Lecture 9:
Consumer Search

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ECON 416-1

Roadmap

- Previously, we discussed two canonical models of variable markups.
  - Nested CES with oligopolistic firms.
  - Non-CES demand.
- Last time, we added two models where markups differed from the Lerner formula.
  - Limit pricing.
  - Cannibalization.
- Today, we will discuss consumer search.
- Consumer search breaks the elasticity of demand into two parts: (i) knowledge of substitutes, (ii) willingness-to-substitute conditional on knowledge.

The Economics of Information (Stigler 1961)

- George Stigler (1961): *“One should hardly have to tell academicians that information is a valuable resource: knowledge is power. And yet it occupies a slum dwelling in the town of economics. Mostly it is ignored: the best technology is assumed to be known; the relationship of commodities to consumer preferences is a datum. And one of the information-producing industries, advertising, is treated with a hostility that economists normally reserve for tariffs or monopolists.”*

- *“Prices change with varying frequency in all markets, and, unless a market is completely centralized, no one will know all the prices which various sellers (or buyers) quote at any given time. A buyer (or seller) who wishes to ascertain the most favorable price must canvass various sellers (or buyers)—a phenomenon I shall term ‘search.’”*

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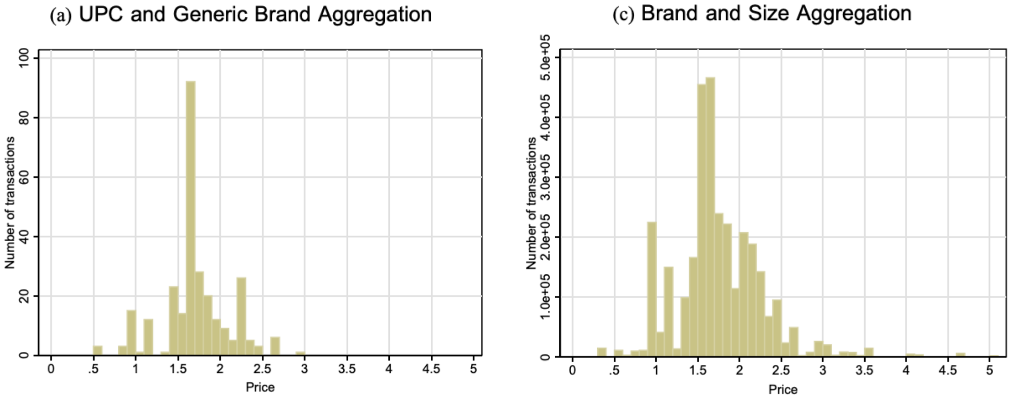
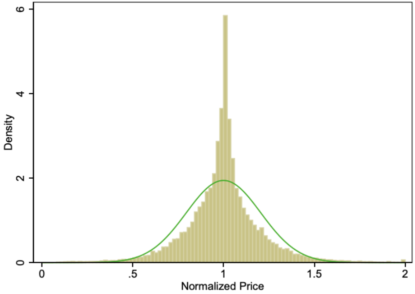
Heinz ketchup (Kaplan and Menzio 2015)



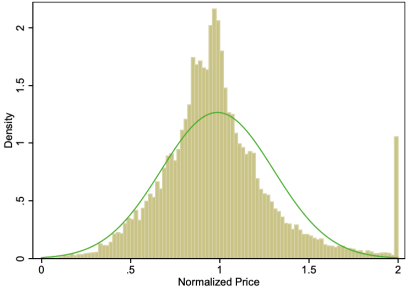
- 36oz bottles of Heinz ketchup purchased in Minneapolis in 2007Q1.
- Among 279 transactions, max price is \$2.99, min price is \$0.50, median is \$1.66.
- Pooling all ketchup products together, coefficient of variation rises from 23% to 29%.

The morphology of price dispersion (Kaplan and Menzio 2015)

(a) UPC



(d) Brand and Size Aggregation



	Aggregation UPC (1)	Generic Brand Aggregation (2)	Brand Aggregation (3)	Brand & Size Aggregation (4)
Standard deviation	0.19	0.21	0.25	0.36
90–10 ratio	1.72	1.79	2.04	2.61
90–50 ratio	1.26	1.29	1.38	1.55
50–10 ratio	1.35	1.38	1.46	1.64

NOTES: All statistics are expenditure weighted across goods, markets, and quarters. Sample includes only transactions at stores with unique identifiers for goods/markets/quarters with at least 25 transactions and coefficient of variation less than 1.

The morphology of price dispersion (Kaplan and Menzio 2015)

- Some price dispersion expected if stores vary in retail services bundled with goods.

- Decompose price of good j at store s in transaction k as

$$p_{j,s,k} = \bar{p}_j + \underbrace{(\bar{p}_s - \bar{p}_j)}_{\text{Store}} + \underbrace{(\bar{p}_{j,s} - \bar{p}_s)}_{\text{Store–good}} + \underbrace{(p_{j,s,k} - p_{j,s})}_{\text{Transaction}},$$

  where

  - $\bar{p}_j$ is avg. price for good j in s's market in quarter t (subscripts m and t omitted).
  - $\bar{p}_s$ is avg. price of store s for all goods relative to other stores in market.
  - $\bar{p}_{j,s}$ is avg. price of good j at store s relative to all other stores in market.

- Intuition: Isolate price variation due to average expensiveness of store.

The morphology of price dispersion (Kaplan and Menzio 2015)

TABLE 3
DECOMPOSITION OF THE VARIANCE OF PRICES

	Substitution UPC (1)	Generic Aggregation (2)	Brand Aggregation (3)	Brand & Size Aggregation (4)
A: Restrict to stores/goods with at least five transactions				
Store-specific component	6%	7%	8%	8%
Store–good specific component	24%	30%	38%	40%
Transaction component	70%	62%	53%	53%
Covar: store, store–good	–1%	0%	1%	–2%
B: Common variance of the transaction component across goods				
Store-specific component	11%	11%	13%	15%
Store–good component	38%	41%	43%	31%
Transaction component	49%	47%	46%	64%
Covar: store, store–good	2%	1%	–2%	–10%

NOTES: All statistics are expenditure weighted across goods, markets and quarters. Sample includes only transactions at stores with unique identifiers for goods/markets/quarters with at least 25 transactions and coefficient of variation less than 1. See text for details of decomposition. May not add to 100% due to rounding.

- Only about 10% of variance owes to expensiveness of stores.

The morphology of price dispersion (Kaplan and Menzio 2015)

- Large amount of dispersion in price of identical good, even across stores that are equally expensive on average.

- Stigler (1961): *"Some automobile dealers might perform more service, or carry a larger range of varieties in stock, and a portion of the observed dispersion is presumably attributable to such differences. But it would be metaphysical, and fruitless, to assert that all dispersion is due to heterogeneity."*

Simple partial-equilibrium model of consumer search

- Suppose consumer i values good at δ_i.
- Knows distribution of prices $F(p)$, but not which store charges which price.
- Cost of getting a price quote is c_i.
- Value function V_i before observing a price is

$$V_i = \mathbb{E}\left[\max\{\delta_i - p, V_i - c_i\}\right].$$

- Consumer will search again if price is above a reservation price, $R_i = \delta_i - (V - c_i)$.

$$R_i = c_i + \mathbb{E}\left[\min\{p, R_i\}\right].$$

Simple partial-equilibrium model of consumer search

- Comparative statics in cost of price quote c_i.
- Reservation price:

$$c_i = \int_0^{R_i} [R_i - p]\, dF(p).$$

$$\Rightarrow \qquad \frac{dR_i}{dc_i} = \frac{1}{\int_0^{R_i} dF(p)} > 0.$$

- Expected price paid:

$$\mathbb{E}[p_i] = \frac{\int_0^{R_i} p dF(p)}{\int_0^{R_i} dF(p)}.$$

$$\Rightarrow \qquad \frac{d\mathbb{E}[p_i]}{dR_i} = \frac{dF(R_i)}{\int_0^{R_i} dF(p)} [R_i - \mathbb{E}[p|p < R_i]] > 0.$$

Simple partial-equilibrium model of consumer search

- Expected number of price quotes received:

$$\mathbb{E}\left[n_i\right] = \sum_{n=1}^{\infty} n\left[1 - F\left(R_i\right)\right]^{n-1} F\left(R_i\right).$$

$$\begin{aligned}
\Rightarrow \qquad \frac{d\mathbb{E}\left[n_i\right]}{dR_i} &= \sum_{n=2}^{\infty} -n\left(n-1\right)\left[1 - F\left(R_i\right)\right]^{n-2} \frac{dF\left(R_i\right)}{dR_i} + \sum_{n=1}^{\infty} n\left[1 - F\left(R_i\right)\right]^{n-1} \frac{dF\left(R_i\right)}{dR_i} \\
&= \sum_{n=2}^{\infty} \left[n\left[1 - F\left(R_i\right)\right]^{n-2}\left(-F\left(R_i\right) - \left(n-2\right)\right)\right] \frac{dF\left(R_i\right)}{dR_i} < 0.
\end{aligned}$$

- So expected price paid is increasing in c_i and number of quotes is decreasing in c_i.

Simple search model: Predictions

TABLE 8
EFFECT OF SHOPPING BEHAVIOR ON HOUSEHOLD PRICE INDEXES

	(1)	(2)	(3)	(4)
No. shopping trips	−0.144**			−0.035**
($\times 10^2$)	(0.005)			(0.004)
No. stores visited		−1.063**		−0.556**
($\times 10^2$)		(0.027)		(0.022)
Fraction of transactions involving coupons			−0.324**	−0.317**
			(0.003)	(0.003)
Observations	880,104	880,104	880,104	880,104
Households	78,758	78,758	78,758	78,758
R^2	0.015	0.021	0.181	0.187

NOTES: All regressions control for time dummies, market dummies, cubic polynomial in the average age of household heads, quadratic polynomial in log household size, and education dummies. Controlling also for expenditure at average prices yields similar results. Dependent variable is household price index using Generic Brand Aggregation definition. Standard errors in parentheses. ** denotes significance at 1%.

- Indeed, more quotes ($\mathbb{E}[n_i]$) $\Rightarrow$ lower prices paid ($\mathbb{E}[p_i]$).

Simple search model: Predictions

TABLE 10

EFFECT OF EMPLOYMENT ON HOUSEHOLD PRICE INDEXES NET OF STORE COMPONENT

	Definition of Good			
	UPC (1)	Generic Aggregation (2)	Brand Aggregation (3)	Brand & Size Aggregation (4)
Panel A: Households with one head				
Not employed	−0.006**	−0.006**	−0.011**	−0.012**
	(0.002)	(0.002)	(0.002)	(0.003)
Observations	132,274	137,079	141,505	145,264
Households	15,023	15,201	15,384	15,523
R^2	0.010	0.009	0.008	0.009
Panel B: Households with two heads				
Both heads not employed	−0.007**	−0.011**	−0.022**	−0.035**
	(0.003)	(0.003)	(0.004)	(0.006)
One head not employed	−0.004**	−0.004**	−0.007**	−0.009**
	(0.001)	(0.001)	(0.001)	(0.002)
Observations	292,634	299,580	306,505	312,950
Households	33,758	34,072	34,436	34,735
R^2	0.007	0.008	0.010	0.007

NOTES: All regressions control for time dummies, market dummies, cubic polynomial in the average age of household heads, quadratic polynomial in log household size, and education dummies. Dependent variable is household price index net of the store component. Standard errors in parentheses. ** denotes significance at 1%.

- Lower opportunity cost of time (c_i) $\Rightarrow$ lower prices paid ($\mathbb{E}[p_i]$).

Simple search model: Predictions (Aguiar Hurst 2007)

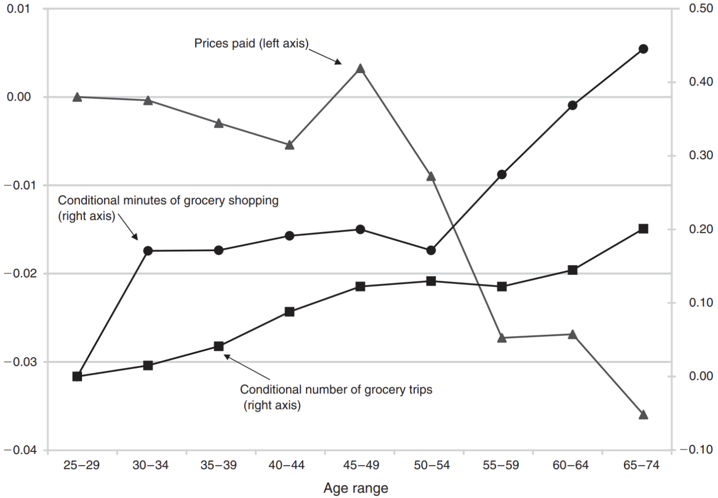


Figure 1. Price Paid and Shopping Frequency over the Life Cycle: Log Deviation from 25- 29-Year-Olds

- Aguiar and Hurst (2007) use change in opportunity cost of time around retirement.

Simple search model: Predictions (Aguiar Hurst 2007)

TABLE 3—THE ELASTICITY OF PRICE WITH RESPECT TO SHOPPING INTENSITY

	I	II	III	IV
Estimated elasticity: α_s	−0.010 (0.006)	−0.189 (0.035)	−0.074 (0.032)	−0.037 (0.013)
Measure of shopping intensity	Shopping trips per month	Shopping trips per month	Shopping trips per month	Minutes spent shopping per month
Regression type	OLS	IV	IV	OLS
Instrument set	None	Age dummies	Income dummies	None

Notes: This table reports the results from a regression of log price on log shopping intensity and controls for household shopping needs. In columns I–III, the measure of shopping intensity is the log number of grocery shopping trips per month, averaged within a household over each year. These regressions are estimated using a sample of 4,854 household-year pairs from the Homescan data. This sample is the same as the sample used in columns I–IV of Table 1. In column IV, the measure of shopping intensity is the minutes per week spent shopping for groceries. The regression in column IV is estimated using data from both Homescan and ATUS. To merge the two datasets, we compute the means of the relevant variables in both datasets for each age-sex-marital status cell. This regression included 27 averaged observations (9 age ranges and 3 sex-marital status groups). See text for full details. The controls for shopping needs were the same as the shopping needs controls used in columns I–IV of Table 1. Columns II and III are estimated via instrumental variables. The instrument set for column II consists of nine age-range dummies. The instrument set for column III consists of four income-category dummies. The F-statistic on the first stage for the instrument sets in columns II and III are 17.7 and 32.1, respectively. Robust standard errors clustered at the household level are included in parentheses.

- Doubling shopping intensity reduces prices paid by 7–10 percent.

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Diamond (1971) paradox

- So far, so good: looks like the predictions of our simple model of sequential search are borne out in the data.
- Not so fast!
- We have taken the price distribution $F(p)$ as exogenous.
- What happens if we solve for firms' prices in equilibrium?
- With identical consumers, we will run into the Diamond (1971) paradox: market can only sustain monopoly price.

Diamond (1971) paradox

- Consumers: Given $F(p)$ and cost of search c, choose reservation price R:

$$c = \int_0^R [R - p] dF(p).$$

- Firms: Given reservation price R, choose distribution of prices $F(p)$.
- Need to choose price $p \leq R$, otherwise no demand.
- No incentive to choose price $p < R$, since demand same for all $p \leq R$.
- Firms charge uniform price R.
- Consumers can't be incentivized to search, so $R = \delta$.

Diamond (1971) paradox

- This is a surprisingly robust result in sequential search.
- When consumers are homogeneous, firms converge to selling at monopoly price.
- Uniform monopoly price is equilibrium because no consumer can be incentivized to search, and therefore no firm has an incentive to deviate.
- Bugbear for literature on price dispersion.

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Diamond (1971) paradox: Solutions

- One way to get around Diamond paradox is to introduce heterogeneity in consumers' reservation prices or heterogeneity across firms.
  - Salop and Stiglitz (1976): Consumers differ in search costs c_i.
  - Wilde and Schwartz (1979): Some consumers love shopping and others hate it.
  - Reinganum (1979): Firms have varying costs and consumers have non-unit demand.
- But these models have the unwelcome property that equilibrium price dispersion depends on the extent of ex ante consumer/firm heterogeneity.
- Price dispersion will not be sustained if this heterogeneity becomes too small.

Ex ante vs. ex post heterogeneity

- Insight: Equilibrium price dispersion can be sustained with *ex post* heterogeneity in consumer information even if consumers are *ex ante* identical.
- Burdett and Judd (1983) offer one such model with nonsequential search. [today]
  - Heterogeneity in reservation prices comes from stochastic set of offers.
- Burdett and Mortensen (1989) offer another such model with sequential search.
  - Heterogeneity in reservation prices comes from current match.

Burdett and Judd (1983): Setup

- Identical firms produce with marginal cost w.
- Consumer values good at δ, pays fixed cost c to retrieve price quotes.
- Number of price quotes received is stochastic and given by $\{q_n\}_{n=1}^{\infty}$.
  - With probability q_1, observe one price quote.
  - With probability q_2, observe two price quotes.
  - And so on...
- Consumer either buys at minimum price among retrieved quotes or draws again.
- If $q_1 = 1$, we are back in the Diamond (1971) world $\Rightarrow p = \delta$.
- If $q_1 = 0$, no firm can be incentivized to keep its price above others $\Rightarrow p = w$.

Burdett and Judd (1983): Dispersed price equilibrium

- What happens for $q_1 \in (0,1)$? For any firm,
  - q_1 customers will observe only the firm's quote.
  - $2q_2$ customers will observe this firm's quote and one other quote.
  - $3q_3$ customers will observe this firm's quote and two other quotes, ...

- Thus, for a firm with price p, profits are given by:

$$\pi(p) = (p-w)\sum_{n=1}^{\infty} nq_n \underbrace{(1-F(p))^{n-1}}_{\text{Prob of lowest quote}} .$$

- Since firms will have positive profits as long as $p \leq R$, reservation price $R \in \text{supp}(F)$.

$$\pi(R) = (R-w)q_1 .$$

Burdett and Judd (1983): Dispersed price equilibrium

- Thus, firms will be indifferent between any price p and R if $\pi(p) = \pi(R)$, i.e.,

$$(R-w)q_1 = (p-w)\sum_{n=1}^{\infty} nq_n \underbrace{(1-F(p))^{n-1}}_{\text{Prob of lowest quote}}.$$

- This gives us the following closed-form expression for $F(p)$,

$$F(p) = \begin{cases} 0 & \text{if } p < \underline{p} \\ 1 - \Psi\left[\left(\frac{R-w}{p-w}\right)\bar{q}_1\right] & \text{if } \underline{p} \leq p \leq R \\ 1 & \text{if } p > R \end{cases}$$

where the lowest price $\underline{p}$ is

$$\underline{p} = w + \frac{\bar{q}_1}{\sum_{n=1}^{\infty} n\bar{q}_n}(R-w),$$

and $\Psi(\cdot)$ is the inverse of the strictly increasing, C^{∞} function $y(x) = \sum_{n=1}^{\infty} n\bar{q}_n x^{n-1}$.

Burdett and Judd (1983): Dispersed price equilibrium

- Reservation price is in turn pinned down by

$$c = \underbrace{\int_0^R \sum_{n=1}^{\infty} q_n \left(1 - (1 - F(p))^n\right) dp}_{\text{Expected benefit of searching again}}.$$

- In equilibrium, all consumers only search once, since no firm prices above R.

- R and $\{q_n\}_{n=1}^{\infty}$ pin down $F(p)$.

- $F(p)$ in turn pins down R.

- Dispersed price equilibrium exists as long as w not too high and $q_1 \in (0, 1)$.

Burdett and Judd (1983): Illustration

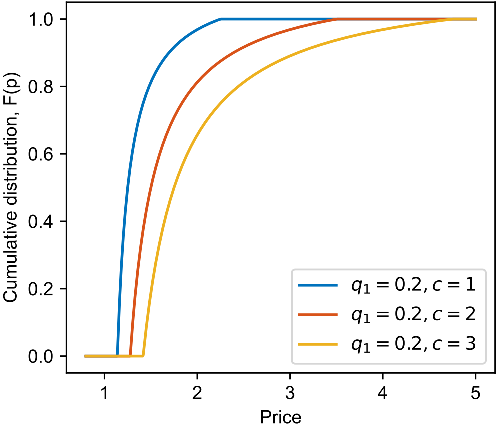
- Suppose customers always receive either one or two quotes, so that $q_2 = 1 - q_1$.
- Simple, closed form expression for $F(p)$:

$$F(p) = 1 - \frac{1}{2}\frac{q_1}{1-q_1}\frac{R-p}{p-w}, \qquad \underline{p} = w + \frac{q_1}{2-q_1}(R-w).$$

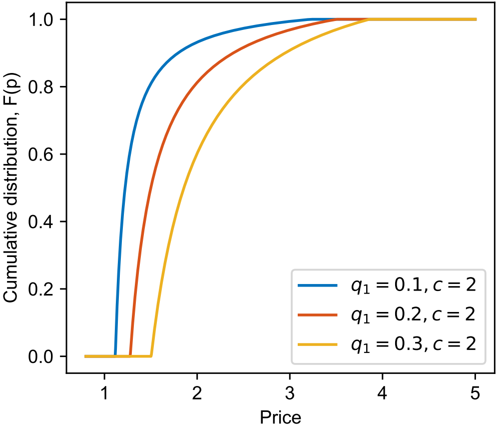
- In turn, reservation price given by

$$R = w + \frac{c}{1-q_1}.$$

Burdett and Judd (1983): Illustration



(a) Varying search cost c.



(b) Varying captive probability q_1.

Burdett and Judd (1983): Illustration

- Does the Lerner formula hold in this model?

Burdett and Judd (1983): Illustration

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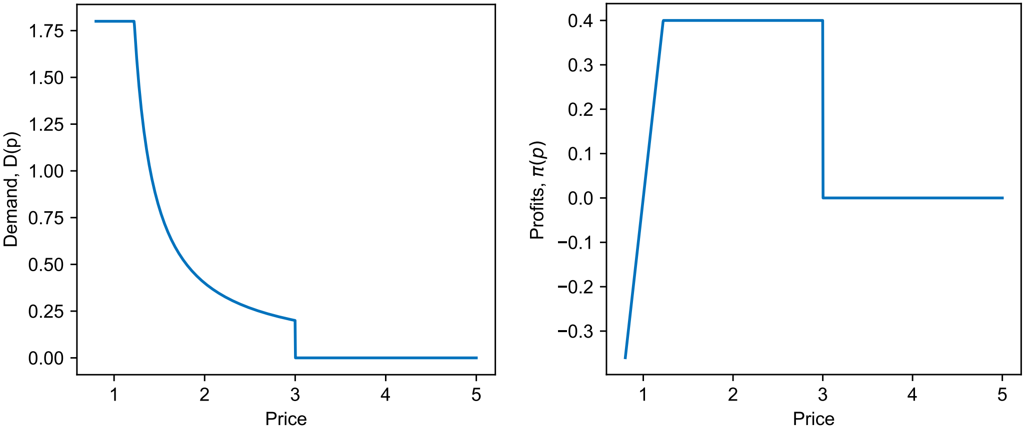

(a) Demand $D(p)$.

(b) Profits $\pi(p)$.

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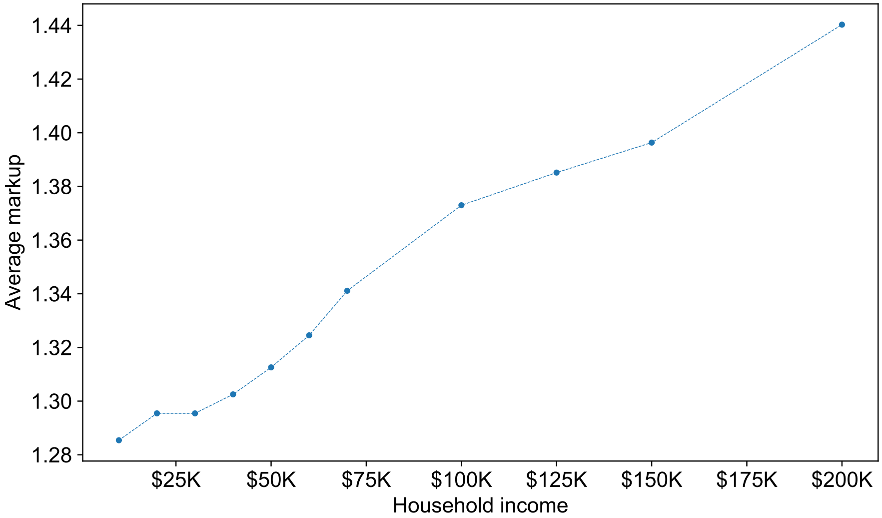
Interaction between income composition and markups

- Last time, we discussed how income inequality and competition interact via cannibalization (Bornstein and Peter, 2025).
- Today, we will discuss how the income distribution can shape markups through strategic interactions in search and firm pricing.
- Will draw from Sangani (2023).
- Key question #1: Are differences in search across households sufficient to explain differences in markups across products?
- Key question #2: How do strategic decisions (search behavior, firm pricing) shape spillovers across households?

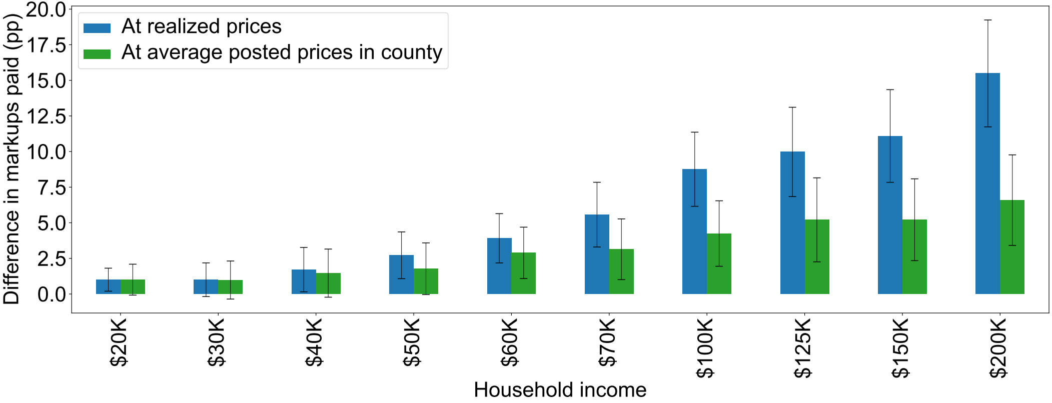
Motivating evidence: Markups paid increase with income

Figure: Aggregate (cost-weighted average) markup paid by income group.



Motivating evidence: 50-50 prices for identical products vs. basket

- Half due to diff. prices paid for identical products; half due to differences in basket.



Search Model of Income and Markups

- 1. Households have different tastes for goods.
  - $\Rightarrow$ Basket composition varies across households.
- 2. Households have different endogenous search intensities.
  - $\Rightarrow$ Price dispersion for identical products (spatial / intertemporal).
- Search choice (Aguiar and Hurst 2007) $+$ firm pricing (Burdett and Judd 1983).
  - In PE, diff prices for identical products $+$ basket composition.
  - In GE, composition of buyers $\rightarrow$ distribution of firm markups.

Basket Composition: Household Preferences Over Goods

- Utility for household i comes from consumption of goods $k = 1, ..., K$:

$$u(\{c_{ik}\}) = \left(\sum_{k=1}^{K} (\beta_{ik} c_{ik})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

  where β_{ik} is a taste shifter for good k and c_{ik} is i's consumption of k.

- Taste shifters β_{ik} determine different basket composition across households.
  - Allowing σ to vary with i is isomorphic in cross-section (but not time series).

Household Search Technology

- For each good, households know the price distribution, but not firms' individual prices.

- Household i buying good k has probability mass function over no. of quotes $\{q_{ik,n}\}_{n=1}^{\infty}$,
  - Observes only one price quote with probability $q_{ik,1}$,
  - Observes two price quotes with probability $q_{ik,2}$, etc.

- For each purchase, households buy iff min price $p \leq$ reservation price R.
  Redraw n quotes costlessly if $p > R$.

Household Search Technology

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  - Observes two price quotes with probability $q_{ik,2}$, etc.
- For each purchase, households buy iff min price $p \leq$ reservation price R.
  Redraw n quotes costlessly if $p > R$.
- Endogenous search decision: Household i chooses search intensity s_{ik} for each k.
- Mapping function from search intensity to probability of observing n price quotes,
  $\mathcal{S} : s_{ik} \mapsto \{q_{ik,n}\}_{n=1}^{\infty}$.

Household Problem

$$\max \; u(\{c_{ik}\}) \qquad \text{s.t.} \qquad \begin{cases} \sum_k t_i(c_{ik}, s_{ik}) + l_i = 1, & \text{(Time constraint)} \\ \sum_k p_{ik} c_{ik} = z_i l_i. & \text{(Budget constraint)} \end{cases}$$

where

- c_{ik} is amount of good k consumed (mass of infinitesimal purchases),
- l_i is time spent working with labor productivity z_i.
- $t_i(c_{ik}, s_{ik})$ is the time it takes i to shop for c_{ik} units with search intensity s_{ik}.
- p_{ik} is the average price paid by i for good k (deterministic over continuum of units).

- Let $t_i(c_{ik}, s_{ik}) = c_{ik} s_{ik} / a_i$.
    - Search productivity a_i can reflect technologies (e.g., car) or returns to scale in shopping.

Household Problem

$$\max\ u(\{c_{ik}\}) \qquad \text{s.t.} \qquad \begin{cases} \sum_k t_i(c_{ik}, s_{ik}) + l_i = 1, & \text{(Time constraint)} \\ \sum_k p_{ik} c_{ik} = z_i l_i. & \text{(Budget constraint)} \end{cases}$$

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- Let $t_i(c_{ik}, s_{ik}) = c_{ik} s_{ik} / a_i$.
  - Search productivity a_i can reflect technologies (e.g., car) or returns to scale in shopping.

- First order condition:

$$\underbrace{-\partial p_{ik} / \partial s_{ik}}_{\text{Marginal savings}} = \underbrace{\phi_i}_{\text{Opportunity cost}}$$

where opportunity cost of increasing search intensity $\phi_i = z_i / a_i$.

Firm Problem

- Mass M_k of firms pay $f_e \cdot w$ to enter market for good k.
- Constant returns production with marginal cost w.
- Define aggregate search behavior for good k as $\bar{q}_k$,

$$\bar{q}_{k,n} = \int_0^\infty q_{k,n}(z) d\Lambda_k(z), \qquad \text{for all } n,$$

  where $H(z) =$ CDF of household incomes, consumption $C_k = \int_0^\infty c_k(z) dH(z)$, and density of buyers' incomes for k is $d\Lambda_k(z) = \frac{c_k(z)}{C_k} dH(z)$.

- Firms set prices to maximize profits, taking as given $\bar{q}_k$ and distribution of prices F_k:

$$\max_p \pi(p) = (p - w) \underbrace{\frac{C_k}{M_k} \sum_{n=1}^{\infty} n\bar{q}_{k,n} (1 - F_k(p))^{n-1}}_{\text{Firm's demand at price } p},$$

Dispersed Price Equilibrium (Burdett and Judd 1983)

- Dispersed price eq: $F_k(p)$ where firms make identical profits for any $p \in \text{supp}(F_k)$.

- Given $\{\bar{q}_n\}_{n=1}^{\infty}$ with $\bar{q}_1 \in (0,1)$, the unique equilibrium price distribution $F(p)$ is

$$F(p) = \begin{cases} 0 & \text{if } p < \underline{p} \\ 1 - \Psi\left[\left(\frac{R-w}{p-w}\right)\bar{q}_1\right] & \text{if } \underline{p} \leq p \leq R \\ 1 & \text{if } p > R \end{cases}$$

  where the lowest price $\underline{p}$ is

$$\underline{p} = w + \frac{\bar{q}_1}{\sum_{n=1}^{\infty} n\bar{q}_n}(R-w),$$

  and $\Psi(\cdot)$ is the inverse of the strictly increasing, C^∞ function $y(x) = \sum_{n=1}^{\infty} n\bar{q}_n x^{n-1}$.

- Mass of firms M_k adjusts to ensure $\pi_k = f_e w$.

Equilibrium

- Equilibrium $(F_k, \{c_k(z), s_k(z)\}, M_k)$ such that (1) $c_k(z), s_k(z)$ maximize utility for all z, (2) F_k is a dispersed price eq. given $\bar{q}_k$, (3) $\pi_k = f_e$, (4) markets clear.
  - Assume all households choose interior $s_k(z)$.
  - Focus on comparative statics of stable equilibrium.

Equilibrium

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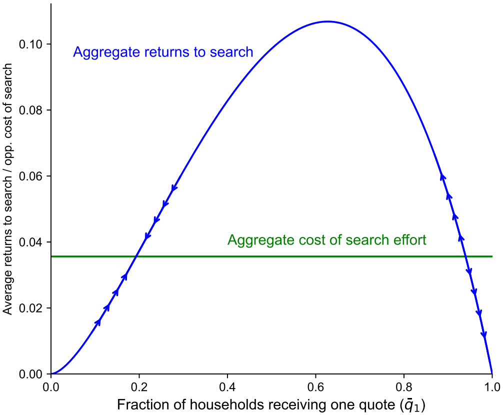
    - Assume all households choose interior $s_k(z)$.
    - Focus on comparative statics of stable equilibrium.

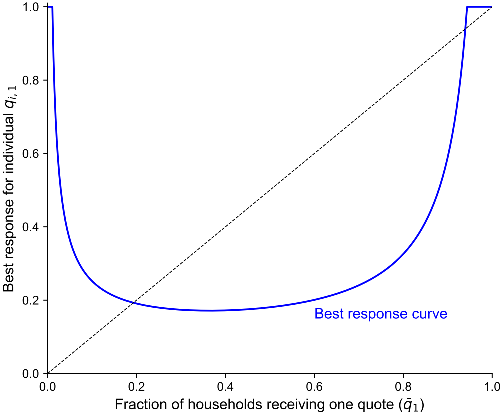
- In paper: Assumptions on search mapping $\mathcal{S} : s \mapsto \{q_n\}$.
    - Ensure price is decreasing and convex in search intensity s, and $q_1''(z) > 0$ if $\phi''(z) > 0$.
    - Conditions satisfied by two common parameterizations:
        - Two quote. (Alessandria and Kaboski 2011; Pytka 2018; Kaplan et al. 2019.)
        - Poisson. (Albrecht, Menzio, and Vroman 2023; Menzio 2023.)

Equilibrium: Graphical Intuition

$$\int -p_s(s(z), \{\bar{q}_n\}) d\Lambda(z) = \int \phi(z) d\Lambda(z),$$



(a) Aggregate equilibrium condition.



(b) Individual best response curve.

Illustration: Micro- and Macro Elasticities with Homogeneous Households

- Consider an economy with initially identical households.

How does household *i*'s markup change if we perturb own income z_i and agg. income z_{-i}:

$$d\mu_i = \left(\frac{d\log\phi}{d\log z}\right)\left(\underbrace{\kappa_1 dz_i}_{\substack{\text{Own income:} \\ \text{Through search choice}}} + \underbrace{\kappa_2 dz_{-i}}_{\substack{\text{Others' incomes:} \\ \text{Through price dist.}}} + \underbrace{\kappa_3 \frac{\partial s_i}{\partial s_{-i}} dz_{-i}}_{\substack{\text{Others' incomes:} \\ \text{Through search choice}}}\right),$$

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with coefficients $\kappa_1, \kappa_2, \kappa_3 > 0$.

- Recall $\phi(z) = z/a(z)$. Race between labor and search productivity leads to either:
    - $a(z)$ rises faster than z: "poverty premium." Caplovitz (1963), Prahalad and Hammond (2002).
    - $a(z)$ rises slower than z: markups paid increase with income.

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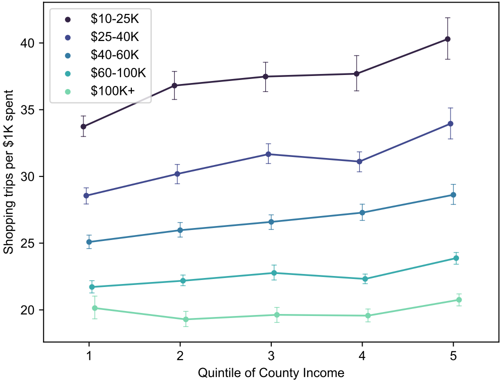
with coefficients $\kappa_1, \kappa_2, \kappa_3 > 0$.

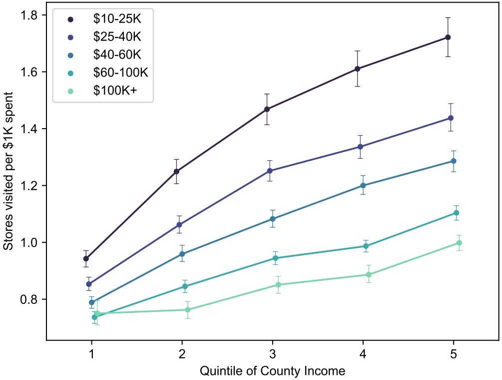
- Effect of others' incomes implies macro elasticity $\neq$ micro elasticity.
- Spillovers from both effects on price distribution and strategic interactions in search.
  - In paper: Theoretical results and evidence on strategic substitutability in search.
  - Strategic interactions in search moderate macro elasticity, while price spillovers amplify.

Data: Strategic substitutability in search

- Model: In stable equilibrium, households' search intensities are strategic substitutes.
- Search intensity (Kaplan and Menzio 2015) falls w/ income, rises w/ county income.



(a) Shopping trips per $1K expenditures.



(b) Unique stores visited per $1K expenditures.

Comparative Statics: Changes in Income Distribution

- Analytic comparative statics for single-good model: $K = 1$.
  - Single distribution of buyers' incomes $\Lambda(z)$.

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Proposition (Shift in Buyers' Incomes)

Aggregate markup (total sales / total costs) weakly increases if

- *First-order stochastic shift in $\Lambda(z)$ and opp. cost of search $\phi(z)$ increasing.*
- *Mean-preserving spread in $\Lambda(z)$ and opp. cost of search $\phi(z)$ increasing and convex.*

- Intuition: Sufficient conditions for change in $\Lambda(z)$ to increase avg. cost of search.

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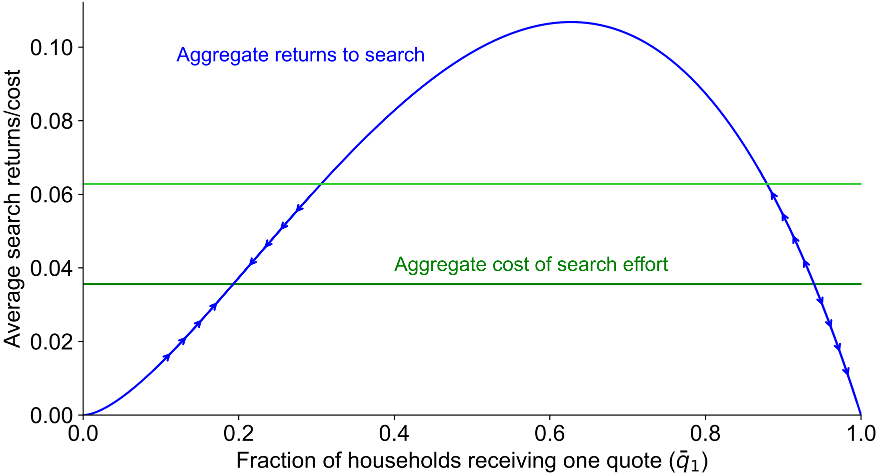
- Intuition: Sufficient conditions for change in $\Lambda(z)$ to increase avg. cost of search.
- Corollary: Balanced growth if search productivity a_i grows 1-for-1 with labor prod z_i.
- In the data, elasticity of markups to income across space $\approx$ over time.

Comparative Statics: Intuition

$$\int -p_s(s(z), \{\bar{q}_n\}) d\Lambda(z) = \int \phi(z) d\Lambda(z),$$



Calibration approach

- Outer loop: Preferences to match spending shares in the data exactly.
- Inner loop: Search behavior to match markups gaps across income groups exactly.
  - Assume households with >$200K income have identical behavior to those with $200K.

Parameter		Value	Source
Number of products	K	$10^{\dagger}$	Increasing $K > 10$ does not change results
Elasticity of substitution	σ	$1^{\dagger}$	Cobb-Douglas
Taste shifters	$\beta_k(z)$	-	Match spending shares exactly
Unit wage	w	1	Numeraire
Reservation price	R	$3.0^{\dagger}$	98th percentile of markups in the data
Search mapping	$\mathcal{S}$	Poisson	Albrecht et al. (2023), Menzio (2023)
Opp. costs of search	$\phi(z)$	-	Match avg. markup paid by income exactly
Search productivity	$a(z)$	-	Solved from $\phi(z) = z/a(z)$

† Paper reports robustness to parameter choice.

Calibration outer loop: Spending shares directly from the data

- Order UPCs by buyer income, split into $K = 10$ groups.
- Note: Similar results if $K = 20$, 50, 100, etc.

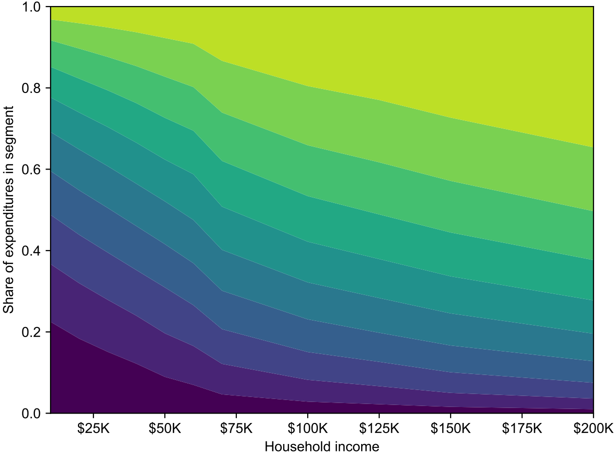
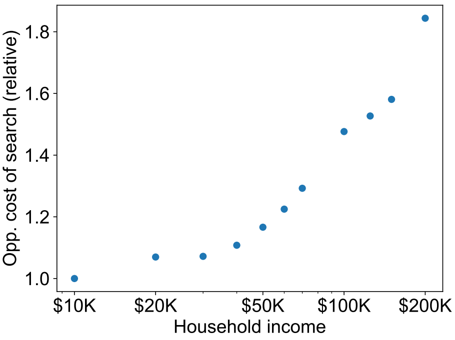
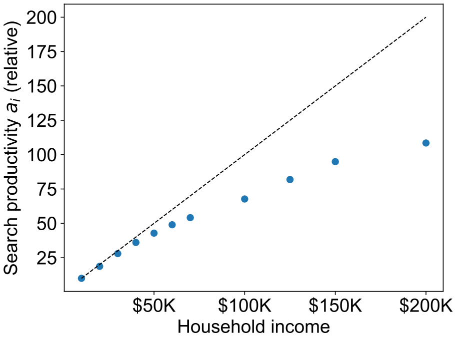


Figure: Spending shares over $K = 10$ groups.

Calibration inner loop: Search parameters to match markups



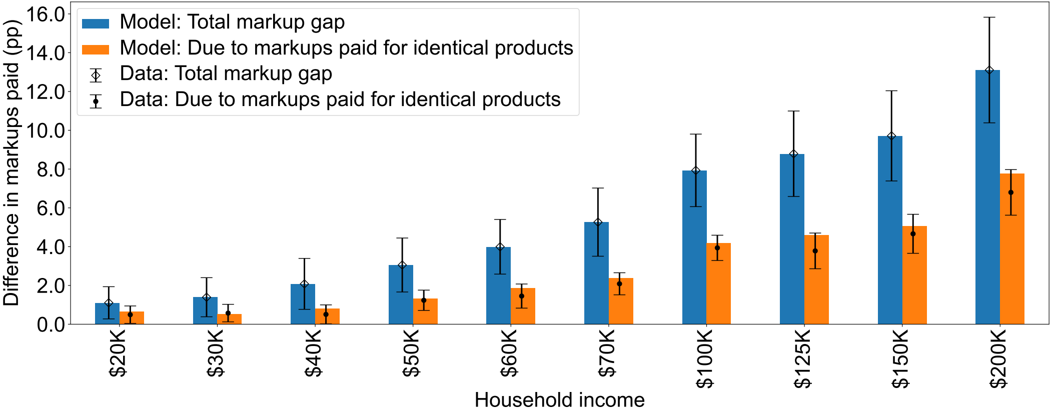
(a) Opportunity cost of search effort $\phi(z)$.



(b) Search productivity $a(z)$.

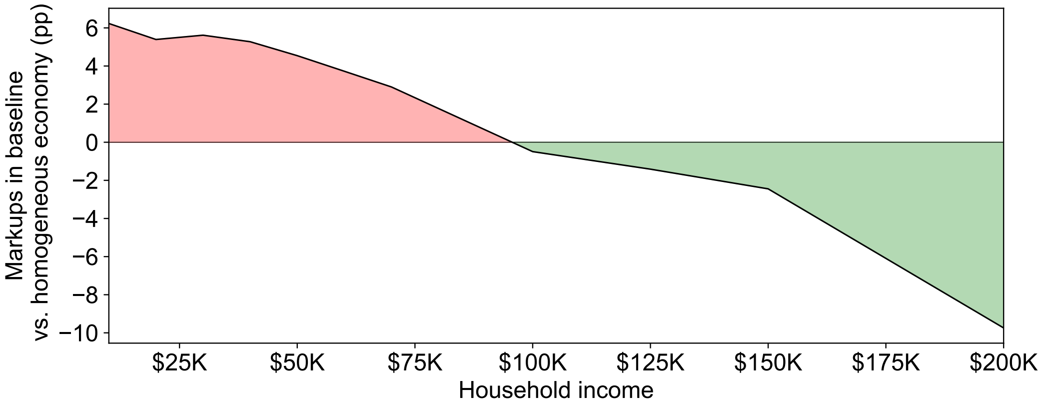
- Doubling search time decreases prices paid 7–9%. (7–10% estimated by Aguiar and Hurst 2007.)
- Elasticity of search intensity to income is -11%. (-12% in McKenzie and Schargrodsky 2005.)

Calibration fit: Decomposition of markup gap across income groups



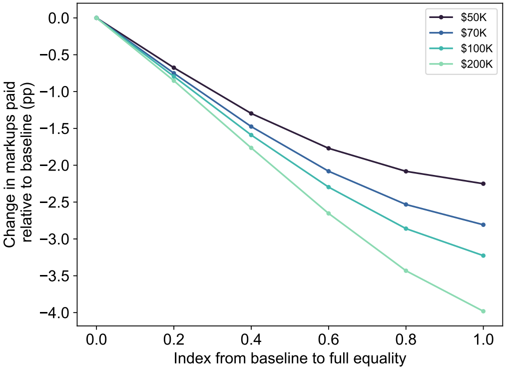
- Untargeted moment: Share of markup gap due to prices paid for identical products.
- Across products, elasticity of product avg. markup to income is **10%** (matches data).
- Don't need differences beyond buyer composition to explain product markups!

Spillovers: Effects of income level

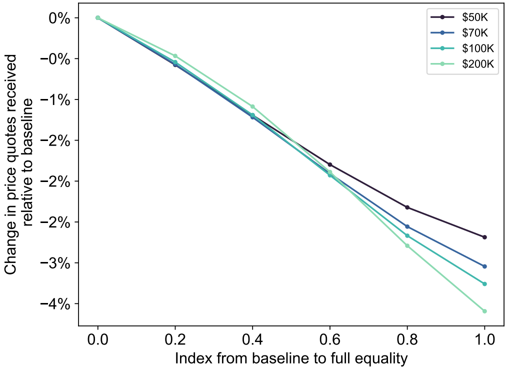


- Low-income pay 6pp higher markups due to presence of high-income shoppers.
- Moving to low-income area would save ≈$250/yr on $5K expenditures.

Spillovers: Gains from reducing income inequality



(a) Markups paid.

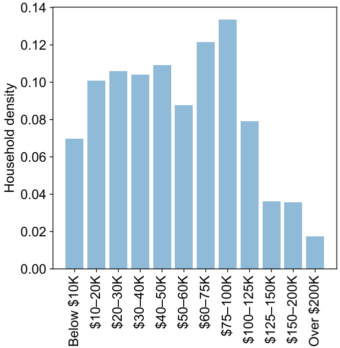


(b) Avg. price quotes received.

- Moving to full equality reduces markups paid 2–4pp and search time 2–4%.
- Note: One channel among several effects of inequality.

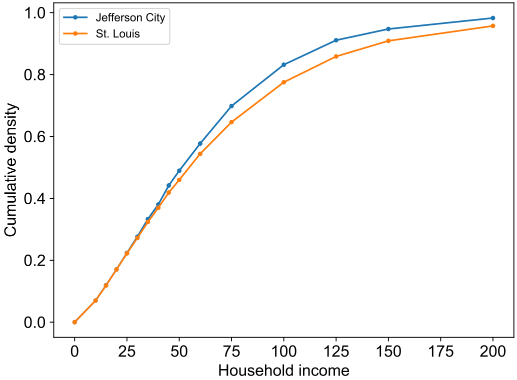
Markups across space: Predicting markups using income dist. of US cities

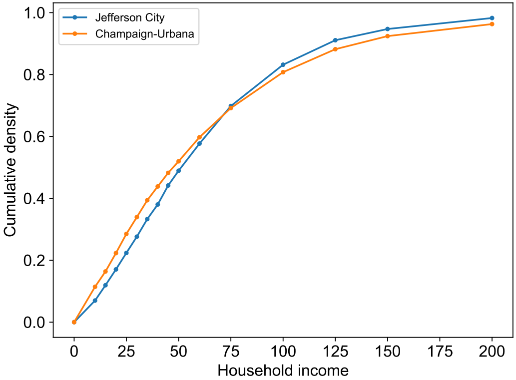
- Predict CBSA markups using income dist. from ACS.
- Compare predicted markups to retail markup data.
- Compare "supply-side" model of markups.
    - Macro literature inferring markups from market shares.
      (e.g., Atkeson and Burstein 2008, Smith and Ocampo 2023.)
    - Nested CES model using retailer market shares.



Example: Income dist. in Jefferson City, MO from ACS 5-year survey.

Model predicts that markups increase with income level and dispersion





CBSA	Average Income	Gini Index	Predicted Markup
Jefferson City	\$33.3K	0.41	34%
St. Louis	\$41.0K	0.46	38%

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Jefferson City	\$33.3K	0.41	34%
Champaign-Urbana	\$33.7K	0.48	37%

Markups across space: Explains 31% of variation in CBSA markups in data

- Outperforms income measures alone and supply-side (nested CES) model.

	Model-Predicted		Data				
Log CBSA Markup	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log CBSA Income	0.086**	0.081**					
	(0.001)	(0.001)					
Gini Index		0.088**					
		(0.011)					
Log Model-Predicted Markup							
Log Nested CES Markup							
N	881	881					
R^2	0.84	0.85					

** is significant at 5%, * at 10%. Regressions weighted by CBSA sales.

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Log CBSA Income	0.086**	0.081**	0.110**	0.102**			
	(0.001)	(0.001)	(0.006)	(0.007)			
Gini Index		0.088**		0.153**			
		(0.011)		(0.057)			
Log Model-Predicted Markup							
Log Nested CES Markup							
N	881	881	881	881			
R^2	0.84	0.85	0.27	0.28			

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Gini Index		0.088**		0.153**			
		(0.011)		(0.057)			
Log Model-Predicted Markup					1.248**		
					(0.063)		
Log Nested CES Markup							
N	881	881	881	881	881		
R^2	0.84	0.85	0.27	0.28	0.31		

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Gini Index		0.088**		0.153**			
		(0.011)		(0.057)			
Log Model-Predicted Markup					1.248**		
					(0.063)		
Log Nested CES Markup						-0.720**	
						(0.072)	
N	881	881	881	881	881	881	
R^2	0.84	0.85	0.27	0.28	0.31	0.10	

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Log CBSA Markup	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log CBSA Income	0.086**	0.081**	0.110**	0.102**			0.019
	(0.001)	(0.001)	(0.006)	(0.007)			(0.015)
Gini Index		0.088**		0.153**			0.075
		(0.011)		(0.057)			(0.058)
Log Model-Predicted Markup					1.248**		0.956**
					(0.063)		(0.169)
Log Nested CES Markup						-0.720**	-0.123*
						(0.072)	(0.074)
N	881	881	881	881	881	881	881
R^2	0.84	0.85	0.27	0.28	0.31	0.10	0.31

** is significant at 5%, * at 10%. Regressions weighted by CBSA sales.

Recap

- Conceptually, price elasticity depends on two things:
  - 1. Availability of alternatives (supply-side)
  - 2. Consumer propensity to switch to alternatives (demand-side).
- Income is one salient characteristic that shapes #2.
- Modeling consumer heterogeneity in price sensitivity can help us make progress toward understanding markups in data.
- Next: Contextualize these models with evidence on labor share / markups over time.

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