CS 512: Design and Analysis of Algorithms

Autumn 2020-2021
Homework # 1
Due Date: 21-09-2020
Total Marks: 20

September 13, 2020

Important

1. Typeset your answers using LaTeX or Word. Upload a pdf file as your submission.

2. Identical answers by two students on the same problem will incur zero marks for both students for the problem.

3. Copying answers from the Internet will also be penalized by awarding zero marks.

4. A plagiarism checker will be used to detect all types of copying.

5. Include your name and roll number at the top of your answer script.

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1. Suppose S is a stable matching for a given instance I of the Stable Matching algorithm, not necessarily the one produced by the Gale-Shapley algorithm.

   (a) Does the matching S *necessarily* become unstable if we reverse the preference list of all the women (and keep the men's preferences intact)?

   (b) What happens when we reverse the preference lists for all the men as well as all the women – does S necessarily become unstable?

   Justify your answer for both cases, *i.e.*, give a proof in case the answer is yes, and a counter-example if the answer is no. (10 marks)

2. Let I be an instance of the stable matching problem, where M and W denotes the set of men and women respectively. Let S be any perfect matching, not necessarily stable. For a person p in I, where p could be either a man or a woman, define the quantity $c_S(p)$, the *cost of S for p* to be the ranking of p's partner q in p's preference list. For example, if woman w's preference list is m_3, m_2, m_1, m_4 and her partner in S is m_1 then

$c_S(w) = 3$ since m_1 is ranked third in w's preference list. Then define the *regret* $r(S)$ of the matching S to be $r(S) = \Sigma_{p \in M \cup W} c_S(p)$, *i.e.*, the sum of all the costs of S for each person. Suppose S is a perfect matching with minimum regret. Is S *necessarily* a stable matching? Justify your answer, *i.e.*, give a proof in case the answer is yes, and a counter-example if the answer is no. (10 marks)