AE-214 TUTORIAL-2 12th February 2014

Question 1. **An innovative way of power generation involves the utilization of geothermal energy-the energy of hot water that exists naturally underground-as the heat source. If a supply of hot water at 140°C is discovered at a location where the environmental temperature is 20°C, determine the maximum thermal efficiency a geothermal power plant built at that location can have.**

Solution 1.

Assumptions The power plant operates steadily.

Analysis The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

$$\eta_{\text{th,max}} = \eta_{\text{th,C}} = 1 - \frac{T_L}{T_H} = 1 - \frac{20 + 273\ \text{K}}{140 + 273\ \text{K}} = 0.291 \text{ or } \mathbf{29.1\%}$$

Question 2. **A Carnot heat pump is to be used to heat a house and maintain it at 20°C in winter. On a day when the average outdoor temperature remains at about 2°C, the house is estimated to lose heat at a rate of 82,000 kJ/h. If the heat pump consumes 8 kW of power while operating, determine (a) How long the heat pump ran on that day; (b) The total heating costs, assuming an average price of 8.5¢/kWh for electricity; and(c) The heating cost for the same day if resistance heating is used instead of a heat pump.**

Solution 2.

Analysis (*a*) The coefficient of performance of this Carnot heat pump depends on the temperature limits in the cycle only, and is determined from

$$\mathrm{COP_{HP,rev}} = \frac{1}{1-(T_L/T_H)} = \frac{1}{1-(2+273\ \mathrm{K})/(20+273\ \mathrm{K})} = 16.3$$

The amount of heat the house lost that day is

$$Q_H = \dot{Q}_H(1\ \mathrm{day}) = (82{,}000\ \mathrm{kJ/h})(24\ \mathrm{h}) = 1{,}968{,}000\ \mathrm{kJ}$$

Then the required work input to this Carnot heat pump is determined from the definition of the coefficient of performance to be

$$W_{\mathrm{net,in}} = \frac{Q_H}{\mathrm{COP_{HP}}} = \frac{1{,}968{,}000\ \mathrm{kJ}}{16.3} = 120{,}736\ \mathrm{kJ}$$

Thus the length of time the heat pump ran that day is

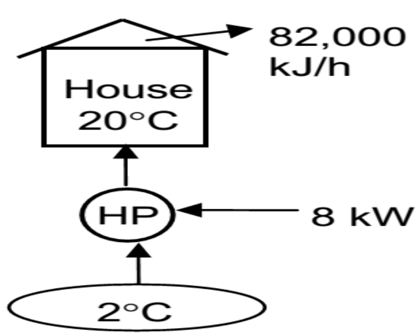
$$\Delta t = \frac{W_{\mathrm{net,in}}}{\dot{W}_{\mathrm{net,in}}} = \frac{120{,}736\ \mathrm{kJ}}{8\ \mathrm{kJ/s}} = 15{,}092\ \mathrm{s} = \mathbf{4.19\ h}$$

(*b*) The total heating cost that day is

$$\mathrm{Cost} = W \times \mathrm{price} = (\dot{W}_{\mathrm{net,in}} \times \Delta t)(\mathrm{price}) = (8\ \mathrm{kW})(4.19\ \mathrm{h})(0.085\ \$/\mathrm{kWh}) = \mathbf{\$2.85}$$

(*c*) If resistance heating were used, the entire heating load for that day would have to be met by electrical energy. Therefore, the heating system would consume 1,968,000 kJ of electricity that would cost

$$\mathrm{New\ Cost} = Q_H \times \mathrm{price} = (1{,}968{,}000\mathrm{kJ})\left(\frac{1\ \mathrm{kWh}}{3600\ \mathrm{kJ}}\right)(0.085\ \$/\mathrm{kWh}) = \mathbf{\$46.47}$$



Question3**. Consider two Carnot heat engines operating in series. The first engine receives heat from the reservoir at 1800 K and rejects the waste heat to another reservoir at temperature T. The second engine receives this energy rejected by the first one, converts some of it to work, and rejects the rest to a reservoir at 300 K. If the thermal efficiencies of both engines are the same, determine the temperature T.**

Solution 3: Two Carnot heat engines operate in series between specified temperature limits. If the thermal efficiencies of both engines are the same, the temperature of the intermediate medium between the two engines is to be determined.

Assumptions: The engines are said to operate on the Carnot cycle, which is totally reversible.

Analysis: The thermal efficiency of the two Carnot heat engines can be expressed as:

$$\eta_{\text{th,I}} = 1 - \frac{T}{T_H} \quad \text{and} \quad \eta_{\text{th,II}} = 1 - \frac{T_L}{T}$$

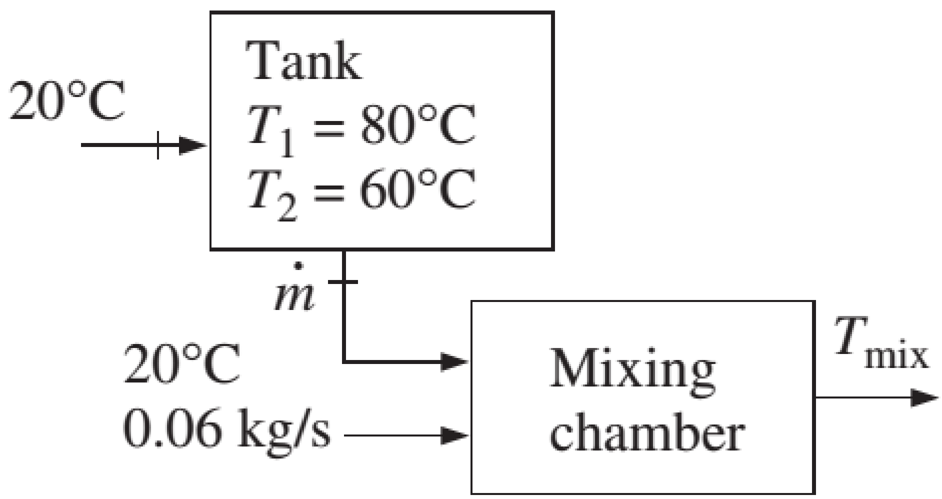
Equating,

$$1 - \frac{T}{T_H} = 1 - \frac{T_L}{T}$$

Solving for T,

$$T = \sqrt{T_H T_L} = \sqrt{(1800 \text{ K})(300 \text{ K})} = \mathbf{735\ K}$$

Question 4. **The hot-water needs of a household are met by a 60-L electric water heater whose heaters are rated at 1.6 kW. The hot-water tank is initially full with hot water at 80°C. Somebody takes a shower by mixing a constant flow of hot water from the tank with cold water at 20°C at a rate of 0.06 kg/s. After a shower period of 8 min, the water temperature in the tank is measured to drop to 60°C. The heater remained on during the shower and hot water withdrawn from the tank is replaced by cold water at the same flow rate. Determine the mass flow rate of hot water withdrawn from the tank during the shower and the average temperature of mixed water used for the shower**.



Solution 4. A water tank is heated by electricity. The water withdrawn from the tank is mixed with cold water in a chamber. The mass flow rate of hot water withdrawn from the tank and the average temperature of mixed water are to be determined.

Assumptions 1: The process in the mixing chamber is a steady-flow process since there is no change with time.2 Kinetic and potential energy changes are negligible. 3 There are no work interactions. 4 The device is adiabatic and thus heat transfer is negligible.

Properties: The specific heat and density of water are taken: Cp= 4.18 kJ/kg.K, ρ= 1000 kg/m$^{3.}$

Analysis: We take the mixing chamber as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as:

Energy balance:

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \quad (\text{since } \dot{Q} \cong \dot{W} \cong \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

or $$\dot{m}_{\text{hot}} c_p T_{\text{tank,ave}} + \dot{m}_{\text{cold}} c_p T_{\text{cold}} = (\dot{m}_{\text{hot}} + \dot{m}_{\text{cold}}) c_p T_{\text{mixture}} \quad (1)$$

Similarly, an energy balance may be written on the water tank as

$$\left[\dot{W}_{\text{e,in}} + \dot{m}_{\text{hot}} c_p (T_{\text{cold}} - T_{\text{tank,ave}})\right] \Delta t = m_{\text{tank}} c_p (T_{\text{tank,2}} - T_{\text{tank,1}}) \quad (2)$$

where $$T_{\text{tank,ave}} = \frac{T_{\text{tank,1}} + T_{\text{tank,2}}}{2} = \frac{80 + 60}{2} = 70°\text{C}$$

and $$m_{\text{tank}} = \rho \boldsymbol{V} = (1000 \text{ kg/m}^3)(0.060 \text{ m}^3) = 60 \text{ kg}$$

Substituting into Eq. (2),

$$\left[1.6\text{ kJ/s} + \dot{m}_{\text{hot}}(4.18\text{ kJ/kg.°C})(20-70)\text{°C}\right](8\times 60\text{ s}) = (60\text{ kg})(4.18\text{ kJ/kg.°C})(60-80)\text{°C}$$

$$\longrightarrow \dot{m}_{\text{hot}} = \mathbf{0.0577\ kg/s}$$

Substituting into Eq. (1),

$$(0.0577\text{ kg/s})(4.18\text{ kJ/kg.°C})(70\text{°C}) + (0.06\text{ kg/s})(4.18\text{ kJ/kg.°C})(20\text{°C})$$

$$= [(0.0577+0.06)\text{kg/s}](4.18\text{ kJ/kg.°C})T_{\text{mixture}}$$

$$\longrightarrow T_{\text{mixture}} = \mathbf{44.5°C}$$