

# MA 214: Introduction to Numerical Analysis, Spring 2014

## Quiz 1 - 7th February 2014, 8:10 P. M. - 9:00 P. M. (D)

### Solution

1. Let

$$x_0 = 1, x_1 = \frac{4}{3}, x_2 = \frac{5}{3}, x_3 = 2$$

and for  $i = 0, 1, 2, 3$ , let  $\ell_i(x)$  be the Lagrange interpolation polynomial of degree 3 such that

$$\ell_i(x_i) = 1, \ell_i(x_j) = 0, \text{ for } i \neq j.$$

Evaluate

$$l_0\left(\frac{3}{2}\right) + l_1\left(\frac{3}{2}\right) + l_2\left(\frac{3}{2}\right) + l_3\left(\frac{3}{2}\right).$$

(1 mark)

**Ans.:** 1.

2. Let  $f : [0, 7] \rightarrow \mathbb{R}$  be such that

$$f(0) = 0, f(1) = 1, f(3) = 9, f(7) = 217.$$

Find

(a) a polynomial of degree  $\leq 2$  which interpolates  $f$  at  $0, 1, 3$ ,

$$\mathbf{Ans.:} \quad x + x(x - 1).$$

(b) a polynomial of degree  $\leq 3$  which interpolates  $f$  at  $0, 1, 3, 7$ .

$$\mathbf{Ans.:} \quad x + x(x - 1) + x(x - 1)(x - 3).$$

(1+1 marks)

3. Let

$$f(x) = 250x^4 + 44x^3 - 13x^2 + 47x + 23.$$

Find the divided difference  $f[1 \ 2 \ 3 \ 4 \ 5]$ . (1 mark)

**Ans.:** 250.

4. Let  $f(x) = \frac{1}{x^2}$ ,  $x \in [1, 3]$  and  $p_2(x)$  be the quadratic polynomial which interpolates  $f$  at 1, 2, 3. Find the best possible upper bound for  $\|f - p_2\|_\infty = \max_{x \in [1, 3]} |f(x) - p_2(x)|$ . (2 marks)

**Ans.:**  $\frac{8}{3\sqrt{3}}$ .

5. Let  $f : [0, 1] \rightarrow \mathbb{R}$  be such that

$$f(0) = 2, \ f'(0) = 2, \ f(1) = 6, \ f'(1) = 8, \ f(2) = 30,$$

where  $f'(x)$  denotes the derivative of  $f$  at  $x$ . Find

(a) the cubic polynomial which interpolates  $f$  and  $f'$  at 0 and at 1.

**Ans.:**  $2 + 2x + 2x^2(x - 1)$ .

(b) the polynomial of degree  $\leq 4$  which interpolates  $f$  at 0, 1, 2 and  $f'$  at 0 and at 1.

**Ans.:**  $2 + 2x + 2x^2(x - 1) + 2x^2(x - 1)^2$ .

( 1 + 1 marks)

6. Evaluate

$$\int_0^6 [2(x-1)(x-3)(x-6) + 3x(x-3)(x-6) + 5x(x-2)(x-5)] dx.$$

(2 marks)

**Ans.:** -36.