Nonlinear Dynamical Systems: Lecture 11

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The differential equation becomes:

$$\ddot{x} + \epsilon(1 - x^2)\dot{x} + x = 0$$

Example

Consider parallel RLC circuit where L > 0, C > 0 resistive element is an active circuit with v-i characteristic i=h(v).
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....Continued

Applying KCL

$$i_C + i_L + i_R = 0.$$

Hence

$$C\frac{dv}{dt} + \frac{1}{L}\int v(t)dt + h(v) = 0$$

Differentiating once and dividing by CL

$$\frac{d^2v}{dt^2} + \frac{v}{LC} + \frac{h'(v)}{C}\frac{dv}{dt} = 0$$

....Continued

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2. When $|v| > 1$ damping is positive and energy is dissipated.

Therefore the dynamics of the circuit are restricted in some area. For a given initial condition(eg.voltage of C),at some point of time in a cycle the energy that is fed in will be equal to the energy dissipated.At that time there will be stable oscillations or the trajectory will be a stable limit cycle.