Real-time Energy Markets

1 Introduction

Real-time energy markets are operated every five minutes. A linear optimization problem is solved to determine lowest-cost operation of a power system (a 'power grid') so as to satisfy demands.

In this project we will study some economic problems using a representation of power systems that ignores, simplifies or approximates several real-world features.

This power system consists of a *network* where *buses* (i.e. nodes) are connected by *branches* (i.e. power lines). Power is produced by *generators* that are located at some of the buses.

- **Generators.** Each generator g, in this project, is described by two quantities: its maximum output, pmax_g and its per unit cost, σ_g. The generator g is located at some bus indicated in the data.

- **Buses.** Each bus i has a *load* (i.e. demand) d_i.

- **Branches.** A branch b connects two buses. By convention, one of the two buses is called the 'from' bus and the other bus is the 'to' bus. However positive power can flow in either direction. The branch b is further described by two physical parameters, x_b (the *reactance*) and u_b (the *branch limit*). Important: 'x' does not mean a variable. **Also important:** there may be several lines that are parallel, i.e. have the same 'from' and 'to' bus as each other.

- **Physical behavior.** The behavior of a power system is fully described once we specify:

  1. The power output of each generator. For a generator g we will denote it by Γ_g
  2. For each bus i a numerical parameter, θ_i (the 'phase angle' at i)

  In particular, given the latter, we can determine the power flow on every branch. Consider a branch b with 'from' bus i and 'to' bus j. The power flow on this line, from i to j, equals

  $$p_b \;=\; \frac{1}{x_b}(\theta_i - \theta_j) \tag{1}$$

  Notice that this quantity can be positive, negative or zero. In other words, our saying that this is power *flowing from i to j* is simply a convention. For safety reasons, a limit on the power flow is imposed:

  $$|p_b| \;\leq\; u_b \tag{2}$$

  Moreover, consider any bus i, and let:

  - $F(i)$ is the set of branches whose 'from' bus is i,
  - $T(i)$ is the set of branches whose 'to' bus is i,
  - $G(i)$ is the set of generators located in bus i

Then, the *power balance equation at i* stipulates the following (explanation follows):

$$\sum_{b \in F(i)} p_b \; - \; \sum_{b \in T(i)} p_b \;\; = \; \textstyle\sum_{g \in G(i)} \Gamma_g \; - \; d_i \tag{3}$$

Let us analyze this equation. On the right hand side of (3), the first term indicates the total generation at generators located in bus i; the second term is the load at i. Hence, the right-hand side is the net amount of power *injected by the bus i into the system.* On the left-hand the first term is power flowing 'away' from i, while the second term is power flowing 'into' i; thus the difference is again the net power being injected into the grid at i. To better understand this equation you should consider a small example where the grid is a triangle and all x values are 1.

See the attached data, for a network with 1814 buses, approximately 2023 branches and 395 generators.

2 Economic dispatch

The nomenclature 'economic dispatch' refers to a linear program that optimally chooses the generation amounts Γ_g and the phase angles θ_i. The objective of the problem is

$$\text{Minimize} \quad \textstyle\sum_g \sigma_g \Gamma_g \tag{4}$$

The constraints are (1) and (2) (for every branch b) and (3) (at every bus i). Additionally, we have the bounds

$$0 \; \leq \; \Gamma_g \; \leq \; \text{pmax}_g, \quad \text{for all generators } g \tag{5}$$

The variables θ_i are free (can be positive or negative) and unbounded. The variables p_b can be of any sign.

2.1 First enhancement

As stated, the problem can be infeasible. In fact in this study we will consider scenarios where data changes (in particular wind variability) causes the problem to become infeasible. To handle that, constraint (3) is modified as follows:

$$\sum_{b \in F(i)} p_b \; - \; \sum_{b \in T(i)} p_b \;\; = \; \textstyle\sum_{g \in G(i)} \Gamma_g \; - \; (d_i - S_i) \tag{6}$$

Here,

$$0 \leq S_i \leq d_i \tag{7}$$

is a new variable; it models demand that is being 'lost' or 'shed'. This quantity should be kept as small as possible, accordingly, we add to the objective (4) a penalty term:

$$\text{Minimize} \quad \textstyle\sum_g \sigma_g \Gamma_g \; + 10^6 \sum_i S_i \tag{8}$$

3 The project

The first component of the project asks you to read the provided data, formulate and solve the LP with objective (8) and constraints (1), (2), (5), (6) and (7).

The second component will investigate the way that grid economics works, i.e. how prices are extracted from this optimization problem and charged to customers.

3.1 Second task: pricing of demand

Suppose we solve the above LP. The way that power grid economics works (in a somewhat simplified manner) is that the public, indirectly, pays

$$\mathbf{cost} = \quad \sum_i \pi_i^* (d_i - S_i) \tag{9}$$

where the sum is over all the buses, and for each bus i,

$$\pi_i^* \quad = \textbf{ -} \text{ Optimal dual variable for constraint (6).}$$

In this part of the project we study the impact **on cost** of wind variability. To that effect,

1. Assume that the 'pmax' quantity for 'wind' generators (indicated as such in 'generators.csv') is in fact the *expectation* of power.

2. Assume that the output of wind generators is multivariate Gaussian, with mean as just indicated, and covariance as per the matrix 'covariance.txt'.

3. Repeat the following experiment at least 1000 times: sample output for the wind farms as just indicated (please remember that output cannot be negative). Solve the resulting LP (you may need to ask me how to change the LP, perhaps). Then compute the cost as indicated above in formula (9).

4. What is the distribution of cost? Is it Gaussian? Discuss.

3.2 Extra credit

(a) The public cost defined above becomes a random variable under the distribution I asked you to simulate. **Estimate the value-at-risk of this quantity at the 95 % level.**

(b) In the default case of the optimization problem introduce above (i.e. with wind output at the original values), it should be the case that some (or many?) generators reach their output limit. This suggests that at least some of these generators are more efficient than others – hence if we could increase their capacity, the public cost would decrease. **Investigate this possibility: double the capacity of every non-wind generator whose output is at the output limit as originally stated before part (a) – does the VAR decrease? By how much?**

(c) Extra-extra credit. Here we return to the original setup with objective (8) and constraints (1), (2), (5), (6) and (7). Suppose that for every non-wind generator g, we have the option to expand its capacity as follows:

- We increase its generation capacity from pmax_g to $2\,\text{pmax}_g$ (double its capacity)
- The cost of this action is $\frac{\sigma_g}{10}\,\text{pmax}_g$

- If the output, Γ_g, of the generator is at most pmax_g the cost is as before: $\sigma_g \Gamma_g$. If the output is between pmax_g and 2pmax_g, the cost is $\sigma_g \text{pmax}_g + 4\sigma_g(\Gamma_g - \text{pmax}_g)^2$.

We are allowed to expand the capacity of at most ten generators.

We are interested in the best choice of such generators! Given a choice of which generators to expand, we evaluate it as follows:

1. First of all, we incur the expansion cost
2. Second, we compute the expectation of the minimum operational cost (8), subject to the original constraints (1), (2), (5), (6) and (7). This expectation is computed (or estimated, rather) over the Gaussian samples for wind output as indicated above.
3. The total we pay is this expectation, plus the expansion cost.

Task: come up with a heuristic to address this problem. You may need to repeatedly solve LPs or quadratic optimization problems, for example.