ASSIGNMENT 2

AE639:CONTINUUM MECHANICS

1. The motion of a continous medium is specified by the component equations

$$x_1 = \frac{1}{2}(X_1 + X_2)e^t + \frac{1}{2}(X_1 - X_2)e^{-t}$$
$$x_2 = \frac{1}{2}(X_1 + X_2)e^t - \frac{1}{2}(X_1 - X_2)e^{-t}$$
$$x_3 = X_3$$

   (a) Show that the Jacobian determinant J does not vanish, and solve for inverse equations X=X(x,t).
   (b) Calculate the velocity and acceleration components in terms of material coordiantes.
   (c) Using the inverse equations developed to express velocity and acceleration component in terms of spatial coordinates
   (d) Derive the displacement field for the motion in eulerian form and by using material derivative operator, verify the velocity and accelration components

2. A velocity field is givenin Lagrangian form by

$$v_1 = 2t + X1, v_2 = X_2e^2, v_3 = X_3 - t$$

   Integrate the equations to obtain x=x(X,t) at t=0,and use results to obtain velocity and accelration component in the eulerian form.

3. With respect to reference configuration material axes X_i and spatial axis x_i, the dispacement field of a continuum body is given by: $x_1 = X_1, x_2 = X_2 + AX_3, x_3 = X_3 + AX_2$ where A is constant.

   (a) Determine the displacement vector components in both material and spatial forms.
   (b) Determine the displaced location of the material particles which orignally comprise
      i. the plane circular surface $X_1 = 0, X_2^2 + X_3^2 = \frac{1}{1-A_2}$;
      ii. the infinitesmal cube with edges along coordiante axes of length $dX_i = dX$.
      iii. Sketch the displaced configuration for above cases if A=1/2.

4. The dispcement vector component of a body is given by $u_1 = 4X_1^2, u_2 = X_2X_3^2, u_3 = X_1X_3^2$. Determine the displaced location of the material particle orignally at (1,0,2)

5. Determine the values of constant k for which $u_1 = k(X_2 - X_1), u_2 = k(X_1 - X_2), u_3 = kX_1X_2$ are possible displacement components for a continous body.

6. An infinitesimal homogenous deformation $u_i = A_{ij}X_j$ is one for which the constant A_{ij} are so small that their products may be neglected. Show that for two sequential infintesimal deformation the total displacement is the sum of the individual displacements regardless of the order in which deformation are apploed.

7. A homogeneous deformation has been described as one for which all of the deformation and strain tensors are independant of the coordinates, and may therefore be expressed in general by displacement field $u_i - A_{ij}X_j$ where the A_{ij} are constants (ot in case of motion, the functions of time. Show that for homogenous deformation:

   (a) plane material surfaces remain plane
   (b) straight line particle element remain straight
   (c) material surfaces which are spherical in the reference configuration become ellipsoidal surfaces in the deformed configuraiton.