Lecture 8: Other Models of Markups

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ECON 416-1

Roadmap

- We covered two canonical models of variable markups.
- In those models, we had single-product firms pricing according to Lerner index.
- Today we will start on three alternative models of markup determination.
  - Limit pricing.
  - Cannibalization.
  - Search. [next class]

Table of Contents

Peters (2020): Setup

- Output is Cobb-Douglas aggregate across markets $i \in [0, 1]$.
  Each market contains perfectly substitutable goods produced by firms $f \in S_{it}$.

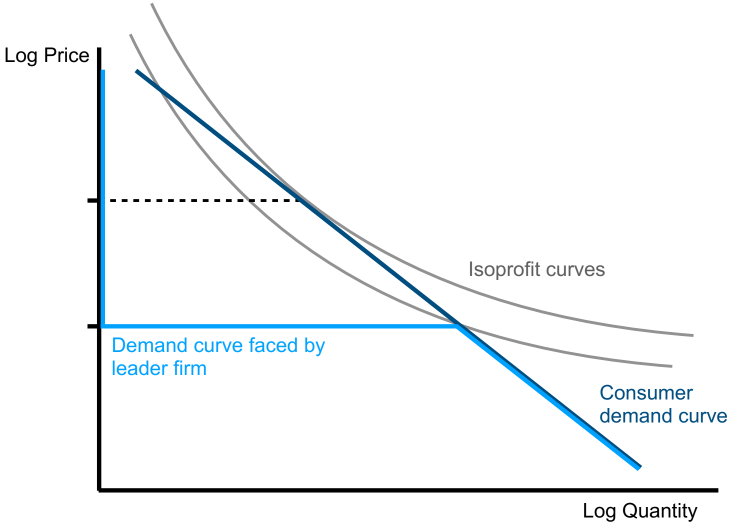
$$\log Y_t = \int_0^1 \log y_{it} di, \qquad \text{where} \qquad y_{it} = \sum_{f \in S_{it}} y_{fit}.$$

- Output of firm f in market i is

$$y_{fi} = A_{fi} L_{fi}.$$

- Let A_{it} and A_{it}^F denote the highest and second highest A_{fit} for $f \in S_{it}$.

Kinked demand curve



- Elasticity of market demand curve $= 1$.
- In the absence of competitor, $\mu_i \to \infty$.
- Firm residual demand curve kinked at price of most productive "follower."

$$\mu_i = \frac{p_i}{mc_i} = \frac{mc_i^F}{mc_i} = \frac{A_i}{A_i^F}.$$

Peters (2020): Misallocation

- Cobb-Douglas preferences imply that

$$p_{it}y_{it} = Y_t,$$

where we take the price of output as the numeraire.

- Thus, the cost of labor used by market i is

$$w_t l_{it} = Y_t / \mu_{it}.$$

- Aggregating across markets gives us usual formula for labor share Λ,

$$\Lambda_t = \frac{Y_t}{\int_0^1 w_t l_{it}} = \int_0^1 \mu_{it}^{-1} di = \frac{w_t L_{Pt}}{Y_t},$$

where L_{Pt} is the total use of production labor.

Peters (2020): Misallocation

- Aggregate output is

$$\log Y_t = \int_0^1 \log A_{it} l_{it} di$$
$$= \int_0^1 \log A_{it} di + \log L_{Pt} + \int_0^1 \log \frac{1}{\mu_{it}} di - \log \int_0^1 \frac{1}{\mu_{it}} di.$$

  Thus,

$$Y_t = A_t L_{Pt} = A_t^{\text{eff}} \mathcal{M}_t L_{Pt},$$

  where

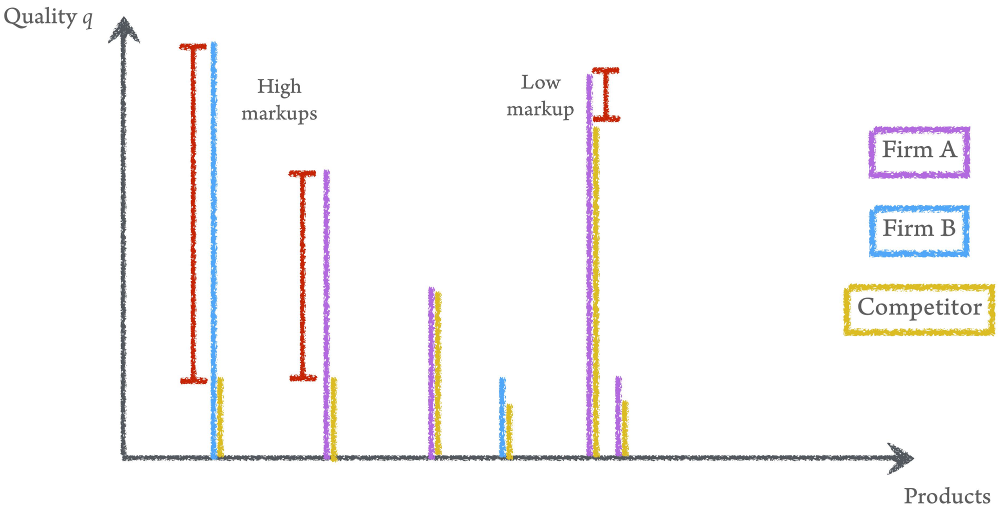
$$\log A_t^{\text{eff}} = \int_0^1 \log A_{it} di, \qquad \text{and} \qquad \mathcal{M}_t = \frac{\exp \int_0^1 \log \mu_{it}^{-1} di}{\int_0^1 \mu_{it}^{-1} di}.$$

- $\mathcal{M}_t \leq 1$. Under what conditions is $\mathcal{M}_t = 1$?

Peters (2020): Quality ladders

- Quality ladder model (Aghion and Howitt 1992, Klette and Kortum 2004).
- For rung r on ladder,
$$A_{r+1} = \lambda A_r.$$
- Thus,
$$\mu_{it} = \frac{A_{it}}{A_{it}^F} = \lambda^{r_{it} - r_{it}^F} = \lambda^{\Delta_{it}},$$
where $\Delta_{it} = r_{it} - r_{it}^F$ is gap in rungs between leader and follower in market i.
- Own innovation vs. creative destruction has different effects on markups:
  - Current producer increases its productivity: Δ_{it} increases by one.
  - Creative destruction by new producer: $\Delta_{it} = 1$.

Peters (2020): Quality ladders



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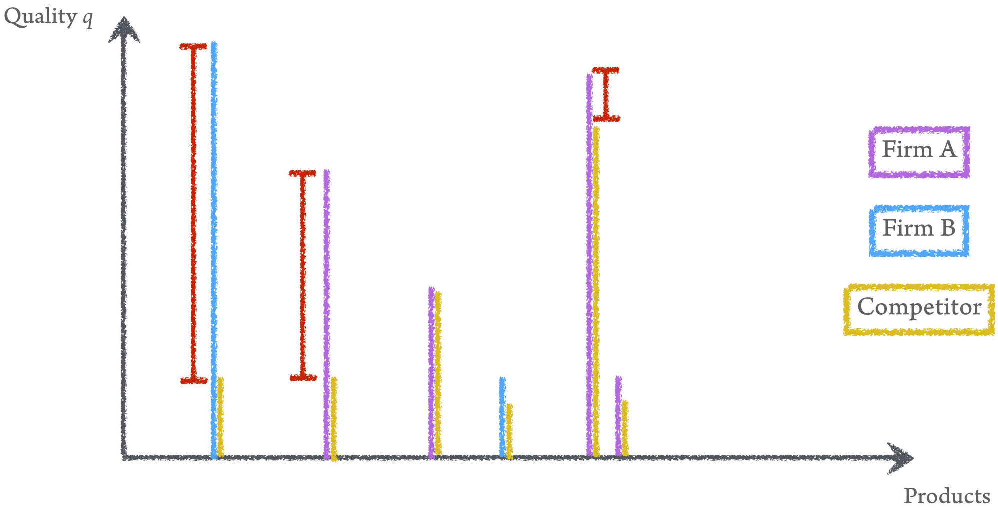

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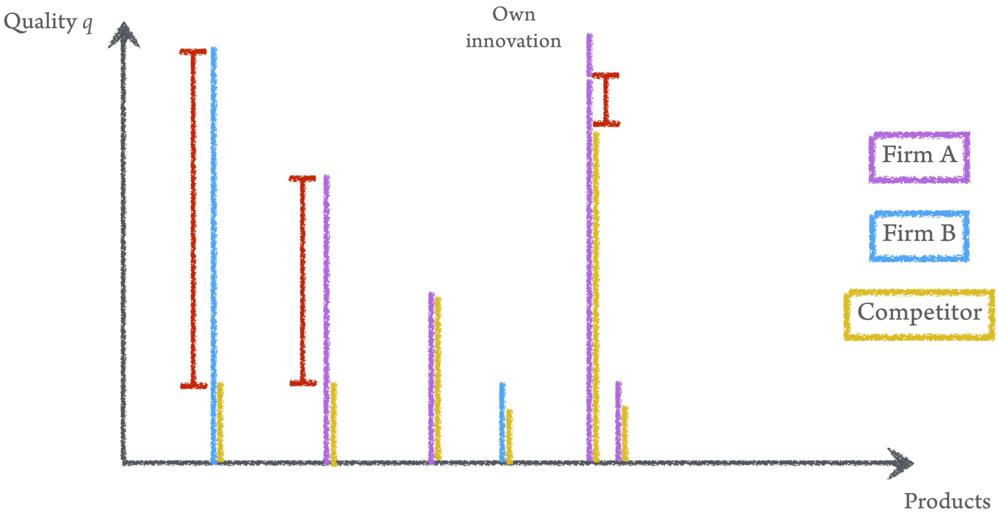

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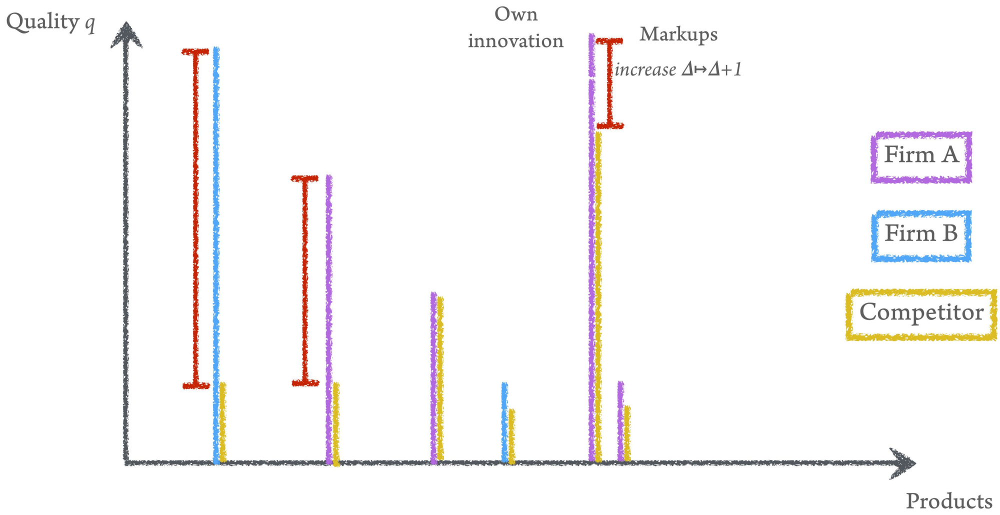

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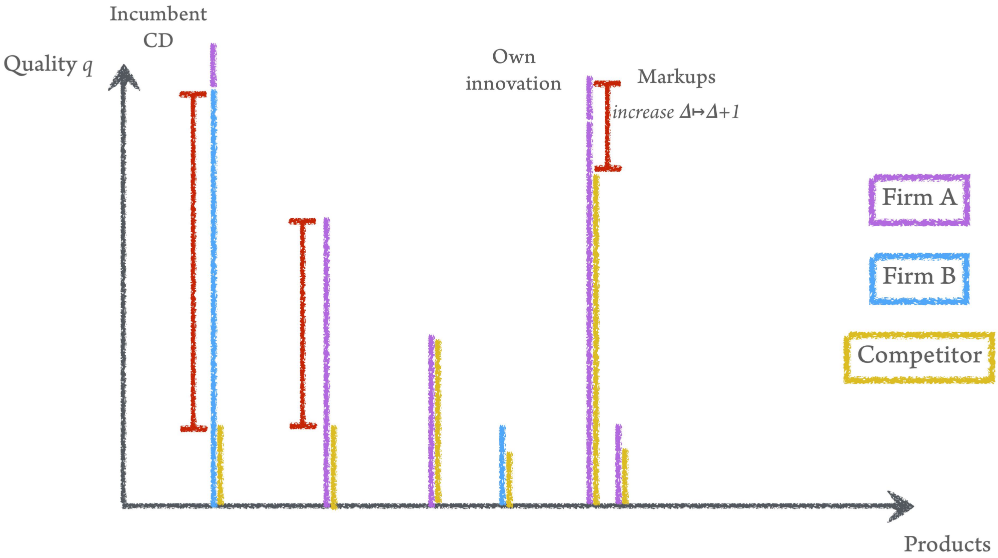

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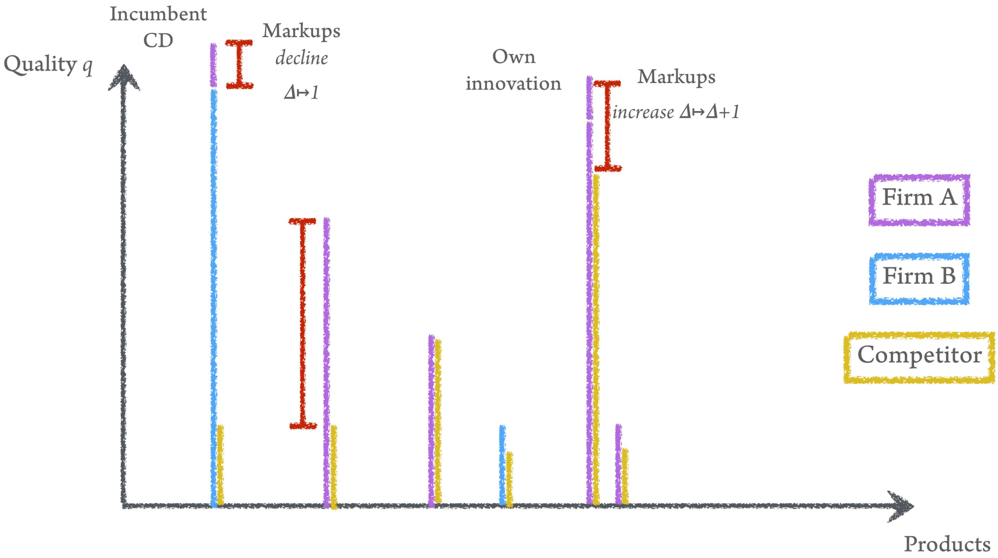

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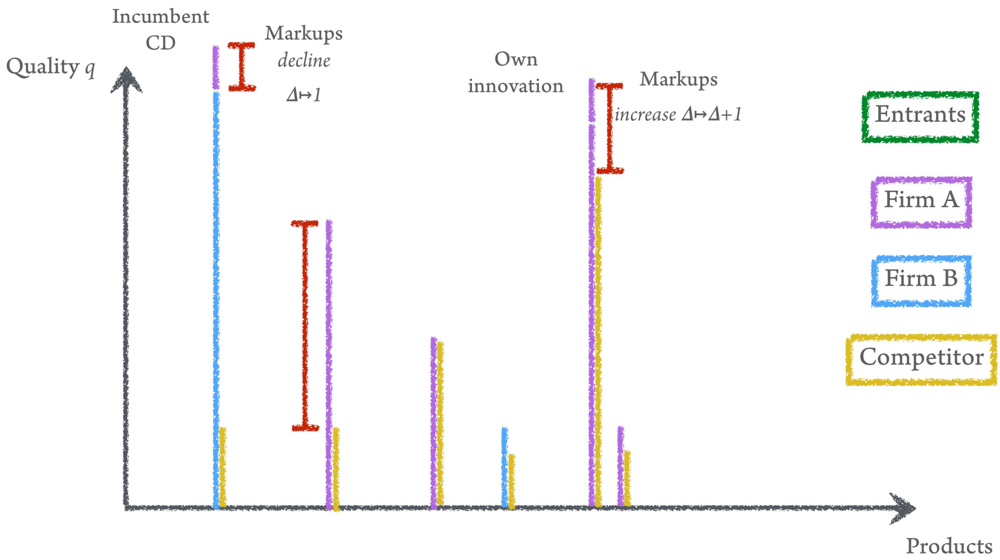

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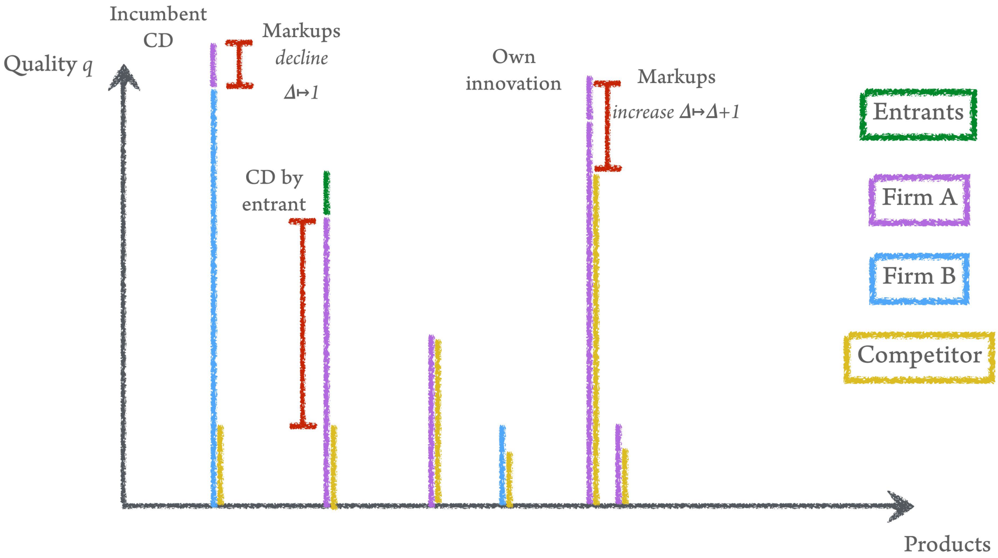

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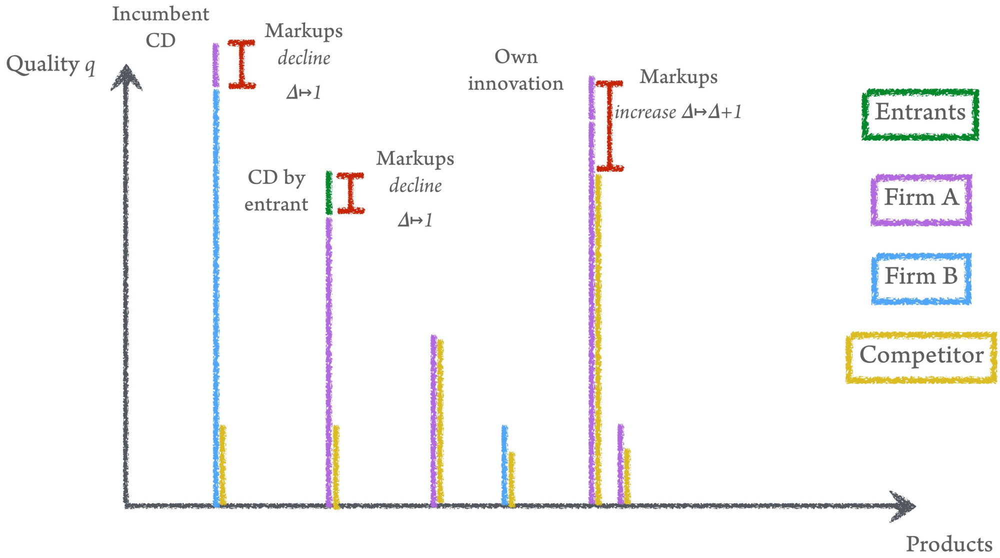

Peters (2020): Endogenous markup distribution

- A_t^{eff} pinned down by distribution of A_{it} across markets.

- $\mathcal{M}_t$ pinned down by distribution of μ_{it}, and thus Δ_{it}, across markets.

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- A_t^{eff} pinned down by distribution of A_{it} across markets.
- $\mathcal{M}_t$ pinned down by distribution of μ_{it}, and thus Δ_{it}, across markets.
- Denote rate of creative destruction by τ and rate of own innovation by I.

$$\Delta_{it+dt} = \begin{cases} \Delta_{it} + 1 & \text{with probability } I dt, \\ 1 & \text{with probability } \tau dt, \\ \Delta_{it} & \text{with probability } (1 - I - \tau) dt, \end{cases}$$

- In steady state, with $\nu(\Delta)$ denoting share of markets with gap Δ,

$$\begin{aligned} &\text{Outflows} && = \text{Inflows} && \\ &(\tau + I)\nu(\Delta) && = I\nu(\Delta - 1) && \text{for } \Delta \geq 2, \\ &I\nu(\Delta) && = \tau(1 - \nu(1)) && \text{for } \Delta = 1. \end{aligned}$$

Peters (2020): Endogenous markup distribution

- Solving yields, steady-state distribution:

$$v(1) = \frac{\tau}{I+\tau}, \qquad v(\Delta) = \frac{I}{\tau+I} v(\Delta-1), \;\; \Delta \geq 2.$$

$$\Rightarrow v(\Delta) = \left(\frac{I}{I+\tau}\right)^{\Delta-1} \frac{\tau}{I+\tau}.$$

- Since $\log \mu_{it} = \Delta_{it} \log \lambda$, CDF of markups is (approximately)

$$G(\mu) = 1 - \mu^{-\theta}, \qquad \text{where} \qquad \theta = \log\left(1 + \frac{\tau}{I}\right) / \log \lambda .$$

(Note: Continuous approximation to discrete Δ. See Peters (2020) appendix A-1.3.)

- Endogenous Pareto tail of markup distribution:
  - More churn from creative destruction ($\tau >> I$), thinner tail.
  - Larger innovation step size λ, thicker tail.

Peters (2020): Endogenous markup distribution

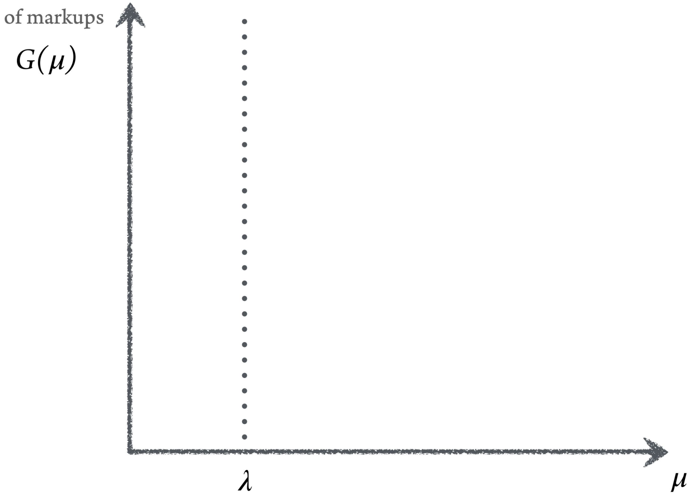
$$G(\mu) = 1 - \mu^{-\theta}, \qquad \text{where} \qquad \theta = \log\left(1 + \frac{\tau}{I}\right) / \log\lambda .$$

- Thus, steady-state labor share and aggregate misallocation,

$$\Lambda = \int_0^1 \mu_{it}^{-1} di = \frac{\theta}{1+\theta},$$
$$\mathcal{M} = \frac{\exp \int_0^1 \log \mu_{it}^{-1} di}{\int_0^1 \mu_{it}^{-1} di} = e^{-1/\theta} \frac{1+\theta}{\theta}.$$

- As $\theta \to \infty$, markups fall toward one, and labor share rises toward one.
- As $\theta \to \infty$, tail becomes thinner, markup dispersion falls.
- Growth rate is $(I + \tau) \log\lambda$, but misallocation only depends on relative intensity!

Peters (2020): Endogenous markup distribution



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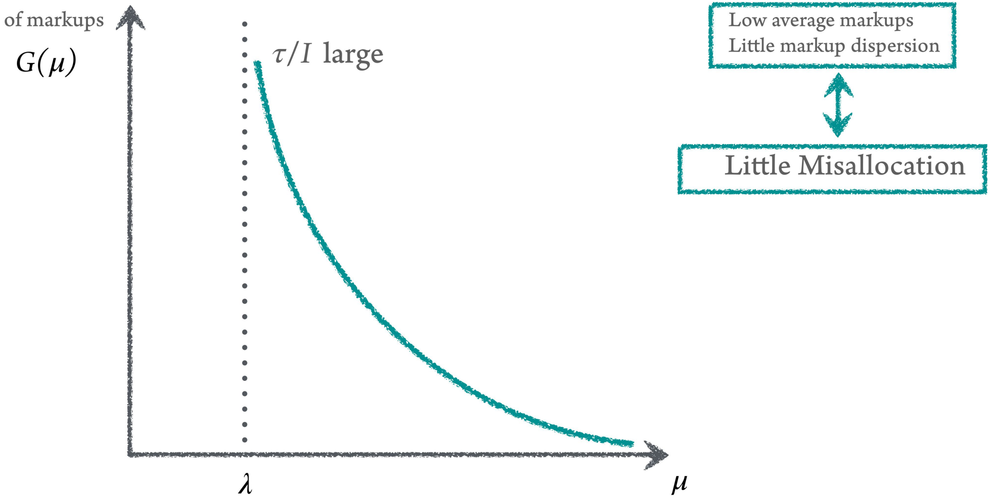

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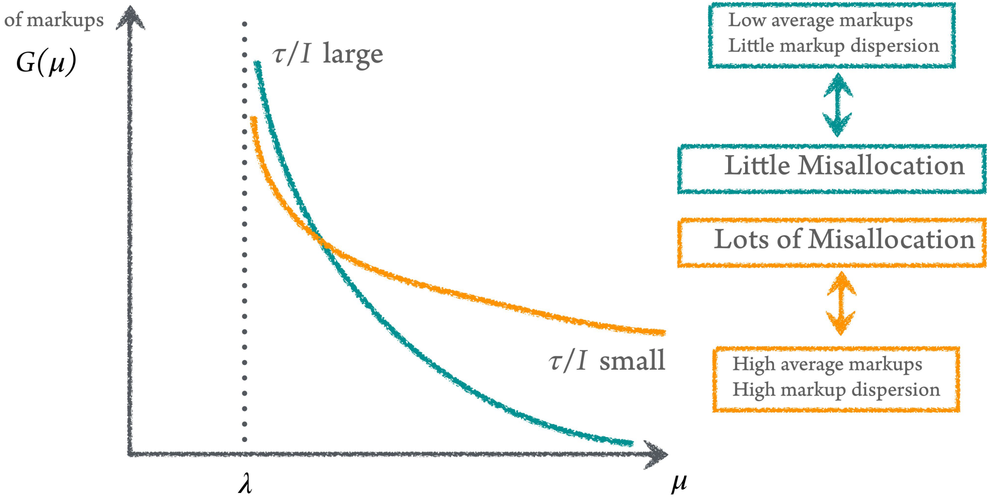

Peters (2020): Innovation problem

- Denote value function of firm with n products and gaps $\{\Delta_{it}\}_{i=1}^{n}$ by $V_t(n,[\Delta_i])$.
- Free entry with linear entry technology $\Rightarrow$ one unit of labor generates φ_z ideas.

$$V(1,1) = w_t/\varphi_z.$$

- Firms choose rates of own innovation I_i and expansion x_i for each market to max:

$$\begin{aligned} r_t V_t(n,[\Delta_i]) = {} & \underbrace{\sum_{j=1}^{n} \pi_t(\Delta_j)}_{\text{Flow profits}} - \underbrace{\sum_{j=1}^{n} \tau_t \left[V_t(n,[\Delta_i]) - V_t(n-1,[\Delta_i]_{i\neq j}) \right]}_{\text{Creative destruction}} \\ & + \max_{I,x} \left\{ \underbrace{\sum_{j=1}^{n} I_j \left[V_t(n,\left[[\Delta_i]_{i\neq j}, \Delta_j + 1\right]) - V_t(n,[\Delta_i]) \right]}_{\text{Own innovation}} \right. \\ & \left. + \underbrace{\sum_{j=1}^{n} x_j \left[V_t(n+1,\left[[\Delta_i]_{i\neq j}, 1\right]) - V_t(n,[\Delta_i]) \right]}_{\text{Expansion}} - \underbrace{\Gamma\left([x_i, I_i]; n; [\Delta_i]\right) w_t}_{\text{Innovation costs}} \right\}. \end{aligned}$$

Peters (2020): Innovation problem

- Suppose innovation costs Γ take on the following additively separable form:

$$\Gamma([x_i, I_i]; n, [\Delta_i]) = \sum_{i=1}^{n} \lambda^{-\Delta_i} \frac{1}{\varphi_I} I_i^{\zeta} + \frac{1}{\varphi_x} x_i^{\zeta}.$$

  - $\zeta > 1$ ensures cost function is convex.
  - $\lambda^{-\Delta}$ ensures innovations are easier as Δ grows, to counteract concavity of profits in Δ.
  - φ_I and φ_x are cost-shifters for own-innovation vs. expansion.

- Then there is a closed-form solution for the balanced growth path with

$$x = \left(\frac{\varphi_x}{\varphi_z}\frac{1}{\zeta}\right)^{\frac{1}{\zeta-1}}, \qquad I = \left(\frac{\lambda-1}{\lambda}\frac{1}{\rho+\tau}\left(\frac{\varphi_I}{\zeta}\frac{Y_t}{w_t} - \frac{\zeta-1}{\zeta}I^{\zeta}\right)\right)^{\frac{1}{\zeta-1}}.$$

  Rate of entrant creative destruction z given by labor market clearing, and $\tau = z + x$.

Peters (2020): Life-cycle of markups

- For any given product, markups increase with age as long as not displaced. But older firms also have more time to expand to new markets and lose high-markup products.

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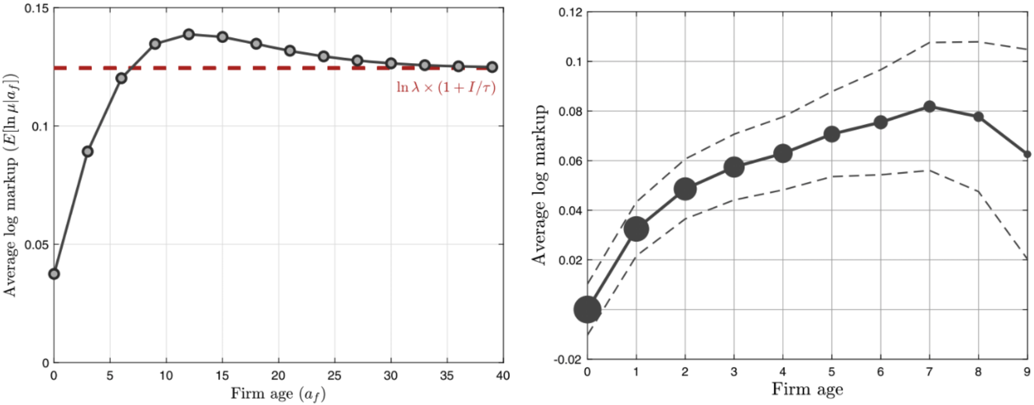

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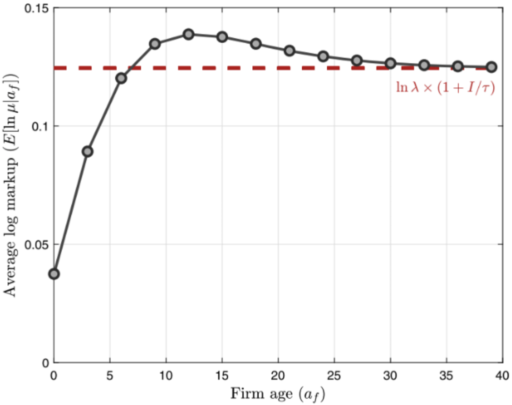
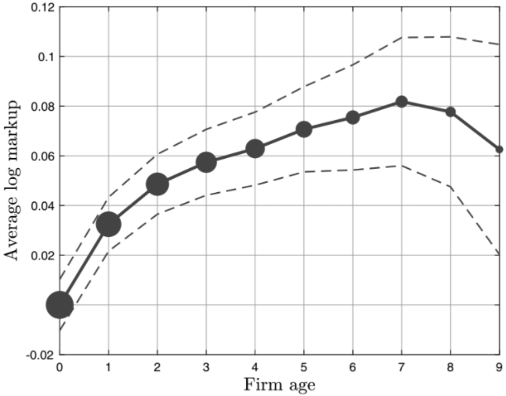




- But in model (unlike data), covariance of markups and size conditional on age is < 0.

Peters (2020): Calibration

- $\theta = \log(1+\frac{\tau}{I})/\log\lambda \approx 9.5$. Thus, $\Lambda = 0.9$ and $\mathcal{M} = 0.995$.
- Reactions to these values?

TABLE III

CALIBRATION: MOMENTS AND STRUCTURAL PARAMETERS[a]

Targets	Data	Model	Calibrated Parameters			Outcomes	
Markup life-cycle	0.082	0.082	φ_I	Innovation efficiency	8.896	I	0.561
Empl. life-cycle	0.494	0.494	φ_x	Expansion efficiency	1.077	x	0.207
Entry rate	10.4%	10.4%	φ_z	Entry efficiency	2.590	z	0.032
Agg. growth	3%	3%	λ	Step size	1.038	τ	0.240
–	–	–	ζ	Cost function	2		
–	–	–	ρ	Discount rate	0.05		

[a]This table contains the calibrated parameters, the targeted data moments, the resulting moments in the model, and the equilibrium outcomes for I, x, and z. Given (I, x, z), all other equilibrium allocations can be calculated. I measure the life-cycle of markups and employment as the log difference between entrants and 7-year-old firms. The two parameters ζ and ρ are set exogenously. As explained in the text, the mapping from the data moments to (I, x, z, λ) does not depend on ζ and ρ.

Peters (2020): Calibration

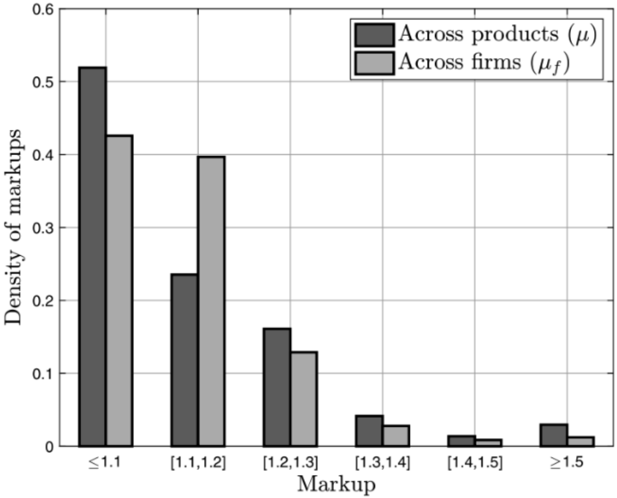
TABLE IV

SENSITIVITY MATRIX[a]

	Change in …				
	φ_I	φ_x	φ_z	λ	Initial Level
	Effects on Endogenous Outcomes				
Rate of own-innovation (I)	4.0%	−1.1%	−0.9%	1.9%	0.561
Incumbent creative destruction (x)	0.0%	5.0%	−4.8%	0.0%	0.2078
Entry (z)	8.1%	−21.9%	38.1%	24.3%	0.0326
Creative destruction (τ)	1.1%	1.4%	1.1%	3.3%	0.2404
	Effects on Equilibrium Moments				
Life-cycle of markups	3.6%	−3.3%	0.6%	5.8%	0.0818
Life-cycle of employment	−1.5%	6.9%	−7.9%	−3.5%	0.494
Entry rate	3.5%	−7.1%	12.9%	10.2%	0.104
Growth rate	3.1%	−0.3%	−0.3%	7.3%	0.03

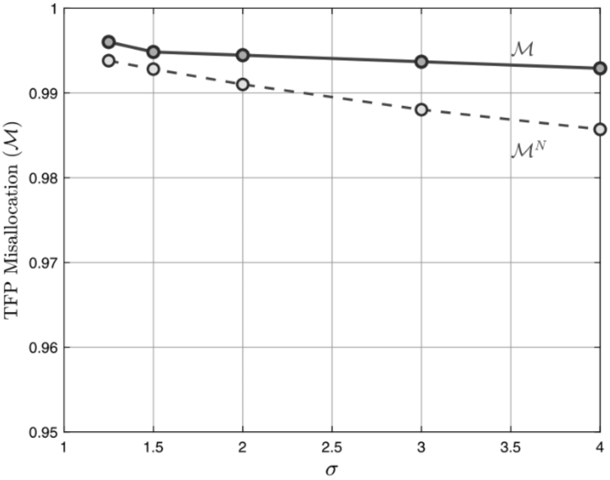
[a]The table reports the effect of a 5% change in the relative innovation efficiencies ($\varphi_I, \varphi_x, \varphi_z$) and the quality increase ($\lambda - 1$) on the endogenous outcomes (top panel) and the equilibrium moments (lower panel).

Misallocation across products vs. across firms



- Relevant measure of markup dispersion is across products.
- Usually we measure markups at firm-level.
- In calibrated model, firm dispersion understates true dispersion by 20%.
- Nevertheless, $1/\mathcal{M} \approx 0.5\%$.

How would misallocation changed if depart from Cobb-Douglas?



- Holding markup distribution fixed, greater elasticity of substitution increases misallocation.
- In model, this force is small.
- Moreover, when $\sigma > 1$,

$$\mu(\Delta) = \min\left\{\frac{\sigma}{\sigma-1}, \lambda^{\Delta}\right\}.$$

$\Rightarrow$ reduces dispersion.

Table of Contents

Multi-product firms

- What if a firm prices a portfolio of products?
- For example, suppose the firm sells two products,

$$\max_{p_1, p_2} (p_1 - c_1) D_1(p_1, p_2) + (p_2 - c_2) D_2(p_1, p_2).$$

- Suppose $c_1 = c_2$, demand symmetric, $\frac{\partial \log D_i}{\partial \log p_i} = -\sigma$, $\frac{\partial \log D_1}{\partial \log p_2} = \frac{\partial \log D_2}{\partial \log p_1} = \eta$.
- Then profit-maximizing markups are

$$\mu_1 = \mu_2 = \frac{\sigma - \eta}{\sigma - \eta - 1}.$$

If $\eta > 0$, markups increase. Setting markup for one product at profit-maximizing level cannibalizes profits of other product.

Quality discrimination

- Mussa and Rosen (1978) show how this cannibalization distortion appears with a monopolist producing a continuum of goods with varying quality levels.
- Today, consider simple two-type adaptation from Tirole (1988) Chapter 3.
- Suppose there are two types of consumers with types θ_h, θ_l.
- Utility of type i is $u_i = \theta_i q - p(q)$, where q is quality.
- Cost of producing good with quality q is $C(q) = \frac{1}{2}q^2$.
- Monopolist chooses which products q to sell and prices $p(q)$ to maximize profits.

Quality discrimination

- Suppose each type is isolated ("islands").
- For each type i, monopolist solves

$$\max_{q,p(q)} p(q) - C(q) \qquad \text{s.t.} \qquad \theta_i q \geq p(q).$$

- Choose price $p(q) = \theta_i q$.

$$\max_q \theta q - C(q) = \theta_i q - \frac{1}{2}q^2.$$

- Choose quality $q = \theta_i$, price $p(q) = q^2 = \theta_i^2$.
- Efficient outcome, firm captures entire surplus.

Quality discrimination

- Now suppose both types are together.
- Monopolist solves

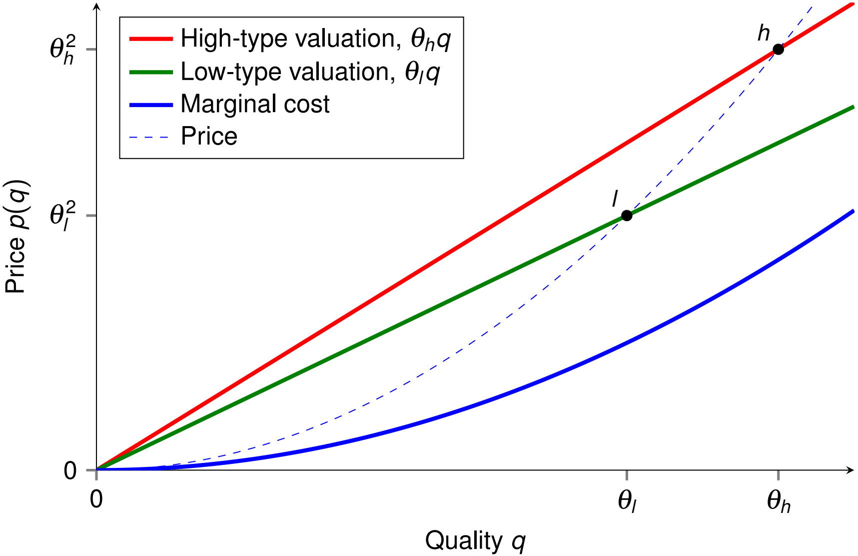
$$\max_{q_l, q_h, p(q)} p(q_h) - C(q_h) + p(q_l) - C(q_l)$$

subject to

$$\begin{aligned}
\theta_h q_h &\geq p(q_h), && \text{(Participation constraint: } h\text{)}\\
\theta_l q_l &\geq p(q_l), && \text{(Participation constraint: } l\text{)}\\
\theta_h q_h - p(q_h) &\geq \theta_h q_l - p(q_l), && \text{(Incentive compatibility: } h\text{)}\\
\theta_l q_l - p(q_l) &\geq \theta_l q_h - p(q_h), && \text{(Incentive compatibility: } l\text{)}
\end{aligned}$$

- Incentive compatibility constraints reflect potential for cannibalization.
- What happens if we try to set “island” qualities / prices?

Quality discrimination



Quality discrimination

- Thus,

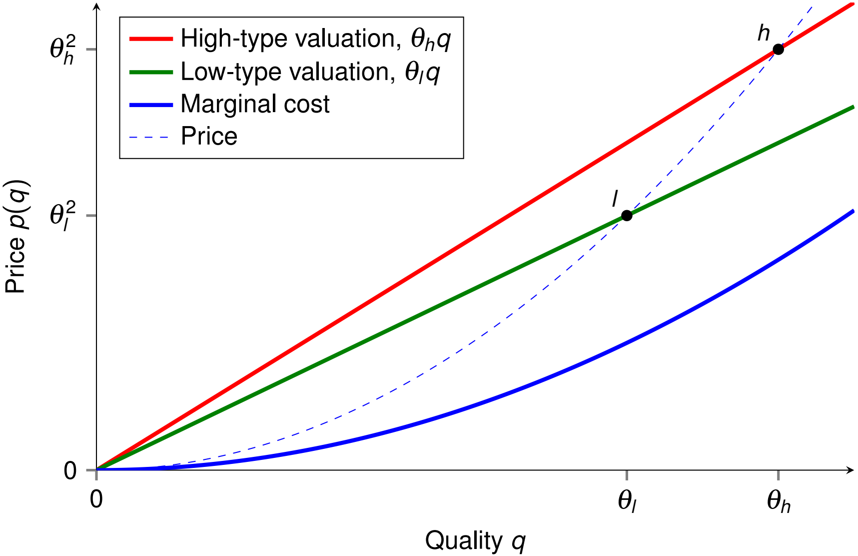
$$\max_{q_l, q_h, p(q)} p(q_h) - C(q_h) + p(q_l) - C(q_l)$$

subject to

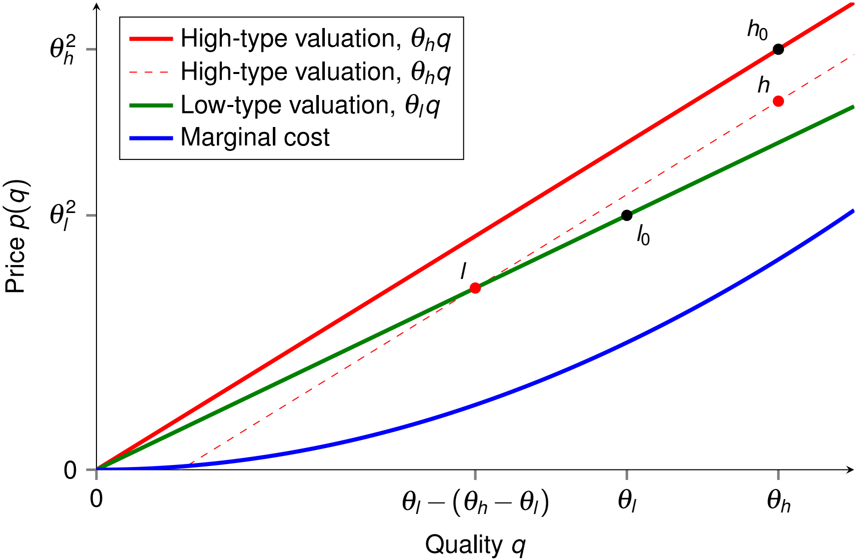
$$\theta_h q_h - p(q_h) \geq 0, \quad \text{(Participation constraint: } h\text{)}$$
$$\theta_l q_l - p(q_l) \geq 0, \quad \text{(Participation constraint: } l\text{)}$$
$$\theta_h q_h - p(q_h) \geq \theta_h q_l - p(q_l), \quad \text{(Incentive compatibility: } h\text{)}$$
$$\theta_l q_l - p(q_l) \geq \theta_l q_h - p(q_h), \quad \text{(Incentive compatibility: } l\text{)}$$

- The term in blue is a cannibalization wedge relative to the island.
- It only depends on q_l! So for qualities, only the choice of q_l is distorted.
- Participation constraint continues to bind for l.

Quality discrimination



Quality discrimination: "No distortion at the top"



Quality discrimination: "No distortion at the top"

- So the monopolist chooses:

$$\begin{aligned} q_h &= \theta_h, \\ q_l &= \theta_l - (\theta_h - \theta_l), \\ p(q_l) &= \theta_l q_l, \\ p(q_h) &= \theta_h q_h - (\theta_h - \theta_l)\, q_l. \end{aligned}$$

- Note that type l households are no worse off.
- Presence of type l households makes type h households strictly better off.
- Deadweight loss is $\theta_l(q_l^{\text{island}} - q_l) = \theta_l(\theta_h - \theta_l)$.

Quality discrimination in GE

- Bornstein and Peter (2025) bring this concept to general equilibrium.
- Motivation: What are distributional effects of market power?
- Income distribution can affect product design.
- Degree to which product design is distorted depends on market structure.
- Interaction between income inequality and competition.

Bornstein and Peter (2025): Setup

- Continuum of households indexed by $i \in [0,1]$ supply labor inelastically.
- Type l: $i \in [0, 0.5]$ has labor endowment and capital share $(1-\alpha)$.
- Type h: $i \in (0.5, 1]$ has labor endowment and capital share $(1+\alpha)$.
- In each sector $s \in (0,1)$, consumers choose a single good $k(s)$ from a single firm $j(s)$,

$$\log C_i = \int_0^1 \log c_i(s) ds,$$

where

$$c_i(s) = \max_{j(s),k(s)} \log(q_{j(s),k(s)}) + \eta_{j(s)} + \frac{1}{\sigma - 1}\varepsilon_{ij(s)},$$

subject to

$$\int_0^1 p_{j(s)k(s)} ds = E_i.$$

Bornstein and Peter (2025): Setup

- In each sector, household buys product from firm that maximizes:

$$\max_j \left[\underbrace{\max_k (\sigma - 1)\left(\log(q_{jk}) - \frac{p_{jk}}{P_i} + \eta_j \right)}_{v_{ij}} + \varepsilon_{ij} \right].$$

  - P_i is Lagrange multiplier on budget constraint for consumer i (marginal price index),
  - v_{ij} is common surplus, depends on brand value η_j and income through P_i,
  - $\varepsilon_{ij} \sim$ Gumbel is taste shock.

- Probability household i buys from firm j is

$$B_{ij} = \frac{\exp(v_{ij})}{\sum_{j'} \exp(v_{ij'})}.$$

Bornstein and Peter (2025): Firm problem

- N firms who differ in brand value η_j (exogenous).
- Face identical linear cost of producing quality κ.
- Maximize profits

$$\max_{\{p_{ij},q_{ij},B_{ij}\}_{i\in\{h,l\}}} B_{lj}(p_{lk}-\kappa q_{lj})+B_{hj}(p_{hj}-\kappa q_{hj}),$$

subject to

$$B_{ij}=\frac{\exp\left[(\sigma-1)\left(\log(q_{ij})-p_{ij}/P_i+\eta_j\right)\right]}{\sum_{j'}\exp\left[(\sigma-1)\left(\log(q_{ij'})-p_{ij'}/P_i+\eta_{j'}\right)\right]} \qquad \text{(Demand)}$$

$$\log(q_{ij})-p_{ij}/P_i\geq\log(q_{i'j})-p_{i'j}/P_i \qquad \text{(Incentive compatibility)}$$

Quality choice in "islands"

- Suppose high- and low-income consumers live on different islands.

- On each island, firms solve

$$\max B_{ij}(p_{ij} - \kappa q_{ij}),$$

subject to

$$B_{ij} = \frac{\exp\left[(\sigma-1)\left(\log(q_{ij}) - p_{ij}/P_i + \eta_j\right)\right]}{\sum_{j'} \exp\left[(\sigma-1)\left(\log(q_{ij'}) - p_{ij'}/P_i + \eta_{j'}\right)\right]}.$$

Quality choice in "islands"

- Suppose high- and low-income consumers live on different islands.
- On each island, firms solve
$$\max B_{ij}(p_{ij} - \kappa q_{ij}),$$
subject to
$$B_{ij} = \frac{\exp\left[(\sigma-1)\left(\log(q_{ij}) - p_{ij}/P_i + \eta_j\right)\right]}{\sum_{j'} \exp\left[(\sigma-1)\left(\log(q_{ij'}) - p_{ij'}/P_i + \eta_{j'}\right)\right]}.$$
- Firms profit margins depend on consumer income and market share
$$p_{ij} = \kappa q_{ij} + \frac{1}{1-B_{ij}} \frac{P_i}{\sigma-1}.$$
- From labor market clearing, quality is
$$q_{ij} = L_i/\kappa.$$

Quality choice in “islands”

- Firms with high η have larger markups.
- Standard misallocation across firms due to markup dispersion (as in Atkeson and Burstein 2008).
- Consumption inequality across h and l types is

$$\log C_h - \log C_l = \log \frac{1+\alpha}{1-\alpha},$$

equal to income inequality.

Quality choice in full economy

- Now suppose all consumers are in one market.

- Incentive compatibility constraint for high-income households can now bind,

$$\log(q_{hj}) - p_{hj}/P_h \geq \log(q_{lj}) - p_{lj}/P_h.$$

- Why might the high-income households buy the low-quality product?

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- Why might the high-income households buy the low-quality product?
- Low-quality products have lower margin, so can pay less to consume “brand” η_j,

$$p_{ij} = \kappa q_{ij} + \frac{1}{1-B_{ij}} \frac{P_i}{\sigma - 1}.$$

- Why does incentive compatibility constraint not always bind?

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- Why does incentive compatibility constraint not always bind?
- Households get positive surplus even in island economy (due to ε_{ij} shocks).

Quality choice in full economy

- For which firms is incentive compatibility constraint more likely to bind?
- Incentive compatibility violated if:

$$p_{hj} - p_{lj} > P_h \left(\log(q_h) - \log(q_l)\right).$$

- This is more likely to be the case if the firm has a high margin,

$$p_{ij} = \kappa q_{ij} + \frac{1}{1 - B_{ij}} \frac{P_i}{\sigma - 1}.$$

- Thus, high brand-value (and high market share) firms are at more risk of cannibalization.
- High “external competition” (i.e., low B_{ij}) makes IC constraint less binding.

Quality choice in full economy

- When there is low external competition, quality choices are

$$q_{hj} = \frac{P_h}{\kappa},$$
$$q_{lj} = \frac{P_l}{\kappa} - \frac{1}{\kappa}\frac{\psi}{B_l}\left(1 - \frac{P_l}{P_h}\right).$$

- Conditional on aggregates (P_h, P_l), quality sold to high-income consumers is efficient.
- Quality offered to low-income consumers is distorted down.
- However, now less labor is required to produce type l goods.
- Depresses wage in equilibrium, which means more than $(1+\alpha)$ share of labor allocated to production of type h good.

Consumer surplus

- Consumer surplus in island economies:

$$v_{hj} = (\sigma - 1)\left(\log q_{hj} - \frac{\kappa q_{hj}}{P_h} + \eta_j\right) - \frac{1}{1 - B_{hj}},$$
$$v_{lj} = (\sigma - 1)\left(\log q_{lj} - \frac{\kappa q_{lj}}{P_l} + \eta_j\right) - \frac{1}{1 - B_{lj}}.$$

- Consumer surplus in full economy:

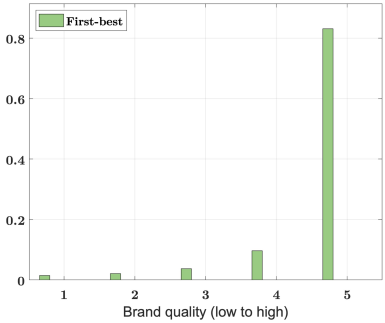
$$v_{hj} = (\sigma - 1)\left(\log q_{hj} - \frac{\kappa q_{hj}}{P_h} + \eta_j\right) - \frac{1}{1 - B_{hj}}\left(1 - \frac{\psi_j}{P_h B_{hj}}\right),$$
$$v_{lj} = (\sigma - 1)\left(\log q_{lj} - \frac{\kappa q_{lj}}{P_l} + \eta_j\right) - \frac{1}{1 - B_{lj}}\left(1 + \frac{\psi_j}{P_h B_{lj}}\right).$$

- Consumption inequality $>$ expenditure inequality.

Consumer surplus

- As in island economy, high η_j firms are too small relative to efficient cross-sectional allocation (higher markups).
- For high η_j firms (for which the incentive compatibility constraint is binding):
  - Lower $q_l \Rightarrow$ even smaller for low-income consumers.
  - Lower $p_h \Rightarrow$ larger for high-income consumers.
- Cannibalization aggravates lack of external competition for low-income consumers.
- Cannibalization alleviates lack of external competition for high-income consumers.

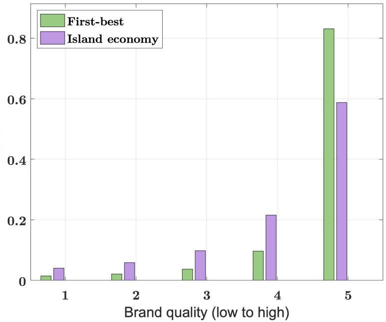
Bornstein and Peter (2025): Numerical Example



Calibration Parameters

- $\alpha = 0.3$ (income ineq. 61 log p)
- $\sigma = 4$ (elasticity of substitution)
- $N = 5$ (number of firms)
- $\chi = 2$ (brand value Pareto shape)

Bornstein and Peter (2025): Numerical Example



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Consumption relative to first-best

	Island
High-income	-0.06
Low-income	-0.06
Inequality	-

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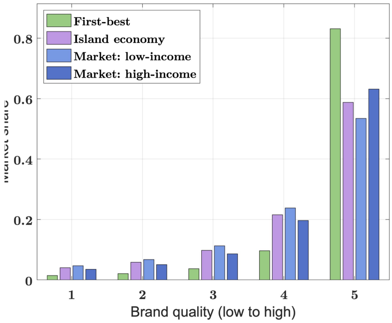

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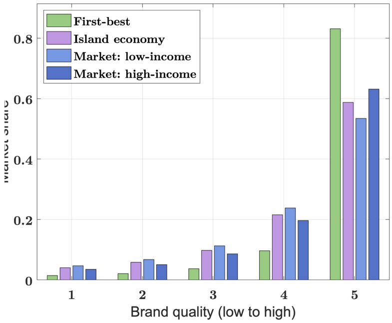

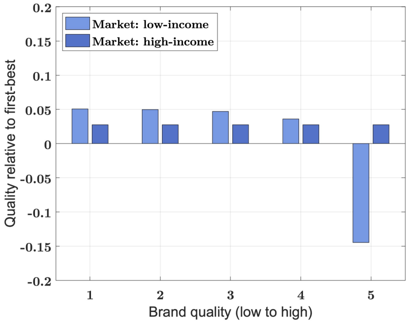

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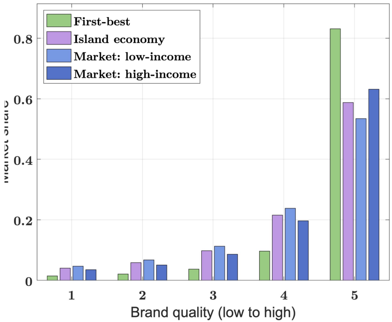

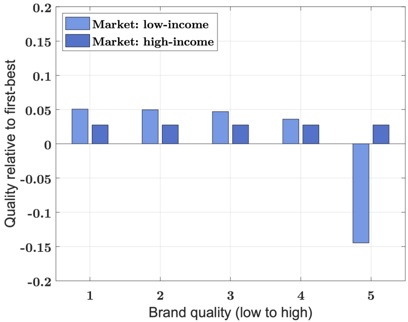

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- $\sigma = 4$ (elasticity of substitution)
- $N = 5$ (number of firms)
- $\chi = 2$ (brand value Pareto shape)

Consumption relative to first-best

	Island	Market
High-income	-0.06	-0.01
Low-income	-0.06	-0.14
Inequality	-	0.13

Bornstein and Peter (2025): Numerical Example

- What are the effects of higher market concentration on inequality?
- Suppose we make the Pareto distribution for η_j thicker-tailed, while keeping consumer surplus in the first-best unchanged.

	Baseline		Higher concentration	
	Island	Market	Island	Market
High-income	-0.06	-0.01	-0.08	-0.02
Low-income	-0.06	-0.14	-0.08	-0.20
Consumption inequality	-	0.13	-	0.18

- More misallocation across firms.
- More within-firm cannibalization. $\Rightarrow$ rise in inequality.