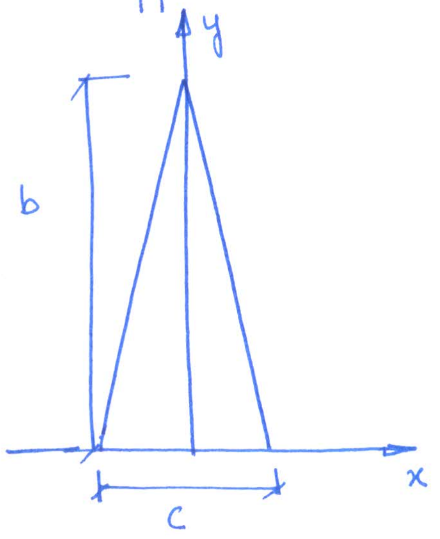
Tutorial 4

Q. Using membrane analogy, show that the approximate solution for narrow triangular c/s is given as,

$$M_t = \frac{1}{12} G\theta b c^3$$



Assuming that the stress function φ is independent of y.

$$\varphi = -G\theta\left(x^2 - \frac{t^2}{4}\right)$$ t = width of the triangle at y.

$$= -G\theta\left[x^2 - \frac{c^2}{4b^2}(b-y)^2\right] dx\,dy.$$

Torque $M_t = 2\iint \varphi \, dx\,dy$

$$= -2G\theta \int_0^b \int_{-\frac{c}{2b}(b-y)}^{\frac{c}{2b}(b-y)} \left[x^2 - \frac{c^2}{4b^2}(b-y)^2\right] dx\,dy$$

$$= -2G\theta \int_0^b \left[\frac{x^3}{3} - \frac{c^2}{4b^2}x(b-y)^2\right]_{-\frac{c}{2b}(b-y)}^{\frac{c}{2b}(b-y)} dy$$

$$= -2G\theta \int_0^b \left\{\frac{c^3}{12b^3}(b-y)^3 - \frac{c^3}{4b^3}(b-y)^3\right\} dy.$$

$$= \frac{2G\theta c^3}{12b^3} \times 2 \int_0^b (b-y)^3 dy$$

$$= \frac{G\theta c^3}{3b^3} \frac{1}{4}(b-y)^4 \Big|_0^b = \boxed{\frac{G\theta c^3 b}{12}}$$