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MATLAB

Unit 3-Lecture 8

BTech (CSBS) -Semester VII

5 August 2022, 09:35AM

*few figures/content have been prepared/referred from internet source/books

Solving Linear Equation

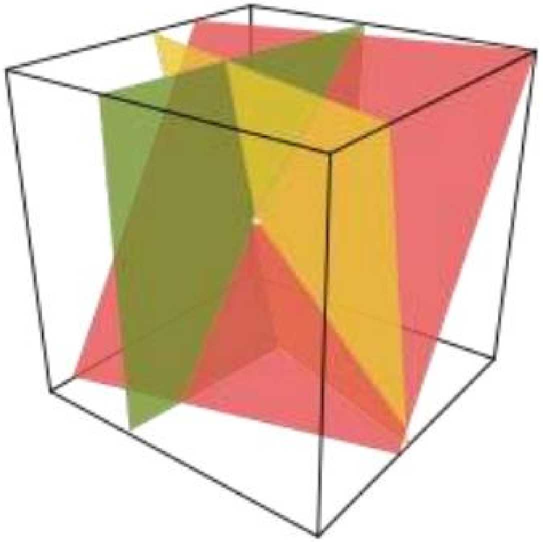
Solve System of Linear Equations Using linsolve:

A system of linear equations

$$
\begin{aligned}
&a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\
&a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\
&\cdots \\
&a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m
\end{aligned}
$$

can be represented as the matrix equation $A.\vec{x} = \vec{b}$ where A is the coefficient matrix,

Solving Linear Equation...contd.



- Ex: Find values $x_1, x_2, x_3 \in \mathbb{R}$ that satisfy
  - $3x_1 + 2x_2 - x_3 - 1 = 0$
  - $2x_1 - 2x_2 + x_3 + 2 = 0$
  - $-x_1 - \frac{1}{2}x_2 - x_3 = 0$

Solution:

- Step 1: write the system of linear equations as a matrix equation

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 2 & -2 & 1 \\ -1 & -\frac{1}{2} & -1 \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, b = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}.$$

- Step 2: Solve for $Ax = b$

Example 1

```
>> A = [5 -3 2; -3 8 4; 2 4 -9];   % Enter matrix A
>> b = [10; 20; 9];                 % Enter column vector b
>> x = A\b                          % Solve for x
x =
    3.4442
    3.1982
    1.1868

>> c = A*x                          % check the solution
c =
    10.0000
    20.0000
     9.0000
```

The backslash (\) or the left division is used to solve a linear system of equations $[\mathbf{A}]\{\mathbf{x}\} = \{\mathbf{b}\}$. For more information, type: `help slash`.

Question 1

- What's the A, x, and b for the following linear equations?
  - $2x_2 \ - 7 = 0$
  - $2x_1 - 3x_3 + 2 = 0$
  - $4x_2 + 2x_3 = 0$
  - $x_1 + 3x_3 = 0$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & -3 \\ 0 & 4 & 2 \\ 1 & 0 & 3 \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, b = \begin{bmatrix} 7 \\ -2 \\ 0 \\ 0 \end{bmatrix}.$$

Question

LinearEquation.m

```
syms x y z
eqn1= 5*x + y + 4*z == 12;
eqn2= -x + y - 2*z == 43;
eqn3= x - y + z==-10;
[A,B]=equationsToMatrix([eqn1,eqn2,eqn3], [x,y,z])
X=linsolve(A,B)
```

Command Window

```
>> LinearEquation

A =

[ 5,  1,  4]
[-1,  1, -2]
[ 1, -1,  1]


B =

 12
 43
-10


X =

167/6
 29/6
  -33
```

Solve System of Linear Equations Using solve

Use **solve** instead of linsolve if you have the equations in the form of expressions and not a matrix of coefficients.

```
%---------using solve function--------
sol=solve([eqn1, eqn2, eqn3], [x,y,z])
xSol = sol.x
ySol = sol.y
zSol = sol.z
```

```
sol =

  struct with fields:

    x: 167/6
    y: 29/6
    z: -33


xSol =

167/6


ySol =

29/6


zSol =

-33
```

Question

1. **Linear algebraic equations:** Find the solution of the following set of linear algebraic equations, as advised below.

$$\begin{aligned} x + 2y + 3z &= 1 \\ 3x + 3y + 4z &= 1 \\ 2x + 3y + 3z &= 2. \end{aligned}$$

- Write the equation in matrix form and solve for $\mathbf{x} = [x\ y\ z]^T$ using the left division \.

Gaussian elimination method

MATLAB has a built-in function, rref, that does precisely this reduction, i.e., transforms the matrix to its row reduced echelon form.

```
A=[1 2 3; 3 3 4; 2 3 3];
B=[1; 1; 2];
%C=B\A
%linsolve(A,B)
%transpose (C)
%------------Gaussian ellimination method
C=[A B];
Cr=rref(C)
```

```
Cr =

    1.0000         0         0   -0.5000
         0    1.0000         0    1.5000
         0         0    1.0000   -0.5000
```

Find eigenvalues and eigenvectors

Step 1: Enter matrix A and type [V, D] = eig(A)

```
>> A = [ 5 -3 2; -3 8 4; 2 4 -9];
>> [V, D] = eig(A)
V =

   -0.1709    0.8729    0.4570
   -0.2365    0.4139   -0.8791
    0.9565    0.2583   -0.1357
D =

  -10.3463         0         0
         0    4.1693         0
         0         0   10.1770
```

Step 2: Extract what you need:

'V' is an 'n x n' matrix whose columns are eigenvectors

D is an 'n x n' diagonal matrix that has the eigenvalues of 'A' on its diagonal.

Question

2. **Eigenvalues and eigenvectors:** Consider the following matrix.

$$\mathbf{A} = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

- Find the eigenvalues and eigenvectors of $\mathbf{A}$.
- Show, by computation, that the eigenvalues of $\mathbf{A}^2$ are square of the eigenvalues of $\mathbf{A}$.
- Compute the square of the eigenvalues of $\mathbf{A}^2$. You have now obtained the eigenvalues of $\mathbf{A}^4$. From these eigenvalues, can you guess the structure of $\mathbf{A}^4$?
- Compute $\mathbf{A}^4$. Can you compute $\mathbf{A}^{-1}$ without using the `inv` function?