Q1. The 60-W fan of a central heating system is to circulate air through the ducts. The analysis of the flow shows that the fan needs to raise the pressure of air by 50 Pa to maintain flow. The fan is located in a horizontal flow section whose diameter is 30 cm at both the inlet and the outlet. Determine the highest possible average flow velocity in the duct.

$\Delta P = 50$ Pa

Air
V m/s

$D = 30$ cm

60 W
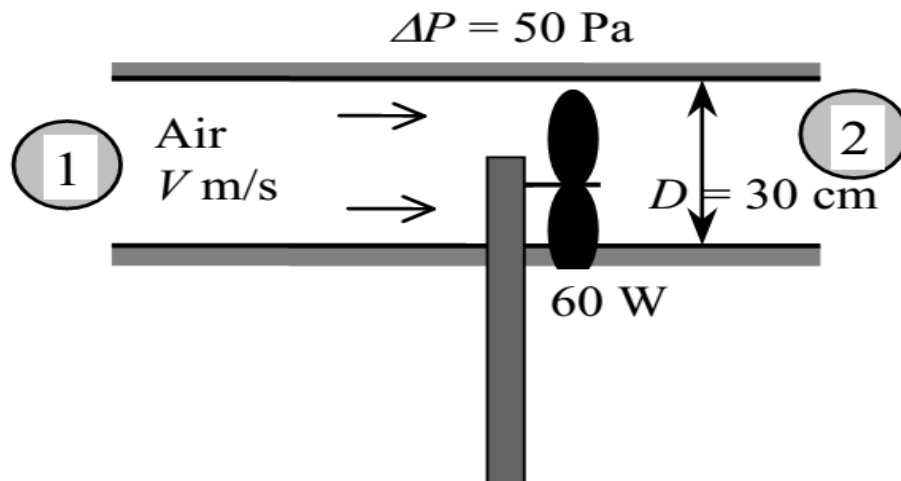
Q2. A piston–cylinder device contains 50 kg of water at 250 kPa and 25°C. The cross-sectional area of the piston is 0.1 m². Heat is now transferred to the water, causing part of it to evaporate and expand. When the volume reaches 0.2 m³, the piston reaches a linear spring whose spring constant is 100 kN/m. More heat is transferred to the water until the piston rises 20 cm more. Determine (a) the final pressure and temperature and (b) the work done during this process. Also, show the process on a P-V diagram.

$A = 0.1$ m²
H_2O
$m = 50$ kg
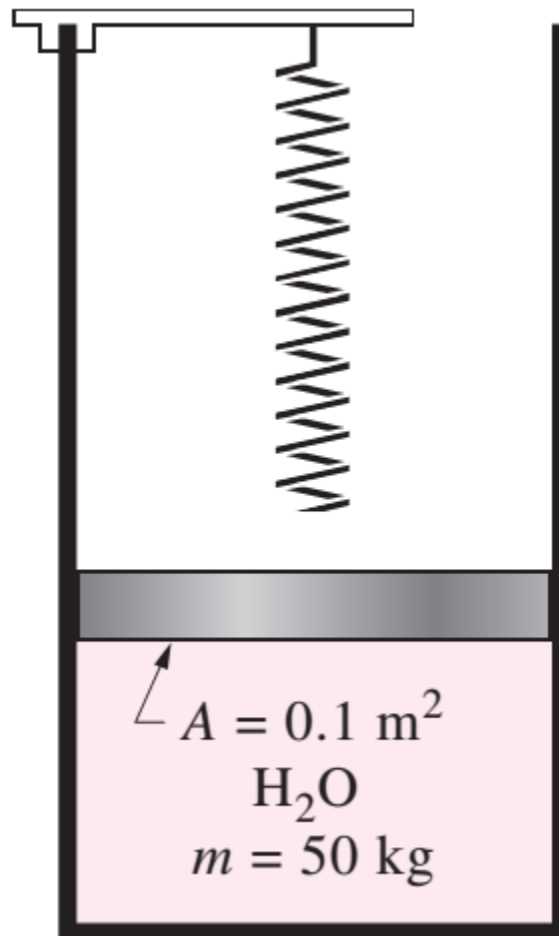
Q3. A piston–cylinder device contains 0.15 kg of air initially at 2 MPa and 350°C. The air is first expanded isothermally to 500 kPa, then compressed polytropically with a polytropic exponent of 1.2 to the initial pressure, and finally compressed at the constant pressure to the initial state. Determine the boundary work for each process and the net work of the cycle.

Q4. Air enters an adiabatic nozzle steadily at 300 kPa, 200°C, and 30 m/s and leaves at 100 kPa and 180 m/s. The inlet area of the nozzle is 80 cm². Determine (a) the mass flow rate through the nozzle, (b) the exit temperature of the air, and (c) the exit area of the nozzle.

$P_1 = 300$ kPa
$T_1 = 200$°C
$V_1 = 30$ m/s
$A_1 = 80$ cm²

AIR
→

$P_2 = 100$ kPa
$V_2 = 180$ m/s
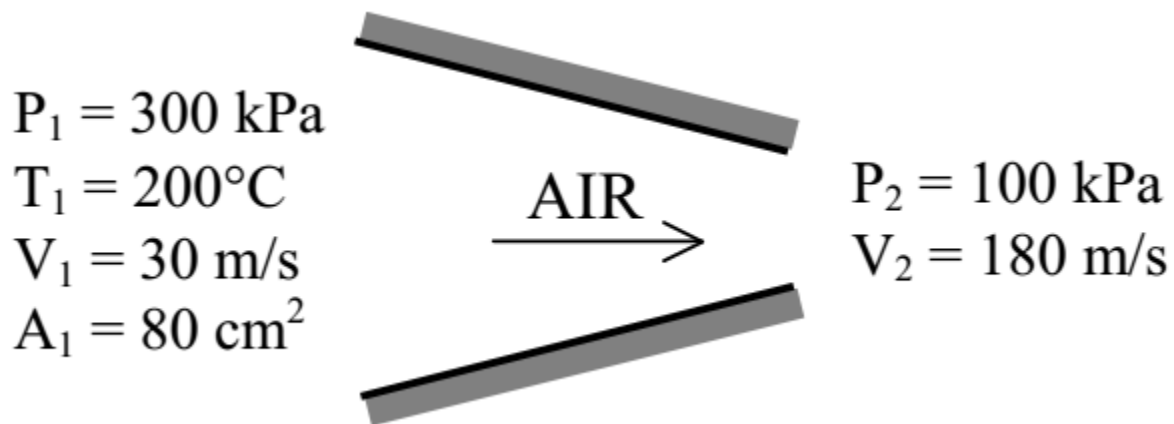
SOLUTIONS

Soln1. Highest possible average flow velocity in the duct =17 m/s

Soln2. (a) Final pressure and temperature 450 kPa, 147.9°C

 (b) Work done = 44.5kJ

Soln3. For the isothermal expansion process: W= 37.18kJ

 For the polytropic compression process: W= - 34.86kJ

For the constant pressure compression process W= - 6.97kJ

 Net work for the cycle = - 4.65kJ

Soln4. (a) Mass flow rate through the nozzle = 0.5304 kg/s

 (b) Temperature = 184.6°C

 (c) Exit area of the nozzle = 38.7 cm^2

<u>**Solution1**</u>. The fan of a central heating system circulates air through the ducts. For a specified pressure rise, the highest possible average flow velocity is to be determined.

Assumptions: 1) The fan operates steadily.

2) The changes in kinetic and potential energies across the fan are negligible.

Analysis: for a control volume that encloses the fan unit, the energy balance can be written as:

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{dE_{system} / dt}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}}^{\nearrow 0 \text{ (steady)}} = 0 \quad \rightarrow \quad \dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{in} + \dot{m}(Pv)_1 = \dot{m}(Pv)_2 \quad \rightarrow \quad \dot{W}_{in} = \dot{m}(P_2 - P_1)v = \dot{V}\,\Delta P$$

since $\dot{m}=(\dot{V}/v)$ and the changes in kinetic and potential energies of gasoline are negligible, Solving for volume flow rate and substituting, the maximum flow rate and velocity are determined to be :

$$\dot{V}_{max} = \frac{\dot{W}_{in}}{\Delta P} = \frac{60 \text{ J/s}}{50 \text{ Pa}}\left(\frac{1\,\text{Pa}\cdot\text{m}^3}{1\,\text{J}}\right) = 1.2 \text{ m}^3/\text{s}$$

$$V_{max} = \frac{\dot{V}_{max}}{A_c} = \frac{\dot{V}_{max}}{\pi D^2 / 4} = \frac{1.2 \text{ m}^3/\text{s}}{\pi (0.30 \text{ m})^2/4} = \textbf{17.0 m/s}$$

Solution2.

Water in a cylinder equipped with a spring is heated and evaporated. The vapor expands until it compresses the spring 20 cm. The final pressure and temperature, and the boundary work done are to be determined, and the process is to be shown on a P-V diagram

Assumptions The process is quasi-equilibrium.

Analysis (a) The final pressure is determined from

$$P_3 = P_2 + \frac{F_s}{A} = P_2 + \frac{kx}{A} = (250 \text{ kPa}) + \frac{(100 \text{ kN/m})(0.2 \text{ m})}{0.1 \text{ m}^2}\left(\frac{1 \text{ kPa}}{1 \text{ kN/m}^2}\right) = \textbf{450 kPa}$$

The specific and total volumes at the three states are

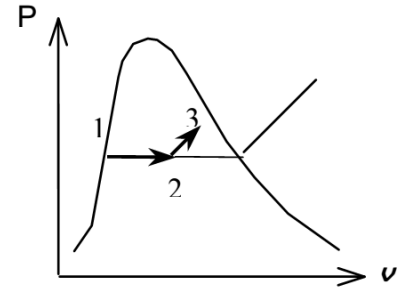$$\left.\begin{array}{l} T_1 = 25°C \\ P_1 = 250 \text{ kPa} \end{array}\right\} v_1 \cong v_{f@25°C} = 0.001003 \text{ m}^3/\text{kg}$$

$$V_1 = m v_1 = (50 \text{ kg})(0.001003 \text{ m}^3/\text{kg}) = 0.05 \text{ m}^3$$

$$V_2 = 0.2 \text{ m}^3$$

$$V_3 = V_2 + x_{23}A_p = (0.2 \text{ m}^3) + (0.2 \text{ m})(0.1 \text{ m}^2) = 0.22 \text{ m}^3$$

$$v_3 = \frac{V_3}{m} = \frac{0.22 \text{ m}^3}{50 \text{ kg}} = 0.0044 \text{ m}^3/\text{kg}$$

At 450 kPa, $v_f = 0.001088 \text{ m}^3/\text{kg}$ and $v_g = 0.41392 \text{ m}^3/\text{kg}$. Noting that $v_f < v_3 < v_g$, the final state is a saturated mixture and thus the final temperature is

$$T_3 = T_{sat@450 \text{ kPa}} = \textbf{147.9°C}$$

(b) The pressure remains constant during process 1-2 and changes linearly (a straight line) during process 2-3. Then the boundary work during this process is simply the total area under the process curve,

$$W_{b,out} = \text{Area} = P_1(V_2 - V_1) + \frac{P_2 + P_3}{2}(V_3 - V_2)$$

$$= \left((250 \text{ kPa})(0.2 - 0.05)\text{m}^3 + \frac{(250 + 450) \text{ kPa}}{2}(0.22 - 0.2)\text{m}^3\right)\left(\frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3}\right)$$

$$= \textbf{44.5 kJ}$$

Solution3.

A piston-cylinder device contains air gas at a specified state. The air undergoes a cycle with three processes. The boundary work for each process and the net work of the cycle are to be determined
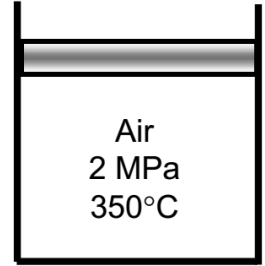
Properties The properties of air are $R = 0.287$ kJ/kg.K , $k = 1.4$ (Table A-2a).

Analysis For the isothermal expansion process:

$$V_1 = \frac{mRT}{P_1} = \frac{(0.15\,\text{kg})(0.287\,\text{kJ/kg.K})(350+273\,\text{K})}{(2000\,\text{kPa})} = 0.01341\,\text{m}^3$$

$$V_2 = \frac{mRT}{P_2} = \frac{(0.15\,\text{kg})(0.287\,\text{kJ/kg.K})(350+273\,\text{K})}{(500\,\text{kPa})} = 0.05364\,\text{m}^3$$

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                Air
               2 MPa
               350°C
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$$W_{b,1-2} = P_1 V_1 \ln\left(\frac{V_2}{V_1}\right) = (2000\,\text{kPa})(0.01341\,\text{m}^3)\ln\left(\frac{0.05364\,\text{m}^3}{0.01341\,\text{m}^3}\right) = \textbf{37.18 kJ}$$

For the polytropic compression process:

$$P_2 V_2^n = P_3 V_3^n \longrightarrow (500\,\text{kPa})(0.05364\,\text{m}^3)^{1.2} = (2000\,\text{kPa})V_3^{1.2} \longrightarrow V_3 = 0.01690\,\text{m}^3$$

$$W_{b,2-3} = \frac{P_3 V_3 - P_2 V_2}{1-n} = \frac{(2000\,\text{kPa})(0.01690\,\text{m}^3)-(500\,\text{kPa})(0.05364\,\text{m}^3)}{1-1.2} = \textbf{-34.86 kJ}$$

For the constant pressure compression process:

$$W_{b,3-1} = P_3(V_1 - V_3) = (2000\,\text{kPa})(0.01341 - 0.01690)\,\text{m}^3 = \textbf{-6.97 kJ}$$

The net work for the cycle is the sum of the works for each process

$$W_{net} = W_{b,1-2} + W_{b,2-3} + W_{b,3-1} = 37.18 + (-34.86) + (-6.97) = \textbf{-4.65 kJ}$$

Solution4.

Air is accelerated in a nozzle from 30 m/s to 180 m/s. The mass flow rate, the exit temperature, and the exit area of the nozzle are to be determined.

Assumptions : 1. This is a steady flow process since there is no change with time 2. Air is an ideal gas with constant specific heats 3. Potential energy changes are negligible 4. The device is adiabatic and thus heat transfer is negligible 5. There are no work interactions.

Gas constant for air = $0.287 \text{kPam}^3/\text{kgK}$. C_p for air at 450K = 1.02kJ/kg°C

Analysis : Since there is only one inlet and one exit $\dot{m}_1 = \dot{m}_2 = \dot{m}$ .Using the ideal gas relation the specific volume and mass flow rate of air are determined to be :

$$\nu_1 = \frac{RT_1}{P_1} = \frac{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(473 \text{ K})}{300 \text{ kPa}} = 0.4525 \text{ m}^3/\text{kg}$$

$$\dot{m} = \frac{1}{\nu_1} A_1 V_1 = \frac{1}{0.4525 \text{ m}^3/\text{kg}} (0.008 \text{ m}^2)(30 \text{ m/s}) = \mathbf{0.5304 \text{ kg/s}}$$

(*b*) We take nozzle as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{system}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + V_1^2/2) = \dot{m}(h_2 + V_2^2/2) \quad (\text{since } \dot{Q} \cong \dot{W} \cong \Delta pe \cong 0)$$

$$0 = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} \longrightarrow 0 = c_{p,ave}(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$

Substituting, $\quad 0 = (1.02 \text{ kJ/kg} \cdot \text{K})(T_2 - 200°\text{C}) + \frac{(180 \text{ m/s})^2 - (30 \text{ m/s})^2}{2} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right)$

It yields $\quad T_2 = \mathbf{184.6°C}$

(*c*) The specific volume of air at the nozzle exit is

$$\nu_2 = \frac{RT_2}{P_2} = \frac{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(184.6 + 273 \text{ K})}{100 \text{ kPa}} = 1.313 \text{ m}^3/\text{kg}$$

$$\dot{m} = \frac{1}{\nu_2} A_2 V_2 \longrightarrow 0.5304 \text{ kg/s} = \frac{1}{1.313 \text{ m}^3/\text{kg}} A_2 (180 \text{ m/s}) \rightarrow A_2 = 0.00387 \text{ m}^2 = \mathbf{38.7 \text{ cm}^2}$$