Lecture 6:
Financial Constraints and Quotas

Kunal Sangani

ECON 416-1

Endogenous Distortions, contd.

- Last class, we discussed models with variable markups.
- Today, we will take a break from markups for two other endogenous distortions.
  1. Financial constraints.
  2. Quotas.

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Kiyotaki and Moore (1997): Setup

- Two types of agents: Farmers (unit mass) and gatherers (mass m).
- Fixed supply of land $\bar{K}$ that can be used to produce fruit.
- Farmers produce fruit from land with constant returns to scale,

$$y_{t+1} = (a+c)k_t.$$

- $c/(a+c)$ assumed to be bruised and nontradable (only farmer can consume it). We will assume $c > (R-1)a$, where R is exogenous interest rate.
- Gatherers (tilde variables) find fruit on land with decreasing returns to scale,

$$\tilde{y}_{t+1} = G(\tilde{k}_t).$$

Kiyotaki and Moore (1997): Setup

- Once farmer has started farming land at date t, no fruit will be produced at $t+1$ if the farmer withdraws her labor.
- Farmer cannot precommit to work.
- $\Rightarrow$ These frictions imply that a farmer could threaten to withdraw her labor.
- Creditors in turn protect themselves by collateralizing the farmer's land.
- Since land yields no fruit without farmer's labor, liquidation value $<$ continuation value.
- Thus, debt b_t farmer can credibly pay back tomorrow capped by value of land tomorrow:

$$Rb_t \leq q_{t+1}k_t.$$

Kiyotaki and Moore (1997): Setup

- Budget constraint for farmers:

$$\underbrace{(a+c)k_{t-1}+b_t-Rb_{t-1}}_{\text{Income and net new debt}} = \underbrace{x_t+q_t(k_t-k_{t-1})}_{\text{Consumption and investment}}.$$

- Budget constraint for gatherers:

$$G(\tilde{k}_{t-1})+\tilde{b}_t-R\tilde{b}_{t-1}=\tilde{x}_t+q_t(\tilde{k}_t-\tilde{k}_{t-1}).$$

Kiyotaki and Moore (1997): Setup

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- Budget constraint for gatherers:

$$G(\tilde{k}_{t-1})+\tilde{b}_t-R\tilde{b}_{t-1}=\tilde{x}_t+q_t(\tilde{k}_t-\tilde{k}_{t-1}).$$

- Claim: Around steady state, farmers consume only nontradable fruit $x_t = ck_{t-1}$ and invest remainder in land. Moreover, borrowing constraint binds, $Rb_t = q_{t+1}k_t$:

$$\underbrace{(q_t-q_{t+1}/R)}_{\substack{u_t\text{=Down payment}\\ \text{per unit of land}}} k_t = \underbrace{(a+q_t)\,k_{t-1}-Rb_{t-1}}_{\text{Net worth at date } t} .$$

i.e., net worth + borrowing capacity $q_{t+1}k_t/R$ to invest q_tk_t.

Kiyotaki and Moore (1997): Aggregates

- Since farmers' k_t and b_t are linear, we can aggregate across unit mass of farmers:

$$K_t = \frac{1}{u_t}[(a+q_t)K_{t-1} - RB_{t-1}].$$
$$B_t = q_{t+1}K_t/R.$$

- Gatherers are not credit constrained, so demand for land sets marginal product of land equal to the opportunity cost (user cost) of holding land,

$$\frac{1}{R}G'(\tilde{k}_t) = u_t.$$

- Thus, in equilibrium, user cost of land pinned down by

$$u_t = u(K_t) = \frac{1}{R}G'\left[\frac{1}{m}(\bar{K} - K_t)\right] = q_t - q_{t+1}/R.$$

Kiyotaki and Moore (1997): Steady state

- We have the following equilibrium conditions:

$$
\begin{aligned}
u_t &= q_t - \frac{q_{t+1}}{R}, && \text{(Definition of user cost)} \\
u_t &= \frac{1}{R} G' \left[\frac{1}{m} (\bar{K} - K_t) \right], && \text{(Gatherer land demand)} \\
K_t &= \frac{1}{u_t} [(a + q_t) K_{t-1} - RB_{t-1}], && \text{(Farmer land demand)} \\
RB_t &= q_{t+1} K_t. && \text{(Farmer borrowing constraint)}
\end{aligned}
$$

Kiyotaki and Moore (1997): Steady state

- In steady state,

$$u^* = \frac{R-1}{R} q^* = a, \qquad \text{(Definition of user cost)}$$

$$u^* = \frac{1}{R} G' \left[\frac{1}{m} (\bar{K} - K^*) \right], \qquad \text{(Gatherer land demand)}$$

$$K^* = \frac{1}{u^*} a K^* \qquad \text{(Farmer land demand)}$$

$$B^* = q^* K^* / R = \frac{a}{R-1} K^*. \qquad \text{(Farmer borrowing constraint)}$$

Kiyotaki and Moore (1997): Steady state

- In steady state,

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K^* &= \frac{1}{u^*} a K^* && \text{(Farmer land demand)} \\
B^* &= q^* K^* / R = \frac{a}{R-1} K^*. && \text{(Farmer borrowing constraint)}
\end{aligned}
$$

- Farmers' tradable output aK^* just enough to cover interest on debt, $(R-1)B^*$.
- Required down payment for land u^* exactly equals tradable output a.
- Farms neither expand nor shrink.

Kiyotaki and Moore (1997): Misallocation

- Output is

$$Y_t = mG\left[\frac{1}{m}(\bar{K} - K_{t-1})\right] + (a+c)K_{t-1}.$$

- Efficient point equates marginal product of land for farmers and gatherers:

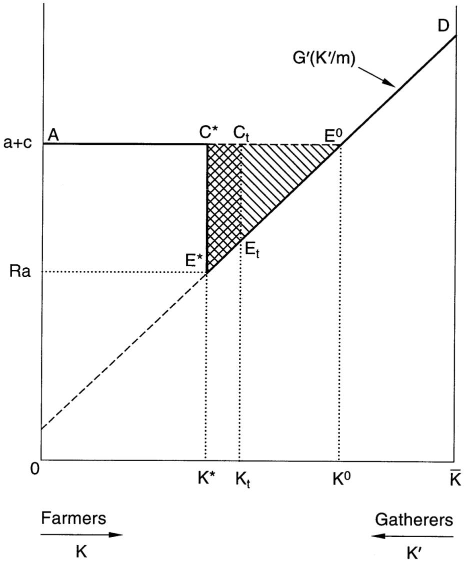
$$G'\left[\frac{1}{m}(\bar{K} - K^{\text{eff}})\right] = a + c.$$

- At decentralized steady-state equilibrium, farmers constrained:

$$G'\left[\frac{1}{m}(\bar{K} - K^{*})\right] = Ra < a + c.$$

Land for farmers has higher marginal value product.

Kiyotaki and Moore (1997): Misallocation



- Deadweight loss to a second order,

$$\mathcal{L} \approx \frac{1}{2}\left(\log \frac{a+c}{Ra}\right)\left(\log \frac{K^{\text{eff}}}{K^*}\right).$$

- Reallocations of land toward farmers will increase allocative efficiency and output.

- I.e., any shock that loosens their constraint.

Kiyotaki and Moore (1997): Misallocation

- Output is

$$Y_t = mG\left[\frac{1}{m}(\bar{K} - K_{t-1})\right] + (a+c)K_{t-1}.$$

- Log-linearizing,

$$d\log Y_t = \left((a+c) - G'\left[\frac{1}{m}(\bar{K} - K)\right]\right)\frac{K}{Y}d\log K_{t-1}.$$

- What is this at efficient point?

- Around decentralized steady state equilibrium,

$$d\log Y_t = \left[\frac{a+c-Ra}{a+c}\right]\frac{(a+c)\,K^*}{Y^*}d\log K_{t-1}.$$

What is $\frac{(a+c)K^*}{Y^*}$? What is $\frac{Ra}{a+c}\frac{(a+c)K^*}{Y^*}$?

Kiyotaki and Moore (1997): Model ingredients

- Three assumptions in this stylized model:
  1. Once farmer has started farming land at date t, if the farmer withdraws her labor between t and $t+1$, no fruit is produced at $t+1$. (Farmer's labor is "inalienable.")
  2. Farmer cannot precommit to work (incomplete contracts).
  3. Fraction $c/(a+c)$ of farmer's output is nontradable.
- Assumptions #1-2 imply inside value $>$ outside value. Outside value limits borrowing.
- Assumption #3: If $c=0$, farmer could borrow up to full value of their expected output.
  - Could leverage to the limit. No accumulation of internal funds / evolution of net worth.
  - Nontradable share ensures farmers can never fully pledge future income, and must accumulate wealth to self-finance some of capital purchases.

Kiyotaki and Moore (1997): Dynamics

- Around steady state, output depends on reallocations to/away from farmers:

$$d\log Y_t = \left[\frac{a+c-Ra}{a+c}\right]\frac{(a+c)\,K^*}{Y^*}d\log K_{t-1}.$$

- What determines K_{t-1}? Let's work through this mechanically:

$$u_t = q_t - \frac{q_{t+1}}{R}, \qquad \text{(Definition of user cost)}$$

$$u_t = \frac{1}{R}G'\left[\frac{1}{m}(\bar{K}-K_t)\right], \qquad \text{(Gatherer land demand)}$$

$$K_t = \frac{1}{u_t}[(a+q_t)K_{t-1} - RB_{t-1}], \qquad \text{(Farmer land demand)}$$

$$RB_t = q_{t+1}K_t. \qquad \text{(Farmer borrowing constraint)}$$

Kiyotaki and Moore (1997): Dynamics

- Log-linearized equations:

$$d\log u_t = \frac{q_t}{q_t - \frac{q_{t+1}}{R}} d\log q_t - \frac{\frac{q_{t+1}}{R}}{q_t - \frac{q_{t+1}}{R}} d\log q_{t+1}, \qquad \text{(Definition of user cost)}$$

$$d\log u_t = \frac{K_t}{\bar{K} - K_t} \frac{\frac{1}{m}\left(\bar{K} - K_t\right) G''\left[\frac{1}{m}\left(\bar{K} - K_t\right)\right]}{G'\left[\frac{1}{m}\left(\bar{K} - K_t\right)\right]} \left(-d\log K_t\right), \quad \text{(Gatherer land demand)}$$

$$\begin{aligned} d\log K_t = {} & -d\log u_t + \frac{a_t K_{t-1}\, d\log a_t + q_t K_{t-1}\, d\log q_t}{(a+q_t)\, K_{t-1} - RB_{t-1}} \\ & + \frac{(a+q_t)\, K_{t-1}\, d\log K_{t-1} - RB_{t-1}\, d\log B_{t-1}}{(a+q_t)\, K_{t-1} - RB_{t-1}}, \qquad \text{(Farmer land demand)} \end{aligned}$$

$$d\log B_t = d\log q_{t+1} + d\log K_t. \qquad \text{(Farmer borrowing constraint)}$$

Kiyotaki and Moore (1997): Dynamics

- Evaluated at steady state,

$$d\log u_t = \frac{R}{R-1} d\log q_t - \frac{1}{R-1} d\log q_{t+1}, \qquad \text{(Definition of user cost)}$$

$$d\log u_t = \frac{1}{\eta} d\log K_t, \qquad \text{(Gatherer land demand)}$$

$$d\log K_t = -d\log u_t + d\log a_t + d\log K_{t-1} + \frac{R}{R-1}\left(d\log q_t + d\log K_{t-1} - d\log B_{t-1}\right), \qquad \text{(Farmer land demand)}$$

$$d\log B_t = d\log q_{t+1} + d\log K_t. \qquad \text{(Farmer borrowing constraint)}$$

where we define $1/\eta = -\frac{K^*}{\bar{K}-K^*} \frac{\frac{1}{m}\left(\bar{K}-K^*\right) G''\left[\frac{1}{m}\left(\bar{K}-K^*\right)\right]}{G'\left[\frac{1}{m}\left(\bar{K}-K^*\right)\right]} > 0.$

Kiyotaki and Moore (1997): Dynamics

- User cost of land depends on price tomorrow vs today.

$$d\log u_t = \frac{R}{R-1} d\log q_t - \frac{1}{R-1} d\log q_{t+1}, \qquad \text{(Definition of user cost)}$$

- More land to farmers increases marginal product for gatherers (decreasing returns).

$$d\log u_t = \frac{1}{\eta} d\log K_t, \qquad \text{(Gatherer land demand)}$$

- Changes in net worth from income on existing land, value of collateral relative to debt.

$$d\log K_t = -d\log u_t + d\log a_t + d\log K_{t-1} + \frac{R}{R-1}\left(d\log q_t + d\log K_{t-1} - d\log B_{t-1}\right), \qquad \text{(Farmer land demand)}$$

- Can borrow more if more pledgable assets.

$$d\log B_t = d\log q_{t+1} + d\log K_t. \qquad \text{(Farmer borrowing constraint)}$$

Kiyotaki and Moore (1997): Response to productivity shock

- Suppose we are in steady state. MIT, temporary shock $d\log a_0$.
- Law of motion for farmer's land:

$$\left(1+\frac{1}{\eta}\right) d\log K_t = d\log a_t + d\log K_{t-1} + \frac{R}{R-1}\left(d\log q_t + d\log K_{t-1} - d\log B_{t-1}\right).$$

- At date zero, net worth increases "directly" due to productivity shock and "indirectly" due to unanticipated rise in land price:

$$\left(1+\frac{1}{\eta}\right) d\log K_0 = d\log a_0 + \frac{R}{R-1}\left(d\log q_0\right).$$

  Farmers leveraged, so large effect of land price increase on net worth.

- For date 1, 2, ..., future land price changes are anticipated, so we have:

$$\left(1+\frac{1}{\eta}\right) d\log K_t = d\log K_{t-1}.$$

Kiyotaki and Moore (1997): Response to productivity shock

$$\left(1+\frac{1}{\eta}\right) d\log K_0 = d\log a_0 + \frac{R}{R-1}\left(d\log q_0\right)$$
$$\left(1+\frac{1}{\eta}\right) d\log K_t = d\log K_{t-1}.$$

- What is the change in the land price, $d\log q_0$?

$$\begin{aligned}
d\log q_t &= \frac{R-1}{R} d\log u_t + \frac{1}{R} d\log q_{t+1} \\
&= \frac{R-1}{R}\sum_{s=0}^{\infty} R^{-s} d\log u_{t+s} \\
&= \frac{1}{\eta}\frac{R-1}{R}\sum_{s=0}^{\infty} R^{-s}\left(1+\frac{1}{\eta}\right)^{-s} d\log K_t \\
&= \frac{1}{\eta}\frac{R-1}{R}\frac{1}{1-\frac{1}{R}\frac{\eta}{1+\eta}} d\log K_t.
\end{aligned}$$

Kiyotaki and Moore (1997): Response to productivity shock

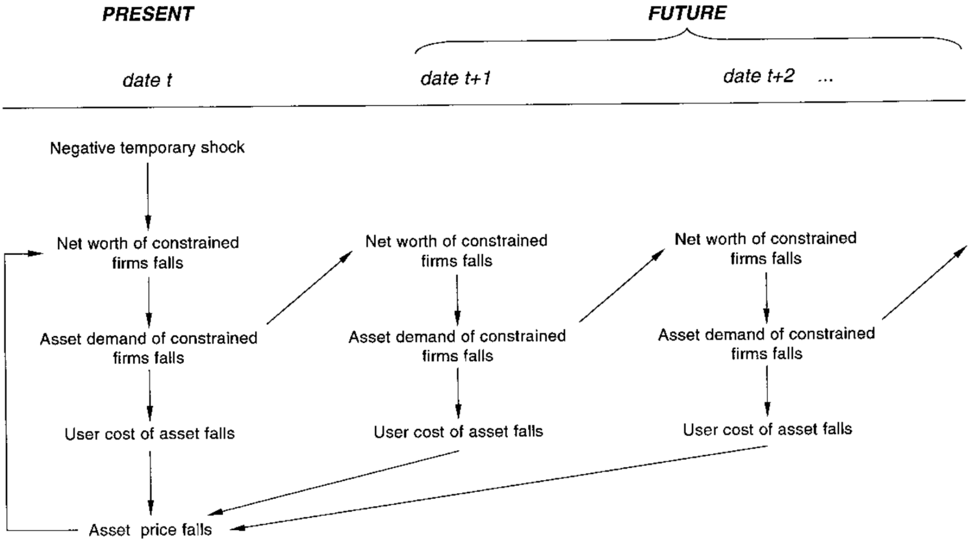
- We get:

$$d \log q_0 = \frac{1}{\eta} d \log a_0.$$

$$d \log K_0 = \frac{1}{1+\frac{1}{\eta}} \left(1 + \frac{R}{R-1} \frac{1}{\eta} \right) d \log a_0.$$

- Land price increases by same magnitude as temporary productivity shock!
- "Indirect effect" of land price on land accumulation by farmers is large due to leverage.
- Output moves more than one-for-one with productivity shock.
- $\Rightarrow$ Large incr. in land value b/c allocated to higher marginal product users into future.
- $\Rightarrow$ Direct effect of productivity shock augmented by endogenous reallocations.

Kiyotaki and Moore (1997): Dynamic multiplier



Net worth channel

- Core idea from Kiyotaki–Moore is that shocks can move asset prices that endogenously alleviate (or exacerbate) financial constraints.
- Bernanke, Gertler, and Gilchrist (1999) develop a related version with an external finance premium that falls with borrower net worth ("financial accelerator").
- Post-2008, literature developed on how banks' / intermediaries' net worth drives credit spreads and asset prices.
  (See e.g., Gertler and Kiyotaki 2010; Gertler and Karadi 2011, 2015; He and Krishnamurthy, 2013.)

Net worth channel: Misallocation

- At core, these are models of misallocation across constrained vs. unconstrained firms.
- Credit markets facilitate allocation of resources to most productive uses.
- Imperfections in credit markets create deadweight losses.
- Nonlinearities stem from the fact that limiting credit supply further deprives constrained producers of resources. (At least around efficient point, we know that output must be concave w.r.t. frictions.)
- How capital moves across users with varying marginal products in response to shocks is a sufficient statistic.

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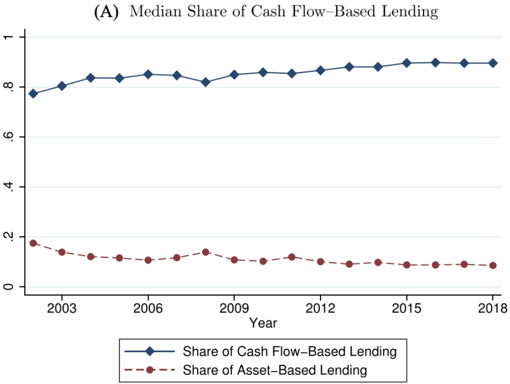
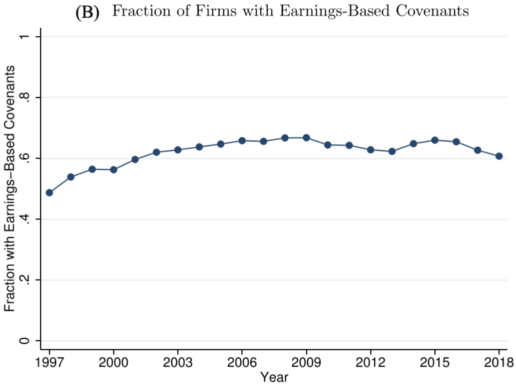
Two tales of debt

- Kiyotaki and Moore (1997) emphasize loans secured by borrower collateral.
- Borrowing constraints depend on liquidation value of physical assets that firms own.
- Some precursors instead modeled borrowing capacity depending on cash flows from firms' operations (e.g., Stiglitz and Weiss 1981, Holmstrom and Tirole 1997).
- Underlying model will determine how misallocation endogenously responds to shocks.
- What is more reasonable?

Two tales of debt: Lian and Ma (2021)





- 80% of debt for U.S. nonfinancial firms is “cash flow–based,” rather than “asset–based.”

Two tales of debt: Lian and Ma (2021)

- Asset–based lending vs. cash flow–based lending central to credit markets in practice.
- **Asset–based:**
  - Debt based on liquidation value of specific assets (physical assets like real estate, equipment, or inventory; and separable assets like receivables or patents).
  - Creditors payoffs in default driven by liquidation value of these assets.
  - Typically secured by specific assets. If value of asset in Ch. 11 falls short of debt claim, then creditors have secured claim equal to collateral value plus unsecured general claim (deficiency claim).
  - At debt issuance, lenders hire specialist appraisers to estimate liquidation values.
  - Examples: Commercial mortgages (backed by commercial real estate), loans based on equipment / inventory / oil and gas reserves, etc.

Two tales of debt: Lian and Ma (2021)

- Asset–based lending vs. cash flow–based lending central to credit markets in practice.
- **Cash flow–based:**
  - Debt based on going-concern value of borrower (cash flows of continuing operations).
  - Can be secured by the corporate entity ("substantially all assets," also called blanket lien), or unsecured.
  - At debt issuance, lenders assess cash flow values. Monitor cash flows over life of loan.
  - Examples: Majority of corporate bonds, most syndicated loans.

Two tales of debt: Lian and Ma (2021)

- Asset–based lending vs. cash flow–based lending central to credit markets in practice.
- **Cash flow–based:**
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  - At debt issuance, lenders assess cash flow values. Monitor cash flows over life of loan.
  - Examples: Majority of corporate bonds, most syndicated loans.
- About 1/3 of all secured debt for public firms is cash flow–based.

Two tales of debt: Lian and Ma (2021)

- A standard form of cash flow–based lending is earnings-based.
- Maximum debt relative to operating earnings: $b_t \leq \phi \pi_t$. (ϕ is 3–4.5).
- Or, maximum interest expenses relative to earnings: $r_t b_t \leq \phi \pi_t$. (ϕ is 0.33–0.5.)
- π_t typically measured as EBITDA (earnings before income taxes, depreciation and amortization); excludes nonoperating income (e.g., windfalls, natural disaster losses, capital gains/losses).
- Earnings-based constraints often imposed using financial covenants in debt contracts.
- Covenant violations place borrowers in "technical default." $\Rightarrow$ renegotiations and fees (rarely bankruptcy).

Two tales of debt: Lian and Ma (2021)

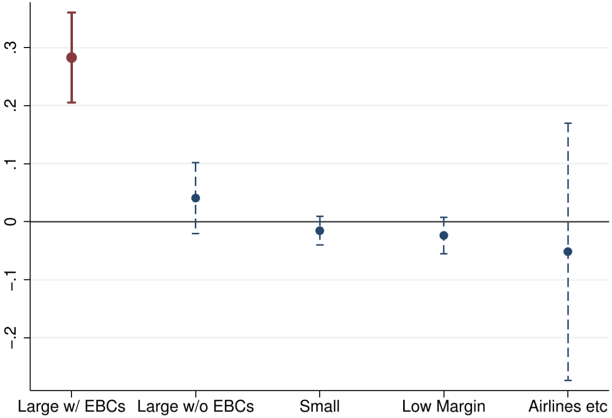

FIGURE IV

Borrowing Sensitivity to Operating Earnings by Firm Group

Figure: Lian and Ma (2021).

- Increasing EBITDA by $1 allows firms to issue $0.28 more debt.
- Does not effect debt issuance for firms that largely use asset–based debt (small/low-margin; airlines).
- IV: FASB rule SFAS 123(r) required firms to start counting option compensation as part of OpEx.
- Real estate values affect issuance of asset–based debt, but not cash flow–based debt.

Revisiting Kiyotaki and Moore

- What happens if we replace farmers' collateral-based borrowing constraint,

$$b_t \leq \frac{(1-\delta)}{R} q_{t+1} k_t,$$

with earnings-based borrowing constraint,

$$b_t \leq \frac{\theta}{R} a k_t.$$

- Choose θ to ensure that farmers have same steady-state leverage.

- Lian and Ma: *"We then compare the equilibrium impact of a shock to productive firms' internal funds in these two scenarios. [...] Using parameters similar to those in Kiyotaki and Moore (1997), we find that the effect on productive firms' capital holding and aggregate output under earnings-based constraints is about one-tenth of that under traditional collateral constraints."*

U.S. vs. Japan

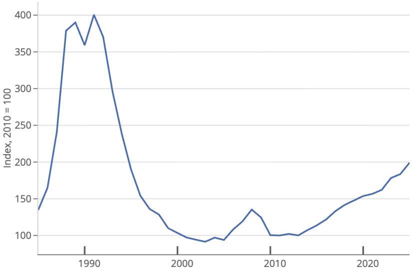

Figure: Tokyo commercial property price index (BIS).

- Corporate debt forms depend on institutions.
- Countries with reorganization-based bankruptcy (e.g., Ch. 11) have more cash flow–based debt.
- Japan: asset–based corporate debt (esp. real estate).
- Comparable regressions show very little sensitivity of net debt issuance to cash flows for Japanese firms.
- Gan (2007) shows that Japanese firms exposed to real estate did worse during 1990s price collapse.
- Lian and Ma (2021) do not find similar results for U.S. firms in Great Recession.

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Quota Distortions

- Many policies / frictions directly constrain quantities without regard to prices.
  - E.g., import quotas, visa caps, zoning restrictions, emissions limits, local content requirements, land use ceilings, taxicab medallions.
  - Missing markets (land markets, credit markets, insurance markets).
- The classic approach to analyzing distortions is to recast them as implicit taxes.
- But mapping quotas to implicit taxes requires detailed info about economy.
- This paper: A general framework for analyzing economies with quota-like distortions.

Preview of Results

- Much like implicit taxes/wedges, quotas can decentralize any feasible allocation.
- But, economies with quotas are constrained eff. and obey macro-envelope conditions.
  - Comparative statics disciplined by simple sufficient statistics.
  - Not subject to Theory of Second Best.

Preview of Results

- Much like implicit taxes/wedges, quotas can decentralize any feasible allocation.
- But, economies with quotas are constrained eff. and obey macro-envelope conditions.
  - Comparative statics disciplined by simple sufficient statistics.
  - Not subject to Theory of Second Best.
- How small quota changes and productivity shocks affect output.
- How large quota changes affect output (i.e., nonlinearities).
- Distance to the efficient frontier (misallocation cost of quotas).

General Framework

- Representative household, N goods indexed by i, F factors indexed by f.
- Real output Y maximizes constant-returns aggregator $\mathcal{D}$,

$$Y = \max_{\{c_1,\ldots,c_N\}} \mathcal{D}(c_1,\ldots,c_N),$$

  subject to the budget constraint,

$$\sum_i^N p_i c_i = \sum_{f=1}^F w_f L_f + \sum_{i=1}^N \Pi_i,$$

  where c_i final demand, p_i prices, L_f factor endowments, w_f wages, and Π_i profits.

General Framework: Quotas

- Each good i produced by competitive firms using constant returns technology,

$$A_i F_i(x_{i1}, \ldots, x_{iN}, L_{i1}, \ldots, L_{iF}),$$

where x_{ij} is use of intermediate good j, L_{if} use of factor f, and A_i productivity shifter.

- A quota restricts the production of good i at a quantity y_i^*,

$$y_i = \min\{y_i^*, A_i F_i(x_{i1}, \ldots, x_{iN}, L_{i1}, \ldots, L_{iF})\}.$$

- Profits for producers of i are revenues less intermediate and factor costs,

$$\Pi_i = p_i y_i - \sum_{i=1}^{N} p_j x_{ij} - \sum_{f=1}^{F} w_f L_{if}.$$

- Take nominal GDP as numeraire, $\sum_i p_i c_i = 1$. Denote Domar weights $\lambda_i \equiv p_i y_i$.

Equilibrium

- Given quotas y_i^*, productivities A_i, production functions F_i, and factor endowments L_f, an equilibrium consists of prices p_i, wages w_f, outputs y_i, final demands c_i, and intermediate / factor input choices x_{ij} and L_{if} such that:
  - Final demands c_i maximizes real output subject to the budget constraint.
  - Each producer maximizes profits, taking prices and quotas as given.
  - For all goods with quotas, $y_i \leq y_i^*$.
  - Resource constraints satisfied:

$$c_i + \sum_{j=1}^{N} x_{ji} \leq y_i \text{ for all } i \qquad \text{and} \qquad \sum_{i=1}^{N} L_{if} \leq L_f \text{ for all } f.$$

Implementing an Allocation Using Quotas

Definition (Feasible allocation)

An allocation $(\{c_i\},\{x_{ij}\},\{L_{if}\})$ is feasible if:

- c_i, x_{ij}, and L_{if} are non-negative for all i, j, and f,
- $y_i \leq A_i F_i(x_{i1},\ldots,x_{iN},L_{i1},\ldots L_{iF})$ for all i,
- Resource constraints are satisfied.

Proposition

Consider some feasible allocation $\mathcal{X}$. Then:

1. *There exists a vector of quotas, $\{y_i^*\}$, such that the equilibrium has allocation $\mathcal{X}$.*
2. *Given these quotas, the allocation $\mathcal{X}$ is efficient.*

Implementing an Allocation Using Quotas

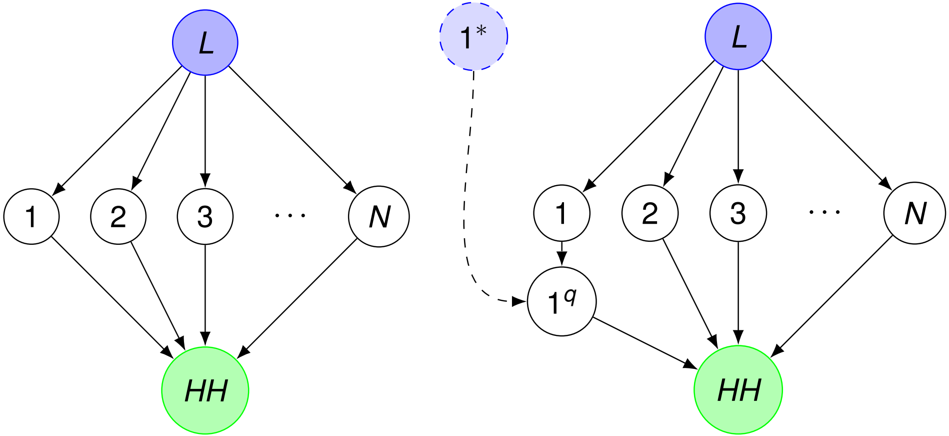
Proposition

Consider some feasible allocation $\mathcal{X}$. Then:

1. *There exists a vector of quotas, $\{y_i^*\}$, such that the equilibrium has allocation $\mathcal{X}$.*
2. *Given these quotas, the allocation $\mathcal{X}$ is efficient.*

- Add nodes/quotas to guarantee that competitive eqm. yields desired allocation.
  - Extends tariff–quota equivalence result in trade to any GE allocation. (e.g., Bhagwati 1965)
- First Welfare Theorem implies allocation is constrained efficient.
  - Can analyze eqm. using tools for efficient economies (e.g., Hulten's Theorem).

Constrained Efficiency: Intuition



(a) Observed economy. (b) Implementation with quota.

- Introduce new factor "rights to produce good 1" with supply y_1^*.
- Introduce new producer 1^q with Leontief production, $y_{1^q} = \min\{y_1, y_1^*\}$.

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Comparative Statics

- Unlike equilibria with wedges, equilibrium with quotas is constrained efficient.
- Comparative statics governed by simple sufficient statistics:

$$\frac{d\log Y}{d\log y_i^*} = \frac{rents_i}{GDP} = \Pi_i, \qquad \frac{d\log Y}{d\log TFP_i} = \frac{sales_i - rents_i}{GDP} = \lambda_i - \Pi_i.$$

where $rents_i$ are excess profits earned by producers that hold quota rights.

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  where $rents_i$ are excess profits earned by producers that hold quota rights.

- If equilibrium efficient, quotas non-binding ($\mathbf{\Pi} = 0$) and we recover Hulten (1978):

$$\frac{d\log Y}{d\log y_i^*} = 0, \qquad \frac{d\log Y}{d\log TFP_i} = \frac{sales_i}{GDP} = \lambda_i.$$

- Holding other quotas fixed, removing a quota always raises output.
  - Holding other wedges fixed, removing a wedge can lower output (Theory of 2nd Best).

Comparison to Wedges/Taxes

- In response to a change in a wedge τ_j, we need

$$d\log Y = \sum_i \Pi_i \frac{\partial \log y_i}{\partial \log \tau_j} d\log \tau_j,$$

- In response to a change in a productivity A_j, we need

$$d\log Y = \sum_i \Pi_i \frac{\partial \log y_i}{\partial \log A_j} d\log A_j + (\lambda_i - \Pi_i)\, d\log A_j,$$

- Need how every y_i responds (which typically requires knowledge of everything).
- Theory of second best means even the sign is unknown.

Simple Example

- Consider a small open economy with balanced trade.
- Trade domestic good for foreign good, domestic consumption is

$$Y = \left(\omega c_d^{\frac{\theta-1}{\theta}} + (1-\omega) c_f^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}},$$

- Consider quota or import tax that generates Π_f revenues.
- An increase TFP of exporters under quotas :

$$\Delta \log Y \approx (\lambda_f - \Pi_f) \Delta \log A_f$$

- An increase TFP of exporters under taxes:

$$\Delta \log Y \approx \left(\lambda_f - \Pi_f + \frac{\Pi_f}{1-\Pi_f} \left[(\lambda_f - \Pi_f) + \theta \left(1 - \lambda_f \right) \right] \right) \Delta \log A_f.$$

- Need to know θ.

Empirical Example 1: Zoning Restrictions on Single-Family Housing

- What are the gains from loosening zoning restrictions on single-family housing?

- To a first order, given by value of rights to build new single-family housing.
  - Gyourko and Krimmel (2021) isolate "zoning taxes" by comparing land value for parcels with rights to build new single-family housing to value of land with existing housing.

- Note: Efficiency gains expressed directly in terms of new units permitted.
  - Wedge approach would require mapping quantities into changes in effective zoning tax.

Empirical Example 1: Zoning Restrictions on Single-Family Housing

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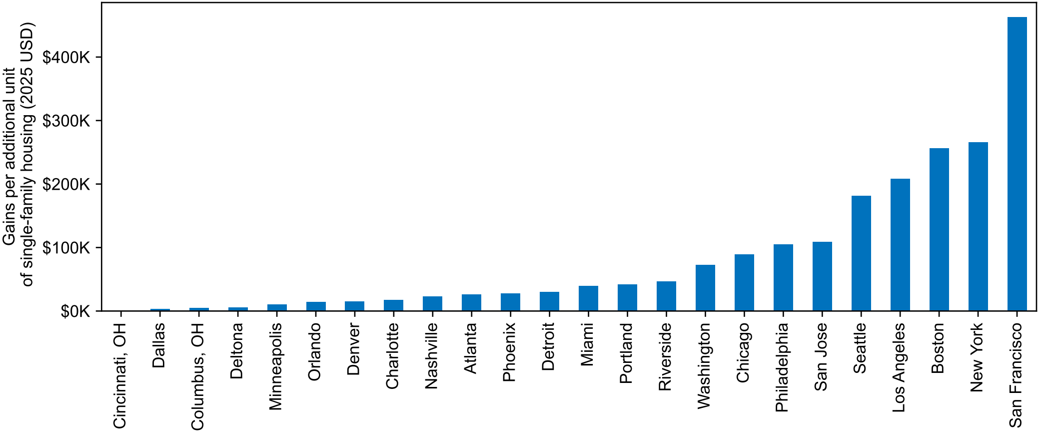

Empirical Example 2: H-1B Visa Quota

- H-1B visa for high-skill foreign workers capped at 85,000 visas/year since mid-2000s.
  - Applications each year exceed cap, so visas awarded through a lottery.
- To a first order, gains from increasing quota equal to rents earned by visa winners:

$$d \log Y = \Pi_i d \log y_{i^*} \approx \frac{\Pi_i}{y_{i^*}} dy_{i^*}.$$

- Clemens (2013) compares earnings of winners vs. losers of 2007 H-1B lottery.
  - Earnings for workers who won lottery were \$18,823 higher (in 2025 USD) two years after.
  - Assuming workers paid their marginal product, difference in earnings isolates excess profits of workers with right to migrate.

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Nonlinearities

- What about the effects of a large liberalization?
- Since first-order effect depends on rents, nonlinearities depend on **change** in rents:

$$\Delta \log Y \approx \Pi_i \Delta \log y_i^* + \frac{1}{2} \underbrace{\frac{d\Pi_i}{d \log y_i^*}}_{\Delta \text{ rents}} (\Delta \log y_i^*)^2.$$

- If rents rise with quota, first-order approx. understates gains from large liberalization.

Illustration of Nonlinearities

- Always concave around efficiency ($\Pi_i = 0$). Rents rise when slack quota tightened.
- Can be convex away from frontier: tightening already-binding quota may lower rents.
- When convex, nonlinearities amplify gains from liberalization / mitigate further harms.

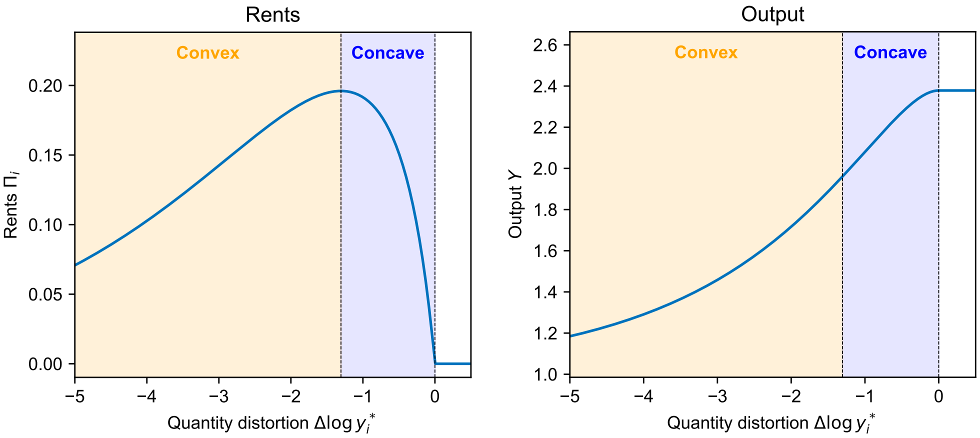
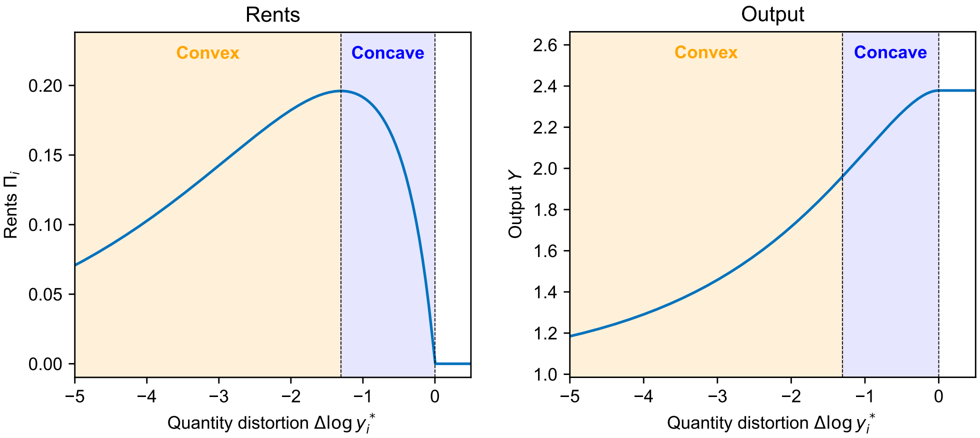


Illustration of Nonlinearities

- Before quota binds, first-order losses are zero.
- Once activity is nearly banned, further tightening does nothing.
- Output responses convex on the wrong side of the Laffer curve.



Special Case: Monopolist

- A tractable special case: Monopolist chooses output quota to maximize real profits.

Proposition (Nonlinearities with a Monopolist)

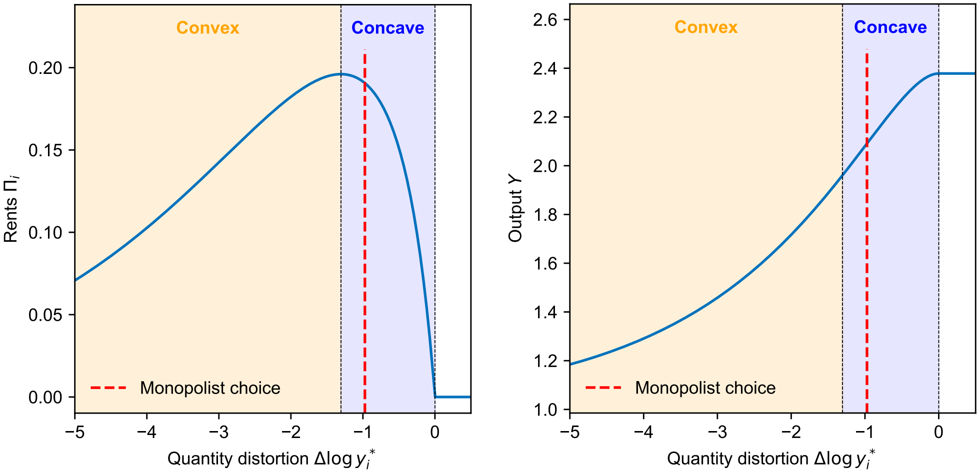
Effect of changes in the monopolist's quantity on output are

$$\Delta \log Y \approx \Pi_i d \log y_i - \frac{1}{2}\Pi_i^2 (d \log y_i)^2.$$

- Intuition: Monopolist's first-order condition determines response of profits to quantity.
- Profits sufficient to calculate how gains from increasing production peter out.

Illustration of Nonlinearities

- Monopolist is never on the wrong side of the Laffer curve.



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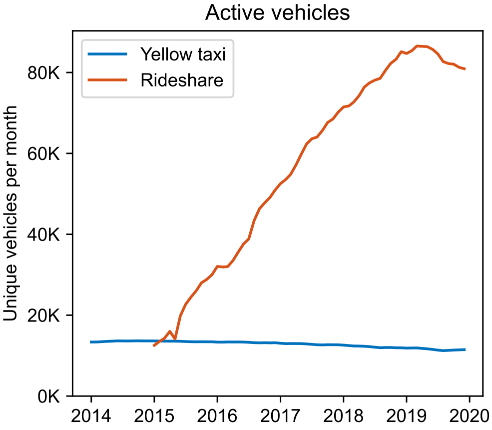
- If rents rise with quota, first-order approx. understates gains from large liberalization.
- Can solve for Δ rents using input-output network & elasticities. (à la Baqaee and Farhi 2019).
- Or obtain Δ rents from ex-post variation: Taxicab medallions in New York.

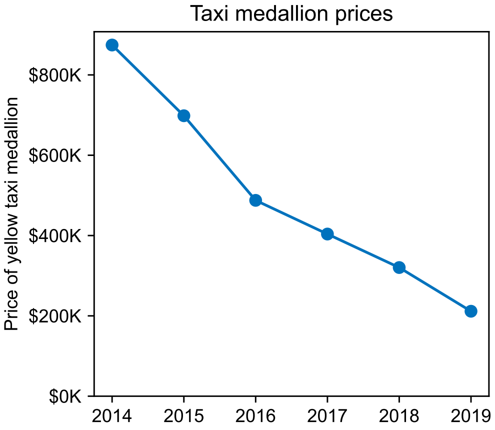
Empirical Example: Taxicab Medallions

- Since 1937, quota on NYC taxicab medallions restricting total supply to $\approx$ 14k.

Empirical Example: Taxicab Medallions

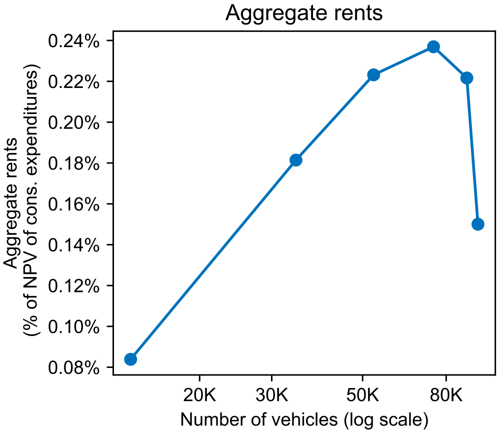
- Since 1937, quota on NYC taxicab medallions restricting total supply to $\approx$ 14k.
- Use arrival of rideshare apps in NYC to quantify gains from relaxing quota on cabs.

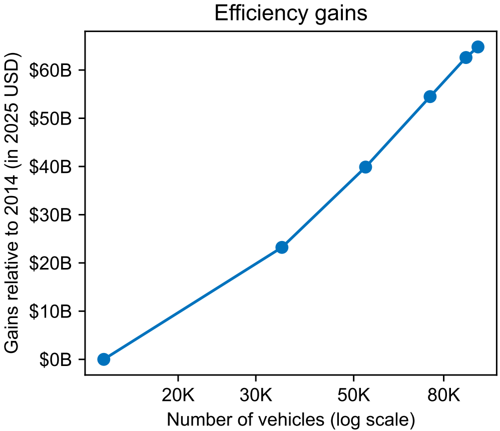




Nonlinearities: Taxicab Medallions

- Assume medallion price reflects NPV of steady-state rents (Π_i = NPV rents / wealth).
- Gains from relaxing taxicab quota are $\Delta \log Y_t \approx \left(\Pi_{it} + \frac{1}{2} d\Pi_{it}\right) \Delta \log y_{it}^*$.





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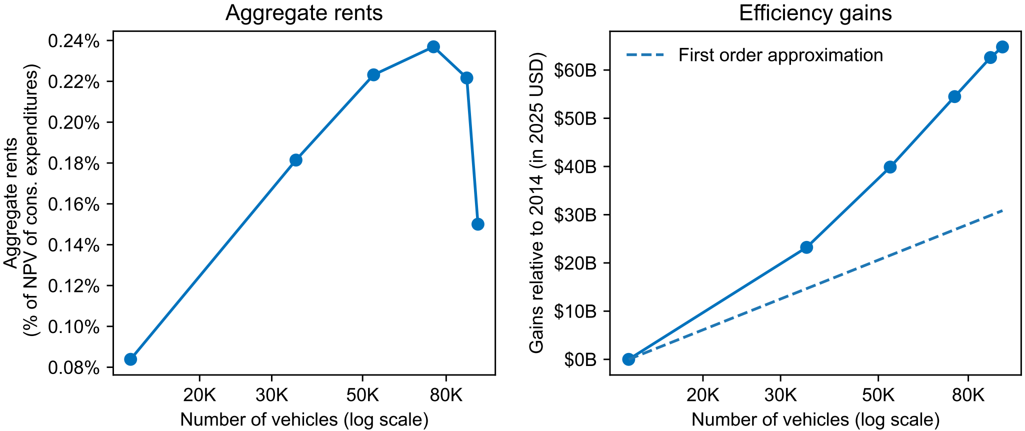

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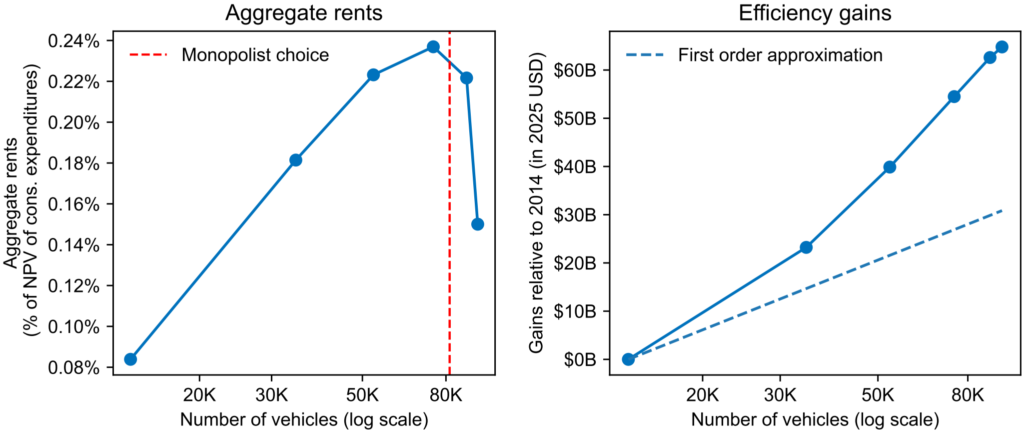

Empirical Example: Taxicab Medallions

- Gains from relaxing quota from 2014 to 2019 steady-state level.
  - Cumulating gains over each year: $\Delta \log Y \approx \sum_t \left(\Pi_{it} + \frac{1}{2} d\Pi_{it}\right) \Delta \log y_{it}^*$.

	Change from 2014–2019
Output gains	\$64.8B
Gains per New York MSA household	\$7,237
% of NPV of transportation expenditures	2.58%

Nonlinearities with Anticipated Path of Quota Changes

- Prior calculation assumes each year is a new steady-state.

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- Suppose initial steady-state quota $\bar{y}$ and shock $\Delta y^*(t)$ that unfolds over time.
- To a first-order,

$$\Delta \log Y \approx P \frac{\int_0^\infty e^{-\int_0^t r(s)ds} \Delta y^*(t)\, dt}{\int_0^\infty e^{-\int_0^t r(s)ds} dt}$$

  where P is the asset price.
- Asset price reflects steady-state rents.

Nonlinearities with Anticipated Path of Quota Changes

- Prior calculation assumes each year is a new steady-state.
- Suppose initial steady-state quota $\bar{y}$ and shock $\Delta y^*(t)$ that unfolds over time.
- To a second-order,

$$\Delta \log Y \approx P \frac{\int_0^\infty e^{-\int_0^t r(s)ds} \Delta y^*(t)\, dt}{\int_0^\infty e^{-\int_0^t r(s)ds} dt} + \frac{1}{2} \Delta P \frac{\int_0^\infty e^{-\int_0^t r(s)ds} \Delta y^*(t)^2\, dt}{\int_0^\infty e^{-\int_0^t r(s)ds} \Delta y^*(t)\, dt},$$

  where P is the asset price, and ΔP is change in asset price.

- Asset price reflects steady-state rents. Change in asset price reflects Δ rents.

Nonlinearities in Dynamic Environment

$$\Delta \log Y \approx P \frac{\int_0^\infty e^{-\int_0^t r(s)ds} \Delta y^*(t)\, dt}{\int_0^\infty e^{-\int_0^t r(s)ds} dt} + \frac{1}{2} \Delta P \frac{\int_0^\infty e^{-\int_0^t r(s)ds} \Delta y^*(t)^2\, dt}{\int_0^\infty e^{-\int_0^t r(s)ds} \Delta y^*(t)\, dt},$$

- If quota changes fully anticipated, use observed $\Delta y^*(t)$ and asset price change.

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- If quota changes fully anticipated, use observed $\Delta y^*(t)$ and asset price change.

- For one-time, persistent change $\Delta y^*(t) = \Delta y^*$, so formula becomes:

$$\Delta \log Y \approx P \Delta y^* + \frac{1}{2} \Delta P \Delta y^* \approx \Pi \Delta \log y^* + \frac{1}{2} \Delta \Pi \Delta \log y^*.$$

Thus, for a sequence of unanticipated, persistent quota changes (indexed by h'):

$$\Delta \log Y \approx \sum_0^h \left(\Pi(h') + \frac{1}{2} \Delta \Pi(h') \right) \Delta \log y^*(h').$$

where $\Pi(h)$ is market value of asset at h (i.e., earlier calculation is special case).

Empirical Example: Taxicab Medallions

- Gains from relaxing quota over 2014–2019.
  - Unanticipated: Cumulate gains over each year: $\Delta \log Y \approx \sum_t \left(\Pi_{it} + \frac{1}{2} d\Pi_{it}\right) \Delta \log y^*_{it}$.

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Distance to the Efficient Frontier

- What are gains from eliminating a quota altogether?
- To a second-order, gains are average of first-order effect at distorted pt and efficient pt:

$$\Delta \log Y \approx \frac{1}{2}\Pi_i(\Delta \log y_i^*) + \frac{1}{2}(0),$$

where $\Delta \log y_i^* = \log y_i^* - \log y_i^{\text{eff}}$ is gap between quota and undistorted level.

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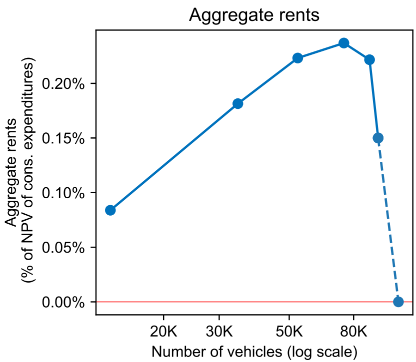

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- Estimate increase in quota necessary to decrease rents to zero.
- If rents fall quickly when quota relaxed, close to efficiency $\Rightarrow$ smaller gains.

Empirical Example: Taxicab Medallions

- Gains from relaxing quota over 2014–2019.
  - Unanticipated: Cumulate gains over each year: $\Delta \log Y \approx \sum_t \left(\Pi_{it} + \frac{1}{2} d\Pi_{it}\right) \Delta \log y^*_{it}$.
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- Not efficient at the end. What is the remaining distance to frontier?
  - Use elasticity of rents to quota in final year: $\Delta \log Y \approx \frac{1}{2} \Pi_i \left[- \frac{d \log \Pi_i}{d \log y_i^*} \right]^{-1}$.

	Change from 2014–2019		Distance
	Unanticipated	Anticipated	to frontier
Output gains	\$64.8B	\$81.2B	\$2.3B
Gains per New York MSA household	\$7,237	\$9,066	\$253
% of NPV of transportation expenditures	2.58%	3.23%	0.09%

Nonlinearities: Multiple Quotas

- Method scales up to multiple interacting quotas

$$\Delta \log Y \approx \mathbf{\Pi}' d \log \mathbf{y}^* + \frac{1}{2} (d \log \mathbf{y}^*)' \frac{d\mathbf{\Pi}}{d \log \mathbf{y}^*} (d \log \mathbf{y}^*),$$

- Quota demand system $\frac{d\mathbf{\Pi}}{d \log \mathbf{y}^*}$ summarizes responses of rents to quotas.

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Illustrative Example: Multiple Quotas

- Consider horizontal economy. Response to changes in quotas on firms 1 and 2:

$$\Delta \log Y \approx \underbrace{\Pi_1 d\log y_{1^*} + \Pi_2 d\log y_{2^*}}_{\text{First order}}$$

$$\underbrace{+(1/2)\left(\frac{\partial \Pi_1}{\partial \log y_1^*}(d\log y_{1^*})^2 + \frac{\partial \Pi_2}{\partial \log y_2^*}(d\log y_{2^*})^2 + 2\frac{\partial \Pi_1}{\partial \log y_2^*}(d\log y_{1^*})(d\log y_{2^*})\right)}_{\text{Second order}}.$$

- $\frac{\partial \Pi_1}{\partial \log y_2^*}$ determines whether relaxing y_{1^*} amplifies/reduces gains from relaxing y_{2^*}.

- In horizontal economy, $\frac{\partial \Pi_1}{\partial \log y_2^*} > 0$ if

$$\theta < 1 - \frac{(\lambda_1 - \Pi_1)(\lambda_2 - \Pi_2)}{(1 - \lambda_1 - \lambda_2)\Pi_1 \Pi_2}.$$

Necessary condition is that $\theta < 1$, i.e., outputs of firms 1 and 2 are complements.

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- Quota demand system $\frac{d\mathbf{\Pi}}{d \log \mathbf{y}^*}$ summarizes responses of rents to quotas.
- Similarly, gains from eliminating quotas simultaneously given by:

$$\Delta \log Y \approx -\frac{1}{2} \mathbf{\Pi}' \left[\frac{d\mathbf{\Pi}}{d \log \boldsymbol{y}^*} \right]^{-1} \mathbf{\Pi}.$$

- If *i*'s rents fall when *j*'s quota relaxed, then

gains from relaxing both quotas $<$ sum of gains from relaxing each.

Empirical Example: China's Textile & Clothing Exports

- 1975–1994 Multi-Fiber Agreement capped China's textile & clothing exports to US, EU.
- Staged phase-out:
  - Jan 2002 (Phase III): Knit fabrics, gloves, dressing gowns, brassieres, etc.
  - Jan 2005 (Phase IV): Silk, wool, and cotton textiles, other apparel categories, etc.
- Obtain quota demand system using initial rents & response of exports to liberalization.
- Use quota auction prices for initial rents: $\Pi_{\text{Phase III}} = \$520\text{M}$, $\Pi_{\text{Phase IV}} = \$1583\text{M}$.

Quota Demand System: China's Textile and Clothing Exports

1. Use quota auction prices for initial quota profits.

2. Use response of export quantities to phase-out to recover quota demand system, H.

3. Use H to estimate gains from relaxing any subset of quotas.

Quota Demand System: China's Textile and Clothing Exports

1. Use quota auction prices for initial quota profits.
   - Market prices for quotas to export in each product category in 2001.
   - Quota profits in 2025 USD, $\Pi_{\text{Phase III}} = \$520\text{M}$, $\Pi_{\text{Phase IV}} = \$1583\text{M}$.
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Quota Demand System: China's Textile and Clothing Exports

1. Use quota auction prices for initial quota profits.

2. Use response of export quantities to phase-out to recover quota demand system, H.

   - Because profits go to zero when quotas are removed, $H = \partial \Pi / \partial \log y$ solves:

$$\begin{aligned} \Pi_{\text{Phase III}} &= (d \log y^{\text{III}}_{\text{Phase III}})\, H_{11}, \\ \Pi_{\text{Phase III}} &= (d \log y^{\text{IV}}_{\text{Phase III}})\, H_{11} + (d \log y^{\text{IV}}_{\text{Phase IV}})\, H_{12}, \\ \Pi_{\text{Phase IV}} &= (d \log y^{\text{III}}_{\text{Phase IV}})\, H_{21} + (d \log y^{\text{IV}}_{\text{Phase IV}})\, H_{22}, \end{aligned}$$

     where $d \log y^{\text{III}}_x$ ($d \log y^{\text{IV}}_x$) is the change in exports for x goods following Phase III (IV).

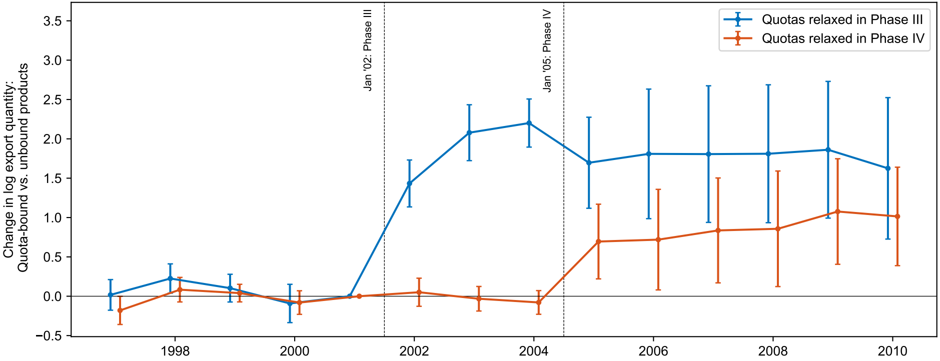
   - Symmetry implies $H_{12} = H_{21}$.

3. Use H to estimate gains from relaxing any subset of quotas.

Quota Demand System: China's Textile and Clothing Exports

$$\log y_{ict} = \beta_t^{\text{Phase III}} (\text{Binding}_c \times 1\{c \text{ quota relaxed in Phase III}\} \times 1\{\text{year} = t\})$$
$$+ \beta_t^{\text{Phase IV}} (\text{Binding}_c \times 1\{c \text{ quota relaxed in Phase IV}\} \times 1\{\text{year} = t\}) + \alpha_t + \delta_i + \varepsilon_{ict},$$

- E.g., $\beta_t^{\text{Phase III}}$ is change in Phase III good exports in t relative to unconstrained goods.



Quota Demand System: China's Textile and Clothing Exports

1. Use quota auction prices for initial quota profits.

2. Use response of export quantities to phase-out to recover quota demand system, H.

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   $$\Pi_{\text{Phase III}} = (d \log y^{\text{IV}}_{\text{Phase III}})\, H_{11} + (d \log y^{\text{IV}}_{\text{Phase IV}})\, H_{12},$$
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Quota Demand System: China's Textile and Clothing Exports

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   - Estimated inverse quota demand system:

$$\Pi = \begin{bmatrix} \Pi_{\text{Phase III}} \\ \Pi_{\text{Phase IV}} \end{bmatrix} = \begin{bmatrix} \$520\text{M} \\ \$1583\text{M} \end{bmatrix}, \qquad \frac{d \log \boldsymbol{\Pi}}{d \log \mathbf{y}^*} = \begin{bmatrix} -0.472 & -0.200 \\ -0.066 & -1.149 \end{bmatrix}.$$

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Intervention	Efficiency gains (2025 USD)
(A) Relaxing Phase III quotas only	\$565M
(B) Relaxing Phase IV quotas only	\$706M
(C) Relaxing both Phase III and IV quotas	\$1075M
Difference: $\text{C} - (\text{A} + \text{B})$	\$196M

Distance to Frontier: Argentina's Capital Controls

- In September 2019, Argentina reimposed capital controls, restricting amount that households and firms could access foreign exchange markets.
- Restrictions led to decline in capital outflows from \$7.2B to $<$\$1.5B per month.
- What are the efficiency losses from these restrictions?
  - Permitted transactions take place at official exchange rate $\bar{e}$.
  - Unconstrained transactions take place at black-market exchange rate e.
  - Gap between $\bar{e}$ and e are profits earned by permit to exchange under controls.

Distance to Frontier: Argentina's Capital Controls

- Option 1: $\Delta \log Y \approx -\frac{1}{2}\Pi_i d \log y_{i^*} \approx -\frac{1}{2}\left(\log e/\bar{e}\right) dy_{i^*}$.
  Measure distortion dy_{i^*} as gap relative to 2016–2019 outflows.
- Option 2: Elasticity of exchange rate to currency purchases, $\log e/\bar{e} = \theta\left(dy_i^*/\mathrm{GDP}\right)$.
  Then, $\frac{\Delta Y}{\mathrm{GDP}} \approx -\frac{1}{2}\frac{1}{\theta}\left(\log e/\bar{e}\right)^2$. (Blanchard et al. 2015, Adler et al. 2019.)

Distance to Frontier: Argentina's Capital Controls

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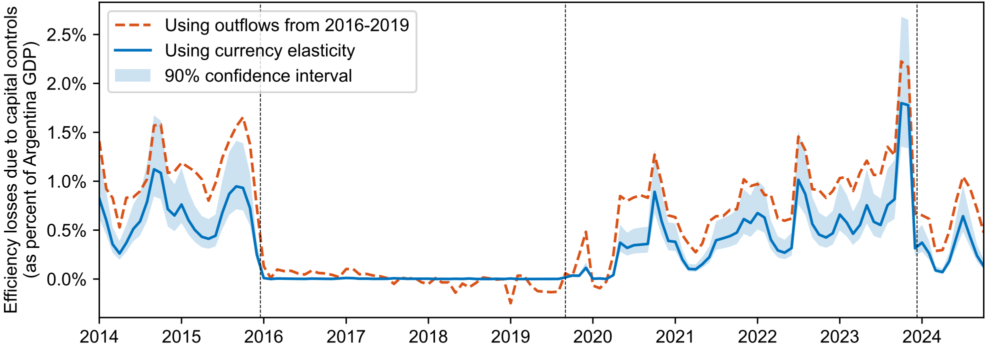

Conclusion

- General framework for analyzing economies with quota distortions.
- Comparative statics simple because of constrained efficiency.
- Nonlinearities, distance to efficient frontier using quota demand system.
- Can be identified with local variation, e.g., response of rents to quota changes.
- Examples of how to apply results (H-1Bs, zoning, taxicabs, export/capital controls).

Adler, G., N. Lisack, and R. C. Mano (2019). Unveiling the effects of foreign exchange intervention: A panel approach. *Emerging Markets Review* (100620).

Baqaee, D. R. and E. Farhi (2019). The macroeconomic impact of microeconomic shocks: Beyond Hulten's theorem. *Econometrica 87*(4), 1155–1203.

Bhagwati, J. (1965). *Trade, growth and the balance of payments*, Chapter On the equivalence between tariffs and quotas, pp. 53–67. Rand McNally.

Blanchard, O., G. Adler, and I. de Carvalho Filho (2015). Can foreign exchange intervention stem exchange rate pressures from global capital flow shocks. IMF Working Paper.

Clemens, M. A. (2013). Why do programmers earn more in Houston than Hyderabad? Evidence from randomized processing of US visas. *American Economic Review 103*(3), 198–202.

Gyourko, J. and J. Krimmel (2021). The impact of local residential land use restrictions on land values across and within single family housing markets. *Journal of Urban Economics 126*, 103374.

Hulten, C. R. (1978). Growth accounting with intermediate inputs. *The Review of Economic Studies 45*(3), 511–518.