AE 230 - **Modeling and Simulation Laboratory**

Generalized approach to modelling

Three empirical laws:

Conservation of mass

Conservation of momentum

Conservation of energy

Above three laws can be used to describe any changes in the physical world

While using above laws, fundamental dependent variables are identified. Theses define the state of the system at all instants.

Generalized approach to modelling

Electrical system is defined by current i and voltage e

Power = voltage x current

Power : rate of change of energy

Power supplied $$\frac{dE_i}{dt} = ei$$

Conservation of Power

Power supplied = Power into storage + Power dissipated

Generalized approach to modelling

Conservation of Power

Power supplied = Power into storage + Power dissipated

$$\frac{dE_i}{dt} = \frac{dE_c}{dt} + \frac{dE_R}{dt}$$

$$ei = eC\frac{de}{dt} + e\frac{e}{R}$$

$$i = C\frac{de}{dt} + \frac{e}{R}$$

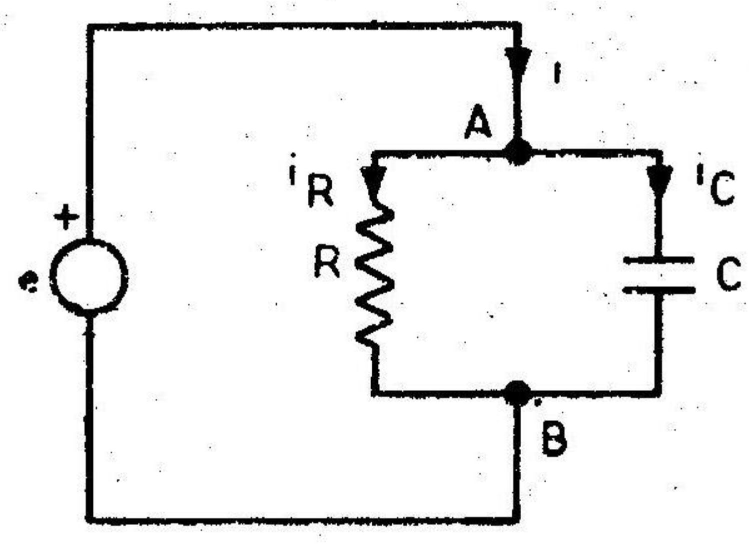


Fig. A simple electrical circuit.

Above equation is Kirchoff's current law

Generalized approach to modelling

Dynamical systems are defined using constitutive relations; compatibility and continuity conditions

i)Equations describing the behavior of elements - constitutive relations

ii) Compatibility conditions applied to across variables

iii) Continuity conditions applied to through variables

Voltage can be called as across variable

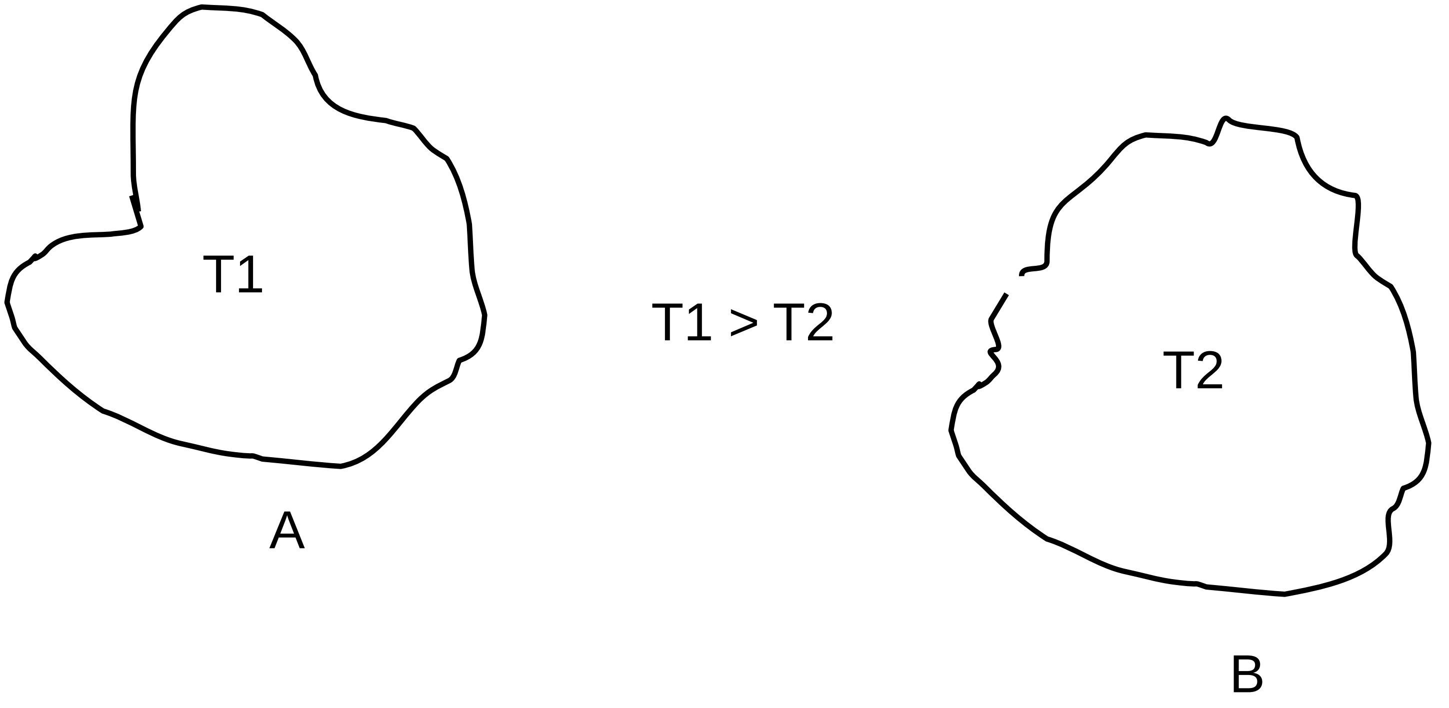
Current can be called as through variables

Generalized approach to modelling

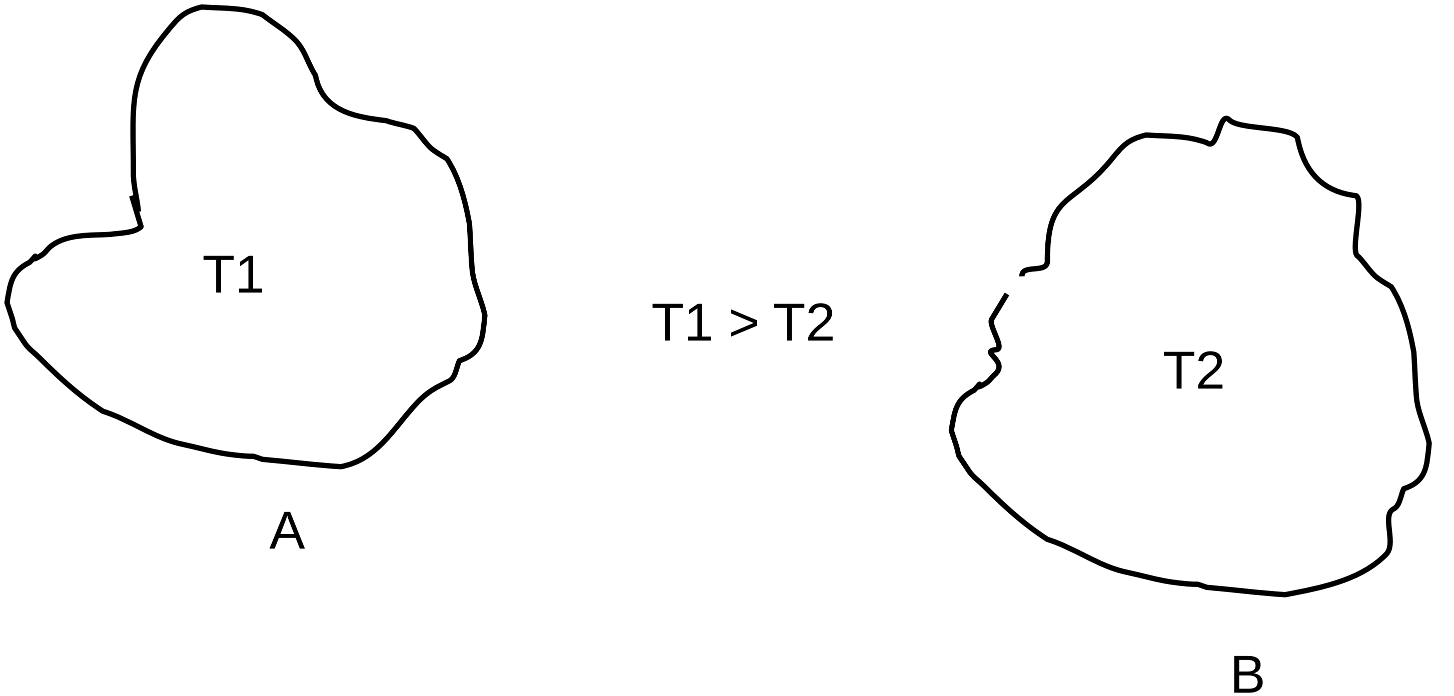
System Type	Through variable	Integrated through variable	Across variable	Integrated across variable
1. Mechanical translational	Force	Translational momentum	Relative velocity	Relative displacement
2. Mechanical rotational	Torque	Angular momentum	Relative angular velocity	Relative angular displacement
3. Electrical	Current	Charge	Voltage difference	Flux linkage
4. Fluid	Fluid flow rate	Volume	Pressure difference	Pressure momentum
5. Thermal	Heat flow rate	Heat energy	Temperature difference	Not used in general

In general power can be expressed as multiplication of through and across variable for fluid and mechanical systems. This is not true for thermal system because through variable is power itself.

Thermal systems



Thermal systems



Heat transfer from A to B till T1 = T2

Thermal systems

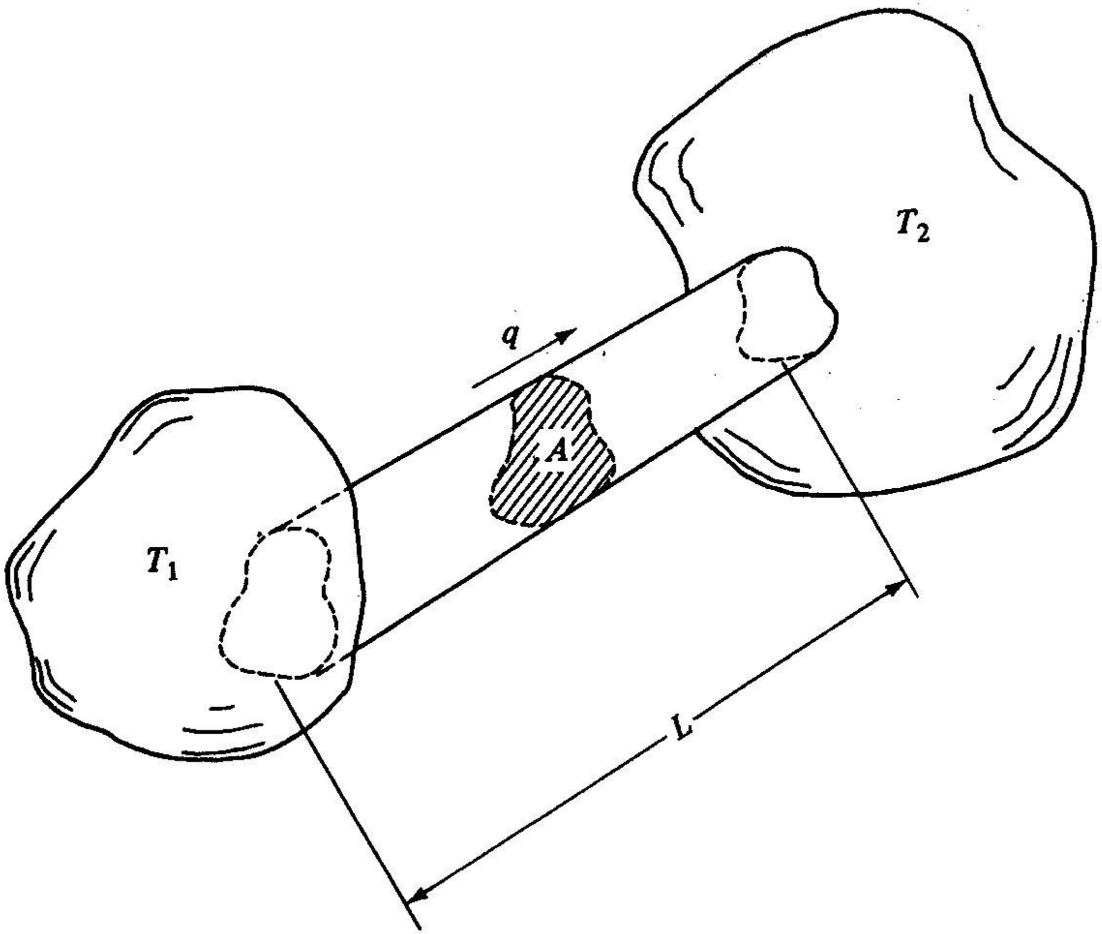


Figure Heat transfer by conduction.

Thermal systems

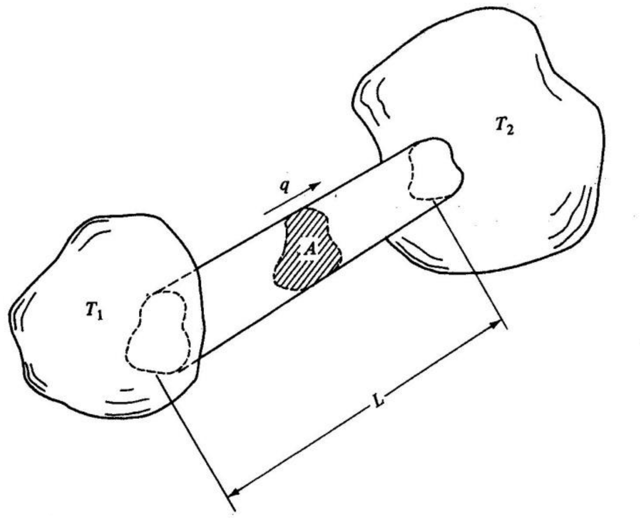


Figure Heat transfer by conduction.

k – Thermal conductivity

A – constant cross-sectional area

L – length of the connecting member

$$\text{Heat transfer rate} = q = -kA\frac{dT}{dl}$$

$$q\int_0^l dl = -kA\int_{T_1}^{T_2} dT$$

$$\text{Heat transfer rate} = q = \frac{kA}{L}(T_1 - T_2) = \frac{kA}{L}(\Delta T)$$

Thermal systems

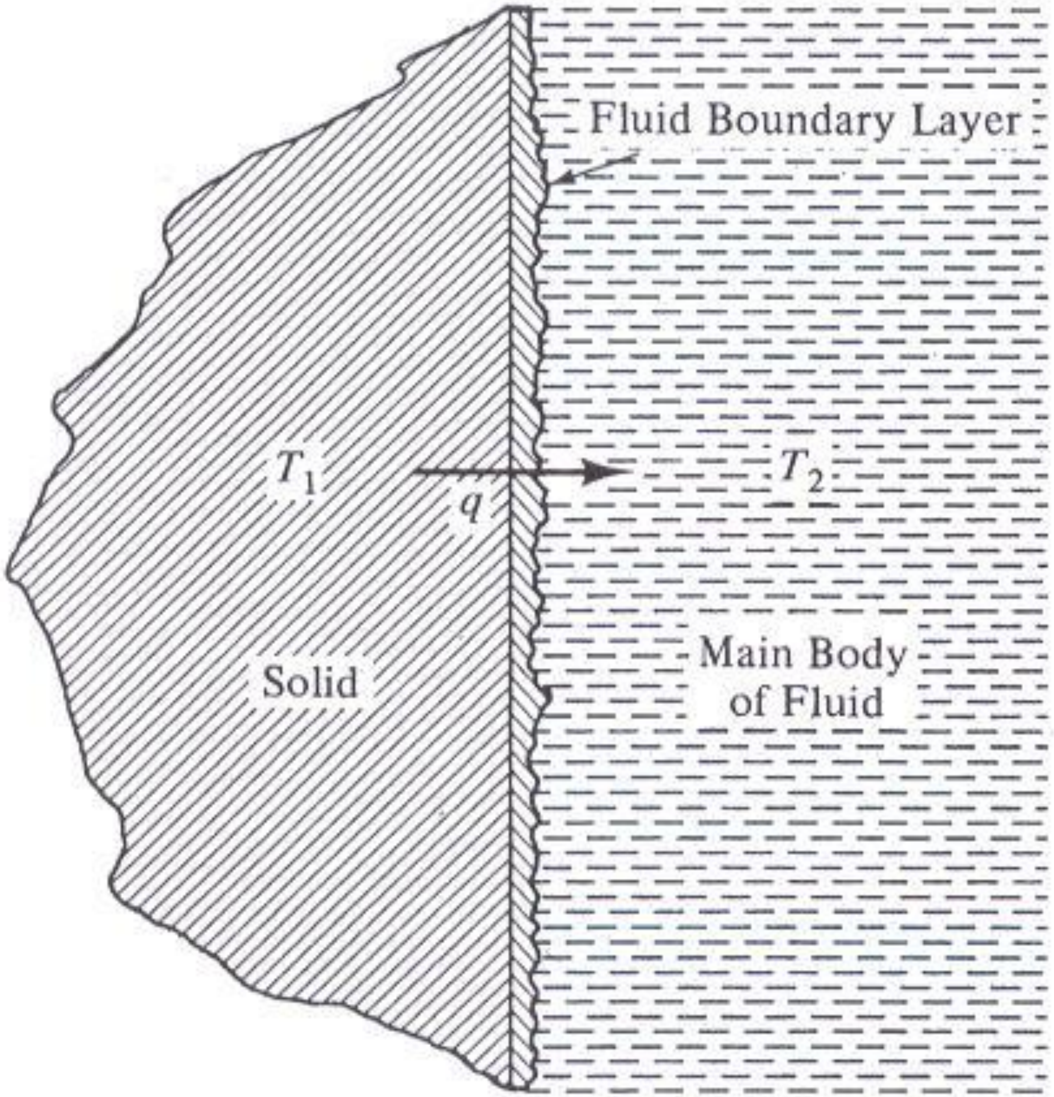


Figure 4-31 Heat transfer by convection.

Thermal systems

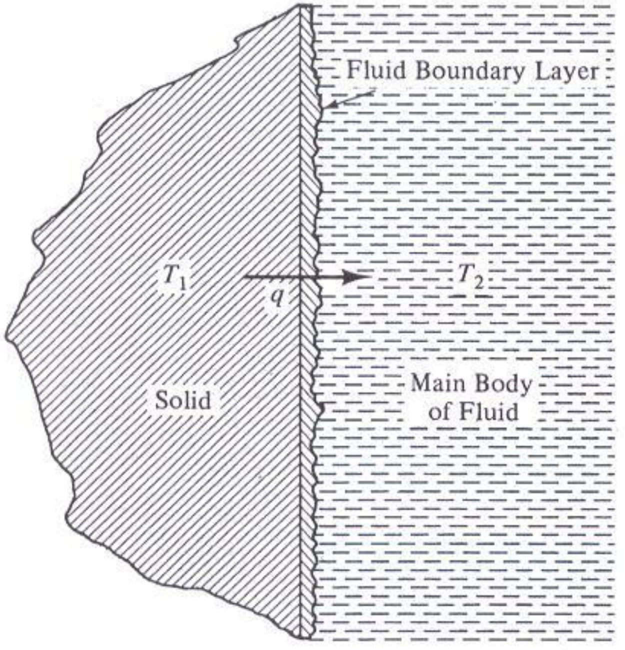


Figure 4-31 Heat transfer by convection.

Heat is carried away by constantly moving particles

- Forced convection

- Free convection

$$\textbf{Heat transfer rate} = q = hA(T_1 - T_2) = hA(\Delta T)$$

h – film coefficient of heat transfer

Thermal systems

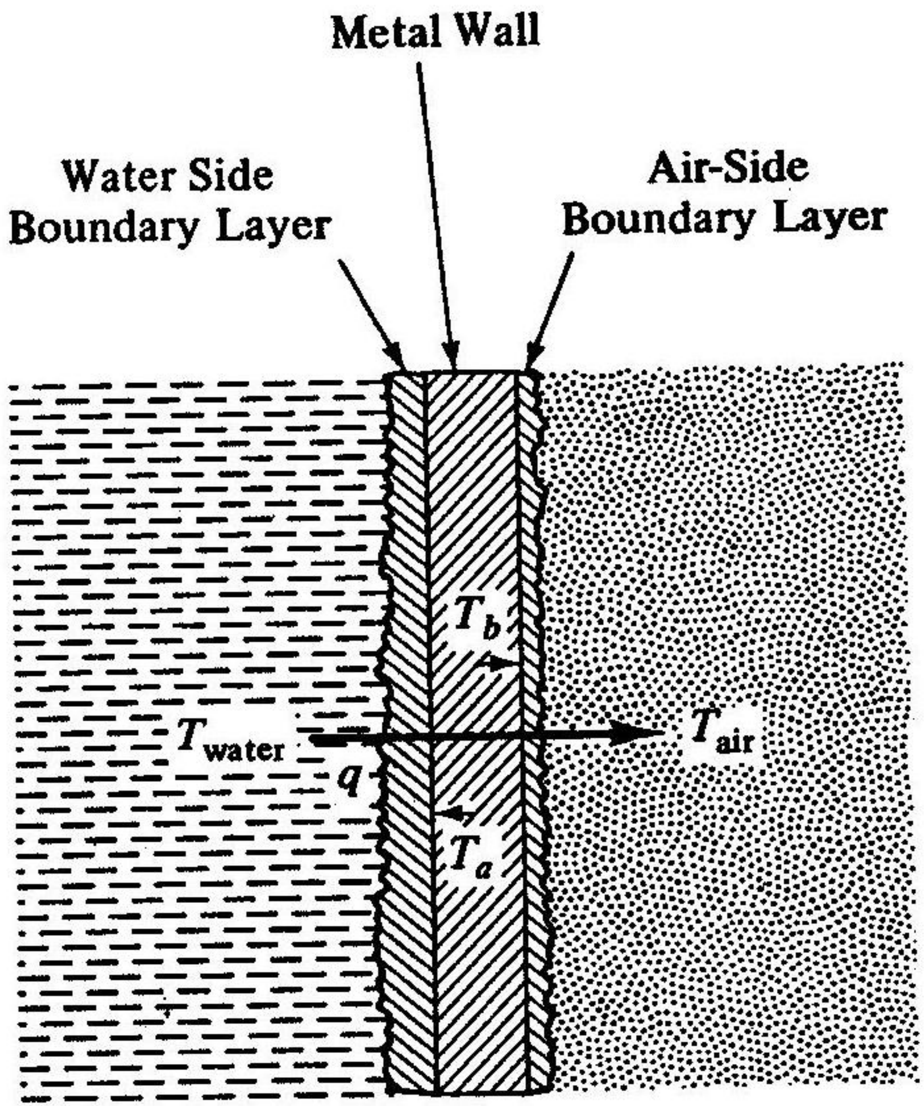


Figure Combined conduction/convection: overall heat transfer.

Thermal systems

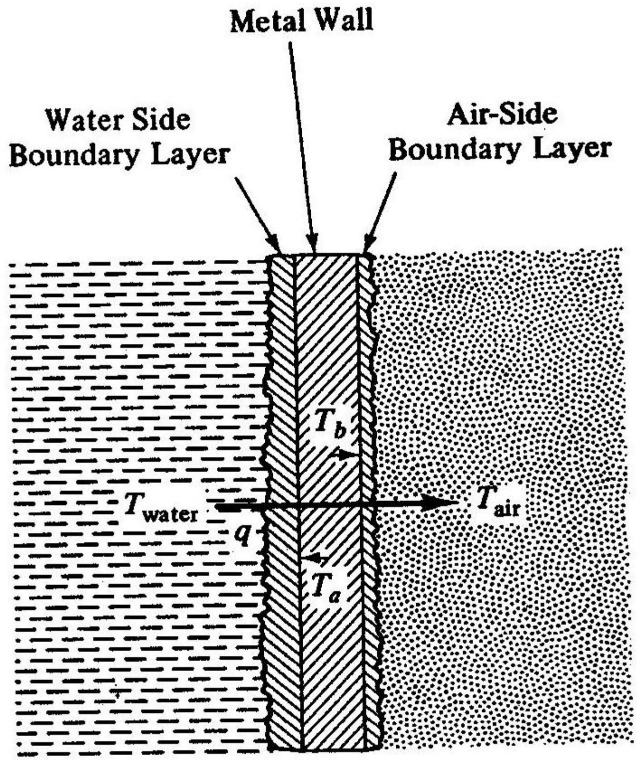


Figure Combined conduction/convection: overall heat transfer.

$$q = \frac{T_W - T_A}{\frac{1}{h_W A} + \frac{L}{kA} + \frac{1}{h_A A}} = \frac{\Delta T}{R_t} = UA\Delta T$$

$$q \frac{1}{h_W A} + q \frac{L}{kA} + q \frac{1}{h_A A} = (T_W - T_A)$$

Thermal systems

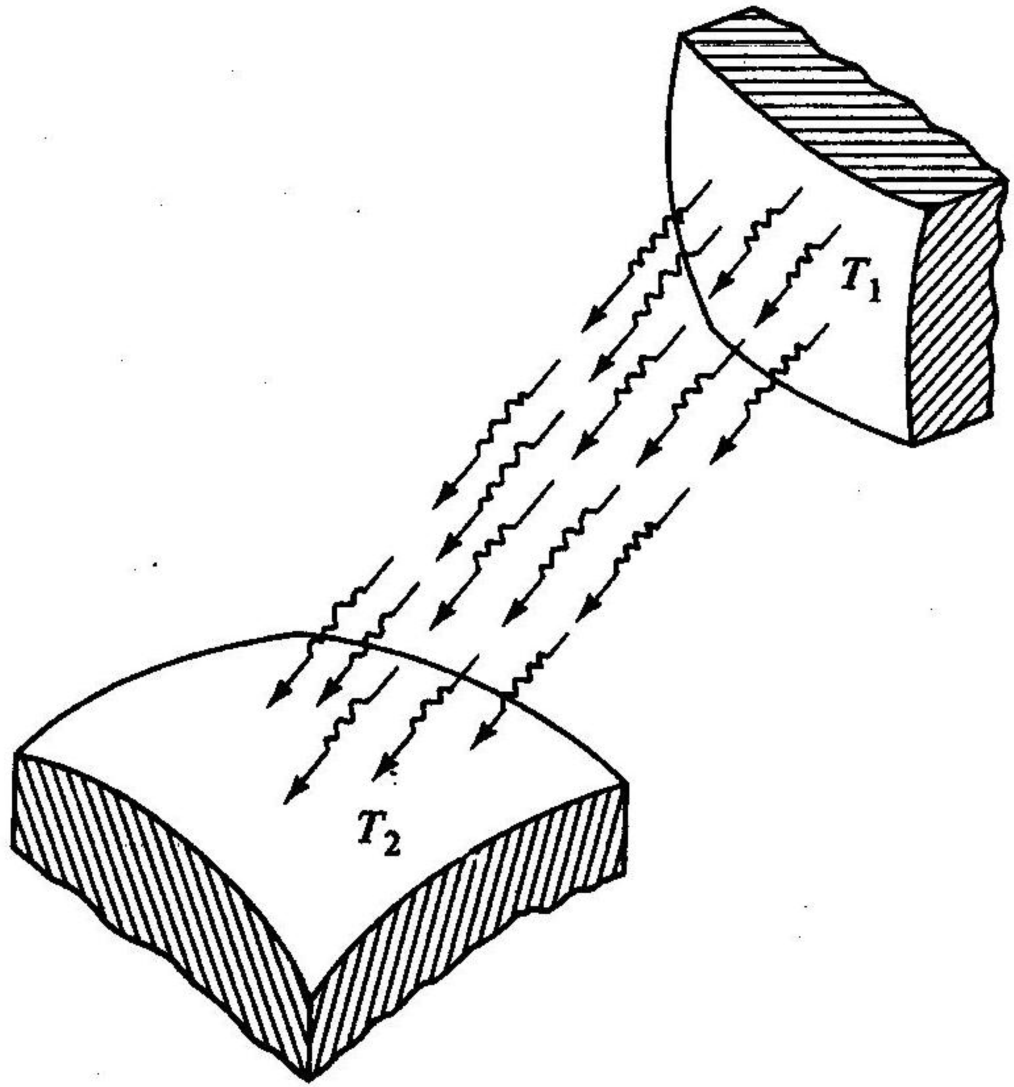


Figure 4-33 Heat transfer by radiation.

Thermal systems

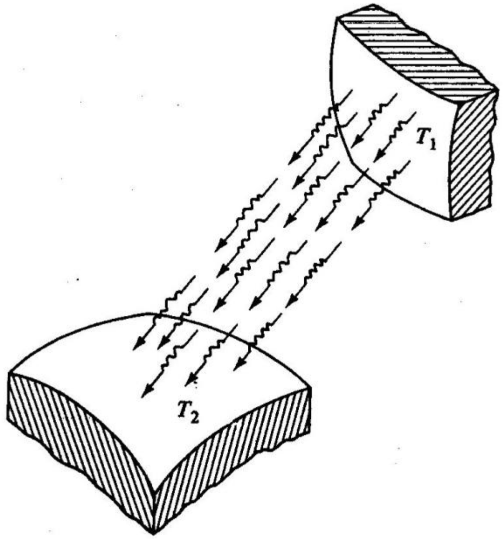


Figure 4-33 Heat transfer by radiation.

$$q = C(T_1^4 - T_2^4)$$

$$q = C(T_1^2 + T_2^2)(T_1 + T_2)(T_1 - T_2)$$

$$R_{\mathbf{t}} = \frac{\Delta T}{q} = \frac{1}{C(T_1^2 + T_2^2)(T_1 + T_2)}$$

$$R_{\mathbf{t}} \approx \frac{1}{C(T_{1,0}^2 + T_{2,0}^2)(T_{1,0} + T_{2,0})}$$

Thermal systems

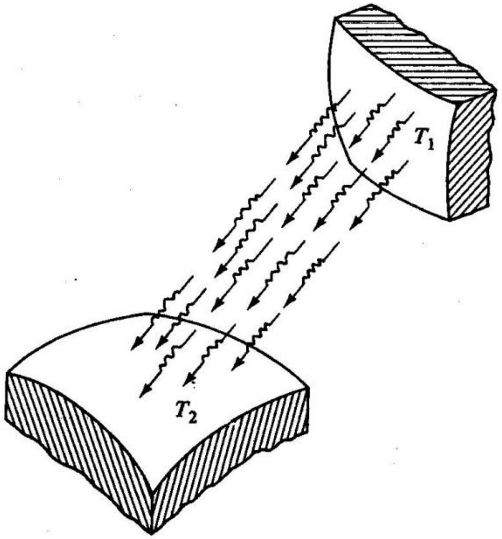


Figure 4-33 Heat transfer by radiation.

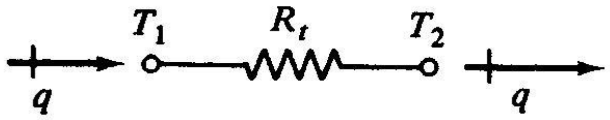
$$q = C(T_1^4 - T_2^4)$$

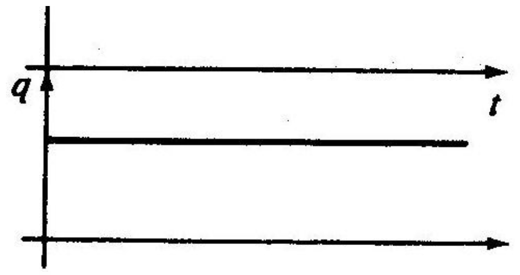
$$q \approx C(T_{1,0}^4 - T_{2,0}^4) + \left[\frac{\partial q}{\partial T_1}\right]_{T_{1,0},T_{2,0}} (T_1 - T_{1,0}) + \left[\frac{\partial q}{\partial T_2}\right]_{T_{1,0},T_{2,0}} (T_2 - T_{2,0})$$

$$q \approx -3CT_{1,0}^4 + 3T_{2,0}^4 + (4CT_{1,0}^3)T_1 - (4CT_{2,0}^3)T_2$$

$$q \approx 4CT^3(T_1 - T_2) \qquad T_{1,0} = T_{2,0} \qquad R_t \approx \frac{\Delta t}{q} = \frac{1}{4CT^3}$$

Thermal systems





Step Response

Thermal systems

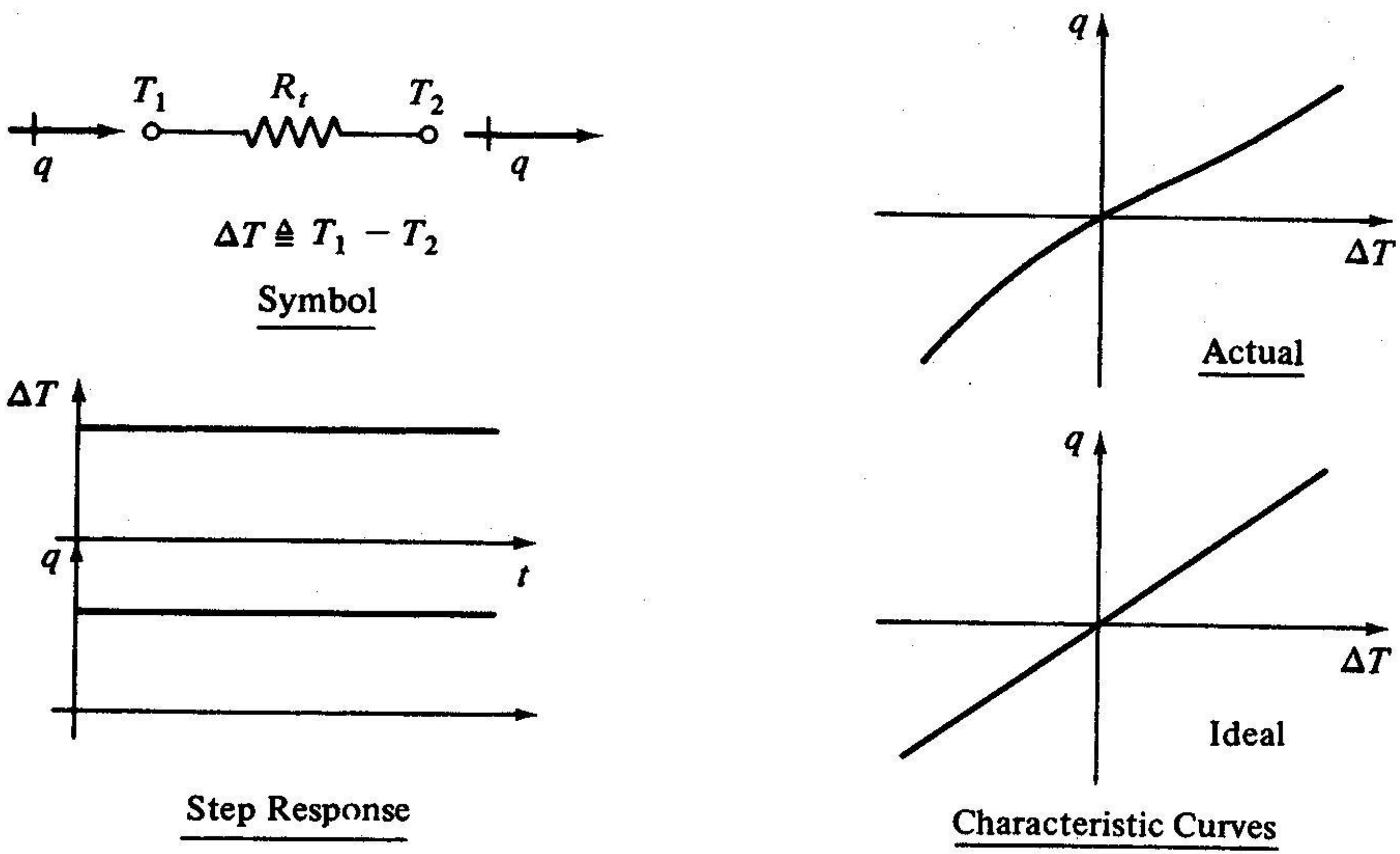
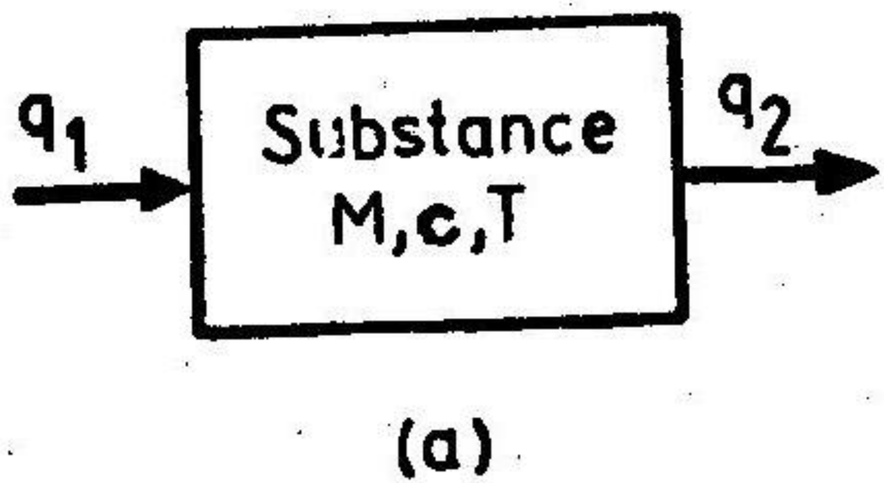


Figure The thermal resistance element (see table in Appendix C).

Thermal capacitance



Heat stored as internal energy $q = q_1 - q_2$

Heat added = $\int q\,dt$ = mass x specific heat x temperature rise

$$q = q_1 - q_2 = Mc\frac{dT}{dt}$$

Thermal capacitance

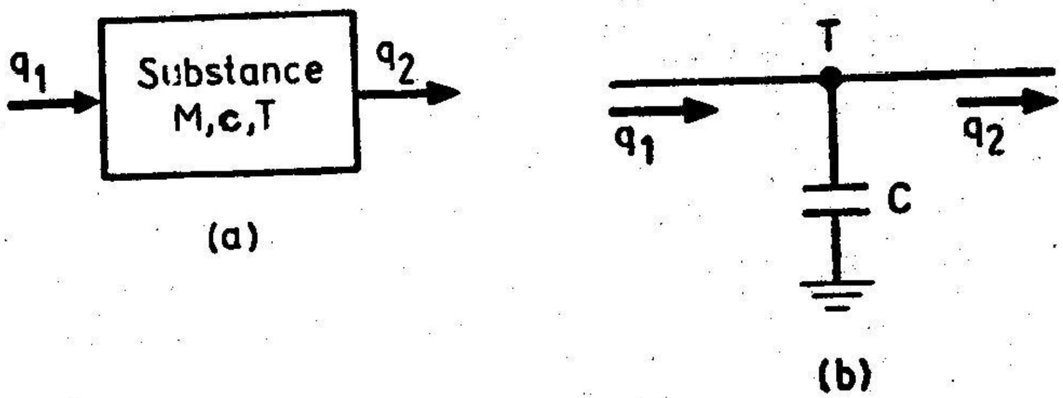


Fig. 3.28 Thermal capacitance.

Thermal capacitance $C = Mc$

Thermal systems

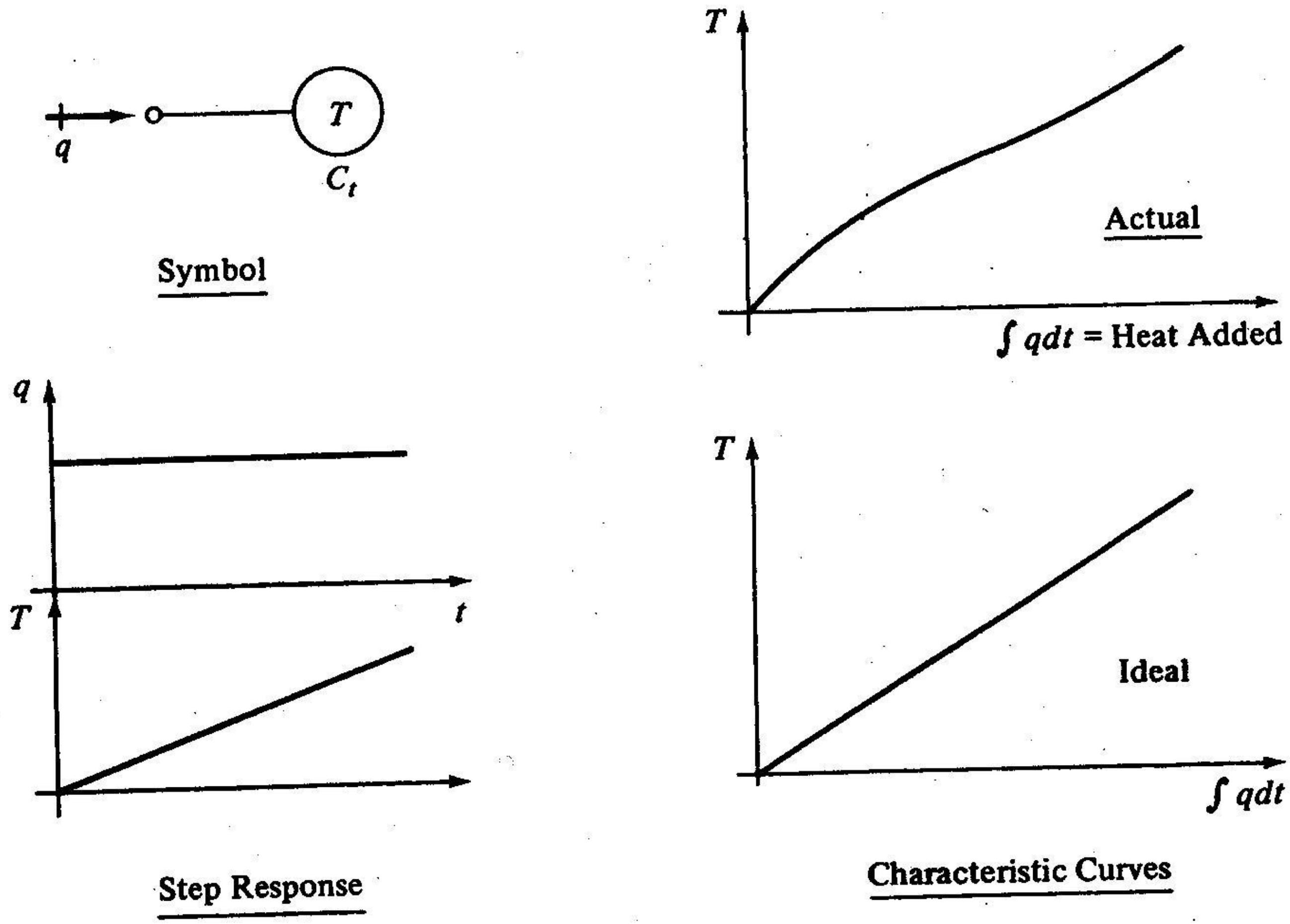


Figure 4-36 The thermal capacitance element (see table in Appendix C).

Thermal inductance?

Is it possible to have analogy of inductance in thermal systems?