Lecture 11: The Rise in Markups

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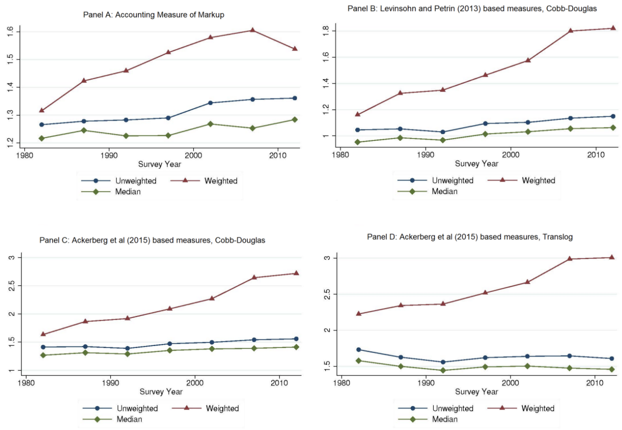
ECON 416-1

Markups

Figure 10: Markup Changes



- Last time we talked about the decline in the labor share.
- At the heart of two of the stories for the decline in the manufacturing labor share had to do with markups.
- Rising concentration and a reallocation to high-markup firms (Autor et al. 2020).
- Product premia among hyper-productive firms (Kehrig and Vincent, 2021).

Markups

Figure: Growth accounting.

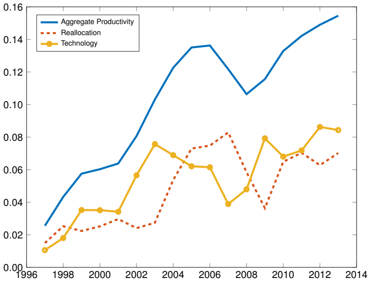


Table: Misallocation costs.

Markup measure	Distance to the frontier (2015)
User Cost (UC)	13%
Accounting (AP)	11%
Production Function (PF)	25%

- Markups were also at the center of other issues in our models of disaggregated production.
- Needed to compute cost-based Domar weights, $\tilde{\lambda}_i$.
- Growth accounting.
- Distance to efficient frontier. Tobin: *"It takes a heap of Harberger triangles to fill an Okun gap."*

Markups

- Even at the center of business cycle questions in New Keynesian model.
- Recall from "distortions" lecture,

$$\Delta u_t = -\frac{1}{2} Y_t^{1-\sigma} \left[\underbrace{\varepsilon \, Var\left(\log \mu_t(i)\right)}_{\substack{\text{Cost of inflation:} \\ \text{Misallocation due to wedge dispersion}}} + \underbrace{(\sigma + \varphi)\left(\Delta \log Y_t\right)^2}_{\substack{\text{Cost of output gap:} \\ \text{Moving off labor-leisure condition}}} \right] + h.o.t.$$

- Recall from "endogenous distortions" lecture,

$$\begin{aligned} d\log Y &= d\log A + d\log L \\ &= \frac{1}{1+\gamma\zeta}\left(d\log\bar{\mu} - \mathbb{E}_\lambda[d\log\mu_\theta]\right) - \frac{\zeta}{1+\gamma\zeta}\mathbb{E}_\lambda[d\log\mu_\theta]. \end{aligned}$$

Markups

- If markups are (theoretically) at the heart of each of these questions...
- This begs the question: How do we measure markups?
- Short answer:

  "Markups are notoriously difficult to measure because marginal costs are generally unobservable. Most empirical studies use structural approaches that rely on assumptions about production functions and market structure to infer marginal costs. This literature, reviewed in depth by Nekarda and Ramey (2020), is divided in its conclusions about the cyclical properties of markups, in part because different studies rely on different structural assumptions."
  —Anderson, Rebelo, Wong, 2023.

Markups

- Gameplan: Today, we will cover the "production function" approach:
  - Pioneered by Hall (1988), popularized by De Loecker, Eeckhout, and collaborators.
  - Ratio estimator is arguably most popular structural approach used in macro today.
  - We will cover methodology, critiques, and headline findings from De Loecker, Eeckhout, and Unger (2020).
- Next class, we will cover alternative approaches:
  - Accounting / user cost approaches.
  - Demand estimation.
  - How well various approaches align.

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Ratio estimator

- Consider firm with production, $y = F(x_1, ..., x_N)$, cost function $C(y, \boldsymbol{w})$. Markup is:

$$\mu = \frac{p}{\frac{\partial C(y,\boldsymbol{w})}{\partial y}} = \frac{py/C(y,\boldsymbol{w})}{\frac{\partial \log C(y,\boldsymbol{w})}{\partial \log y}}.$$

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- Using any input i, we can rewrite the elasticity of costs to output as

$$\frac{\partial \log C(y,\boldsymbol{w})}{\partial \log y} = \frac{\partial \log C(y,\boldsymbol{w})}{\partial \log x_i} \left(\frac{\partial \log y}{\partial \log x_i} \right)^{-1}.$$

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- If firm takes input prices $\boldsymbol{w}$ as given, cost minimization implies (Shephard's lemma):

$$\frac{\partial \log C(y,\boldsymbol{w})}{\partial \log x_i} = \frac{w_i x_i}{C(y,\boldsymbol{w})}.$$

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$$\frac{\partial \log C(y,\boldsymbol{w})}{\partial \log x_i} = \frac{w_i x_i}{C(y,\boldsymbol{w})}.$$

- So,

$$\mu = \frac{py/C(y,\boldsymbol{w})}{\frac{\partial \log C(y,\boldsymbol{w})}{\partial \log y}} = \frac{py/C(y,\boldsymbol{w})}{\frac{w_i x_i}{C(y,\boldsymbol{w})} \left(\frac{\partial \log F}{\partial \log x_i} \right)^{-1}} = \frac{\frac{\partial \log F}{\partial \log x_i}}{\frac{w_i x_i}{py}} = \frac{\theta_i}{\chi_i}.$$

Ratio estimator

- So to measure a firm's markup, we only need (i) one input's costs as share of revenue, and (ii) the elasticity of output with respect to that input.
- What are the underlying assumptions?
- Shephard's Lemma:
  - Firm is static cost minimizer.
  - Prod. fn. $F(x_1, ..., x_N)$ is continuous and nondecreasing in inputs, quasi-concave.
  - Cost function $C(\boldsymbol{w}, y)$ is homog. degree one in $\boldsymbol{w}$ (i.e., firm is **price taker**).
- We will return to threats to these assumptions... First, some examples.

Ratio estimator: Examples

- Suppose we have DRS production in labor, $Y = AL^{\alpha}$ and exog. markup μ.

$$\theta_L = \alpha.$$

$$\chi_L = \frac{wL}{pY} = \frac{w\left(\frac{Y}{A}\right)^{\frac{1}{\alpha}}}{\mu \frac{w}{\alpha}\left(\frac{Y}{A}\right)^{\frac{1}{\alpha}}} = \frac{\alpha}{\mu}.$$

- Suppose we have IRS production due to a fixed cost, $Y = A(L - \bar{L})$.

$$\theta_L = \frac{L}{L - \bar{L}}.$$

$$\chi_L = \frac{wL}{pY} = \frac{wL}{\mu \frac{w}{A} A(L - \bar{L})} = \frac{1}{\mu}\frac{L}{L - \bar{L}}.$$

- Aside: θ_L depends on level of L. Suppose we estimate $\hat{\theta}_L$ using variation in L. Will our estimate of μ be biased up or down the firm is using lower-than-avg L?

Ratio estimator: Examples

- Suppose firms indexed by i, with prod. fn. $Y_i = \left((A_i K_i)^{\frac{\theta-1}{\theta}} + L_i^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$.

$$\theta_{iL} = \frac{wL_i}{wL_i + rK_i} = \frac{w^{1-\theta}}{w^{1-\theta} + A_i^{\theta-1} r^{1-\theta}}.$$

$$\chi_{iL} = \frac{1}{\mu} \frac{wL_i}{wL_i + rK_i}.$$

- Suppose $\theta > 1$. Is the output elasticity w.r.t. labor θ_{iL} increasing or decreasing in A_i?
- Suppose we estimate $\hat{\theta}_L$ across all firms. Is $\hat{\theta}_L \lessgtr \theta_{iL}$ for high-A_i firms?
- Are our estimates of markups biased up or down for high-A_i firms?

Ratio estimator: Examples

- Suppose firms indexed by i, with prod. fn. $Y_i = \left((A_i K_i)^{\frac{\theta-1}{\theta}} + L_i^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$.

$$\theta_{iL} = \frac{wL_i}{wL_i + rK_i} = \frac{w^{1-\theta}}{w^{1-\theta} + A_i^{\theta-1} r^{1-\theta}}.$$

$$\chi_{iL} = \frac{1}{\mu} \frac{wL_i}{wL_i + rK_i}.$$

- Suppose we measure a translog production function, so that

$$\log Y_i = \alpha_K \log K_i + \alpha_L \log L_i + \frac{1}{2}\alpha_{KK}(\log K_i)^2 + \frac{1}{2}\alpha_{LL}(\log L_i)^2 + \alpha_{KL}(\log K_i)(\log L_i).$$

$$\Rightarrow \hat{\theta}_{iL} = \alpha_L + \alpha_{KL}(\log K_i) + \alpha_{LL}(\log L_i).$$

Could we fit α_{KL}, α_{LL} constants to match true output elasticities?

The central role of the output elasticity

- Clear that measure of output elasticity is crucial.
- To make this explicit: suppose all firms CRS, so that true $\theta_{iL} = wL_i / C(Y_i)$.
- Measured $\chi_{iL} = \theta_{iL} / \mu_i$.
- If we do not correctly measure heterogeneity in output elasticities, markup estimates influenced by variation in cost shares.
- Not so different from “accounting approach”...
  Maybe even worse if we only use one input and discard information on other costs.

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Challenges to measuring output elasticity

1. Simultaneity and selection problems.
2. Revenue data in lieu of output data.
3. Adjustment costs, demand-generating inputs, multi-product firms.

1. Simultaneity and selection problems

- Suppose output is log-linear in labor and capital, with Hicks-neutral productivity A.

$$\log y_{it} = \log A_{it} + \alpha_L \log L_{it} + \alpha_K \log K_{it}.$$

- Suppose we estimate regression:

$$\log y_{it} = \beta_0 + \beta_L \log L_{it} + \beta_K \log K_{it} + \varepsilon_{it}.$$

- Concern 1: Firms choose inputs observing productivity shock A_{it}, which are not observed by econometrician (simultaneity).

- E.g., if firms that have positive productivity shock grow and hire more labor, then

$$Cov(\varepsilon_{it}, \log L_{it}) = Cov(\log A_{it}, \log L_{it}) > 0.$$

Omitted variable bias increases estimate of β_L.

1. Simultaneity and selection problems

- Clearly, bias will be more severe for inputs that respond quickly to productivity shock A_{it}, such has labor (or materials, if we included it).

- But will also bias estimates of other production parameters. E.g., with prod. fn.

$$\log y_{it} = \log A_{it} + \alpha_L \log L_{it} + \alpha_K \log K_{it}.$$

  OLS estimates are

$$\hat{\beta}_L = \alpha_L + \frac{\sigma_{kk}\sigma_{l\varepsilon} - \sigma_{lk}\sigma_{k\varepsilon}}{\sigma_{ll}\sigma_{kk} - \sigma_{lk}^2},$$
$$\hat{\beta}_K = \alpha_K + \frac{\sigma_{ll}\sigma_{k\varepsilon} - \sigma_{lk}\sigma_{l\varepsilon}}{\sigma_{ll}\sigma_{kk} - \sigma_{lk}^2}.$$

- Even if capital is fixed (i.e., $\sigma_{k\varepsilon} \approx 0$), if $\sigma_{l\varepsilon} > 0$ and $\sigma_{lk} > 0$, then β_L will be overestimated and β_K will be underestimated.

1. Simultaneity and selection problems

- Suppose we just have labor in production:

$$\log y_{it} = \log A_{it} + \alpha_L \log L_{it},$$

  and we estimate the regression,

$$\log y_{it} = \beta_0 + \beta_L \log L_{it} + \varepsilon_{it}.$$

- But now suppose we test this regression with an unbalanced panel that includes endogenous entry and exit.

- Let $\chi = 1$ indicate survival. We estimate

$$\mathbb{E}[y_{it} | L_{it}, \chi_{it} = 1] = \beta_0 + \beta_L \log L_{it} + \mathbb{E}[\varepsilon_{it} | L_{it}, \chi_{it} = 1].$$

1. Simultaneity and selection problems

$$\mathbb{E}[y_{it}|L_{it},\chi_{it}=1]=\beta_0+\beta_L\log L_{it}+\mathbb{E}[\varepsilon_{it}|L_{it},\chi_{it}=1].$$

- Suppose survival model: $\chi_{it}=1$ if and only if $y_{it}\geq\bar{y}$.
- Then, $\chi_{it}=1$ if $\varepsilon_{it}=\log A_{it}\geq\log\bar{y}-\log L_{it}$.

$$\mathbb{E}[y_{it}|L_{it},\chi_{it}=1]=\beta_0+\beta_L\log L_{it}+\underbrace{\mathbb{E}[\varepsilon_{it}|L_{it},\varepsilon_{it}\geq\log\bar{y}-\log L_{it}]}_{\text{Decreasing in } L_{it}}.$$

  Intuitively, large firms are less likely to exit conditional on the same negative productivity shock as a small firm.

- Error term is negatively correlated with L_{it}, so selection leads to downward bias in β_L.

1. Simultaneity and selection problems – Solutions

- Luckily, IO literature has spent a lot of time thinking about these issues.
- Control function approach developed by Olley and Pakes (1992).
- Improvements by Petrin and Levinsohn (2003); Ackerberg, Caves, and Frazer (2015).
- Today: Brief overview of main ideas.

1. Simultaneity and selection problems – Olley and Pakes (1992)

- Olley and Pakes (1992) study the telecommunication equipment industry.
- AT&T procured 90% of equipment from subsidiary, Western Electric.
- “Carterphone” decision of 1968 and subsequent rulings by FCC made it possible for other equipment to be connected to network.
- Simultaneously, technological advancement (multiplexers, modems, faxes, etc.) due to transition from electromechanical to fully electronic technology.

1. Simultaneity and selection problems – Olley and Pakes (1992)

TABLE I

CHARACTERISTICS OF THE DATA

Year	Plants	Firms	Shipments (billions 1982 $)	Employment
1963	133	104	5.865	136899
1967	164	131	8.179	162402
1972	302	240	11.173	192248
1977	405	333	13.468	192259
1982	473	375	20.319	222058
1987	584	481	22.413	184178

- If we ignored exit, would lose 60% of plants and 40% of shipments in 1972.
- If we ignored entry, would lose 79% of plants and 30% of shipments in 1987.

1. Simultaneity and selection problems – Olley and Pakes (1992)

- Idea: At the beginning of each period, agent decides whether to continue operations, and if so chooses investment and variable inputs. In equilibrium, we will have:
  - Some exit rule $\chi_t(k_{it}, A_{it})$ in terms of state variables (capital) and productivity.
  - Some investment policy $i_t(k_{it}, A_{it})$ in terms of state variables and productivity.
- To deal with simultaneity,
  - (Under some conditions) can invert investment policy function $A_{it} = i_t^{-1}(k_{it}, i_{it})$.
  - Then, substitute this into production function to get consistent estimate of β_L,

$$y_{it} = \beta_L l_{it} + \underbrace{\phi_t(k_{it}, i_{it})}_{\beta_0 + \beta_K k_{it} + i_t^{-1}(k_{it}, i_{it})} + \varepsilon_{it},$$

- To deal with selection:
  - Estimate exit probability $Pr(\chi_{it+1} | k_{t+1}, A_{it+1}) = Pr(A_{it+1} > \bar{A}_{it+1}(k_{t+1}) | A_{it}) = P(k_{it}, i_{it})$.
  - With some assumptions on productivity process, can estimate, $y_{it+1} - \hat{\beta}_L l_{t+1} = \beta_k k_{t+1} + g(P_{it}, \hat{\phi}_{it} - \beta_K k_{it}) + \varepsilon_{it+1}$.

1. Simultaneity and selection problems – Olley and Pakes (1992)

TABLE VI

ALTERNATIVE ESTIMATES OF PRODUCTION FUNCTION PARAMETERS[a]
(STANDARD ERRORS IN PARENTHESES)

Sample:	Balanced Panel		Full Sample[c, d]						
								Nonparametric F_ω	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Estimation Procedure	Total	Within	Total	Within	OLS	Only P	Only h	Series	Kernel
Labor	.851	.728	.693	.629	.628			.608	
	(.039)	(.049)	(.019)	(.026)	(.020)			(.027)	
Capital	.173	.067	.304	.150	.219	.355	.339	.342	.355
	(.034)	(.049)	(.018)	(.026)	(.018)	(.02)	(.03)	(.035)	(.058)
Age	.002	−.006	−.0046	−.008	−.001	−.003	.000	−.001	.010
	(.003)	(.016)	(.0026)	(.017)	(.002)	(.002)	(.004)	(.004)	(.013)
Time	.024	.042	.016	.026	.012	.034	.011	.044	.020
	(.006)	(.017)	(.004)	(.017)	(.004)	(.005)	(.01)	(.019)	(.046)
Investment	—	—	—	—	.13	—	—	—	—
					(.01)				
Other Variables	—	—	—	—	—	Powers of P	Powers of h	Full Polynomial in P and h	Kernel in P and h
# Obs.[b]	896	896	2592	2592	2592	1758	1758	1758	1758

1. Simultaneity and selection problems – Solutions

- Olley and Pakes (1992) provide a way to use investment and survival probabilities to control for simultaneity and selection problems.
- Levinsohn and Petrin (2003) point out that investment behavior is lumpy.
- Static intermediate inputs will also be monotonically increasing in productivity.

TABLE 2

Percent of Usable Observations, 1979-85

Industry	Investment	Fuels	Materials	Electricity
Metals	44.8	63.1	99.9	96.5
Textiles	41.2	51.2	99.9	97.0
Food Products	42.7	78.0	99.8	88.3
Beverages	44.0	73.9	99.8	94.1
Other Chemicals	65.3	78.4	100	96.5
Printing & Pub.	39.0	46.4	99.9	96.8
Wood Products	35.9	59.3	99.7	93.8
Apparel	35.2	34.5	99.9	97.2

- Ackerberg, Caves, and Frazer (2015) point out that if you can use materials

$$y_{it} = \beta_L l_{it} + \phi_t(k_{it}, m_{it}) + \varepsilon_{it}.$$

1. Simultaneity and selection problems – Solutions

- Ackerberg, Caves, and Frazer (2015) point out that OP/LP take the form:

$$y_{it} = \beta_L l_{it} + \phi_t(k_{it}, i_{it}) + \varepsilon_{it}. \qquad \text{(Olley-Pakes)}$$

$$y_{it} = \beta_L l_{it} + \phi_t(k_{it}, m_{it}) + \varepsilon_{it}. \qquad \text{(Levinsohn-Petrin)}$$

- But if you can use $m_{it}(k_{it}, A_{it})$, why not $l_{it}(k_{it}, A_{it})$?
- Both labor and materials are determined by cost minimization and conditional on state (simultaneously chosen).
- But then, perfect collinearity $\rightarrow$ cannot identify β_L.
- Solutions: Instrument z_{it} that impacts labor choice but independent of state (k_{it}, A_{it}):
  - E.g., i.i.d. input price differences across plants, or I.i.d. random draws to environment (e.g., strikes, machine breakdown, maintenance periods).
  - Helps with OP, not with LP (if materials chosen conditional on labor).

Challenges to measuring output elasticity

1. Simultaneity and selection problems.
   - Difficult problems, but “off the shelf” techniques from IO to deal with these biases.
2. Revenue data in lieu of output data.
3. Adjustment costs, demand-generating inputs, multi-product firms.

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2. Revenue vs. output elasticities

- In practice, we rarely observe output quantity y.
- Often revenue py used as a proxy.
- Suppose instead of output elasticity θ_i, we estimate "revenue elasticity" θ_i^R:

$$\theta_i^R = \frac{\partial \log(py)}{\partial \log x_i} = \left(1 - \frac{1}{\sigma}\right) \frac{\partial \log y}{\partial \log x_i} = \frac{\sigma - 1}{\sigma} \theta_i,$$

where $\sigma = -\frac{\partial \log y}{\partial \log p}$ is the elasticity of demand.

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  where $\sigma = -\frac{\partial \log y}{\partial \log p}$ is the elasticity of demand.

- Then, estimated markup will be

$$\hat{\mu}^R = \frac{\theta_i^R}{\chi_i} = \frac{\sigma - 1}{\sigma} \mu = 1,$$

  where the last equality imposes firm profit maximization.

2. Revenue vs. output elasticities (Bond et al. 2021)

- This critique is raised by Bond, Hashemi, Kaplan, and Zoch (2021).
- Klette and Griliches (1996) point out that revenue elasticity $<$ output elasticity when demand curves are downward-sloping.
  - Often cited as reason why markups obtained with θ_i^R are downward-biased.
- Problem is more fundamental: If firm is profit-maximizing given downward-sloping demand, bias in θ_i^R vs. θ_i removes all information content for markup!
- Moreover, if you estimate $\mu^R \neq 1$, then how should you interpret your estimates?

2. Aside: Markdowns

- Yeh, Macaluso, and Hershbein (2022) show that markdowns on labor can be recovered using a similar ratio estimator.

- First, estimate markup using a statically-chosen input that firm is a price-taker for:

$$\mu = \frac{\theta_M}{\chi_M}.$$

  Then, estimate markdown on labor ν using

$$\nu = \frac{\theta_L}{\chi_L}\mu^{-1} = \frac{\theta_L}{\chi_L}\frac{\chi_M}{\theta_M}.$$

- Even if we use θ_L^R and θ_M^R, bias $(1 - 1/\sigma)$ will cancel.

- Concerns about variation driven by χ_L, χ_M remain.

2. Revenue vs. output elasticities: Solutions

- De Ridder, Grassi, and Morzenti (2025) consider the bias from using revenue data to estimate markups.
- Suppose firms share Cobb-Douglas production function,

$$y_{it} = \alpha l_{it} + a_{it},$$

  where lowercase variables are logs, and we assume A_{it} are i.i.d. shocks.

- If we have output data, given assumption of i.i.d. A_{it}, we can overcome simultaneity bias using l_{it-1} as instrument for l_{it}.
  - Idea: a_{it} is unknown at $t-1$, so l_{it-1} orthogonal to a_{it}.
  - Relevance: either need persistence in input price w_t or persistence in set of competitors.
- So, IV-GMM estimator $\mathbb{E}[(y_{it} - \hat{\alpha} l_{it}) l_{it-1})] = 0$ will yield consistent estimate $\hat{\alpha}$ of α.

2. Revenue vs. output elasticities: Solutions

- Now suppose we instead observe log revenue $r_{it} = p_{it} + y_{it}$.
- Analogous estimator is
$$\mathbb{E}[(r_{it} - \hat{\alpha} l_{it}) l_{it-1}] = 0.$$
- Revenue-based estimate of $\hat{\alpha}$ is now
$$\hat{\alpha} = \alpha + \frac{\mathbb{E}[p_{it} l_{it-1}]}{\mathbb{E}[l_{it} l_{it-1}]} \neq \alpha.$$
- Log-linear approximation of $p_{it} = -\varepsilon_{it} y_{it}$, where ε_{it} is inverse demand elasticity.
$$\begin{aligned}\hat{\alpha} &= \alpha + \frac{\mathbb{E}[-\varepsilon_{it}[\alpha l_{it} + a_{it}] l_{it-1}]}{\mathbb{E}[l_{it} l_{it-1}]} \\ &= \alpha \left(1 - \frac{\mathbb{E}[\varepsilon_{it} l_{it} l_{it-1}]}{\mathbb{E}[l_{it} l_{it-1}]} \right).\end{aligned}$$

2. Revenue vs. output elasticities: Solutions

- So now our markup estimated with the revenue elasticity is

$$\mu_{it}^R = \frac{\alpha\left(1 - \frac{\mathbb{E}[\varepsilon_{it} l_{it} l_{it-1}]}{\mathbb{E}[l_{it} l_{it-1}]}\right)}{\chi_{it}^L} = \mu_{it} \underbrace{\left(1 - \frac{\mathbb{E}[\varepsilon_{it} l_{it} l_{it-1}]}{\mathbb{E}[l_{it} l_{it-1}]}\right)}_{\text{Constant across } i,t}.$$

- If $\varepsilon_{it} = 1/\sigma$, then we get

$$\mu_{it}^R = \mu_{it}\left(1 - \frac{1}{\sigma}\right) \to 1, \qquad \text{if } \mu_{it} = \frac{\sigma}{\sigma - 1}.$$

- But if ε_{it} differs across firms, then this will no longer be the case.

- Constant $\left(1 - \frac{\mathbb{E}[\varepsilon_{it} l_{it} l_{it-1}]}{\mathbb{E}[l_{it} l_{it-1}]}\right)$ will bias level of average markups, but not estimates of relative markups.

2. Revenue vs. output elasticities: Solutions

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- Constant $\left(1 - \frac{\mathbb{E}[\varepsilon_{it} l_{it} l_{it-1}]}{\mathbb{E}[l_{it} l_{it-1}]}\right)$ will bias level of average markups, but not estimates of relative markups. ...Reactions?

2. Revenue vs. output elasticities: Solutions

Table 5: Overview - Translog Log Markup Estimates

	Correlation ln $\widehat{\mu}_{iht}$ with true markup	Log Markup Moments			
		Mean	St. Dev.	Median	IQR
True values	1.00	0.258	0.065	0.213	0.367
Quantity					
Full first stage (preferred)	1.00	0.266	0.064	0.220	0.346
Basic first stage	1.00	0.263	0.067	0.218	0.370
No first stage	0.66	0.300	0.102	0.225	0.621
Revenue					
Full first stage	0.69	0.286	0.108	0.206	0.713
Basic first stage	0.73	0.286	0.105	0.208	0.677
No first stage	0.40	0.312	0.201	0.184	1.543

- De Ridder, Grassi, and Morzenti (2025) simulate industries with translog production functions and Atkeson–Burstein preferences.
- Output elasticity- and revenue elasticity-based estimates are well correlated.

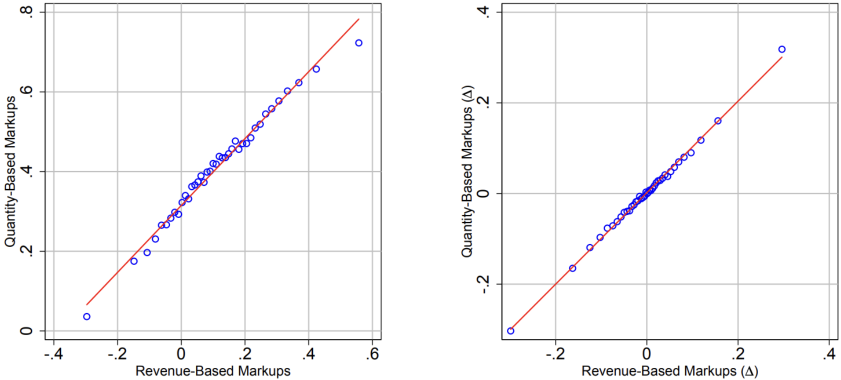
2. Revenue vs. output elasticities: Solutions

Table 6: Correlations across Simulated Specifications - Log Markups

	True	Full - Q	Full - R	Basic - Q	Basic - R	None - Q	None - R
				Pearson Correlation			
True	1.00	1.00	0.69	1.00	0.73	0.66	0.40
Full First Stage - Quantity	1.00	1.00	0.75	0.99	0.79	0.72	0.48
Full First Stage - Revenue	0.69	0.75	1.00	0.66	1.00	1.00	0.94
Basic First Stage - Quantity	1.00	0.99	0.66	1.00	0.70	0.62	0.36
Basic First Stage - Revenue	0.73	0.79	1.00	0.70	1.00	0.99	0.92
No First Stage - Quantity	0.66	0.72	1.00	0.62	0.99	1.00	0.95
No First Stage - Revenue	0.40	0.48	0.94	0.36	0.92	0.95	1.00
				Spearman Rank Correlation			
True	1.00	0.99	0.65	1.00	0.69	0.61	0.39
Full First Stage - Quantity	0.99	1.00	0.72	0.98	0.76	0.69	0.48
Full First Stage - Revenue	0.65	0.72	1.00	0.60	1.00	1.00	0.94
Basic First Stage - Quantity	1.00	0.98	0.60	1.00	0.64	0.56	0.33
Basic First Stage - Revenue	0.69	0.76	1.00	0.64	1.00	0.99	0.92
No First Stage - Quantity	0.61	0.69	1.00	0.56	0.99	1.00	0.96
No First Stage - Revenue	0.39	0.48	0.94	0.33	0.92	0.96	1.00

- Correlation across different GMM specifications.

2. Revenue vs. output elasticities: Solutions



(a) Log-Markups (b) Log-Differenced Markups

- Conduct estimation on French FARE/EAP manufacturing data. FARE covers accounting data and EAP is product-level survey with prices at 10-digit level.
- Different level of markups, but trends / cross-section correlated.

2. Revenue vs. output elasticities: Solutions

Table 10: Correlations across Specifications - Log Markups

	Full - Q	Full - R	Basic - Q	Basic - R	None - Q	None - R
			Pearson Correlations			
Full first stage - Quantity data	1.00	0.28	0.60	0.29	0.67	0.28
Full first stage - Revenue data	0.28	1.00	0.43	0.98	0.31	0.74
Basic first stage - Quantity data	0.60	0.43	1.00	0.46	0.58	0.31
Basic first stage - Revenue data	0.29	0.98	0.46	1.00	0.32	0.73
No first stage - Quantity data	0.67	0.31	0.58	0.32	1.00	0.22
No first stage - Revenue data	0.28	0.74	0.31	0.73	0.22	1.00
			Spearman Rank Correlations			
Full first stage - Quantity data	1.00	0.33	0.71	0.34	0.81	0.33
Full first stage - Revenue data	0.33	1.00	0.49	0.99	0.37	0.86
Basic first stage - Quantity data	0.71	0.49	1.00	0.51	0.73	0.41
Basic first stage - Revenue data	0.34	0.99	0.51	1.00	0.38	0.84
No first stage - Quantity data	0.81	0.37	0.73	0.38	1.00	0.32
No first stage - Revenue data	0.33	0.86	0.41	0.84	0.32	1.00
			Spearman Rank Correlations - Within Sectors			
Full first stage - Quantity data	1.00	0.50	0.74	0.50	0.76	0.52
Full first stage - Revenue data	0.50	1.00	0.62	0.99	0.51	0.86
Basic first stage - Quantity data	0.74	0.62	1.00	0.63	0.75	0.54
Basic first stage - Revenue data	0.50	0.99	0.63	1.00	0.52	0.86
No first stage - Quantity data	0.76	0.51	0.75	0.52	1.00	0.46
No first stage - Revenue data	0.52	0.86	0.54	0.86	0.46	1.00

- Useful reference for effect of assumptions. See paper for translog vs. Cobb-Douglas.

Challenges to measuring output elasticity

1. Simultaneity and selection problems.
   - Difficult problems, but “off the shelf” techniques from IO to deal with these biases.
2. Revenue data in lieu of output data.
   - Perhaps we can still identify variation in markups, if not level.
3. Adjustment costs, demand-generating inputs, multi-product firms.

Challenges to measuring output elasticity

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3. Adjustment costs and demand-generating inputs (Bond et al. 2021)

- Suppose the variable input we use for the ratio estimator is not perfectly flexible.
- For input x_i, baseline quantity $\bar{x}_i$. If firm chooses $x_i \neq \bar{x}_i$, faces adjustment costs $\kappa(x_i)$.
- Then measured output elasticity and markup are

$$\hat{\mu} = \frac{\hat{\theta}_i}{\chi_i} = \mu \left(1 + \frac{d\kappa(x_i)}{dx_i}\right).$$

- Suppose input i can be used for both production (x_i^y) or generating demand (x_i^D),

$$p = P(y, \{x_i^D\}), \qquad \text{and} \qquad x_i^y + x_i^D = x_i.$$

- Then measured output elasticity and markup are

$$\hat{\mu} = \mu \left(\frac{\frac{\partial \log x_i^y}{\partial \log x_i}}{1 + \frac{x_i^d}{x_i^y}} \right).$$

3. Multi-product firms

- Many large firms whose market power we want to study have large product portfolios.
- Ariel Pakes: *"Work by De Loecker and co-authors provides a way of separating revenue growth from output growth from production data given a set of assumptions. This enables one to study markups without the detail needed [for demand system estimation]. Assumptions are questionable; most production is by multi-product firms and the model does not allow this, require a perfectly variable input, ..."*
- *"...We will go over this, but whatever one thinks, it is clear from De Loecker, Eeckhout, and Unger (2020) that the profits and the size (measured in sales) of the largest 10% of firms have gone up, and labor share has gone down more generally in the economy. This has fed into the public debate about inequality. Their assumptions do not hold for the firms they are dealing with, but the analysis has generated a lot of interest as it dovetails with the literatures on the increasing dominance of a few large firms in the economy."*

3. Multi-product firms – Solutions

- De Loecker, Goldberg, Khandelwal, and Pavcnik (2016) show how one can deal with these issues using selection-style approach.
- Idea: Estimate production parameters using single-product firms.
- Explicitly model selection into becoming a multi-product firm.

Table 5: Output Elasticities, Input Price Variation and Sample Selection

	Estimates without Correcting for Input Price Variation				Estimates without Correcting for Sample Selection			
Sector	Labor (1)	Materials (2)	Capital (3)	Returns to Scale (4)	Labor (1)	Materials (2)	Capital (3)	Returns to Scale (4)
15 Food products and beverages	0.66	0.21	1.71	2.62	0.19	0.64	0.24	1.05
17 Textiles, Apparel	-0.02	0.58	-0.05	0.41	0.12	0.75	0.13	1.01
21 Paper and paper products	0.03	-0.09	-0.19	-0.48	0.27	0.62	0.03	0.92
24 Chemicals	0.71	0.67	-0.74	0.57	0.21	0.75	0.15	1.11
25 Rubber and Plastic	0.03	0.02	0.13	0.38	0.19	0.76	0.11	1.04
26 Non-metallic mineral products	0.27	0.30	0.92	1.43	0.24	0.48	0.18	0.90
27 Basic metal	-0.27	0.92	0.02	0.85	0.12	0.74	0.12	0.99
28 Fabricated metal products	-1.28	-0.67	2.18	0.26	0.16	0.65	0.08	0.90
29 Machinery and equipment	0.05	0.22	-0.21	0.29	0.29	0.56	0.34	1.15
31 Electrical machinery, communications	-1.49	-0.08	0.31	-0.26	0.08	0.76	0.05	0.97
34 Motor vehicles, trailers	0.02	-0.46	1.63	0.84	0.26	0.53	0.31	1.07

Notes: The left table reports the median output elasticities from production function estimations that do not account for input price variation. The right panel reports the median output elasticities from production function estimations that do not account for sample selection (transition from single-product to multi-product firms).

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DLEU data

- Main results use publicly listed firms from 1950–2016 (account for 30% of US employment).
- Use cost of goods sold (COGS) as variable input.
- Will also compare markups to overall profitability using SG&A costs and user cost of capital $r_t K_t$, where
  - K_t is gross property, plants, and equipment (PPEGT) deflated by industry-level input price deflators,
  - $r_t = i_t + \pi_t - \delta_t$.

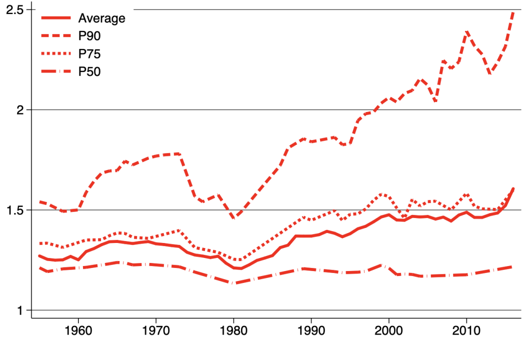
DLEU markups

(A) Constant elasticity

(B) Input weighted (total cost)

- Headline result: Markups increase from 1.21 in 1980 to 1.61 in 2016.
- Many critiques about sales-weighted headline results.
- In response, right panel shows μ weighted by total cost (COGS $+$ SG&A $+$ rK).

DLEU markups



(B) Percentiles markup distribution (revenue weight)

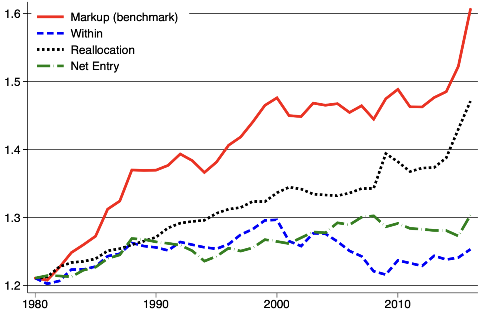
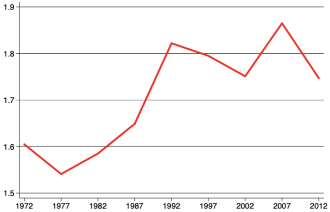


FIGURE IV

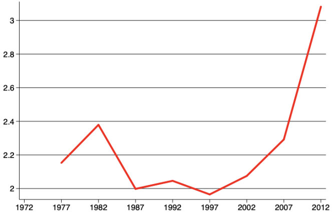
Decomposition of Markup Growth at the Firm Level

- Markup of median firm largely flat.
- 2/3 due to reallocation, 1/3 due to entry / within-firm markup changes.
- 86% of rise due to changes within (broad) sectors, not across.

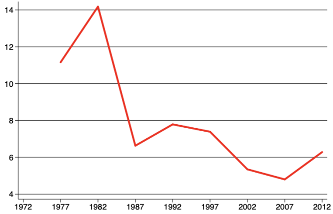
DLEU markups



(A) Manufacturing: average



(C) Retail: average



(E) Wholesale: average

- Concerns about selection with public firms only.
- DLEU also estimate markups in Census data.
- For Census of Manufacturers, observe material and labor costs.
- Retail and wholesale censuses do not observe cost shares, so *"In the absence of information on cost shares, we infer the output elasticities of labor using the cost-share approach in Compustat. In particular for each two-digit NAICS sector (s), we compute the median labor cost share, by year, for the sample of active firms."*

DLEU markups and fixed costs

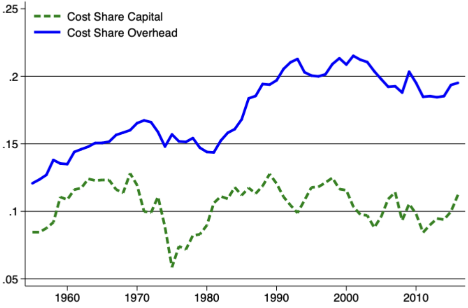
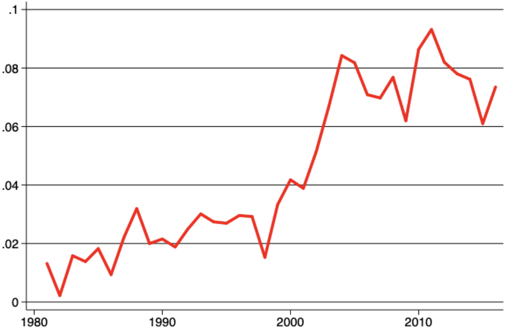


FIGURE VII

Aggregate Overhead and Capital Cost Shares of Total Cost



(A) Average profit rate (revenue weighted)

- Some of the increase in markups has been offset by an increase in fixed costs.
- But argue that profits net of fixed costs (capital, SG&A) have also risen.

DLEU markups and market value / dividends

TABLE II

FIRM-LEVEL REGRESSIONS: MARKET VALUES AND DIVIDENDS ON MARKUPS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\ln\left(\frac{\text{market value}}{\text{sales}}\right)$				ln(market value)			
ln(markup)	0.71	0.64	0.56	0.17	0.71	0.65	0.58	0.27
	(0.03)	(0.02)	(0.02)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)
ln(sales)					0.81	0.81	0.83	0.68
					(0.00)	(0.00)	(0.00)	(0.01)
Year fixed effects		Y	Y	Y		Y	Y	Y
Sector fixed effects			Y				Y	
Firm fixed effects				Y				Y
R^2	0.05	0.13	0.21	0.68	0.68	0.71	0.73	0.89
	$\ln\left(\frac{\text{dividends}}{\text{sales}}\right)$				ln(dividends)			
ln(markup)	1.05	0.97	0.80	0.26	1.03	0.93	0.78	0.26
	(0.04)	(0.03)	(0.04)	(0.05)	(0.04)	(0.04)	(0.04)	(0.05)
ln(sales)					0.94	0.92	0.93	0.76
					(0.01)	(0.01)	(0.01)	(0.02)
Year fixed effects		Y	Y	Y		Y	Y	Y
Sector fixed effects			Y				Y	
Firm fixed effects				Y				Y
R^2	0.06	0.11	0.17	0.70	0.66	0.68	0.70	0.89

Note: Standard errors clustered by firm are in brackets.

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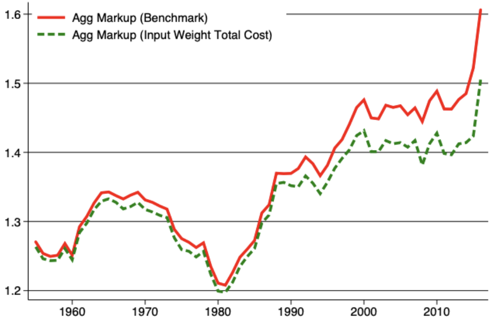
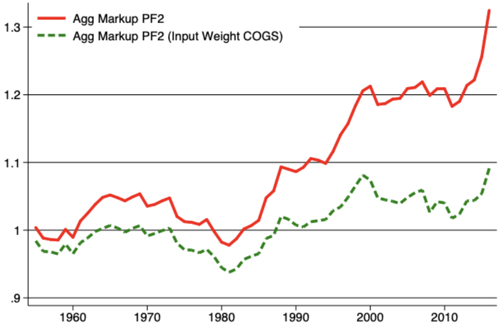
Sales- vs. cost-weighted markups (Edmond, Midrigan, and Xu 2015)



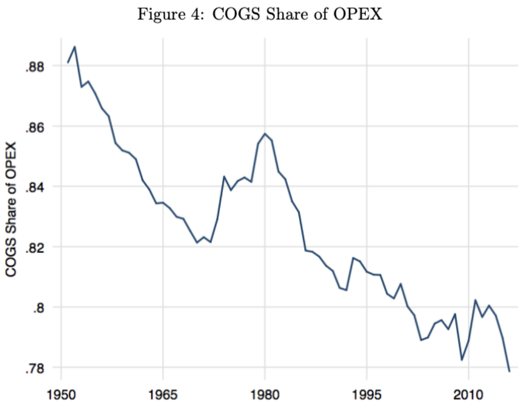
(B) Input weighted (total cost)

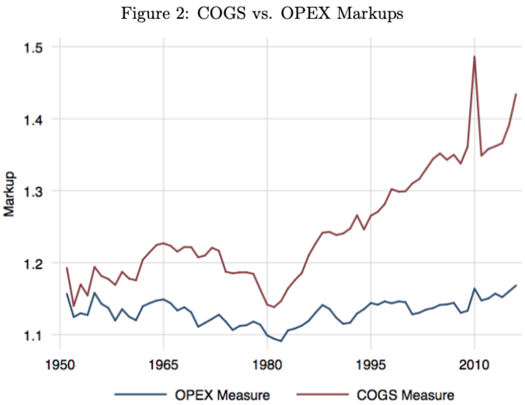


(B) COGS, PF2

- Edmond, Midrigan, and Xu (2015) calculate that cost-weighted average markup increases from 1.15 to 1.25 from 1950 to 2016.
- Why is cost-weighted markup the correct measure?

Critiques of the DLEU markups: Traina (2018) on SG&A





- Whether firms count expenses as costs of goods sold vs. sales, general, and administrative expenses depends more on accounting rules/practices and norms.

Estimates of profit rate and returns to scale (Basu 2019)

- We can write profit rate as

$$\pi = \frac{py - C(y)}{py} = 1 - \frac{C(y)}{\mu \frac{\partial C}{\partial y} y} = 1 - \frac{1}{\mu} \frac{ac}{mc}.$$

- Basu (2019) critique: Most empirical evidence suggests roughly constant returns.

- If $ac/mc \approx 1$, then markup of $\mu = 1.61$ implies profit rate

$$\pi = 1 - \frac{1}{1.61} = 38\%.$$

  This is way higher than 8% profit rate DLEU find.

- In order to match profit rate of 8%, would need $ac/mc \approx 1.48$.

Estimates of profit rate and returns to scale (Basu 2019)

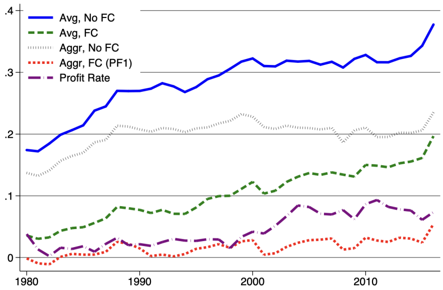
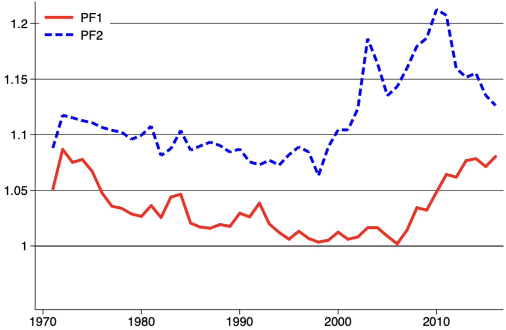


FIGURE XI

Decomposition of Equation (15) due to Overhead Costs and Aggregation



(A) Returns to scale (sum of output elasticities) of estimated PF1 and PF2; revenue weighted

- Combination of fixed costs and composition effects.
- Roughly constant returns to scale "on the margin."

Critiques of the DLEU markups: Benkard, Miller, Yurukoglu (2025)

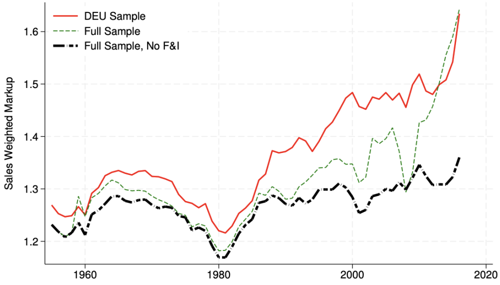


Figure 1: Markups Obtained with the DEU Sample and the Full Sample

Notes: The figure plots estimates of the sales-weighted average markup (the ratio of price to marginal cost) over time. The solid red line is a replication of Figure I of DEU, which uses a restricted sample. The dashed green line uses the full sample. The black dash - dot line uses the full sample except for the F&I sector.

Figure: Benkard, Miller, and Yurukoglu (2025).

- *"[Sample] restrictions excluded an additional 27% of the observations from the Compustat data used for the main results."*
- *"[...] Unlike the results in DEU, which are robust to the exclusion of individual sectors, in the full sample, the increase in markups at the end of the sample is driven almost entirely by a single sector: Finance and Insurance (F&I)."*

Critiques of the DLEU markups: Benkard, Miller, Yurukoglu (2025)

- DLEU response: *“Here we show that the findings in BMY are entirely driven by outliers in one four-digit NAICS industry 3254 (Pharmaceutical and Medicine Manufacturing). BMY estimate output elasticities that are affected by the inclusion of extremely small firms that have negligible revenue because they produce no output. This severely impact the production function estimation and biases the estimates for the entire sample of firms in the broader industry (here NAICS 32).”*

- *“One key requirement of production function estimation is that there be production. [....] The key features of these small firms are: 1. They have very low sales, some costs, no SGA, and negative profits; 2. There is a substantial number of them; 3. There is a massive change over time, from virtually none in 1990 to more than half of the firms in this 4 digit industry; 4. These small firms jointly have a small market share in industry 3254, less than 3% of revenue.”*

- *“By including firms that have revenue that is orders of magnitude smaller than their costs, the regression coefficients of the production function are unreliable. [...] These firms are negligible in sector 32, yet they have equal weight as large firms in the estimation because production function estimation typically does not weigh the observations. However, in the estimation, a small biotech startup for example has equal weight as Pfizer.”*

Recap

- DLEU have been influential in shaping consensus on rising markups.
- Seems to accord with trends, e.g., falling labor share, labor market dynamism.
- Ratio estimator subject to many critiques and caveats.
  - De Loecker and collaborators have addressed several of these limitations in previous work in more stylized settings (De Loecker and Warzynski, 2012 on Slovenia; De Loecker, Goldberg, Khandelwal, and Pavcnik 2016 on India).
  - But limitations seem especially salient for U.S. public firms, and debate ongoing.
- Next class, we will cover alternative approaches:
  - Accounting / user cost approaches.
  - Demand estimation.
  - How well various approaches align.