Shear Center

Open c/s thin-walled beam

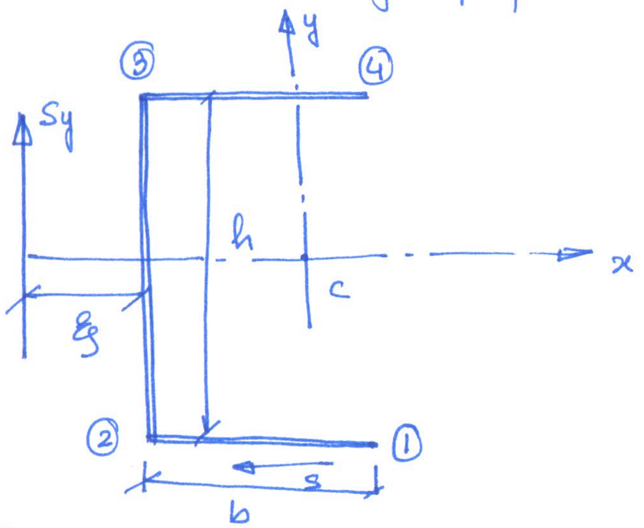
SC is the point through which the shear loads should act to produce no twisting (no torsional moment in the c/s of the beam)

(i) For a symmetric (singly/doubly) c/s, shear center lies on the axis of symmetry

(ii) Any shear load can be represented by shear load through the shear center and a torque.

(iii) The stresses produced by the separate actions of torsion and shear load can be added (by superposition theorem)

Ex



The section is symmetric about x-axis and hence the shear center will lie on the x-axis. Therefore we need to find only ξ and $S_x = 0$

For $S_x = 0$ and $I_{xy} = 0$.

$$q_s - q_0 = -\frac{S_y t}{I_{xx}} \int_0^s y\,ds \qquad \text{where, } I_{xx} = \frac{h^3 t}{12}\left(1 + \frac{6b}{h}\right)$$

For bottom flange,

$$q_{12} = \frac{-S_y t}{I_{xx}}\left(-\frac{h}{2}\right)s = \frac{S_y t h}{2 I_{xx}} s$$

Moment about ③

$$S_y \xi = \int_0^b q_{12}\, h\, ds = \frac{S_y t h^2}{2 I_{xx}} \cdot \frac{s^2}{2} = \frac{S_y t h^2 b^2}{4 I_{xx}} = \frac{3 S_y b^2}{h\left(1 + \frac{6b}{h}\right)}$$

$$\Rightarrow \xi = \frac{3b^2}{h\left(1 + \frac{6b}{h}\right)} \quad \text{(shear center)}$$

Shear center for a closed c/s thin-walled beam

Shear stress distribution in thin walled closed c/s beam

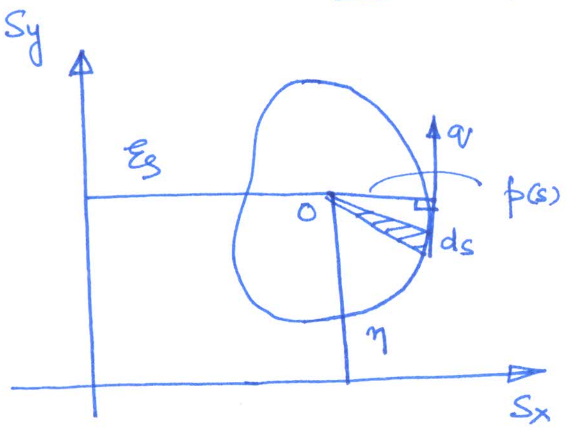
$$q_s - q_0 = -\left[\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right]\int_0^s t x \, ds - \left[\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right]\int_0^s t y \, ds.$$

For a closed c/s beam

$$\int \tau \, ds = 2G\theta A \Rightarrow \int q \, ds = 2G\theta t A$$

In case, the shear forces are passing through the shear center, $\theta = 0$ (no torsion)

$$\therefore \int \frac{q \, ds}{t(s)} = 0 \quad \text{for shear stress due to shear force.} \qquad \text{——— (1)}$$



Taking moment about O

$$S_y \xi - S_x \eta$$

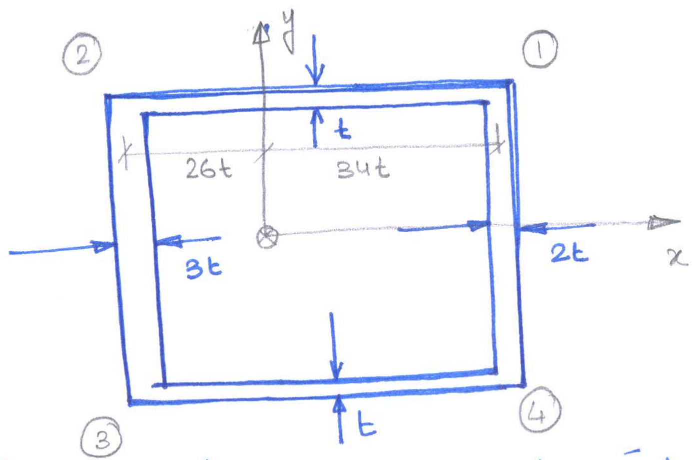
$$= \int p(s) q(s) \, ds$$

$$= \int p(s) q_b(s) \, ds + q_0 \int p(s) \, ds$$

$$= \int p(s) q_b(s) \, ds + 2A q_0 \qquad \text{——— (ii)}$$

Solving eqns. (1) and (2), we get q_0 and location of shear center.

Ex



Find the shear center for the thin-walled closed c/s beam as shown above. and shear flow distribution

$$I_{xx} = 2\times 60t \times t \times (24t)^2 + \frac{1}{12}(48t)^3(2t) + \frac{1}{12}(48t)^3 \times 3t$$

$$= 115200t^4$$

$$q_s - q_0 = \frac{S_y}{I_{xx}}\int ty\,ds$$

The section is symmetric about x-axis, hence $I_{xy}=0$ and the sc will lie on the x-axis & $S_x = 0$, $S_y = F$

For flange 12

$$q_{12} = q_0 + \frac{F}{I_{xx}}\int t(24t)\,ds \qquad q_0 = \text{shear flow at } ①$$

$$= q_0 + \frac{F}{I_{xx}}24t^2 s = q_0' + \frac{F}{I_{xx}}24t^2(34t - x)$$

and $q_2 = q_0 + \frac{F}{I_{xx}}1440t^3$.

For web 23

$$q_{23} = q_2 + \frac{F}{I_{xx}}\int 3t(24t - s)\,ds$$

$$= q_2 + \frac{F}{I_{xx}}3t\left(24ts - \frac{s^2}{2}\right) = q_2 + \frac{F}{I_{xx}}3t(24t - y)\left(12t + \frac{y}{2}\right)$$

$$= q_2 + \frac{F}{I_{xx}}3t\left(288t^2 - \frac{y^2}{2}\right)$$

$$= q_2 + \frac{F}{I_{xx}}1.5t(576t^2 - y^2)$$

$$q_3 = q_0 + \frac{F}{I_{xx}}1440t^3$$

For flange 34

$$q_{34} = q_2 + \frac{F}{I_{xx}} \int t(-24t)\, ds$$

$$= q_2 - \frac{F}{I_{xx}} 24t^2 s$$

$$= q_2 - \frac{F}{I_{xx}} 24t^2 (x+26t) = q_2 - \frac{F}{I_{xx}} 24t^2 (x+26t)$$

$$q_4 = q_2 - \frac{F}{I_{xx}} 24t^2 \times 60t \cdot = q_0$$

For web 41

$$q_{41} = q_0 + \frac{F}{I_{xx}} \int 2ty\, ds$$

$$= q_0 + \frac{F}{I_{xx}} \int 2t(s-24t)\, ds$$

$$= q_0 + \frac{F}{I_{xx}} 2t\left(\frac{s^2}{2} - 24ts\right)$$

$$= q_0 + \frac{F}{I_{xx}} 2t\,(y+24t)\left(\frac{y}{2} - 12t\right)$$

$$= q_0 + \frac{F}{I_{xx}} \left(\frac{y^2}{2} - 288t^2\right) 2t$$

$$= q_0 + \frac{F}{I_{xx}} (y^2 - 576t^2)\, t.$$

$$q_{12} = q_0 + \frac{F}{I_{xx}} 24t^2 (34t - x)$$

$$q_{23} = q_0 + \frac{F}{I_{xx}} 1440t^3 + \frac{1.5tF}{I_{xx}} (576t^2 - y^2)$$

$$q_{34} = q_0 + \frac{F}{I_{xx}} 1440t^3 - \frac{24t^2F}{I_{xx}} (26t+x).$$

$$q_{41} = q + \frac{tF}{I_{xx}} (y^2 - 576t^2).$$

For the shear force $S_y = F$ passing through the shear center.

$$\int \frac{q(s)}{t(s)}\,ds = 0$$

$$\int \frac{q_{12}(s)}{t}\,ds = \int \left[\frac{q_0}{t} - \frac{F}{I_{xx}}\,24t\,(34t - x)\,dx\right]$$

$$= \frac{q_0\,60t}{t} - \frac{F}{I_{xx}}\,24t\left(34tx - \frac{x^2}{2}\right)\Bigg|_{-34t}^{26t}$$

$$= 60q_0 - \frac{24tF}{I_{xx}}\,60t\left(34t - \frac{60}{2}t\right)$$

$$= 60q_0 - \frac{24\times 4\times 60t^3F}{I_{xx}} \Rightarrow 60q_0 - \frac{5760t^3F}{I_{xx}}$$

$$\int \frac{q_{23}(s)\,ds}{t(s)} \Rightarrow \int\left[\frac{q_0}{3t} + \frac{F\times 1440t^3}{3t\,I_{xx}} + \frac{1.5tF}{I_{xx}\,3t}\left(576t^2 - y^2\right)\right]ds$$

$(ds = -dy)$

$$= \frac{48t\,q_0}{3t} + \frac{48t\,F\times 1440t^2}{3t\,I_{xx}} + \frac{1.5tF}{3t\,I_{xx}}\left(576t^2y - \frac{y^3}{3}\right)\Bigg|_{-24t}^{24t}$$

$$= \overset{16}{\cancel{24}}q_0 + \overset{16}{\cancel{12}}\times 1440t^3\frac{F}{I_{xx}} + \frac{1.5}{\cancel{7}3}\,\frac{F}{I_{xx}}\left(576\times 48t^3\right)$$

$$= \overset{16}{\cancel{24}}q_0 + \frac{\overset{16}{\cancel{12}}\times 1440t^3F}{I_{xx}} + \frac{\cancel{3}\;\overset{24}{\cancel{\times 12}}\times 576t^3F}{I_{xx}}$$

$$\int \frac{q_{34}(s)\,ds}{t(s)} = \frac{q_0}{t}\times 60t + \frac{F}{I_{xx}}\,1440\times 60t^3 - \frac{24tF}{I_{xx}}\left(26t + \frac{x}{2}\right)x\Bigg|_{-26t}^{34t}$$

$$= 60q_0 + \frac{F}{I_{xx}}\,1440\times 60t^3 - \frac{24\times 60\times 56t^3F}{I_{xx}}$$

$$\int \frac{q_{41}\,ds}{t(s)} = \frac{q_0\,48t}{2t} + \frac{1}{2}\,\frac{F}{I_{xx}}\left(\frac{y^3}{3} - 576t^2y\right)\Bigg|_{-24t}^{24t}$$

$$= 24q_0 + \frac{F}{I_{xx}}\,24\times 576t^3$$

$\therefore$ $\Rightarrow$ ~~$160q_0$~~ Adding $\int \frac{q\,ds}{t(s)}$ and equating it to zero, we can obtain the value of q_0