CS 512: Design and Analysis of Algorithms

Autumn 2020-2021
Homework # 3
Due Date: 2-11-2020
Total Marks: 20

October 21, 2020

Important

1. Typeset your answers using LaTeX or Word. Upload a pdf file as your submission.

2. Identical answers by two students on the same problem will incur zero marks for both students for the problem.

3. Copying answers from the Internet will also be penalized by awarding zero marks.

4. A plagiarism checker will be used to detect all types of copying.

5. Include your name and roll number at the top of your answer script.

---

1. Given two sequences $X = x_1, x_2, \ldots, x_m$ and $Y = y_1, y_2, \ldots, y_n$ of lengths m and n of elements from a given set, consider the following Bellman equation, where $0 \le i \le m$ and $0 \le j \le n$.

$$\text{OPT}(i, j) = \begin{cases} 0 & \text{if } i = 0 \text{ or } j = 0 \\ 1 + \text{OPT}(i-1, j-1) & \text{if } i > 0, j > 0 \text{ and } x_i = y_j \\ \max\{\text{OPT}(i, j-1), \text{OPT}(i-1, j)\} & \text{otherwise.} \end{cases}$$

Explain in English what $\text{OPT}(m, n)$ computes, and justify your answer. (10 marks)

2. We are given a directed graph $G = (V, E)$ with integer edge weights ℓ_{uv} for each edge $(u, v) \in E$, two distinguished vertices s and t in V and an integer $k \ge 0$. Note that the graph G may have edges with negative weights as well as negative cycles. Then consider the following Bellman equation where $u, v \in V$ and $j \ge 0$:

$$\text{OPT}(u, v, j) = \begin{cases} 0 & \text{if } j = 0 \\ \max\{\text{OPT}(u, v, j-1), \max_{(u,w) \in E}\{\ell_{uw} + \text{OPT}(w, v, j-1)\}\} & \text{otherwise} \end{cases}$$

Explain in English what $\text{OPT}(s, t, k)$ computes, and justify your answer. (10 marks)