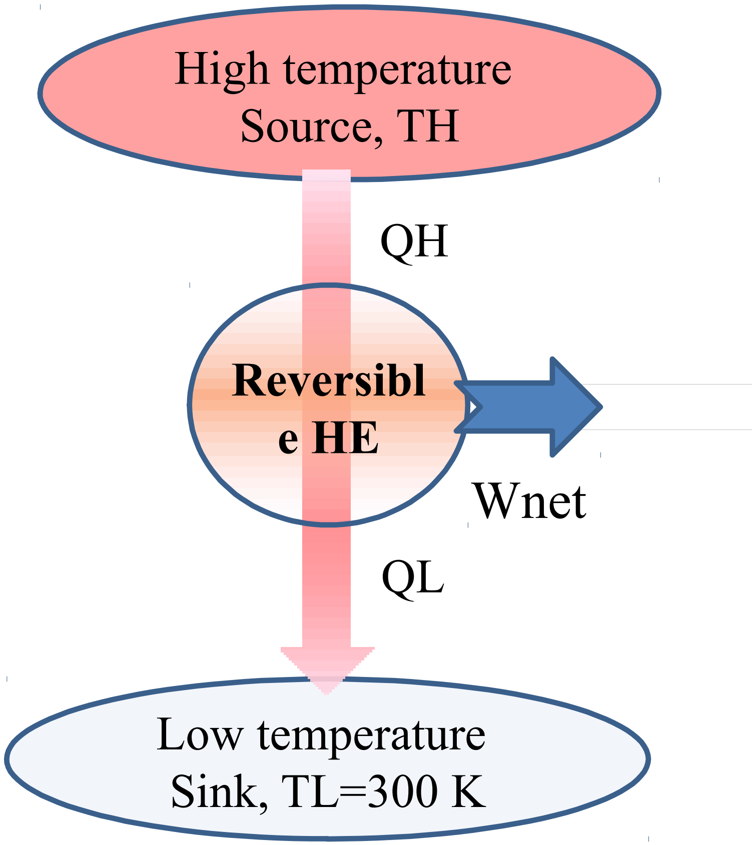
- Recap: Lecture 12: 6th Feb 2014, 0930-1030 hrs.
  - Carnot principles
  - Proof of the Carnot principles
  - Thermodynamic temperature scale

Quality of energy



TH, K	ηth, %
1000	70
700	57.1
500	40
400	25
350	14.3

Quality of energy

- Energy has quality as well as quantity.
- More of the high-temperature thermal energy can be converted to work.
- The higher the temperature, the higher the quality of the energy.
- Work is a high quality form of energy than heat since 100 percent of work can be converted to heat, but only a fraction of heat can be converted to work.

Carnot refrigerator and heat pump

- Operates on a reversed Carnot cycle.
- The coefficients of performance are:

$$COP_R = \frac{1}{Q_H / Q_L - 1} \qquad COP_{HP} = \frac{1}{1 - Q_L / Q_H}$$

$$or, \quad COP_R = \frac{1}{T_H / T_L - 1} \qquad COP_{HP} = \frac{1}{1 - T_L / T_H}$$

- These are the highest coefficients of performance that a refrigerator or a heat pump operating between the temperature limits of *TL* and *TH* can have.

Carnot refrigerator and heat pump

$$COP_{R/HP}\begin{cases} < COP_{R/HP,\,reversible} & \text{Irreversible} \\ = COP_{R/HP,\,reversible} & \text{Reversible} \\ > COP_{R/HP,\,reversible} & \text{Impossible} \end{cases}$$

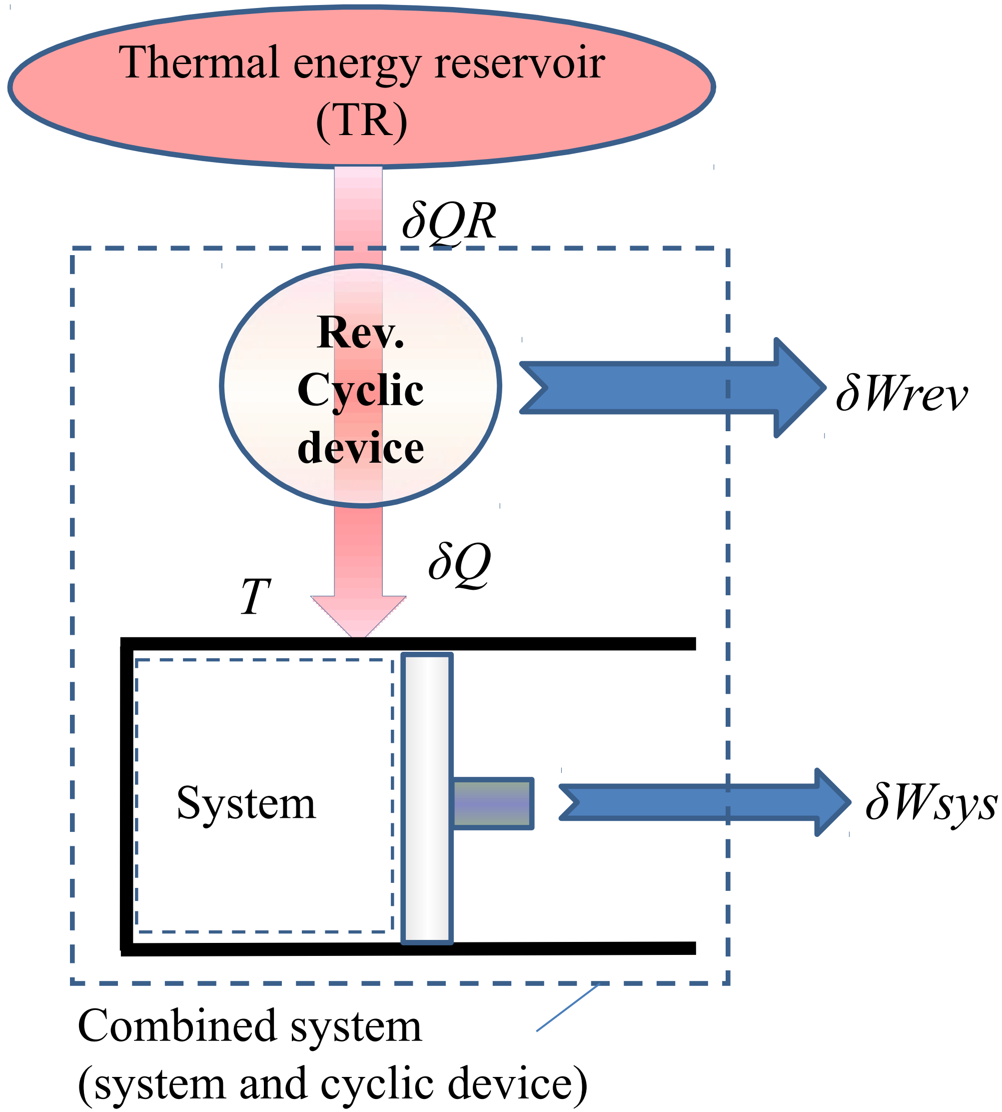
From thermodynamic temperature scale

- Lord Kelvin proposed $\phi(T) = T$ to define a thermodynamic scale as

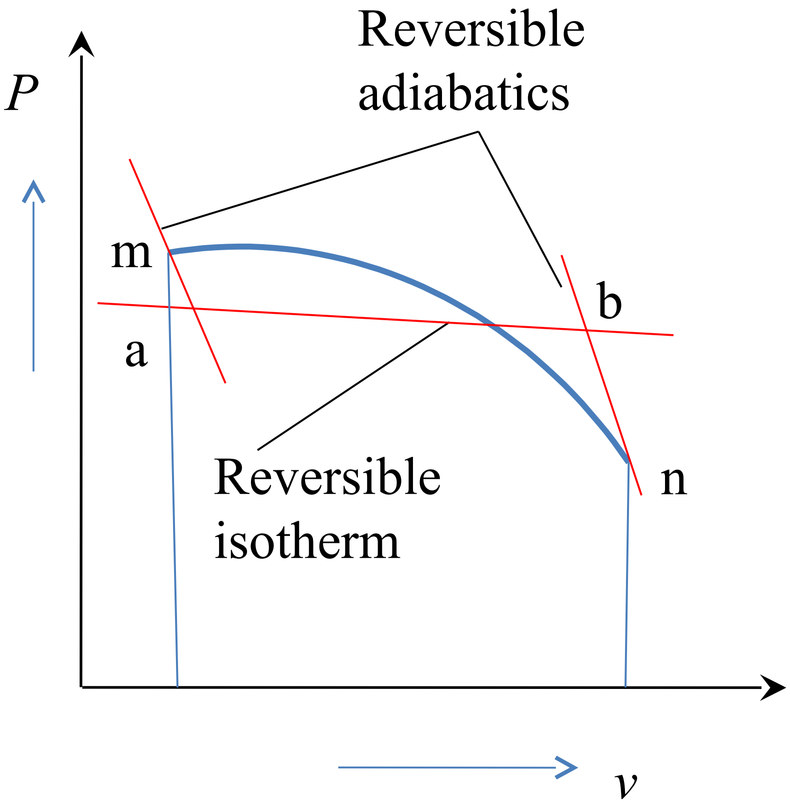
$$\left(\frac{Q_H}{Q_L}\right)_{rev} = \frac{T_H}{T_L}$$

$$\frac{Q_H}{T_H} = \frac{Q_L}{T_L}$$

Clausius inequality



Reversible adiabatics



<u>Process m-n</u>

$$Q_{m\text{-}n} = U_n - U_m + W_{mn}$$

<u>Process m-a-b-n</u>

$$Q_{m\text{-}a\text{-}b\text{-}n} = U_n - U_m + W_{m\text{-}a\text{-}b\text{-}n}$$

Since, $W_{m\text{-}a\text{-}b\text{-}n} = W_{mn}$

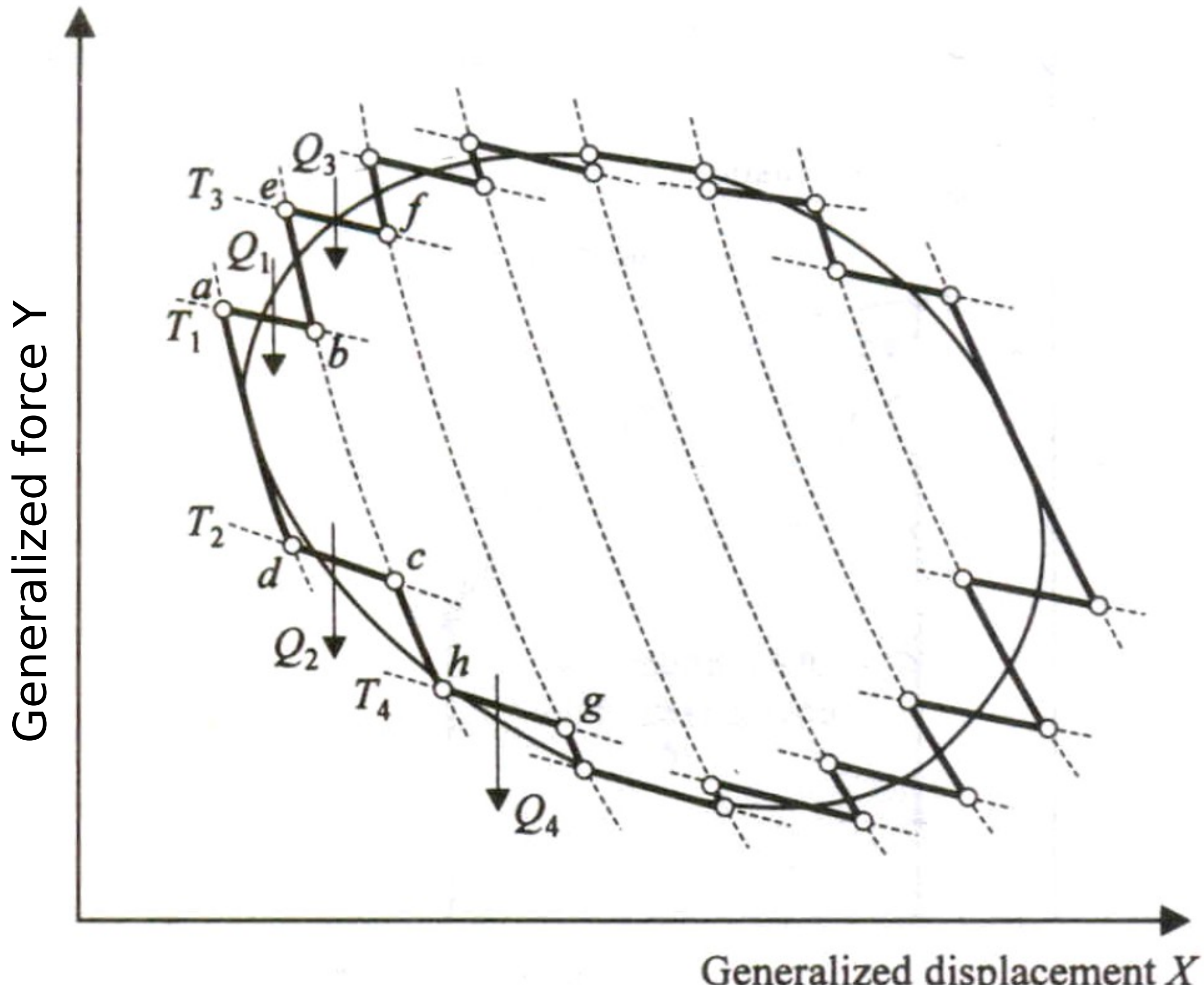
$$Q_{m\text{-}n} = Q_{m\text{-}a\text{-}b\text{-}n}$$

$$= Q_{m\text{-}a} + Q_{a\text{-}b} + Q_{b\text{-}n}$$

Since $Q_{m\text{-}a} = 0,\ Q_{b\text{-}n} = 0$

$$Q_{m\text{-}n} = Q_{a\text{-}b}$$

Reversible path can be substituted by two reversible adiabatics and a reversible isotherm


Generalized force Y
Generalized displacement X
T_1
T_2
T_3
T_4
Q_1
Q_2
Q_3
Q_4
a
b
c
d
e
f
g
h

$$\frac{|Q_2|}{T_2} = \frac{|Q_1|}{T_1}$$

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0$$

or

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \frac{Q_3}{T_3} + \frac{Q_4}{T_4} + \dots\dots\dots = 0$$

$$\sum_j \frac{Q_j}{T_j} = 0$$

Clausius inequality

- Applying the energy balance to the combined system identified by dashed lines yields: $\delta WC = \delta QR - dEC$
- where δWC is the total work of the combined system ($\delta Wrev+\delta Wsys$) and dEC is the change in the total energy of the combined system.
- Considering that the cyclic device is a reversible one

$$\frac{\delta Q_R}{T_R} = \frac{\delta Q}{T}$$

Clausius inequality

- From the above equations:

$$\delta W_C = T_R \frac{\delta Q}{T} - dE_C$$

- Let the system undergo a cycle while the cyclic device undergoes an integral number of cycles

$$W_C = T_R \oint \frac{\delta Q}{T}$$

- Since the cyclic integral of energy is zero.

Clausius inequality

- The combined system is exchanging heat with a single thermal energy reservoir while involving (producing or consuming) work *W*C during a cycle. Hence *WC* cannot be a work output, and thus it cannot be a positive quantity.
- Considering TR to be a positive quantity,

$$\oint \frac{\delta Q}{T} \leq 0$$

- This is the Clausius inequality.

Clausius inequality

- Clausius inequality is valid for all thermodynamic cycles, reversible or irreversible, including the refrigeration cycles.
- If no irreversibilities occur within the system as well as the reversible cyclic device, then the cycle undergone by the combined system is internally reversible.

$$\oint \left(\frac{\delta Q}{T} \right)_{\mathrm{int.}rev} = 0$$

Clausius inequality

- Clausius inequality provides the criterion for the irreversibility of a process.

$$\oint \frac{\delta Q}{T} = 0, \text{the process is reversible.}$$

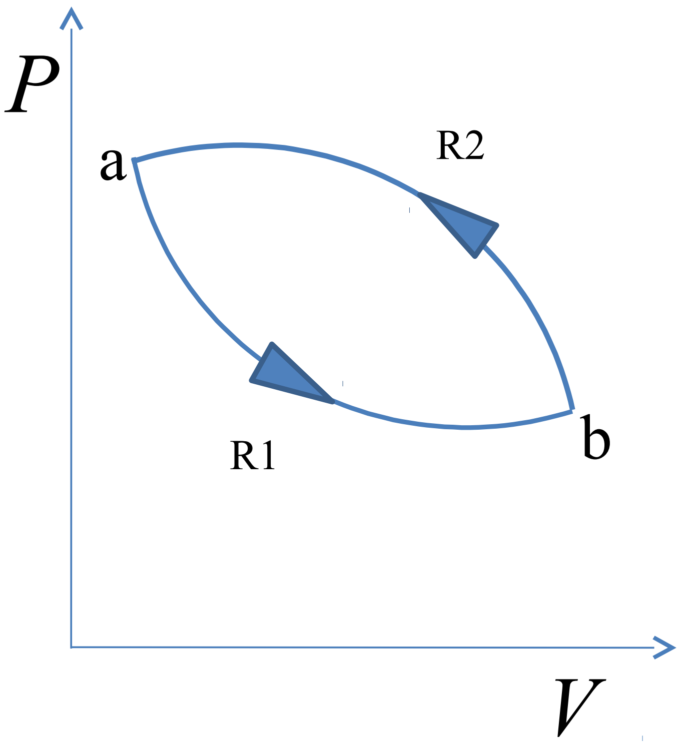
$$\oint \frac{\delta Q}{T} < 0, \text{the process is irreversible and possible.}$$

$$\oint \frac{\delta Q}{T} > 0, \text{the process is impossible.}$$

Clausius inequality and entropy

- The cyclic integral of work and heat are not zero.
- However, the cyclic integral of volume (or any other property) is zero.
- Conversely, a quantity whose cyclic integral is zero depends on the state only and not the process path, and thus it is a property
- Clausius realized in 1865 that he had discovered a new thermodynamic property, and he chose to name this property entropy.

The property of entropy



$$\oint_{R_1 R_2} \frac{dQ}{T} = 0$$

$$\int_{a\,(R1)}^{b} \frac{\delta Q}{T} + \int_{b\,(R2)}^{a} \frac{\delta Q}{T} = 0$$

or, $$\int_{a\,(R1)}^{b} \frac{\delta Q}{T} = -\int_{b\,(R2)}^{a} \frac{\delta Q}{T}$$

Since R_2 is a reversible path,

$$\int_{a\,(R1)}^{b} \frac{\delta Q}{T} = \int_{a\,(R2)}^{b} \frac{\delta Q}{T}$$

The property of entropy

- ${}_R\int_a^b \frac{\delta Q}{T}$ is independent of the reversible path connecting a and b.
- This property whose value at the final state minus the initial state is equal to ${}_R\int_a^b \frac{\delta Q}{T}$ is called entropy, denoted by S.

$$ {}_R\int_a^b \frac{\delta Q}{T} = S_b - S_a $$

- When the two equilibrium states are infinitesimally near,

$$ \frac{\delta Q_R}{T} = dS $$

Entropy

- Entropy is an extensive property of a system and sometimes is referred to as total entropy. Entropy per unit mass, designated *s*, is an intensive property and has the unit kJ/kg·K
- The entropy change of a system during a process can be determined by

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{int. } rev.} \qquad \text{(kJ/kg)}$$

Entropy

- Entropy is a property, and like all other properties, it has fixed values at fixed states.

- Therefore, the entropy change dS between two specified states is the same no matter what path, reversible or irreversible.

Temperature-entropy plot

$$dS = \frac{\delta Q_{rev}}{T}$$

If the process is reversible and adiabatic, $\delta Q_{rev} = 0$

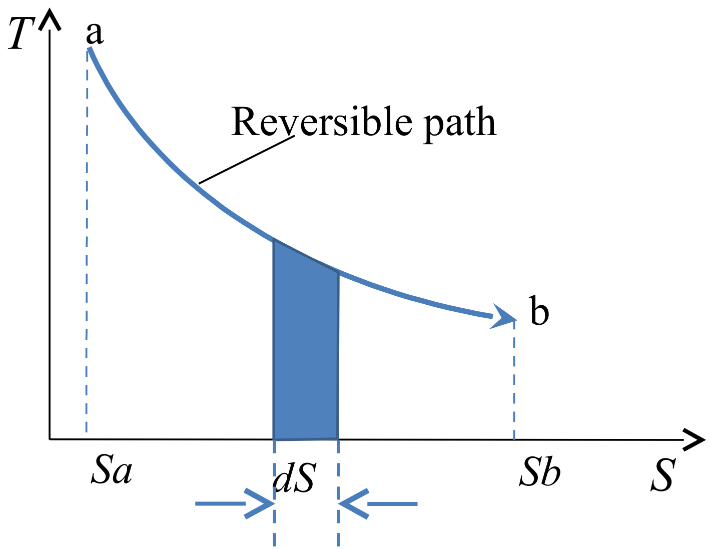
$$\therefore\ dS = 0 \quad or \quad S = \text{constant}$$

- A reversible adiabatic process is, therefore, and isentropic process.

$$\delta Q_{rev} = TdS$$

$$or,\ Q_{rev} = \int TdS$$

Temperature-entropy plot



$$Q_{rev} = \int_a^b TdS = T\left(S_b - S_a\right)$$

- The area under the reversible path on the T-S plot represents heat transfer during that process.

Isentropic processes

- A process where, $\Delta s=0$
- An isentropic process can serve as an appropriate model for actual processes.
- Isentropic processes enable us to define efficiencies for processes to compare the actual performance of these devices to the performance under idealized conditions.
- A reversible adiabatic process is necessarily isentropic, but an isentropic process is not necessarily a reversible adiabatic process.