2. Show that the lightest cantilever beam of length L and square cross-section (free to choose the area) that will support a tip load F without yielding is made of material with largest value of $\frac{\sigma_y^{\frac{2}{3}}}{\rho}$, where, σ_y is the yield strength. **Marks 4**

Mass of the beam $m = \rho A L = \rho t^2 L$ (t = thickness of the square c/s)

Area moment of inertia $= \frac{1}{12} t^4$ (for square c/s)

Maxm. moment M due to tip load F $= M = FL$

Maxm stress due to M $= \sigma_y = \frac{FLt}{2 \cdot \frac{1}{12} t^4} = \frac{6FL}{t^3}$ — (2)

$\Rightarrow t = \left(\frac{6FL}{\sigma_y}\right)^{1/3}$

$\therefore$ Mass of the beam $m = \rho \left(\frac{6FL}{\sigma_y}\right)^{2/3} L$

$= \left[\frac{\rho}{\sigma_y^{2/3}}\right] (6F)^{2/3} L^{5/3}$

Therefore, for given F and L, m will be minimum when $\frac{\rho}{\sigma^{2/3}}$ is smallest or $\frac{\sigma^{2/3}}{\rho}$ is largest. — (2)