Algorithms Assignment #1

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Question 1)

A) **Given**: S is a stable matching for a given instance I.

To find: S becomes unstable if the preference list of women is reversed keeping the men list intact.

Solution: No, S doesn't necessarily become unstable by reversing the preference list of women. Here is a counter example which proves my statement.
Suppose we consider a stable matching with N = 3 i.e. 3 men (X, Y, Z) and 3 women (A, B, C). Let the preference list of men and women be in such a way that they get their preferred choice (final match) in the first iteration itself. For example

X	A	B	C
Y	C	A	B
Z	B	C	A

Table 1: Men's Preference List

A	X	Y	Z
B	Z	X	Y
C	Y	Z	X

Table 2: Women's Preference List

Here when men propose women according to their preference order, we get a stable matching S as **X-A Y-C Z-B**. Now if we reverse the women's preference list, the list will become as follows.

A	Z	Y	X
B	Y	X	Z
C	X	Z	Y

Table 3: Reverse Preference List of Women

Now, if we try to make a stable matching pair between men's preference list and reverse of women's preference list, we can see that the matching **X-A Y-C Z-B** is still stable with women receiving the worst possible choice. Hence, we found one counter example to disprove the fact given in question.

Conclusion: If we reverse the preference list of women, S does **not** necessarily become unstable.

B) **Given**: S is a stable matching for a given instance I.

To find: S becomes unstable if the preference list of both men and women is reversed.

Solution: No, S doesn't necessarily become unstable by reversing the preference list of each man and woman. Here is a counter example which proves my statement. Suppose we consider a stable matching with N = 3 i.e. 3 men (X, Y, Z) and 3 women (A, B, C). Let the preference list of men and women be in such a way that every man has the same preference list as of other men and every woman has the same preference list as of other women. Reason for choosing this kind of example was that after reversing the lists the preferences of men and women will change by equal factor thus neutralizing the effect of change. For example,

X	A	B	C
Y	A	B	C
Z	A	B	C

Table 4: Men's Preference List

A	X	Y	Z
B	X	Y	Z
C	X	Y	Z

Table 5: Women's Preference List

Here if we see, when men propose, X proposes to A so X-A is a pair, Y proposes to A but A already has higher preference so it rejects Y's proposal and then Y proposes to B, B accepts Y's proposal so Y-B is another pair and similarly Z-C is another pair. Hence the stable matching S is X-A Y-B Z-C. Now if we reverse the preference list of both men and women, the lists will become as follows.

X	C	B	A
Y	C	B	A
Z	C	B	A

Table 6: Reversed Men's Preference List

A	Z	Y	X
B	Z	Y	X
C	Z	Y	X

Table 7: Reversed Women's Preference List

Now, if we try to make a stable matching pair between Table 6 and Table 7, we get X-A Y-B Z-C. We can see that the matching S is still stable after reversing the men's and women's preference list. Hence, we found one counter example to disprove the fact given in question.

Conclusion: If we reverse the preference list of each man and woman, S does **not** necessarily become unstable.

Question 2)

Given: S is a perfect matching with minimum regret

To find: If S is a perfect matching with minimum regret, then is S necessarily stable?

Solution: No, S need not be stable matching. Here is a counter example which proves my statement. Suppose we consider a stable matching with N = 3 i.e. 3 men (X, Y, Z) and 3 women (A, B, C). Let the preference list of men and women be in such a way that 2 men have same preference list, and two women have the same preference list keeping one person always at the end for every man and woman (here Y and C are least preferred). For example

X	B	A	C
Y	B	A	C
Z	A	B	C

Table 8: Men's Preference List

A	X	Z	Y
B	Z	X	Y
C	X	Z	Y

Table 9: Women's Preference List

Now, total number of perfect matching possible for N = 3 is 3! = 6. These matchings are as follows:

1) X-A Y-B Z-C
2) X-A Y-C Z-B
3) X-B Y-A Z-C
4) X-B Y-C Z-A
5) X-C Y-A Z-B
6) X-C Y-B Z-A

Out of these perfect matchings, there exists only 1 stable matching which is **X-B Y-C Z-A**. If we calculate the regret values of each of the perfect matching, we get

1) 2(X-A) + 1(Y-B) + 3(Z-C) + 2(A-X) + 3(B-Y) + 3(C-Z) = 12
2) 2(X-A) + 3(Y-C) + 2(Z-B) + 2(A-X) + 3(C-Y) + 1(B-Z) = 13
3) 1(X-B) + 2(Y-A) + 3(Z-C) + 2(B-X) + 3(A-Y) + 2(C-Z) = 13
4) 1(X-B) + 3(Y-C) + 1(Z-A) + 2(B-X) + 3(C-Y) + 2(A-Z) = 12
5) 3(X-C) + 2(Y-A) + 2(Z-B) + 1(C-X) + 3(A-Y) + 1(B-Z) = 12
6) 3(X-C) + 1(Y-B) + 1(Z-A) + 1(C-X) + 3(B-Y) + 2(A-Z) = 11

Here we can see that the sixth matching (X-C Y-B Z-A) has the minimum regret equal to 11 but is not a stable matching as in this matching X prefers B over its current partner(C) and B prefers X over its current partner(Y). The minimum regret for stable matching pair i.e. fourth pair (X-B Y-C Z-A) is 12 which is greater than 11. Hence, we found one counter example to disprove the fact given in question.

Conclusion: If S is a perfect matching with minimum regret, then S is **not** necessarily stable.