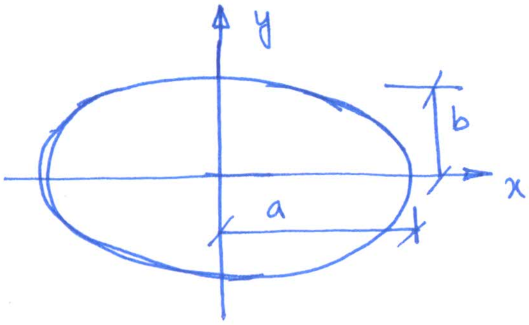
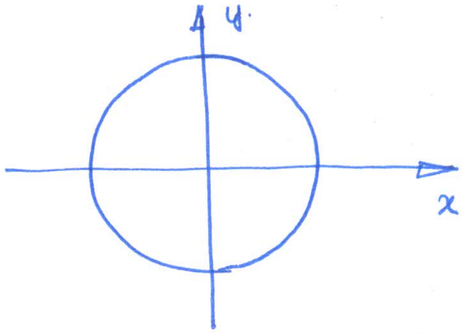
Tutorial 3

Q. Show that for the same twist, the elliptic section has a greater shearing stress than the inscribed circular section (radius equal to the minor axis b of the ellipse). Which takes the greater torque for the same allowable stress.





$$\varphi = k\left[\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right] \quad \& \quad \frac{d\varphi}{ds} = 0$$

$$k = -\frac{G\theta a^2 b^2}{(a^2+b^2)}$$

$$\therefore \quad \varphi = -\frac{G\theta a^2 b^2}{a^2+b^2}\left[\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right]$$

$$\tau_{xz} = -\frac{\partial \varphi}{\partial y} = -\frac{2G\theta a^2 y}{a^2+b^2}$$

By membrane analogy, max^m stress occurs at the minor axis

$$\tau_{max} = \tau_{xz\,(@ \pm b)} = \pm\frac{2G\theta a^2 b}{(a^2+b^2)}$$

$$\therefore \quad \boxed{\tau_{max}^{ellipse} > \tau_{max}^{circle.}}$$ for same twist.

Torque $M_t = 2\iint \varphi\, dx\,dy$

$$= \frac{\pi G\theta a^3 b^3}{(a^2+b^2)}$$

$$= \frac{\pi a b^2}{2}\tau_{max}$$

$$\varphi = k(x^2+y^2-b^2) \quad \& \quad \frac{d\varphi}{ds} = 0$$

$$k = -\frac{G\theta}{2}.$$

$$\Rightarrow \quad \varphi = -\frac{G\theta}{2}(x^2+y^2-b^2)$$

$$\tau_{xz} = -G\theta y.$$

$$\tau_{max} = \tau_{xz\,@\pm b} = \pm G\theta b.$$

(by membrane analogy)

$$\text{Torque} = M_t = 2\iint \varphi\, dx\,dy$$

$$= \frac{G\theta\pi b^4}{2} = \frac{\pi b^3}{2}\tau_{max}$$

$$\therefore \quad M_t^{ellipse} > M_t^{circle.}$$