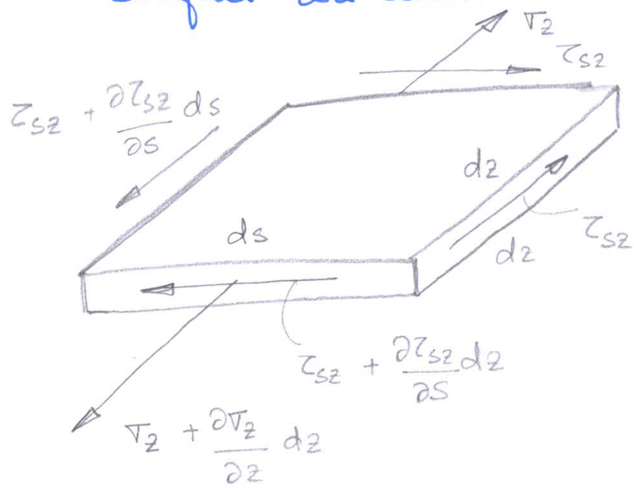
Shear stress distribution in thin-walled beam due to shear load

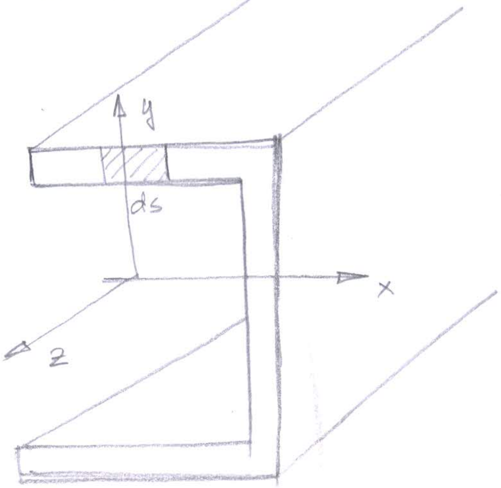
Assumptions

(1) axial constraint effects are negligible

(2) shear stresses normal to the beam surfaces may be neglected since they are zero at each surface and the wall is thin

(3) direct and shear stresses on planes normal to the surface are constant across the thickness





Force balance in z-dirc

$$\left(\sigma_z + \frac{\partial \sigma_z}{\partial z} dz\right) t\, ds - \sigma_z t\, ds + \left(\tau_{sz} + \frac{\partial \tau_{sz}}{\partial s} ds\right) t\, dz - \tau_{sz} dz\, t = 0$$

$$\Rightarrow \boxed{t \frac{\partial \sigma_z}{\partial z} + \frac{\partial q}{\partial s} = 0}$$

$$\sigma_z = \left[\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] x + \left[\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] y.$$

$$t \frac{\partial \sigma_z}{\partial z} = \left[\frac{(\partial M_y/\partial z) I_{xx} - \left(\frac{\partial M_x}{\partial z}\right) I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] t x + \left[\frac{\frac{\partial M_x}{\partial z} I_{yy} - \frac{\partial M_y}{\partial z} I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] t y$$

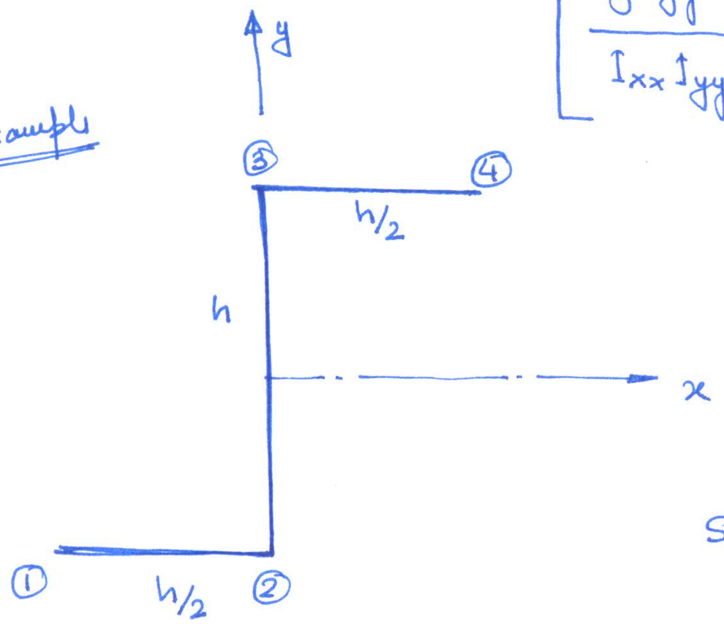
$$= \left[\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] t x + \left[\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] t y$$

$$\frac{\partial q}{\partial s} = -\left[\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] t x - \left[\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] t y.$$

$$\int_0^s \frac{\partial q}{\partial s}\, ds = q_s - q_0 = -\left[\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] \int t x\, ds$$

$$- \left[\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] \int t y\, ds.$$

Example



$$I_{xx} = \frac{h^3 t}{3}$$

$$I_{yy} = \frac{h^3 t}{12}$$

$$I_{xy} = \frac{h^3 t}{8}$$

$S_x = 0$ Find the shear flow distribution for a given S_y.

$$q_s - q_0 = \frac{S_y \cancel{I_{yy}} I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \int_0^s t x\, ds - \frac{S_y I_{yy}}{I_{xx} I_{yy} - I_{xy}^2} \int_0^s t y\, ds$$

Substituting the values for I_{xx}, I_{yy} & I_{xy}, we get,

$$q_s - q_0 = \frac{S_y}{h^3} \int_0^s (10.32x - 6.84y)\, ds$$

For flange 12, $0 \le s \le \frac{h}{2}$, $y = -\frac{h}{2}$, $x = -\frac{h}{2} + s$

$$q_s = q_{12} \underset{\substack{-q_1 \\ =0}}{=} \frac{S_y}{h^3} \int_0^s 10.32\left(-\frac{h}{2} + s\right) ds - \frac{S_y}{h^3} \int_0^s \frac{6.84h}{2}\, ds$$

$$\Rightarrow \quad q_{12} = \frac{S_y}{h^3}\left(5.16s^2 - 1.74hs\right)$$

$$q_2 = \frac{0.42 S_y}{h^3}$$

For web 23

$0 \leq s \leq h$ $$q_{23} - q_2 = \frac{S_y}{h^3} \int \left[10.23\, s - 6.84\, y\right] ds$$

$x = 0$ and $y = s - \frac{h}{2}$

$$q_{23} - q_2 = \frac{S_y}{h^3} \int_0^s \left[10.33\, s - 5.16\, h\right] ds$$

$$q_{23} = \frac{0.42\, S_y}{h} + \frac{S_y}{h^3}\left[\frac{10.32 s^2}{2} - 5.16\, hs\right]$$

$$= \frac{0.42\, S_y}{h} + \frac{S_y}{h^3}\left[5.16 s^2 - 5.16\, hs\right]$$

$$q_3 = \frac{0.42\, S_y}{h}$$

Using anti-symmetry