Lecture 2:
The Aggregate Production Function

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ECON 416-1

Agenda

1. Aggregate production functions.

   (Baqaee & Farhi, JEEA 2019)

2. Disaggregation and Hulten's Theorem.

   (Hulten 1973)

3. Applications of Hulten's Theorem.

   (Gabaix 2011; Acemoglu et al. 2012)

4. Nonlinearities.

   (Baqaee & Farhi, ECMA 2019)

Aggregate production functions

- Aggregate production functions at the heart of modern macroeconomics.
- Solow (1956) on growth accounting,

$$Y = AF(K, L).$$

- Katz and Murphy (1992) on the skill premium,

$$Y = F(H, L).$$

- Macro phenomena depend critically on the properties of these aggregate functions:
  - Convergence depends on returns to scale of F.
  - Labor share depends on elasticity between labor and capital.
  - Skill premium depends on elasticity between high- and low-skilled labor.

The Cambridge-Cambridge Controversy

- Are these aggregate production functions a good description of the economy?
- In 1950-60s, Joan Robinson, Pierro Sraffa emphatically argued: No!
    - Debate pitting Robinson, Sraffa, Pasinetti vs. Solow, Samuelson, Hahn, ...
- Robinson (1953): *"The production function has been a powerful instrument of miseducation. The student of economic theory is taught to write $Q = f(L, K)$ where L is a quantity of labor, K a quantity of capital and Q a rate of output of commodities. He is instructed to assume all workers alike, and to measure L in man-hours of labor; he is told something about the index-number problem in choosing a unit of output; and then he is hurried on to the next question, in the hope that he will forget to ask in what units K is measured. Before he ever does ask, he has become a professor, and so sloppy habits of thought are handed on from one generation to the next."*

The Cambridge-Cambridge Controversy

- Samuelson (1966) summed up "parables" of the neoclassical treatment of capital:
  - The interest rate is equal to the marginal product of capital, $r = F_k$.
  - Diminishing returns: As K/Y, K/L increase, r falls.
  - Substitution: As the interest rate falls, the capital-labor ratio K/L increases.
  - Income shares: Distribution of income determined by relative scarcity of K vs. L.
- Depend on interpreting capital as a physical quantity, even though in reality heterogeneous capital goods cannot be aggregated in physical units (must be aggregated in valuation units).
  - Samuelson (1962): *"Suppose labor and a homogeneous capital jelly (physical not dollar jelly!) produce a flow of homogeneous net national product, which can consist of consumption goods or of net capital (i.e., jelly) formation, the two being infinitely substitutable. [...] Such a Ramsey model, if it held, could justify all of Solow's statistical manipulations with full rigor."*

Breaking the parables: Re-switching / capital reversing

- Re-switching / capital reversing model (Hayek, Böhm-Bawerk) highlights aggregation problems.

- Samuelson (1966) summarizes. Suppose champagne can be produced two ways:

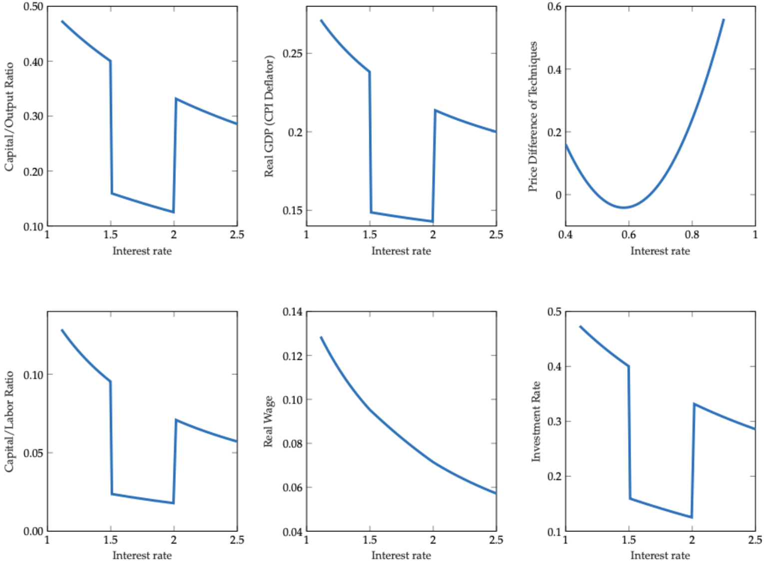
  1. In period $t-1$, 7 units of labor produce brandy, which ferments into champagne.
  2. In period $t-2$, 2 units of labor make grapejuice, which ferments into wine; in period t, 6 more units of labor convert wine to champagne.

- Prefer technology (1) when $r < 50\%$ (lower total labor requirement) and $r > 100\%$ (delays expensive).

- Prefer technology (2) when $r \in (50\%, 100\%)$.

- Re-switching leads to violation of Samuelson's parables.

Re-switching / capital reversing



Does it matter?

- There is not one "elasticity between capital and labor," capital intensity can in fact increase with interest rate, r can increase with K/Y and K/L.
- Solow (1963): *"There is a highbrow answer to this question and a lowbrow one. [...] Where the highbrow approach tends to be technical, disaggregated, and exact, the lowbrow view tends to be pecuniary, aggregative, and approximate. A middlebrow like myself sees virtue in each of these ways of looking at capital theory."*
- Is the re-switching / capital reversing example a curiosity or a deep pathology?
- Did not impede use of aggregate production function for empirics.
- RBC revolution leads to quasi-universal adoption of aggregate production functions, shift of focus from "heterogeneity and aggregation" to "dynamics and expectations."

Resurgence of micro-data and interest

- Increasing availability of industry-, firm-, and household-level data.
- Parallel to research on aggregate consumption function:
  - Vast heterogeneity in wealth, preferences that affect spending/saving behavior.
  - Need to model distributions to understand aggregate consumption behavior.
  - For free, also get distributional implications.
- Disaggregating the production side of the economy requires different tools:
  - Different interactions: Competition, inter-firm transactions, finance/credit, innovation.
  - Different shocks: Technology/productivity changes, input price changes, demand conditions, policy shocks.

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2. Disaggregation and Hulten's Theorem.
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Disaggregated model of production

- Consider economy with arbitrary number of factors F.
  - High-skilled labor, low-skilled labor, machines, buildings, land, oil, ...

- Arbitrary number of producers N with neoclassical production functions,

$$y_i = A_i F_i(x_{i1}, ..., x_{iN}, L_{i1}, ..., L_{iF}).$$

  where x_{ij}, L_{if} are producer i's use of intermediate j, factor f.

- Representative consumer with utility

$$U(c_1, ..., c_N).$$

Disaggregated model of production: What is Real GDP

- Denote rep. consumer expenditure shares $b_i = p_i c_i / \sum_j p_j c_j$.
- Changes in real GDP are *defined* to be

$$d \log Y = \sum_i b_i d \log c_i,$$

or, for longer intervals,

$$\Delta \log Y = \int_0^T \sum_i b_i(t) d \log c_i(t) dt.$$

- In the data, we can approximate this as:

$$\Delta \log Y \approx \sum_i b_{it-1} \Delta \log c_{it}.$$

$$\Delta \log Y \approx \sum_i b_{it} \Delta \log c_{it}.$$

$$\Delta \log Y \approx \sum_i \frac{1}{2} (b_{it-1} + b_{it}) \Delta \log c_{it}.$$

Disaggregated model of production: What is Real GDP

- In practice, quantities hard to measure, so define changes in GDP deflator by

$$d \log P = \sum_i b_i d \log p_i.$$

  Since $d \log PY = d \log P + d \log Y$, we can construct change in real GDP $d \log Y$ using

$$d \log Y = d \log PY - d \log P.$$

- Likewise,

$$\Delta \log Y = \Delta \log PY - \Delta \log P.$$

  Can construct first- and second-order approximations to $\Delta \log Y$ using first- and second-order approximations to $\Delta \log P$.

Disaggregated model of production: What is Real GDP

- So changes in real GDP are *defined* (and measured by statistical agencies) as

$$d\log Y = d\log PY - \sum_i b_i d\log p_i.$$

- Why is real GDP defined this way? Let $e(p,u)$ be the expenditure function of some consumer given utility u and prices p. One popular measure of the change in welfare is equivalent variation,

$$\mathrm{EV} = \frac{e(p_0,u_1)}{e(p_0,u_0)} \approx \frac{\partial \log e}{\partial \log u} d\log u.$$

Using

$$\underbrace{d\log e}_{d\log PY} = \sum_i \underbrace{\frac{\partial \log e}{\partial \log p_i}}_{b_i} d\log p_i + \underbrace{\frac{\partial \log e}{\partial \log u} d\log u}_{\mathrm{EV}},$$

we get that $EV = d\log PY - \sum_i b_i d\log p_i = d\log Y$.

Disaggregated model of production: What is Real GDP

- So, under certain conditions, Δ real GDP $=$ Δ welfare. But not always.
- Heterogeneous consumers.
  - Even if you define social welfare function with weights proportional to expenditures.
- New goods.
  - Unless they enter continuously from a choke point, real GDP does not measure infra-marginal surplus they generate.
- Quality changes.
  - GDP price deflator measurement relies on continuing varieties.
- We will return to some of these later.

Disaggregated model of production

- Now we have a model of production that encompasses the flexibility that the Cambridge, UK side demanded.
    - Arbitrary F factors can represent different types of labor and capital goods...
    - Arbitrary N producers and input-output linkages can be chosen to match data...
    - Aggregate production function "elasticities of substitution" will clearly depend on the properties of the underlying, disaggregated economic units.

1. Is this disaggregated model of production general enough to address the concerns raised in the Cambridge-Cambridge debates?

2. Is the model too general to say anything of value?

Disaggregated model of production: General enough?

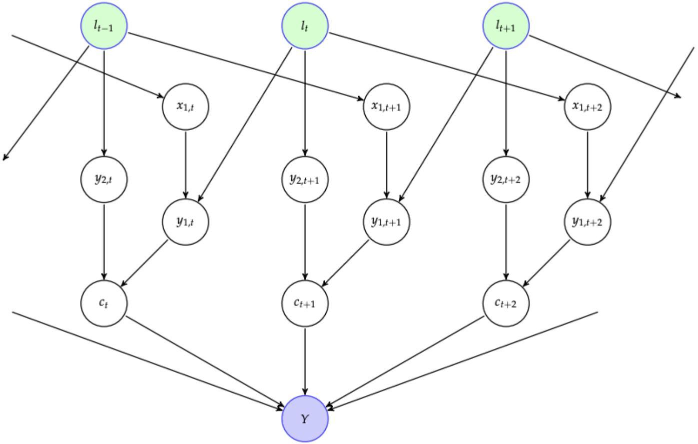

FIGURE 2. The production network underlying Samuelson's reswitching example. The arrows indicate the flow of goods. The green nodes are primary factors, and Y is aggregate output in this economy, which is perfectly substitutable across consumption units at different dates.

Disaggregated model of production: General enough?

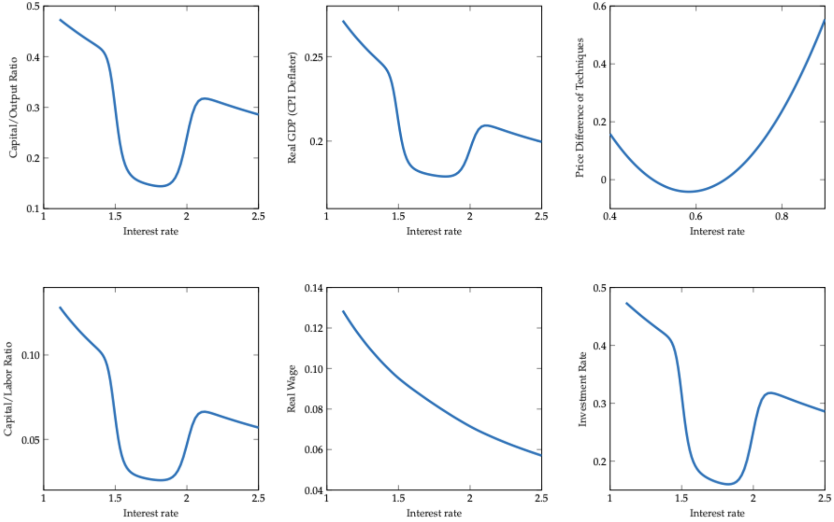

FIGURE 4. Samuelson's smoothed-out reswitching example

Disaggregated model of production: Too general?

- Is the framework still useful for predictions?

- For growth accounting, shock propagation, etc., we want to characterize comparative statics of aggregates (e.g., output, inflation) w.r.t. underlying microeconomic changes.

- What is the effect of a productivity change for producer i on real GDP?

$$\frac{d \log Y}{d \log A_i} = ?$$

  Any ideas?

Disaggregated model of production: Too general?

- Is the framework still useful for predictions?

- For growth accounting, shock propagation, etc., we want to characterize comparative statics of aggregates (e.g., output, inflation) w.r.t. underlying microeconomic changes.

- What is the effect of a productivity change for producer i on real GDP?

$$\frac{d\log Y}{d\log A_i} = \frac{p_i y_i}{PY}$$

  Any ideas?

- When markets perfectly competitive, given by sales share of i: Hulten's Theorem (1978).

Hulten's Theorem: Simple example

- Let's start with a simple example.

- N firms with DRS production technologies in labor

$$y_i = A_i L_i^{\alpha_i}.$$

- Household with Cobb-Douglas preferences over firms' products,

$$\max_{c_i} \prod_i c_i^{\lambda_i}.$$

- Inelastic supply of labor,

$$\sum_i L_i = L.$$

Hulten's Theorem: Simple example – Manual approach

- Demand curve:

$$\frac{c_i}{\sum_k p_k c_k} = \frac{1}{p_i}\lambda_i.$$

- Supply curve:

$$p_i = \frac{\partial C(y_i)}{\partial y_i} = \frac{\partial}{\partial y_i}\left[w\left(\frac{y_i}{A_i}\right)^{\frac{1}{\alpha_i}}\right] = \frac{1}{\alpha_i} w \frac{y_i^{\frac{1}{\alpha_i}-1}}{A_i^{\frac{1}{\alpha_i}}}.$$

- Combining (and taking nominal GDP as numeraire):

$$c_i = A_i\left(\frac{\alpha_i\lambda_i}{w}\right)^{\alpha_i}.$$

- Labor market clearing:

$$L = \sum_i L_i = \sum_i \frac{\alpha_i\lambda_i}{w} \qquad \Rightarrow \qquad w = \sum_i \frac{\alpha_i\lambda_i}{L}.$$

$$Y = \prod_i c_i^{\lambda_i} = \prod_i \left[A_i\left(\frac{\alpha_i\lambda_i}{\sum_k \alpha_k\lambda_k}L\right)^{\alpha_i}\right]^{\lambda_i}. \qquad \Rightarrow \qquad \frac{\partial \log Y}{\partial \log A_i} = \lambda_i.$$

Hulten's Theorem: Simple example – A faster approach

- First welfare theorem holds, so

$$\log Y = \max_{c_i, L_i} \log\left(\prod_i c_i^{\lambda_i}\right) - \mu_i\left(c_i - A_i L_i^{\alpha_i}\right) - \kappa\left(\sum_i L_i - L\right).$$

- Envelope theorem:

$$\frac{\partial \log Y}{\partial A_i} = \mu_i L_i^{\alpha_i} \qquad \Rightarrow \qquad \frac{\partial \log Y}{\partial \log A_i} = \mu_i y_i.$$

- First order condition:

$$\lambda_i \frac{1}{c_i} - \mu_i = 0. \qquad \Rightarrow \qquad \mu_i = \frac{\lambda_i}{c_i} = \frac{p_i}{\sum_k p_k c_k}. \qquad \Rightarrow \qquad \frac{\partial \log Y}{\partial \log A_i} = \lambda_i.$$

Hulten's Theorem: General case

- Decentralized equilibrium with perfect competition.

- Resource constraints:

$$c_i + \sum_j x_{ji} = A_i F_i(x_{i1}, \ldots, x_{iN}, L_{i1}, \ldots, L_{iF}).$$

$$\sum_i L_{if} = L_f.$$

- Profit maximization, taking prices as given:

$$\max_{x_{ij}, L_{if}} p_i A_i F_i(\{x_{ij}\}, \{L_{if}\}) - \sum_j p_j x_{ij} - \sum_f w_f L_{if}.$$

$$\Rightarrow \quad A_i \frac{\partial F_i}{\partial x_{ij}} = \frac{p_j}{p_i}.$$

Hulten's Theorem

- Log-linearizing resource constraints, $c_i + \sum_j x_{ji} = y_i$.

$$d\log c_i = \frac{y_i}{c_i} d\log y_i - \sum_j \frac{x_{ji}}{c_i} d\log x_{ji}.$$

- Log-linearizing production functions, $y_i = A_i F_i(x_{i1}, \ldots, x_{iN}, L_{i1}, \ldots, L_{iF})$,

$$d\log y_i = d\log A_i + \sum_j \frac{\partial \log F}{\partial \log x_{ij}} d\log x_{ij} + \sum_f \frac{\partial \log F}{\partial \log L_{if}} d\log L_{if}.$$

- Under perfect competition,

$$A_i \frac{\partial F_i}{\partial x_{ij}} = \frac{p_j}{p_i} \qquad \Rightarrow \qquad \frac{\partial \log F_i}{\partial \log x_{ij}} = \frac{p_j x_{ij}}{p_i y_i}.$$

$$\Rightarrow \quad d\log c_i = \frac{y_i}{c_i} \left[d\log A_i + \sum_j \frac{p_j x_{ij}}{p_i y_i} d\log x_{ij} + \sum_f \frac{w_f L_{if}}{p_i y_i} d\log L_{if} \right] - \sum_j \frac{x_{ji}}{c_i} d\log x_{ji}.$$

Hulten's Theorem

$$d\log c_i = \frac{y_i}{c_i}\left[d\log A_i + \sum_j \frac{p_j x_{ij}}{p_i y_i} d\log x_{ij} + \sum_f \frac{w_f L_{if}}{p_i y_i} d\log L_{if}\right] - \sum_j \frac{x_{ji}}{c_i} d\log x_{ji}.$$

- Plug this into our definition of the change in real GDP,

$$\begin{aligned}
d\log Y &= \sum_i \frac{p_i c_i}{PY} d\log c_i \\
&= \sum_i \frac{p_i c_i}{PY}\left[\frac{y_i}{c_i}\left[d\log A_i + \sum_j \frac{p_j x_{ij}}{p_i y_i} d\log x_{ij} + \sum_f \frac{w_f L_{if}}{p_i y_i} d\log L_{if}\right] - \sum_j \frac{x_{ji}}{c_i} d\log x_{ji}\right] \\
&= \sum_i \frac{p_i y_i}{PY} d\log A_i + \sum_i \sum_j \frac{p_j x_{ij}}{PY} d\log x_{ij} + \sum_i \sum_f \frac{w_f L_{if}}{PY} d\log L_{if} - \sum_i \sum_j \frac{p_i x_{ji}}{PY} d\log x_{ji} \\
&= \sum_i \frac{p_i y_i}{PY} d\log A_i + \sum_f \frac{w_f L_f}{PY} d\log L_f.
\end{aligned}$$

Hulten's Theorem: Social planner approach

- Alternative approach: Perfect competition equilibrium, so first welfare theorem applies.

- Optimum in this economy solves

$$\max_{c_i, x_{ij}, L_{if}} U(c_1, ..., c_N),$$

s.t.

$$c_i + \sum_j x_{ji} = A_i F_i(x_{i1}, ..., x_{iN}, L_{i1}, ..., L_{iF}).$$

$$\sum_i L_{if} = L_f.$$

- Lagrangian,

$$\mathscr{L} = U(c_1, ..., c_N) + \sum_i p_i \left[A_i F_i(...) - c_i - \sum_j x_{ji} \right] + \sum_f w_f \left[L_f - \sum_i L_{if} \right].$$

Hulten's Theorem: Social planner approach

$$\mathscr{L} = U(c_1, ..., c_N) + \sum_i p_i \left[A_i F_i(...) - c_i - \sum_j x_{ji} \right] + \sum_f w_f \left[L_f - \sum_i L_{if} \right].$$

- Given changes in each producers productivities $d \log A_i$,

$$d\mathscr{L} = \sum_i \frac{\partial \mathscr{L}}{\partial \log A_i} d \log A_i = \sum_i \left[p_i A_i F_i(...) \right] d \log A_i = \sum_i p_i y_i d \log A_i.$$

- We can also write,

$$d\mathscr{L} = \sum_i \frac{\partial U}{\partial c_i} dc_i = \sum_i p_i c_i d \log c_i = (PY) \, d \log Y.$$

- Missing step: Why can we assume Lagrange multiplier for resource i is p_i?

Hulten's Theorem: Comments

- Let us denote "Domar weights," $\lambda_i = p_i y_i / PY$ and $\Lambda_f = w_f L_f / PY$. Then:

$$d \log Y = \sum_i \lambda_i d \log A_i + \sum_f \Lambda_f d \log L_f.$$

- Despite disaggregation, we can say quite a bit about real GDP, productivity, ...
- Two examples for intuition.
  1. Vertical supply chain.
  2. Round-about producer.

Vertical supply chain: Why sales instead of value added?

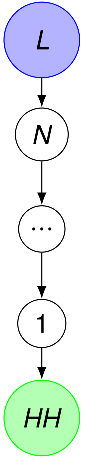

Figure: Vertical supply chain.

- Linear technology: $y_k = A_k y_{k+1}$.
- Each firm has zero value add!
- But sales share for each firm is $\lambda_k = 1$.
- Easy to verify that $Y = \prod_{k=1}^{N} A_k$.

$$\frac{\partial \log Y}{\partial \log A_k} = 1.$$

Round-about economy & intermediate input cycles

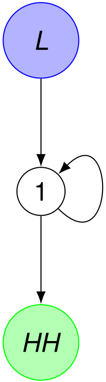

Figure: Round-about economy.

- Round-about firm uses its own output in production:

$$y_1 = A_1 F_1 \left(L, x_{11}\right) = c_1 + x_{11}.$$

- Increase in A_1 increases y_1, which makes more x_{11} available, which further increases y_1, ...
- Multiplier exactly given by $\lambda_1 = p_1 y_1 / p_1 c_1 > 1$.
- Micro shocks w/ reasonable volatility can still drive aggregate fluctuations.

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Applications of Hulten's Theorem

1. Solow residual and granular shocks. (Gabaix 2011).

2. Network irrelevance? (Acemoglu et al. 2012, 2017).

1. Granular shocks

- Solow (1957), in accounting for growth from 1909–1949, writes *"Gross output per man hour doubled over that interval, with 87 1/2 per cent of the increase attritable to technical change and the remaining 12 1/2 per cent to increased use of capital."*

$$d\log\frac{Y}{L} = d\log A + \Lambda_K d\log\frac{K}{L}.$$

- Abramovitz (1956) famously wrote, *"Since we know little about the causes of productivity increase, the indicated importance of this element may be taken to be some sort of measure of our ignorance about the causes of economic growth."*

- Now we can write (under the assumption of efficiency):

$$d\log A = d\log Y - \sum_f \Lambda_f d\log L_f = \sum_i \lambda_i d\log A_i.$$

One place to look: micro shocks.

1. Granular shocks: Problem

- Can micro shocks really explain aggregate movements in Solow residual?
- Lucas (1977): *“In a complex modern economy, there will be a large number of such shifts in any given period, each small in importance relative to total output. There will be much ‘averaging out’ of such effects across markets. Cancellation of this sort is, I think, the most important reason why one cannot seek an explanation of the general movements we call business cycles in the mere presence, per se, of unpredictability of conditions in individual markets.”*

1. Granular shocks

- Gabaix (2011) takes up this challenge:
- Main argument: When firm size distribution is fat tailed, idiosyncratic shocks do not wash out, and granular shocks to large firms matter greatly for aggregate volatility.
- Motivating facts:
  - In 2000, Nokia contributed 1.6 percentage points of Finland's GDP growth.
  - In Dec 2004, a $24B one-time Microsoft dividend boosted growth in personal income from 0.6% to 3.7%.
  - In U.S., sales to top 50 firms is 20–25% of GDP.

1. Granular shocks

- Suppose firm sales follow power law distribution, $P(S_i > c) = ac^{-\zeta}$.

- n'th moment only exists if $\zeta > n$. E.g., variance of firm sizes is finite only if $\zeta > 2$.

- When $\zeta > 2$, given $d\log A_i \sim \mathcal{N}(0, \sigma^2)$,

$$
\begin{aligned}
Var\left(d\log Y\right) &= Var\left(\frac{\sum_i S_i d\log A_i}{\sum_i S_i di}\right) \\
&= \sigma^2 \sum_i \left(\frac{S_i}{\sum_j S_j}\right)^2 \\
&= \sigma^2 \frac{1}{N} \frac{\mathbb{E}\left[S_i^2\right]}{\left(\mathbb{E}\left[S_i\right]\right)^2}.
\end{aligned}
$$

So $\text{std}(d\log Y) \propto N^{-1/2}$, as Lucas predicted. But what if $\zeta \leq 2$?

1. Granular shocks

- If $\zeta \leq 2$, $\mathbb{E}[S_i^2]$ not well defined, and last step of calculation fails.
- Gabaix (2011) shows that

$$\text{std}(d\log Y) \propto \begin{cases} \sigma/N^{1/2} & \text{if } \zeta > 2 \\ \sigma/N^{(1-1/\zeta)} & \text{if } 1 < \zeta < 2 \\ \sigma/\ln N & \text{if } \zeta = 1 \end{cases}$$

  Note connection to Acemoglu et al. (2017) results on fat-tailed Domar weights.
- Using Census data, Axtell (2001) estimates firm sizes follow power law with $\zeta = 1.059 \pm 0.054$.

1. Granular shocks: Proof-of-concept

- Gabaix (2011) provides a proof-of-concept that going to micro data "works" for understanding aggregate movements.

- Measure "granular residual" using top 100 firms

$$\Gamma_t = \sum_{i=1}^{100} \frac{S_{it-1}}{\sum_j S_{jt-1}} \hat{\varepsilon}_{it}, \qquad \text{where} \qquad \hat{\varepsilon}_{it} = g_{it} - \beta' X_{it}.$$

  Idiosyncratic shocks constructed by purging firm growth rate of observable predictors.

1. Granular shocks: Proof-of-concept

TABLE I

EXPLANATORY POWER OF THE GRANULAR RESIDUAL[a]

	GDP Growth$_t$		Solow$_t$	
(Intercept)	0.018**	0.017**	0.011**	0.01**
	(0.0026)	(0.0025)	(0.002)	(0.0021)
Γ_t	1.8*	2.5**	2.1**	2.3**
	(0.69)	(0.69)	(0.54)	(0.57)
Γ_{t-1}	2.6**	2.9**	1.2*	1.3*
	(0.71)	(0.67)	(0.55)	(0.56)
Γ_{t-2}		2.1**		0.65
		(0.71)		(0.59)
N	56	55	56	55
R^2	0.266	0.382	0.261	0.281
Adj. R^2	0.239	0.346	0.233	0.239

[a] For the year $t = 1952$ to 2008, per capita GDP growth and the Solow residual are regressed on the granular residual Γ_t of the top 100 firms (equation (33)). The firms are the largest by sales of the previous year. Standard errors are given in parentheses.

- Γ_t explains 26% of variation in Solow residual.

1. Granular shocks: What are the shocks?

TABLE V

NARRATIVE[a]

Year	Firm	Share of GDP $\frac{S_{i,t-1}}{Y_{t-1}}$ in %	Labor Prod. Growth g_{it} [$\Delta \ln S_{it}, \Delta \ln L_{it}$]	Demeaned Growth $g_{it} - \bar{g}_{I_i t}$ in %	Gran. Res. Γ_{it} in %	Share of GR $\frac{\Gamma_{it}}{\Gamma_t}$	Direct Share of g_{Y_t} $\frac{\Gamma_{it}}{g_{Y_t}}$	Imputed Share of g_{Y_t} $\frac{\mu\Gamma_{it}}{g_{Y_t}}$	Brief Explanation
1952	U.S. Steel	1.03	−10.75 [−13.10, −2.35]	−3.56	−0.037	0.061	−0.810	−2.430	Strike
1953	U.S. Steel	0.87	17.06 [19.51, 2.45]	5.86	0.051	3.985	0.060	0.180	Rebound from strike
1955	GM	2.58	14.00 [21.89, 7.88]	17.84	0.461	0.808	0.142	0.426	Boom in car production: New models and price war
1956	Ford	1.35	−20.95 [−21.96, −1.01]	−20.72	−0.270	0.407	0.145	0.435	End of price war
1956	GM	3.00	−13.55 [−17.61, −4.06]	−13.32	−0.400	0.603	0.215	0.645	End of price war
1957	GM	2.47	0.36 [−1.50, −1.85]	−12.38	−0.305	2.201	0.167	0.501	End of price war (aftermath)
1961	Ford	1.00	25.12 [23.64, −1.48]	27.03	0.199	4.131	−0.147	−0.441	Success of compact Falcon (rebound from Edsel failure)
1965	GM	2.56	7.45 [18.06, 10.61]	11.10	0.284	0.600	0.092	0.276	Boom in new-car sales
1967	Ford	1.55	−19.84 [−18.23, 1.61]	−14.91	−0.232	2.461	0.379	1.137	Strike
1970	GM	2.47	−17.85 [−31.06, −13.20]	−20.52	−0.493	0.757	0.165	0.495	Strike
1971	GM	1.81	25.58 [36.15, 10.57]	23.35	0.361	0.516	7.344	22.032	Rebound from strike

1. Granular shocks: What are the shocks?

TABLE V—*Continued*

Year	Firm	Share of GDP $\frac{S_{i,t-1}}{Y_{t-1}}$ in %	Labor Prod. Growth g_{it} [$\Delta \ln S_{it}$, $\Delta \ln L_{it}$]	Demeaned Growth $g_{it} - \overline{g}_{I_i t}$ in %	Gran. Res. Γ_{it} in %	Share of GR $\frac{\Gamma_{it}}{\Gamma_t}$	Direct Share of g_{Y_t} $\frac{\Gamma_{it}}{g_{Y_t}}$	Imputed Share of g_{Y_t} $\frac{\mu \Gamma_{it}}{g_{Y_t}}$	Brief Explanation
1972	Chrysler	0.71	16.76 [15.64, −1.13]	17.80	0.126	0.234	0.058	0.174	Rush of sales for subcompacts (Dodge Dart and Plymouth Valiant)
1972	Ford	1.46	14.18 [16.36, 2.18]	15.22	0.222	0.411	0.103	0.309	Rush of sales for subcompacts (Ford Pinto)
1974	GM	2.59	−11.31 [−21.28, −9.97]	−15.23	−0.394	0.913	0.115	0.345	Cars with poor gas mileage hit by higher oil price
1983	IBM[b]	1.06	10.46 [11.76, 1.29]	10.52	0.111	0.177	0.071	0.213	Launch of the IBM PC
1987	GE[b]	0.79	25.62 [8.33, −17.29]	21.46	0.158	1.110	0.357	1.071	Moving out of manufacturing and into finance and high-tech
1988	GE[b]	0.83	21.42 [20.08, −1.33]	16.55	0.137	0.441	0.117	0.351	Moving out of manufacturing and into finance and high-tech
1996	AT&T	1.08	38.97 [−44.11, −83.08]	32.45	0.215	0.471	0.446	1.338	Spin-off of NCR and Lucent
2000	GE	1.20	20.56 [12.29, −8.27]	33.04	0.239	9.934	0.468	1.404	Sales topped $111bn, expansion of GE Medical Systems
2002	Walmart	2.16	8.61 [9.83, 1.22]	6.39	0.138	3.219	−0.099	−0.297	Success of lean distribution model

1. Granular shocks: What are the shocks?

- Romer (2016): *"While 'demand' shocks such as the* ***aether****, exogenous spending, and investment-specific* ***phlogiston*** *shocks explain a significant fraction of the short-run forecast variance in output, both the* ***troll's wage mark-up*** *(or* ***caloric****) and, to a lesser extent, output-specific* ***phlogiston*** *shocks explain most of its variation in the medium to long run. ... Third, inflation developments are mostly driven by the* ***gremlin's price mark-up*** *shocks in the short run and the* ***troll's wage mark-up*** *shocks in the long run."*

- Strikes, spin-offs, changes in distribution model, production booms...

2. Network irrelevance

- Hulten's Theorem often described as a "network irrelevance" result. Do you agree?

2. Network irrelevance

- Hulten's Theorem often described as a "network irrelevance" result. Do you agree?
- Domar weight is a sufficient statistic for the effect of microeconomic shocks.
- Any network structure in which i's sales share is λ_i will have same effect of $d \log A_i$!
- But λ_i's determined endogenously by network structure.
- E.g., a network without any cycles cannot exhibit $\sum_i \lambda_i > 1$.

2. Network irrelevance: Acemoglu et al. (2012), Acemoglu et al. (2017)

- Suppose we take BEA input-output table with $N = 405$ detailed industries.
- Each industry's productivity is i.i.d. lognormal random variable, $d\log A_i \sim \mathcal{N}(0, \sigma^2)$.
- If no cycles and industries equally sized,

$$\mathrm{std}(d\log Y) = \sqrt{Var(\sum_i \lambda_i d\log A_i)} = \frac{1}{\sqrt{N}}\sigma \approx \frac{\sigma}{20}.$$

- Lucas (1977) argument.

2. Network irrelevance: Acemoglu et al. (2012), Acemoglu et al. (2017)

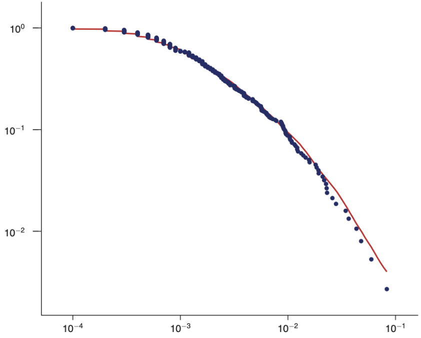

FIGURE 5. THE EMPIRICAL COUNTERCUMULATIVE DISTRIBUTION OF SECTORAL DOMAR WEIGHTS

Figure: Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017).

- Instead, variation in size and interdependencies between industries.
- Given tail coefficient 1.45, Acemoglu et al. (2017) show that aggregate volatility is

$$\mathrm{std}(d\log Y) \approx \frac{\sigma}{N^{1-1/1.45}} \approx \frac{\sigma}{6.4}.$$

- Same-sized industry shocks, $>$3x the size of aggregate fluctuations predicted by Lucas.
- To explain $\mathrm{std}(d\log Y) = 2\%$, $\sigma = 40\% \to 13\%$.

Hulten's Theorem: Recap

- Hulten's Theorem provides sufficient statistic for how micro shocks affect output.

$$\frac{\partial \log Y}{\partial \log A_i} = \frac{p_i y_i}{PY} = \lambda_i.$$

  Applies to efficient economies, regardless of production technologies, details of input-output structure, etc.

- Taking into account empirical distribution of firm sizes / input-output network, micro shocks can play meaningful role in macro aggregates.

- ...and so far we've only considered i.i.d. productivity shocks and efficient economies.

Hulten's Theorem vs. "Evidence of the eyes"

- Summers (2013): *"Now, think about the period after the financial crisis. I always like to think of these crises as analogous to a power failure or analogous to what would happen if all the telephones were shut off for some time. Consider such an event. The network would collapse. The connections would go away. And output would, of course, drop very rapidly. There would be a set of economists who would sit around explaining that electricity was only 4% of the economy, and so if you lost 80% of electricity, you couldn't possibly have lost more than 3% of the economy. Perhaps in Minnesota or Chicago there would be people writing such a paper, but most others would recognize this as a case where the evidence of the eyes trumped the logic of straightforward microeconomic theory.*

  And we would understand that somehow, even if we didn't exactly understand it in the model, that when there wasn't any electricity, there wasn't really going to be much economy. Something similar was true with respect to financial flows and financial interconnection. And that's why it is so important to get the lights back on, and that's why it's so important to contain the financial system."