Chapter 7

Valuation of Bonds and Stocks

Learning Objectives

After studying this chapter you should be able to:

- ✓ Distinguish between various valuation concepts.
- ✓ Estimate the value of a bond.
- ✓ Calculate various measures of bond yield.
- ✓ Read bond and stock quotations.
- ✓ Value a preference stock.
- ✓ Calculate the value of a stock using the dividend discount model and the P/E ratio approach.
- ✓ Show the relationship between E/P ratio, expected return, and growth.

In Chapter 6 we discussed the basic methods used to value future cash flows. In this chapter we will apply those methods for valuing bonds and stocks. In addition, we will introduce some of the terminology used in these areas and describe how the financial press reports the prices of these assets.

We assume that the appropriate discount rate is known. The question of how risk determines the appropriate discount rate is very important and we will discuss this issue in the following chapter. For now, our focus will be on what the relevant cash flows of financial assets are and how to value them, given an appropriate discount rate.

The objective of financial management is to maximise the value of the firm. Hence managers must know how stocks and bonds are valued. Knowing how to value securities (bonds and stocks, in the main) is as important for investors as it is for managers. Current and prospective investors must understand how to value bonds and stocks. Such knowledge is helpful to them in deciding whether they should buy or hold or sell securities at the prices prevailing in the market.

This chapter discusses the basic discounted cash flow valuation model and its application to bonds and stocks.

7.1 VALUATION CONCEPTS

The term value is used in different senses. Hence, let us briefly review the differences that exist among the major concepts of value.

Liquidation Value versus Going Concern Value The **liquidation value** is the amount that can be realised when an asset, or a group of assets representing a part or even the whole of a firm, is sold separately from the operating organisation to which it belongs. In contrast, the **going concern value** represents the amount that can be realised if the firm is sold as a continuing operating entity.

In general, security valuation models assume a going concern, an operating business entity that generates cash flows to its security holders. When the going concern assumption is not appropriate as in the case of an impending bankruptcy, liquidation value of assets is more relevant in determining the worth of the firm's financial securities.

Book Value versus Market Value The **book value** of an asset is the accounting value of the asset, which is simply the historical cost of the asset less accumulated depreciation or amortisation as the case may be. The book value of a firm's equity is equal to the book value of its assets minus the book value of its liabilities. Because book value reflects a historical accounting value it may diverge significantly from market value. However, under IFRS accounting, which is expected to be adopted in India, more assets are likely to be reported at "fair values." So, the traditional divergence between book value and market value may diminish.

The **market value** of an asset is simply the market price at which the asset trades in the market place. Often the market value is greater than the book value.

Market Value versus Intrinsic Value As the nomenclature suggests, the market value of a security is the price at which the security trades in the financial market.

The **intrinsic value** of a security is the present value of the cash flow stream expected from the security, discounted at a rate of return appropriate for the risk associated with the security. Put differently, intrinsic value is economic value. If the market is reasonably efficient, the market price of the security should hover around its intrinsic value. The focus of this chapter is on establishing a security's intrinsic value.

7.2 BOND VALUATION

A bond represents a contract under which a borrower promises to pay interest and principal on specific dates to the holders of the bond.

Bonds are issued by a variety of organisations. The principal issuers of bonds in India are the central government, state governments, public sector undertakings, private sector companies, and municipal bodies.

Bonds issued by the central government are called *Treasury bonds*. These are bonds which have maturities ranging upto 20 years. These bonds generally pay interest semi-annually. Presently, Treasury bonds dominate the Indian bond market in terms of market capitalisation, liquidity, and turnover.

State government bonds are issued by the state governments. These bonds have maturities that generally range from 3 to 20 years and pay interest semi-annually.

Bonds issued by companies are classified into two types: PSU (public sector undertakings) bonds and private sector bonds. *PSU bonds* are bonds issued by companies in which the central or state governments have an equity stake in excess of 50 percent. Some of these bonds enjoy a tax-free status whereas others are taxable.

Private sector bonds are bonds issued by private sector companies. Bonds issued by companies, PSU bonds as well as private sector bonds, generally have maturity ranging from 1 year to 15 years and pay interest semi-annually.

Terminology In order to understand the valuation of bonds, we need familiarity with certain bond related terms.

Par Value This is the value stated on the face of the bond. It represents the amount the firm borrows and promises to repay at the time of maturity. Usually the par or face value of bonds issued by business firms is ₹ 100. Sometimes it is ₹ 1,000.

Coupon Rate and Interest A bond carries a specific interest rate which is called 'the coupon rate'. The interest payable to the bond holder is simply: par value of the bond × coupon rate. Most bonds pay interest semi-annually. For example, a government security which has a par value of ₹ 1,000 and a coupon rate of 11 percent pays an interest of ₹ 55 every six months.

Maturity Period Typically bonds have a maturity period of 1-15 years; sometimes they have longer maturity. At the time of maturity the par (face) value plus perhaps a nominal premium is payable to the bondholder.

Valuation Model The value of a bond - or any asset, real or financial - is equal to the present value of the cash flows expected from it. Hence determining the value of a bond requires:

- An estimate of expected cash flows
- An estimate of the required return

To simplify our analysis of bond valuation we will make the following assumptions:

- The coupon interest rate is fixed for the term of the bond.
- The coupon payments are made annually and the next coupon payment is receivable exactly a year from now.
- The bond will be redeemed at par on maturity.

Given these assumptions, the cash flow for a non-callable bond (a bond that cannot be prematurely retired) comprises of an annuity of a fixed coupon interest and the principal amount payable at maturity. Hence the value of the bond is:

$$P = \sum_{t=1}^{n} \frac{C}{(1+r)^t} + \frac{M}{(1+r)^n} \tag{7.1}$$

where P is the value (in rupees), n is the number of years to maturity, C is the annual coupon payment (in rupees), r is the periodic required return, M is the maturity value, and t is the time when the payment is received.

Since the stream of coupon payments is an ordinary annuity, we can apply the formula for the present value of an ordinary annuity. Hence the bond value is given by the formula:

$$P = C \times \text{PVIFA}_{r,n} + M \times \text{PVIF}_{r,n} \tag{7.1a}$$

To illustrate how to compute the price of a bond, consider a 10-year, 12% coupon bond with a par value of ₹ 1,000. Let us assume that the required yield on this bond is 13%. The cash flows for this bond are as follows:

- 10 annual coupon payments of ₹ 120
- ₹ 1000 principal repayment 10 years from now

The value of the bond is:

$$P = 120 \times \text{PVIFA}_{13\%,10\text{yrs}} + 1{,}000 \times \text{PVIF}_{13\%,10\text{yrs}}$$
$$= 120 \times 5.426 + 1{,}000 \times 0.295$$
$$= 651.1 + 295 = ₹\ 946.1$$

Bond Values with Semi-annual Interest Most of the bonds pay interest semi-annually. To value such bonds, we have to work with a unit period of six months, and not one year. This means that the bond valuation equation has to be modified along the following lines:

- The annual interest payment, *C*, must be divided by two to obtain the semi-annual interest payment.
- The number of years to maturity must be multiplied by two to get the number of half-yearly periods.
- The discount rate has to be divided by two to get the discount rate applicable to half-yearly periods.

With the above modifications, the basic bond valuation becomes:

$$P = \sum_{t=1}^{2n} \frac{C/2}{(1+r/2)^t} + \frac{M}{(1+r/2)^{2n}}$$
$$= C/2\,(\text{PVIFA}_{r/2,2n}) + M(\text{PVIF}_{r/2,2n}) \qquad (7.2)$$

where *P* is the value of the bond, *C*/2 is the semi-annual interest payment, *r*/2 is the discount rate applicable to a half-year period, *M* is the maturity value, and 2*n* is the maturity period expressed in terms of half-yearly periods.

As an illustration, consider an 8 year, 12 percent coupon bond with a par value of ₹ 100 on which interest is payable semi-annually. The required return on this bond is 14 percent.

Applying Eq.(7.2), the value of the bond is:

$$P = \sum_{t=1}^{16} \frac{6}{(1.07)^t} + \frac{100}{(1.07)^{16}}$$
$$= 6(\text{PVIFA}_{7\%,16}) + 100(\text{PVIF}_{7\%,16})$$
$$= ₹\ 6(9.447) + ₹\ 100(0.339) = ₹\ 90.6$$

Let us recalculate the above using the Excel financial function PRICE (settlement, maturity, rate, yield, redemption, frequency, basis), as follows:

	A	B	C	D	E	F	G	H	I	J
1	Settlement	1/1/2015	This is the date of purchase. If not specified fill in any date							
2	Maturity	30/12/2022	The formula in this case is = B1+ 365*8, as the maturity period is 8 years							
3	Rate	12%	The annual coupon rate							
4	Yield	14%	The required return per annum							
5	Redemption	100	Fill in the redemption value as a percentage of the par value							
6	Frequency	2	This represents the number of times interest is paid in a year							
7	Basis	3	3 represents the day count convention, actual no. of days/365, in interest calculation							
8	Price	90.55	To get the result in B8, use the function = PRICE(B1, B2, B3, B4, B5, B6, B7)							
9	Bond price is obtained per ₹ 100 of the face value of the bond. Here, the redemption value being ₹ 100, the price would be ₹ 90.55 × 100/100 = ₹ 90.55									

Relationship between Coupon Rate, Required Yield, and Price A basic property of a bond is that its price varies inversely with yield. The reason is simple. As the required yield decreases, the present value of the cash flow increases; hence the price increases. Conversely, when the required yield increases, the present value of the cash flow decreases.

The price-yield relationship may be illustrated with an example. Consider a bond carrying a coupon rate of 14 percent issued 3 years ago for ₹ 1000 (its par value) by Signal Corporation. The original maturity of the bond was 10 years, so its residual maturity now is 7 years. The interest rate has fallen in the last 3 years and investors now expect a return of 10 percent from this bond. The price of this bond now would be

$$P_0 = \sum_{t=1}^{7} \frac{140}{(1.10)^t} + \frac{1000}{(1.10)^7} = ₹\,1194.7$$

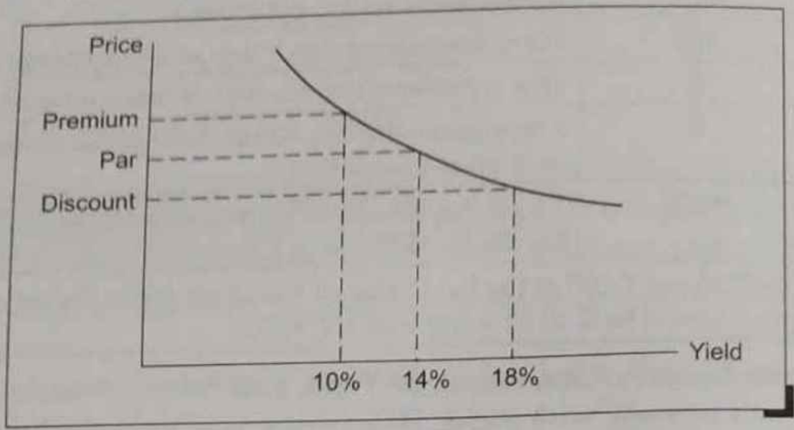
The arithmetic of the bond price increase is clear. What is the logic behind it? The fact that the required return on such a bond has fallen to 10 percent means that if you had ₹ 1,000 to invest, you can buy new bonds like Signal's except that these new bonds would pay ₹ 100, rather than ₹ 140, by way of interest. Naturally, as an investor you would prefer ₹ 140 to ₹ 100, so you would be willing to pay more than ₹ 1,000 for a Signal bond to enjoy its higher coupons. All investors would behave similarly and consequently the bond of Signal would be bid up in price to ₹ 1194.7. At that price it would provide a return of 10 percent, the rate the new bonds offer.

Now let us look at what happens when the interest rate rises after the bond has been issued. Assume that because of a rise in interest rates, investors now expect a return of 18 percent from the Signal bond. The price of the bond would be:

$$P_0 = \sum_{t=1}^{7} \frac{140}{(1.18)^t} + \frac{1000}{(1.18)^7} = ₹\ 847.5$$

The graph of the price-yield relationship for the bond has a convex shape as shown in Exhibit 7.1.

Exhibit 7.1 **Price–Yield Relationship**



To sum up, the relationship between the coupon rate, the required yield, and the price is as follows:

Coupon rate > Required yield ←——→ Price > Par (Premium bond)
Coupon rate = Required yield ←——→ Price = Par
Coupon rate < Required yield ←——→ Price > Par (Discount bond)

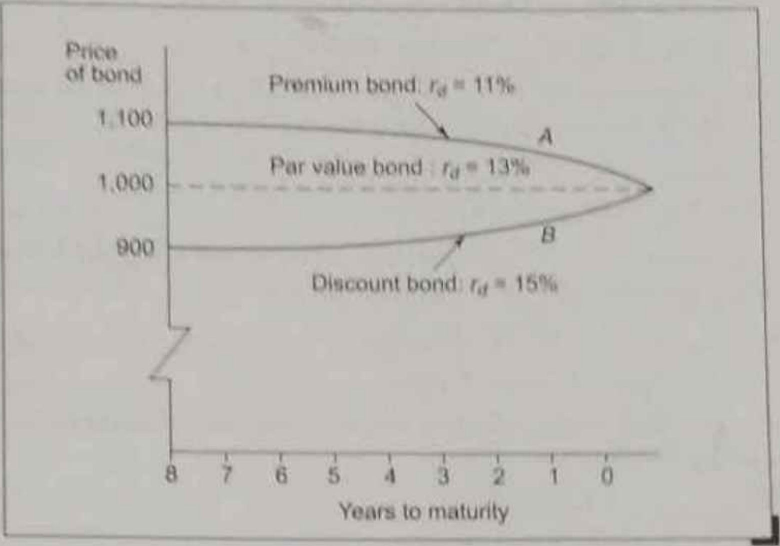
Relationship between Bond Price and Time Since the price of a bond must typically be equal to its par value at maturity (assuming that there is no risk of default), the bond price changes with time. For example, a bond that is redeemable for ₹ 1,000 (which is its par value) after 5 years when it matures, will have a price of ₹ 1,000 at maturity, no matter what the current price is. If its current price is, say, ₹ 1,100, it is said to be a premium bond. If the required yield does change between now and the maturity date, the premium will decline over time as shown by curve A in Exhibit 7.2. On the other hand, if the bond has a current price of say ₹ 900, it is said to be a discount bond. The discount too will disappear over time as shown by curve B in Exhibit 7.2. Only when the current price is equal to par value - in such a case the bond is said to be a par bond - there is no change in price as time passes, assuming that the required yield does not change between now and the maturity date. This is shown by the dashed line in Exhibit 7.2.

7.3 BOND YIELDS

In the previous section we learned how to determine the price of a bond and discussed how price and yield were related. We now discuss various yield measures.

The commonly employed yield measures are: current yield, yield to maturity, and yield to call. Let us examine how these yield measures are calculated.

Exhibit 7.2 **Price Changes with Time**



Current Yield The current yield relates the annual coupon interest to the market price. It is expressed as:

$$\text{Current yield} = \frac{\text{Annual interest}}{\text{Price}}$$

For example, the current yield of a 10 year, 12 percent coupon bond with a par value of ₹ 1000 and selling for ₹ 950 is 12.63 percent.

$$\text{Current yield} = \frac{120}{950} = 0.1263 \text{ or } 12.63 \text{ percent}$$

The current yield calculation reflects only the coupon interest rate. It does not consider the capital gain (or loss) that an investor will realise if the bond is purchased at a discount (or premium) and held till maturity. It also ignores the time value of money. Hence it is an incomplete and simplistic measure of yield.

Yield to Maturity The yield to maturity (YTM) of a bond is the interest rate that makes the present value of the cash flows receivable from owning the bond equal to the price of the bond. Mathematically, it is the interest rate (r) which satisfies the equation:

$$P = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \ldots + \frac{C}{(1+r)^n} + \frac{M}{(1+r)^n} \tag{7.3}$$

where P is the price of the bond, C is the annual interest (in rupees), M is the maturity value (in rupees), and n is the number of years left to maturity.

The computation of YTM requires a trial and error procedure. To illustrate this, consider a ₹ 1,000 par value bond, carrying a coupon rate of 9 percent, maturing after 8 years. The bond is currently selling for ₹ 800. What is the YTM on this bond? The YTM is the value of r in the following equation:

$$800 = \sum_{t=1}^{n} \frac{90}{(1+r)^t} + \frac{1000}{(1+r)^8}$$

$$= 90\,(\text{PVIFA}_{r,8\text{yrs}}) + 1{,}000\,(\text{PVIF}_{r,8\text{yrs}})$$

Let us begin with a discount rate of 12 percent. Putting a value of 12 percent for r we find that the right-hand side of the above expression is:

$$₹\,90\,(\text{PVIFA}_{12\%,8\text{yrs}}) + ₹\,1{,}000\,(\text{PVIF}_{12\%,8\text{yrs}})$$

$$= ₹\,90(4.968) + ₹\,1{,}000(0.404) = ₹\,851.0$$

Since this value is greater than ₹ 800, we may have to try a higher value for r. Let us try $r = 14$ percent. This makes the right-hand side equal to:

$$₹\,90\,(\text{PVIFA}_{14\%,8\text{yrs}}) + ₹\,1{,}000\,(\text{PVIF}_{14\%,8\text{yrs}})$$

$$= ₹\,90\,(4.639) + ₹\,1{,}000\,(0.351) = ₹\,768.1$$

Since this value is less than ₹ 800, we try a lower value for r. Let us try $r = 13$ percent. This makes the right-hand side equal to:

$$₹\,90\,(\text{PVIFA}_{13\%,8\text{yrs}}) + ₹\,1{,}000\,(\text{PVIF}_{13\%,8\text{yrs}})$$

$$= ₹\,90\,(4.800) + ₹\,1{,}000\,(0.376) = ₹\,808$$

Thus r lies between 13 percent and 14 percent. Using a linear interpolation in the range 13 percent to 14 percent, we find that r is equal to 13.2 percent.

$$13\% + (14\% - 13\%)\frac{808 - 800}{808 - 768.1} = 13.2\%$$

An Approximation If you are not inclined to follow the trial-and-error approach described above, you can employ the following formula to find the approximate YTM on a bond:

$$\text{YTM} \simeq \frac{C + (M - P)/n}{0.4M + 0.6P} \tag{7.4}$$

where YTM is the yield to maturity, C is the annual interest payment, M is the maturity value of the bond, P is the present price of the bond, and n is the years to maturity.

To illustrate the use of this formula, let us consider the bond discussed above. The approximate YTM of the bond works out to:

$$\text{YTM} = \frac{90 + (1000 - 800)/8}{0.4 \times 1000 + 0.6 \times 800} = 13.1\%$$

Thus, we find that this formula gives a value which is very close to the true value (13.2 percent). Hence it is very useful.

The YTM calculation considers the current coupon income as well as the capital gain or loss the investor will realise by holding the bond to maturity. In addition, it takes into account the timing of the cash flows.

Yield to Call Some bonds carry a call feature that entitles the issuer to call (buy back) the bond prior to the stated maturity date in accordance with a call schedule (which specifies a call price for each call date). For such bonds, it is a practice to calculate the yield to call (YTC) as well as the YTM.

The yield to maturity for the above example may also be obtained using an Excel spread-eet, either using the RATE function or the YIELD function, as shown below.

	A		B	C
			Formula used	
	Price of the bond at present(PV)	₹		800
	Par value/Maturity value of the bond	₹		1,000
	Coupon rate			9%
	Coupon amount payable per period(PMT)	₹	=C3*C4	90
	No. of periods(NPER)			8
	Yield to Maturity(RATE)		= RATE(C6,C5,-C2,C3,0)	13.20%
	Yield to maturity of a bond can also be obtained using the Yield formula in Excel, as shown below			
10			Formula used	
11	Settlement		As the date is not given, use any date	1/1/2015
12	Maturity		= C11+365*8	30/12/2022
13	Rate			9%
14	Redemption			100
15	Frequency			1
16	Basis			3
17	Price		= 800/10	80
18	Yield to maturity		= YIELD(C11, C12, C13, C17, C14, C15, C16)	13.20%
	Note: The parameters are the same as those used in the spreadsheet illustration for 'PRICE'.			

The procedure for calculating the YTC is the same as that for the YTM. Mathematically, the YTC is the value of r in the following equation:

$$P = \sum_{t=1}^{n^*} \frac{C}{(1+r)^t} + \frac{M^*}{(1+r)^{n^*}} \qquad (7.5)$$

where M^* is the call price (in rupees) and n^* is the number of years until the assumed call date.

7.4 BOND MARKET

Bonds are bought and sold in large quantities. The Indian bond market has grown rapidly since the mid 1990s. With a daily turnover of about ₹ 5,000 crore in mid-2003, it is one of the largest in Asia. The growth in the bond market has been stimulated by a host of reforms such as the increased functional autonomy of the RBI, improved institutional infrastructure, technology-related initiatives, and consolidation and creation of benchmark securities.

Most trading in bonds takes place over the counter. This means that the transactions are privately negotiated and they don't take place through the process of matching of orders on an organised exchange. This is a characteristic of bond markets all over the world, not just in India. Because the bond market is largely over the counter, it lacks transparency. A financial market is transparent if you can easily observe its prices and volumes.

The National Stock Exchange has a Wholesale Debt Market (WDM) segment. The WDM segment is a market for high value transactions in government securities, PSU bonds, commercial papers, and other debt instruments. The quotations of this segment mostly reflect over the counter transactions that are privately negotiated over the phone or computer and registered with the exchange for reporting purposes.

An illustrative quotation from the WDM segment of NSE pertaining to no–repo (NR) trades on 31/12/2014 in 8.40% government securities (GS) issued by central government (CG) and 9.85% bank bonds (BB) issued by SBI is given below.

Date	*Security Type*	*Security Name*	*Issue Name*	*Trade Type*	*No. of Trades*	*Traded Value*	*Low Price/ Rate*	*High Price/ Rate*	*Last Price*	*YTM*
31-Dec-14	GS	CG2024	8.40%	NR	2	125	103.57	103.6	103.57	7.86
31-Jul-14	BB	SBI16	9.85%	NR	1	15	101.446	101.446	101.446	8.96

The retail trade in corporate debt securities is done mostly on the capital market segment of the NSE and the debt segment of the BSE.

7.5 VALUATION OF PREFERENCE STOCK

Preference stock generally pays regular, fixed dividends. Preference dividends are not increased when the profits of the firm rise, nor are they lowered or suspended unless the firm faces financial difficulties. If preference dividends are cut or suspended for some time, the firm is normally required to pay the arrears before paying equity dividends.

Preference stock may be perpetual or redeemable. While the former has no maturity period, the latter is expected to be redeemed after its limited life. Preference stock in India is typically redeemable.

If we assume that the preference stock pays fixed annual dividends during its life and the principal amount on maturity, its value is given as follows.

$$P_o = \sum_{t=1}^{n} \frac{D}{(1+r_p)^t} + \frac{M}{(1+r_p)^n} \tag{7.6}$$

where P_o is the current price of the preference stock, D is the annual dividend, n is the residual life of the preference stock, r_p is the required rate of return on the preference stock, and M is the maturity value.

Since the stream of dividends is an ordinary annuity, we can apply the formula for the present value of an ordinary annuity. Hence the value of the preference stock is:

$$P_0 = D \times \text{PVIFA}r_{p,n} + M \times \text{PVIF}r_{p,n}$$

To illustrate how to compute the value of a preference stock, consider an 8 year, 10 percent preference stock with a par value of ₹ 1000. The required return on this preference stock is 9 percent.

The value of the preference stock is

$$P = 100 \times \text{PVIFA}_{9\%,\,8\text{yrs}} + 1000 \times \text{PVIF}_{9\%,\,8\text{yrs}}$$
$$= 100 \times 5.535 + 1000 \times 0.502 = ₹\ 1055.5$$

7.6 EQUITY VALUATION: DIVIDEND DISCOUNT MODEL

According to the dividend discount model, the value of an equity share is equal to the present value of dividends expected from its ownership plus the present value of the sale price expected when the equity share is sold. For applying the dividend discount model, we will make the following assumptions: (i) dividends are paid annually; and (ii) the first dividend is received one year after the equity share is bought.

Single-period Valuation Model Let us begin with the case where the investor expects to hold the equity share for one year. The price of the equity share will be:

$$P_0 = \frac{D_1}{(1+r)} + \frac{P_1}{(1+r)} \tag{7.7}$$

where P_0 is the current price of the equity share, D_1 is the dividend expected a year hence, P_1 is the price of the share expected a year hence, and r is the rate of return required on the equity share.

Example Prestige's equity share is expected to provide a dividend of ₹ 2.00 and fetch a price of ₹ 18.00 a year hence. What price would it sell for now if investors' required rate of return is 12 percent? The current price will be:

$$P_0 = \frac{2.0}{(1.12)} + \frac{18.00}{(1.12)} = ₹\ 17.86$$

What happens if the price of the equity share is expected to grow at a rate of g percent annually? If the current price, P_0, becomes $P_0(1+g)$ a year hence, we get:

$$P_0 = \frac{D_1}{(1+r)} + \frac{P_0(1+g)}{(1+r)} \tag{7.8}$$

Simplifying Eq.(7.8) we get:

$$P_0 = \frac{D_1}{r-g} \qquad (7.9)^1$$

[1] The steps in simplification are:

$$P_0 = \frac{D_1}{(1+r)} + \frac{P_0(1+g)}{(1+r)} \tag{1}$$

$$P_0 = \frac{D_1 + P_0(1+g)}{(1+r)} \tag{2}$$

$$P_0(1+r) = D_1 + P_0(1+g) \tag{3}$$

$$P_0(1+r) - P_0(1+g) = D_1 \tag{4}$$

$$P_0(r-g) = D_1 \tag{5}$$

$$P_0 = \frac{D_1}{r-g} \tag{6}$$

Example The expected dividend per share on the equity share of Roadking Limited is ₹ 2.00. The dividend per share of Roadking Limited has grown over the past five years at the rate of 5 percent per year. This growth rate will continue in future. Further, the market price of the equity share of Roadking Limited, too, is expected to grow at the same rate. What is a fair estimate of the intrinsic value of the equity share of Roadking Limited if the required rate is 15 percent?

Applying Eq.(7.9) we get the following estimate:

$$P_0 = \frac{2.00}{0.15 - .05} = ₹\ 20.00$$

Expected Rate of Return In the preceding discussion we calculated the intrinsic value of an equity share, given information about (i) the forecast values of dividend and share price, and (ii) the required rate of return. Now we look at a different question: What rate of return can the investor expect, given the current market price and forecast values of dividend and share price? The expected rate of return is equal to:

$$r = D_1 / P_0 + g \tag{7.10}$$

Example The expected dividend per share of Vaibhav Limited is ₹ 5.00. The dividend is expected to grow at the rate of 6 percent per year. If the price per share now is ₹ 50.00, what is the expected rate of return?

Applying Eq. (7.10), the expected rate of return is:

$$r = 5/50 + 0.06 = 16 \text{ percent}$$

Multi-period Valuation Model Having learnt the basics of equity share valuation in a single-period framework, we now discuss the more realistic, and also the more complex, case of multi-period valuation.

Since equity shares have no maturity period, they may be expected to bring a dividend stream of infinite duration. Hence the value of an equity share may be put as:

$$P_0 = \frac{D_1}{(1+r)} + \frac{D_2}{(1+r)^2} + \ldots + \frac{D_\infty}{(1+r)^\infty} = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t} \tag{7.11}$$

where P_0 is the price of the equity share today, D_1 is the dividend expected a year hence, D_2 is the dividend expected two years hence,.... D_∞ is the dividend expected at the end of infinity, and r is the expected return.

Equation (7.11) represents the valuation model for an infinite horizon. Is it applicable to a finite horizon? Yes. To demonstrate this, consider how an equity share would be valued by an investor who plans to hold it for n years and sell it thereafter for a price of P_n. The value of the equity share to him is:

$$P_0 = \frac{D_1}{(1+r)^1} + \frac{D_2}{(1+r)^2} + \ldots + \frac{D_n}{(1+r)^n} + \frac{P_n}{(1+r)^n}$$

$$= \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n} \qquad (7.12)$$

Now, what is the value of P_n in Eq.(7.12)? Applying the dividend capitalisation principle, the value of P_n would be the present value of the dividend stream beyond the nth year, evaluated as at the end of the nth year. This means:

$$P_n = \frac{D_{n+1}}{(1+r)^1} + \frac{D_{n+2}}{(1+r)^2} + \ldots + \frac{D_\infty}{(1+r)^\infty} \qquad (7.13)$$

Substituting this value of P_n in Eq. (7.12) we get:

$$P_0 = \frac{D_1}{(1+r)^1} + \frac{D_2}{(1+r)^2} + \ldots + \frac{D_n}{(1+r)^n}$$

$$+ \frac{1}{(1+r)^n}\left[\frac{D_{n+1}}{(1+r)} + \frac{D_{n+2}}{(1+r)^2} + \ldots + \frac{D_\infty}{(1+r)^\infty}\right]$$

$$= \frac{D_1}{(1+r)} + \frac{D_2}{(1+r)^2} + \ldots + \frac{D_n}{(1+r)^n} + \frac{D_n}{(1+r)^{n+1}} + \ldots + \frac{D_\infty}{(1+r)^\infty}$$

$$= \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t} \qquad (7.14)$$

This is the same as Eq.(7.11) which may be regarded as a generalised multi-period valuation formula. Eq.(7.11) is general enough to permit any dividend pattern — constant, rising, declining, or randomly fluctuating. For practical applications it is helpful to make simplifying assumptions about the pattern of dividend growth. The more commonly used assumptions are as follows:

- The dividend per share remains constant forever, implying that the growth rate is nil (the zero growth model).
- The dividend per share grows at a constant rate per year forever (the constant growth model).
- The dividend per share grows at a constant rate for a finite period, followed by a constant normal rate of growth forever thereafter (the two-stage model).
- The dividend per share, currently growing at an above-normal rate, experiences a gradually declining rate of growth for a while. Thereafter, it grows at a constant normal rate (the *H* model).

Zero Growth Model If we assume that the dividend per share remains constant year after year at a value of D, Eq.(7.11) becomes :

$$P_0 = \frac{D}{(1+r)} + \frac{D}{(1+r)^2} + \ldots + \frac{D}{(1+r)^n} + \ldots \infty \qquad (7.15)$$

Equation (7.15), on simplification, becomes:

$$P_0 = \frac{D}{r} \qquad (7.16)$$

This is an application of the present value of perpetuity formula.

Constant Growth Model One of the most popular dividend discount models assumes that the dividend per share grows at a constant rate (g). The value of a share, under this assumption, is:

$$P_0 = \frac{D_1}{(1+r)} + \frac{D_1(1+g)}{(1+r)^2} + \cdots \frac{D_1(1+g)^n}{(1+r)^{n+1}} + \cdots \tag{7.17}$$

Applying the formula for the sum of a geometric progression, the above expression simplifies to:

$$P_0 = \frac{D_1}{r-g} \tag{7.18}$$ [2]

Example Ramesh Engineering Limited is expected to grow at the rate of 6 percent per annum. The dividend expected on Ramesh's equity share a year hence is ₹ 2.00. What price will you put on it if your required rate of return for this share is 14 percent?

The price of Ramesh's equity share would be:

$$P_0 = \frac{2.00}{0.14 - 0.06} = ₹\ 25.00$$

What Drives Growth Most stock valuation models are based on the assumption that dividends grow over time. What drives this growth? The two major drivers of growth are: (a) ploughback ratio and (b) return on equity (ROE). To see why this is so let us consider an example. Omega Limited has an equity (net worth) base of 100 at the beginning of year 1. It earns a return on equity of 20 percent. It pays out 40 percent of its equity earnings and ploughs back 60 percent of its equity earnings. Its financials for a 3 year period are shown in Exhibit 7.3, from which we find that dividends grow at a rate of 12 percent. The growth figure is a product of: Ploughback ratio × Return on equity = 0.6 × 20% = 12%

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[2] Start with

$$P_0 = \frac{D_1}{(1+r)} + \frac{D_2}{(1+r)^2} + \ldots + \frac{D_\infty}{(1+r)^\infty} = \frac{D_1}{(1+r)} + \frac{D_1(1+g)}{(1+r)^2} + \ldots \tag{1}$$

Multiplying both the sides of (1) by $[(1+g)/(1+r)]$ gives:

$$P_0\left[\frac{1+g}{1+r}\right] = \frac{D_1(1+g)}{(1+r)^2} + \frac{D_1(1+g)^2}{(1+r)^3} + \ldots + \frac{D_1(1+g)^{n+1}}{(1+r)^{n+2}} \quad (n \to \infty) \tag{2}$$

Subtracting (2) from (1) yields:

$$\frac{P_0(r-g)}{(1+r)} = D_1\left[\frac{1}{(1+r)} - \frac{(1+g)^{n+1}}{(1+r)^{n+2}}\right] \quad (n \to \infty) \tag{3}$$

$$\text{As } (n \to \infty),\ \frac{(1+g)^{n+1}}{(1+r)^{n+2}} \to 0 \text{ because } g < r$$

Hence (2) becomes:

$$\frac{P_0(r-g)}{(1+r)} = \frac{D_1}{(1+r)} \tag{4}$$

This means:

$$P_0 = \frac{D_1}{r-g} \tag{5}$$

Exhibit 7.3 **Financials of Omega Limited**

	Year 1	Year 2	Year 3
▪ Beginning equity	100	112	125.44
▪ Return on equity	20%	20%	20%
▪ Equity earnings	20	22.4	25.1
▪ Dividend payout ratio	0.4	0.4	0.4
▪ Dividends	8	8.96	10.04
▪ Ploughback ratio	0.6	0.6	0.6
▪ Retained earnings	12	13.44	15.06

Two Stage Growth Model The simplest extension of the constant growth model assumes that the extraordinary growth (good or bad) will continue for a finite number of years and thereafter the normal growth rate will prevail indefinitely.

Assuming that the dividends move in line with the growth rate, the price of the equity share will be:

$$P_0 = \left[\frac{D_1}{(1+r)} + \frac{D_1(1+g_1)}{(1+r)^2} + \frac{D_1(1+g_1)^2}{(1+r)^3} \ldots + \frac{D_1(1+g_1)^{n-1}}{(1+r)^n}\right] + \frac{P_n}{(1+r)^n} \quad (7.19)$$

where P_0 is the current price of the equity share, D_1 is the dividend expected a year hence, g_1 is the extraordinary growth rate applicable for n years, and P_n is the price of the equity share at the end of year n.

The first term on the right hand side of Eq.(7.19) is the present value of a growing annuity. Its value is equal to:

$$D_1\left[\frac{1-\left[\frac{1+g_1}{1+r}\right]^n}{r-g_1}\right] \quad (7.20)$$

Remember that this is a straightforward application of Eq.(6.7) developed in the previous chapter.

Hence

$$P_0 = D_1\left[\frac{1-\left[\frac{1+g_1}{1+r}\right]^n}{r-g_1}\right] + \frac{P_n}{(1+r)^n} \quad (7.21)$$

Since the two-stage growth model assumes that the growth rate after n years remains constant, P_n will be equal to:

$$\frac{D_{n+1}}{r-g_2} \quad (7.22)$$

where D_{n+1} is the dividend for year $n+1$ and g_2 is the growth rate in the second period.

D_{n+1}, the dividend for year $n+1$, may be expressed in terms of the dividend at the end of the first stage and growth rate in the second stage:

$$D_{n+1} = D_1 (1+g_1)^{n-1} (1+g_2) \quad (7.23)$$

Substituting the above expression, we have:

$$P_0 = D_1 \left[\frac{1 - \left[\frac{1+g_1}{1+r} \right]^n}{r - g_1} \right] + \left[\frac{D_1 (1+g_1)^{n-1} (1+g_2)}{r - g_2} \right] \left[\frac{1}{(1+r)^n} \right] \quad (7.24)$$

Example The current dividend on an equity share of Vertigo Limited is ₹ 2.00. Vertigo is expected to enjoy an above-normal growth rate of 20 percent for a period of 6 years. Thereafter the growth rate will fall and stabilise at 10 percent. Equity investors require a return of 15 percent. What is the intrinsic value of the equity share of Vertigo?

The inputs required for applying the two-stage model are:

$g_1 =$ 20 percent

$g_2 =$ 10 percent

$n =$ 6 years

$r =$ 15 years

$D_1 = D_0 (1+g_1) =$ ₹ 2(1.20) = 2.40

Plugging these inputs in the two-stage model, we get the intrinsic value estimate as follows:

$$P_0 = 2.40 \left[\frac{1 - \left[\frac{1.20}{1.15} \right]^6}{.15 - .20} \right] + \left[\frac{2.40\,(1.20)^5\,(1.10)}{.15 - .10} \right] \left[\frac{1}{(1.15)^6} \right]$$

$$= 2.40 \left[\frac{1 - 1.291}{-0.05} \right] + \left[\frac{2.40\,(2.488)(1.10)}{.05} \right] [0.432]$$

$$= 13.96 + 56.80$$

$$= ₹\ 70.76$$

The Excel spreadsheet for the two stage growth model is as under:

	A	B	C	D	E
1	g_1	g_2	n(years)	r	D_0(₹)
2	20%	10%	6	15%	2
3	P_0(₹)	Formula used = E2*(1 + A2)*(1–((1 + A2)/(1 + D2))^C2)/(D2 – A2) +E2* (1 +A2)* (1 + A2)^(C2 – 1)* (1 + B2)/(D2 – B2)/(1 + D2)^C2			70.76

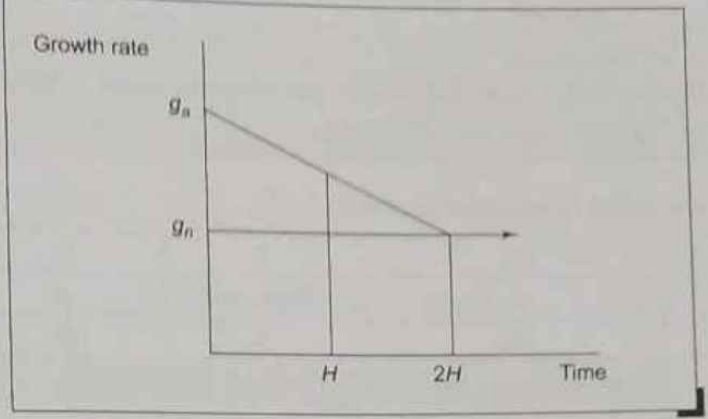
H Model The *H* model of equity valuation is based on the following assumptions:

- While the current dividend growth rate, g_a, is greater than g_n, the normal long-run growth rate, the growth rate declines linearly for 2*H* years.

- After $2H$ years the growth rate becomes g_n.

The graphical representation of the dividend growth rate pattern for the H-model is shown in Exhibit 7.4.

Exhibit 7.4 **Dividend Growth Rate Pattern for the *H* model**



While the derivation of the H model is rather complex, the valuation equation for the H model is quite simple:

$$P_0 = \frac{D_0[(1+g_n)+H(g_a-g_n)]}{r-g_n} \tag{7.25}$$

where P_0 is the intrinsic value of the share, D_0 is the current dividend per share, r is the rate of return expected by investors, g_n is the normal long-run growth rate, g_a is the current growth rate, and H is one-half of the period during which g_a will level off to g_n.
Equation (7.25) may be re-written as:

$$P_0 = \frac{D_0(1+g_n)}{r-g_n} + \frac{D_0H(g_a-g_n)}{r-g_n} \tag{7.26}$$

Expressed this way, the H model may be interpreted in a simple, intuitive manner. The first term on the right hand side of Eq. (7.26)

$$\frac{D_0(1+g_n)}{r-g_n}$$

represents the value based on the normal growth rate, whereas the second term

$$\frac{D_0H(g_a-g_n)}{r-g_n}$$

reflects the premium due to abnormal growth rate.

Example The current dividend on an equity share of International Computers Limited is ₹ 3.00. The present growth rate is 50 percent. However, this will decline linearly over a period of 10 years and then stabilise at 12 percent. What is the intrinsic value per share of International Computers Limited, if investors require a return of 16 percent?

The inputs required for applying the *H*-model are:

$D_0 =$ ₹ 3.00

$g_a =$ 50 percent

$H =$ 5 years

$g_n =$ 12 percent

$r =$ 16 percent

Plugging these inputs in the *H*-model we get the intrinsic value estimate as follows:

$$P_0 = \frac{300[(1.12) + 5(0.50 - 0.12)]}{.16 - .12} = ₹\ 226.5$$

The Excel illustration of the *H*-model is under:

	A	B	C	D	E
1	g_a	g_n	H(years)	r	D_0(₹)
2	50%	12%	5	16%	3
3	P_0(₹)	Formula used = E2* ((1+B2) + C2*(A2–B2))/(D2-B2)			226.50

Impact of Growth on Price, Returns, and P/E Ratio The expected growth rates of companies differ widely. Some companies are expected to remain virtually stagnant or grow slowly; other companies are expected to show normal growth; still others are expected to achieve supernormal growth rate.

Assuming a constant total required return, differing expected growth rates mean differing stock prices, dividend yields, capital gains yields, and price-earnings ratios. To illustrate this, consider three cases:

	Growth rate (%)
Low growth firm	5
Normal growth firm	10
Supernormal growth firm	15

The expected earnings per share and dividend per share of each of the three firms for the following year are ₹ 3.00 and ₹ 2.00 respectively. Investors' required total return from equity investment is 20 percent.

Given the above information, we may calculate the stock price, dividend yield, capital gains yield, and price-earnings ratio for the three cases as shown in Exhibit 7.5.

The results in Exhibit 7.5 suggest the following points:

1. As the expected growth in dividend increases, other things being equal, the expected return[3] depends more on the capital gains yield and less on the dividend yield.
2. As the expected growth rate in dividend increases, other things being equal, the price-earnings ratio increases.

[3] Note that total return is the sum of the dividend yield and capital gain yield:

$$\frac{D_t + P_t - P_{t-1}}{P_{t-1}} = \frac{D_t}{P_{t-1}} + \frac{P_t - P_{t-1}}{P_{t-1}}$$

Total return = Dividend yield + Capital gains yield

3. High dividend yield and low price-earnings ratio imply limited growth prospects.
4. Low dividend yield and high price-earnings ratio imply considerable growth prospects.

Exhibit 7.5 **Price, Dividend Yield, Capital Gains Yield, and Price-Earnings Ratio under Differing Growth Assumptions**

	Price $P_0 = \frac{D_1}{r-g}$	Dividend yield (D_1/P_0)	Capital gains yield $(P_1-P_0)/P_0$	Price earnings ratio (P/E)
Low growth firm	$P_0 = \frac{₹\,2.00}{0.20-0.05} = ₹\,13.33$	15.0%	5.0%	4.44
Normal growth firm	$P_0 = \frac{₹\,2.00}{0.20-0.10} = ₹\,20.00$	10.0%	10.0%	6.67
Supernormal growth firm	$P_0 = \frac{₹\,2.00}{0.20-0.15} = ₹\,40.00$	5.0%	15.0%	13.33

Is the Stock Market Shortsighted? Many managers believe that the stock market is myopic and obsessed with short-term performance. Let's test this assertion with the help of the constant growth model. Sun Pharma was quoting at ₹ 591.80 on May 26, 2014. Sun Pharma's most recent dividend was ₹ 10 per share and the dividend growth rate in the previous five years was 19 percent per year. If we assume that the dividend per share continues to grow at the same rate for the next five years and apply a discount rate of 13 percent, the present value of the projected dividends for the following five years would be:

$$PV = \frac{₹\,10(1.19)}{(1.13)} + \frac{₹\,10(1.19)^2}{(1.13)^2} + \frac{₹\,10(1.19)^3}{(1.13)^3} + \frac{₹\,10(1.19)^4}{(1.13)^4} + \frac{₹\,10(1.19)^5}{(1.13)^5}$$

$$= 10.53 + 11.09 + 11.68 + 12.30 + 12.95 = ₹\,58.55$$

Recall that Sun Pharma's stock price was ₹ 591.80. Therefore, less than 10 percent of the current stock price is attributable to the projected cash flows (by way of dividends) for the following five years. This means that Sun Pharma managers should focus on increasing long-term cash flows. This is true for most companies. Indeed, many researchers and consultants have found that for a typical company more than 80 percent of current stock price is accounted for by cash flows beyond five years.

If long-term cash flows account for the bulk of a stock's value, why are managers and analysts obsessed with quarterly earnings? The primary reason seems to be the informational content of short-term earnings. While the quarterly earnings by themselves may not be important, the information they convey about long-term prospects may be very significant. If the quarterly earnings are lower than expected because the new products launched by the company have failed, the long–term cash flows of the company would be negatively impacted. On the other hand, if the reason is that the company has significantly increased its

SUMMARY

- The term value is used in different senses. Liquidation value, going concern value, book value, market value, and intrinsic value are the most commonly used concepts of value.
- The **intrinsic value** of any asset, real or financial, is equal to the present value of the cash flows expected from it. Hence, determining the value of an asset requires an estimate of expected cash flows and an estimate of the required return.
- The value of a bond is:

$$P = \sum_{t=1}^{n} \frac{C}{(1+r)^t} + \frac{M}{(1+r)^n}$$

- A basic property of a bond is that its price varies inversely with yield.
- The relationship between coupon rate, required yield, and bond price is as follows:
  Coupon rate < Required yield $\longleftrightarrow$ Price < Par (Discount bond)
  Coupon rate = Required yield $\longleftrightarrow$ Price = Par
  Coupon rate > Required yield $\longleftrightarrow$ Price > Par (Premium bond)
- The **current yield** on a bond is defined as: Annual interest / Price
- The **yield to maturity** (YTM) on a bond is the rate of return the investor earns when he buys the bond and holds it till maturity. It is the value of r in the bond valuation model. For estimating YTM readily, the following approximation may be used:

$$\text{YTM} = \frac{C + (M - P)/n}{0.4M + 0.6P}$$

- According to the **dividend discount model**, the value of an equity share is equal to the present value of dividends expected from its ownership.

- If the dividend per share remains constant the value of the share is:

$$P_0 = D / r$$

- If the dividend per share grows at a constant rate, the value of the share is:

$$P_0 = D_1 / (r - g)$$

- The two key drivers of dividend growth are (a) ploughback ratio and (b) return on equity.
- The value per share, according to the H model is:

$$P_0 = \frac{D_0(1+g_n)}{r-g_n} + \frac{D_0 H(g_a - g_n)}{r-g_n}$$

- An approach to valuation, practised widely by investment analysts, is the P/E ratio approach. The value of an equity share, under this approach, is estimated as follows:

$$P_0 = E_1 \times P_0/E_1$$

- The stock price may be considered as the capitalised value of the earnings under the assumption of no growth plus the present value of growth opportunities.
- The stock market consists of a primary segment and a secondary segment. The principal bourses are the National Stock Exchange and the Bombay Stock Exchange, accounting for the bulk of the trading on the Indian stock market.

QUESTIONS

1. Describe briefly the various concepts of value.
2. Discuss the basic bond valuation.
3. State the formula for a bond which pays interest semi-annually.
4. What is the relationship between coupon rate, required yield, and price?
5. Explain and illustrate the following yield measures: current yield, yield to maturity, and yield to call.
6. State and illustrate the formula to find the approximate YTM on a bond.
7. Discuss the constant growth dividend discount model.
8. Explain the two stage dividend discount model.
9. Discuss the H model.
10. What is the impact of growth on price, dividend yield, capital gains yield, and price-earnings ratio?
11. Discuss the P/E ratio approach to stock valuation.
12. How is the E/P linked to the required return and the present value of growth opportunities?
13. Explain how the price-earnings ratio is related to growth, dividend payout ratio, and the required return.
14. Discuss the transformation of the Indian stock market from mid 1990s.
15. Discuss the salient features of the National Stock Exchange.
16. Discuss the salient features of the Bombay Stock Exchange.
17. How is stock price reported?

SOLVED PROBLEMS

7.1 A ₹ 100 par value bond bearing a coupon rate of 12 percent will mature after 5 years. What is the value of the bond, if the discount rate is 15 percent?