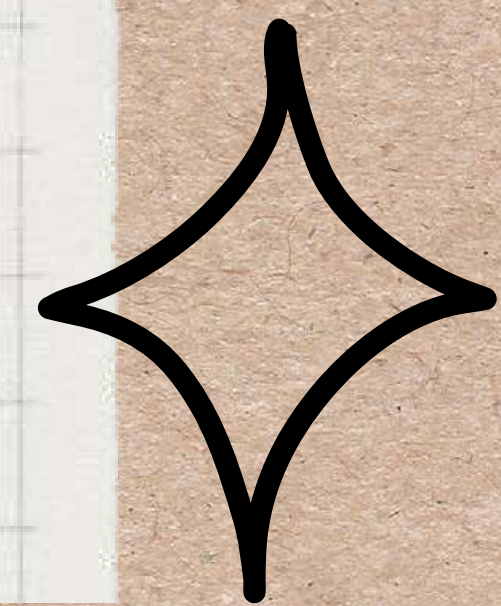
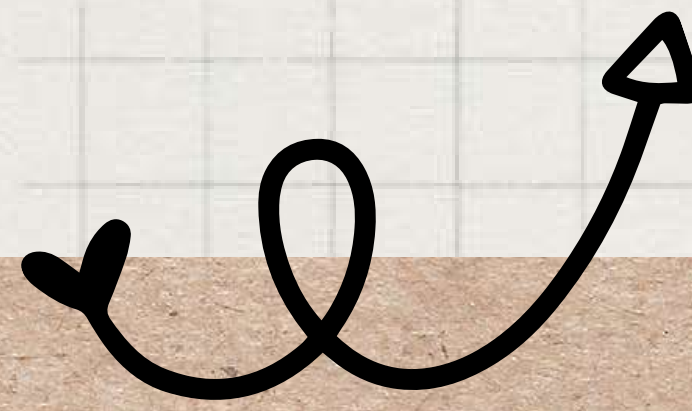




GAME THEORY

Kunal SINGH





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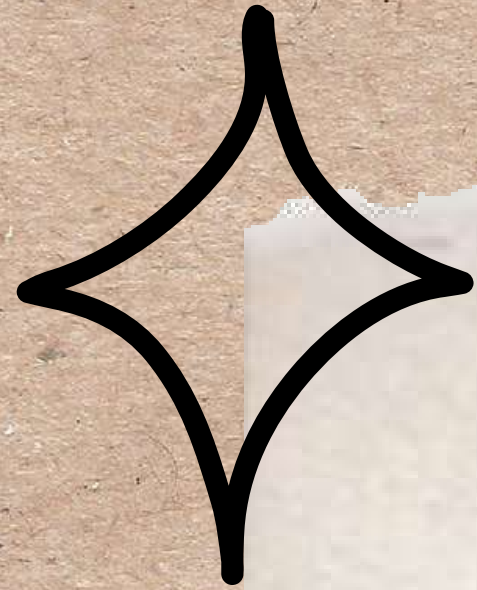
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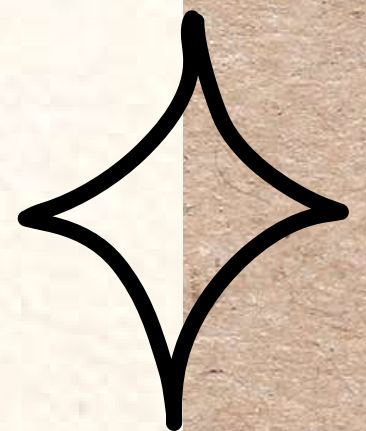
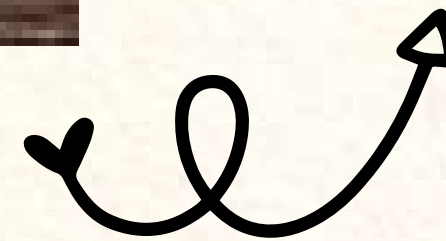


ABOUT



*This Project involves
experimentation of Game
theory.*

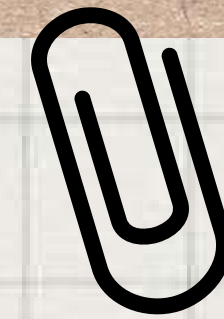
*And analysis of different
auction types.*





WHAT IS AN AUCTION?

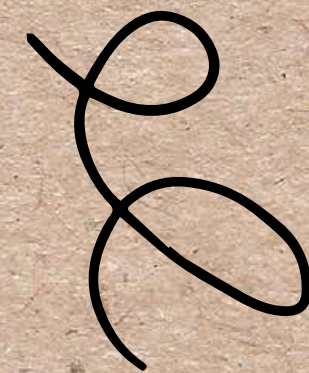
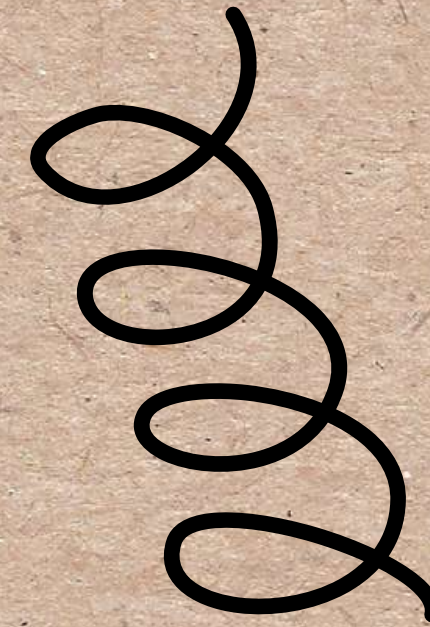
An auction is a sales event wherein potential buyers place competitive bids on assets or services either in an open or closed format



THE TYPES

1. First price auction, Sealed-Bid Auction

2. The second-Price, Sealed-Bid (Vickrey) Auction.

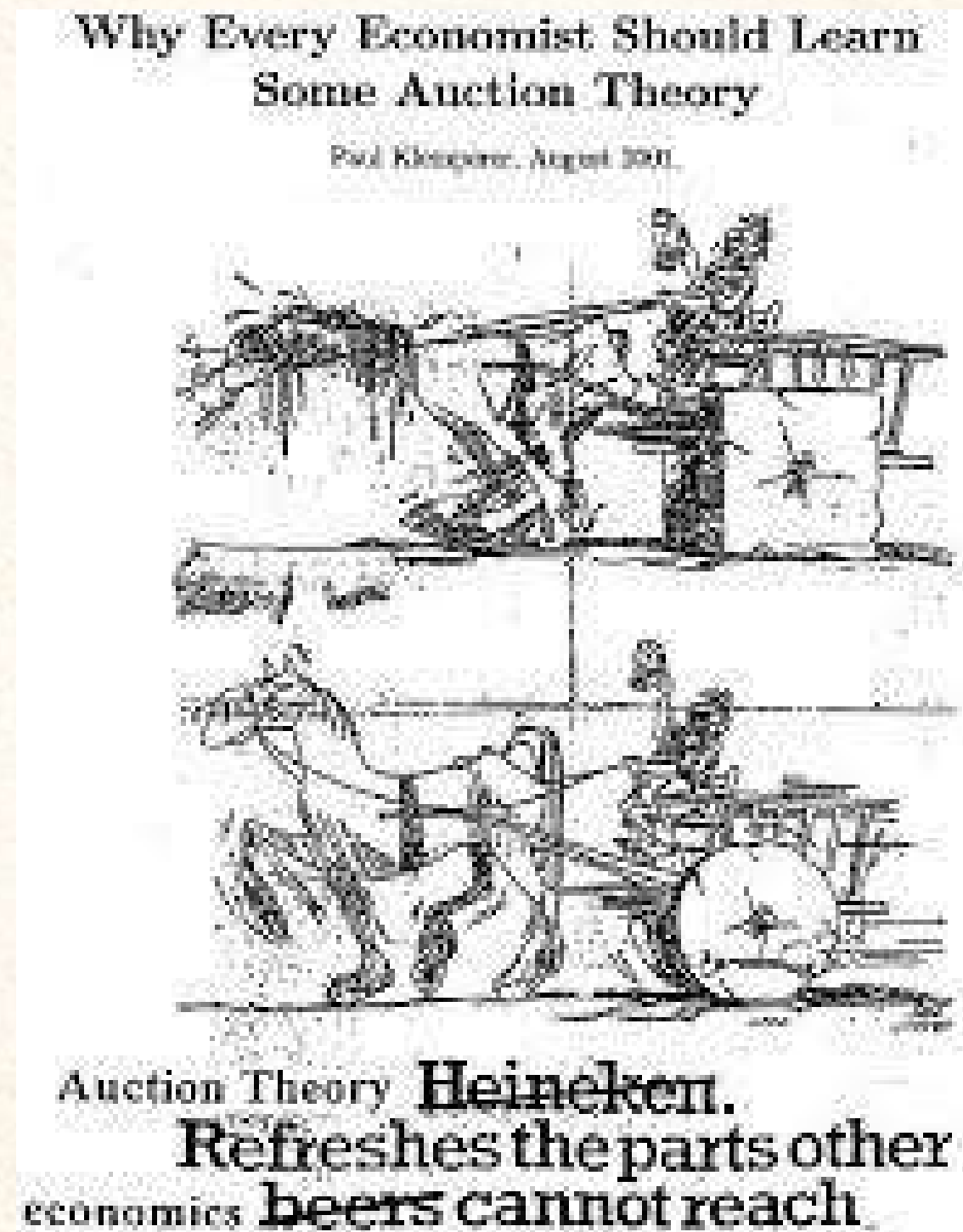


AUCTION MODEL & ASSUMPTIONS

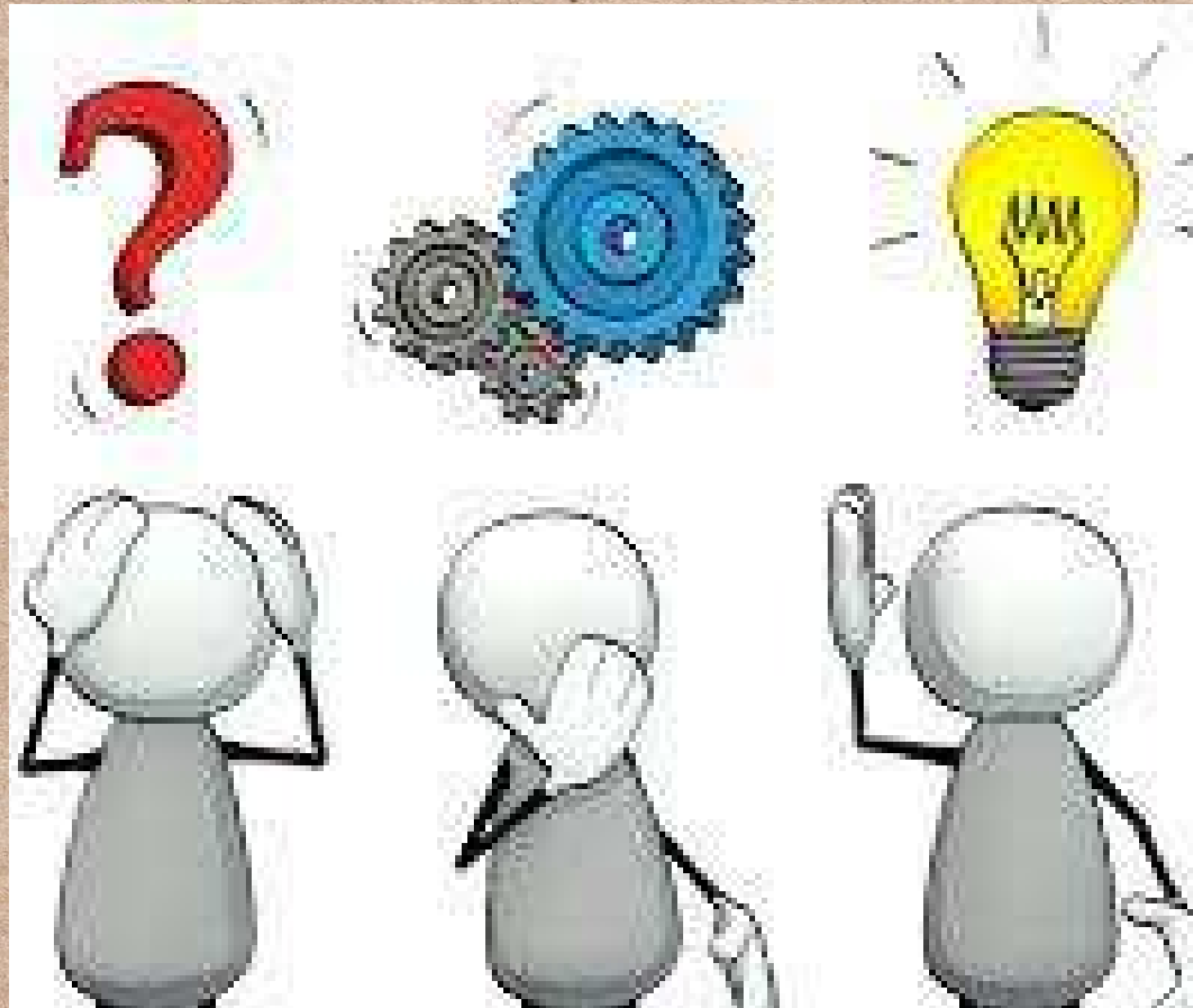
As per Game theory every player want to maximize their payoffs and would like to choose best response against other palyers strategies.

The basic rules of every game are:

- game include players
- strategies for every player
- payoffs.



Assumptions



- Each bidder knows his valuation and no of participants, but not the valuation of others.
- Each buyer knows Upper and lower signal of object.
- The bidder would want to maximize their payoffs given the expected payoff of other bidders.



MODEL & MATHS

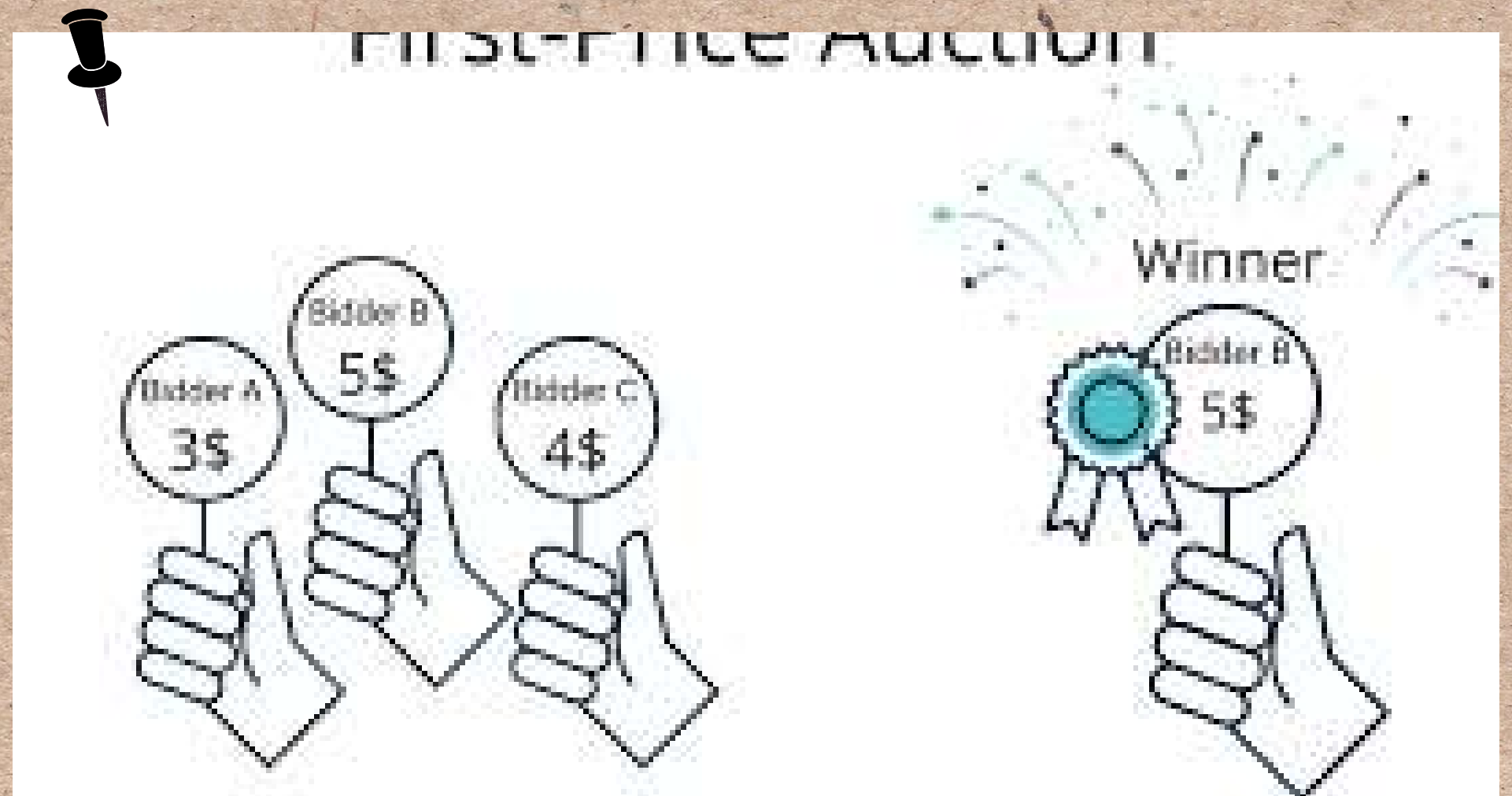


First-Price, Sealed-Bid Auction

Bidders simultaneously submit sealed bids

The highest bidder wins and pays his bid.

All others pay nothing.



Mathematics

Bidders submit b_1, b_2, \dots, b_n .

We solve for symmetric Bayesian Nash equilibrium which means a strategy profile that maximizes the expected payoff for each player given their beliefs and given the strategies played by the other players.

Assume that $b(\cdot)$ is strictly increasing, continuous and differentiable.

Assume bidders $j \neq i$ has identical strategy $b_j = b(S_j)$.

Bidders i 's expected payoff

$$U_i(b_i, b_{-i}, S_i) = (S_i - b_i) * \Pr[b_j = b(S_j) \leq b_i, \forall j \neq i]$$



MODEL & MATHS



Second-Price, Sealed-Bid (Vickrey) Auction

Bidders simultaneously submit sealed bids b_1, b_2, \dots, b_n . The highest bidder wins and pays the second highest bid. All others pay nothing.

These auctions are called Vickrey auctions in honor of William Vickrey, who wrote the first game-theoretic analysis of auctions.

Bidding truthfully is optimal in the Vickrey auction!

- What should a bidder with value v bid?

Option 1: Win the item at price b , get utility $v - b$

Option 2: Lose the item, get utility 0

Would like to win if and only if $v - b > 0$ – but bidding truthfully accomplishes this!

We say the Vickrey auction is **strategy-proof**

Mathematics

In a second-price auction, it is a weakly dominant strategy to bid one's value $V(S_i)=S_i$.


In a second-price auction, it is a weakly dominant strategy to bid one's value $V(S_i)=S_i$.

$b_i \in \mathbb{R}^+$ $S_i \in [S, \bar{S}]$ where S is lower signal of object and \bar{S} is upper signal of object.

$$U_i(b_i, b_{-i}, S_i) = \begin{cases} V(S_i) - \max_{j \neq i} b_j, & \text{if } b_i > \max_{j \neq i} b_j \\ 0, & \text{otherwise} \end{cases}$$

U_i = utility of player i .





Game 1:

Average out payoff



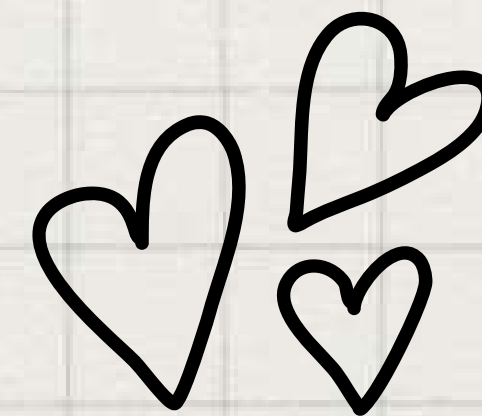


Explanation of the game

"Best of X" in this game the bidder who wins most rounds out of X rounds is declared winner. The object had upper signal and every player had random fund allotted.

He has to pay sum equal to average of his bids in X rounds.

If there is tie in Xth round winner decided by random toss and if after Xth round there is a tie, the winner is decided by whose payoffs are less hence maximizing profit for auction house.

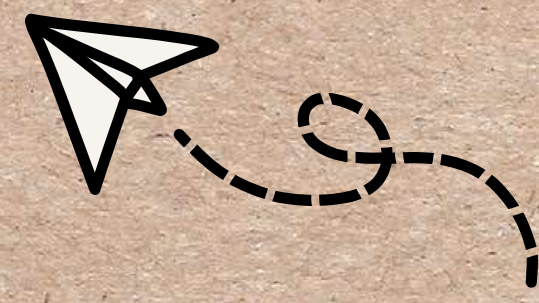


Details of Game

- ▶No of times game played :4
- ▶Sample size of participants:3
- ▶Sample size total:12

| Round/ Player | A | B | C |
|------------------|-----|-----|-----|
| 1 | 120 | 150 | 50 |
| 2 | 180 | 150 | 50 |
| 3 | 160 | 250 | 210 |
| 4 | 160 | 50 | 250 |
| 5 | 50 | 50 | 150 |

Player C won because his average is highest.



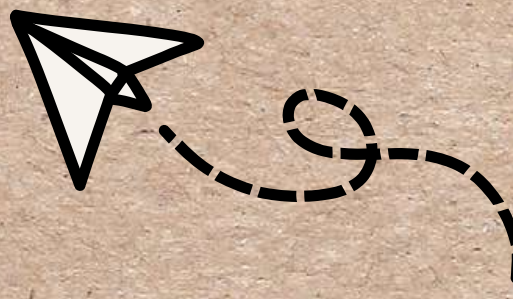


Outcomes

In first price sealed bid auction each buyer's bid depended upon max signal of object and their true valuation.

The winner in the game didn't depended on their valuation as they tried to maximize their payoffs.

Most of the results were in conclusion of were in line with modern auction theory though there were some deviation.





LINK TO ALL DATSETS





Game 2:

Reach the target

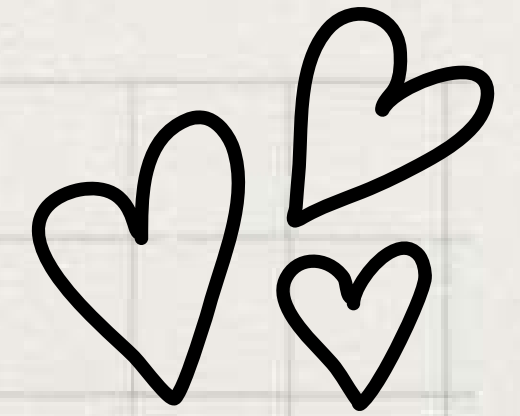


Explanation of the game

"In this game in each round a random no is chosen for a player and if a sum of those number for any player crosses a pre-decided number the game ends."

RULES:

- Highest bidder gets to choose number.
- If in a round 2 players having highest bid are same then winner decided at random capital alphabet.
- If in a round 2nd highest bid are same then winner decided by random small alphabet.
- If winner has lowest average then he will pay lowest signal of object.





Details of Game

- No of times game played :4
- Sample size of participants:3
- Sample size total:12

ROLLS:

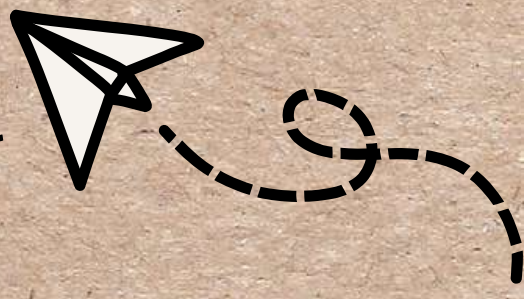
A-4,3,6

B-5,4,6

C-4,3

| Round/ Player | A | B | C |
|------------------|-----|-----|-------|
| 1 | 150 | 150 | 170 |
| 2 | 200 | 200 | 220 |
| 3 | 250 | 250 | 270 |
| Avg | 200 | 200 | 195.5 |

Player C won because he won more number of rounds and has to pay average of second highest in this case lower signal of object.



Outcomes

MOST OF THE RESULTS WERE IN LINE WITH THE SECOND PRICE AUCTION THEORY In a Vickrey auction, the individual is bidding their true value and are not trying to assess what OTHER is going to bid.

Therefore, in a Vickrey auction, the individual is bidding the maximum amount they are willing to pay and are not disadvantaged by it.

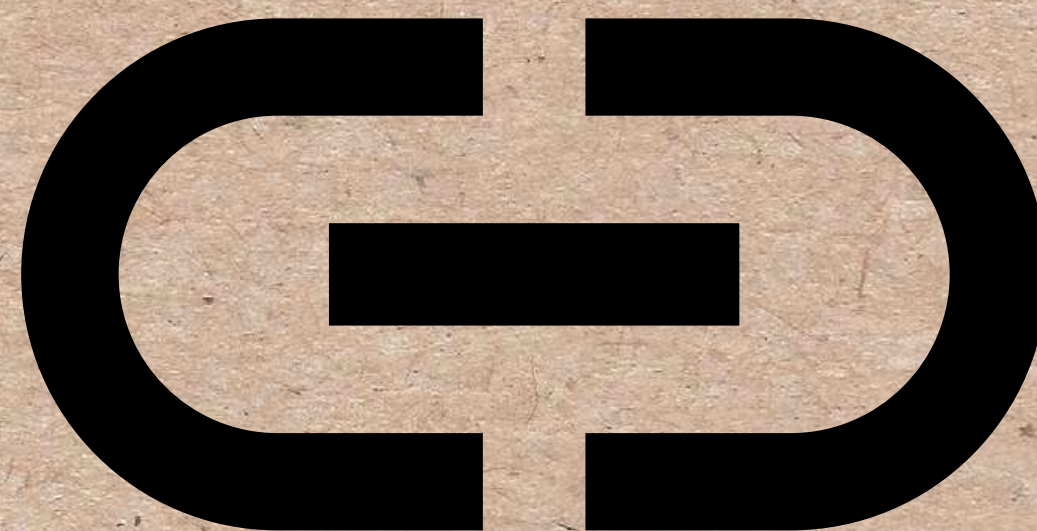
THERE WERE STILL SOME DEVIATIONS, ONE OF THEM WAS the winner has to Pay an amount equalling the average of all the bids placed by the other bidder



OUTCOMES



LINK TO ALL DATSETS

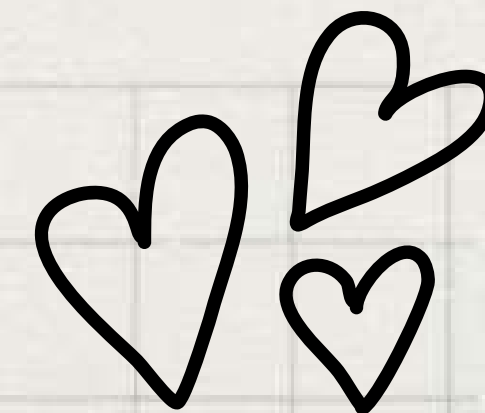


Link to toss generator and
winner decider





Behavioral Analysis



Thus far, we have assumed that bidders' values for the item being auctioned are independent. Each bidder knows his/her own value for the item and is not concerned with how much it is worth to anyone else.

A purely superficial comparison of the first-price and second-price sealed-bid auctions might suggest that the seller would get more money for the item if he ran a first price auction.

After all, he'll get paid the highest bid rather than the second-highest bid.



FINANCE & ECONOMICS CLUB

