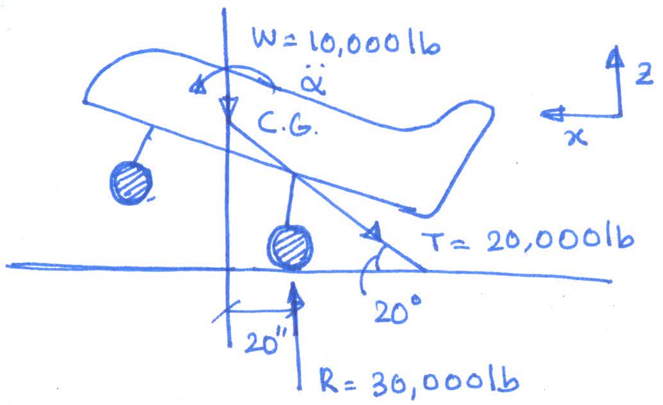
Lecture 2: Inertial Moments



$I_{c.g} = 7800$ slug-ft^2

A UAV is landing, a tail-hook grabs an arresting wire to stop the airplane. Tension in the wire is 20,000 lb

1) Find the load factor n_z and n_x

$$n_x = \frac{-T\cos 20}{W} = -1.967$$

$$n_z = \frac{R - T\cos 70}{W} = 2.653$$

2) Find the apparent weight in the z-dirc of the 1 lb items located at three positions in the airplane.

a) at the c.g. of the plane.

b) 100" ahead of the c.g.

c) 200" aft of the c.g.

Unbalance moment at C.G

$= 30{,}000 \times 20$ lb-in

$= 600{,}000$ lb-in

$$\ddot{\alpha} = \frac{600{,}000}{12 \times 7800} = 6.41 \text{ rad/s}^2$$

(a) at the c.g. : $W_{app} = 2.653W = 2.653$ lb

(b) 100" ahead of the c.g. (a) $= -100 \times 6.41 + 2.653g$.

App. weight $= (-641 + 2.653g)\frac{w}{g}$

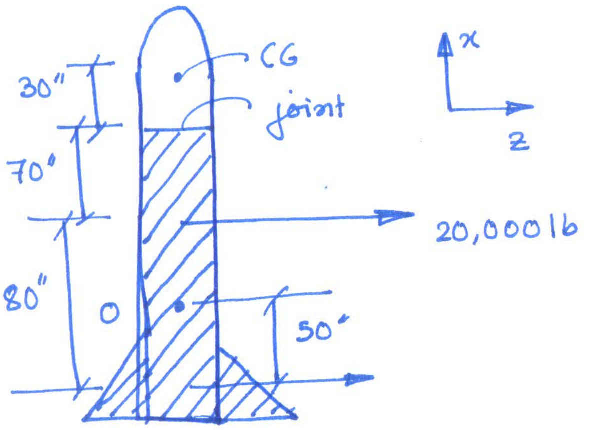
$$= \frac{-641}{12 \times 32.2} + 2.653 = 0.904 \text{ lb}$$

(c) 200" aft of c.g

acceler $a = 200 \times 6.41 + 2.653g$

App. weight $= (1282 + 2.653)\frac{w}{g}$

$$= \frac{1282}{12 \times 32.2} + 2.653 = 5.977 \text{ lb}$$



Given:

$n_z = 1$

$\ddot{\alpha} = 1.08 \text{ rad/s}^2$ (about c.g. of the missile)

$W_{aft} = 3000\text{ lb}$

$I_0 = 16,670 \text{ slug-ft}^2$ (about c.g of the shaded portion)

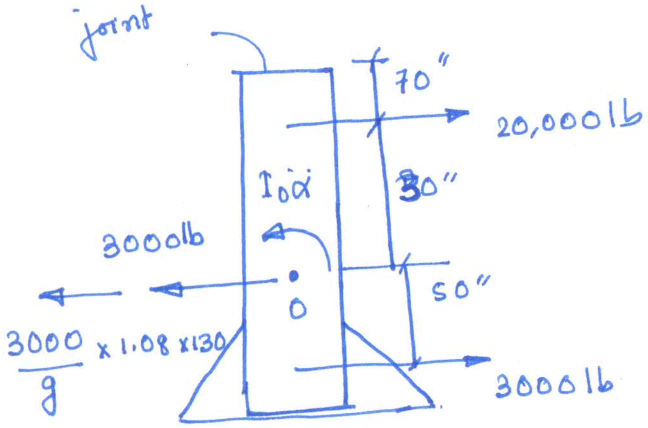
Find the BM and SF at the joint

$n_z = 1 \Rightarrow a_z = g$

translation accelr. of the c.g. of the shaded portion due to rotation

$a_\alpha = 1.08 \times 130 \text{ in/s}.$

FBD of the aft portion



Shear force at the joint:

$$SF = 20,000 + 3000 - 3000 - \frac{3000}{g} \times 1.08 \times 130$$

$$= 20,000 - \frac{3000 \times 1.08 \times 130}{12 \times 32.2}$$

$$= 18910 \text{ lb}$$

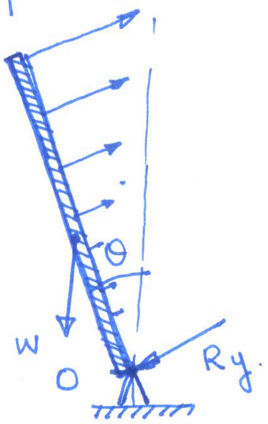
Bending moment at the joint

$$BM = 20000 \times 70 + 3000 \times 150 - 4090 \times 100 + I_0\ddot{\alpha} \; (= 16.670 \times 1.08)$$

$$= 1,225,000 \text{ lb-in}$$

A smokestack is supported on a pin-joint O and undergoing a rotational motion about O. Find the BM and SF at a section at distance l from O

Given:
weight of the smokestack = W
Length = L
uniformly distributed mass



Angular acceler: $\ddot{\theta}$: $I_O\ddot{\theta} = W\frac{L}{2}\sin\theta$

$$\Rightarrow \frac{1}{3}\frac{W}{g}L^2\ddot{\theta} = \frac{WL}{2}\sin\theta$$

$$\Rightarrow \boxed{\ddot{\theta} = \frac{3}{2}\frac{g}{L}\sin\theta}$$

From force equilm, we get,

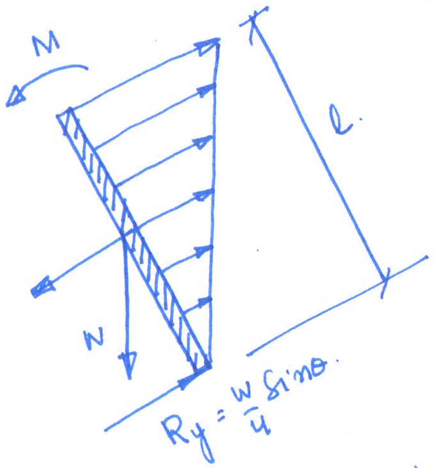
$$R_y + W\sin\theta - \cancel{\frac{W}{2gL}L^2\ddot{\theta}} \int_0^L \frac{W}{gL}dx\, x\ddot{\theta} = 0$$

$$\Rightarrow R_y + W\sin\theta - \frac{W}{2gL}L^2\ddot{\theta} = 0$$

$$\Rightarrow R_y + W\sin\theta - \frac{WL}{2g}\cdot\frac{3g}{2L}\sin\theta = 0$$

$$\Rightarrow R_y + W\sin\theta - \frac{3W}{4}\sin\theta = 0$$

$$\Rightarrow \boxed{R_y = -\frac{W}{4}\sin\theta}$$



Moment balance:

$$M + \frac{W}{4}l\sin\theta - \frac{Wl}{L}\sin\theta\frac{l}{2} + \int_0^l \frac{W}{gL}dx\, x\ddot{\theta}(l-x) = 0$$

$$M + \frac{W}{4}l\sin\theta - \frac{Wl^2}{2L}\sin\theta + \frac{W}{gL}\ddot{\theta}\left(\frac{lx^2}{2} - \frac{x^3}{3}\right)\Big|_0^l = 0$$

$$\Rightarrow M + \frac{Wl}{4}\sin\theta - \frac{Wl^2}{2L}\sin\theta + \frac{Wl^3}{6gL}\ddot{\theta} = 0$$

$$\Rightarrow M + \frac{Wl}{4}\sin\theta - \frac{Wl^2}{2L}\sin\theta + \frac{Wl^3}{3gL}\cdot\frac{3}{2}\frac{g}{L}\sin\theta = 0$$

$$\Rightarrow M + \frac{Wl}{4}\sin\theta - \frac{Wl^2}{2L}\sin\theta + \frac{Wl^3}{4L^2}\sin\theta = 0$$

$$\Rightarrow M = -\frac{WL\sin\theta}{4}\left(\frac{l}{L} - \frac{l^2}{2L^2} + \frac{l^3}{L^3}\right)$$