# MA 214 Quiz 1

**Code A**

1) $\sum_{i=0}^{3} l_i(x) = 1$.

2) **Divided Difference Table**

| $x$ | $f(x)$ | | | |
|---|---|---|---|---|
| 0 | 1 | | | |
| | | 3 | | |
| 1 | 4 | | 5 | |
| | | 18 | | 1 |
| 3 | 40 | | 12 | |
| | | 90 | | |
| 7 | 400 | | | |

$$P_3(x) = \underbrace{1 + 3x + 5x(x-1)}_{P_2(x)} + x(x-1)(x-3)$$

4) $f(x) = \frac{1}{x}$, $f(x) - p_2(x) = f[1,2,3,x]\,\omega(x)$

$$= \frac{f^{(3)}(c_x)}{6}\,\omega(x),$$

where $\omega(x) = (x-1)(x-2)(x-3)$.

$$\|f - p_2\|_\infty \leq \frac{\|f^{(3)}\|_\infty}{6}\,\|\omega\|_\infty .$$

$$\max_{x\in[1,3]} |\omega(x)| = \max_{y\in[-1,1]} |(y+1)\,y\,(y-1)| \quad (y = x-2)$$

$g(y) = y(y^2-1)$, $g'(y) = 3y^2 - 1 = 0 \Rightarrow y = \pm\frac{1}{\sqrt{3}}$

$$\|\omega\|_\infty = \left|\frac{1}{\sqrt{3}}\left(\frac{1}{3}-1\right)\right| = \frac{2}{3\sqrt{3}}$$

$$f(x) = \frac{1}{x}, \quad f'(x) = -\frac{1}{x^2}, \quad f''(x) = \frac{2}{x^3}$$

$$f^{(3)}(x) = -\frac{6}{x^4}, \quad \frac{\|f^{(3)}\|_\infty}{6} = 1.$$

$$\|f - p_2\|_\infty = \frac{2}{3\sqrt{3}}$$

6) $\displaystyle\int_0^6 f(x)\,dx = \frac{6}{6}\left(f(0) + 4f(3) + f(6)\right)$ : Simpson Rule.

$f(0) = -18, \quad f(3) = -6, \quad f(6) = 24$

Integral $= -18 - 24 + 24 = -18$

5)

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | | | | |
| | | 3 | | | |
| 0 | 1 | | 3 | | |
| | | 6 | | 1 | |
| 1 | 7 | | 4 | | 1 |
| | | 10 | | 3 | |
| 1 | 7 | | 10 | | |
| | | 20 | | | |
| 2 | 27 | | | | |

$$p_4(x) = \underbrace{1 + 3x + 3x^2 + x^2(x-1)}_{p_3(x)} + x^2(x-1)^2$$

## Code B

2)

| | | | | |
|---|---|---|---|---|
| 0 | 2 | | | |
| | | 2 | | |
| 1 | 4 | | 3 | |
| | | 11 | | 1 |
| 3 | 26 | | 10 | |
| | | 71 | | |
| 7 | 310 | | | |

$$P_3(x) = \underbrace{2 + 2x + 3x(x-1)}_{P_2(x)} + x(x-1)(x-3)$$

4) $f(x) - p_2(x) = f[1,3,5,x]\,\omega(x)$

$\omega(x) = (x-1)(x-3)(x-5)$

$$\max_{x\in[1,5]} |\omega(x)| = \max_{y\in[-2,2]} |(y+2)\,y\,(y-2)|$$

$(y = x-3)$

$g(y) = y(y^2-4)$, $\quad g'(y) = 3y^2-4 = 0$

$\Rightarrow y = \pm \frac{2}{\sqrt{3}}$

$$\|\omega\|_\infty = \left|\frac{2}{\sqrt{3}}\left(\frac{4}{3}-4\right)\right| = \frac{16}{3\sqrt{3}}$$

$$\|f'''\|_\infty = 6 \Rightarrow \|f-p_2\|_\infty = \frac{16}{3\sqrt{3}}$$

5)

$$
\begin{array}{cccccc}
0 & 1 & & & & \\
 & & 1 & & & \\
0 & 1 & & 1 & & \\
 & & 2 & & 1 & \\
1 & 3 & & 2 & & 1 \\
 & & 4 & & 3 & \\
1 & 3 & & 8 & & \\
 & & 12 & & & \\
2 & 15 & & & &
\end{array}
$$

$$p_4(x) = \underbrace{1 + x + x^2 + x^2(x-1)}_{p_3(x)} + x^2(x-1)^2$$

6) $f(0) = -8$ , $f(2) = 0$ , $f(4) = 8$

$$\int_0^4 f(x)\,dx = \frac{4}{6}(-8+8) = 0$$

## Code C

2)

| | | | | |
|---|---|---|---|---|
| 0 | 1 | | | |
| | | 2 | | |
| 1 | 3 | | 3 | |
| | | 11 | | 1/7 |
| 3 | 25 | | 4 | |
| | | 35 | | |
| 7 | 165 | | | |

$$p_3(x) = \underbrace{1 + 2x + 3x(x-1)}_{p_2(x)} + \frac{1}{7}x(x-1)(x-3)$$

4) $f(x) - p_2(x) = f[1,3,5,x]\,\omega(x)$,

where $\omega(x) = (x-1)(x-3)(x-5)$

$$\max_{x\in[1,5]} |\omega(x)| = \frac{16}{3\sqrt{3}}$$

$f(x) = \frac{1}{x^2}$, $f'(x) = -\frac{2}{x^3}$, $f''(x) = \frac{6}{x^4}$,

$f^{(3)}(x) = -\frac{24}{x^4}$, $\|f^{(3)}\|_\infty = 24$

$$\|f - p_2\|_\infty = \frac{64}{3\sqrt{3}}$$

5)

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | | | | |
| | | 2 | | | |
| 0 | 1 | | 3 | | |
| | | 5 | | 4 | |
| 1 | 6 | | 7 | | 5 |
| | | 12 | | 14 | |
| 1 | 6 | | 35 | | |
| | | 47 | | | |
| 2 | 53 | | | | |

$$p_4(x) = \underbrace{1 + 2x + 3x^2 + 4x^2(x-1)}_{p_3(x)} + 5x^2(x-1)^2$$

6) $f(0) = -32, \quad f(4) = 0, \quad f(8) = 160$

$$\int_0^8 f(x)\,dx = \frac{8}{6}(-32+160)$$

$$= \frac{512}{3}$$

## Code D

2)

| | | | | |
|---|---|---|---|---|
| 0 | 0 | | | |
| | | 1 | | |
| 1 | 1 | | 1 | |
| | | 4 | | 1 |
| | | | 8 | |
| 3 | 9 | | | |
| | | 52 | | |
| 7 | 217 | | | |

$$p_3(x) = \underbrace{x + x(x-1)}_{p_2(x)} + x(x-1)(x-3)$$

4) $f(x) = \frac{1}{x^2}$ , $x \in [1,3]$

$\| f^{(3)} \|_\infty = 24$

$\max\limits_{x \in [1,3]} |(x-1)(x-2)(x-3)| = \frac{2}{3\sqrt{3}}$

$\| f - p_2 \|_\infty \leq \frac{24}{6} \times \frac{2}{3\sqrt{3}} = \frac{8}{3\sqrt{3}}$

5)

| | | | | | |
|---|---|---|---|---|---|
| 0 | 2 | | | | |
| | | 2 | | | |
| 0 | 2 | | 2 | | |
| | | 4 | | 2 | |
| 1 | 6 | | 4 | | 2 |
| | | 8 | | 6 | |
| 1 | 6 | | 16 | | |
| | | 24 | | | |
| 2 | 30 | | | | |

$$p_4(x) = \underbrace{2 + 2x + 2x^2 + 2x^2(x-1)}_{p_3(x)} + 2x^2(x-1)^2$$

6) $f(0) = -36$ , $f(3) = -30$ , $f(6) = 120$

$$\int_0^6 f(x)\,dx = \frac{6}{6}(-36 - 120 + 120)$$

$$= -36 .$$