



Find the shear center for the given isosceles triangle.

$$I_{xx} = \frac{1}{12} (30t)^3 t + 2 \int y^2 dA = 2250t^4 + 2 \int_0^{30t} s^2 \sin^2 \theta ds$$

$$= 2250t^4 + \frac{2}{3} t^2 \sin^2 \theta s^3 \Big|_0^{30t} = 2250t^4 + \frac{2}{3} \sin^2 \theta (30t)^3 t$$

$$= 8100t^4$$

Since $I_{xy} = 0$ and $S_x = 0$

$$q - q_0 = \frac{-S_y}{I_{xx}} \int_0^s ty ds$$

Assuming $q = q_0$ at ①, we get,

For portion 12

$$q_{12} - q_0 = \frac{-S_y t}{I_{xx}} \int_0^s y ds$$

$$= \frac{-S_y t}{I_{xx}} \int_0^s s \sin \theta ds = \frac{-S_y t \sin \theta}{2 I_{xx}} s^2$$

$$\Rightarrow q_{12} = \frac{-S_y t s^2 \sin \theta}{2 I_{xx}} + q_0 = \frac{-S_y t s^2}{2 I_{xx}} \left(\frac{15}{39} \right) + q_0$$

$$q_2 = \frac{-S_y t (30t)^2 \left(\frac{15}{39} \right)}{2 I_{xx}} + q_0 = \frac{-S_y t^3}{I_{xx}} \frac{1407.5}{292.5} + q_0$$

For portion 23

$$q_{23} - q_2 = \frac{-S_y t}{I_{xx}} \int_0^s (15t - s) ds + \frac{S_y t^3}{I_{xx}} \frac{292.5}{1407.5} + q_0$$

$$q_3 = \frac{-S_y t (15ts - \frac{s^2}{2})}{I_{xx}} \Big|_0^{30t} - \frac{292.5 S_y t^3}{I_{xx}} + q_0$$

$$q_3 = -292.5 \frac{S_y t^3}{I_{xx}} + q_0$$

For portion 31

$$q_{31} - q_3 = -\frac{S_y}{I_{xx}} \int ty ds = -\frac{S_y t}{I_{xx}} \int_0^s \left(\frac{15s}{39} - 15t\right) ds$$

$$= -\frac{S_y t}{I_{xx}} \left[\frac{15s^2}{2 \times 39} - 15ts \right]$$

$$q_{31} = -\frac{S_y t}{I_{xx}} \left[\frac{5}{26} s^2 - 15ts \right] - \frac{292.5 S_y t^3}{I_{xx}} + q_0$$

$$q_1 = -\frac{S_y t}{I_{xx}} [-292.5 t^2] - \frac{292.5 S_y t^3}{I_{xx}} + q_0 = q_0$$

Assuming that S_y is acting through the shear center,

$$\int q ds = 0$$

$$\Rightarrow q_0 (2 \times 39t + 30t) - \frac{S_y t}{2 I_{xx}} \cdot \frac{5}{13} \int_0^{39t} s^2 ds - \frac{S_y t^3}{I_{xx}} 292.5 \int_0^{30t} ds$$

$$- \frac{S_y t}{I_{xx}} \int_0^{30t} \left(15ts - \frac{s^2}{2}\right) ds - \frac{292.5 S_y t^3}{I_{xx}} \int_0^{39t} ds - \frac{S_y t}{I_{xx}} \int_0^{39t} \left(\frac{5}{26} s^2 - 15ts\right) ds = 0$$

$$\Rightarrow 108tq_0 - \frac{S_y t}{I_{xx}} \cdot \frac{5}{6 \times 13} s^3 \Big|_0^{39t} - 292.5 \times 30t \frac{S_y t^3}{I_{xx}} - \frac{S_y t}{I_{xx}} \left(\frac{15ts^2}{2} - \frac{s^3}{6}\right) \Big|_0^{30t}$$

$$- \frac{292.5 S_y t^3}{I_{xx}} 39t - \frac{S_y t}{I_{xx}} \left(\frac{5s^3}{3 \times 26} - \frac{15ts^2}{2}\right) \Big|_0^{39t} = 0$$

$$\Rightarrow 108tq_0 - \frac{S_y t^4}{I_{xx}} \left[3802.5 + 8775 + 6750 - 4500 + 114075 + 3802.5 - 11407.5 \right] = 0$$

$$\Rightarrow 108tq_0 = \frac{18630}{8100} S_y \Rightarrow q_0 = \frac{0.0213 S_y}{t}$$

Shear Center Taking moment about ①

$$\begin{aligned} S_y (e_y + 36t) &= \frac{S_y t}{I_{xx}} \int_0^{30t} \left(15ts - \frac{s^2}{2} \right) 36t ds \\ &\quad + \frac{292.5 S_y t^3}{I_{xx}} \int_0^{30t} 36t ds - q_0 \int_0^{30t} 36t ds \\ &= \frac{S_y t}{I_{xx}} 36t \left(\frac{15ts^2}{2} - \frac{s^3}{6} \right) \Big|_0^{30t} + \frac{292.5 S_y t^3}{I_{xx}} 36t \times 30t \\ &\quad - q_0 36t \cdot 30t \\ &= 25.996 S_y t \end{aligned}$$

$$\Rightarrow \boxed{e_y = -10t}$$