

1.

(a)

```
#include<bits/stdc++.h>
using namespace std;
int main(){
    vector<int> v = {1,1,2,4,1};
    cout << count(v.begin(),v.end(),v[0]);
}
```

(b)

```
#include<bits/stdc++.h>
using namespace std;
#define r 3
#define c 2
void transpose(int a[][c] ){
    int b[c][r];
    for(int i=0;i<r;i++){
        for(int j=0;j<c;j++){
            b[j][i] = a[i][j];
        }
    }
    for(int i=0;i<c;i++){
        for(int j=0;j<r;j++){
            cout << b[i][j] << " ";
        }
        cout << endl;
    }
}
int main(){
    int a[r][c];
    for(int i=0;i<r;i++){
        for(int j=0;j<c;j++){
            cin >> a[i][j];
        }
    }
    transpose(a);
}
```

2.

a)

$f(n) = \Theta(g(n))$ means there are positive constants c_1, c_2 , and k , such that $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all $n \geq n_0$. The values of c_1, c_2 , and n_0 must be fixed for the function f and must not depend on n .

$$\Sigma i^2 = 0^2 + 1^2 + 2^2 + \dots + n^2$$

$$\Sigma i^2 = n^2 + (n-1)^2 + (n-2)^2 + \dots + 0^2$$

$$2 \times \Sigma i^2 = (n^2 + 0^2) + ((n-1)^2 + 1^2) + ((n-2)^2 + 2^2) + \dots + (0^2 + n^2)$$

$$\Sigma i^2 = \frac{((n^2 + 0^2) + ((n-1)^2 + 1^2) + ((n-2)^2 + 2^2) + \dots + (0^2 + n^2))}{2}$$

$$\Sigma i^2 = \frac{((n^2 + 0^2) + ((n-1)^2 + 1^2) + ((n-2)^2 + 2^2) + \dots + (0^2 + n^2))}{2}$$

$$\geq \frac{(n^2 + n^2 + \dots + n^2)}{2} \geq \frac{(n * (n^2))}{2} \geq 0.5 n^3$$

$$\Sigma i^2 = 0^2 + 1^2 + 2^2 + \dots + n^2 \leq n^2 + n^2 + \dots + n^2 \leq n * n^2 \leq n^3$$

so, $0.5 n^3 \leq \Sigma i^2 \leq n^3$ with constants $c_1 = 0.5, c_2 = 1$ and $k = 1$

so, we've proved that $\Sigma i^2 = \Theta(n^3)$, using the definition of Theta (Θ)

b)

$f(n) = O(g(n))$ means there are positive constants c and n_0 , such that $0 \leq f(n) \leq c g(n)$ for all $n \geq n_0$

$$n! = O(n^n)$$

$$\Rightarrow n! \leq c(n^n)$$

Let's assume $c = 1$

$$\Rightarrow n! \leq c(n^n)$$

$$\Rightarrow n! \leq 1(n^n)$$

$$\Rightarrow n! \leq n^n$$

$$\Rightarrow n * (n-1) * (n-2) * \dots * 1 \leq n * n * n * n * \dots * n$$

it's true for all $n \geq 1$

so, $n! = O(n^n)$ given $c = 1, n_0 = 1$, using the definition of Big - O

3.

a)

$f(n) = O(g(n))$ means there are positive constants c and n_0 , such that $0 \leq f(n) \leq c g(n)$ for all $n \geq n_0$

$$10n^2 + 9 = O(n)$$

$$\Rightarrow 10n^2 + 9 \leq c(n)$$

$$\Rightarrow \frac{10n^2}{n} + \frac{9}{n} \leq c$$

$$\Rightarrow 10n + \frac{9}{n} \leq c$$

$$\Rightarrow 10n \leq c$$

for $f(n) = O(g(n))$ then c must be a constant. here $c \geq$

$10n$ which is not a constant **(dependent on n)**

so, we can say that $10n^2 + 9$ is not $O(n)$ using definition of Big - O

b)

$$\frac{n^2}{\log(n)} = \theta(n^2)$$

for $f(n) = \Theta(g(n))$, then $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$ must both be true

so, let's first check whether $f(n) = \Omega(g(n))$ is true or not

$f(n) = \Omega(g(n))$ means there are positive constants c and n_0 , such that $f(n) \geq cg(n)$ for all $n \geq n_0$

$$\frac{n^2}{\log(n)} = \Omega(n^2)$$

$$\Rightarrow \frac{n^2}{\log(n)} \geq c(n^2)$$

$$\Rightarrow \frac{1}{\log(n)} \geq c$$

as n grows larger, then $\frac{1}{\log(n)}$ gets closer to 0

$$\Rightarrow 0 \geq c$$

for $f(n) = \Omega(g(n))$ then c must be a positive constant. here c is found to be non-negative

so, we can say that $n^2 / \log(n)$ is not $\Omega(n^2)$ using definition of Big - Ω

since $n^2 / \log(n)$ is not $\Omega(n^2)$ we can say that $n^2 / \log(n)$ is not $\Theta(n^2)$

4.

```
#include<bits/stdc++.h>
using namespace std;
class Complex{
    float a,b;
    public:
        Complex(){
            a = 0,b = 0;
            cout << "Default Constructor: "<<a<<"+"<<b<<"i"<<endl;
        }
};
int main(){
    Complex ans;
```

```
}
```

5.

```
#include<bits/stdc++.h>
using namespace std;
class Quadratic{
    public:
        int a,b,c;
        Quadratic operator+(Quadratic &d){
            Quadratic func;
            func.a=a+d.a;
            func.b=b+d.b;
            func.c=c+d.c;
            return func;
        }
        void output(){
            cout<<a<<"x^2"<<"+"<<b<<"x"<<"+"<<c<<"="<<"0"<<endl;
        }
};

int main(){
    Quadratic Q1,Q2,Q3;
    cout<<"Enter first polynomial: "<<endl;
    cin>>Q1.a>>Q1.b>>Q1.c;
    cout <<endl<<"Enter second polynomial: "<<endl;
    cin>>Q2.a>>Q2.b>>Q2.c;
    Q3=Q1+Q2;
    cout <<endl<<"First polynomial is: "<<endl;
    Q1.output();
    cout <<endl<<"Second polynomial is: "<<endl;
    Q2.output();
    cout <<endl<<"Addition of two polynomial: "<<endl;
    Q3.output();
}
```

6.

```
#include<bits/stdc++.h>

template <typename T> class bag
{
    protected:
```

```

        T *arr;
        int capacity;
        int top;
public:
    bag(int bagCapacity = 10){
        capacity = 10;
        arr = new T[10];
        top=0;
    };

    virtual int Size() const{
        return top;
    }
    virtual bool IsEmpty() const{
        return top==0;
    }
    virtual T Element() const{
        return arr[top];
    }

    virtual void Push(const T &x){
        arr[top++] = x;
    }
    virtual void Pop(){
        --top;
    }

};

template <typename TT> class Queue : public bag<TT>{
private:
    int l , r;

public:
    Queue(int n = 10){
        l = 0 , r = -1;
    }
};

```

```

    }
    virtual void Pop(){
        ++l;
    }
    virtual void Push(const TT &x){
        this->arr[++r] = x;
    }
    virtual TT Element(){
        return this->arr[l];
    }
    virtual int Size(){
        return r-l+1;
    }
    virtual int IsEmpty(){
        return r-l+1==0;
    }
};

int main(){
    Queue<int> q;
    q.Push(3);
    q.Push(17);
    q.Push(2);
    std::cout << q.Element() << '\n';
    q.Pop();
    std::cout << q.Element() << '\n';
}

```

7.

$A * B * C = *C*AB$

8.

```

#include<bits/stdc++.h>
using namespace std;
class sparseMatrix{
public:
    sparseMatrix(int _r,int _c){

```

```

        r = _r; c = _c;
        m = new int*[r];
        for(int i=0;i<r;i++){
            m[i] = new int[c];
        }
        for(int i=0;i<r;i++){
            for(int j=0;j<c;j++){
                m[i][j] = 0;
            }
        }
        for(int i=0;i<r;i++){
            for(int j=0;j<c;j++){
                if(i==j){
                    m[i][j] = 2;
                }else if(i==0 && j==(c-1)){
                    m[i][j] = 7;
                }else{
                    m[i][j] = 0;
                }
            }
        }
    }

    sparseMatrix(const sparseMatrix &M){
        r = M.r;
        c = M.c;
        m = new int* [r];
        for(int i=0;i<r;i++){
            m[i] = new int[c];
        }
        for(int i=0;i<r;i++){
            for(int j=0;j<c;j++){
                m[i][j] = M.m[i][j];
            }
        }
    }

    void print(){
        for(int i=0;i<r;i++){

```

```

        for(int j=0;j<c;j++){
            cout << m[i][j] << " ";
        }
        cout << endl;
    }
}

private:
    int** m;
    int r,c;
};

int main(){
    sparseMatrix sparsematrix_1(12,12);
    sparseMatrix sparsematrix_2(sparsematrix_1);
    cout << "First sparsematrix" << endl;
    sparsematrix_1.print();
    cout << endl;
    cout << "Copy sparsematrix" << endl;
    sparsematrix_2.print();
}

```

The computing time of our copy constructor is approximately $9n^2 + 5n + 3$
 $O(n^2)$

9.

```

#include<bits/stdc++.h>
using namespace std;
void printArray(int a[],int len){
    for(int i=0;i<len;i++){
        cout<<a[i]<<" ";
    }
    cout<<endl;
}

void Sort(int a[],int len){
    for(int i=1;i<len;i++){
        int temp = a[i];
        int j = i-1;
        while(temp < a[j] && j>=0){
            a[j+1] = a[j];
            j--;
        }
        a[j+1] = temp;
    }
}

```



```

        }
        a[j+1] = temp;
    }
}
int main(){
    int a[5] = {3,2,4,1,5};
    cout<<"original:\n";
    printArray(a,5);
    Sort(a,5);
    cout<<"sorted:\n";
    printArray(a,5);
}

```

10.

```

#include<bits/stdc++.h>
using namespace std;
int main(){
    string s;
    cin >> s;
    int len = s.size();
    bool isP = 1;
    for(int l=0,r=s.size()-1;l<=r;l++,r--){
        if(s[l]!=s[r]){
            isP = 0;
        }
    }
    if(isP==1){
        cout<<"Yes";
    }else cout<<"No";
}

```