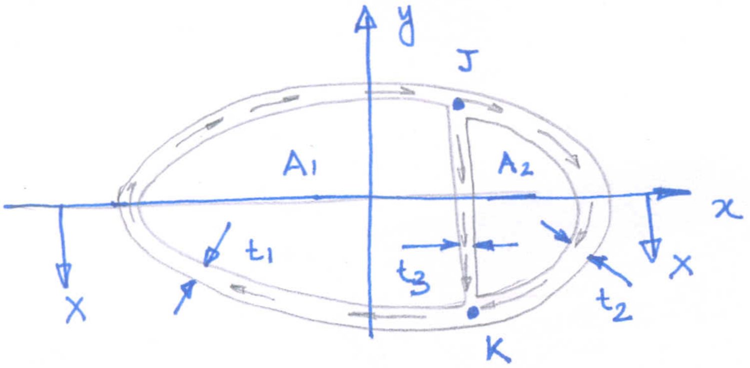
Torsion of thin hollow shaft (multiple cells)



Shear flow:

$q = \tau t$ → thickness

q: shear flow; τ: shear stress

(i) No change happens in shear flow between two junctions (e.g. K, J)

(iii) At any junction, say K

$$\tau_3 t_3 + \tau_2 t_2 = \tau_1 t_1$$

$$q_3 + q_2 = q_1$$

i.e. total input shear flow = total output shear flow

Section X-X of the deformed membrane

$$\tau_1 = \frac{h_1}{t_1}, \quad \tau_2 = \frac{h_2}{t_2} \quad \text{and} \quad \tau_3 = \frac{h_1 - h_2}{t_3} = \frac{\tau_1 t_1 - \tau_2 t_2}{t_3}$$

$$\Rightarrow \boxed{\underbrace{\tau_3 t_3}_{q_3} = \underbrace{\tau_1 t_1}_{q_1} - \underbrace{\tau_2 t_2}_{q_2}} \Rightarrow \boxed{q_3 + q_2 = q_1} \quad \text{——(i)}$$

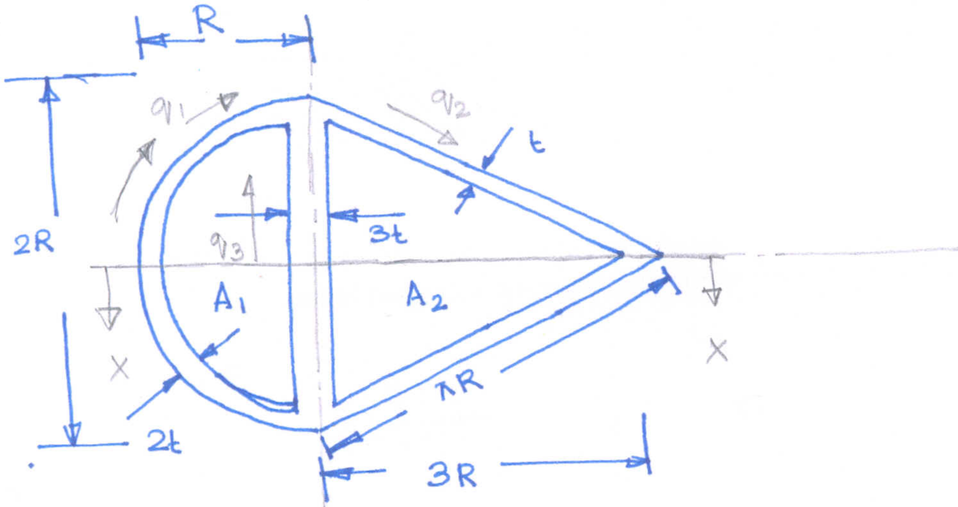
Torque = 2 × vol of the deformed membrane

$$T = 2(A_1 h_1 + A_2 h_2) = 2(A_1 \tau_1 t_1 + A_2 \tau_2 t_2) \quad \text{——(ii)}$$

$$\int \tau ds = 2GA\theta \Rightarrow \tau_1 S_1 + \tau_3 S_3 = 2G\theta A_1$$

$$\text{and} \quad \tau_2 S_2 - \tau_3 S_3 = 2G\theta A_2 \quad \text{——(iii)}$$

Solving Eqns. (i), (ii) and (iii), T can be expressed in terms of $G\theta$ and τ_1, τ_2 & τ_3 can be obtained in terms of $G\theta$ or T.



X–X

$$\tau_1 = \frac{h_1}{2t} \qquad \tau_2 = \frac{h_2}{t} \qquad \tau_3 = \frac{h_2 - h_1}{3t} = \frac{\tau_2 t - 2\tau_1 t}{3t}$$

$$\Rightarrow \boxed{3\tau_3 = \tau_2 - 2\tau_1} \quad \text{—(i)} \qquad \& \quad q_3 = q_2 - q_1$$

$$\Rightarrow \boxed{q_1 + q_3 = q_2}$$

$$T = 2(A_1h_1 + \cancel{2}A_2h_2) = 2\left[\frac{\pi R^2}{2}\tau_1 2\times t + \frac{1}{2}3R\times 2R\times\tau_2\times t\right]$$

$$= \left[2\pi R^2 t\tau_1 + 6R^2 t\tau_2\right] \quad \text{—(ii)}$$

$$\int \tau\, ds = 2GA\theta \Rightarrow \tau_1 S_1 - \tau_3 S_3 = 2G\theta A_1$$

$$\Rightarrow \tau_1 \times \pi R - \tau_3 \times 2R = 2G\theta \frac{\pi R^2}{2}$$

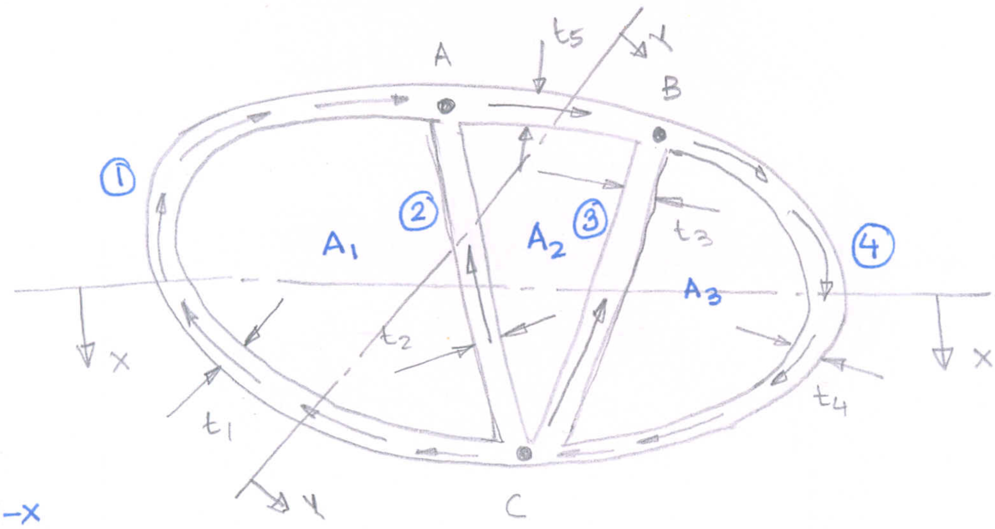
$$\Rightarrow \boxed{\pi\tau_1 - 2\tau_3 = G\theta\pi R} \quad \text{—(iii)}$$

$$\tau_2 S_2 + \tau_3 S_3 = 2G\theta A_2$$

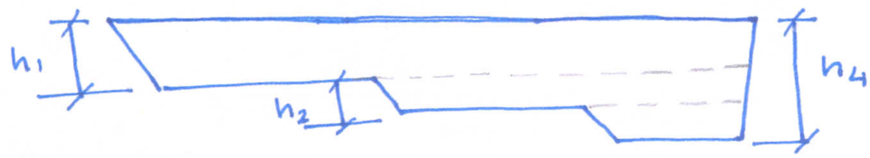
$$\Rightarrow \tau_2 \times 2\pi R + 2R\tau_3 = 2G\theta \times \frac{1}{2} 3R\times 2R$$

$$\Rightarrow \boxed{\pi\tau_2 + \tau_3 = 3G\theta R} \quad \text{—(iv)}$$

Solving Eqns. (i), (iii) and (iv) τ_1, τ_2 & τ_3 can be obtained in terms of $G\theta$.
and Eqn (ii) gives T in terms of $G\theta$.



X–X

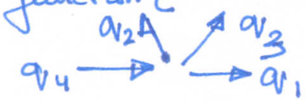


$\tau_1 = \dfrac{h_1}{t_1}$ $\quad \tau_2 = \dfrac{h_2}{t_2}$, $\tau_4 = \dfrac{h_4}{t_4}$ $\quad \tau_3 = \dfrac{h_4 - h_2 - h_1}{t_3}$

$\boxed{\tau_3 t_3 = \tau_4 t_4 - \tau_2 t_2 - \tau_1 t_1}$

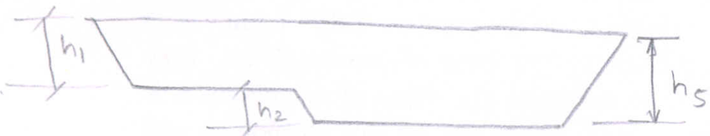
$q_3 = q_4 - q_2 - q_1$

∴ at junction C



$\Rightarrow \boxed{q_3 + q_1 + q_2 = q_4}$ —— (i)

Y–Y



$\tau_5 = \dfrac{h_5}{t_5} = \dfrac{h_1 + h_2}{t_5} \Rightarrow \tau_5 t_5 = \tau_1 t_1 + \tau_2 t_2$

$\Rightarrow \boxed{q_5 = q_1 + q_2}$ —— (ii)

Torque $T = \{2h_1(A_1 + A_2 + \cancel{A_3}) + 2h_2(A_2 + \cancel{A_3}) + 2(h_4 - \cancel{h_2} - \cancel{h_1})A_3\}$

$= 2(h_1 + h_2)A_2 + 2h_1 A_1 + 2h_4 A_3$

$= 2[(h_1 + h_2)A_2 + h_1 A_1 + h_4 A_3]$

$\int \tau ds = 2G\theta A \Rightarrow \tau_1 S_1 - \tau_2 S_2 = 2G\theta A_1$

$\tau_5 S_5 - \tau_3 S_3 + \tau_2 S_2 = 2G\theta A_2$

$\tau_4 S_4 + \tau_3 S_3 = 2G\theta A_3$