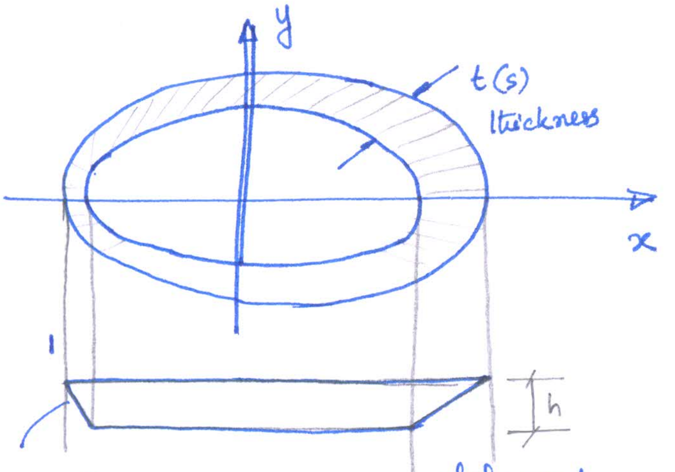
Torsion of thin-walled closed c/s shaft

Membrane analogy



The slope of the membrane is assumed to be linear.

deformed shape of the membrane.

$\tau = \frac{h}{t}$ ($= \frac{dw}{dn}$ slope of the deformed membrane) ——(1)

$= \frac{d\varphi}{dn}$ [φ is analogous to w]

resultant shear stress in tangential dirc of the contour lines.

Torque = 2 x Volume of the membrane.

$= 2Ah$ ——(2)

mean area enclosed by the membrane in the x-y plane.

From eqns (1) and (2), we get,

$$T = 2A\tau t \Rightarrow \tau = \frac{T}{2At} \Rightarrow q = \tau t = \frac{T}{2A}$$

shear flow.

For uniqueness

$$\int \tau ds = 2GA\theta \Rightarrow \frac{T}{2A}\int \frac{ds}{t(s)} = 2GA\theta$$

$$\Rightarrow \theta = \frac{T}{4GA^2}\int \frac{ds}{t(s)}$$

For uniform thickness shaft

$$\boxed{\theta = \frac{Ts}{4GA^2 t}}$$

$s \rightarrow$ length of the centerline of the c/s.

Ex

A lengthwise slit annular tube and a closed annular tube have the same mean diameter D and same tube thickness t, where $D = 16t$. If both tubes are subjected to the same torque, what is the ratio of twist and the ratio of maximum c/s stresses.

For the slitted annular section

$$\text{Torque} = T_S = G\theta \frac{1}{3} \pi t^3 D$$
$$= T$$

$$= \frac{\pi\, 16 G\theta}{3} t^4 \Rightarrow \theta_s = \frac{3T}{16\pi G t^4}$$

For the closed annular section

$$\text{Torque} = T_c = \frac{4G\theta A^2 t}{S} = \frac{4G\theta \pi^2 D^4 t}{16\pi D} = 1024\, G\theta \pi t^4$$
$$= T$$

$$\Rightarrow \theta_c = \frac{T}{1024 \pi G t^4}$$

$$\therefore \frac{\theta_s}{\theta_c} = \frac{3T}{16\pi G t^4} \cdot \frac{1024 \pi G t^4}{T} = 192 \quad \underline{\underline{\text{Ans}}}$$

Maximum shear stress

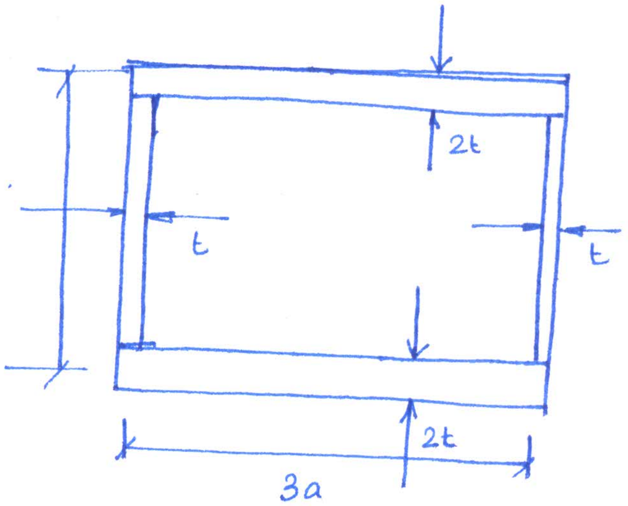
$$\tau^s_{max} = G\theta_s t = \frac{3T}{16\pi t^3}$$

$$\tau^c_{max} = \tau = \frac{2G\theta_c A}{S} = \frac{2G\theta_c \pi D^2}{4\pi D} = \frac{G\theta_c D}{2} = 8G\theta_c t$$

$$= \frac{8T}{1024\pi t^3}$$

$$\therefore \frac{\tau^s_{max}}{\tau^c_{max}} = \frac{3T}{16\pi t^3} \cdot \frac{1024\pi t^3}{8T} = 24 \quad \underline{\underline{\text{Ans}}}$$

Example



A doubly symmetric box beam as shown is subjected to a torque M_t. Find the twist per unit length for this torque.

Torque $M_t = 2Ah$

$= 2A\tau t(s)$

$= 2 \times 3a^2 \tau t(s)$

$$\Rightarrow \tau = \frac{M_t}{6a^2 t(s)} \qquad \text{---------} \quad (1)$$

$$\int \tau \, ds = 2GA\theta = 6Ga^2\theta$$

$$\Rightarrow \theta = \frac{M_t}{36Ga^3}\int \frac{ds}{t(s)} = \frac{M_t}{36Ga^3}\left[\frac{2a}{t} + \frac{6a}{2t}\right]$$

$$= \frac{M_t}{36Ga^3}\left[\frac{5a}{t}\right] = \frac{5M_t}{36Ga^2 t}$$ Ans