ASSIGNMENT 1

AE639:CONTINUUM MECHANICS

1. Verify the following:

   (a) $\vec{u}.\vec{v} = u_i v_i$
   (b) $\vec{u} \times \vec{v} = \epsilon_{ijk} e_i u_j v_k$
   (c) $(\vec{u} \times \vec{v}).\vec{w} = \vec{u}.(\vec{v} \times \vec{w})$
   (d) $(\vec{u} \times \vec{v}) \times \vec{w} = (\vec{u}.\vec{w})\vec{v} - (\vec{v}.\vec{w})\vec{u}$
   (e) $(\vec{u} \times \vec{v})^2 = u^2 v^2 - (\vec{u}.\vec{v})^2$
   where $u^2 = |\vec{u}|^2 and\, v^2 = |\vec{v}|^2$

2. Let A be 3×3 matrix with enteries A_{ij}. Verify

   (a) $Det[A] = \epsilon_{ijk} A_{1i} A_{2j} A_{3k} = \epsilon_{ijk} A_{i1} A_{j2} A_{k3}$
   (b) $\epsilon_{lmn} Det[A] = \epsilon_{ijk} A_{il} A_{jm} A_{kn}$
   (c) $Det[A] = \dfrac{1}{6} \epsilon_{ijk} \epsilon_{lmn} A_{il} A_{jm} A_{kn}$

3. Write in expanded form:
   $A_{ii}, B_{ijj}, R_{ij}, a_i T_{ij}, a_i b_j S_{ij}$

4. Suppose B is skew-symmetric Matrix for which the vector $b_i = \dfrac{1}{2} \epsilon_{ijk} B_{jk}$,show that $B_{pq} = \epsilon_{pqi} b_i$.

5. If B is skew symmetric and A is symmetric, show that $A_{ij} B_{ij} = 0$.

6. Suppose that $T_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}$, where $\mu\, and \lambda$ are positive constants. Solve for ε in terms of T_{ij}. Note that no term involving ε_{ij} should appear on the right hand side of the equation $\varepsilon_{ij} =$ ......

7. Express in indicial notation:

   (a) A
   (b) A^2
   (c) AB
   (d) BA
   (e) $A^T B$
   (f) $A^T B A$

8. Prove that δ_{ij} and ϵ_{ijk} are isotropic tensors.

9. Evaluate:

   (a) $\delta_{ij} \delta_{jk} \delta_{kl} \delta_{il}$
   (b) $\epsilon_{ijk} \delta_{jk}$
   (c) $\epsilon_{jk2} \epsilon_{k2j}$
   (d) $\epsilon_{23i} \epsilon_{2i3}$