ASSIGNMENT 3
AE639:CONTINUUM MECHANICS

1. For the deformation field

$$x_1 = \sqrt{3}X_1 + X_2$$
$$x_2 = 2X_2$$
$$x_3 = X_3$$

determine

(a) the Matrix representation of rotation tensor R

(b) the right strettensor U and the left stretch tensor V, then show that the proincipal values of U and V are equal

(c) the direction of the axis of rotation and the magnitude of the angle of the roation

2. A Deformation field is expressed by

$$x_1 = \mu(X_1 cos\beta X_3 + X_2 sin\beta X_3)$$
$$x_2 = \mu(-X_1 sin\beta X_3 + X_2 cos\beta X_3)$$
$$x_3 = \nu X_3$$

where $\mu, \beta \, and \, \nu$ are constants.

(a) Determine the relationship between these constants if the deformation is to be a possible one for an incompressible medium.

(b) If the above deformation is applied to the circular cylinder of length L and radius a with X_3 in axial direction, determine

   i. the deformed length l of an element of the lateral surface which has unit length and is parallel to the cylinder axis in refernce configuration

   ii. the initial length L of a line element on the lateral surface which has unit length and is parallel to cylinder axis after deformation

3. A velocity field is defined in terms of the spatial coordinates and time by the equations,

$$v_1 = 2tx_1 sinx_3, \; v_2 = 2tx_2 cosx_3, \; v_3 = 0$$

At the point (1,-1,0) at the time t=1, determine

(a) the rate of deformation tensor and vorticity tensor

(b) the stretch rate per unit length in direction of normal $\hat{n} = (\hat{e_1} + \hat{e_2} + \hat{e_3})/\sqrt{3}$

(c) the maximum stretch rate per unit length an dthe direction in which it occurs

(d) the maximum shear strain rate

4. In a ceratin region of flow the velocity components are

$$v_1 = (x_1^3 + x_1 x_2^2)e^{-kt},\ v2 = (x_2^3 - x_1^2 x_2)e^(-kt),\ ; v_3 = 0$$

where k is constant, and t is time in s. Determine at the point (1,1,1) when t=0,

(a) the components of acceleration

(b) the principal values of the rate of deformation tensor

(c) the maximum shear rates of deformation

5. The velocity field is given in spatial form by

$$v_1 = x_1 x_3,\ v_2 = x_2^2 t,\ v3 = x_2 x_3 t$$

(a) Determine the voricity tensor **W** and the vorticity vector **w**

(b) Verify the equation $\epsilon_{pqi} w_i = W_q p$ for the results of part (a)

(c) Show that at the point (1,0,1) when t=1, the vorticity tensor has only one real root.

6. Show that for any velocity field **v** derived from a vector potential ψ by v=curlψ, the flow is isochoric. Also for velocity field

$$v_1 = ax_1x_3 - 2x_3, v_2 = -bx_2x_3, v_3 = 2x_1x_2$$

determine the relationship between the constants a and b if flow is isochoric