs, transportation and storage, hospitality, information, finance, real estate, business services, other services, public sector, education, health). The export share is the fraction of exports out of total revenue and the import share is the fraction of imports out of total costs, both averaged across firms in the county for 2006. Landesbank in crisis is a dummy for whether the county's Landesbank suffered losses in the financial crisis. Population density, total population (ln), and GDP per capita (ln) are from 2000. Debt index is a 2003 measure of county household leverage, calculated by credit rating agency Schufa (Privatverschuldungsindex). The regressions are weighted by year 2000 population. Standard errors are clustered at the level of 42 quantiles of the county's industrial production share (GDP share of mining, manufacturing, utilities, recycling, construction). The GDP and employment data include the years 2000 to 2012. Migration data for all counties are only available for the years 2008 to 2012. R^2 is the within-county R^2.

- Diff-in-diff at county level.
- Rules out effects on household debt and migration out of county.
- Do counties with high and low CB dependence differ?
- IV using county distance to nearest office (Dusseldorf, Frankfurt, or Hamburg) set up after WWII.
- Can control for distance to each office.

Huber (2018): Spillovers

TABLE 10—THE DIRECT AND INDIRECT EFFECTS ON FIRM EMPLOYMENT GROWTH

	(1)	(2)
Firm *CB dep*	−0.030	−0.036
	(0.009)	(0.009)
CB dep of other firms in county	−0.166	−0.170
	(0.076)	(0.082)
Observations	48,101	48,101
R^2	0.012	0.017
Firm controls	Yes	Yes
County controls	No	Yes

Notes: This table reports estimates from cross-sectional firm OLS regressions. The outcome is the symmetric growth rate of firm employment from 2008 to 2012. *CB dep* of other firms in county is the average firm Commerzbank dependence of all the other firms in the county. The firm control variables are the same as in Table 4, except there are no county fixed effects. The county controls and the standard error calculations are the same as in Table 8.

- Alternative approach: Measure spillovers in panel using leave-one-out measure of county CB exposure.
- Indirect effects 5x as large as direct effects!

Huber (2018): Spillovers

TABLE 11—THE IMPLIED COUNTY EMPLOYMENT CHANGE BASED ON DIFFERENT ESTIMATES

	Estimate from section	Estimator	Dataset	Estimated effect	Point estimate	95 percent CI	
						Lower	Upper
1.	IVB	OLS	Firm panel	Only direct	−0.32	−0.49	−0.14
2.	VB	OLS	County panel	Direct & indirect	−0.83	−1.31	−0.34
3.	VC	IV	County panel	Direct & indirect	−1.25	−2.58	−0.09
4.	VIA	OLS	Firm cross section	Direct & indirect	−1.24	−2.17	−0.29

Notes: This table reports different estimates of the county employment loss from increasing county Commerzbank dependence by a standard deviation (6 percentage points). Row 1 uses the estimate of the direct effect from column 3 of Table 6. Row 2 uses the county OLS estimate from Table 8, column 4. Row 3 uses the county IV estimate from Table 9, column 6. Row 4 uses the sum of direct and indirect effects from column 2 of Table 10.

- Both approaches (estimating effect in county-aggregated data and estimating spillovers directly) yield similar estimates.

Huber (2018): Heterogeneity in spillovers

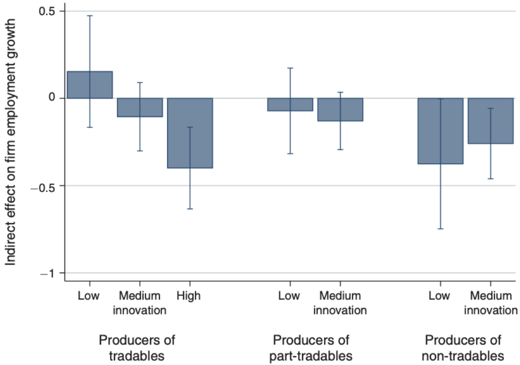

FIGURE 7. THE SIZE OF THE INDIRECT EFFECT BY INDUSTRY TYPE

Notes: This figure illustrates heterogeneity in the indirect effect by industry type. The plotted point estimates are the effect of the Commerzbank dependence of all other firms in the county on the symmetric growth rate of firm employment between 2008 and 2012. The estimates are from a single regression that controls for the firm's direct Commerzbank dependence and the other control variables from Table 10. The vertical lines are 90 percent confidence intervals.

- What mechanisms underlie indirect effects?
- 1. Direct effects reduce local agglomeration spillovers?
  - $\Rightarrow$ High innovation industries particularly exposed.
- 2. Employment losses reduced household demand?
  - $\Rightarrow$ Nontradables particularly exposed.

Summary

- Questions of interest in macro naturally lend themselves to SUTVA violations.
  - SUTVA-micro violations: Spillovers on other units (e.g., migration, prices).
  - Aggregation: Even de minimis spillovers can be economically relevant in aggregate.
  - SUTVA-macro violations: Response of aggregate policies (e.g., monetary/fiscal policy).
- Sometimes we want to ensure that GE effects do not dampen or eliminate an effect (e.g., production subsidies).
- Other times, we might be interested in GE effects themselves! (Paradox of thrift, economically large spillovers of lending cuts to other firms).
- Next time: More issues in measuring spillovers, more techniques for estimating GE.