Lecture 5:
Models of Endogenous Distortions

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ECON 416-1

Recall: Aggregation Results

- In efficient economies, first-order effects of micro shocks given by Hulten's Theorem:

$$d\log Y = \sum_i \lambda_i d\log A_i + \sum_f \Lambda_f d\log L_f.$$

To compute nonlinearities ex ante, need structural model for $d\lambda_i$ in terms of shocks.

- In inefficient economies, macro-envelope conditions no longer hold. Yet, first-order effects of micro shocks given by cost-based Domar weights:

$$d\log Y = \underbrace{\sum_i \tilde{\lambda}_i d\log A_i + \sum_f \tilde{\Lambda}_f d\log L_f}_{\Delta \text{ Technology}} \underbrace{- \sum_i \tilde{\lambda}_i d\log \mu_i - \sum_i \tilde{\Lambda}_f (d\log \Lambda_f - d\log L_f)}_{\Delta \text{ Allocative Efficiency}}.$$

To compute ex ante, need structural model for $d\log \Lambda_f$ in terms of shocks.

Table of Contents

Misallocation in the New Keynesian Model

- Per-period utility:

$$u_t(Y_t, L_t) = \frac{Y_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi}, \qquad \text{where} \qquad Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

  subject to budget constraint,

$$P_t Y_t = W_t L_t + T_t, \qquad \text{where} \qquad P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}.$$

- Production:

$$Y_t(i) = L_t(i).$$

- Prices including optimal subsidy τ and wedges from price rigidities, $\mu_i(t)$:

$$P_t(i) = \mu_t(i) \frac{\varepsilon}{\varepsilon - 1} \tau W_t.$$

  Optimal subsidy $\tau = 1 - \varepsilon^{-1}$ sets $\frac{P_t Y_t}{W_t L_t} = 1$.

Misallocation in the New Keynesian Model

- Household optimality:

$$Y_t^{1-\sigma} = \frac{P_t Y_t}{W_t L_t} L_t^{1+\varphi}.$$

- Differentiating per-period utility to a second-order in shocks,

$$\Delta u_t = Y_t^{1-\sigma} (\Delta \log Y_t) - L_t^{1+\varphi} (\Delta \log L_t)$$
$$+ \frac{1}{2}(1-\sigma) Y_t^{1-\sigma} (\Delta \log Y_t)^2 - \frac{1}{2}(1+\varphi) L_t^{1+\varphi} (\Delta \log L_t)^2 + h.o.t.$$

- Change in output due to change in inputs and misallocation:

$$\Delta \log Y_t = \Delta \log L_t - \frac{\varepsilon}{2} Var(\log \mu_t(i)) + h.o.t.$$

- Plugging in and evaluating at optimal point,

$$\Delta u_t = -\frac{1}{2} Y_t^{1-\sigma} \left[\varepsilon Var(\log \mu_t(i)) + (\sigma+\varphi)(\Delta \log Y_t)^2 \right] + h.o.t.$$

Misallocation in the New Keynesian Model

$$\Delta u_t = -\frac{1}{2} Y_t^{1-\sigma} \left[\underbrace{\varepsilon \, Var\left(\log \mu_t(i)\right)}_{\substack{\text{Cost of inflation:} \\ \text{Misallocation due to price dispersion}}} + \underbrace{(\sigma+\varphi)\left(\Delta \log Y_t\right)^2}_{\substack{\text{Cost of output gap:} \\ \text{Moving off labor-leisure condition}}} \right] + h.o.t.$$

- $Var(\log \mu_t(i))$ depends on inflation due to price rigidity.
- Simple e.g.: Suppose wage jumps permanently by $\Delta \log W_t$. With Calvo friction δ,

$$Var(\log \mu_t(i)) \approx \delta(1-\delta)(\Delta \log W_t)^2 = \frac{1-\delta}{\delta}(\Delta \log \pi_t)^2.$$

More generally, Woodford (2003, ch. 6) shows

$$\sum_{t=0}^{\infty} \beta^t Var(\log \mu_t(i)) = \frac{1-\delta}{\delta(1-\beta(1-\delta))} \sum_{t=0}^{\infty} \beta^t (d \log \pi_t)^2.$$

Misallocation in the New Keynesian Model

- Costs of inflation in the canonical NK model are misallocation costs.
- Due to inefficient price/markup dispersion, which arises from price-setting frictions.
- Nakamura, Steinsson, Sun, and Villar (2018) on "The Elusive Costs of Inflation":
  - *In standard New Keynesian models [...] the consumption-equivalent welfare loss of moving from 0% inflation to 12% inflation is roughly 10%."*
  - Conclusions from measuring (a proxy for) price dispersion during the 1970s Great Inflation: *"There is thus no evidence that prices deviated more from their optimal level during the Great Inflation period when inflation was running at higher than 10% a year than during the more recent period when inflation has been close to 2% a year. We conclude from this that the main costs of inflation in the New Keynesian model are completely elusive in the data."*

Does inflation lead to inefficient price dispersion and misallocation?

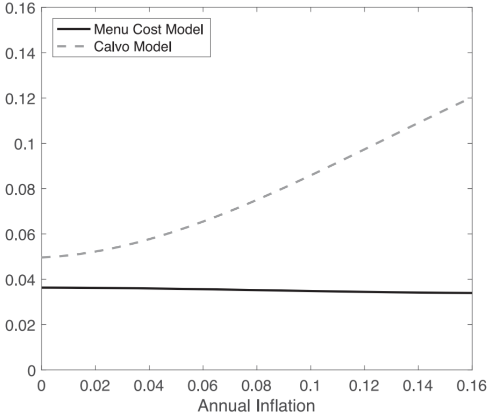


FIGURE V

Inefficient Price Dispersion

(a) Price dispersion vs. inflation.

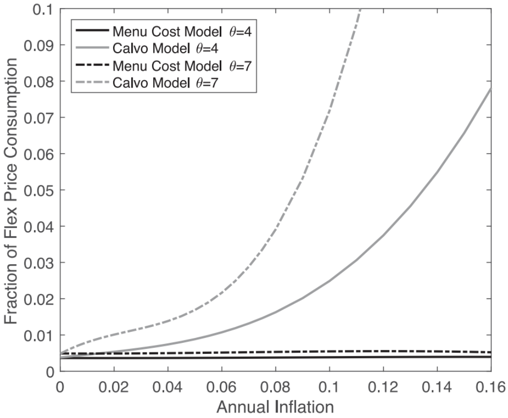


FIGURE II

Welfare Loss

(b) Welfare loss vs. inflation.

Source: Nakamura, Steinsson, Sun, and Villar (2018).

Does inflation lead to inefficient price dispersion and misallocation?

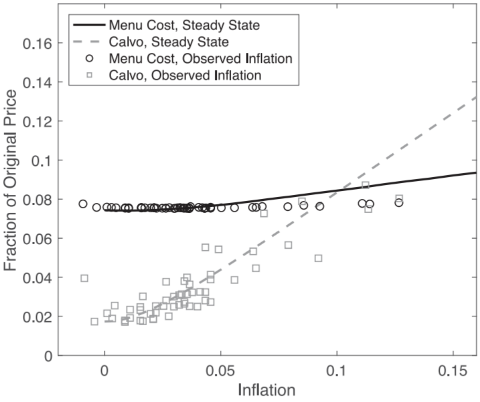


FIGURE VIII

Mean Absolute Size of Price Changes in Sticky-Price Models

(a) Absolute size of price changes: Model.

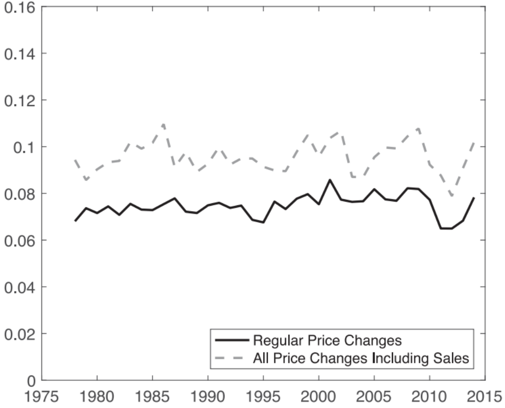


FIGURE IX

Absolute Size of Price Changes in U.S. Data

(b) Absolute size of price changes: Data.

Source: Nakamura, Steinsson, Sun, and Villar (2018).

Misallocation, revisited with data from Argentina's hyperinflation

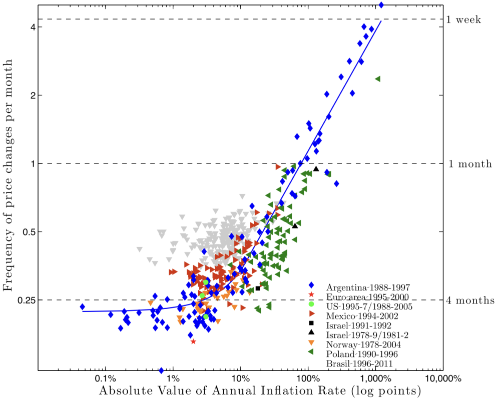


FIGURE VI

The Frequency of Price Changes (λ) and Expected Inflation: International Evidence

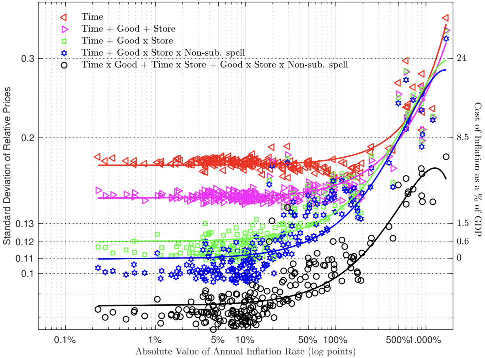


FIGURE X

Cross-Sectional Standard Deviation of Prices and Costs of Price Dispersion versus Inflation

(a) Frequency of price changes.

(b) Price dispersion.

Source: Alvarez, Beraja, Gonzalez-Rozada and Neumeyer (2019).

Price dispersion: Micro to macro

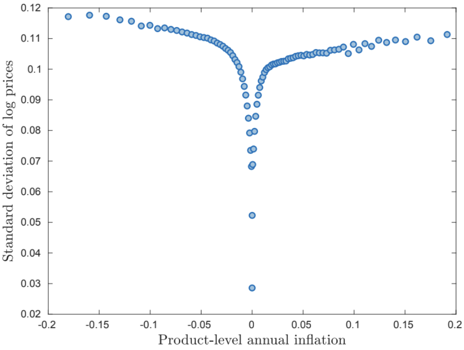


Figure 1: Product-level price dispersion and inflation, raw data

Figure: Sara-Zaror (2024).

- Nakamura et al. (2018) find little relationship between agg. inflation and price dispersion.
- At micro level, steep relationship between inflation and price dispersion!
- Aggregation over non-linear relationship.
- Alvarez et al. (2019) model with idiosyncratic productivity shocks and menu costs.

Table of Contents

Endogenous Markups

- Today: Models of endogenous markups.
- One type of wedge. Rich implications for price levels and pass-through.
- Lerner index holds throughout.

In such cases (where the cost curve is horizontal) the ratio of monopoly revenue to total receipts coincides exactly with the ratio of the divergence of price from marginal cost to price, and it is this latter formula that I wish to put forward as the measure of monopoly power. If P = price and C = marginal cost, then the index of the degree of monopoly power is $\frac{P-C}{P}$.

It will be observed that this formula looks like the inverse of the formula for the elasticity of demand. It differs from it only in that the item marginal cost replaces the item marginal receipts. In equilibrium as normally conceived marginal costs coincide with marginal receipts so that our formula becomes identical with the inverse of the elasticity of demand. It will be best to consider this as a special case.

In this special case we can find the degree of monopoly power via the elasticity of demand. The determination of this elasticity of demand is not to be confused with that of Pigou and Schultz in finding the elasticity of demand (as part of the demand function) for a materially (physically) defined commodity on a market. What we want here is the elasticity of demand for the product of a particular firm. This is much easier to obtain, for it is only when he knows the shape of the demand curve for his product that any entrepreneur can obtain his maximum profit; and he is, therefore, always applying himself energetically to obtaining as accurate an estimate as possible of this elasticity. This does not mean that the entrepreneur will be able to fill in the elasticity of demand on a questionnaire form. He will rarely

Figure: Lerner (1934).

Endogenous Markups

- When (1) demand is smooth, and (2) conduct is Nash-in-prices, firms' profit-maximizing markups are given by the Lerner index, $1/(1-1/\sigma_i)$.

$$\max_p \pi(p) = pD_i(p, p_{-i}, E) - C_i(D_i(p, p_{-i}, E)).$$

$$p\frac{\partial D_i}{\partial p} + D(p) - mc\frac{\partial D_i}{\partial p} = 0.$$

$$p = mc + \frac{D_i}{-\partial D_i/\partial p}.$$

$$\Rightarrow \mu = \frac{p}{mc} = \frac{\sigma_i}{\sigma_i - 1}, \qquad \text{where} \qquad \sigma_i = -\frac{\partial D_i}{\partial p}\frac{p}{D_i}.$$

  - What is the relevant horizon for calculating $D_i(p)$?
  - Why might smooth demand be violated?

Endogenous Markups: CES

- Canonical New Keynesian model assumes monopolistic competition so that

$$D_i(p,P,E) = \left(\frac{p}{P}\right)^{-\varepsilon}\frac{E}{P}, \qquad \text{where} \qquad P = \left(\int_0^1 p_j^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}}.$$

- Thus, $\sigma_i = \varepsilon$ for all firms, and

$$\sigma_i = -\frac{\partial \log D_i}{\partial \log p} = \varepsilon, \qquad \Rightarrow \qquad p_i = \frac{\varepsilon}{\varepsilon - 1}\frac{w}{A_i}.$$

- Marginal product of labor equated across uses,

$$\text{MRPL}_i = p_i\frac{\partial y_i}{\partial L_i} = \frac{\varepsilon}{\varepsilon - 1}w.$$

$\Rightarrow$ allocation of labor across firms is efficient.

- In reality, firms exhibit substantial dispersion in markups.

Atkeson and Burstein (2008)

- Model of heterogeneous markups and incomplete pass-through with oligopoly.

- Suppose representative household utility from consuming goods from sectors $s \in [0, 1]$.

$$Y = \left(\int_0^1 Y_s^{\frac{\eta-1}{\eta}} ds \right)^{\frac{\eta}{\eta-1}}.$$

- Sectoral output Y_s is an aggregate of N_s varieties produced by firms,

$$Y_s = \left(\sum_{i=1}^{N_s} y_{si}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

- η is the elasticity of substitution across sectors, ε is elasticity of substitution across firms within a sector.

Atkeson and Burstein (2008): Bertrand competition

- Demand curve for firm i in sector s,

$$\frac{y_{si}}{Y} = \left(\frac{p_{si}}{P_s}\right)^{-\varepsilon} \left(\frac{P_s}{P}\right)^{-\eta},$$

  where

$$P = \left(\int_0^1 P_s^{1-\eta}\, ds\right)^{\frac{1}{1-\eta}}, \qquad \text{and} \qquad P_s = \left(\sum_{i=1}^{N_s} p_{si}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}.$$

- If i sets its price taking prices of all other firms as given (Bertrand competition),

$$\sigma_{si} = -\frac{d\log y_{si}}{d\log p_{si}} = \varepsilon + (\eta - \varepsilon)\frac{d\log P_s}{d\log p_{si}} = \varepsilon + (\eta - \varepsilon)\lambda_{si}.$$

  where $\lambda_{si} = p_{si} y_{si} / P_s Y_s$ is the sales share of i in sector s.

Atkeson and Burstein (2008): Bertrand competition

$$\sigma_i = -\frac{d\log y_{si}}{d\log p_{si}} = \varepsilon + (\eta - \varepsilon)\frac{d\log P_s}{d\log p_{si}} = \varepsilon + (\eta - \varepsilon)\lambda_{si}.$$

- Following Lerner formula, firm *i*'s markup in sector *s* is

$$\mu_{si} = \frac{\varepsilon(1-\lambda_{si}) + \eta\lambda_{si}}{\varepsilon(1-\lambda_{si}) + \eta\lambda_{si} - 1}.$$

- Realistically, elasticities of substitution within a sector $\varepsilon > \eta$ across sector.

- For $\lambda_{si} = 0$, with $\mu_{si} = \frac{\varepsilon}{\varepsilon-1}$. For $\lambda_{si} = 1$, $\mu_{si} = \frac{\eta}{\eta-1}$.

Atkeson and Burstein (2008): Bertrand competition

- Markups μ_{si} increasing in within-sector sales shares λ_{si}.
- What determines within-sector sales share, λ_{si}?

$$\lambda_{si} = \frac{p_{si} y_{si}}{P_s Y_s} = \left(\frac{p_{si}}{P_s}\right)^{1-\varepsilon} = \frac{(\mu_{si}/A_{si})^{1-\varepsilon}}{\sum_{k=1}^{N_s} (\mu_{sk}/A_{sk})^{1-\varepsilon}}.$$

- This is a fixed point, since μ_{si} each depend on sales shares
- Increase in A_{si} leads to decrease in price and higher market share (though partially offset by endogenous increase in markup).

Atkeson and Burstein (2008): Cournot competition

- Aside: Before we assumed that firms compete á la Bertrand.

- Under Cournot, firms instead take others' quantity decisions as given. Use inverse demand curve:

$$\frac{y_{si}}{Y} = \left(\frac{p_{si}}{P_s}\right)^{-\varepsilon} \left(\frac{P_s}{P}\right)^{-\eta},$$

$$\Rightarrow \frac{p_{si}}{P} = \left(\frac{y_{si}}{Y_s}\right)^{-1/\varepsilon} \left(\frac{Y_s}{Y}\right)^{-1/\eta}.$$

$$d\log p_{si} = -\frac{1}{\varepsilon} d\log y_{si} - \left(\frac{1}{\eta} - \frac{1}{\varepsilon}\right)\frac{\partial \log Y_s}{\partial \log y_{si}} d\log y_{si}.$$

$$\sigma_{si} = -\frac{d\log y_{si}}{d\log p_{si}} = \frac{1}{\frac{1}{\varepsilon} + \lambda_{si}\left(\frac{1}{\eta} - \frac{1}{\varepsilon}\right)}.$$

- Qualitatively similar, though shape of markup function differs.

Atkeson and Burstein (2008): Pass-through

- Suppose Bertrand competition.

$$p_{si} = \mu_{si} \frac{w}{A_i}, \qquad \text{where} \qquad \mu_{si} = \frac{\varepsilon(1-\lambda_{si}) + \eta\lambda_{si}}{\varepsilon(1-\lambda_{si}) + \eta\lambda_{si} - 1}.$$

- How does a cost shock to firm i change its price (i.e., $d\log mc_i = -d\log A_i$)?

Atkeson and Burstein (2008): Pass-through

- Suppose Bertrand competition.

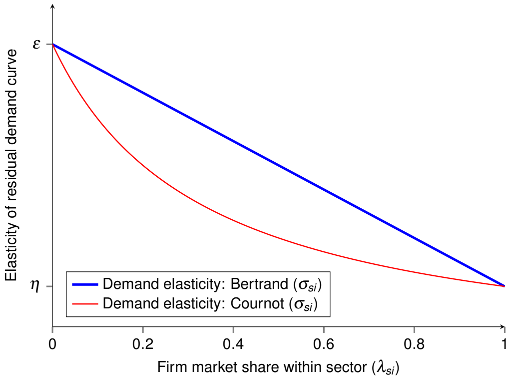
$$p_{si} = \mu_{si} \frac{w}{A_i}, \qquad \text{where} \qquad \mu_{si} = \frac{\varepsilon(1-\lambda_{si}) + \eta\lambda_{si}}{\varepsilon(1-\lambda_{si}) + \eta\lambda_{si} - 1}.$$

- How does a cost shock to firm i change its price (i.e., $d\log mc_i = -d\log A_i$)?

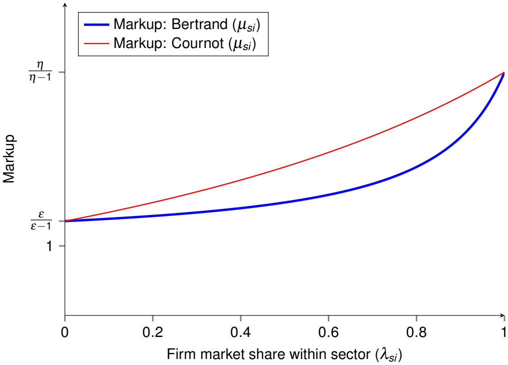
$$\begin{aligned}
\rho_{si} = \frac{d\log p_{si}}{d\log mc_{si}} &= \frac{d\log \mu_{si}}{d\log mc_{si}} + 1 \\
&= -\frac{1}{\sigma_{si}(\sigma_{si}-1)} \frac{d\sigma_{si}}{d\log mc_{si}} + 1 \\
&= \frac{(\varepsilon-\eta)}{\sigma_{si}(\sigma_{si}-1)} \frac{d\lambda_{si}}{d\log mc_{si}} + 1
\end{aligned}$$

Since $d\lambda_{si}/d\log mc_{si} < 0$, we have $\rho_{si} < 1$. Incomplete pass-through.

Atkeson and Burstein (2008): Illustration



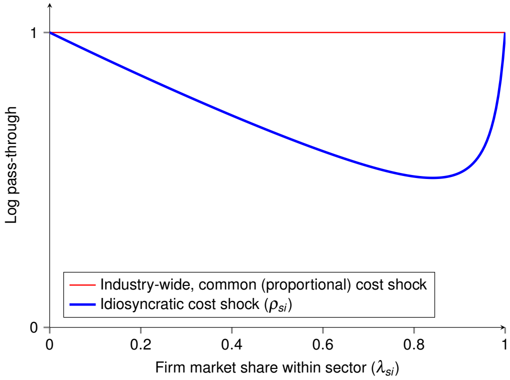
(a) Demand elasticity σ_{si}.



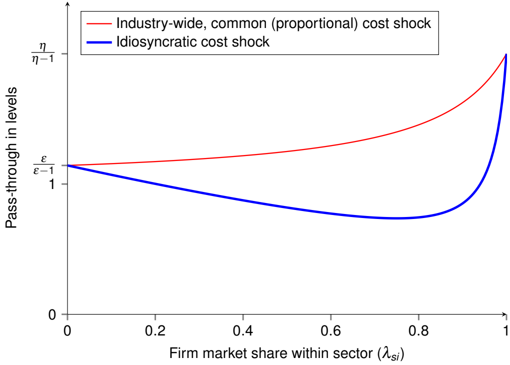
(b) Markup μ_{si}.

- Calibration with $\varepsilon = 8$, $\eta = 2$.

Atkeson and Burstein (2008): Illustration



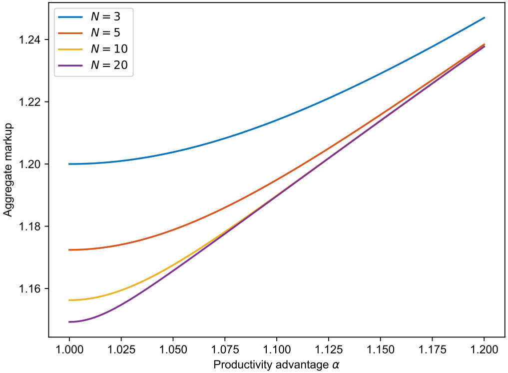
(a) Log pass-through ρ_{si}.



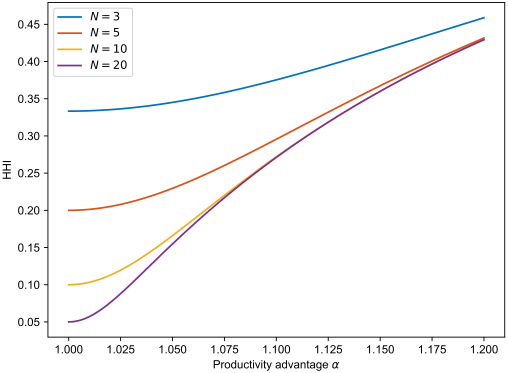
(b) Pass-through in levels.

- Calibration with $\varepsilon = 8$, $\eta = 2$.

Atkeson and Burstein (2008): Illustration



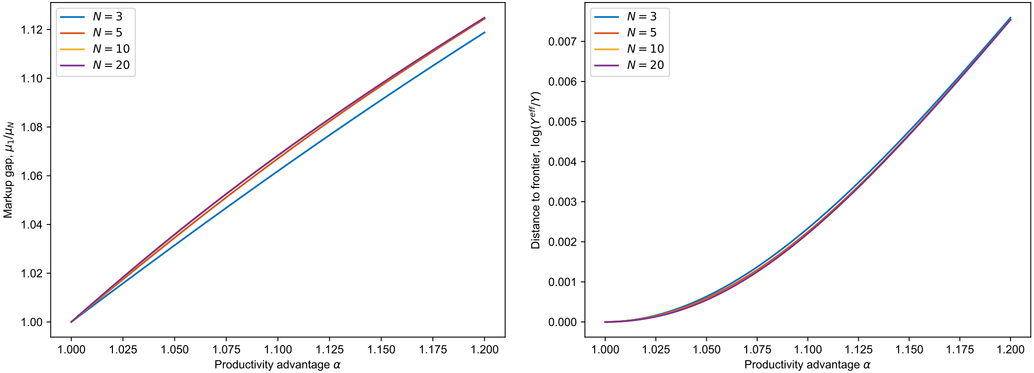
(a) Aggregate markup.



(b) Herfindahl-Hirschman Index.

- Calibration with N firms, with $A_i = 1/\alpha^i$, $\varepsilon = 8$, $\eta = 2$.

Atkeson and Burstein (2008): Illustration



(a) Markup gap. (b) Distance to the frontier.

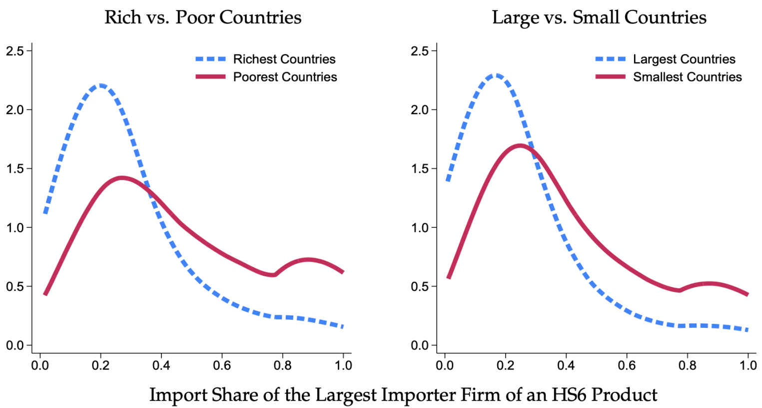
- Calibration with N firms, with $A_i = 1/\alpha^i$, $\varepsilon = 8$, $\eta = 2$.

Cautionary note

- Nested CES has proliferated in macro to model oligopoly and oligopsony power.
- The model imposes strong assumptions on where variation in markups, firm size, and concentration come from (e.g., exogenous number of firms, productivity differences).
- Number of firms in a market, concentration are *endogenous* equilibrium outcomes.

Cautionary note: Example

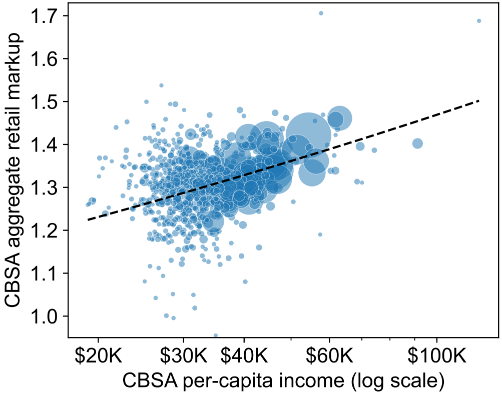
Figure 1: Distribution of the Import Share of the Largest Importer Firm in an HS6 Product



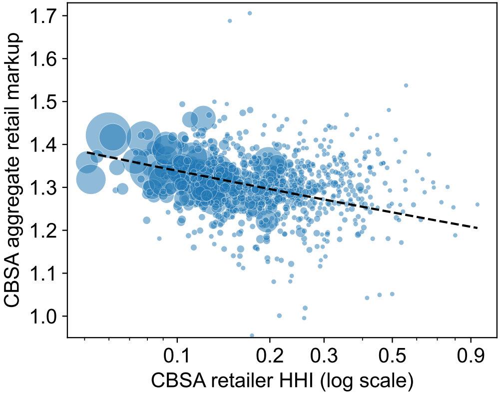
- Adao et al. (2024) find importer concentration higher in **poorer** and **smaller** countries.
- Use nested CES model to infer importer markups are higher in these settings.

Cautionary note: Example

- In pset 1, you will show that HHI–markups correlation depends on source of variation.
- E.g., across U.S. cities, retail markups *negatively* correlated with HHI.



(a) Markups vs. per-capita income.



(b) Markups vs. retailer HHI.

Cautionary note: Example

- As for importers, retailer market concentration is higher in **poorer** and **smaller** cities.
- But we would be wrong to associate this with higher markups!

	Retailer HHI		*Log Agg. Retail Markup*		
	(1)	(2)	(3)	(4)	(5)
Log Income / Capita	−0.163**	−0.048**	0.110**	0.095**	
	(0.018)	(0.015)	(0.016)	(0.020)	
Log Population		−0.020**		0.003	
		(0.002)		(0.003)	
Retailer HHI					−0.266**
					(0.048)
N	881	881	881	881	881
R^2	0.26	0.38	0.27	0.28	0.17

Note: Unit of observation is a CBSA. Retailer HHI and retail markups from Sangani (2023). Robust SEs.

Table of Contents

Monopolistic competition beyond CES (Matsuyama and Ushchev)

- Atkeson and Burstein model requires granular firms.
- Strategic interactions between finite firms difficult to analyze.
- Tractable alternative is monopolistic competition, where atomistic firms compete relative to market aggregates.
- Invaluable guide: “Beyond CES” by Matsuyama and Ushchev (2017).

Monopolistic competition beyond CES (Matsuyama and Ushchev)

- Introduce three classes of homothetic demand systems that generalize CES.
- Homothetic with Direct Implicit Additivity (HDIA), which generalizes Kimball (1995), defines output Y implicitly as
$$\int_0^1 \Upsilon_i(\frac{y_i}{Y})di = 1.$$
- Homothetic with Indirect Implicit Additivity (HIIA), defines the ideal price index P^Y implicitly as
$$\int_0^1 \theta_i(\frac{p_i}{P^Y})di = 1.$$
- Homothetic with a Single Aggregator (HSA), with single aggregator P, defined implicitly as
$$p_i y_i = s_i(\frac{p_i}{P}), \qquad \text{s.t.} \qquad \int_0^1 s_i(\frac{p_i}{P})di = 1.$$

Monopolistic competition beyond CES (Matsuyama and Ushchev)

- Each of these classes of demand systems nest CES as special case.
- Moreover, these classes are mutually disjoint except for CES.
- Today, we will walk through an example using HDIA (generalized Kimball) demand.
- When we talk about entry, we will return to an example with HSA demand.

Table of Contents

Supply-Side Effects of Demand Shocks

- Do demand shocks change aggregate productivity?
- Common view: No.

  Procyclicality of TFP is caused by mismeasurement.
- This paper: Yes.

  With realistic firm heterogeneity, we should expect procyclicality.

Transmission of Monetary Policy

- Monetary expansion can raise allocative efficiency if:

  1. initial allocation of resources is inefficient;
  2. expansion systematically reallocates resources from low to high marginal revenue product firms.

- This naturally happens if

  1. firms have variable markups;
  2. firms with high markups partly absorb shocks (incomplete pass-through).

- New misallocation channel of monetary policy.

Misallocation Channel of Monetary Policy

- Easing raises TFP and generates endogenous supply shock.
- Implications:
    - Amplifies impact and persistence of monetary shocks.
    - Aggregate TFP falls after contraction without technical regress.
    - Firm TFPR dispersion rises in recessions without uncertainty shocks.
    - Flatter Phillips curve, slope linked to industrial concentration.
- Reallocations consistent with the data.

Model Elements

- Monopolistic competition: (Generalized) Kimball demand.
- Heterogeneity: marginal costs, markups, pass-throughs.
- Nominal Rigidities: Calvo pricing.

Two-Period Model

$t = 0$
Initial equilibrium:
Firms maximize profits,
households maximize utility,
resource constraints hold.

$t = 1/2$
Monetary authority
chooses nominal
price.

$t = 1$
New equilibrium:
Flex-price firms reset prices,
households adjust,
resource constraints hold.

Households

- Unit mass of identical households maximize

$$\frac{Y^{1-\gamma}-1}{1-\gamma}-\frac{L^{1+\frac{1}{\zeta}}}{1+\frac{1}{\zeta}},$$

  where output is

$$\int_0^1 \Upsilon_\theta\left(\frac{y_\theta}{Y}\right)d\theta = 1,$$

  subject to the budget constraint

$$\int_0^1 p_\theta y_\theta d\theta = wL + \Pi.$$

- If $\Upsilon_\theta(x) = x^{\frac{\sigma-1}{\sigma}}$, recover CES.

Demand Curves

- Demand curve for each variety:

$$\frac{p_\theta}{P} = \Upsilon'_\theta\left(\frac{y_\theta}{Y}\right),$$

  with aggregators P and Y.

- Note: Price aggregator P is not equal to ideal price index. Instead,

$$P = \frac{P^Y}{\int_0^1 \Upsilon'_\theta\left(\frac{y_\theta}{Y}\right)\frac{y_\theta}{Y}d\theta},$$

  where P^Y is the ideal price index.

- If Υ is isoelastic (CES), then $P = P^Y$.

Some Notation

- Price elasticity of demand:
$$\sigma_\theta\left(\frac{y}{Y}\right) = \frac{\Upsilon'_\theta(\frac{y}{Y})}{-\frac{y}{Y}\Upsilon''_\theta(\frac{y}{Y})}.$$
- Desired markup:
$$\mu_\theta\left(\frac{y}{Y}\right) = \frac{1}{1-\frac{1}{\sigma_\theta(\frac{y}{Y})}}.$$
- Desired pass-through of marg. cost into price, in partial eq., is
$$\frac{\partial \log p_\theta}{\partial \log mc_\theta} = \rho_\theta\left(\frac{y}{Y}\right) = \frac{1}{1+\frac{\frac{y}{Y}\mu'_\theta(\frac{y}{Y})}{\mu_\theta(\frac{y}{Y})}\sigma_\theta(\frac{y}{Y})}.$$

Empirical Evidence on Pass-through

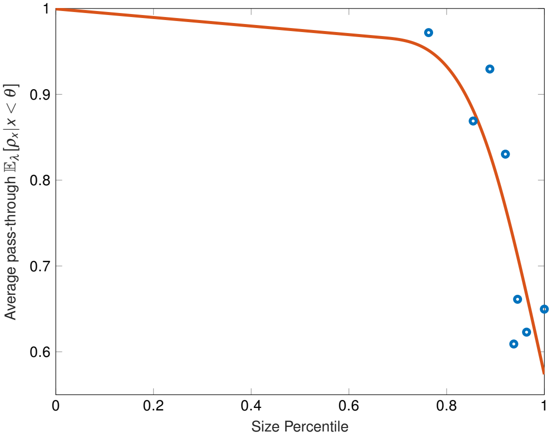


Figure: Pass-through ρ_θ from Amiti et al (2019) estimates.

- Pass-through is incomplete, declines in size. (Marshall's strong 2nd law of demand.)

Some Notation

- Sales share density

$$\lambda_\theta = \frac{p_\theta y_\theta}{P^Y Y}.$$

- For some variable z_θ, define

$$\mathbb{E}_\lambda [z_\theta] = \int_0^1 \lambda_\theta z_\theta d\theta$$

.

- Define aggregate markup

$$\bar{\mu} = \left(\mathbb{E}_\lambda \left[\mu_\theta^{-1}\right]\right)^{-1}.$$

Why is this the aggregate markup? What is the relationship to the labor share Λ_L?

Producers

- Each variety supplied by single producer subject to Calvo friction.
- With prob. $1 - \delta_\theta$, set price before observing nominal shock.
- Production is linear in labor with marginal cost w/A_θ.
- Firms set their prices to maximize expected profits:

$$\max_{p_\theta} \left(p_\theta y_\theta - \frac{w}{A_\theta} y_\theta \right)$$

s.t.

$$\frac{p_\theta}{P} = \Upsilon'_\theta\left(\frac{y_\theta}{Y}\right).$$

Equilibrium

- Bank chooses a nominal price (via implicit money demand).
- Households choose consumption and labor to maximize utility.
- Flex price firms set price to maximize profits.
- Sticky price firms set price to maximize profits assuming no monetary shock.

Transmission of Monetary Policy

- Consider perturbation in nominal wage $d \log w$.
- Output can move for two reasons:

$$d \log Y = d \log L + [d \log Y - d \log L],$$

$$= \underbrace{d \log L}_{\text{Employment}} + \underbrace{d \log A}_{\text{Productivity}}.$$

Changes in Allocative Efficiency

- Change in allocative efficiency,

$$d\log A = -\mathbb{E}_\lambda[d\log \mu_\theta] - d\log \Lambda_L = d\log \bar{\mu} - \mathbb{E}_\lambda[d\log \mu_\theta].$$

- Rearranging,

$$\begin{aligned} d\log A &= d\log \bar{\mu} - \mathbb{E}_\lambda\left[d\log \mu_\theta\right] \\ &= \mathbb{E}_\lambda\left[\left(\bar{\mu}/\mu_\theta^{-1} - 1\right) d\log \mu_\theta\right] - \mathbb{E}_\lambda\left[\left(\bar{\mu}/\mu_\theta^{-1}\right) d\log \lambda_\theta\right] \\ &= -\mathbb{E}_\lambda\left[\left(\bar{\mu}/\mu_\theta^{-1} - 1\right) d\log \lambda_\theta/\mu_\theta\right] \\ &= -Cov_\lambda\left[\bar{\mu}/\mu_\theta^{-1}, d\log \lambda_\theta/\mu_\theta\right]. \end{aligned}$$

- If $\mu_\theta(\frac{y_\theta}{Y}) = \mu$ in initial equilibrium, then regardless of $d\log \mu_\theta$,

$$d\log A = 0.$$

Changes in Allocative Efficiency

- Using $d\log y_\theta - d\log Y = -\sigma_\theta(d\log p_\theta - d\log P)$,

$$d\log A = \bar{\mu}\, Cov_\lambda[\sigma_\theta/\mathbb{E}_\lambda[\sigma_\theta], d\log p_\theta].$$

- Average change in price for firm type θ is

$$d\log p_\theta = \delta_\theta[d\log p_\theta^{\text{flex}}] = \delta_\theta[\rho_\theta d\log w + (1-\rho_\theta)d\log P].$$

Firm places ρ_θ weight on own cost and $(1-\rho_\theta)$ weight on price aggregator.

- Change in price aggregator in term depends on price changes of all firms,

$$d\log P = \mathbb{E}_{\lambda\sigma}[d\log p_\theta] = \frac{\mathbb{E}_\lambda[\delta_\theta\sigma_\theta\rho_\theta]}{\mathbb{E}_\lambda[\delta_\theta\sigma_\theta\rho_\theta] + \mathbb{E}_\lambda[\sigma_\theta(1-\delta_\theta)]} d\log w.$$

Price aggregator moves less than 1:1 with wage due to rigidity, pricing-to-market.

Misallocation Channel of Monetary Policy

Proposition

In response to a nominal wage shock,

$$\frac{d\log A}{d\log w} = \underbrace{\kappa_\rho \, Cov_\lambda\left[\rho_\theta, \sigma_\theta | flex\right]}_{\substack{\text{Reallocation due to} \\ \text{heterogenous pass-through}}} + \underbrace{\kappa_\delta \, Cov_\lambda\left[\delta_\theta, \sigma_\theta\right]}_{\substack{\text{Reallocation due to} \\ \text{heterogenous price-stickiness}}}$$

where

$$\kappa_\rho = \frac{\bar{\mu}\mathbb{E}_\lambda[\delta_\theta]\mathbb{E}_\lambda\left[1-\delta_\theta\right]}{\mathbb{E}_\lambda\left[\left[\delta_\theta\rho_\theta + (1-\delta_\theta)\right]\sigma_\theta\right]} > 0,$$

$$\kappa_\delta = \frac{\bar{\mu}\mathbb{E}_{\lambda\delta}\left[\rho_\theta\right]}{\mathbb{E}_\lambda\left[\left[\delta_\theta\rho_\theta + (1-\delta_\theta)\right]\sigma_\theta\right]} > 0.$$

Misallocation Channel of Monetary Policy

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- Two reasons why $d\log A \neq 0$:
    1. Heterogeneous desired passthrough.
    2. Heterogeneous price stickiness.
       (Baqaee & Farhi 2017, Meier & Reinelt 2020)
- $d\log A > 0$ implies dispersion in firm-level TFPR is countercyclical.

Heterogeneous Passthrough

Corollary

Suppose homogeneous Calvo $\delta_\theta = \delta$. Then,

$$\frac{d \log A}{d \log w} = \kappa_\rho Cov_\lambda \left[\rho_\theta , \sigma_\theta\right].$$

- Marshall's strong second law of demand sufficient but not necessary for

$$\frac{d \log A}{d \log w} > 0.$$

- Pass-through could negatively covary with markup for other reasons (e.g. quality, nicheness, etc.)
  (e.g. Chen & Juvenal, 2016; Auer et al. 2018)

Heterogeneous Price-Flexibility

Corollary

Suppose homogeneous pass-through $\rho_\theta = \rho$. Then,

$$\frac{d\log A}{d\log w} = \kappa_\delta Cov_\lambda\left[\delta_\theta, \sigma_\theta\right],$$
$$\propto \mathbb{E}_\lambda[\sigma_\theta | flex] - \mathbb{E}_\lambda[\sigma_\theta | sticky].$$

- If high markup firms are more sticky, then

$$\frac{d\log A}{d\log w} > 0.$$

Changes in Employment (Standard Channel)

- Household maximization implies

$$Y^{1-\gamma} = \bar{\mu} L^{1+\frac{1}{\zeta}}.$$

- Log-linearizing,

$$(1-\gamma) d\log Y = d\log\bar{\mu} + \left(1+\frac{1}{\zeta}\right) d\log L.$$

$$d\log A = d\log\bar{\mu} - \mathbb{E}_{\lambda}[d\log\mu_{\theta}].$$

$$\Rightarrow d\log Y = \frac{1}{1+\gamma\zeta} d\log A - \frac{\zeta}{1+\gamma\zeta}\mathbb{E}_{\lambda}[d\log\mu_{\theta}].$$

Transmission of Monetary Policy

Proposition

The response of output to $d\log w$ is

$$d\log Y = \underbrace{-\frac{\zeta}{1+\gamma\zeta}\mathbb{E}_\lambda\left[d\log\mu_\theta\right]}_{\textit{Demand-Side Effect}} + \underbrace{\frac{1}{1+\gamma\zeta}d\log A}_{\textit{Supply-Side Effect}},$$

where

$$\mathbb{E}_\lambda\left[d\log\mu_\theta\right] = -\left[\underbrace{\mathbb{E}_\lambda[1-\delta_\theta]}_{\textit{Sticky prices}} + \underbrace{\frac{\mathbb{E}_\lambda\left[\delta_\theta(1-\rho_\theta)\right]\mathbb{E}_\lambda\left[\sigma_\theta(1-\delta_\theta)\right]}{\mathbb{E}_\lambda\left[\left[\delta_\theta\rho_\theta+(1-\delta_\theta)\right]\sigma_\theta\right]}}_{\textit{Real rigidities}}\right]d\log w.$$

$d\log A > 0$ strengthens monetary policy.

Phillips Curve

Proposition

The wage Phillips curve is

$$d\log w = (1+\gamma\zeta)\frac{1}{\left[\frac{d\log A}{d\log w} - \zeta\mathbb{E}_\lambda\left[\frac{d\log \mu_\theta}{d\log w}\right]\right]} d\log Y.$$

The price Phillips curve is

$$d\log P^Y = (1+\gamma\zeta)\frac{1+\mathbb{E}_\lambda\left[\frac{d\log \mu_\theta}{d\log w}\right]}{\left[\frac{d\log A}{d\log w} - \zeta\mathbb{E}_\lambda\left[\frac{d\log \mu_\theta}{d\log w}\right]\right]} d\log Y.$$

$\frac{d\log A}{d\log w} > 0 \Rightarrow$ reallocation channel flattens the Phillips curve.

E.g. CES with heterogeneous productivity

- Suppose $\Upsilon(x) = x^{\frac{1-\sigma}{\sigma}}$.
- Hence,
$$\sigma_\theta = \sigma, \quad \mu_\theta = \frac{1}{1-\frac{1}{\sigma}}, \quad \rho_\theta = 1.$$
- By envelope theorem,
$$d\log A = 0,$$
and
$$d\log P^Y = \frac{(1+\gamma\zeta)}{\zeta}\frac{\delta}{(1-\delta)}d\log Y$$
- "Traditional" Phillips curve: lower δ, flatter Phillips curve.

E.g. non-CES with representative firm

- Suppose $A_\theta = 1$ and $\delta_\theta = \delta$.

- Hence,

$$\sigma_\theta = \sigma, \quad \mu_\theta = \frac{1}{1 - \frac{1}{\sigma}}, \quad \rho_\theta < 1.$$

- Then,

$$d \log A = 0,$$

and

$$d \log P^Y = \frac{(1 + \gamma\zeta)}{\zeta} \frac{\delta}{(1 - \delta)} \rho \, d \log Y.$$

- Phillips curve with real rigidities: lower ρ, flatter Phillips curve.

E.g. non-CES with het. firms and constant pass-through

- Suppose $\delta_\theta = \delta$ and $\rho_\theta = \rho \leq 1$.

- We still have
$$d \log A = 0,$$
even though markups are dispersed.

- Hence,
$$d \log P^Y = \frac{(1+\gamma\zeta)}{\zeta} \frac{\delta}{(1-\delta)} \rho \, d \log Y.$$

- Phillips curve with real rigidities: lower ρ, flatter Phillips curve.

E.g. non-CES with heterogeneous firms and pass-through

- Suppose two firm types: H and L with $\mu_H > \mu_L$.

$$d\log A = \lambda_H \left(1 - \frac{\bar{\mu}}{\mu_H}\right) \left(d\log l_H - d\log l_L\right).$$

- Hence,

$$d\log P^Y = \frac{1+\gamma\zeta}{\zeta} \frac{\delta}{1-\delta} \frac{\delta\lambda_H\lambda_L\left(\sigma_L - \sigma_H\right)\left(\rho_L - \rho_H\right) + \mathbb{E}_\lambda\left[\sigma_\theta\right]\mathbb{E}_\lambda\left[\rho_\theta\right]}{\delta\left(1+\frac{\bar{\mu}}{\zeta}\right)\lambda_H\lambda_L\left(\sigma_L - \sigma_H\right)\left(\rho_L - \rho_H\right) + \mathbb{E}_\lambda\left[\sigma_\theta\right]} d\log Y$$

- Slope decreasing in $\left(\sigma_L - \sigma_H\right)\left(\rho_L - \rho_H\right)$.

Dynamic Model

- Taylor rule:

$$d \log i_t = \phi_\pi d \log \pi_t + \phi_y d \log Y_t + v_t;$$

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$$d\log Y_t = d\log Y_{t+1} - \frac{1}{\gamma}\left(d\log i_t - d\log \pi_{t+1}\right);$$

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- Misallocated NK Phillips Curve:

$$d\log \pi_t = \mathbb{E}_\lambda\left[\rho_\theta\right]\varphi d\log Y_t + \beta d\log \pi_{t+1} - \alpha d\log A_t;$$

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- Endog. TFP:

$$d\log A_t = \frac{1}{\kappa_A} d\log A_{t-1} + \frac{\beta}{\kappa_A} d\log A_{t+1} + \frac{\varphi}{\kappa_A}\frac{Cov_\lambda\left(\rho_\theta,\sigma_\theta\right)}{\mathbb{E}_\lambda\left[\sigma_\theta\right]} d\log Y_t.$$

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Corollary

Static model is a special case of this when $\beta = 0$.

Calibration

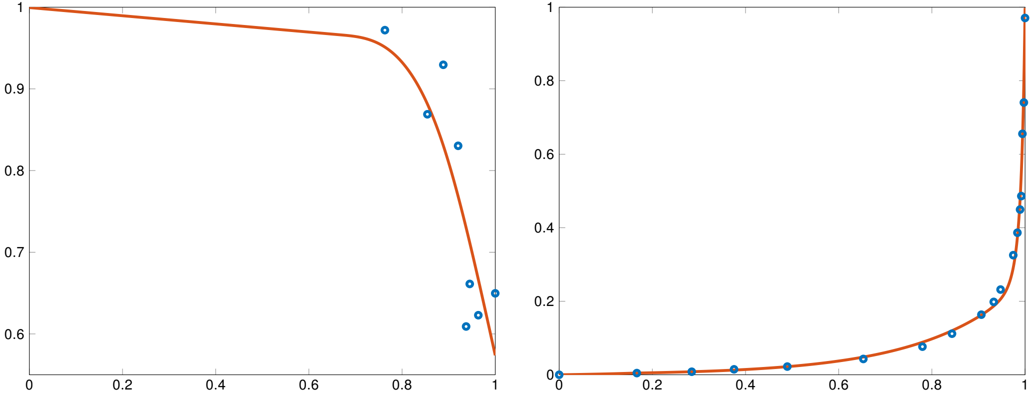
- Sufficient statistics:

$$\mathbb{E}_\lambda[\rho_\theta], \quad \mathbb{E}_\lambda[\sigma_\theta], \quad \bar{\mu}, \quad Cov_\lambda(\rho_\theta, \sigma_\theta).$$

- Do not use off-the-shelf functional form.
- Sales and pass-throughs by firm size for Belgian manufacturers (price and quantity data) from Amiti et al. (19).
- Back out sufficient statistics non-parametrically.

Data

- Let $\theta \in [0, 1]$ be firm size percentile.



(a) Cumulated Pass-through. (b) Cumulated sales share.

- Assume $\Upsilon_\theta\left(\frac{y_\theta}{Y}\right) = \Upsilon\left(B_\theta \frac{y_\theta}{Y}\right)$ (allow for taste-shifters). Solve for Υ that matches above.

Non-Parametric Calibration (Key Equations)

- Compared to θ, $\theta + d\theta$:
  - Has higher taste-adjusted productivity $d \log B_\theta A_\theta / d\theta$,
  - lower taste-adjusted price $\rho_\theta d \log B_\theta A_\theta / d\theta$,
  - and higher sales $(\sigma_\theta - 1)\rho_\theta d \log B_\theta A_\theta / d\theta$
  - Using $\sigma_\theta - 1 = 1/(\mu_\theta - 1)$:

$$\frac{d \log \lambda_\theta}{d\theta} = \frac{\rho_\theta}{\mu_\theta - 1} \frac{d \log B_\theta A_\theta}{d\theta}.$$

- Compared to θ, $\theta + d\theta$:
  - Has higher markup $(1 - \rho_\theta) d \log B_\theta A_\theta / d\theta$.

$$\frac{d \log \mu_\theta}{d\theta} = (1 - \rho_\theta) \frac{d \log B_\theta A_\theta}{d\theta}.$$

Non-Parametric Calibration

- Combining, we get

$$\frac{d\log\mu_\theta}{d\theta} = \frac{(\mu_\theta - 1)(1-\rho_\theta)}{\rho_\theta}\frac{d\log\lambda_\theta}{d\theta} \quad \text{s.t.} \quad \mathbb{E}_\lambda[\mu_\theta^{-1}]^{-1} = \bar{\mu}.$$

- From this, we get

$$\sigma_\theta = \frac{\mu_\theta}{\mu_\theta - 1}.$$

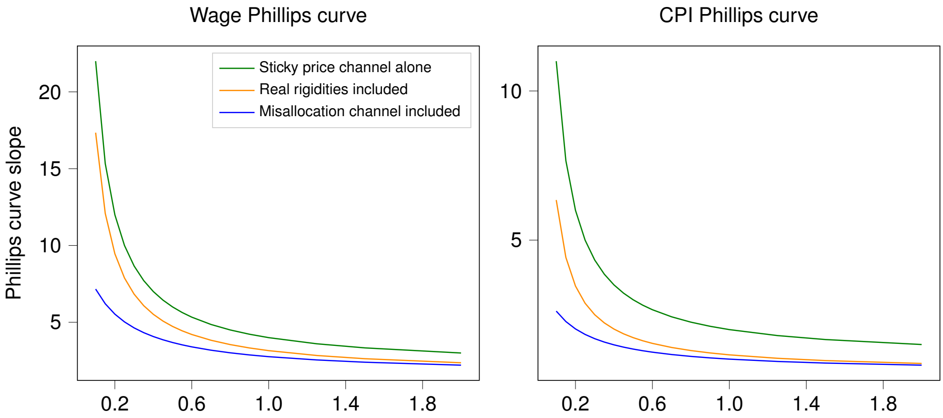
Slope of Phillips Curve Using Static Model

- For quantitative results: $\bar{\mu} = 15\%$; $\delta = 0.5$; $\zeta = 0.2$.

Model	$\frac{d \log w}{d \log Y}$	$\frac{d \log P^Y}{d \log Y}$
Sticky price alone	12.00	6.00
Real rigidities included	9.10	3.10
Misallocation channel included	5.53	2.02

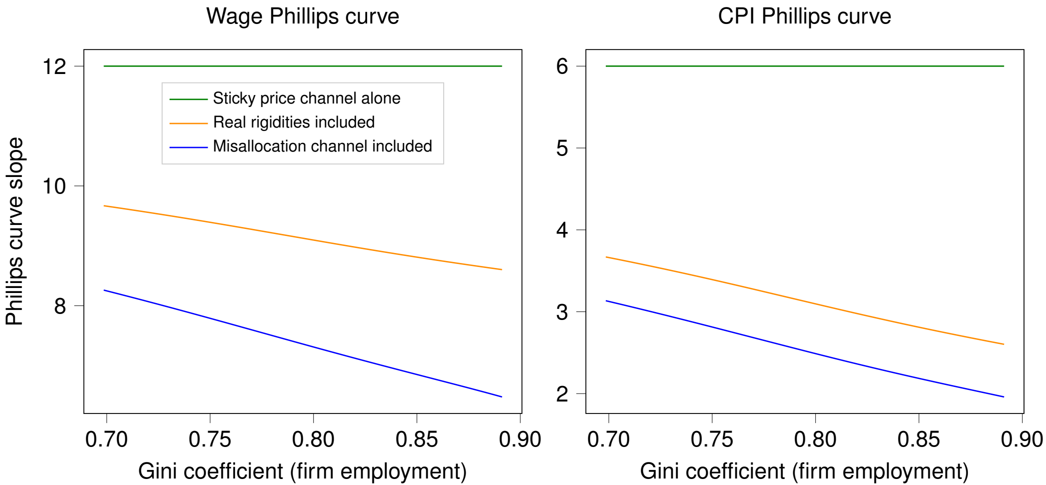
- Supply-side channel roughly as powerful as real rigidities.

Importance of Frisch – Static Model



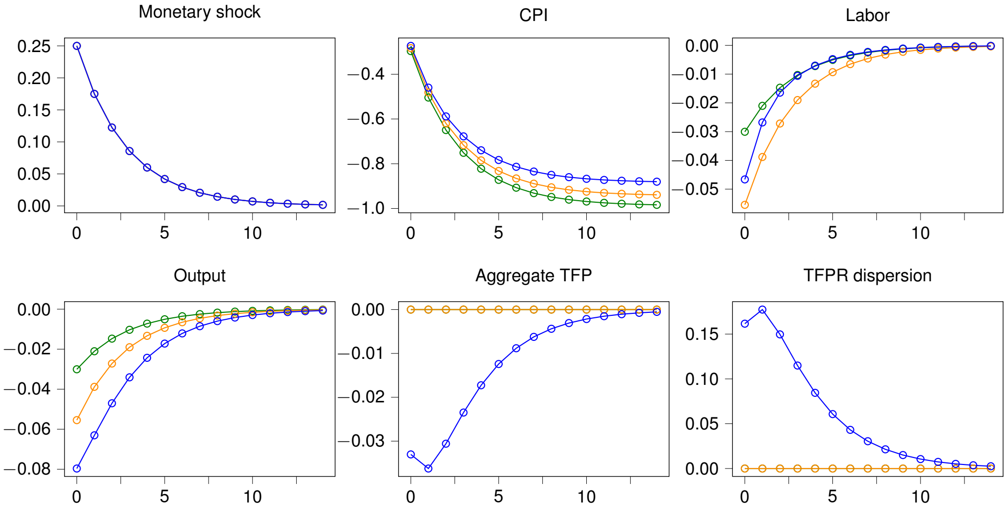
- Supply-side channel more powerful when Frisch is low.

Importance of Industrial Concentration – Static Model

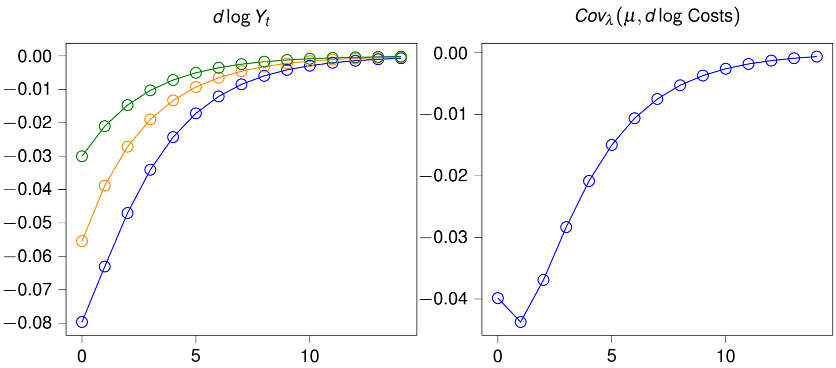


- Match Δ Gini coefficient of retail sector over 1978–2018 flattens Phillips curve by 41%.

IRF for Interest Rate Shock



Monetary Non-Neutrality



Model	Impact	Half life	Cumulative output
CES	-0.14	1.95	-0.47
With real rigidities	-0.22	1.95	-0.75
Full model	-0.27	2.39	-1.02

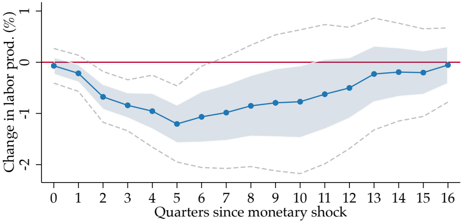
Predictions

- Macroeconomic:
  - Aggregate TFP is procyclical and falls after contractionary monetary shocks.
- Microeconomic:
  - Firm-level TFPR dispersion is countercyclical (Kehrig, 2011).
  - Reallocations to high-markup firms in expansions.

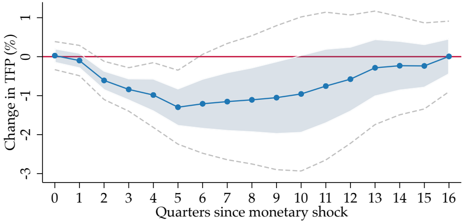
Aggregate TFP is Procyclical

Cycle measure	Labor productivity			Solow residual			Cost-based Solow residual		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Unemp.	-0.355**			-0.465**			-0.477**		
	(0.126)			(0.141)			(0.142)		
Recession		-0.878**			-2.114**			-2.082**	
		(0.394)			(0.414)			(0.500)	
$\%\Delta GDP$			0.221**			0.209*			0.354**
			(0.087)			(0.106)			(0.097)
Period	1948-2020			1948-2020			1961-2014		

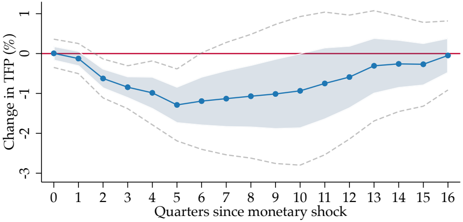
Productivity Responds to Romer & Romer Monetary Shocks



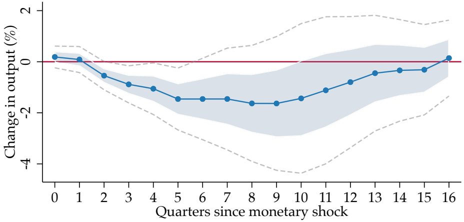
(a) Labor productivity



(b) Solow residual

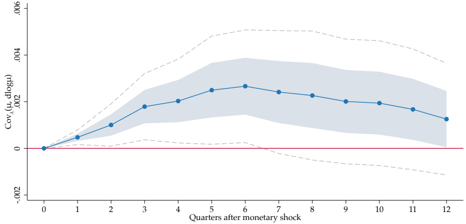


(c) Cost-based Solow residual

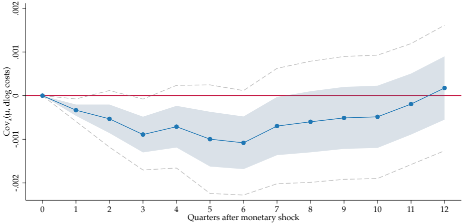


(d) Output

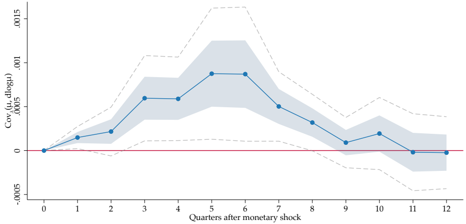
Micro-Level Reallocations (using Compustat data)



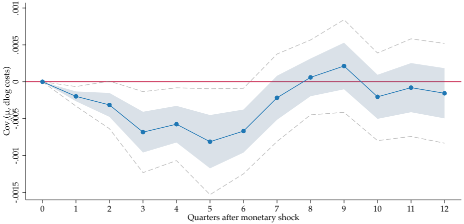
(a) $Cov_\lambda(\mu, \Delta \log \mu)$



(b) $Cov_\lambda(\mu, \Delta \log \text{Costs})$



(c) $Cov_\lambda(\mu, \Delta \log \mu)$ within NAICS-3



(d) $Cov_\lambda(\mu, \Delta \log \text{Costs})$ within NAICS-3

Recap

- Two models where firms face varying demand elasticities and set varying markups.

1. Nested CES with oligopolistic firms.
2. Non-CES demand with monopolistic competition.

- Decentralized equilibrium features misallocation across firms.
- Productivity can change due to endogenous reallocations.

Derivation for NKPC

- Optimal reset price for firm θ:

$$d\log p^*_{\theta,t} = [1-\beta(1-\delta_\theta)] \sum_{k=0}^{\infty} \beta^k (1-\delta_\theta)^k \left[\rho_\theta d\log w_{t+k} + (1-\rho_\theta) d\log P_{t+k}\right].$$

- Implies type-specific inflation equation:

$$d\log \pi_{\theta,t} = \varphi \left[\mathbb{E}\left[d\log p_{\theta,t}\right] + \rho_\theta d\log w_t + (1-\rho_\theta) d\log P_t\right] + \beta d\log \pi_{\theta,t+1}.$$

- Taking a sales weighted average yields NKPC.

Derivation for TFP equation

- Changes in TFP are given by

$$d\log A_t = d\log\bar{\mu} - \mathbb{E}_\lambda\left[d\log\mu_\theta\right].$$

- Changes in markups by subtracting $d\log w_t$ from

$$d\log\pi_{\theta,t} = \varphi\left[\mathbb{E}\left[d\log p_{\theta,t}\right] + \rho_\theta d\log w_t + (1-\rho_\theta)d\log P_t\right] + \beta d\log\pi_{\theta,t+1}.$$

- Combining gives second-order difference equation for TFP.