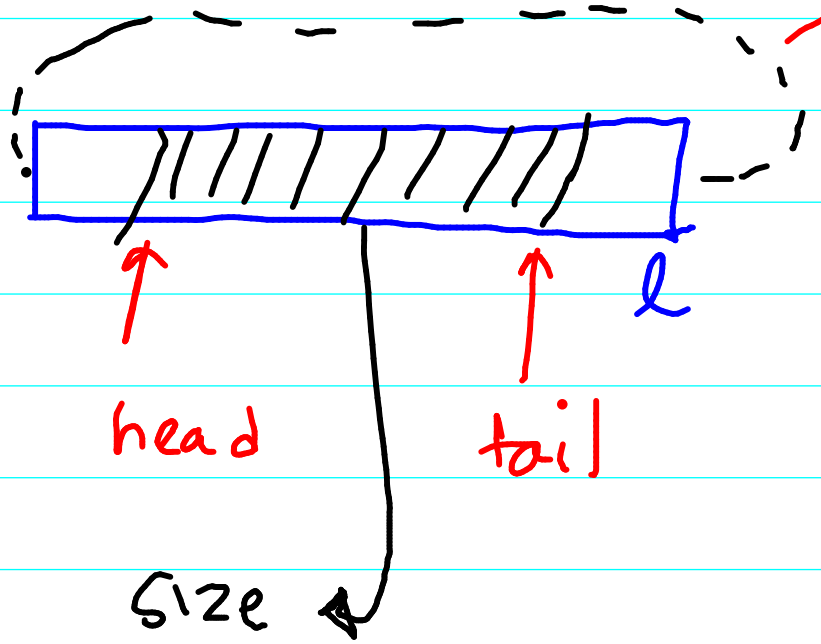
Q: Suppose you want to implement a "list", not as Linked list but as an array of Nodes, i.e

```
Node[ ] nodeList
```

so that I could (a) insert at head
(b) insert at tail
(c) delete at head
(d) delete at tail.

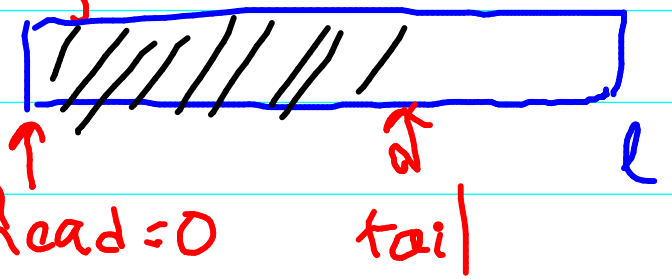
Design a data structure that uses Node[ ] to implement above

One soln




Without this "circularity" you need to shift many elements if head = 0 or tail = l even if size ≠ l

Eg:




Challenge when size = l :

- Create new array of length 2l
- Copy over the elements
- For insertion in middle, more "shifting" work.

# Amortized analysis

Complexity of insertion at the end (tail) in the d.s designed on previous slide, summed up across n insertions, starting with no element in the ArrayList

Without loss assume $n = 2^k$

Doubling when overflow occurs

Time: $1 + 2 + 4 + 8 + \cdots + 2^k + n$

Assuming initialisation with size = 1

$$= 1 * \left(\frac{2^{k+1} - 1}{2 - 1}\right)$$

$$= 2 * n$$

$$= O(n)$$

Cost of insertion w/o overflow

$$\sum_{i=0}^{n} \alpha r^i = \alpha \left(\frac{r^{n+1} - 1}{r - 1}\right)$$

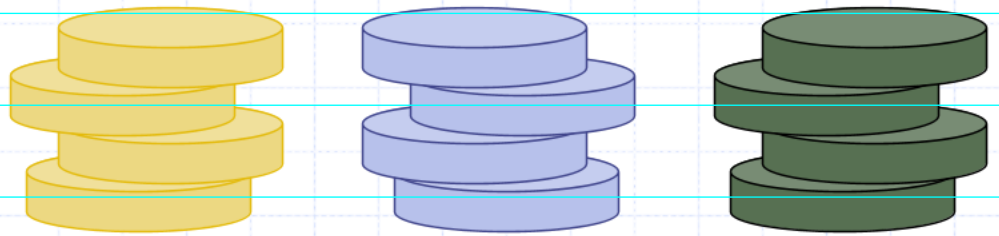
You can do amortized analysis for each of following scenarios separately

(1) $n_1$ insertions followed by $n_2$ deletions

(2) $n_1$ insertions interspersed with $n_2$ deletions

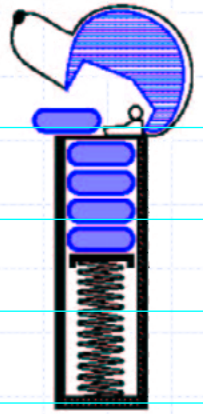
& so on...

# Stacks



FILO
LIFO

# The Stack ADT (§4.2)



- The Stack ADT stores arbitrary objects
- Insertions and deletions follow the last-in first-out scheme
- Think of a spring-loaded plate dispenser
- Main stack operations:
  - push(object): inserts an element
  - object pop(): removes and returns the last inserted element
- Auxiliary stack operations:
  - object top(): returns the last inserted element without removing it
  - integer size(): returns the number of elements stored
  - boolean isEmpty(): indicates whether no elements are stored

# Stack Interface in Java

- Java interface corresponding to our Stack ADT
- Requires the definition of class EmptyStackException
- Different from the built-in Java class java.util.Stack

```
public interface Stack {

  public int size();

  public boolean isEmpty();

  public Object top()
      throws EmptyStackException;

  public void push(Object o);

  public Object pop()
      throws EmptyStackException;
}
```

Like the .h file in C or C++

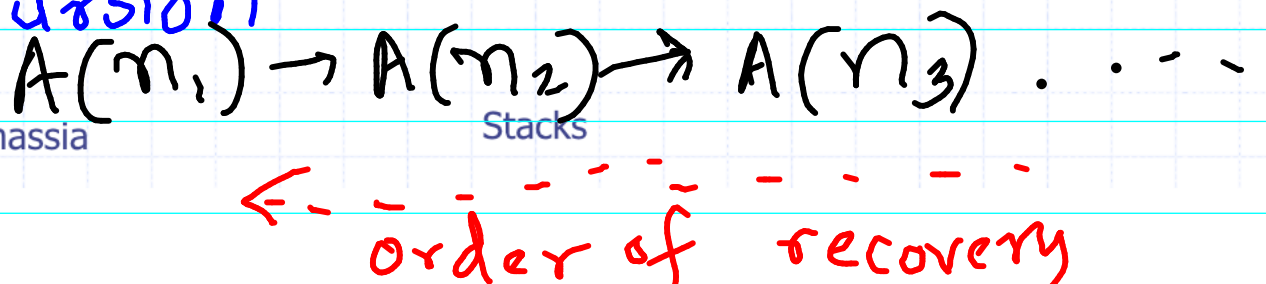
# Exceptions

- Attempting the execution of an operation of ADT may sometimes cause an error condition, called an exception
- Exceptions are said to be "thrown" by an operation that cannot be executed
- In the Stack ADT, operations pop and top cannot be performed if the stack is empty
- Attempting the execution of pop or top on an empty stack throws an EmptyStackException

# Applications of Stacks

- Direct applications
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
  - Chain of method calls in the Java Virtual Machine
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

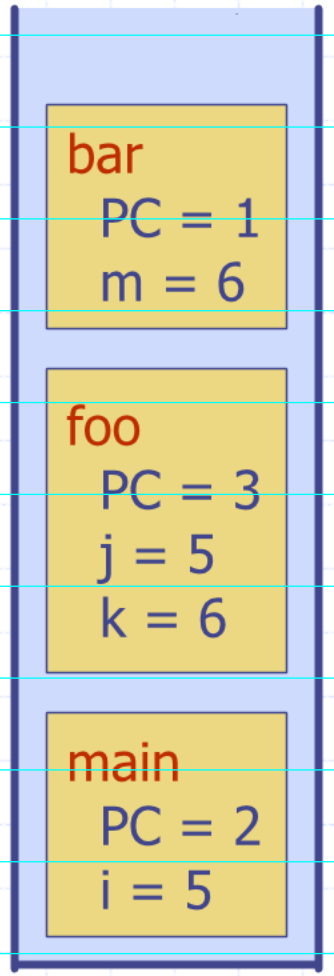
- Recursion



# Method Stack in the JVM

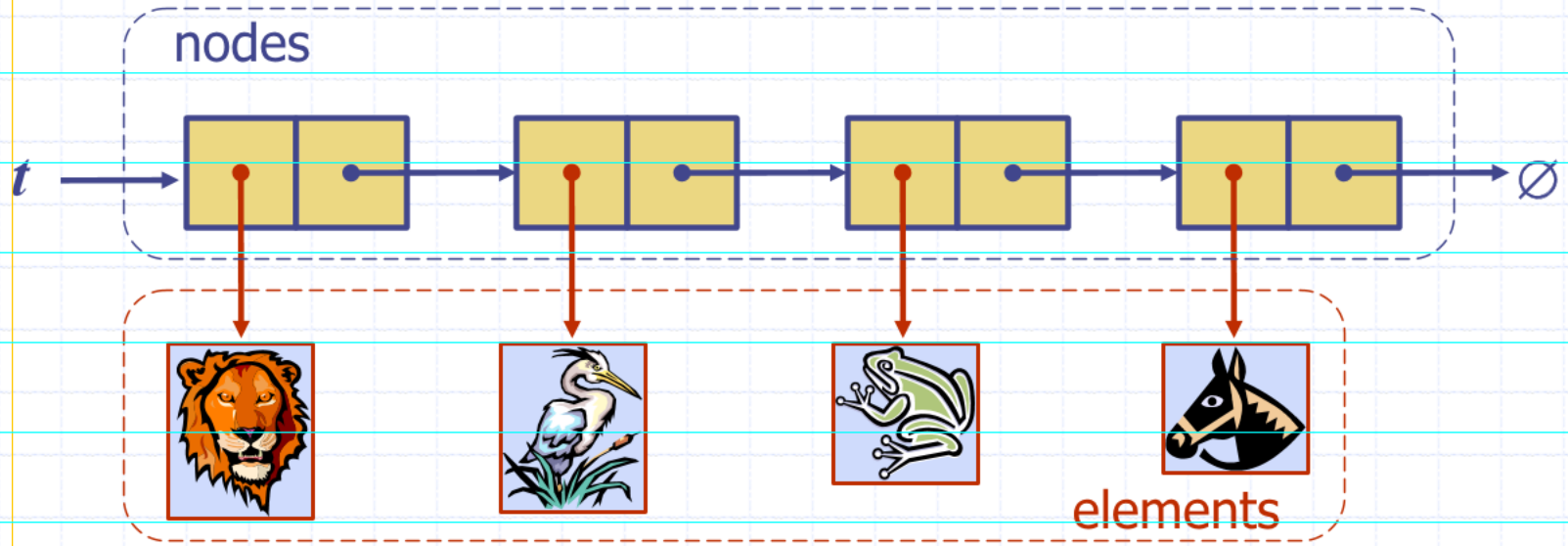
- The Java Virtual Machine (JVM) keeps track of the chain of active methods with a stack
- When a method is called, the JVM pushes on the stack a frame containing
  - Local variables and return value
  - Program counter, keeping track of the statement being executed
- When a method ends, its frame is popped from the stack and control is passed to the method on top of the stack
- Allows for **recursion**

```
main() {
  int i = 5;
  foo(i);
  }

foo(int j) {
  int k;
  k = j+1;
  bar(k);
  }

bar(int m) {
  …
  }
```



# Stack with a Singly Linked List

- We can implement a stack with a singly linked list
- The top element is stored at the first node of the list
- The space used is $O(n)$ and each operation of the Stack ADT takes $O(1)$ time



# Array-based Stack

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable keeps track of the index of the top element

```
Algorithm size()
  return t + 1

Algorithm pop()
  if isEmpty() then
    throw EmptyStackException
  else
    t ← t − 1
    return S[t + 1]
```

when pop() is unable to execute




Push Problem: when t = last element: Double size & copy over. Amortized analysis gives O(n)

# Recurring tradeoff:

(next) Pointer:
[Linked List]

✓ Flexibility of extension

✗ Extra memory to store "next" pointer

✗ Increased access time for non-stack operations

Contiguous block
[Array]

✓ Less memory (no need of next pointer) & contiguous block of mem

✗ Memory wastage

✗ Increased extension costs

# Array-based Stack (cont.)

- The array storing the stack elements may become full
- A push operation will then throw a FullStackException
  - Limitation of the array-based implementation
  - Not intrinsic to the Stack ADT

**Algorithm** *push*($o$)
  **if** $t = S.length - 1$ **then**
    **throw** *FullStackException*
  **else**
    $t \leftarrow t + 1$
    $S[t] \leftarrow o$

can be avoided by creation of new array on demand

# Performance and Limitations

- Performance
  - Let $n$ be the number of elements in the stack
  - The space used is $O(n)$
  - Each operation runs in time $O(1)$
- Limitations
  - The maximum size of the stack must be defined a priori and cannot be changed
  - Trying to push a new element into a full stack causes an implementation-specific exception

# Array-based Stack in Java

```
public class ArrayStack
    implements Stack {

  // holds the stack elements
  private Object S[ ];

  // index to top element
  private int top = -1;

  // constructor
  public ArrayStack(int capacity) {
     S = new Object[capacity]);
  }
```

```
public Object pop()
     throws EmptyStackException {
  if isEmpty()
     throw new EmptyStackException
        ("Empty stack: cannot pop");
  Object temp = S[top];
  // facilitates garbage collection
  S[top] = null;
  top = top – 1;
  return temp;
}
```

# Parentheses Matching

- Each “(”, “{”, or “[” must be paired with a matching “)”, “}”, or “[”
  - correct: ( )(( )){([( )])}
  - correct: ( )(( )){([( )])}
  - incorrect: )(( )){([( )])}
  - incorrect: ({[ ])}
  - incorrect: (

Useful in "parsing" program (code) files to check for syntactic correctness

# W/o stack [Typical answers]

1) Keep 3 counters of open brackets, one for each type

2) Keep a flag storing type of last open bracket

3) On encountering "closed" bracket match its "type" with flag in 2

4) If match in (3) decrement corresponding counter in 1.....

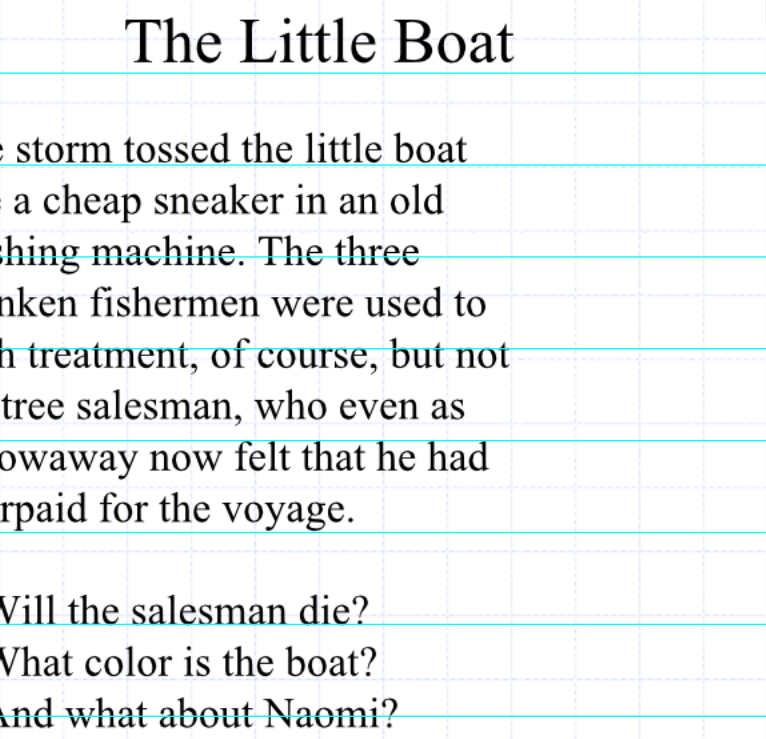
# Parentheses Matching Algorithm

**Algorithm** ParenMatch($X$,$n$):
***Input:*** An array $X$ of $n$ tokens, each of which is either a grouping symbol, a variable, an arithmetic operator, or a number
***Output:*** **true** if and only if all the grouping symbols in $X$ match
Let $S$ be an empty stack
**for** $i$=0 to $n$-1 **do**
    **if** $X[i]$ is an opening grouping symbol **then**
        $S$.push($X[i]$)
    **else if** $X[i]$ is a closing grouping symbol **then**
        **if** $S$.isEmpty() **then**
            **return false** {nothing to match with}
        **if** $S$.pop() does not match the type of $X[i]$ **then**
            **return false** {wrong type}
**if** $S$.isEmpty() **then**
    **return true** {every symbol matched}
**else**
    **return false** {some symbols were never matched}

# HTML Tag Matching

- For fully-correct HTML, each <name> should pair with a matching </name>

```
<body>
<center>
<h1> The Little Boat </h1>
</center>
<p> The storm tossed the little
boat like a cheap sneaker in an
old washing machine. The three
drunken fishermen were used to
such treatment, of course, but
not the tree salesman, who even as
a stowaway now felt that he
had overpaid for the voyage. </p>
<ol>
<li> Will the salesman die? </li>
<li> What color is the boat? </li>
<li> And what about Naomi? </li>
</ol>
</body>
```



# Tag Matching Algorithm

- Is similar to parentheses matching:

```
import java.util.StringTokenizer;
import datastructures.Stack;
import datastructures.NodeStack;
import java.io.*;
/** Simpli.ed test of matching tags in an HTML document. */
public class HTML { /** Nested class to store simple HTML tags */
        public static class Tag { String name; // The name of this tag
                boolean opening; // Is true i. this is an opening tag
                public Tag() { // Default constructor
                        name = "";
                        opening = false;
                }
                public Tag(String nm, boolean op) { // Preferred constructor
                name = nm;
                opening = op;
                }
                /** Is this an opening tag? */
                public boolean isOpening() { return opening; }
                /** Return the name of this tag */
                public String getName() {return name; }
        }
        /** Test if every opening tag has a matching closing tag. */
        public boolean isHTMLMatched(Tag[ ] tag) {
                Stack S = new NodeStack(); // Stack for matching tags
                for (int i=0; (i <tag.length) && (tag[i] != null); i++) {
                        if (tag[i].isOpening())
                        S.push(tag[i].getName()); // opening tag; push its name on the stack
                        else {
                                if (S.isEmpty()) // nothing to match
                                        return false;
                                if (!((String) S.pop()).equals(tag[i].getName())) // wrong match
                                        return false;
                        }
                }
                if (S.isEmpty())
                    return true; // we matched everything
                return false; // we have some tags that never were matched
        }
```

# Tag Matching Algorithm, cont.

```
public final static int CAPACITY = 1000;              // Tag array size upper bound
  /* Parse an HTML document into an array of html tags */
  public Tag[ ] parseHTML(BufferedReader r) throws IOException {
      String line; // a line of text
      boolean inTag = false ;                         // true iff we are in a tag
       Tag[ ] tag = new Tag[CAPACITY]; // our tag array (initially all null)
      int count = 0 ;                                 // tag counter
      while ((line = r.readLine()) != null) {
            // Create a string tokenizer for HTML tags (use < and > as delimiters)
            StringTokenizer st = new StringTokenizer(line,"<> \t",true);
            while (st.hasMoreTokens()) {
                  String token = (String) st.nextToken();
                  if (token.equals("<")) // opening a new HTML tag
                        inTag = true;
                  else if (token.equals(">"))  // ending an HTML tag
                        inTag = false;
                  else if (inTag) { // we have a opening or closing HTML tag
                        if ( (token.length() == 0) || (token.charAt(0) != '/') )
                        tag[count++] = new Tag(token, true); // opening tag
                        else  // ending tag
                        tag[count++] = new Tag(token.substring(1), false); // skip the
                  } // Note: we ignore anything not in an HTML tag
            }
      }
      return tag; // our array of tags
  }
   /** Tester method */
  public static void main(String[ ] args) throws IOException {
      BufferedReader stdr;          // Standard Input Reader
      stdr = new BufferedReader(new InputStreamReader(System.in));
      HTML tagChecker = new HTML();
      if (tagChecker.isHTMLMatched(tagChecker.parseHTML(stdr)))
            System.out.println("The input file is a matched HTML document.");
      else
            System.out.println("The input file is not a matched HTML document.");
      }
}
```

Given an an array X, the span S[i] of X[i] is the maximum number of consecutive elements X[j] immediately preceding X[i] (and including j= i) such that X[j] X[i]. Given an array X[1...n], present algorithms for computing the array of spans S[1...n] with and without making use of the stack data structure.