1.

Applied torque $M_t = M_t^1 + M_t^2 + M_t^3$

$$= \frac{1}{3} b t^3 G\theta + \frac{1}{3} a t^3 G\theta + \frac{1}{3} a t^3 G\theta$$

$$= \frac{1}{3}\left(b t^3 + 2 a t^3\right) G\theta.$$

$\therefore$ Rate of twist $\theta = \cancel{3M_t} \dfrac{3 M_t}{G(2a+b)t^3}$ 3 + 1

Max^m shear stress using membrane analogy

$$= \tau_{max} = G\theta t$$

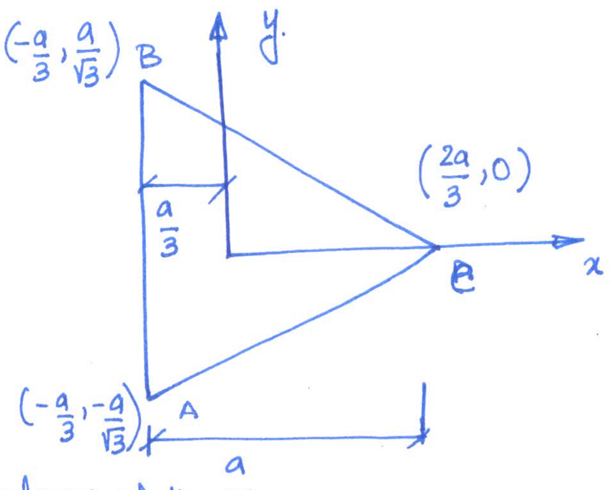
$$= \frac{3 M_t t}{(2a+b)t^3}$$

$$= \frac{3 M_t}{(2a+b)t^2}$$

1

1

Q.2



Eqns. of the boundary of the c/s

(i) AB $\longrightarrow$ $x + \frac{a}{3} = 0$

(ii) AC $\longrightarrow$ $y - \frac{x}{\sqrt{3}} + \frac{2a}{3\sqrt{3}} = 0$

(iii) BC $\longrightarrow$ $y + \frac{x}{\sqrt{3}} - \frac{2a}{3\sqrt{3}} = 0$

Condn 1

The expression satisfying the above equations,

$$\left(x+\frac{a}{3}\right)\left(y-\frac{x}{\sqrt{3}}+\frac{2a}{3\sqrt{3}}\right)\left(y+\frac{x}{\sqrt{3}}-\frac{2a}{3\sqrt{3}}\right)=0 \quad \text{———} \quad 5$$

$$\Rightarrow \left(x+\frac{a}{3}\right)\left[y^2-\left(\frac{x}{\sqrt{3}}-\frac{2a}{3\sqrt{3}}\right)^2\right]=0$$

$$\Rightarrow \frac{2a}{3}\left[\frac{1}{2}(x^2+y^2)-\frac{1}{2a}(x^3-3xy^2)-\frac{2a^2}{27}\right]=0$$

$\varphi = 0$ is satisfied on the boundary.

Condn 2

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2} = -G\theta\frac{\partial}{\partial x}\left[x-\frac{3}{2a}x^2\right] - G\theta\frac{\partial}{\partial y}\left[y+\frac{3\times 2}{2a}xy\right]$$

$$= -G\theta\left[1-\frac{6}{2a}x+1+\frac{6x}{2a}\right] = -2G\theta.$$

$$\Rightarrow \frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2} = -2G\theta \quad \text{———} \quad 1$$

Stress distribution

$$\tau_{xz} = \frac{\partial\varphi}{\partial y} = -G\theta\left[y+\frac{6}{2a}xy\right] = -G\theta\left[y+\frac{3}{a}xy\right]$$

$$\tau_{yz} = -\frac{\partial\varphi}{\partial x} = G\theta\left[x-\frac{3x^2}{2a}+\frac{3y^2}{2a}\right] \quad \text{———} \quad 1$$

Applied torque $M_t = 2\iint \varphi \, dx\,dy$

$$= -2G\theta \iint \left[\frac{1}{2}(x^2+y^2) - \frac{1}{2a}(x^3-3xy^2) - \frac{2a^2}{27}\right]dx\,dy$$

$$= \frac{2G\theta . 3}{2a}\iint \left(x+\frac{a}{3}\right)\left(y-\frac{x}{\sqrt{3}}+\frac{2a}{3\sqrt{3}}\right)\left(y+\frac{x}{\sqrt{3}}-\frac{2a}{3\sqrt{3}}\right)dx\,dy$$

$$= \frac{3G\theta}{a}\int_{-\frac{a}{3}}^{\frac{2a}{3}}\int_{-\left(\frac{x}{\sqrt{3}}-\frac{2a}{3\sqrt{3}}\right)}^{\left(\frac{x}{\sqrt{3}}-\frac{2a}{3\sqrt{3}}\right)} \left(x+\frac{a}{3}\right)\left(y^2-\left(\frac{x}{\sqrt{3}}-\frac{2a}{3\sqrt{3}}\right)^2\right)dx\,dy$$

limits written correctly (2)

$$= \frac{3G\theta}{a}\int_{-\frac{a}{3}}^{2a/3}\left(x+\frac{a}{3}\right)\left(\frac{1}{3}y^3-\left(\frac{x}{\sqrt{3}}-\frac{2a}{3\sqrt{3}}\right)^2 y\right)\Bigg|_{-\left(\frac{x}{\sqrt{3}}-\frac{2a}{3\sqrt{3}}\right)}^{\frac{x}{\sqrt{3}}-\frac{2a}{3\sqrt{3}}} dx$$

$$= \frac{3G\theta}{a}\int_{-\frac{a}{3}}^{\frac{2a}{3}}\left(x+\frac{a}{3}\right)\left(-\frac{4}{3}\right)\left(\frac{x}{\sqrt{3}}-\frac{2a}{3\sqrt{3}}\right)^3 dx = -\frac{4G\theta}{a}\int_{-\frac{a}{3}}^{\frac{2a}{3}}\left(x+\frac{a}{3}\right)\left(\frac{x}{\sqrt{3}}-\frac{2a}{3\sqrt{3}}\right)^3 dx$$

Integrating by parts

$$\underbrace{\cancel{-\frac{4G\theta}{a}\left(x+\frac{a}{3}\right)\frac{\sqrt{3}}{4}\left(\frac{x}{\sqrt{3}}-\frac{2a}{3\sqrt{3}}\right)^4\Bigg|_{-a/3}^{\frac{2a}{3}}}}_{=0} + \frac{4G\theta}{a}\int_{-\frac{a}{3}}^{\frac{2a}{3}}\frac{\sqrt{3}}{4}\cdot\frac{\sqrt{3}}{5}\left(\frac{x}{\sqrt{3}}-\frac{2a}{3\sqrt{3}}\right)^5 dx$$

$$\Rightarrow \frac{3G\theta}{5a} - \left(\frac{-a}{3\sqrt{3}}-\frac{2a}{3\sqrt{3}}\right)^5 = \frac{3G\theta}{5a}\,\frac{a^5}{9\sqrt{3}} = \frac{G\theta a^4}{15\sqrt{3}}$$

$$\Rightarrow \boxed{\theta = \frac{15\sqrt{3}M_t}{Ga^4}}$$ rate of twist.

(3)

Warping of cross-section

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = -\theta\left[y + \frac{3}{a}xy\right]$$

Considering
$u = -\theta yz$
$V = \theta xz$ —— 1

$$\Rightarrow \frac{\partial w}{\partial x} = \frac{-3\theta}{a}xy. \Rightarrow w(x,y) = \frac{-3\theta x^2 y}{2a} + f_1(y)$$ —— (1) 1

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \theta\left[x - \frac{3x^2}{2a} + \frac{3y^2}{2a}\right]$$

$$\Rightarrow \theta\cancel{x} + \frac{\partial w}{\partial y} = \theta\left[\cancel{x} - \frac{3x^2}{2a} + \frac{3y^2}{2a}\right]$$

$$\Rightarrow \frac{\partial w}{\partial y} = \frac{-3\theta x^2}{2a} + \frac{3\theta y^2}{2a}.$$

$$\Rightarrow w = \frac{-3\theta x^2 y}{2a} + \frac{3\theta y^3}{6a} + f_2(x)$$

$$= \frac{-3\theta x^2 y}{2a} + \frac{\theta y^3}{2a} + f_2(x)$$ —— 1 —— (2)

∴ Equating (1) and (2), we get

$$f_2(x) = c$$

and $f_1(y) = \frac{\theta y^3}{2a} + c.$

$$\therefore w(x,y) = \frac{-3\theta x^2 y}{2a} + \frac{\theta y^3}{2a} + c$$ —— 1 + 1 —— (3)

Assuming ~~w = 0 at x, y, z = 0~~
no rigid body mode $c = 0$

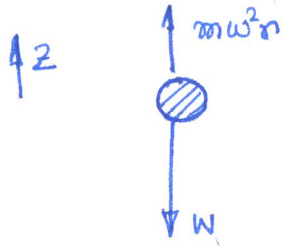
$$\therefore \boxed{w(x,y) = \frac{-3\theta x^2 y}{2a} + \frac{\theta y^3}{2a}}$$

Max^m shear stress will occur at the mid-point of each side.

—— 1

3. <u>Case 1</u>

The aircraft is in the top horizontal position of the vertical circle.



$$n_z = \frac{W - m\omega^2 r}{W}$$

$$= \frac{W - \frac{W}{g}\omega^2 r}{W}$$

$$= 1 - \frac{\omega^2 r}{g}$$

$$= 1 - \frac{0.615^2 \times 595}{32.2}$$

$$= -5.99$$

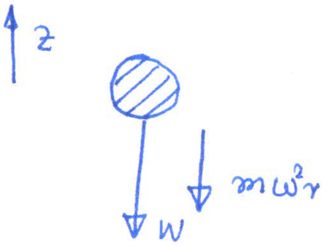
$$\omega = \frac{v}{r} = \frac{366}{595} = 0.615$$

— 3

<u>Case 2</u>

The aircraft is in the bottom horizontal position of the vertical circle

ase 1
N
ase2
or
both done correctly
will fetch 3 marks.



$$n_z = \frac{W + m\omega^2 r}{W}$$

$$= \frac{W + \frac{W}{g}\omega^2 r}{W}$$

$$= 1 + \frac{\omega^2 r}{g}$$

$$= 6.99$$

— 3

4. $u = -\theta y z$ and $v = \theta x z$.

Shear stresses for a circular c/s shaft

$$\tau_{xz} = -G\theta y \quad \text{and} \quad \tau_{yz} = G\theta x \quad \text{—— (1)}$$

$$\therefore \quad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = -\theta y \Rightarrow -\theta y + \frac{\partial w}{\partial x} = -\theta y$$

$$\Rightarrow \frac{\partial w}{\partial x} = 0 \Rightarrow w(x,y) = f_1(y) \quad \text{—— (1)}$$

(1)

Similarly,

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \theta x \Rightarrow +\theta x + \frac{\partial w}{\partial y} = \theta x$$

$$\Rightarrow \frac{\partial w}{\partial y} = \theta x \Rightarrow w(x,y) = f_2(x) \quad \text{—— (2)}$$

(1)

$$\therefore \quad f_1(y) = f_2(x) = \text{const}$$

(1)

$\Rightarrow \quad w(x,y) = C = 0$ (assuming no rigid body mode)