AE 230 - **Modeling and Simulation Laboratory**

Dynamic System

Input and output can be related in simplified form:

$$a_n \frac{d^n q_0}{dt^n} + a_{n-1} \frac{d^{n-1} q_0}{dt^{n-1}} + \dots + a_1 \frac{dq_0}{dt} + a_0 q_0 =$$

$$b_m \frac{d^m q_i}{dt^m} + b_{m-1} \frac{d^{m-1} q_i}{dt^{m-1}} + \dots + b_1 \frac{dq_i}{dt} + b_0 q_i$$

q_0 = Output quantity

q_i = Input quantity

a's, b's = system physical parameters assumed constant

Solution of above expression can be found out using standard mathematical techniques.

Most of the engineering systems can be simplified. No need to have such a complicated differential equation. Closed form or numerical methods can be used for solutions.

First order systems

Many physical system behave like first order system

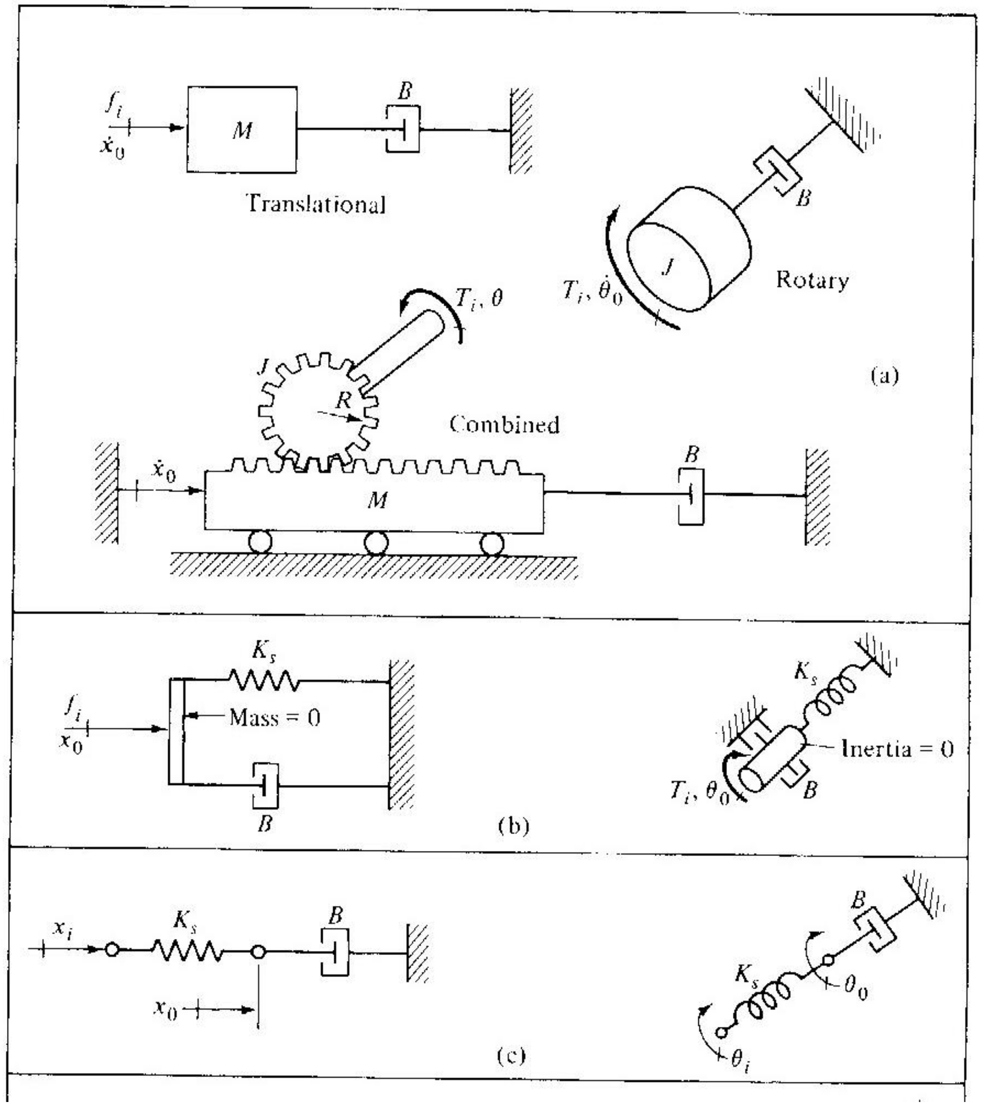
- Simple to understand
- Information about system time constant and amplification

$$a_1 \frac{dq_o}{dt} + a_o q_o = b_1 \frac{dq_i}{dt} + b_o q_i$$

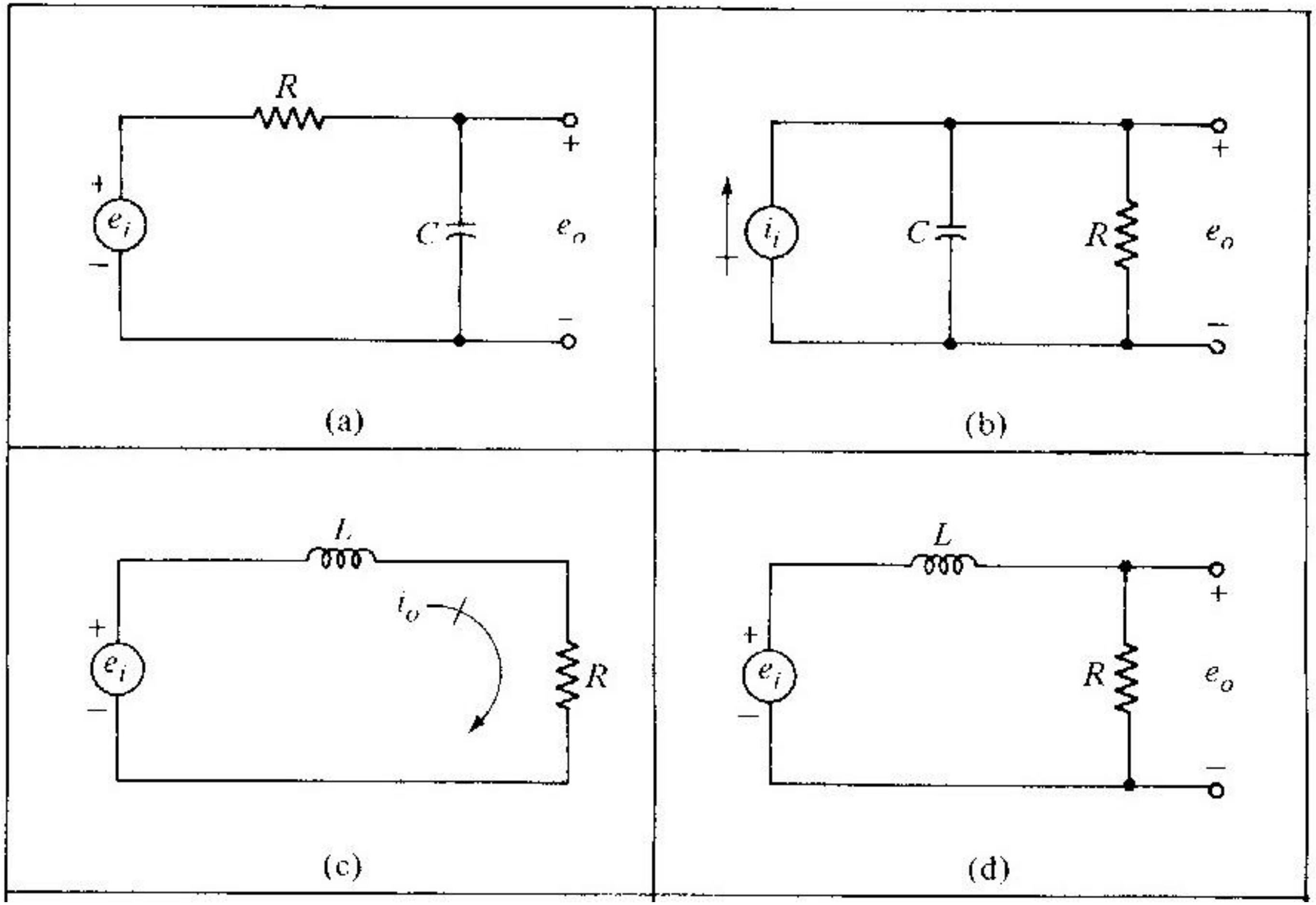
Generic first order system

- q_o is output and q_i is input
- a_1, a_0, b_1, b_0 constants

First order systems – Mechanical



First order systems – Electrical



First order systems

$$\frac{a_1}{a_o}\frac{dq_o}{dt} + q_o = \frac{b_o}{a_o}q_i$$

$$\tau = \frac{a_1}{a_0} = \text{system time constant}$$

$$K = \frac{b_0}{a_0} = \text{System steady state gain}$$

$$\tau\frac{dq_o}{dt} + q_o = Kq_i$$

Standard first order system equation

First order systems

$$\tau \frac{dq_o}{dt} + q_o = Kq_i$$

Standard first order system equation

Two parameters are required to completely describe the first order system, i.e. τ and K

τ - units will be time (sec)

K – units will be ratio of output quantity to input quantity

First order systems

$$\tau \frac{dq_o}{dt} + q_o = Kq_i$$

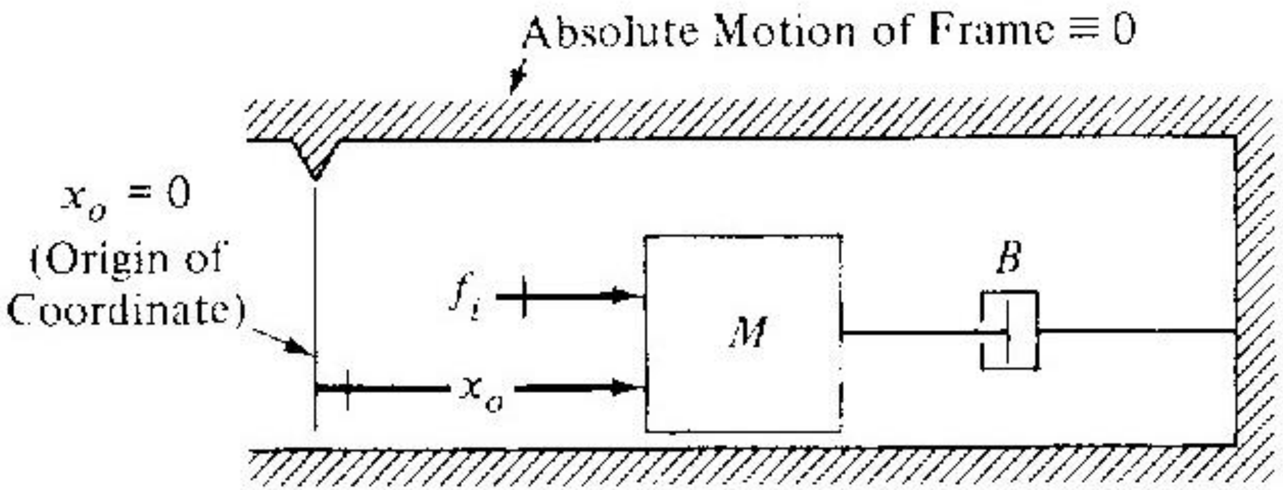
Standard first order system equation

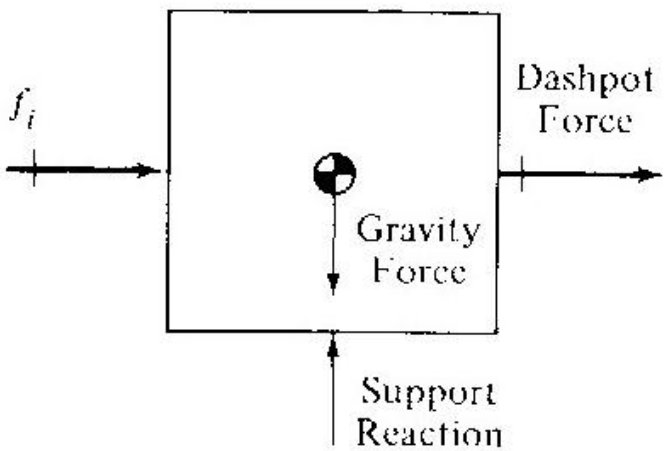
$$Q_0(s) = \frac{Kq_{is}}{s(\tau s + 1)}$$

$$q_o(t) = Kq_{is}(1 - e^{-t/\tau})$$

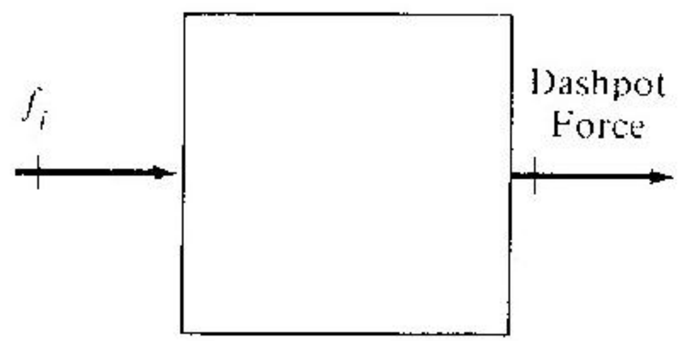
Step response of first order system

First order systems



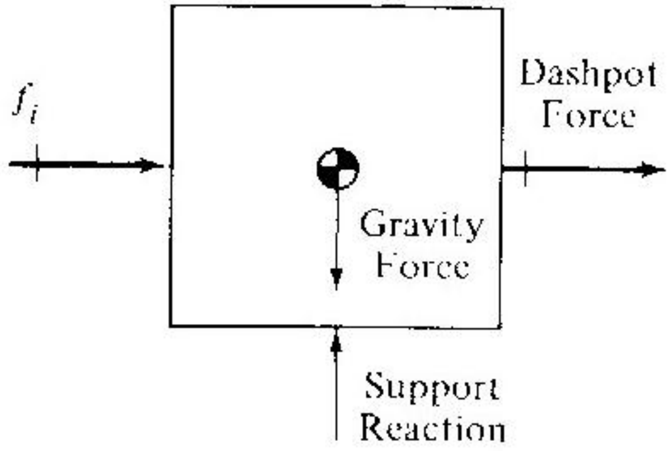


(a)



(b)

First order systems



(a)

$$\sum forces = (mass)(acceleration)$$

$$f_i - B\dot{x} = f_i - Bv_o = M\ddot{x}_o = M\ \dot{v}_o$$

$$M\ \frac{dv_o}{dt} + Bv_o = f_i$$

First order systems

$$M \frac{dv_o}{dt} + Bv_o = f_i$$

$$\frac{M}{B} \frac{dv_o}{dt} + v_o = \frac{1}{B} f_i$$

Standard first order system equation

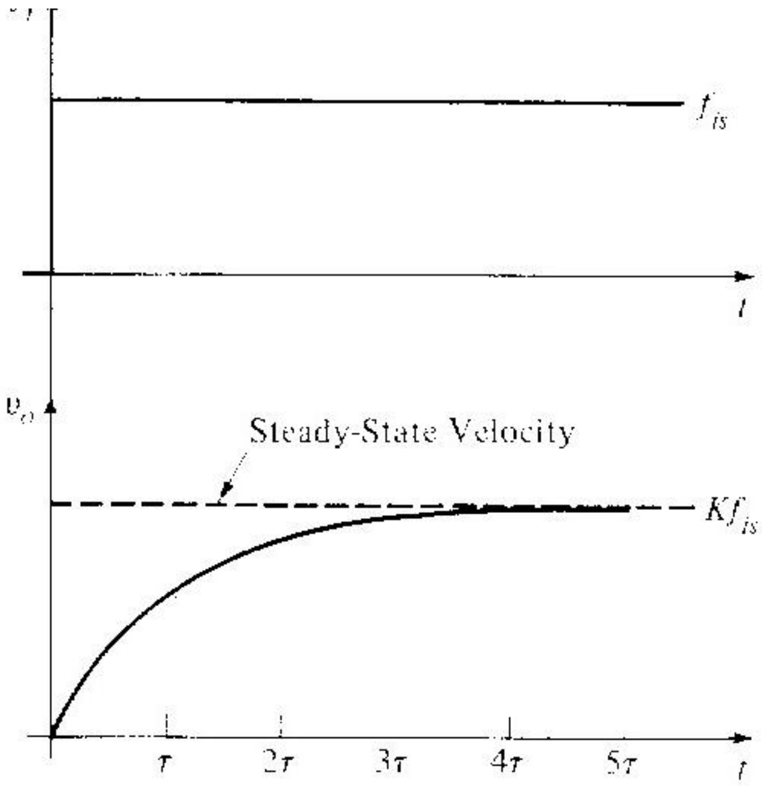
$$\tau = \frac{M}{B} = \frac{kg}{N/(m/\sec)} = \sec$$

$$K = \frac{1}{B} = \frac{m/\sec}{N}$$

$$v_o(t) = Kf_{is}(1 - e^{-t/\tau})$$

Step response of first order system

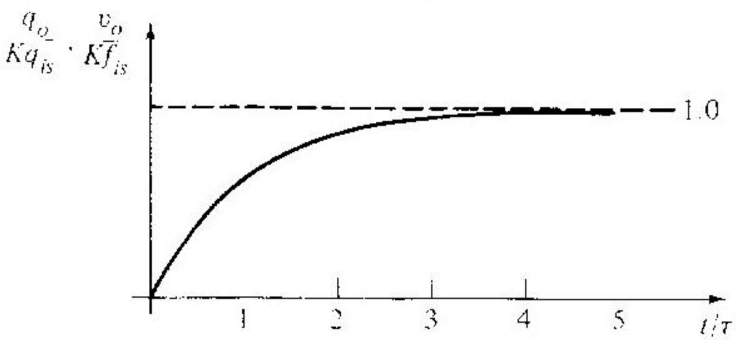
First order systems



(a)

$$v_o(t) = Kf_{is}(1 - e^{-t/\tau})$$

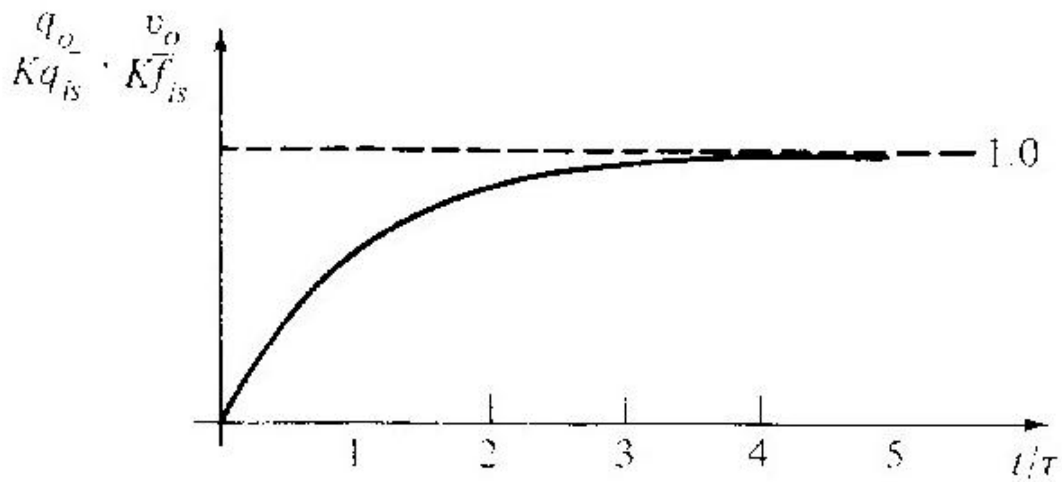
Step response of first order system



(b)

t/τ	q_o/Kq_{is}
0	0.0
1	0.632
2	0.865
3	0.950
4	0.982
∞	1.000

First order systems



(b)

t/τ	q_o/Kq_{is}
0	0.0
1	0.632
2	0.865
3	0.950
4	0.982
∞	1.000

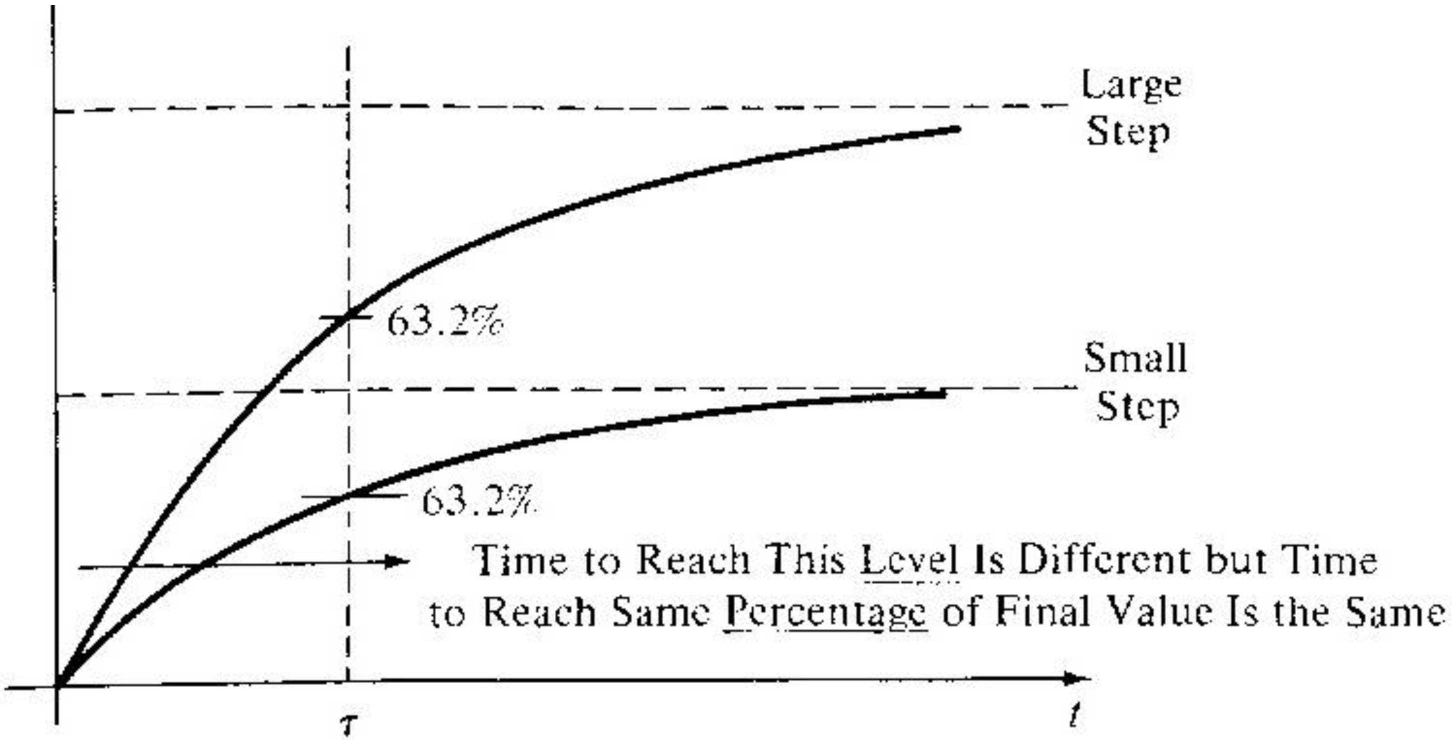
5% of final steady state value in 3 time constants, it is independent of step size

Smaller the time constant faster the system. When time constant approaches zero, the system is zeroth order and it is an algebraic equation.

$$q_o = Kq_i$$

First order systems

Significance of K, steady state gain



K has no effect on how rapidly the steady state is achieved. Steady state is entirely dependent on time constant.

First order systems

$$\frac{M}{B}\frac{dv_o}{dt} + v_o = \frac{1}{B} f_i$$

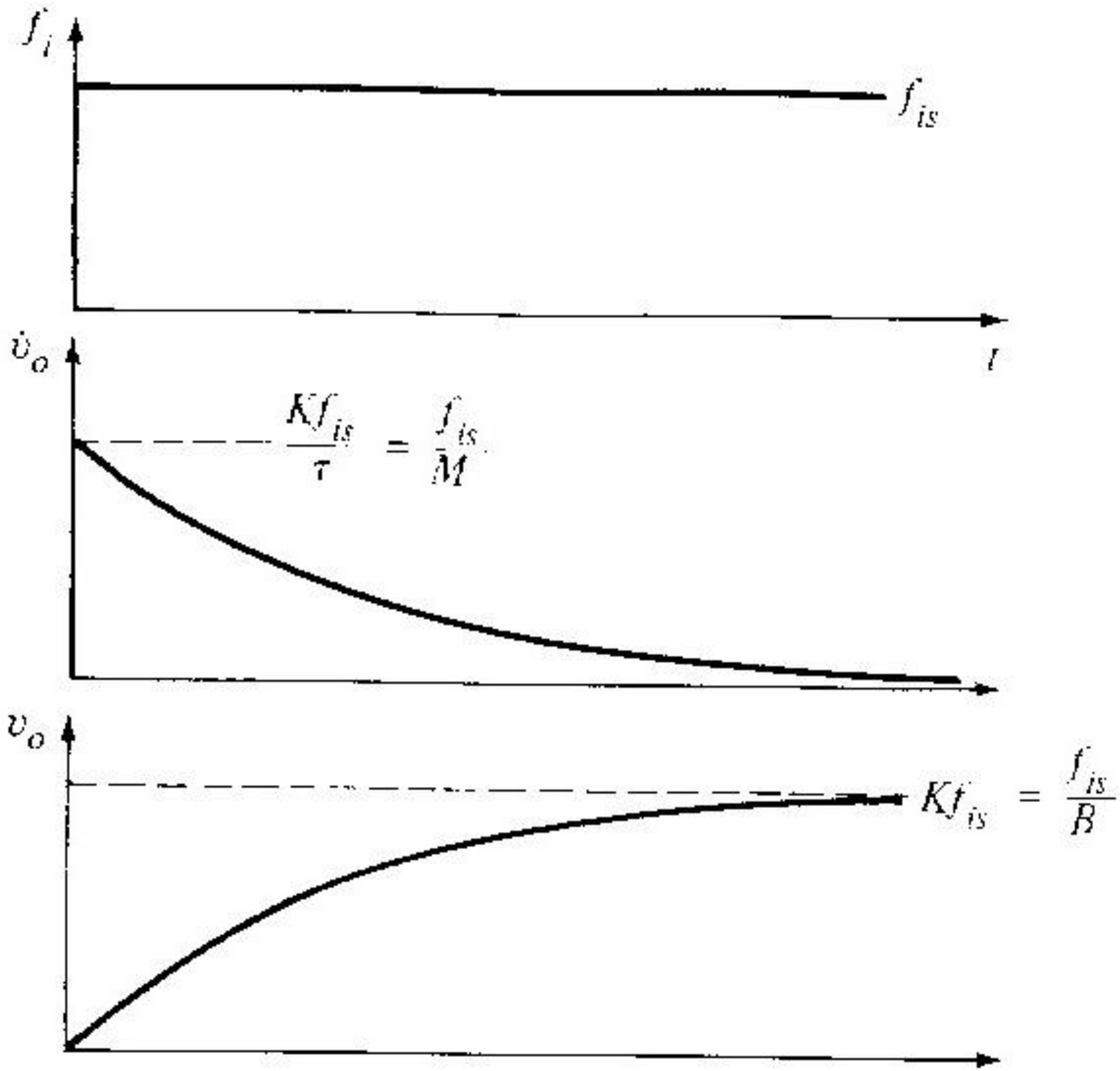
$$\tau \frac{dq_o}{dt} + q_o = Kq_i$$

$$\tau = \frac{M}{B} = \frac{kg}{N/(m/\sec)} = \sec$$

$$K = \frac{1}{B} = \frac{m/\sec}{N}$$

Steady state velocity is only dependent on B. Where as speed of response or time constant is dependent on both M and B

First order systems



First order systems

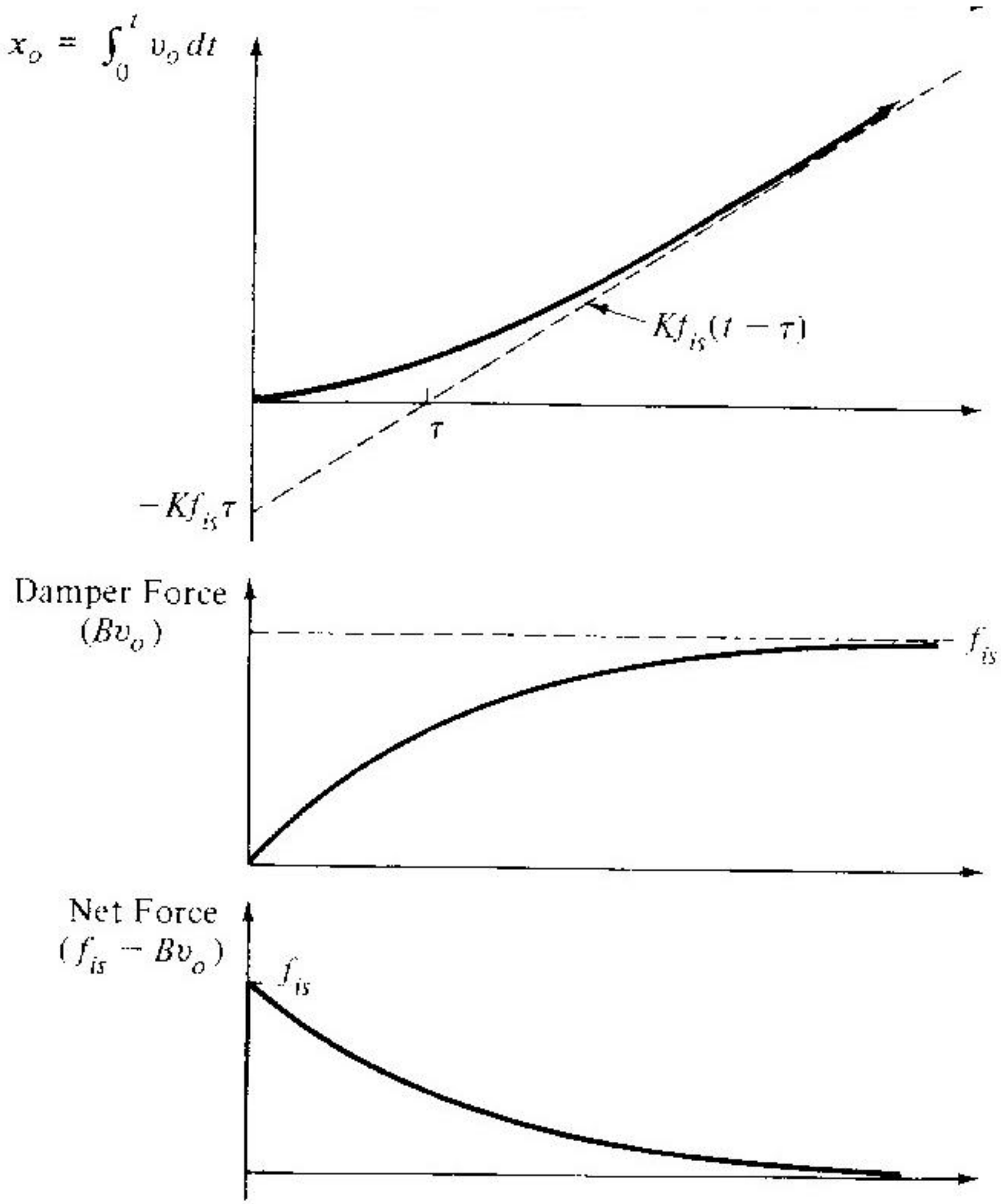


Figure 7-7 Mass/damper system step response.

First order systems

$$q_o(t) = Kq_{is}(1 - e^{-t/\tau})$$

$$\frac{dq_0}{dt} = \frac{Kq_{is}}{\tau} e^{-t/\tau}$$

Slope of the above equation t = 0

$$\frac{\mathrm{dq}_0}{\mathrm{dt}} = \frac{\mathrm{Kq}_{\mathrm{is}}}{\tau}$$

Line with above slope passing through origin

$$q_1 = \frac{Kq_{is}}{\tau} t \qquad \mathrm{eqn}(1)$$

Steady state value of output is $= \mathrm{Kq}_{\mathrm{is}}$

Eqn (1) will cut the steady state line at time τ

First order systems

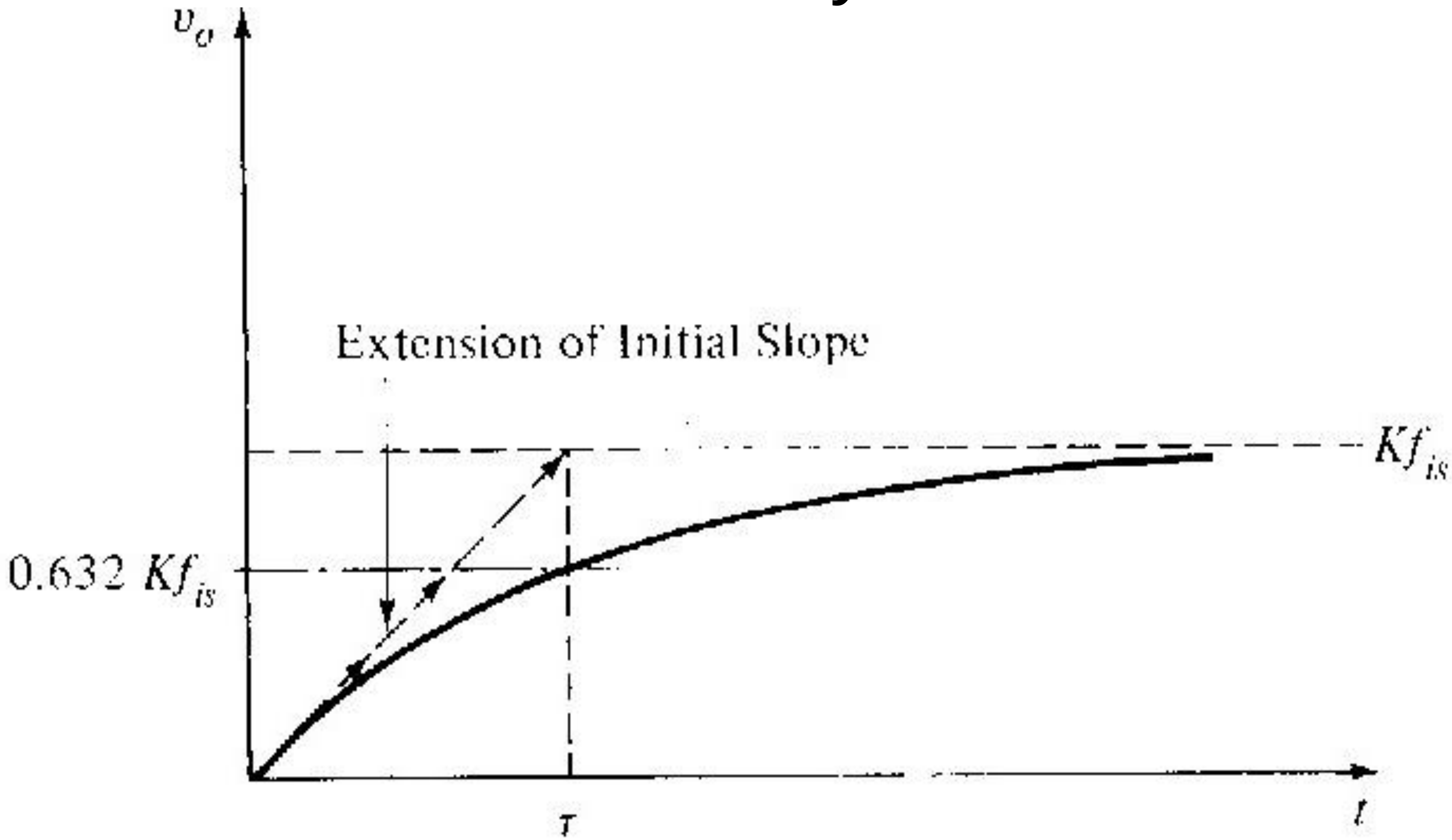


Figure 7-8 First-order system response characteristics.

$$q_o(t) = Kq_{is}(1 - e^{-t/\tau})$$

First order systems

$$q_o(t) = Kq_{is}(1 - e^{-t/\tau})$$

$$Z = \log_e(1 - \frac{q_0}{Kq_{is}}) = \log(e^{-t/\tau}) = -\frac{t}{\tau}$$

Plot of Z versus time t is a plot of slope -1/τ

First order svstems

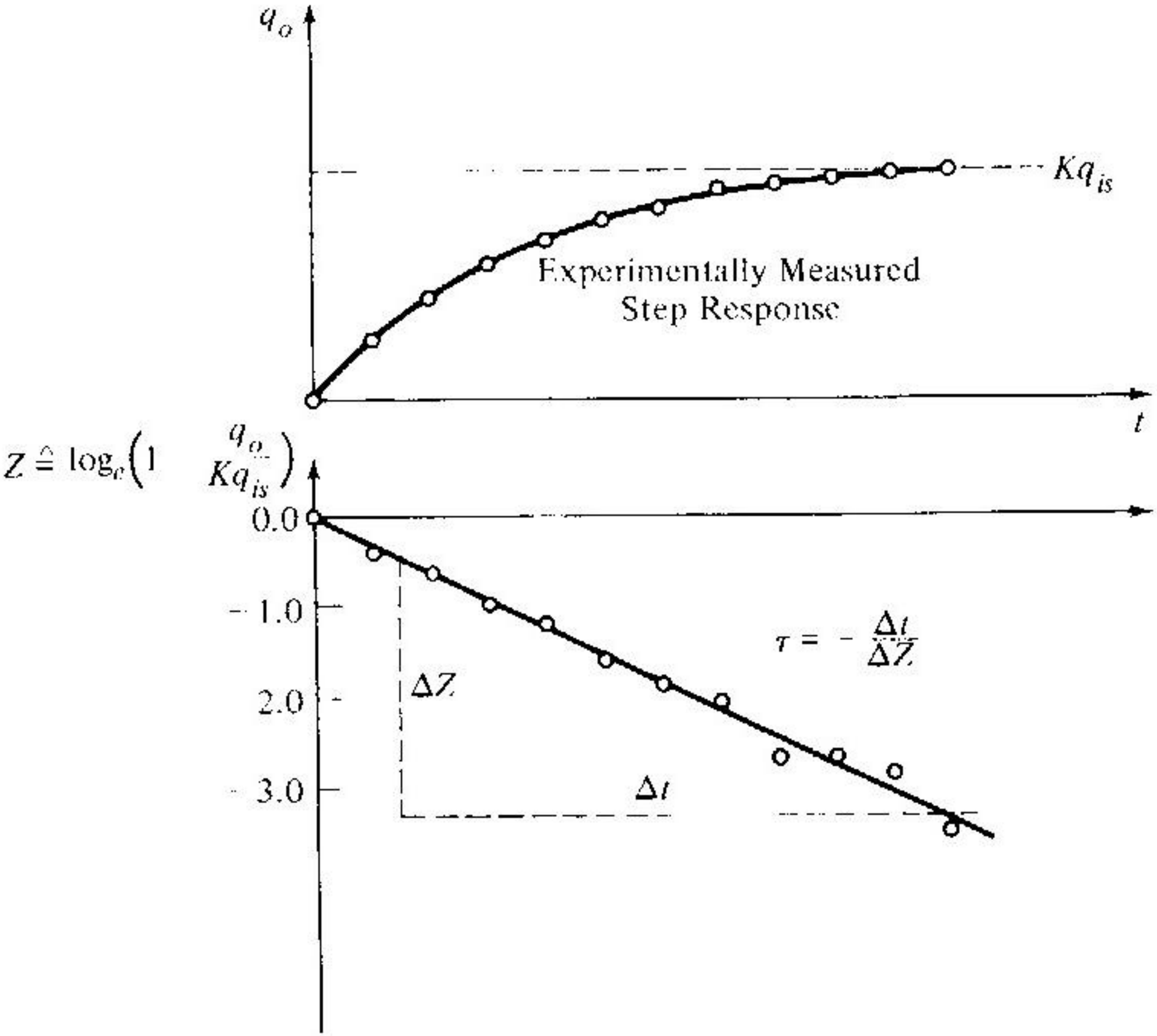


Figure 7-9 Experimental modeling by step testing.

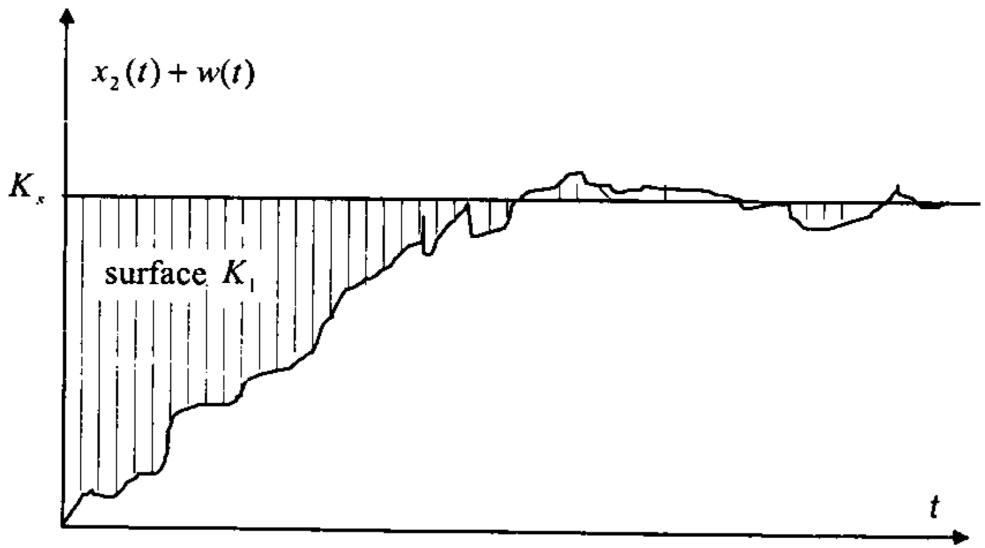


Fig. 6.9 Surface method in presence of a disturbances $w(t)$

$$q_o(t) = Kq_{is}(1 - e^{-t/\tau})$$

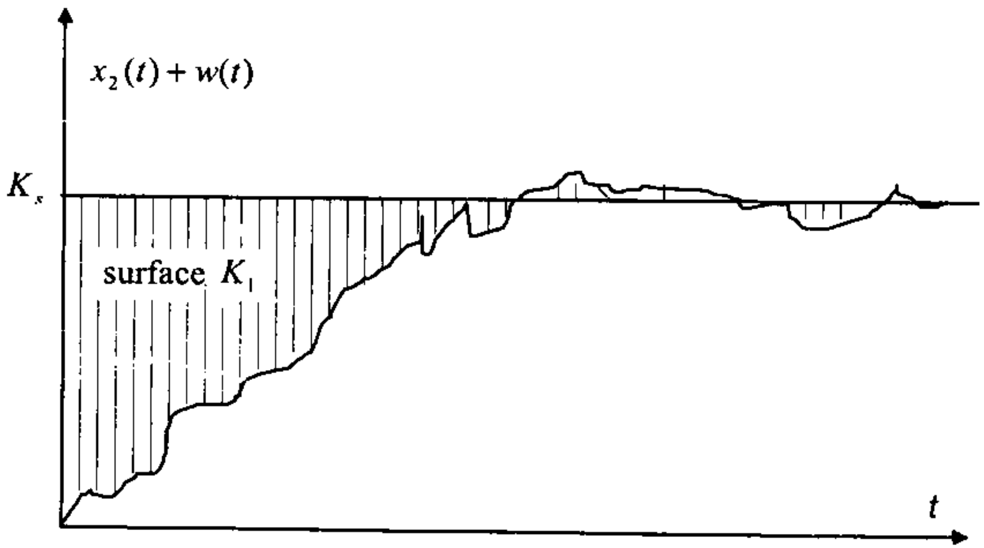


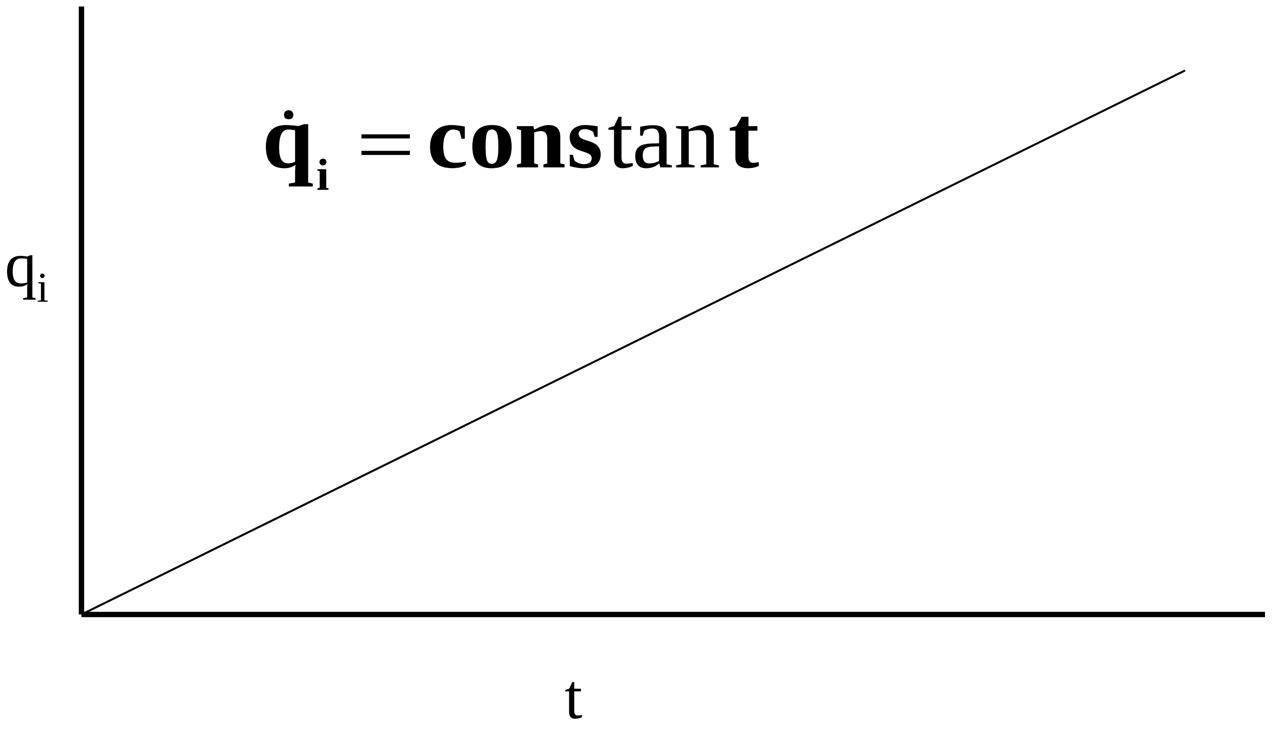
Fig. 6.9 Surface method in presence of a disturbances $w(t)$

$$q_o(t) = Kq_{is}(1 - e^{-t/\tau})$$

$$K_s = \lim x_2(t),\ \mathrm{t} \to \infty$$

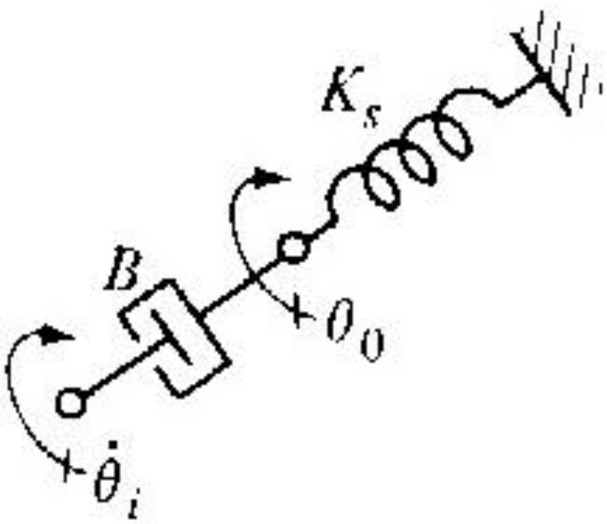
$$T_s = \frac{K_1}{K_s}$$

First order systems – Ramp input



Ramp – Signal which starts at a fixed level (usually taken as zero) and grows at a fixed (usually positive) rate.

First order systems – Ramp input



$$\mathbf{BD}(\theta_{\mathbf{i}} - \theta_{\mathbf{o}}) - \mathbf{K_s}\theta_{\mathbf{o}} = \mathbf{JD}^2\theta_{\mathbf{o}} = 0$$

$$\frac{\mathbf{B}}{\mathbf{K_s}}\frac{\mathbf{d}\theta_{\mathbf{o}}}{\mathbf{dt}} + \theta_{\mathbf{o}} = \frac{\mathbf{B}}{\mathbf{K_s}}\dot{\theta}_{\mathbf{i}}$$

Rotary inertia J = 0

$$\tau = \frac{\mathbf{B}}{\mathbf{K_s}} \qquad \mathbf{K} = \frac{\mathbf{B}}{\mathbf{K_s}} \qquad (\tau\mathbf{D} + 1)\theta_{\mathbf{o}} = \mathbf{K}\dot{\theta}_{\mathbf{i}} = \mathbf{K}\omega_{\mathbf{i}}$$

For constant speed ω_i

First order systems – Ramp input

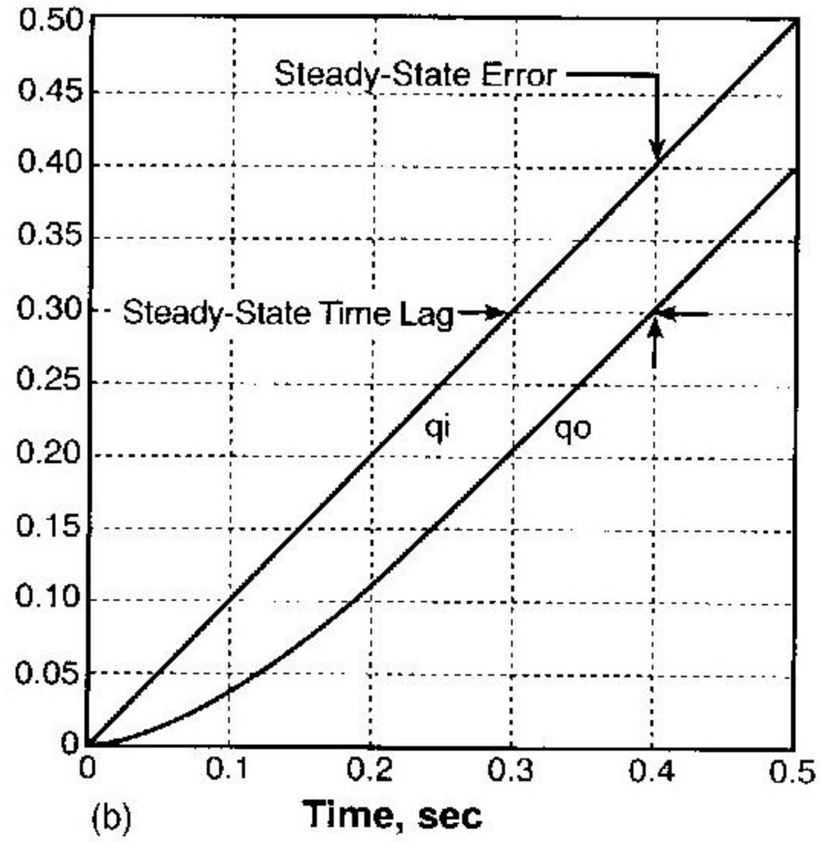
$$(\boldsymbol{\tau}\mathbf{D}+1)\boldsymbol{\theta}_{\mathbf{o}} = \mathbf{K}\dot{\boldsymbol{\theta}}_{\mathbf{i}} = \mathbf{K}\boldsymbol{\omega}_{\mathbf{i}} \qquad (\tau D+1)\theta_o = K\omega_i = K\alpha_i t$$

For speed ω_i varying linearly with time = α_i t

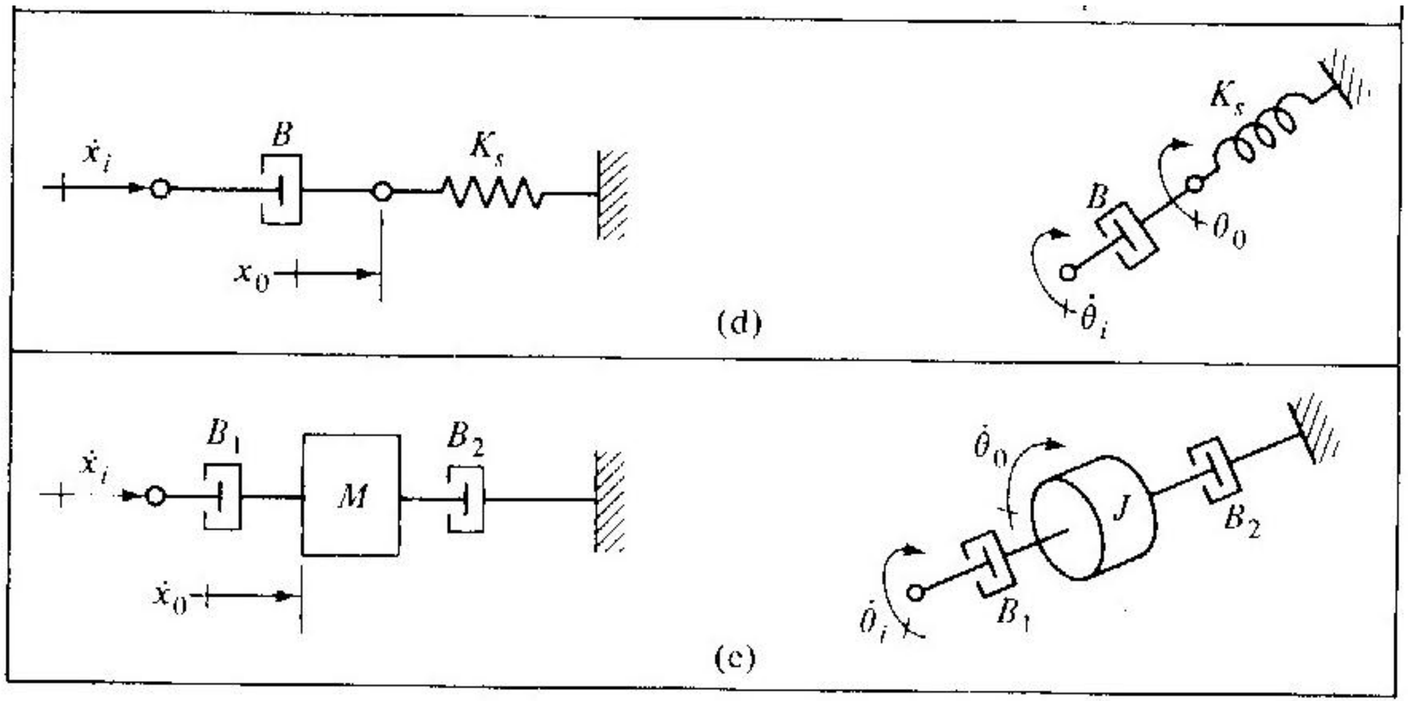
$$\boldsymbol{\theta}_{\mathbf{o}} = \mathbf{K}\boldsymbol{\alpha}_{\mathbf{i}}\boldsymbol{\tau}\mathbf{e}^{-\mathbf{t}/\tau} + \mathbf{K}\boldsymbol{\alpha}_{\mathbf{i}}(\mathbf{t}-\boldsymbol{\tau})$$

For τ = 0 output will be= $K\alpha_i$ t

For $\tau \neq 0$ steady state error will be = - α_i τ and with lag = τ



First order systems – Mechanical



First order systems – Electrical