MA 214: Introduction to Numerical Analysis, Spring 2014

Quiz 1 - 7th February 2014, 8:10 P. M. - 9:00 P. M. (C)

Solution

1. Let
$$x_0 = 1,\ x_1 = \frac{5}{4},\ x_2 = \frac{7}{4},\ x_3 = 2$$
and for $i = 0, 1, 2, 3$, let $\ell_i(x)$ be the Lagrange interpolation polynomial of degree 3 such that
$$l_i(x_i) = 1,\ l_i(x_j) = 0,\ \text{for } i \neq j.$$
Evaluate
$$l_0\left(\frac{7}{8}\right) + l_1\left(\frac{7}{8}\right) + l_2\left(\frac{7}{8}\right) + l_3\left(\frac{7}{8}\right).$$
(1 mark)

Ans.: 1.

2. Let $f : [0, 7] \to \mathbb{R}$ be such that
$$f(0) = 1,\ f(1) = 3,\ f(3) = 25,\ f(7) = 165.$$
Find

(a) a polynomial of degree ≤ 2 which interpolates f at $0, 1, 3$,

Ans.: $1 + 2x + 3x(x-1)$.

(b) a polynomial of degree ≤ 3 which interpolates f at $0, 1, 3, 7$.

Ans.: $1 + 2x + 3x(x-1) + \frac{1}{7}x(x-1)(x-3)$.

(1+1 marks)

3. Let
$$f(x) = 100x^4 + 44x^3 - 13x^2 + 47x + 23.$$
Find the divided difference $f[1 \quad 2 \quad 3 \quad 4 \quad 5]$. (1 mark)

Ans.: 100.

4. Let $f(x) = \dfrac{1}{x^2},\ x \in [1, 5]$ and $p_2(x)$ be the quadratic polynomial which interpolates f at 1, 3, 5. Find the best possible upper bound for $\|f - p_2\|_\infty = \max\limits_{x\in[1,5]} |f(x) - p_2(x)|$. (2 marks)

Ans.: $\frac{64}{3\sqrt{3}}$.

5. Let $f : [0, 1] \to \mathbb{R}$ be such that
$$f(0) = 1,\ f'(0) = 2,\ f(1) = 6,\ f'(1) = 12,\ f(2) = 53,$$
where $f'(x)$ denotes the derivative of f at x. Find

(a) the cubic polynomial which interpolates f and f' at 0 and at 1.

Ans.: $1 + 2x + 3x^2 + 4x^2(x - 1)$.

(b) the polynomial of degree ≤ 4 which interpolates f at 0, 1, 2 and f' at 0 and at 1.

Ans.: $1 + 2x + 3x^2 + 4x^2(x - 1) + 5x^2(x - 1)^2$.

(1 + 1 marks)

6. Evaluate
$$\int_0^8 [(x - 1)(x - 4)(x - 8) + x(x - 4)(x - 8) + x(x - 4)(x - 3)]\, dx.$$
(2 marks)

Ans.: $\frac{512}{3}$.