2 /07/2025

S 6160 ryptology Lectu
Introduction & Logistic

Maria Francis

July 28 2025

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Welcome to Cryptology

Introductory course on Cryptography and Cryptar

It is a theoretical course with a few practical assi

We will cover the basics of modern cryptography
functions, provable security, reductionist argumen
key cryptography and public key cryptography (P

Prerequisites

No prerequisites other than discrete mathematics
required.

But it is good to know basics of probability (e.g:
variable, discrete probability distributions) and co
complexity (e.g: NP, NP completeness, polynomi
reductions)

The number theory required will be covered in cl
a good basic resource: `http://www.hyperellip`
`tanja/teaching/crypto 13/nt.pdf`

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Syllabus

Classic cryptosystems, perfect secrecy, one-way functi
random generators, private and public key cryptograph
resistant hashing, PKI, digital signatures.

Introduction to Cryptanalysis, ttacks on block cipher
search, time-space tradeoffs, differential & linear crypt
meet in the middle.

References

Introduction to Modern ryptography - J. K
Lindell. new textbook with a very modern tak

Graduate Course in pplied Cryptography - D.
Shoup. More practical engineering!

Other References

Lecture notes by

- Shafi Goldwasser and Mihir Bellare (collected in document but slightly old),
- Rafael Pass and bhi Shelat (collected into a si document), and many many more.

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    document but slightly old),
  - Rafael Pass and bhi Shelat (collected into a si
    document), and many many more.
- Explore and utilize all resources -
  - lecture notes from reputed universities and profe
  - video lectures (from reputed sources like univers
    Simons Institute, Institute for dvanced Study, e

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Classes

We meet during B slot, Mon (10 : 00   10 : 55),
(9 : 00   9 : 55) and Thurs (11 : 00   11 : 55).

T s for this course: Reisha   li, Supreet Shukla,
and Kanchan Bisht

Evaluation!!

- Exams - 50 (Previously announced, 2 of them)
- Programming ssignments - 30
- Tutorial Exams – 20

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We will eliminate the lowest 3 tutorial marks in t

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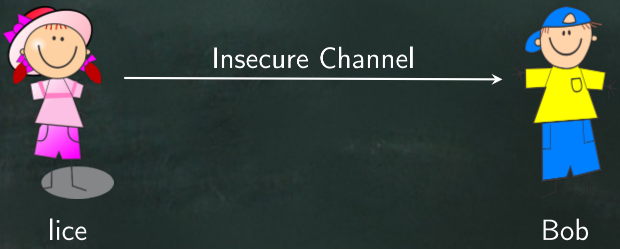

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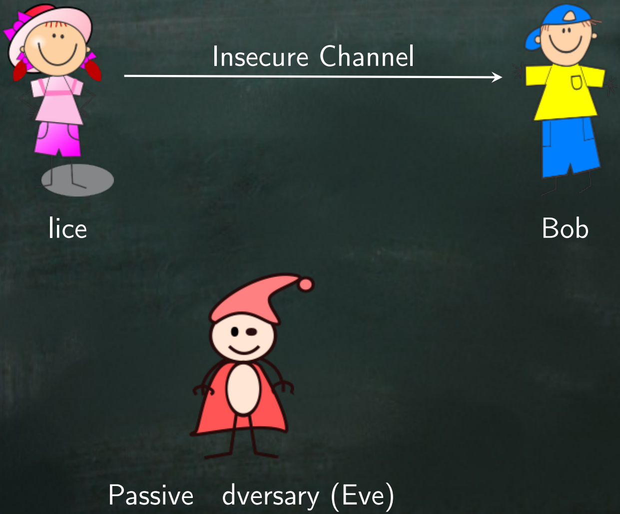

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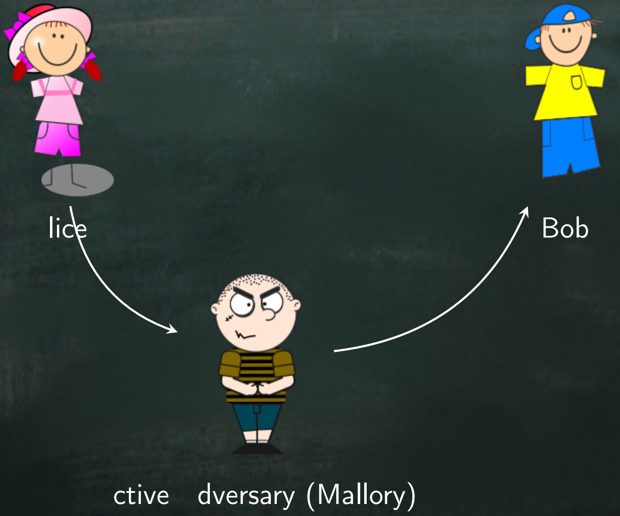

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The need for cryptography

- Communicating or computing over a channel whe adversaries.
- There are information systems(PCs, cellphones, computers, TMs, cars, smart grids, etc.)
- Our aim is to control access to the information – looking to see if we can control who sees and mo information.
- Some examples of such rules :
  - Only XX can read the contents of the file
  - The contents of this file has not been changed a
  - The recipient of this email can authenticate the

Our ims and the Tools at Hand

What we aim to achieve:

- Data Confidentiality
- Data Integrity
- uthentication –
- Non-repudiation – the sender cannot claim that not send it

How do we achieve them? Tools such as:

- Encryption
- Hash Functions
- Digital Signatures
- Zero knowledge Proofs

What about the dversary?

- Could be anybody – insider/outsider
- ssume he knows system design including implen details
- Resources – powerful computers, ability to interc ability to collude with some participants
- Generally generous assumptions about adversary' we assume a computationally bounded adversary.

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Simple Solution

lice

Key k
Encrytion lgorithm *Enc*
Decryption lgorithm *Dec*

ll previously agreed upon

Simple Solution

$$c = Enc_k(m)$$

lice

Bob

Retrieving the mes
$m = Dec_k(c$

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Questions: Which of these are secret here?
How are the secrets agreed upon? Parties may not ev
before?

Formalizing the Solution

Symmetric key encryption (SKE) scheme consists of:

- $\mathcal{M}$: a set of possible plaintexts
- $\mathcal{C}$: a set of possible ciphertexts
- $\mathcal{K}$: a set of possible keys.
- Gen (called the key generation algorithm) is a ra
  algorithm that returns a key k such that $k \in \mathcal{K}$.
  - family of encryption functions, $Enc_k : \mathcal{M} \to \mathcal{C}$
  - family of decryption functions, $Dec_k : \mathcal{C} \to \mathcal{M}$

such that $Dec_k(Enc_k(m)) = m$ for all $m \in \mathcal{M}$ an

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Timeline of Cryptography as a fiel

0

ncient times to 1900s:
Classical
Ciphers

1900s :
Mechanical
Ciphers

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-
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Classical Ciphers

The first idea that comes up when you need secret co

Popular upto 1900s :

- Shift/Ceaser cipher
- Substitution cipher
- Vigenère cipher, etc

S	E	N	D	R	E	I	N	F	O	R	C	E	M	E	N
V	I	G	E	N	E	R	E	V	I	G	E	N	E	R	E
N	M	T	H	E	I	Z	R	A	W	X	G	R	Q	V	R

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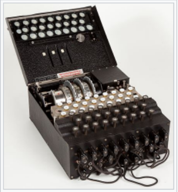
Mechanical Ciphers - Enigma, Pur

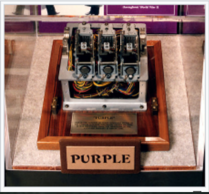
Motor devices

Electromechanical devices

Cryptography performed by (typically, rotor) mac

lan Turing and others at Bletchley Park, Willian
and others in the US – all helped break these ci





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The main idea was security by obscurity.

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Not really! – Kerchoff's principle (1884)

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Dec should be assumed to be public.

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First step to formally defining the security of enc
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If so, then Eve would be able to compute everyth
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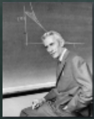
To prevent this we require Gen to be randomized

Perfect Secrecy (C. Shannon (1916



Can we perfect secrecy (information theoretic sec

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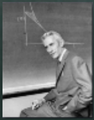

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adversary has infinite computational resources an
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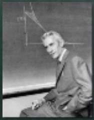

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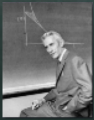

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- Key is perfectly random
- Key is at least as long as the message
- Key is never re-used!

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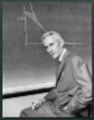

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One Time Pad/ Vernam Cipher - unbreakable!

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One Time Pad - n Overview

$Enc_{k=k_1 k_2 \quad k_n}(m \;\; m_2 \quad m_n) = c \;\; c_2 \quad c_n,$ where $c_i =$

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This is not practical! Why?

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So we make a reasonable assumption – Eve has probabilistic polynomial time (PPT) computing p fair, we restrict lice and Bob to PPT as well.

Computational Complexity Theor

systematic study of what computationally boun
can and cannot do

Started with lan Turing but formal study with C
ward '82), Karp (Turing ward '85) and Blum (
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Notions that are critical in this area – polynomia
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The foundation of modern cryptography.

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and an integer $n_0 \in$ such that

$$|f(n)| \leq \quad |g(n)| \text{ for all } n \geq n_0,$$

and one writes $f = \mathcal{O}(g)$ or $f \in \mathcal{O}(g)$.

Big-O Notation

We say that f has linear, quadratic, cubic or poly
growth in n if $f = \mathcal{O}(n)$, $f = \mathcal{O}(n^2)$, $f = \mathcal{O}(n^3)$
$f = \mathcal{O}(n^k)$, for some $k \in$, respectively.

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Let $f(n) = 100 + \frac{20}{n}$. Set $= 101$ and $n_0 = 1$
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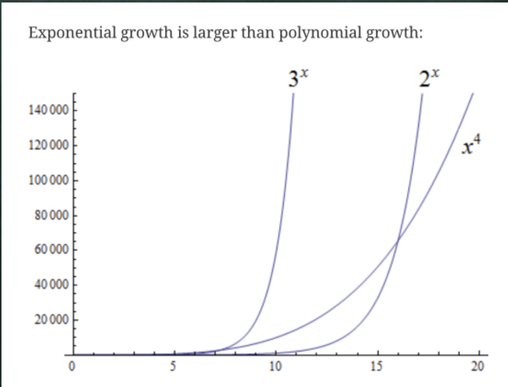
Growth of Functions



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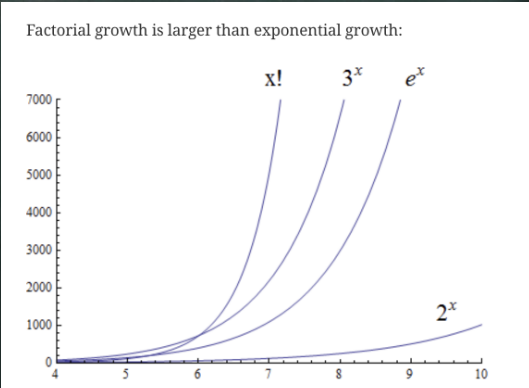

Pic courtesy – `https://ggc-discrete-math.github.io/growth_functi`

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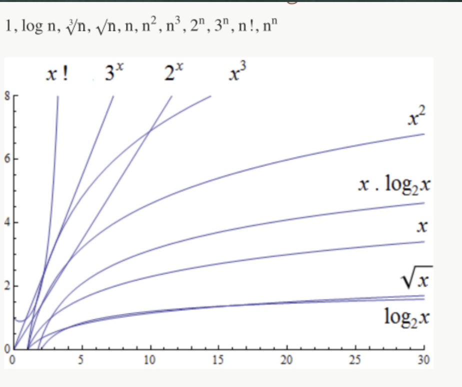

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That is every algorithm solving the problem will h
super-polynomial running time *in the worst case*

P vs NP

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adversaries. More on that later!

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So it requires average complexity of algorithms to that breaking the scheme will be hard for most o i.e. for randomly picked instances.

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$$\forall c > 0,\ lim_{k \to \infty} f(k)k^c = 0$$

Exercises/Tutorials

- Is $2^{n} = \mathcal{O}(2^n)$? Is $2^{2n} = \mathcal{O}(2^n)$?
- Show that $2^{\;k}$, $k^{\;\mathit{logk}}$ are $\mathrm{n\;gl}$ but $1/k^{\;000}$ is
- Let $\mathrm{n\;gl}\;\;\mathrm{n\;gl}_2$ be two negligible functions. Prov
  sum $\mathrm{n\;gl} + \mathrm{n\;gl}_2$ is negligible.
- Negligible function stays negligible even after pol
  $\mathrm{poly}(k) \cdot \mathrm{n\;gl}(k) = \mathrm{n\;gl}(k)$.

Security Parameter

It is a measure of both the resource requirements
cryptographic protocol/algorithm as well as the p
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- Increasing k we get higher security but a degrade
What is the correct value of k?

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1970s - Public Key Revolution

Merkle, and independently Hellman and Diffie, in
notion of public-key cryptography.

In November 1976, Diffie and Hellman published
Directions in Cryptography, proclaiming *We are a*
a revolution in cryptography.

PKE - Diffie/Hellman Ideas

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Digital Signatures – sign with SK and verify wi

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Public Key Encryption

c

lice

B

Encrypting
using Bob's public key:
$c = Enc_{PK_B}(m)$

Decry
using Bob's
$m = De$

Digital Signatures using Public Ke

(m, s)

Bob

Signing
using Bob's private key:
$s = Sign_{SK_B}(m)$

Ver
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$Verify_P$

Public Key Encryption – RS

How to implement Diffie/Hellman ideas? – Rivest, Sh dleman in 1977 came with the RS algorithm.

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dleman in 1977 came with the RS algorithm.

Note: In 1999, it was revealed that Ellis, Cocks and W
invented PKC in the British secret service, before thei
outside.

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Idea : reduce the problem of finding the secret ke
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Public Key Encryption – RS

- Idea : reduce the problem of finding the secret ke
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- RS security relies (in part) on the conjectured h
factoring n, a product of two very large primes p
- What we look for in any PKC are one-way functi
trapdoor:
  - Function f must be invertible so as to decrypt e
messages.
  - Efficient to encrypt
  - Difficult to invert so that Eve cannot compute m
  - Has a trapdoor: given some information (SK)
easy given $f(m)$.

PKC revolution

Huge implications - exponential rise in study and
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- More Turing award winners in theoretical crypto
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- More interesting ideas like for e.g:
  - secret sharing : collaboration between distrustin
  - zero knowledge proofs (ZKPs) : revealing nothi
validity of the statement
  - fully homomorphic encryption : computation on
- I CR - Intl. ssn. for Crypto Research. Sponsors
crypto conferences – Crypto, Eurocrypt and sia

Principles of Modern Cryptograph

The aim is to move away from art of solving cod
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It helps analyze and evaluate the scheme and son
security too!

Formal Definitions - n example

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What is prior knowledge? What is leak?

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ssumptions give us a way of comparing two sch
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If an assumption is broken (like factoring on a qu
computer) the schemes built on the assumption (
ECDS) will break!

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Pegasus shows us that well-resourced targeted at
impossible to prevent! Read :
`https://blog.cryptographyengineering.co`
`20/a-case-against-security-nihilism/.`

The future of the field

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- ttacks on different cryptographic primitives like
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nd of course, there is cryptanalysis!

Question for Tutorial

If $f(n)$ is non-negligible and $g(n)$ is negligible, then $h(n) = f(n) \quad g(n)$ is non-negligible.