Risk and Return

OUTLINE

- Risk and Return of a Single Asset
- Risk and Return of a Portfolio
- Measurement of Market Risk
- Relationship between Risk and Return

Past Returns:

$$\text{Holding Period Return} = \frac{\text{Dividend} + (\text{Price at the End} - \text{Price at the beginning})}{\text{Price at the beginning}}$$

$$\text{Annualised Return} = \frac{\text{Holding Period Return}}{\text{Holding period in months}} \times \text{12 months}$$

RISK AND RETURN OF A SINGLE ASSET

$$\text{Rate of Return} = \underbrace{\frac{\text{Annual income}}{\text{Beginning price}}}_{\text{Current yield}} + \underbrace{\frac{\text{Ending price-Beginning price}}{\text{Beginning price}}}_{\text{Capital gains /loss yield}}$$

Example

- Consider the following information for an equity stock
- Price at the beginning of the year : Rs. 60
- Dividend paid at the end of the year : Rs. 2.40
- Price at the end of the year : Rs. 69
- Calculate the total return on this stock

- Soln : =(2.40/60 + (69-60)/60) *100 = 19%

AVERAGE ANNUAL RETURNS

The arithmetic mean is defined as:

$$\bar{R} = \frac{\sum_{i=1}^{n} R_i}{n} \qquad (8.$$

where $\bar{R}$ is the arithmetic mean , R_i is the i the value of the total return (i= 1,...n) n is the number of total returns.

To illustrate, suppose the total returns from stock A over a five year period are as follows:

Year	Total return (percentage)
1	19.0
2	14.0
3	22.0
4	-12.0
5	5.0

The arithmetic mean return for stock A is:

$$\bar{R} = \frac{19 + 14 + 22 - 12 + 5}{5} = 9.6 \text{ percent}$$

AVERAGE ANNUAL RETURNS

To calculate the average compound rate of growth over a period of time, the **geometric mean** is used. The geometric mean is defined as follows:

$$GM = \left[(1+R_1)(1+R_2)\ldots(1+R_n)\right]^{1/n} - 1 \qquad (8.3)$$

where GM is the geometric mean return, R_i is the total return for period $i(i=1,\ldots,n)$, and n is the number of time periods.

To illustrate, consider the total return relative $(1+R_i)$ for stock A over a 5- year period:

Year	Total return (%)	Return relative $(1+R_i)$
1	19	1.19
2	14	1.14
3	22	1.22
4	-12	0.88
5	5	1.05

The geometric mean of the returns over the 5 year period is:

$$GM = \left[(1.19)(1.14)(1.22)(0.88)(1.05)\right]^{1/5} - 1$$

$= 1.089 - 1 = 0.089$ or 8.9 percent

DATA ON THE NIFTY INDEX

Year ending	*NIFTY*	*Annual return (%)*	*Year ending*	*NIFTY*	*Annual return (%)*
1990	331	-	2002	1094	3.25
1991	559	68.84	2003	1880	71.90
1992	761	36.28	2004	2081	10.68
1993	1043	36.95	2005	2837	36.34
1994	1182	13.40	2006	3966	39.83
1995	909	-23.15	2007	6139	54.77
1996	899	-1.04	2008	2959	-51.79
1997	1079	20.05	2009	5201	75.76
1998	884	-18.08	2010	6135	17.95
1999	1480	67.42	2011	4624	-24.62
2000	1264	-14.65	2012	5905	27.70
2001	1059	-16.18	2013	6304	6.76
			2014	8284	31.41

DATA ON THE NIFTY INDEX

The return for the year ended 1991 is 559/331- 1 = 68.88 percent. The returns for other years have been calculated the same way. The means of the returns are calculated as under:

Arithmetic Mean = (68.84 + 36.28 +-------------------+ 6.76 + 31.41)/ 24 = 19.57 percent

Geometric Mean=$(1.6884 \times 1.3628 \text{----------} \times 1.0676 \times 1.3141)^{1/24} - 1$ = (25.0378)1/24-1 = 14.36 percent

VARIANCE OF RETURNS

$$\sigma^2 = \frac{\sum_{i=1}^{n} (R_i - \overline{R})^2}{n-1} \qquad (8.4)^{1}$$

where σ^2 is the variance of return, σ is the standard deviation of return, Ri is the return from the stock in period i (i =1,…., n), R is the arithmetic return, and n is the number of periods.

VARIANCE OF RETURNS

To illustrate, consider the returns from a stock over a 6 year period:

$R_1 = 15\%$, $R_2 = 12\%$, $R_3 = 20\%$, $R_4 = -10\%$, $R_5 = 14\%$, and $R_6 = 9\%$

The variance and standard deviation of returns are calculated below:

Period	Return	Deviation	Square of deviation
		$(R_i - \overline{R})$	$(R_i - \overline{R})^2$
1	15	5	25
2	12	2	4
3	20	10	100
4	-10	-20	400
5	14	4	16
6	9	-1	1
	$\sum R_i = 60$ $\overline{R} = 10$		$\sum (R_i - R)^2 = 546$

$$\sigma^2 = \left(\frac{\sum (R_i - \overline{R})^2}{n-1} \right) = 109.2 \quad \sigma = \left(\frac{\sum (R_i - \overline{R})^2}{n-1} \right)^{1/2} = \left(\frac{546}{6-1} \right)^{1/2} = 10.45$$

PROBABILITY DISTRIBUTION AND EXPECTED RATE OF RETURN

		Rate of Return (%)	
State of the Economy	*Probability of Occurrence*	*Bharat Foods*	*Oriental Shipping*
Boom	0.30	16	40
Normal	0.50	11	10
Recession	0.20	6	-20

$$E(R) = \sum_{i=1}^{n} p_i R_i \qquad (8.1)$$

Similarly, the expected rate of return on Oriental Shipping stock is:

$E(Ro) = (0.30)\ (40\%) + (0.50)\ (10\%) + (0.20)\ (-20\%) = 13.0\%$

STANDARD DEVIATION

Illustration of the Calculation of Standard Deviation

Bharat Foods Stock

i. State of the Economy	p_i	R_i	p_iR_i	R_i-$E(R)$	$(R_i$-$E(R))^2$	$p_i(R_i$-$E(R))^2$
1. Boom	0.30	16	4.8	4.5	20.25	6.075
2. Normal	0.50	11	5.5	-0.5	0.25	0.125
3. Recession	0.20	6	1.2	-5.5	30.25	6.050

$\Sigma p_iR_i = 11.5$ $\Sigma p_i(R_i - E(R))^2 = 12.25$

$\sigma = [\Sigma p_i\,(R_i\text{-}E(R))^2]^{1/2} = (12.25)^{1/2} = 3.5\%$

Oriental Shipping Stock

i. State of the Economy	p_i	R_i	p_iR_i	R_i-$E(R)$	$(R_i$-$E(R))^2$	$p_i(R_i$-$E(R))^2$
1. Boom	0.30	40	12.0	27.0	729.0	218.7
2. Normal	0.50	10	5.0	-3.0	0.25	4.5
3. Recession	0.20	-20	-4.0	-33.0	1089.00	217.8

$\Sigma p_iR_i = 13.0$ $\Sigma p_i\,(R_i - E(R))^2 = 441.0$

$\sigma = [\Sigma p_i(R_i\text{-}E(R))^2]^{1/2} = (441.0)^{1/2} = 21.0\%$

RISK AVERSION AND REQUIRED RETURNS

The relationship of a person's certainty equivalent to the expected monetary value of a risky investment defines his attitude toward risk. If the certainty equivalent is less than the expected value, the person is *risk-averse*; if the certainty equivalent is equal to the expected value, the person is *risk-neutral*; finally, if the certainty equivalent is more than the expected value, the person is *risk-loving*.

In general, investors are risk-averse. This means that risky investments must offer higher expected returns than less risky investments to induce people to invest in them. Remember, however, that we are talking about *expected* returns; the *actual* return on a risky investment may well turn out to be less than the *actual* return on a less risky investment.

Put differently, risk and return go hand in hand. This indeed is a well-established empirical fact, particularly over long periods of time.

EXPECTED RETURN ON A PORTFOLIO

- The expected return on a portfolio is simply the weighted average of the expected returns on the assets comprising the portfolio. For example, when a portfolio consists of two securities, its expected return is:
- $E(R_p) = w_1 E(R_1) + (1-w_1) E(R_2)$

- Where $E(R_p)$ = Expected return on a portfolio,
- $E(R_1)$ = expected return on a security 1
- $(1-w_1)$ = the proportion of portfolio invested in security 2
- $E(R_2)$ = expected return on a security 2

EXPECTED RETURN ON A PORTFOLIO

- Illustration – consider a portfolio consisting of five securities with the following expected returns
- $E(R_1) = 10\%$, $E(R_2) = 12\%$, $E(R_3) = 15\%$, $E(R_4) = 18\%$, $E(R_5) = 20\%$. The portfolio proportions invested in these securities are w1 = 0.1, w2 = 0.2, w3= 0.3, w4 = 0.2, w5 = 0.2. What is the expected return on portfolio?

$$E(R_p) = \Sigma\, w_i\, E(R_i)$$

$$= 0.1 \times 10 + 0.2 \times 12 + 0.3 \times 15 + 0.2 \times 18 + 0.2 \times 20$$

$$= 15.5 \text{ percent}$$

DIVERSIFICATION AND PORTFOLIO RISK

Probability Distribution of Returns

State of the Econcmy	*Probability*	*Return on Stock A*	*Return on Stock B*	*Return on Portfolio*
1	0.20	15%	-5%	5%
2	0.20	-5%	15	5%
3	0.20	5	25	15%
4	0.20	35	5	20%
5	0.20	25	35	30%

Expected Return

Stock A : 0.2(15%) + 0.2(-5%) + 0.2(5%) +0.2(35%) + 0.2(25%) = 15%

Stock B : 0.2(-5%) + 0.2(15%) + 0.2(25%) + 0.2(5%) + 0.2(35%) = 15%

Portfolio of A and B : 0.2(5%) + 0.2(5%) + 0.2(15%) + 0.2(20%) + 0.2(30%) = 15%

Standard Deviation

Stock A : $\sigma^2_A = 0.2(15-15)^2 + 0.2(-5-15)^2 + 0.2(5-15)^2 + 0.2(35-15)^2 + 0.20\,(25-15)^2$

$= 200$

$\sigma_A = (200)^{1/2} = 14.14\%$

Stock B : $\sigma^2_B = 0.2(-5-15)^2 + 0.2(15-15)^2 + 0.2(25-15)^2 + 0.2(5-15)^2 + 0.2\,(35-15)^2$

$= 200$

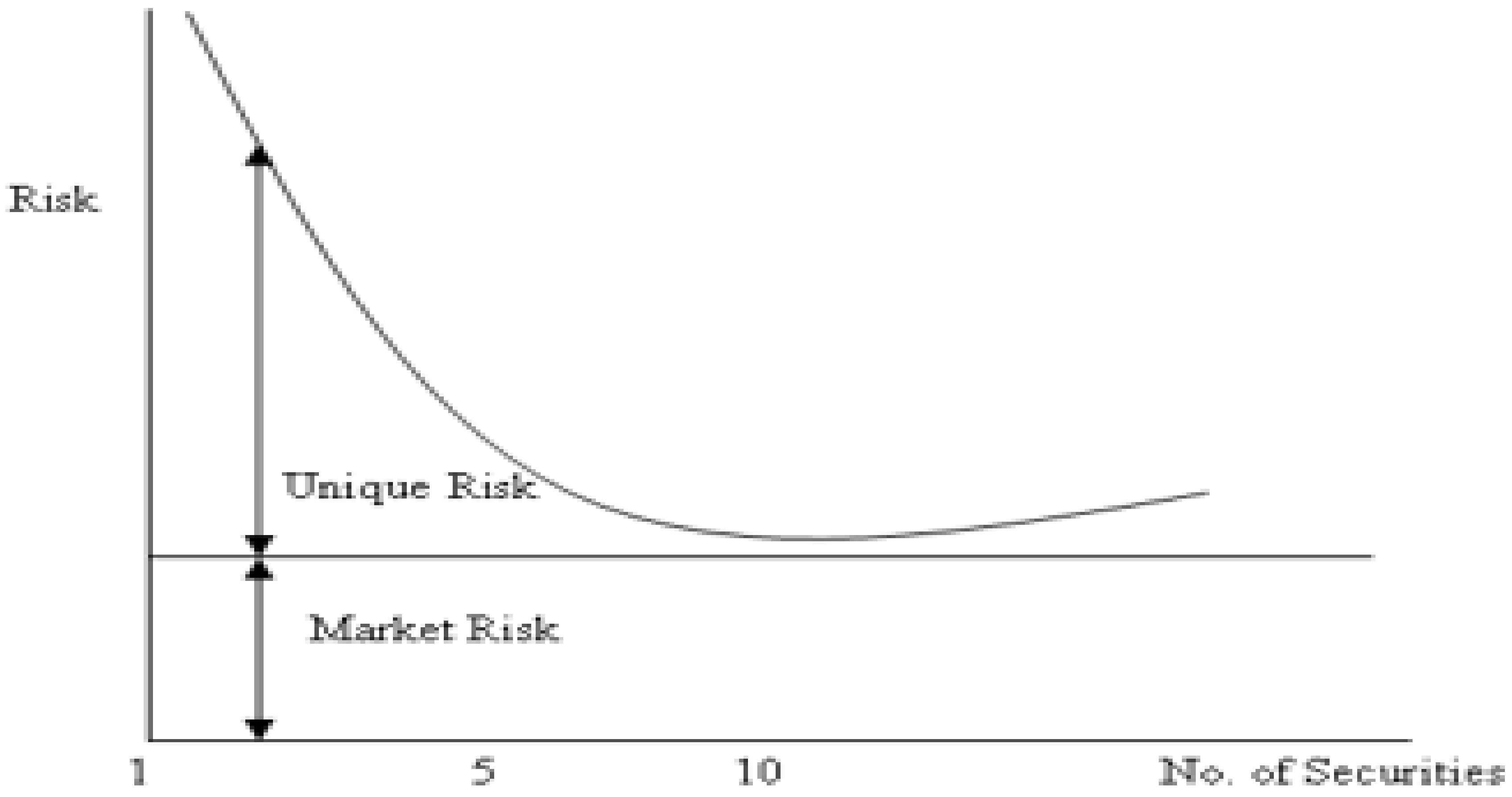
$\sigma_B = (200)^{1/2} = 14.14\%$

Portfolio : $\sigma^2_{(A+B)} = 0.2(5-15)^2 + 0.2(5-15)^2 + 0.2(15-15)^2 + 0.2(20-15)^2 + 0.2(30-15)^2$

$= 90$

$\sigma_{A+B} = (90)^{1/2} = 9.49\%$

RELATIONSHIP BETWEEN DIVERSIFICATION AND RISK



MARKET RISK VS UNIQUE RISK

Total Risk = Unique risk + Market risk

Unique risk of a security represents that portion of its total risk which stems from company-specific factors.

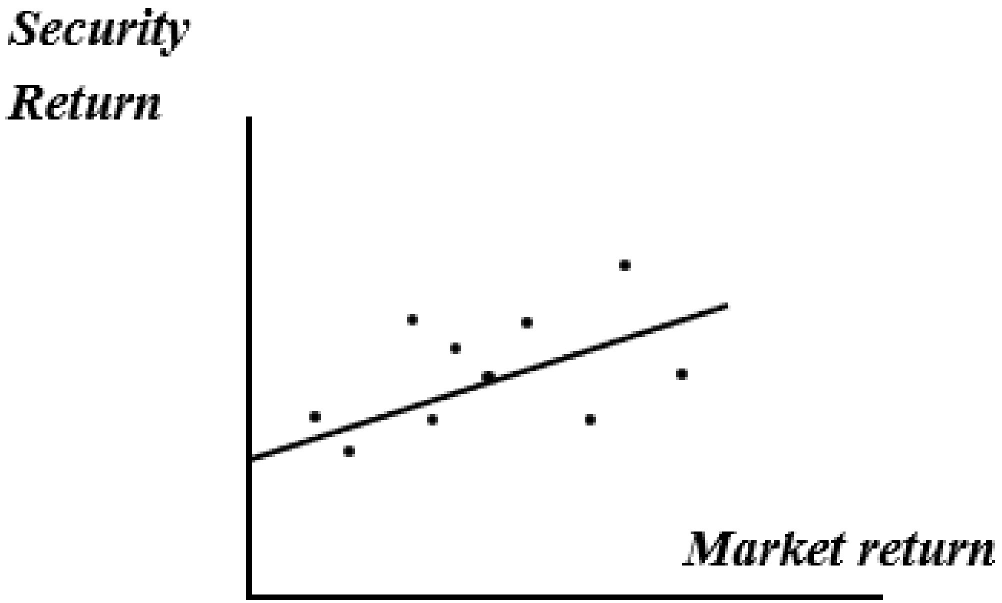
Market risk of security represents that portion of its risk which is attributable to economy –wide factors.

MEASUREMENT OF MARKET RISK

The sensitivity of a security to market movements is called beta .

Beta reflects the slope of a linear regression relationship between the return on the security and the return on the portfolio

Relationship between Security Return and Market Return



CALCULATION OF BETA

For calculating the beta of a security, the following market model is employed:

$$R_{jt} = \alpha_j + \beta_j R_{Mt} + e_j$$

where	R_{jt}	=	**return of security *j* in period *t***
	α_j	=	**intercept term alpha**
	β_j	=	**regression coefficient, beta**
	R_{Mt}	=	**return on market portfolio in period *t***
	e_j	=	**random error term**

Beta reflects the slope of the above regression relationship. It is equal to:

$$\beta_j = \frac{\text{Cov}\,(R_j, R_M)}{\sigma^2_M} = \frac{\rho_{jM}\,\rho_j\,\sigma_M}{\sigma^2_M} = \frac{\rho_{jM}\,\sigma_j}{\sigma_M}$$

where Cov = covariance between the return on security *j* and the return on market portfolio *M*. It is equal to:

$$\sum_{i=1}^{n} (R_{jt} - \bar{R}_j)(R_{Mt} - \bar{R}_M)/(n-1)$$

CALCULATION OF BETA

Historical Market Data

Year	R_{jt}	R_{Mt}	$R_{jt}-\bar{R}_j$	$R_{Mt}-\bar{R}_M$	$(R_{jt}-\bar{R}_j)(R_{Mt}-\bar{R}_M)$	$(R_{Mt}-\bar{R}_M)^2$
1	10	12	-2	-1	2	1
2	6	5	-6	-8	48	64
3	13	18	1	5	5	25
4	-4	-8	-16	-21	336	441
5	13	10	1	-3	-3	9
6	14	16	2	3	6	9
7	4	7	-8	-6	48	36
8	18	15	6	2	12	4
9	24	30	12	17	204	289
10	22	25	10	12	120	144

$\Sigma R_{jt} = 120$ $\Sigma R_{Mt} = 130$

$\bar{R}_j = 12$ $\bar{R}_M = 13$

$\Sigma (R_{jt} - \bar{R}j)(R_{Mt} - \bar{R}_M) = 778$ $\Sigma(R_{Mt} - \bar{R}_M)^2 = 1022$

$Cov\ (R_{jt}, R_{Mt}) = 778/9 = 86.4$ $\sigma^2_M = 1022/9 = 113.6$

$$\text{Beta : } \beta j = \frac{Cov\ (R_{jt}, R_{Mt})}{\sigma^2_M} = \frac{86.4}{113.6} = 0.76$$

$$\text{Alpha : } a_j = \bar{R}_j - \beta_j \bar{R}_M = 12 - (0.76)(13) = 2.12\%$$

RECAPITULATION OF THE STORY SO FAR

- Securities are risky because their returns are variable.
- The most commonly used measure of risk or variability in finance is standard deviation.
- The risk of a security can be split into two parts: unique risk and market risk.
- Unique risk stems from firm-specific factors, whereas market risk emanates from economy-wide factors.
- Portfolio diversification washes away unique risk, but not market risk. Hence, the risk of a fully diversified portfolio is its market risk.
- The contribution of a security to the risk of a fully diversified portfolio is measured by its beta, which reflects its sensitivity to the general market movements.

IMPLICATIONS

- Diversification is important. Owning a portfolio dominated by a small number of stocks is a risky proposition.
- While diversification is desirable , an excess of it is not. There is hardly any gain in extending diversification beyond 10 to 12 stocks.
- The performance of well –diversified portfolio more or less mirrors the performance of the market as a whole.
- In a well ordered market, investors are compensated primarily for bearing market risk, but not unique risk. To earn a higher expected rate on return, one has to bear a higher degree of market risk.

SUMMARY

- Risk is present in virtually every decision. Assessing risk and incorporating the same in the final decision is an integral part of financial analysis.
- The **rate of return** on an asset for a given period (usually a period of one year) is defined as follows:

$$\text{Rate of return} = \frac{\text{Annual income} + (\text{Ending price} - \text{Beginning price})}{\text{Beginning price}}$$

- Based on the probability distribution of the rate of return, two key parameters may be computed: expected rate of return and standard deviation.
- The **expected rate of return** is the weighted average of all possible returns multiplied by their respective probabilities. In symbols,

$$\mathrm{E}(R) = \Sigma\, p_i\, R_i$$

SUMMARY

- Risk refers to the dispersion of a variable. It is commonly measured by the variance or the standard deviation.
- The **variance** of a probability distribution is the sum of the squares of the deviations of actual returns from the expected return, weighted by the associated probabilities. In symbols,

$$\sigma^2 = \Sigma\, p_i\, (R_i - R)^2$$

- **Standard deviation** is the square root of variance.
- The expected return on a portfolio is simply the weighted average of the expected returns on the assets comprising the portfolio. In general, when the portfolio consists of n securities, its expected return is:

$$E(R_{p)} = \Sigma w_i\, \mathrm{E}(\mathrm{R}_\mathrm{i})$$

- If returns on securities do not move in perfect lockstep, diversification reduces risk.

SUMMARY

- As more and more securities are added to a portfolio, its risk decreases, but at a decreasing rate. The bulk of the benefit of **diversification** is achieved by forming a portfolio of about 10 securities.
- The following relationship represents a basic insight of **modern portfolio theory**:

  Total risk = Unique risk + Market risk
- The **unique risk** of a security represents that portion of its total risk which stems from firm-specific factors. It can be washed away by combining it with other securities. Hence, unique risk is also referred to as diversifiable risk or **unsystematic risk.**
- The **market risk** of a security represents that portion of its risk which is attributable to economy-wide factors. It is also referred to as **systematic risk** (as it affects all securities) or non-diversifiable risk (as it cannot be diversified away).
- The market risk of a security reflects its sensitivity to market movements. It is called **beta.**