

Recent advances in active control of turbulent boundary layers[†]

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In this article, we review the recent progress in active control of a turbulent boundary layer for skin-friction drag reduction. Near-wall coherent structures, which are closely associated with large skin-friction drag and are thus often the target to be manipulated, are discussed briefly, providing a rationale of various control strategies. Open- and closed-loop controls are extensively reviewed, largely focusing on techniques and drag-reduction mechanisms. Finally, some concluding remarks are given.

turbulence, boundary layer, active control

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1 Introduction

The lasting high oil price highlights greatly the necessity and urgency of searching for methods to conserve energy and hence drag reduction for air, sea and land vehicles, for pipelines and for other industrial devices. Many classes of transport and multifarious important applications stand to reap great rewards from the successful development of viscous drag reduction technologies. For example, the annual fuel cost exceeds one billion dollars for the existing railway transport (MTR) in Hong Kong and is expected to be huge for the planned high-speed train service to the Mainland of China. This cost is several billion dollars for all commercial airlines in Hong Kong solely. At subsonic cruising speeds, approximately half of the total drag of aircrafts is due to skin friction. Even a modest reduction in viscous friction drag will lead to tremendous saving of fuel cost.

Turbulence control is a field in fluid mechanics where flow is manipulated in order to improve the efficiency of thermofluid systems such as reducing skin-friction drag,

enhancing heat transfer and flow mixing [1]. Most of turbulence control methods take the advantage of, either explicitly or implicitly, the coherent structures of turbulent shear flows. For the purpose of reducing skin-friction drag in a turbulent boundary layer, passive control method such as riblets [2–4], which requires no external energy input, has achieved limited success. Active control methods involving the input of external energies have been extensively investigated, with various skin-friction drag reduction techniques developed and great advances made in the understanding of control mechanisms and underlying physics [5–9]. Active control can be open- or closed-loop. The signals of flow information such as wall shear stresses are fed back to a controller in a closed-loop scheme, but not in an open-loop scheme [6]. These active control methods, either in an open-loop scheme or in a closed-loop scheme, include modification of the fluid viscosity by heating the wall, use of compliant wall, suction and/or blowing from the wall, spanwise wall-oscillation, oscillating Lorentz forces, and transverse travelling wave, etc.

In this paper, we review the recent progress in the active control of turbulent boundary layers for skin-friction drag reduction, focusing on techniques *per se* and underlying drag-reduction mechanisms.

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2 Dynamics of near-wall coherent structures

It is now well known that quasi-streamwise vortices and large wall shear stress are closely associated with each other. The production of the mean Reynolds stress (subsequently viscous drag) is linked directly to the dynamics of the vortices in the wall region [10]. The vortices are generally located immediately above and displaced laterally from high skin-friction-drag regions [11,12]. The well-known events, i.e. sweeps and ejections, bursts and streak-like structures, in the boundary layer are all related to the vortices [13–18]. A conceptual model illustrating the coherent structures in a turbulent boundary layer was proposed by Robinson [19], as shown in Figure 1. Both ejection and sweep are induced by the vortices. The ejection is the process when slow-moving fluid is lifted up from the wall on the updraught side of the vortices, resulting in a low-speed streak. The lifted slow-moving fluid induces an inflection in the mean velocity profile. When intensified, the inflection results in a secondary instability and a subsequent burst of Reynolds stress, which transfers energy from large- to small-scales, and produces turbulent fluctuations. The ejection is the central mechanism for energy, momentum, and vorticity transfer between the inner and outer layers [20–22]. On the other hand, the sweep is the downdraught on the opposite side of the vortex, producing a high-speed streak. The sweep is responsible for large wall friction [9,11,12,23] and is therefore particularly important for drag reduction. The sweep and the ejection account for about 80% of the turbulent energy production [24].

Possible linear mechanisms may exist in the dynamic evolutions of near-wall coherent structures. The coherent structures could grow or decay linearly on average given an adequately short time, as advocated by Hunt and Carruthers [25] based on a rapid distortion theory and by Landahl [26] based on a simplified model. This proposition is supported by Johansson et al.'s [27] experimental investigation, where

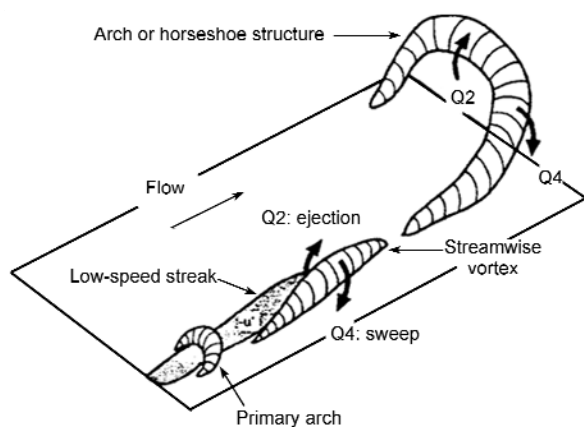


Figure 1 Conceptual model of coherent structures in a turbulent boundary layer proposed by Robinson [19].

conditionally sampled turbulent statistics scaled linearly with the threshold values. Kim and Lim [28] investigated numerically the influence of the coupling terms in Navier-Stokes equations on the near-wall turbulent structures and suggested that the coherent structures, albeit essentially nonlinear, be maintained by a transient linear process. Active control strategies, in particular the closed-loop schemes, may take advantage of the linear mechanisms to interrupt or suppress the near-wall turbulence [29,30].

3 Open-loop control

The key to skin-friction-drag reduction is to control locally quasi-streamwise vortices. Sweeps and ejections occur within 15 wall units from the wall [31] and are closely connected to the bursting process [20,21]. Manipulating the near-wall region will directly influence ejections and subsequently sweeps. Therefore, it is feasible to implement wall-based control schemes for drag reduction, such as spanwise wall-oscillation, oscillating Lorentz force, travelling wave, and suction and/or blowing.

Jung et al. [32] were the first to carry out a direct numerical simulation (DNS) on flow over a spanwise oscillating wall. Given a normalized oscillation period of 100, their DNS data indicated about 40% reduction in skin-friction drag, along with a drastic suppression of near-wall turbulence activities. The experimental demonstration of this technique was conducted by Choi et al. [33], who achieved a maximum of 45% drag reduction. The crucial parameters for the spanwise wall-oscillation are the oscillation period (or frequency) and amplitude, which may be combined into the so-called wall velocity, $w^+ = (\Delta z / 2) \omega / u_\tau$, where Δz and ω are the peak-to-peak amplitude and the angular velocity of the oscillating wall, respectively, and u_τ is the friction velocity. In this review, superscript “+” denotes normalization by wall units. Figure 2 shows that friction drag due to spanwise oscillating wall is closely correlated with w^+ ; it decreases with increasing w^+ , achieving about 45% drag reduction at $w^+ = 15$. Detailed investigation [34] into the modified near-wall flow structures unveiled that a net spanwise vorticity was created by the periodic Stokes layer over the spanwise oscillating wall. Located at the viscous sublayer edge, this net spanwise vorticity may lead to a reduction in the mean velocity gradient and consequently an upward-shifted logarithmic velocity profile. Based on particle image velocimetry measurements, Cicca et al. [35] found that near-wall coherent structures were weakened and subsequently stabilized by the transversal Stokes layer induced by the oscillating wall. As a result, the regeneration of streamwise vortices was attenuated. Quadrio and Ricco [36] studied numerically and systematically the influence of different parameters on the effectiveness of spanwise wall-oscillation and observed that the drag reduction was

correlated with the maximum wall velocity and the oscillation period. They further pointed out that the efficiency, defined as the ratio of the saved power to that consumed, of spanwise wall-oscillation was low, only about 7%, because of the auxiliary mechanical movements involved.

Spanwise oscillating Lorentz force may be considered as a variant of spanwise oscillating wall. Berger et al. [37] found that, with this technique, skin-friction drag could be reduced by 40% in a turbulent channel flow, though the control was impractical because of a very low efficiency, in the order of 10^{-3} . Pang and Choi [38] confirmed experimentally that a drag reduction up to 40% could be achieved by means of the spanwise oscillating Lorentz force. It was observed that low-speed streaks were periodically forced into the spanwise direction by the oscillating Lorentz forces, similar to the scenario over a spanwise oscillating wall.

A travelling wave in the near-wall region can modify the near-wall turbulence structures and thus reduce the friction drag. Du and Karniadakis [39] and Du et al. [40] investigated the effectiveness of a transverse travelling wave along the spanwise direction, induced by a spanwise force within the viscous sublayer, on the control of a turbulent channel flow based on DNS data and observed a drag reduction exceeding 50%. The near-wall streaks were largely eliminated by the travelling wave (Figure 3). The maximum drag reduction was achieved when the penetration depth of the wave was comparable to the thickness of viscous sublayer. Karniadakis and Choi [9] discussed the drag-reduction mechanisms of transverse motions, including spanwise wall-oscillation and travelling wave in a turbulent boundary layer, pointing out that the key to reducing friction drag is to stabilize the near-wall streamwise vortices and to weaken the near-wall streaky structures.

A streamwise travelling wave formed by a wavy or deformed wall surface was numerically investigated by Shen et al. [41], focusing on the influence of phase speed on the near-wall turbulence structures. It was found that turbulence intensity and turbulent shear stress were reduced sig-

nificantly when the phase speed was comparable to the free-stream velocity and the drag force decreased proportionally to the increased phase speed. However, flow separation and drag increase were induced by a negative phase speed. Quadrio et al. [43] proposed to generate numerically a streamwise travelling wave, formed by the spanwise movement of the wall surface, for the control of a turbulent boundary layer. This wave imposes on the wall surface a spanwise velocity, which is varied in time and modulated in space along the streamwise direction; furthermore, it can be stationary, travelling longitudinally forward or backward, given a zero, positive or negative phase speed, respectively. Such a wave was observed to change the friction drag significantly, given appropriate wavenumber and phase speed. Whilst drag reduction was always obtained with a backward-travelling wave, the opposite effect, i.e., drag increase, was observed with the wave travelling forward at a speed comparable to the convection velocity of the near-wall turbulence structures. Quadrio et al.'s [43] idea was demonstrated experimentally by Auteri et al. [44] in a pipe flow. The pipe wall was subdivided into thin slabs, which rotated independently in the azimuthal direction to form a streamwise travelling wave on the wall surface. They managed to achieve a drag reduction up to 33%. In their manipulation of a turbulent boundary layer, Huang et al. [45] investigated numerically a transverse wave travelling along the streamwise direction and observed the suppressed near-wall turbulence structures and a maximum friction drag reduction by 42%.

Kim and Sung [46] and Xu et al. [47] are among other significant works, achieving a skin-friction drag reduction by as much as 70% by means of unsteady blowing and a localized steady force, respectively. It is worth mentioning that the effectiveness of unsteady blowing has been confirmed physically. Blowing periodically through a spanwise slot, Tardu and Doche [48] achieved experimentally a local maximum drag reduction of 50% in a turbulent boundary layer.

Wall deformation created by wall-mounted actuators is a technique frequently used by experimentalists. Segawa et al. [49] devised an array of 8 piezo-electric actuators to create wall-normal oscillation and observed a decreased regularity in streak-like structures. Itoh et al. [50] used a loudspeaker to excite a flexible sheet, flush with a flat wall surface, to form a transverse travelling wave. They achieved a drag reduction by 7.5%, as estimated by the growth rate of momentum thickness. In their turbulent boundary layer control investigation, Zhang et al. [51] deployed a total of 16 piezo-ceramic actuators, aligned in the spanwise direction, to generate wall-normal oscillations and, given a phase shift between two adjacent actuators, a transverse travelling wave. Local skin-friction drag, estimated from the slope of streamwise mean velocity profile in the linear region, exhibits a strong dependence on the control parameters (i.e. wavelength, oscillation amplitude and frequency). A maxi-

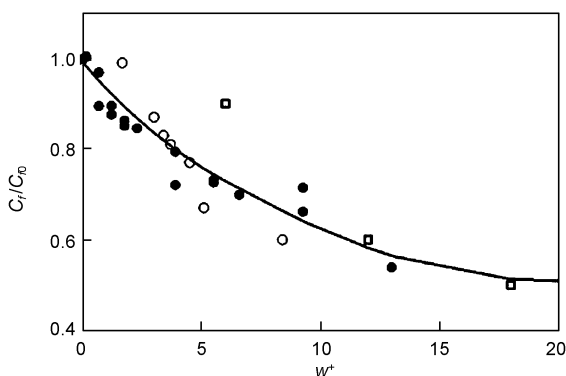


Figure 2 Dependence of friction drag on the normalized wall velocity w^+ [34]. C_f and C_{f0} are the friction drag coefficients with and without control, respectively. Symbols: •, Choi et al. [23]; o, Laadhari et al. [42]; □, Jung et al. [32].

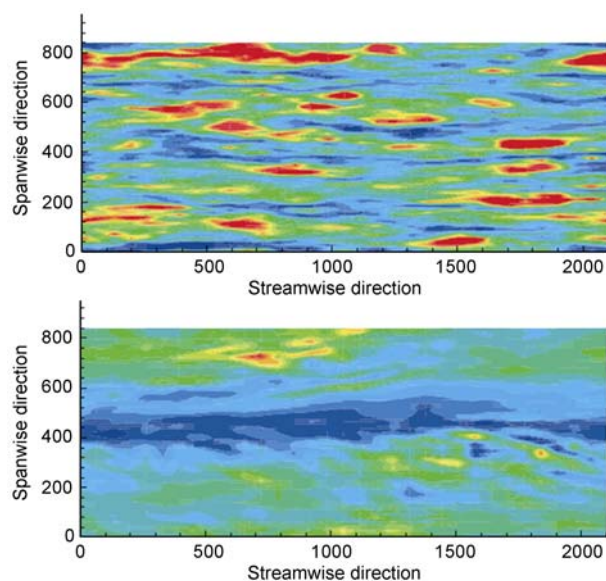


Figure 3 Instantaneous flow visualization of wall streaks (top) without control and (bottom) with control [39]. Parameters of the travelling wave: excitation amplitude is unit; wavelength and period are 840 and 50, respectively, both in wall units. Blue indicates low-speed streaks and yellow-red indicates high-speed streaks. Flow is left to right.

mum drag reduction of 50% was achieved at 17 wall units downstream of the actuator tip, given the wavelength, oscillation amplitude and frequency at 416, 1.94 and 0.39, respectively, all in wall units. The near-wall flow structures were altered greatly; a comparison is made in Figure 4 between flow structures with and without control.

4 Closed-loop control

Turbulence control can be significantly more effective if a closed-loop control is applied [6]. This was confirmed both

numerically [8,52] and experimentally [29,30,53,54]. Control action must interact with turbulent fluctuations in a boundary layer, and the random aspects of the fluctuations reduce the effectiveness of an open-loop control. The sensor-feedback loop may treat effectively the random-phase problem in turbulence dynamics [53,55]. Kim and Bewley [56] and Kasagi et al. [57] provided excellent compendiums on works along this line.

Numerical simulations of the closed-loop control of turbulent boundary layers usually assume a dense and uniform network of both sensors and actuators on the wall. Choi et al. [4] developed numerically an out-of-phase or opposition feedback control, which used the wall-normal and spanwise fluctuating velocities and their combination as the feedback signal to actuate a wall-based blowing/suction for canceling wall-normal and/or spanwise velocities associated with the quasi-streamwise vortices. A schematic diagram in Figure 5 illustrates the working principle of this opposition feedback control. A 20%–30% skin-friction-drag reduction was achieved in a turbulent channel flow. With the same control strategy, Kang and Choi [58] replaced the blowing/suction actuation by wall deformation and observed 13%–17% drag reduction; Kang et al. [59] intervened directly the outer-layer large-scale hairpin structures. The opposition control was physically demonstrated in a real-time wind-tunnel experiment by Rebbeck and Choi [60], who detected and subsequently cancelled the high-speed downwash of sweep events near the wall using the signal from an upstream wall-mounted hotwire to actuate a pulsed wall-normal jet. Jacobson and Reynolds [61] developed a combined feedforward and feedback control system, with the feedforward and feedback signals from two arrays of hot-film sensors, placed upstream and downstream of actuators to measure streamwise wall shear stress, respectively. They obtained 10% and 80% reductions in the mean and rms wall shear stresses, respectively, when the control

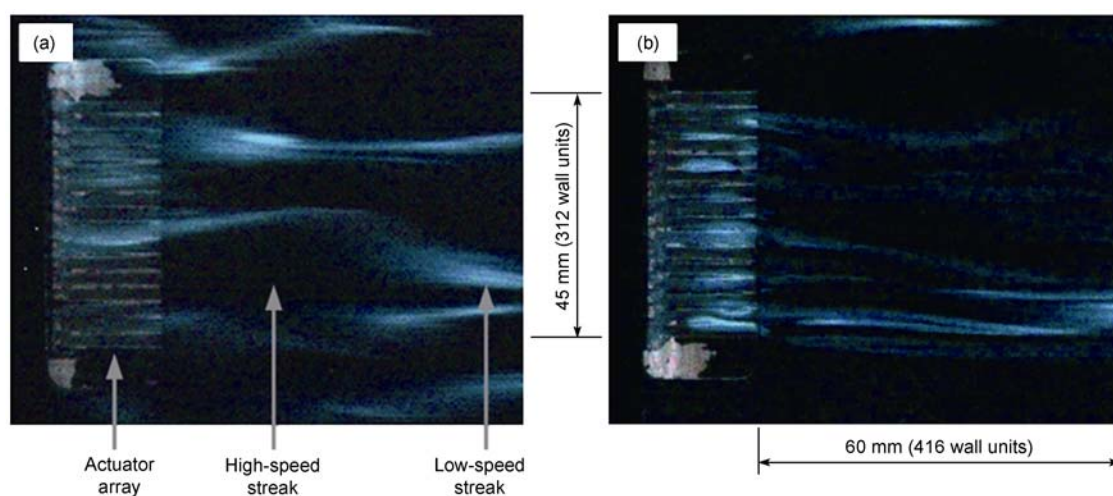


Figure 4 Typical photographs of instantaneous flow structures from smoke-wire flow visualization [51]. (a) Uncontrolled; (b) controlled. Control parameters: wavelength, oscillation amplitude and frequency are 416, 2.22 and 0.65, respectively, all in wall units. Flow is left to right.

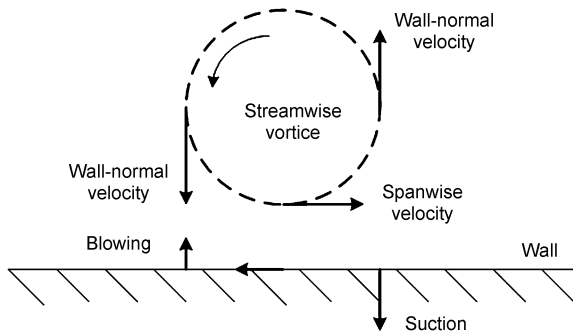


Figure 5 Schematic diagram of the opposition feedback control scheme (after Choi et al. [4]). Flow is Change it into 'normal' to the paper.

system space in between to a laminar boundary layer under the perturbation of periodic pulsed suction. Li and Gaster [62] devised a multi-input and multi-output feedforward control system to cancel the oncoming disturbance waves and to suppress the instabilities in a laminar boundary layer. Transfer functions bridging the signals from upstream wall-mounted hotwires and those driving actuating synthetic jets were determined based on numerical modeling. It was demonstrated that a very simple transfer function could provide almost the same result as a full causal transfer function. Lundell [63] developed a feedforward control system, consisting of upstream wall-mounted hotwires and downstream actuators (suctions), to suppress the random disturbances in a laminar boundary layer, and managed to delay transition from laminar to turbulent flow. Rathnasingham and Breuer [29,30] incorporated Wiener-filter-based system identification into their feedforward control scheme to predict the downstream characteristics of the streamwise velocity fluctuation, reducing this fluctuation by 30% and the bursting frequency by 23%. Only a simple adaptation algorithm was attempted in their investigation to minimize the root mean square value of the error signal by varying the gain and lag of the filter. Reducing skin-friction drag was not the control objective. An interesting observation from this work is the impressive performance of a linear controller in a highly nonlinear turbulent boundary layer, which conforms to the proposition that the structures in a turbulent boundary layer respond to the mean shear in a linear manner [26]. The linear control system was employed in all the above-mentioned works.

Linear models do not capture the non-linear "cascade" of energy over a range of length and time scales [56]. Attempts have been made to develop a nonlinear controller. Lee et al. [52] applied an adaptive controller based on neural networks to control numerically a turbulent channel flow for drag reduction and demonstrated that the nonlinear network performed better than the linear in view of retaining a stable coherent pattern in the control system, suggesting that a certain amount of nonlinearity in the controller be critical to capture the coherent structures in a turbulent boundary layer.

Lee et al. [64] proposed a suboptimal control scheme to minimize a cost function related to the quasi-streamwise vortices, with the aid of wall pressure and wall shear stress. They achieved a drag reduction of 22% and 16% when taking the spanwise wall shear stress and wall pressure as the control input, respectively. Based on wall shear stress information, Lorang et al. [65] employed a compact neural network to estimate and control the near-wall flow, achieving about 13% drag reduction. The controllers used in previous numerical investigations performed extremely well in terms of minimizing the given cost function but unfortunately did not lead to significant drag reduction. This may indicate an inherent limitation of these simplified controllers for turbulent boundary layer or simply calls for different cost functions [8].

5 Concluding remarks

This paper is a compendium of previous investigations on the active control of a turbulent boundary layer, largely focusing on friction-drag reduction techniques. In spite of past great progress in this area, a practical active control technique for turbulent boundary layers remains to be explored or developed. It has been demonstrated both numerically and experimentally that large friction drag reduction is obtainable under open loop control schemes. However, most open-loop control methods suffer from a very low efficiency, which hampers their practical applications. As such, search continues for both effective and efficient open-loop control techniques.

The closed-loop control can capitalize the natural instabilities in turbulent boundary layers and hence may improve greatly the control efficiency, reducing energies consumed by control action and achieving a large net energy saving. There has been a limited success in developing numerically closed-loop control schemes. However, previous DNS-based closed-loop control of wall turbulence has so far been performed in the friction Reynolds number (Re_τ) range of 100–180, where the low- Re_τ effects can be significant. No friction-drag reduction has been demonstrated physically or experimentally in a turbulent boundary layer [57], perhaps due to difficulties in measuring reliably wall shear stresses and implementing practically complicated control systems. As such, many important issues need to be addressed before an effective closed-loop control scheme could be physically developed. For example, fluctuating streamwise velocity, wall pressure and shear stresses have been frequently used for the feed-forward or feedback signals in the DNS-based closed-loop control of friction drag. Which is feasible and best in a physical control system? Furthermore, how effective could a linear control technique be physically in reducing friction drag? To what extent could the control performance be improved given the deployment of a nonlinear controller, e.g. model-based neural

networks with near-wall turbulence dynamics incorporated? These issues, along with the goal to develop a practical active control technique for turbulent boundary layers, warrant future investigations.

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- 1 Liepmann H W, Narasimha R. (eds.) *Turbulence Management and Relaminarisation*. Berlin: Springer-Verlag, 1988. 1–524
- 2 Wash M J. Riblets as a viscous drag reduction technique. *AIAA J*, 1983, 21: 485–486
- 3 Bechert D W, Bartenwerfer M. The viscous flow on surface with longitudinal ribs. *J Fluid Mech*, 1989, 206: 105–129
- 4 Choi H, Moin P, Kim J. Active control for drag reduction in wall-bounded flows. *J Fluid Mech*, 1994, 262: 75–110
- 5 Gad-el-Hak M. Modern development in fluid control. *Appl Mech Rev*, 1996, 49: 365–379
- 6 Gad-el-Hak M. *Flow Control: Passive, Active, and Reactive Flow Management*. Cambridge: Cambridge University Press, 2000
- 7 Lumley J, Blossey P. Control of turbulence. *Ann Rev Fluid Mech*, 1998, 30: 311–327
- 8 Kim J. Control of turbulent boundary layers. *Phys Fluids A*, 2003, 15: 1093–1105
- 9 Karniadakis G E, Choi K S. Mechanisms on transverse motions in turbulent wall flows. *Ann Rev Fluid Mech*, 2003, 35: 45–62
- 10 Bernard P S, Thomas J M, Handler R A. Vortex dynamics and the production of Reynolds stress. *J Fluid Mech*, 1993, 253: 385–419
- 11 Kravchenko A G, Choi H, Moin P. On the generation of near-wall streamwise vortices to wall skin friction in turbulent boundary layers. *Phys Fluids A*, 1993, 5: 3307–3309
- 12 Orlandi P, Jimenez J. On the generation of turbulent wall friction. *Phys Fluids A*, 1994, 6: 634–641
- 13 Kim J. On the structure of wall-bounded turbulent flows. *Phys Fluids*, 1983, 26: 2088–2097
- 14 Kim J. Turbulence structures associated with the bursting event. *Phys Fluids*, 1985, 28: 52–58
- 15 Swearingen D, Blackwelder R. The growth and breakdown of streamwise vortices in the presence of a wall. *J Fluid Mech*, 1987, 182: 255–290
- 16 Waleffe F. On a self-sustaining process in shear flows. *Phys Fluids A*, 1997, 9: 883–900
- 17 Jiménez J, Pinelli A. The autonomous cycle of near-wall turbulence. *J Fluid Mech*, 1999, 389: 335–359
- 18 Schoppa W, Hussain F. Coherent structure generation in near-wall turbulence. *J Fluid Mech*, 2002, 453: 57–108
- 19 Robinson S K. Coherent motions in the turbulent boundary layer. *Ann Rev Fluid Mech*, 1991, 23: 601–639
- 20 Kline S J, Reynolds W C, Schraub F A, et al. The structure of turbulent boundary layers. *J Fluid Mech*, 1967, 30: 741–773
- 21 Kim J, Moin P, Moser R. Turbulence statistics in fully developed channel flow at low Reynolds number. *J Fluid Mech*, 1987, 177: 133–166
- 22 Johansson A V, Alfredsson P H, Kim J. Evolution and dynamics of shear-layer structures in near-wall turbulence. *J Fluid Mech*, 1991, 224: 579–599
- 23 Choi K S. Near-wall structure of a turbulent boundary layer with riblets. *J Fluid Mech*, 1989, 208: 417–458
- 24 Lu S S, Willmarth W W. Measurements of the structure of the Reynolds stress in a turbulent boundary layer. *J Fluid Mech*, 1973, 60: 481–511
- 25 Hunt J, Carruthers J. Rapid distortion theory and the problems of turbulence. *J Fluid Mech*, 1990, 212: 497–532
- 26 Landahl M T. On sublayer streaks. *J Fluid Mech*, 1990, 212: 593–614
- 27 Johansson A V, Her J, Haritonidis J H. On the generation of high amplitude wall-pressure peaks in turbulent boundary layers and spots. *J Fluid Mech*, 1987, 175: 119–142
- 28 Kim J, Lim J. A linear process in wall-bounded turbulent shear flows. *Phys Fluids*, 2000, 12: 1885–1888
- 29 Rathnasingham R, Breuer K S. System identification and control of a turbulent boundary layer. *Phys Fluids*, 1997, 9: 1867–1869
- 30 Rathnasingham R, Breuer K S. Active control of turbulent boundary layers. *J Fluid Mech*, 2003, 495: 209–233
- 31 Wallace J M, Eckelmann H, Brodkey R S. The wall region in turbulent shear flow. *J Fluid Mech*, 1972, 54: 39–48
- 32 Jung W J, Mangiavacchi N, Akhavan R. Suppression of turbulence in wall-bounded flows by high-frequency spanwise oscillations. *Phys Fluids A*, 1992, 4: 1605–1607
- 33 Choi K S, DeBisschop J R, Clayton B R. Turbulent boundary-layer control by means of spanwise-wall oscillation. *AIAA J*, 1998, 36: 1157–1163
- 34 Choi K S. Near-wall structure of turbulent boundary layer with spanwise-wall oscillation. *Phys Fluids*, 2002, 14: 2530–2542
- 35 Cicca G M D, Iuso P G S, Onorato M. Particle image velocimetry investigation of a turbulent boundary layer manipulated by spanwise wall oscillations. *J Fluid Mech*, 2002, 467: 41–56
- 36 Quadrio M, Ricco P. Critical assessment of turbulent drag reduction through spanwise wall oscillations. *J Fluid Mech*, 2004, 521: 251–271
- 37 Berger T W, Kim J, Lee C, et al. Turbulent boundary layer control utilizing the Lorentz force. *Phys Fluids*, 2000, 12: 631–649
- 38 Pang J, Choi K S. Turbulent drag reduction by Lorentz force oscillation. *Phys Fluids*, 2004, 16: L35–38
- 39 Du Y Q, Karniadakis G E. Suppressing wall turbulence by means of a transverse traveling wave. *Science*, 2000, 288: 1230–1234
- 40 Du Y Q, Symeonidis V, Karniadakis G E. Drag reduction in wall-bounded turbulence via a transverse traveling wave. *J Fluid Mech*, 2002, 457: 1–34
- 41 Shen L, Zhang X, Yue D, et al. Turbulent flow over a flexible wall undergoing a streamwise travelling wave motion. *J Fluid Mech*, 2003, 484: 197–221
- 42 Laadhari F, Skandaji L, Morel R. Turbulence reduction in a boundary layer by a local spanwise oscillating surface. *Phys Fluids*, 1994, 6: 3218–3220
- 43 Quadrio M, Ricco P, Viotti C. Streamwise-travelling waves of spanwise wall velocity for turbulent drag reduction. *J Fluid Mech*, 2009, 627: 161–178
- 44 Auteri F, Baron A, Belan M, et al. Experimental assessment of drag reduction by travelling waves in a turbulent pipe flow. *Phys Fluids*, 2010, 22: 115103
- 45 Huang L, Fan B, Dong G. Turbulent drag reduction via a transverse wave travelling along streamwise direction induced by Lorentz force. *Phys Fluids*, 2010, 22: 015103
- 46 Kim K, Sung H J. Effects of unsteady blowing through a spanwise slot on a turbulent boundary layer. *J Fluid Mech*, 2006, 557: 423–450
- 47 Xu J, Dong S, Maxey M, et al. Turbulent drag reduction by constant near-wall forcing. *J Fluid Mech*, 2007, 582: 79–101
- 48 Tardu S F, Doche O. Active control of the turbulent drag by a localized periodical blowing dissymmetric in time. *Exps Fluids*, 2009, 47: 19–26
- 49 Segawa T, Kawaguchi Y, Kikushima Y, et al. Active control of streak structures in wall turbulence using an actuator array producing inclined wavy disturbances. *J Turbulence*, 2002, 3: 1–15
- 50 Itoh M, Tamano S, Yokota K, et al. Drag reduction in a turbulent boundary layer on a flexible sheet undergoing a spanwise traveling wave motion. *J Turbulence*, 2006, 7: 1–17
- 51 Zhang W G, Zhou Y, Bai H L. Turbulent Drag Reduction Using an Array of Piezo-ceramic Actuators. In: the 17th Australasian Fluid

- Mechanics Conference. Auckland, New Zealand, 2010. Paper No. 64 (CDROM)
- 52 Lee C, Kim J, Babcock D, et al. Application of neural networks to turbulence control for drag reduction. *Phys Fluids*, 1997, 9: 1740–1747
- 53 Zhang M M, Cheng L, Zhou Y. Closed-loop-controlled vortex shedding and vibration of a flexibly supported square cylinder under different schemes. *Phys Fluids*, 2004, 16: 1439–1448
- 54 Zhang M M, Cheng L, Zhou Y. Closed-loop controlled vortex-airfoil interactions. *Phys Fluids*, 2006, 18: 046102
- 55 Bushnell D M, McGinley C B. Turbulence control in wall flows. *Ann Rev Fluid Mech*, 1989, 21: 1–20
- 56 Kim J, Bewley T R. A linear system approach to flow control. *Ann Rev Fluid Mech*, 2007, 39: 373–417
- 57 Kasagi N, Suzuki Y, Fukagata K. Microelectromechanical systems-based feedback control of turbulence for skin friction reduction. *Ann Rev Fluid Mech*, 2009, 41: 231–251
- 58 Kang S, Choi H. Active wall motions for skin-friction drag reduction. *Phys Fluids*, 2000, 12: 3301–3304
- 59 Kang Y D, Choi K S, Chun H H. Direct intervention of hairpin structures for turbulent boundary-layer control. *Phys Fluids*, 2008, 20: 101517
- 60 Rebbeck H, Choi K S. A wind-tunnel experiment on real-time opposition control of turbulence. *Phys Fluids*, 2006, 18: 035103
- 61 Jacobson S A, Reynolds W C. Active control of streamwise vortices and streaks in boundary layers. *J Fluid Mech*, 1998, 360: 179–211
- 62 Li Y, Gaster M. Active control of boundary-layer instabilities. *J Fluid Mech*, 2006, 550: 185–205
- 63 Lundell F. Reactive control of transition induced by free-stream turbulence: an experimental demonstration. *J Fluid Mech*, 2007, 585: 41–71
- 64 Lee C, Kim J, Choi H. Suboptimal control of turbulent channel flow for drag reduction. *J Fluid Mech*, 1998, 358: 245–258
- 65 Lorang L V, Podvin B, Quere P L. Application of compact neural network for drag reduction in a turbulent channel flow at low Reynolds numbers. *Phys Fluids*, 2008, 20: 045104