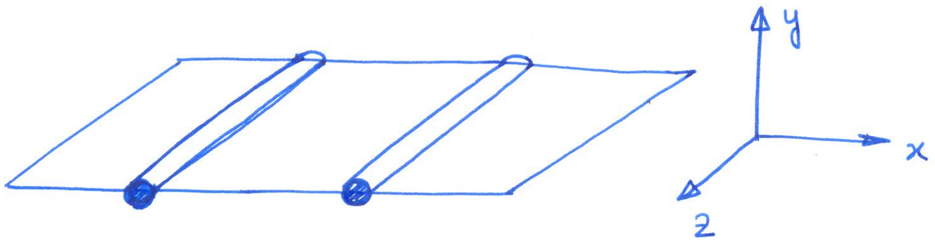
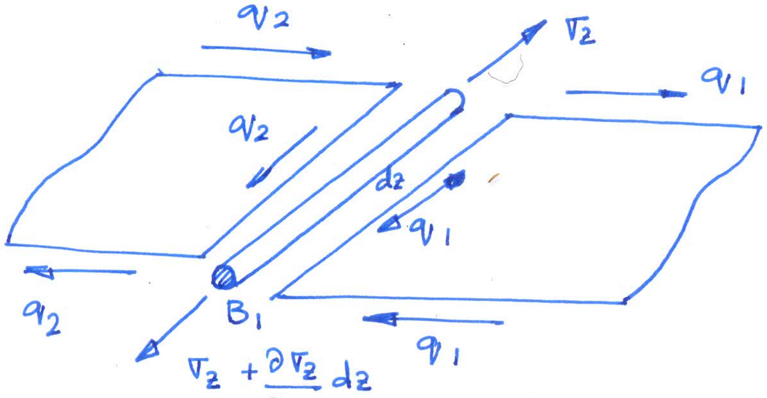
Shear of open section (skin-stringer) idealized beam





Considering force equilbm in z-dirc.

$$\left(\sigma_z + \frac{\partial \sigma_z}{\partial z} dz\right) B_1 - \sigma_z B_1 - q_1 dz + q_2 dz = 0.$$

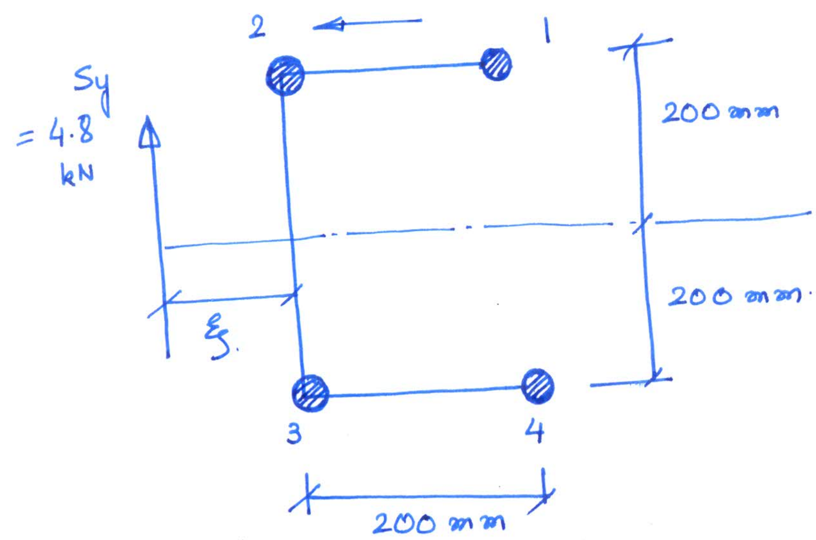
$$\Rightarrow \quad q_2 - q_1 = -\frac{\partial \sigma_z}{\partial z} B_1$$

$$\Rightarrow \quad q_2 - q_1 = -\left[\frac{\frac{\partial M_y}{\partial z} I_{xx} - \frac{\partial M_x}{\partial z} I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] B_1 x - \left[\frac{\frac{\partial M_x}{\partial z} I_{yy} - \frac{\partial M_y}{\partial z} I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] B_1 y.$$

$$= -\left[\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] B_1 x - \left[\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] B_1 y.$$

Ex

Shear flow in open c/s beam



Area of each boom = 300 mm^2

$I_{xx} = 4 \times 300 \times (200)^2$

$= 48 \times 10^6 \text{ mm}^4$

$$q_2 - q_1 = -\left[\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] B_r x_r - \left[\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] B_r y_r$$

$I_{xy} = 0$ and $S_x = 0$

$$q_2 - q_1 = -\frac{S_y}{I_{xx}} B_r y_r .$$

For flange 12

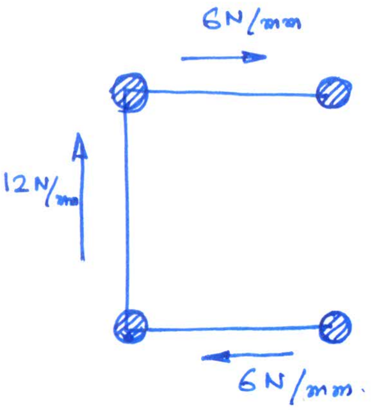
$$q_{12} = \frac{-4.8 \times 10^3}{48 \times 10^6} \times 300 \times 200 = -6 \text{ N/mm}.$$

For web 23

$$q_{23} = \frac{-4.8 \times 10^3}{48 \times 10^6} \times 300 \times 200 - 6 = -12 \text{ N/mm}.$$

For flange 34

$$q_{34} = \frac{-4.8 \times 10^3}{48 \times 10^6} \times 300 \times (-200) - 12 = -6 \text{ N/mm}$$

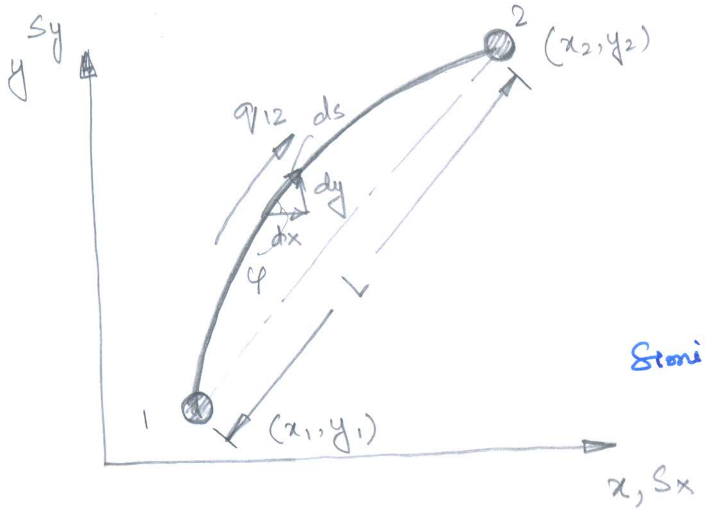


Shear center Let us assume that shear center is at a distance ξ_s from the web as shown.

Taking moment about 3, we get

$$S_y \xi_s = 6 \times 200 \times 400 = 480000$$

$$\Rightarrow \xi_s = 100 \text{ mm}.$$



$$S_x = \int q_{12}\, ds \cos\varphi$$

$$= q_{12}\int_{x_1}^{x_2} dx = q_{12}(x_2 - x_1)$$

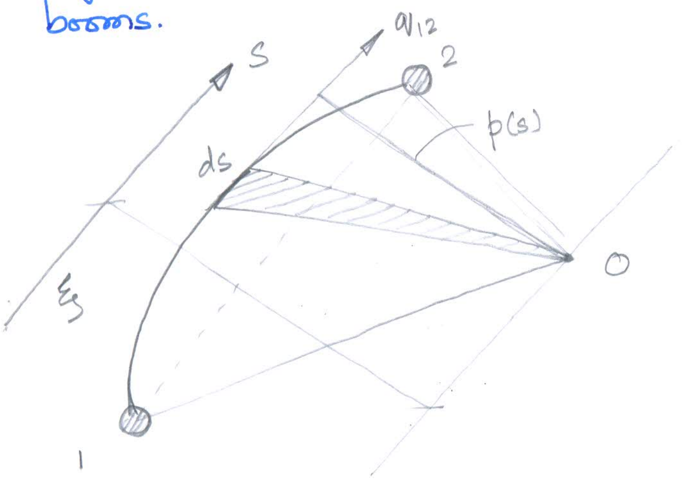
Similarly,

$$S_y = q_{12}(y_2 - y_1)$$

$$S = \sqrt{S_x^2 + S_y^2} = q_{12}\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= q_{12} L$$

∴ The resultant of the shear flow is equal to the shear flow multiplied by the straight line length between the booms.



Moment about O

$$\int q_{12}\, ds\, p(s) = 2 q_{12} A$$

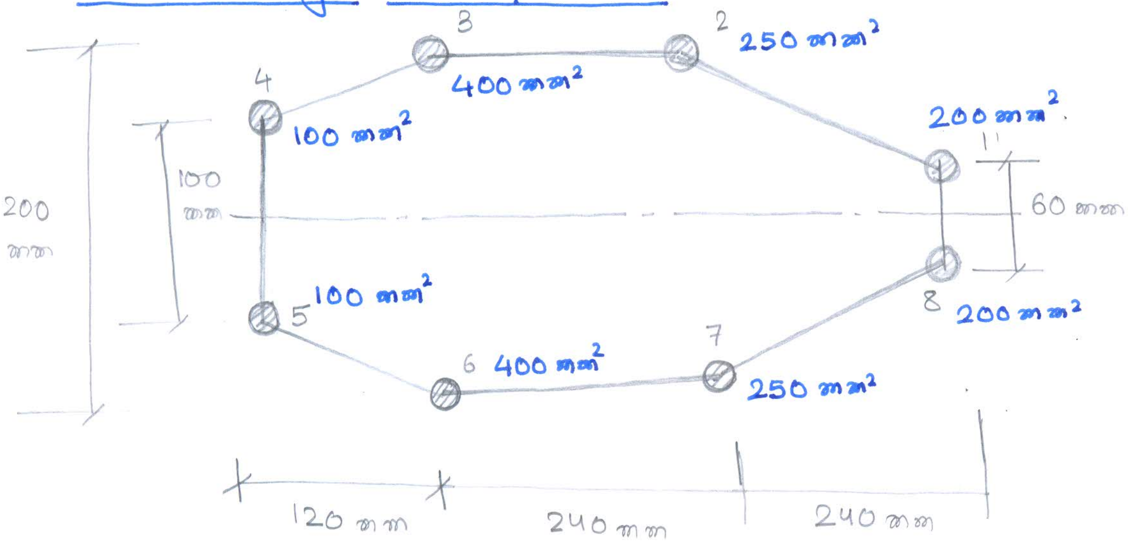
Shear center

$$S \xi = 2 q_{12} A$$

$$= \frac{2S}{L} A$$

$$\Rightarrow \boxed{\xi = \frac{2A}{L}}$$

Shear loading in closed c/s beam



Find the shear flow for $S_y = 10\,kN$ and the location of shear center

$$I_{xx} = 2\left[200\times(30)^2 + 250\times(100)^2 + 400\times(100)^2 + 100\times(50)^2\right]$$

$$= 13.86\times10^{6}\ mm^4$$

$$q_2 - q_1 = -\frac{S_y}{I_{xx}} B_r y_r.$$ Assuming $q_{23} = q_0$

$$q_{34} = \frac{-10\times10^3}{13.86\times10^6}(400\times100) + q_{23} = -28.9\ N/mm + q_0$$

$$q_{45} = \frac{-10\times10^3}{13.86\times10^6}(100\times50) + q_{34} = \frac{-5\times10^7}{13.86\times10^6} - 28.9 + q_0 = -32.5 + q_0$$

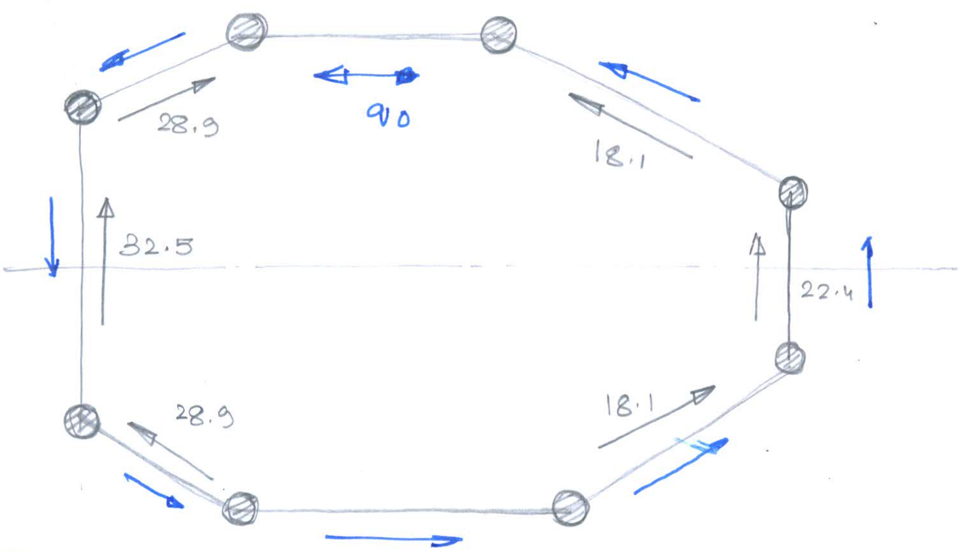
$$q_{56} = \frac{-10\times10^3}{13.86\times10^6}(100\times-50) + q_{45} = -28.9 + q_0$$

$$q_{67} = \frac{-10\times10^3}{13.86\times10^6}(400\times-100) + q_{56} = q_0$$

$$q_{78} = \frac{-10\times10^3}{13.86\times10^6}(250\times-100) + q_{67} = \cancel{18.3}\ 18.1 + q_0$$

$$q_{81} = \frac{-10\times10^3}{13.86\times10^6}(200\times-30) + q_{78} = 22.4 + q_0$$

$$q_{12} = 18.1 + q_0$$



$$\int z\,ds = 0$$

$$\Rightarrow q_0\left[2\times 240 + 60 + 100 + 2\sqrt{120^2+50^2} + 2\times\sqrt{70^2+240^2}\right]$$
$$- 28.9\times 2\times\sqrt{120^2+50^2} - 32.5\times 100 + 22.4\times 60 + 2\times 18.1\sqrt{70^2+240^2} = 0$$

$$\Rightarrow q_0\left[480+160+2\times 130+2\times 250\right] - 28.9\times 2\times 130 - 32.5\times 100$$
$$+ 22.4\times 60 + 2\times 18.1\times 250 = 0$$

$$\Rightarrow 1400 q_0 - 7514 - 3250 + 1344 + 9050 = 0$$

$$\Rightarrow q_0 = 0.264 \text{ N/mm}$$