MA 214: Introduction to Numerical Analysis, Spring 2014

Extra Problems

1. For $x \in \mathbb{R}^n$, prove the following inequalities.

   (a) $\|x\|_\infty \leq \|x\|_2 \leq \sqrt{n}\|x\|_\infty$.

   (b) $\|x\|_\infty \leq \|x\|_1 \leq n\|x\|_\infty$.

   (c) $\|x\|_2 \leq \|x\|_1 \leq \sqrt{n}\|x\|_2$.

   Hence show that

   $$\|x_k - x\|_1 \to 0 \Leftrightarrow \|x_k - x\|_2 \to 0 \Leftrightarrow \|x_k - x\|_\infty \to 0 \text{ as } k \to \infty.$$

2. Let A be a diagonal matrix with the ith diagonal entry equal to d_i. Find

   $$\|A\|_1, \ \|A\|_\infty \text{ and } \|A\|_2.$$

3. Find all the diagonal matrices of size $n \times n$ such that the condition number $\kappa_\infty(A) = 1$.

4. Let A be a real $n \times n$ matrix such that $\kappa_2(A) = 1$. Show that all the eigenvalues of A^TA are equal.

5. Let A be an invertible matrix and B be a singular matrix. Prove the following inequality.

   $$\frac{1}{\|A - B\|} \leq \|A^{-1}\|.$$