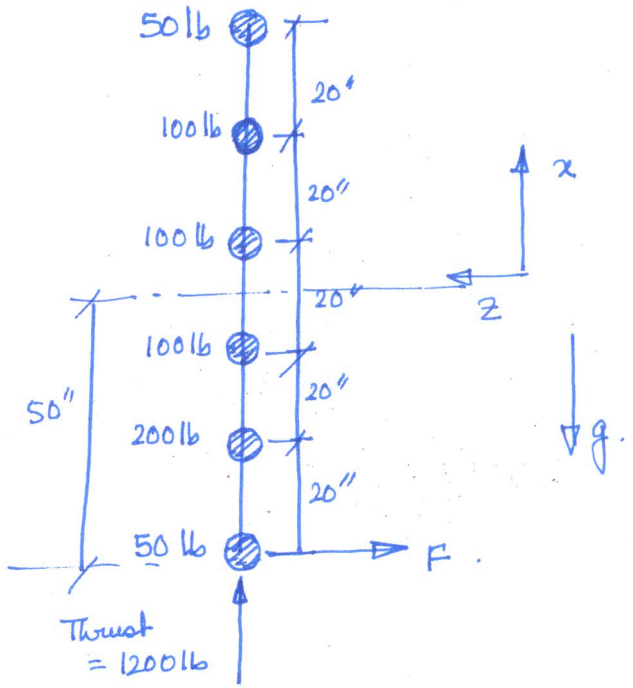
Sample Solution: Tutorial 2

A missile in flight, climbing out of the Earth's atmosphere, is propelled by a 1200 lb thrust engine. Suddenly a side force, F, due to malfunction appears.

The c.g. and mass moment of inertia (about the missile c.g.) are provided by the dynamics group, $I_0 = 113.22$ slug-ft^2. The c.g. is located 45" from the bottom.

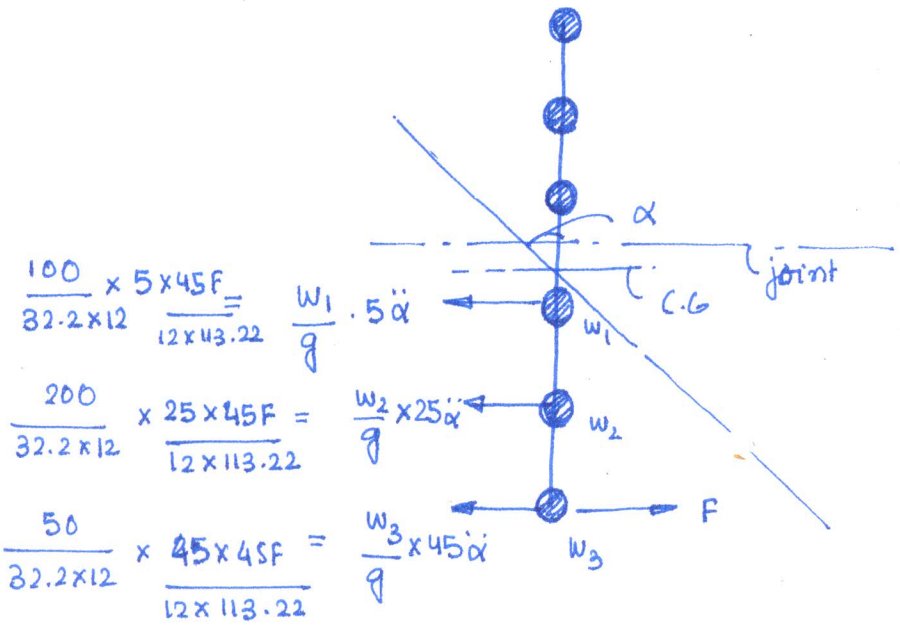
A critical joint on the missile, located at a position 50" from the bottom of the missile (or 5" from the c.g.) can sustain a bending moment of 25,000 lb-in (in either direction) before it fails.

Find the value of F required to cause a bending moment just large enough to cause the joint failure.



Soln Angular acceleration about c.g. $\ddot{\alpha} = \frac{M_{cg}}{I_{cg}} = \frac{45F}{12 \times I_{cg}} = \frac{45F}{12 \times 113.22}$ rad/s²

Inertial forces on the masses below the critical joint due to rotation

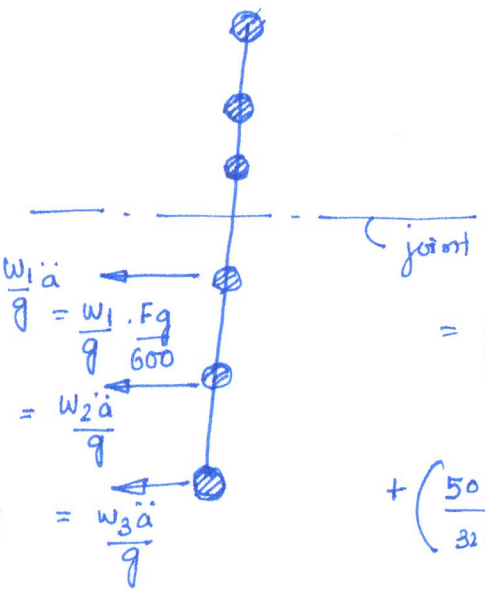


Acceleration in -ve z-dirc. due to unbalance force F

$$\ddot{a} = \frac{F}{\sum M_i} = \frac{Fg}{600} \text{ in/s}^2$$

Inertial forces on the masses below the critical joint due translational accelev. $\ddot{a}$

$$= \frac{F \times 32.2 \times 12}{600} \text{ in/s}$$



Moment about the critical joint

$$\left(\frac{w_1}{g} \cdot 5\ddot{\alpha} \times 5 + \frac{w_1}{g} \cdot \ddot{a}\right) \times 10 + \left(\frac{w_2}{g} 25\ddot{\alpha} + \frac{w_2 \ddot{a}}{g}\right) \times 30$$

$$+ \left(\frac{w_3}{g} \cdot 45\ddot{\alpha} + \frac{w_3}{g} \ddot{a}\right) \times 50 - 50F$$

$$= \left(\frac{5 \times 100 \times 45F}{32.2 \times 12 \times 12 \times 113.22} + \frac{100}{g} \cdot \frac{Fg}{600}\right) 10 + \left(\frac{25 \times 200 \times 45F}{32.2 \times 12 \times 12 \times 113.22} + \frac{200Fg}{g \times 600}\right) \times 30$$

$$+ \left(\frac{50 \times 45 \times 45F}{32.2 \times 12 \times 12 \times 113.22} + \frac{50}{g} \cdot \frac{Fg}{600}\right) \times 50 - 50F$$

$$\Rightarrow (0.0429 + 0.1667) \times 10F + (0.429 + 0.333) \times 30F + (0.193 + 0.0833) \times 50F - 50F = 25000$$

$$\Rightarrow 2.096F + 22.86F + 13.815F - 50F = 25000$$

$$\Rightarrow -11.229F = 25000 \Rightarrow F = -2226.4 \text{ lb}$$

∴ The lateral force $F = \pm 2226.4$ lb .