AE 230
Fluid Systems

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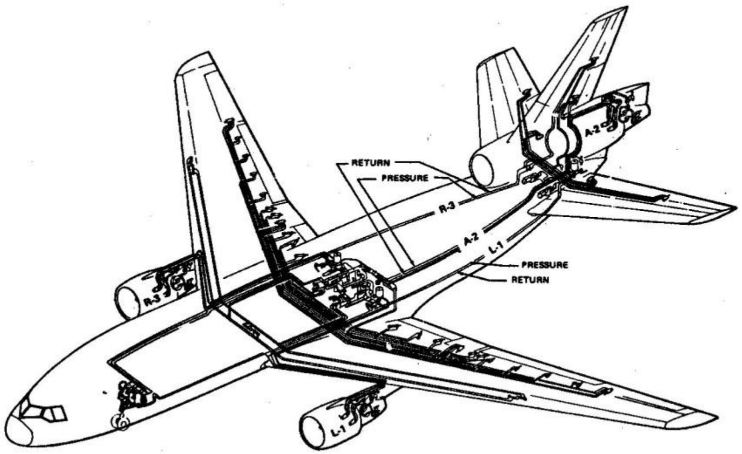
Fluid Systems

- Water supply in home, city etc.
- Transportation of gas, oil etc in pipe lines
- Flow of chemicals in the industry
- Flow of air in industry
- Flow of oil in aircraft
- Flow of fuel in aircraft
- Flow water in hydraulic power plant

HYDRAULIC POWER SYSTEM AND PIPING



HYDRAULIC SYSTEM

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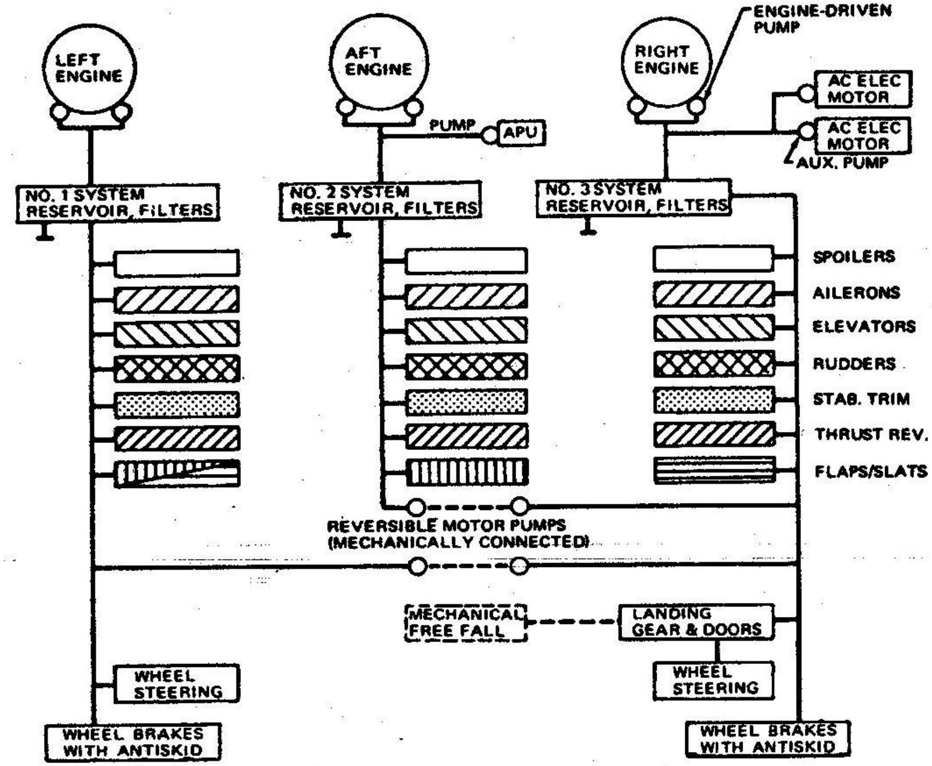
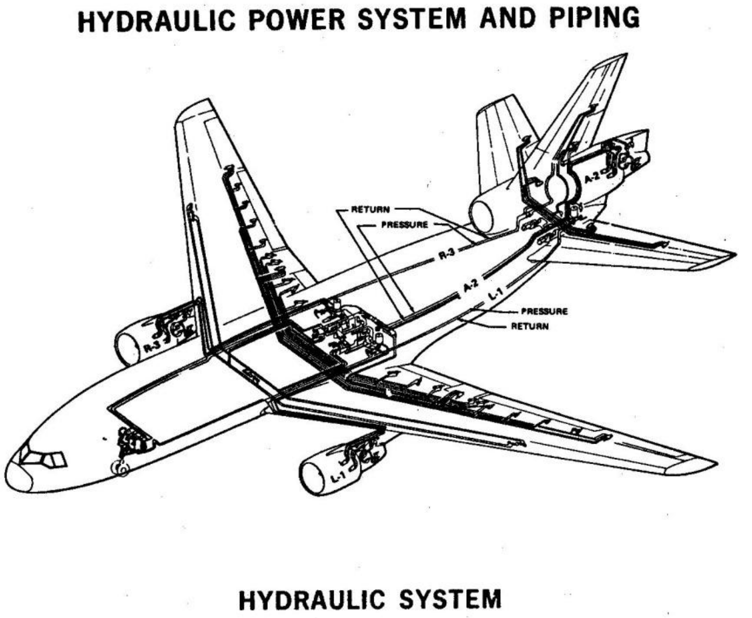


Aircraft hydraulic system.

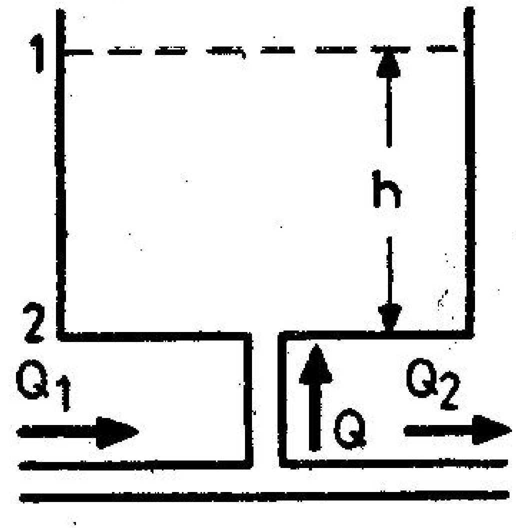


- Pipes carrying liquid
- Valves to control the flow
- Pumps to pressurise the fluid
- Heat exchanger to remove heat
- Tanks to balance the weight

Fluid systems - hydraulic capacitance

Storage tank of constant area A

$$P_2 - P_1 = P = \rho g h \qquad h = \frac{P}{\rho g}$$



(a) Storage tank

$\rho =$ Mass density of the fluid

$h =$ Height of the fluid level in the tank

$A =$ Cross-sectional area of the tank

Volume flow rate $Q_1 - Q_2 = Q = A\dfrac{dh}{dt} = A\dfrac{d}{dt}\left(\dfrac{P}{\rho g}\right) = \left(\dfrac{A}{\rho g}\right)\dfrac{dP}{dt}$

Q is equivalent of current and P is equivalent to voltage

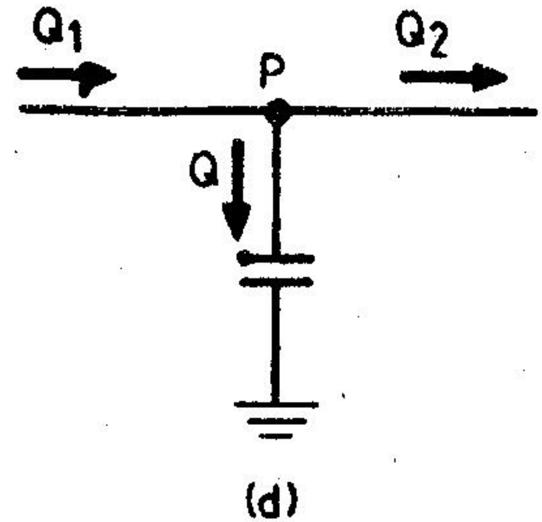
$$Q = \left(\frac{A}{\rho g}\right)\frac{dP}{dt} = C\frac{dP}{dt}$$

C is equivalent to hydraulic capacitance

Fluid systems – hydraulic capacitance

Storage tank of constant area A

$$P_2 - P_1 = P = \rho g h \qquad h = \frac{P}{\rho g}$$



Hydraulic capacitance.

$\rho = \text{Mass density of the fluid}$

$h = \text{Height of the fluid level in the tank}$

$A = \text{Cross-sectional area of the tank}$

Volume flow rate $Q_1 - Q_2 = Q = A\dfrac{dh}{dt} = A\dfrac{d}{dt}\left(\dfrac{P}{\rho g}\right) = \left(\dfrac{A}{\rho g}\right)\dfrac{dP}{dt}$

Q is equivalent of current and P is equivalent to voltage

$Q = \left(\dfrac{A}{\rho g}\right)\dfrac{dP}{dt} = C\dfrac{dP}{dt}$ C is equivalent to hydraulic capacitance

Fluid systems - hydraulic capacitance

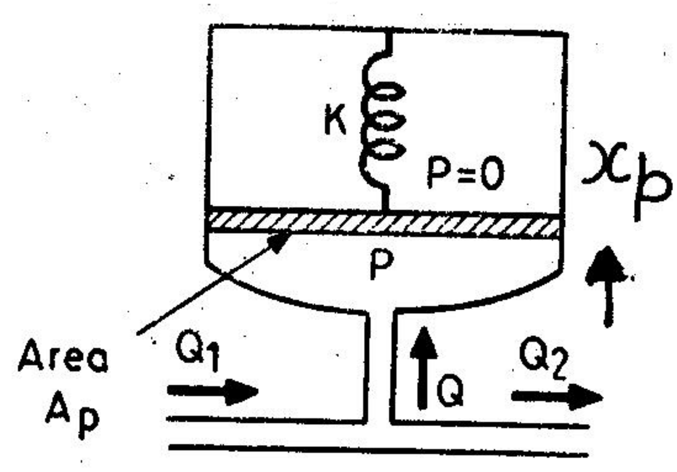
Accumulator

$$A_p P = K x_p$$

A_P = **Cross sectional area of the plate**

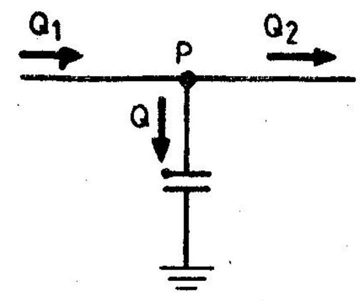
x_P = **displacement of the plate**

K = **Coefficient of the stiffness of the spring**



(b) Accumulator

$$A_p P = K \int_{-\infty}^{t} \frac{Q}{A_P} dt$$



(d)

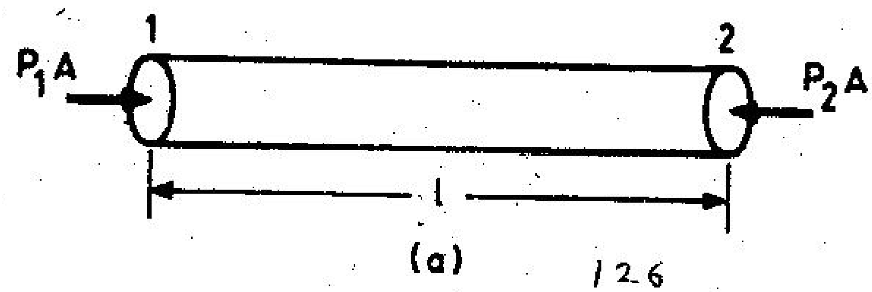
Hydraulic capacitance.

Differentiating above equation

$$A_p \frac{dP}{dt} = K \frac{Q}{A_P} \qquad Q_1 - Q_2 = Q = \frac{A_P^2}{K} \frac{dP}{dt} \qquad C = \frac{A_P^2}{K}$$

Fluid systems - hydraulic inertance

Related to inertia of the fluid



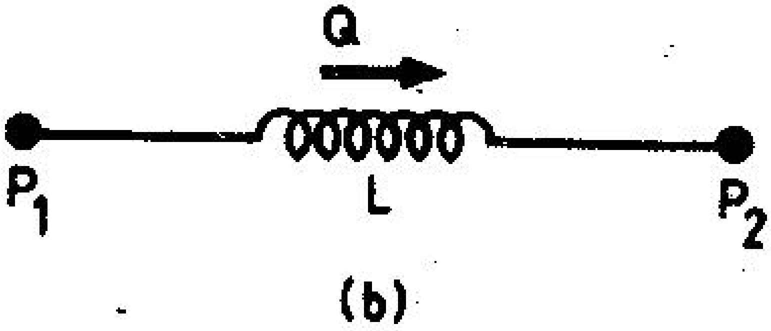
$$P_1 A - P_2 A = \rho A l \frac{dv}{dt}$$

$A = \text{Cross-sectional area of the pipe}$

$l = \text{length of the pipe}$

$v = Q / A = \text{average fluid velocity in pipe}$

$$P_1 - P_2 = \frac{\rho l}{A} \frac{dQ}{dt} \qquad L = \frac{\rho l}{A}$$

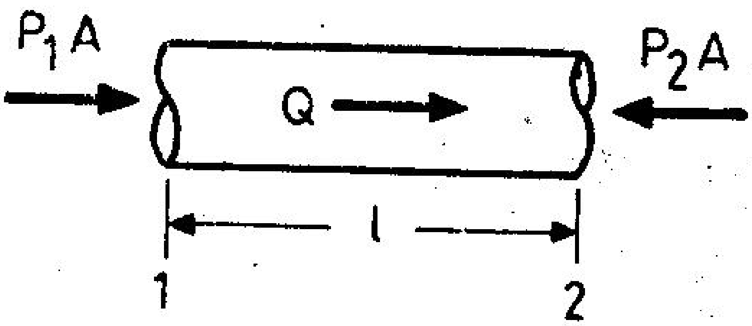


Fluid systems - hydraulic resistance

Related to resistance of the fluid

$$P_1 - P_2 = P = \frac{128 l \mu}{\pi D^4} Q \text{ (for laminar flow)}$$

$$P_1 - P_2 = P = \frac{8 K_t l \rho}{\pi^2 D^5} Q^2 = K_T Q^2 \text{ (for turbulent flow)}$$



$D =$ Diameter of the pipe

$l =$ length of the pipe

$\mu =$ viscocity of the fluid

$K_t =$ Constant to be determined experimentally

$P = Q\,R$ analogus to $e = iR$

Fluid systems - hydraulic resistance

$$P_1 - P_2 = P = \frac{8K_t l\rho}{\pi^2 D^5} Q^2 = K_T Q^2 \text{(for turbulent flow)}$$

Resistance is non-linear; Can be liberalized about an operating point

Steady state $$P_0 = K_T Q_0^2$$

Retaining only the first order terms in Taylor series expansion

$$P = P_0 + \frac{dP}{dQ}_{(P_0 - Q_0)} (Q - Q_0)$$

$$P - P_0 = 2K_T Q_0 (Q - Q_0)$$

$$\tilde{P} = \tilde{Q}\, R \qquad R = 2K_T Q_0$$

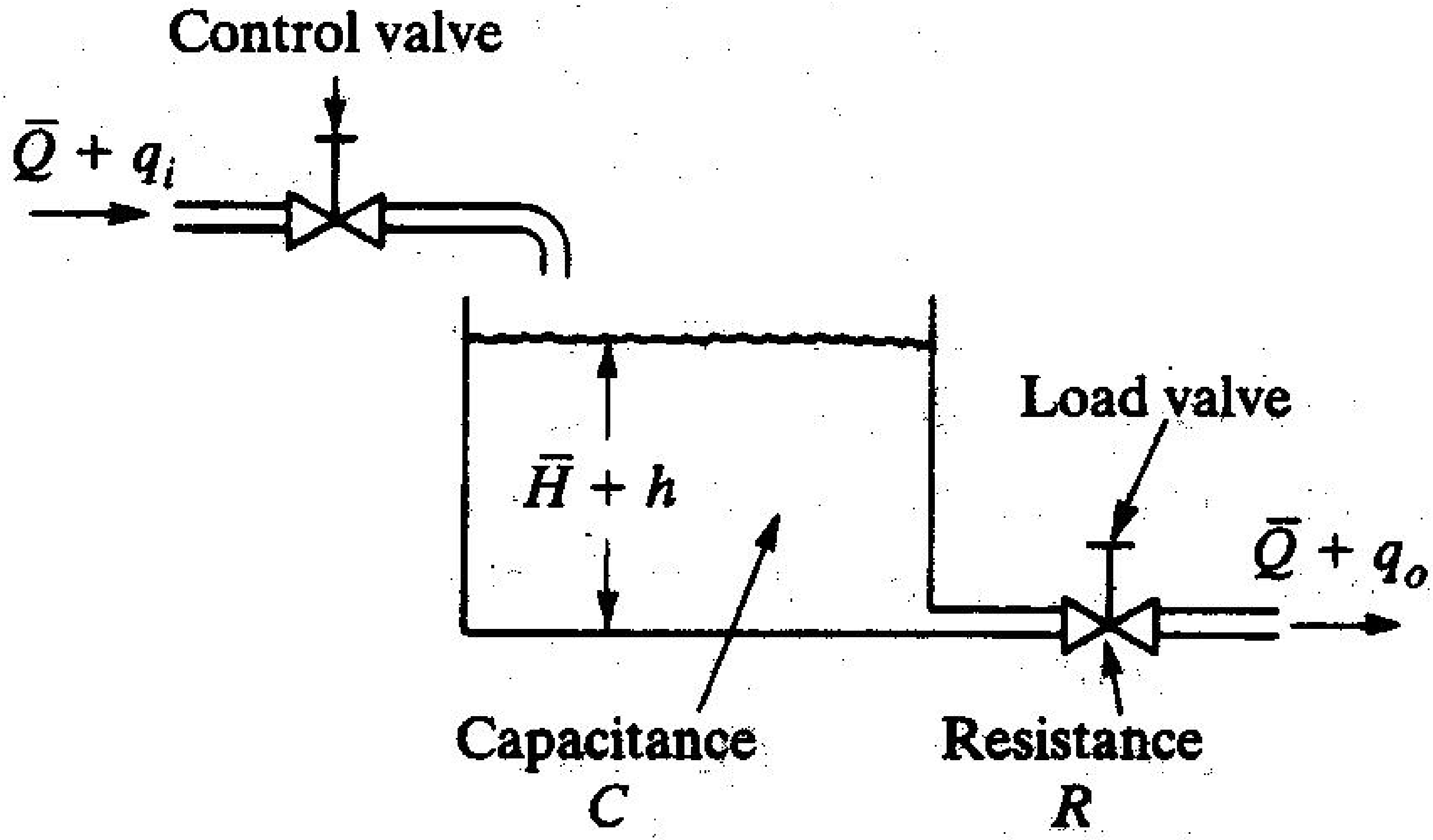
Fluid systems – Capacitance, inertance resistance

Capacitance : Capacitance of a tank is defined as the change in quantity of stored liquid necessary to cause a unit change in potential or head.

Inertance: Change in potential required to make a unit change in flow rate, velocity or current.

Resistance: Change in level difference (potential) necessary to cause a unit change in flow

Fluid systems – Fluid tank problem

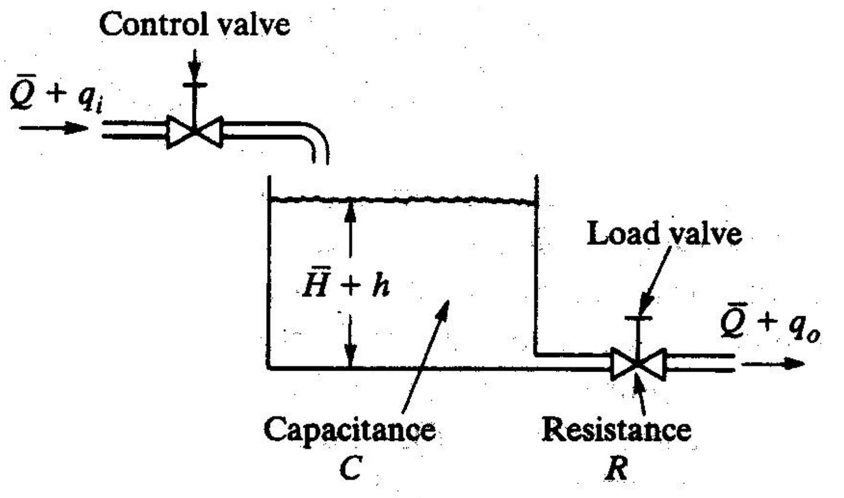


(a)

Fluid systems – Fluid tank problem

Flow in the valve can be laminar or turbulent

For laminar flow $Q = K_l H$



(a)

$Q =$ Steady state flow rate

$K_l =$ Constant

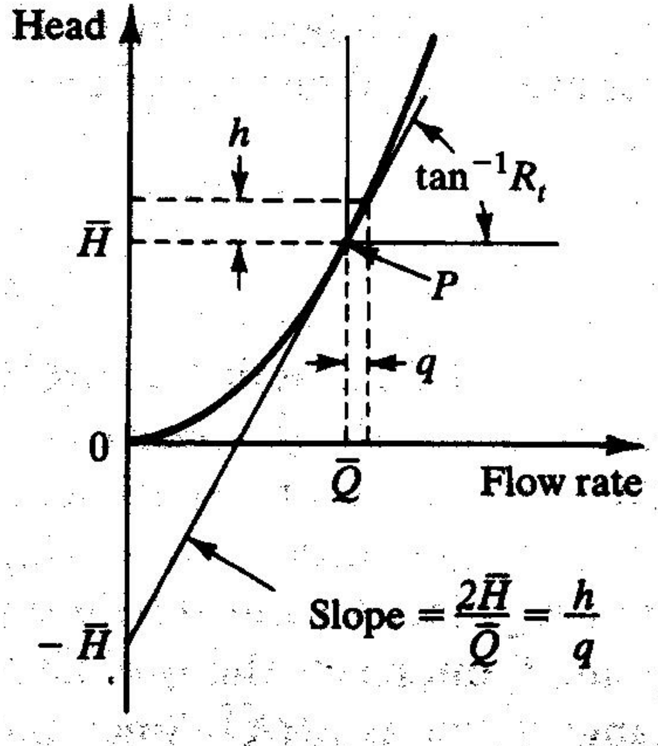
$H =$ Steady state head

For laminar flow resistance $R_l = \dfrac{dH}{dQ} = \dfrac{1}{K_l} = \dfrac{H}{Q}$

Fluid systems – Fluid tank problem

Flow in the valve can be laminar or turbulent

For turbulent flow $Q = K_t \sqrt{H}$



(b)

Q = Steady state flow rate

K_t = Constant

H = Steady state head

For turbulent flow resistance $R_t = \dfrac{dH}{dQ}$

$$dQ = \frac{K_t}{2\sqrt{H}} dH \qquad R_t = \frac{2H}{Q}$$

Fluid systems – Fluid tank problem

$\overline{H}$ **= Steady state head**

h **= Small deviation from Steady state head**

$\overline{Q}$ **= Steady state flow rate**

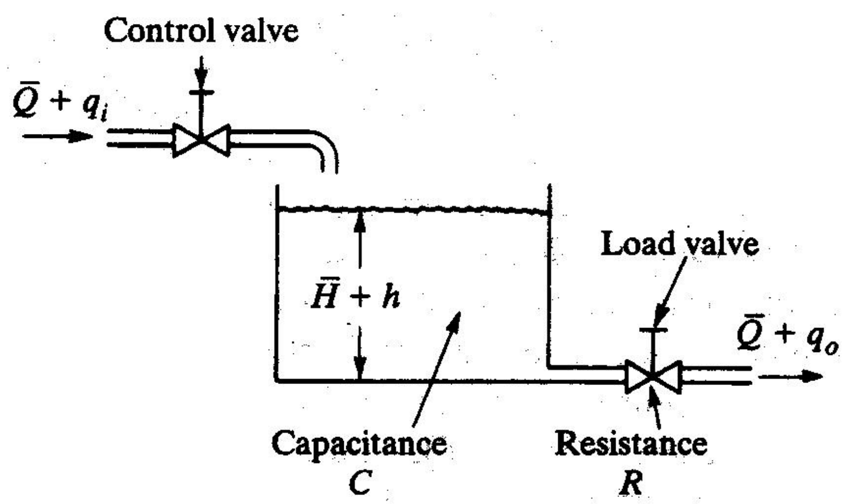
q_i **= Small deviation of inflow rate from its steady state value**

q_o **= Small deviation of outflow rate from its steady state value**

$$C dh = (q_i - q_o) dt$$

$$R = \frac{dH}{dQ} = \frac{h}{q_o}$$

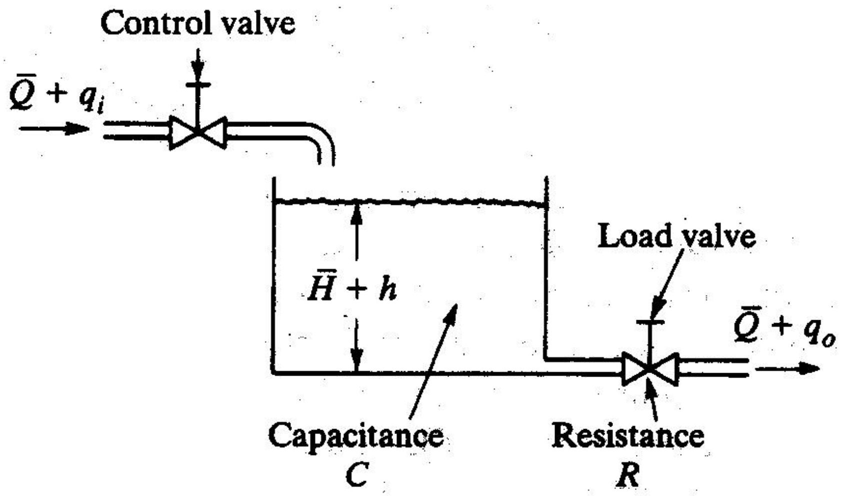
$$C\frac{dh}{dt} = q_i - \frac{h}{R}$$



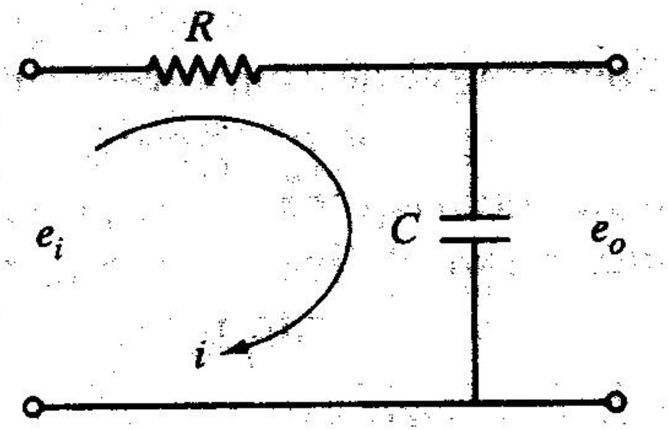
(a)

$$RC\frac{dh}{dt} + h = Rq_i \qquad RC\frac{dq_0}{dt} + q_0 = q_i$$

Fluid systems – Fluid tank problem

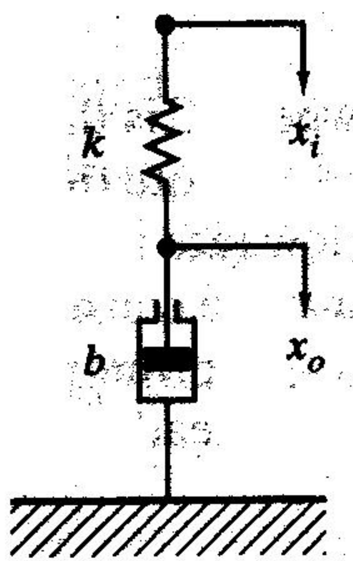


(a)



$$RC\frac{dh}{dt} + h = Rq_i$$

$$RC\frac{dq_0}{dt} + q_0 = q_i$$



(b)

$$RC\frac{de_0}{dt} + e_0 = e_i$$

$$\frac{b}{k}\frac{dx_0}{dt} + x_0 = x_i$$