System: When a number of elements or components are connected in a sequence to perform a specific function. the group formed is called a system.

Control System: In a system, when the output quantity is controlled by varying the i/p quantity then the system is called control system. The o/p quantity is called controlled variable or response & the input is called command signal or Exitation.

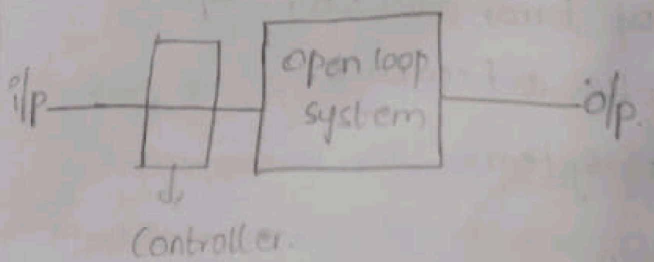
* There are two types of controlled system
1. Open loop control system.
2. closed loop control system.

Open loop control System: Any physical system which doesnot automatically correct the variation in o/p is called open loop system

(or)

Control system in which the o/p quantity has no effect upon the input quantity are called open loop control system.

⇒ In this control system the feedback is abscent.
(or) There is no feedback in this type of system.

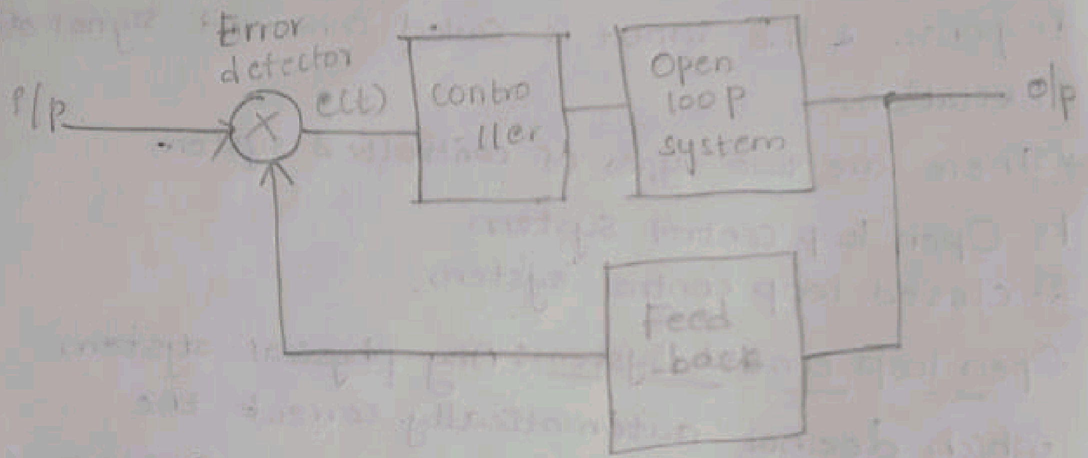


⇒ In open loop system the o/p can be ~~value~~ varied by varying the o/p but due to external disturbances the system output may changes.

⇒ The changes in o/p corrected by varying the i/p manually.

Closed loop systems:

Control system in which the o/p has an effect upon the i/p quantity in order to maintain the desired o/p value are called closed loop system.



⇒ The open loop system can be modified as closed loop system by providing a feedback

⇒ the feedback automatically corrects the changes in output due to the disturbances so the Closed loop systems is called automatic control system.

It consist of Error detector, open loop system, controller & Feedback.

Examples of control system

1. Traffic control system
2. Temperature Control System.
3. Numerical control system.

1. Traffic control system:

i, Open loop traffic control system:

* An open loop traffic control system depends on equal distributed time to each direction. An open loop control system each direction have equal time interval.

* Open loop control system has drawback that heavy traffic direction & it contains controller and the traffic lights.

* It cannot adapt to sudden traffic jams, inefficient during heavy flow, accuracy relies on good calibration.

ii, Closed-Loop Traffic Control System.

* In this type of system, Uses sensors to measure traffic volum and density, feeding this data back to a computer that adjusts signal timings in real-time.

* Components are controller, lights, sensors, feed back element, optimizes traffic flow.

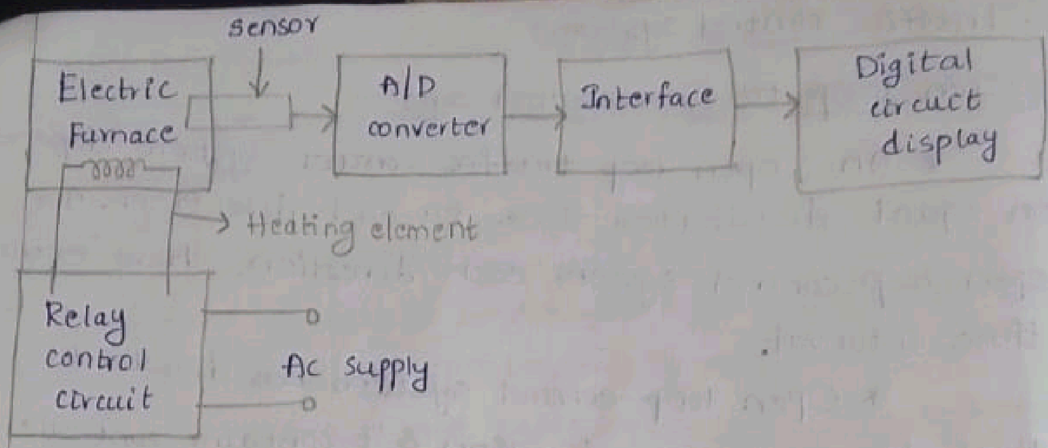
* Examples are smart traffic systems like adaptive signal control) in busy cities, or even autonomous veehicles adjusting speed to maintain gaps.

2. Temperature control system

i, Open loop temperature control system.

The system operates based on a pre-set input without checking the actual temperature.

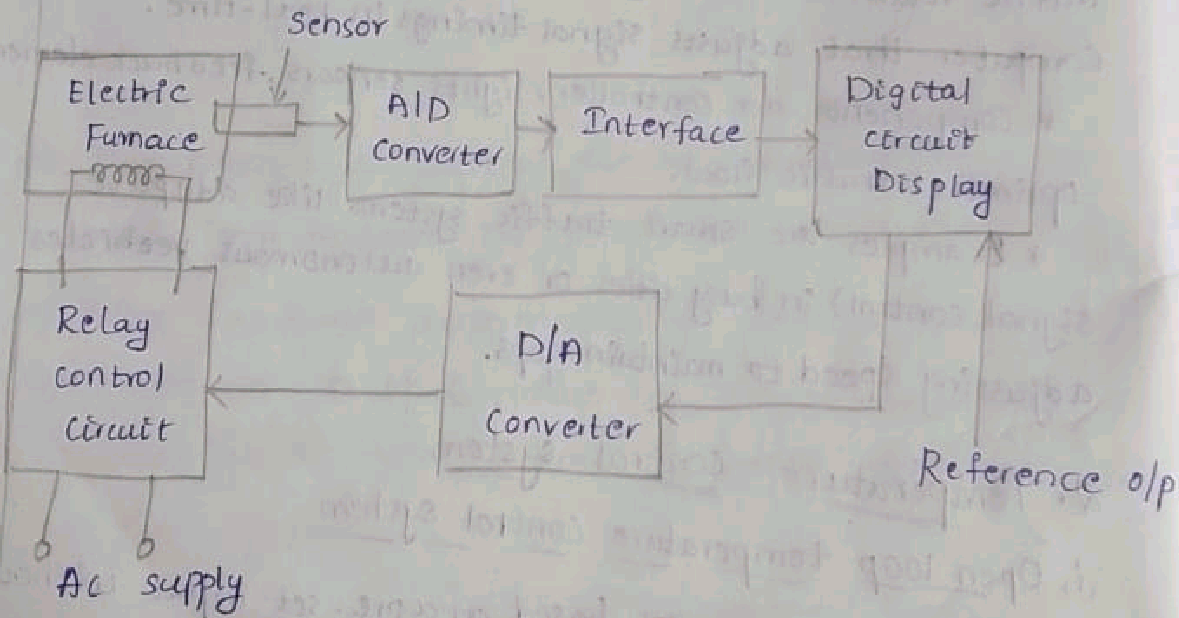
components are input, controller, Relay circuit, Electric furnace, Sensor, Analog to Digital converter, Interface & Digital Display.



Closed-Loop temperature control system

This system uses a sensor to measure the actual temperature, compares it to the desired setpoint, & adjust the heating/cooling as needed.

Components are Relay circuit, electric furnace, Sensor, A/D converter, interface, Digital Display, D/A converter.

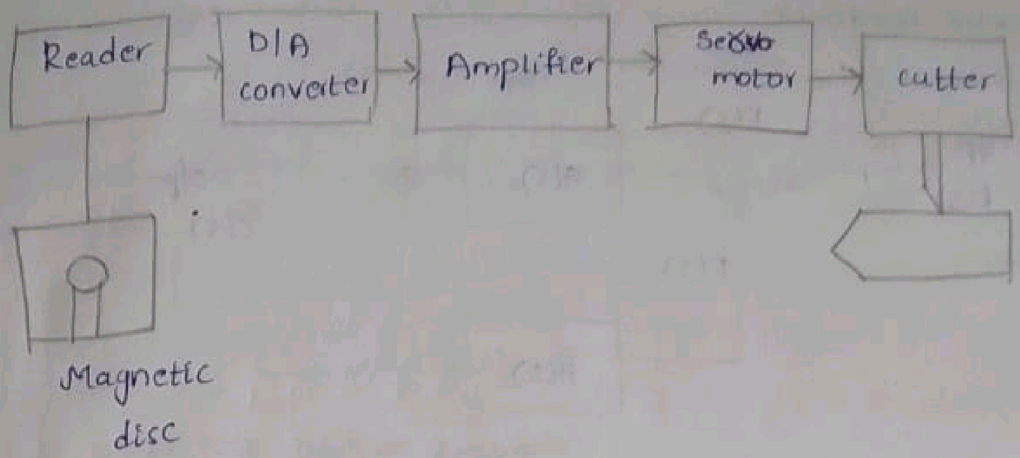


3. Numerical Control system.

Open loop Numerical control system

It doesn't contain feedback, commands are sent, & the system assumes the movement occured correctly.

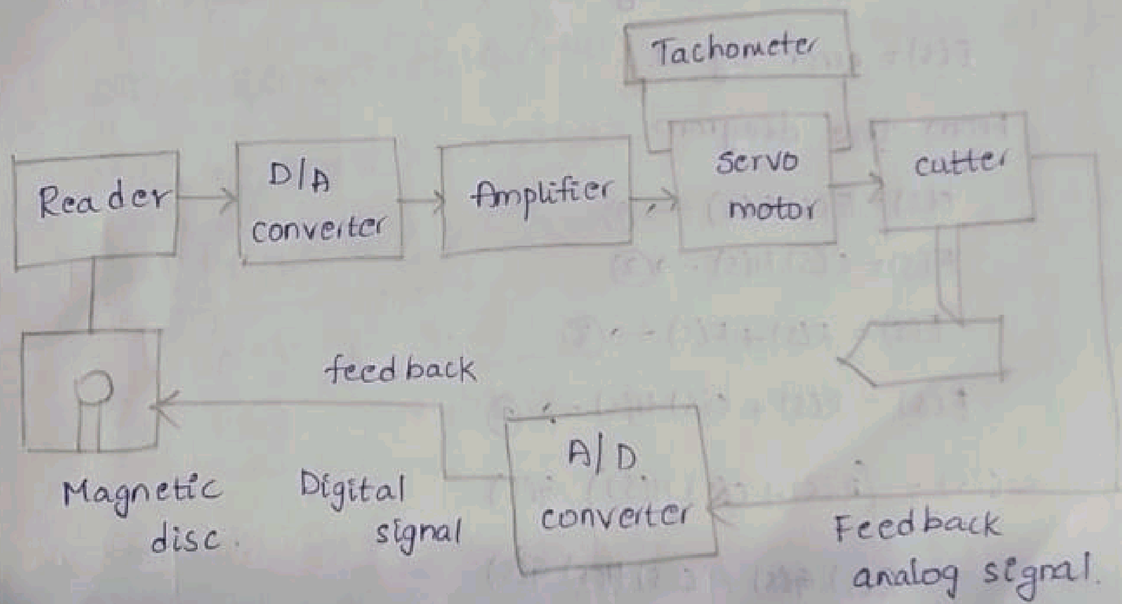
Components are magnetic disc, Reader, D/A converter, Amplifier, Sensor motor & cutter.



Closed loop Numerical control system.

This type of system uses feedback from encoders or resolvers to compare actual position with comparing it to the command & adjusting motor power to eliminate errors.

Components are Magnetic disc, Reader, D/A converter, Amplifier, servo motor, cutter & feed back [A/D converter].



Feedback in the control system.

There two types

(i) +ve feedback

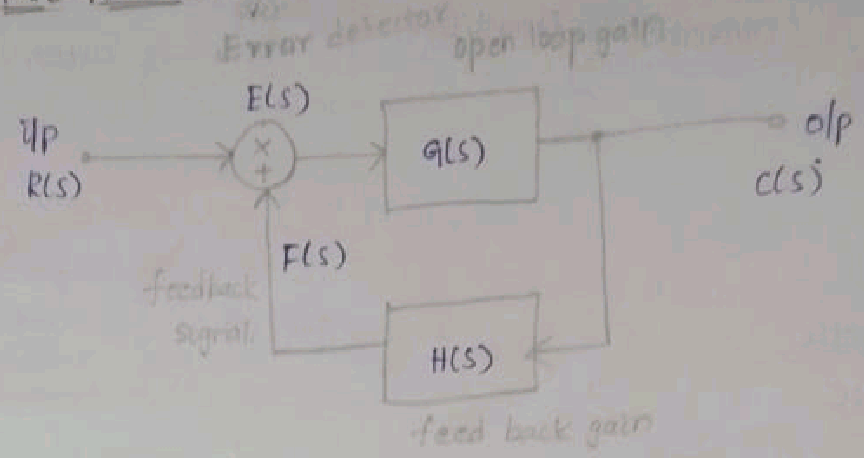
(ii) -ve feedback.

Both are available in closed loop system only

If we connect +ve it is called +ve feed back

If we connect -ve it is called -ve feedback

+ve feedback



$$E(s) = R(s) + F(s)$$

$R(s)$ = Input quantity of system

$C(s)$ = output quantity of system

$G(s)$ = Gain of open loop system

$H(s)$ = gain of feedback

$F(s)$ = Feed back signal

$E(s)$ = error signal.

From the diagram,

$$C(s) = E(s)G(s) \rightarrow (1)$$

$$F(s) = C(s)H(s) \rightarrow (2)$$

$$E(s) = R(s) + F(s) \rightarrow (3)$$

$$E(s) = R(s) + C(s)H(s) \rightarrow (4)$$

$$C(s) = (R(s) + C(s)H(s))\,G(s)$$

$$= R(s)G(s) + C(s)H(s)G(s)$$

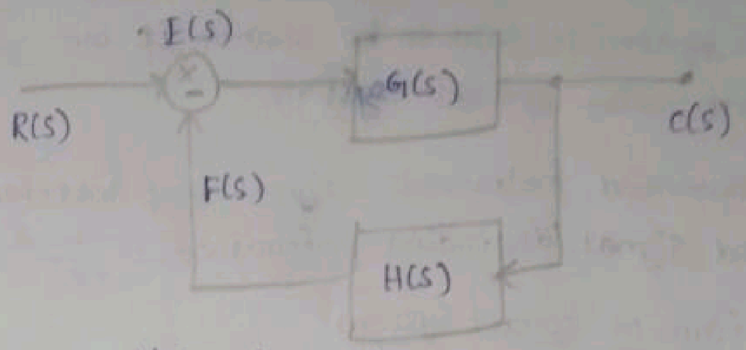
$$C(s) - C(s)G(s)H(s) = G(s)R(s)$$

$$C(s)\left[1 - G(s)H(s)\right] = G(s)R(s)$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)}}$$

Transfer function

-ve feedback



From the above diagram

$$C(s) = E(s)\,G(s) \rightarrow (1)$$

$$F(s) = C(s)\,H(s) \rightarrow (2)$$

$$E(s) = R(s) - F(s) \rightarrow (3)$$

$$E(s) = R(s) - C(s)\,H(s) \rightarrow (4)$$

$$C(s) = [R(s) - C(s)\,H(s)]\,G(s)$$

$$C(s) = R(s)G(s) - C(s)G(s)H(s)$$

$$C(s) + C(s)G(s)H(s) = R(s)G(s)$$

$$C(s)[1 + G(s)H(s)] = R(s)G(s)$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}}$$

Effects of Feedback.

There are 4 type of effects

1. Gain o/p / i/p
2. Senstivity
3. Stability
4. Noise

1. **Gain:** The ratio of the magnitude of the output signal to the magnitude of the input signal

(or)

The ratio of o/p to the i/p.

2. <u>Sensitivity</u>: How much the system's output changes in response to variations in its parameters or input signals.

3. <u>Stability</u>: The system is said to be stable if we get desired output otherwise unstable system.

4. <u>Noise</u>: Any unwanted, extraneous signal that interferes with the desired signal, degrading performance.

Mathematical form of control system.

They are three types
1. Differential equation
2. Transfer function
3. State space model.

⇒ Mathematical expression of control system is used to design a control system. & analysis of the o/p (or) representing the control system using mathematical expression.

⇒ The mathematical model of a system used for the purpose designing the Control system & for the analysis of the system.

There are three types of mathematical models
1. Differential equation model [time domain]
2. Transfer function model [frequency domain]
3. State space model [Matrix form]

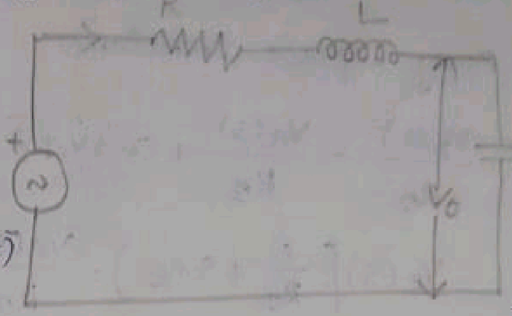
1. Differential equation model:

Differential equation model is a time domain mathematical model of a control system.

Transfer function model:

It is an S-domain mathematical model of a Control System. The transfer function Can be obtain by taking the ratio of Laplace transform of output to the laplace transform of input with using zero initial Conditions.

Examples: Obtain the differential equation model & transfer function model for the following electrical system.



$$V_i(t) = R\,i(t) + L\frac{d}{dt}i(t) + V_o$$

$$i(t) = C\frac{d}{dt}v(t)$$

$$V_i(t) = RC\frac{d}{dt}V_o(t) + LC\frac{d^2}{dt^2}V_o(t) + V_o(t)$$

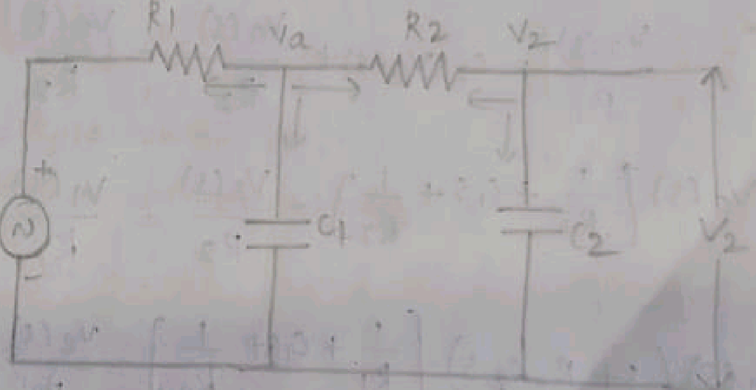
$$V_i(t) = LC\left(\frac{d^2}{dt^2}V_o(t) + \frac{R}{L}\frac{d}{dt}V_o(t) + \frac{V_o(t)}{LC}\right] \rightarrow ①$$

$$V_i(S) = LC\left(s^2 V_o(S) + \frac{R}{L}S\cdot V_o(S) + \frac{1}{LC}V_o(S)\right]$$

$$= V_o(S)\left[s^2 LC + RCS + 1\right]$$

$$\boxed{\frac{V_o(s)}{V_i(S)} = \frac{1}{s^2 LC + sRC + 1}}$$

Obtain the transfer function model for the electrical system.



At node V_a.

$$\frac{V_a(t)}{R_1} + C_1\frac{d}{dt}V_a(t) + \frac{V_a - V_2}{R_2} = \frac{V_i(t)}{R_1}$$

$$\frac{V_a(t)}{R_1} + C_1\frac{d}{dt}V_a(t) + \frac{V_a}{R_2} - \frac{V_2}{R_2} = \frac{V_1(t)}{R_1} \rightarrow ①$$

At node V_2.

$$\frac{V_2 - V_a}{R_2} + C_2\frac{d}{dt}V_2(t) = 0$$

$$\frac{V_2(t)}{R_2} - \frac{V_a(t)}{R_2} + C_2 \frac{d}{dt} V_2(t) = 0$$

Apply Laplace transform

$$\frac{V_2(s)}{R_2} - \frac{V_a(s)}{R_2} + C_2 s V_2(s) = 0$$

$$V_2(s)\left[\frac{1}{R_2} + s \cdot C_2\right) = \frac{V_a(s)}{R_2}$$

$$\frac{V_2(s)(1 + R_2 C_2 s)}{\cancel{R_2}} = \frac{V_a(s)}{\cancel{R_2}}$$

$$V_a(s) = V_2(s)(1 + R_2 C_2 s) \rightarrow (2)$$

Consider eqn (1)

$$\frac{V_a(t)}{R_1} + C_1 \frac{d}{dt} V_a(t) + \frac{V_a(t)}{R_2} - \frac{V_2(t)}{R_2} = \frac{V_1(t)}{R_1}$$

Apply Laplace transform

$$\frac{V_a(s)}{R_1} + C_1 s V_a(s) + \frac{V_a(s)}{R_2} - \frac{V_2(s)}{R_2} = \frac{V_1(s)}{R_1}$$

$$V_a(s)\left[\frac{1}{R_1} + C_1 s + \frac{1}{R_2}\right] - \frac{V_2(s)}{R_2} = \frac{V_1(s)}{R_1}$$

sub (2)

$$V_2(s)(1 + R_2 C_2 s)\left[\frac{1}{R_1} + C_1 s + \frac{1}{R_2}\right] - \frac{V_2(s)}{R_2} = \frac{V_1(s)}{R_1}$$

$$V_2(s)\left[(1 + R_2 C_2 s)\left[\frac{1}{R_1} + C_1 s + \frac{1}{R_2}\right) - \frac{1}{R_2}\right] = \frac{V_1(s)}{R_1}$$

$$\frac{V_2(s)\left[(R_1 + R_2 R_1 C_2 s)(1 + R_1 C_1 s + \frac{R_1}{R_2}) - \frac{R_1}{R_2}\right]}{\cancel{R_1}} = \frac{V_1(s)}{\cancel{R_1}}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{1}{(R_1 + R_2 R_1 C_2 s)(1 + R_1 C_1 s + \frac{R_1}{R_2}) - \frac{R_1}{R_2}}$$

Mechanical system

It is classified in to two types.

i, Mechanical translation system

ii) Mechanical rotational system.

Mechanical translation system

The basic elements are

i, mass

ii, Dash pot or Damper

iii, Spring.

Parameters:

⇒ Force (F), acceleration $a = \frac{dv}{dt}$.

acceleration in terms of displacement

$$a = \frac{d^2x}{dt^2}$$

Velocity $v = \frac{dx}{dt}$

displacement is 'x'

Symbols of mechanical translation system

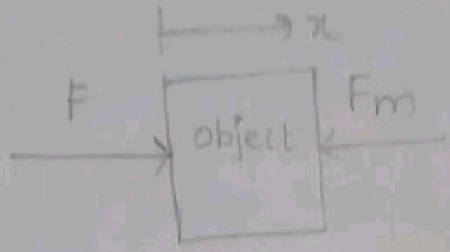
mass (m), Force (F),

Dashpot (B), acceleration (a)

Spring (K), Velocity (v)

Displacement (x).

i, Mass: consider an ideal mass with negligible elasiticity & friction. Let the force applied is 'F' the opposing force from the mass is equal to the applied force is directly propotional to acceleration.



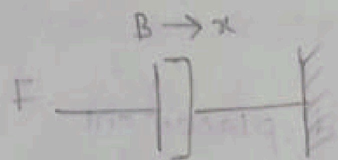
$F = F_m \propto a$

$F = F_m = Ma$. [M = consta

In terms of displacement is

$$F = M \frac{d^2x}{dt^2}$$

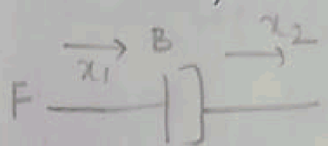
(ii) Dash pot or Damper: consider a Damper with negligible mass & elasticity. Let the force is applied is 'F'. The opposing force from the Damper is equal to the applied force is directly propotional to velocity. It is represented with B.

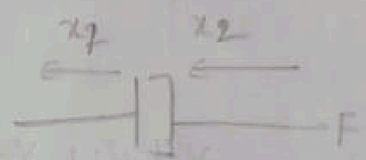


$F = F_B \propto V$

$$F = F_B = B\frac{dx}{dt}$$

→ If Damper is not fixed



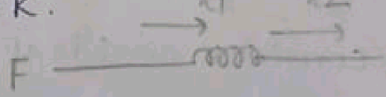


$$F = F_B = B\frac{d}{dt}(x_1 - x_2)$$

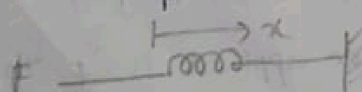
$$F = F_B = B\frac{d}{dt}(x_2 - x_1)$$

(iii) Spring: consider a spring with neglible mass & friction. Let the force is applied is 'F'. The opposing force from the spring is equal to the applied force is directly propotional to displacement.

It is represented with K.

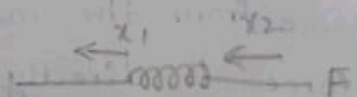


$F = F_K = K(x_1 - x_2)$



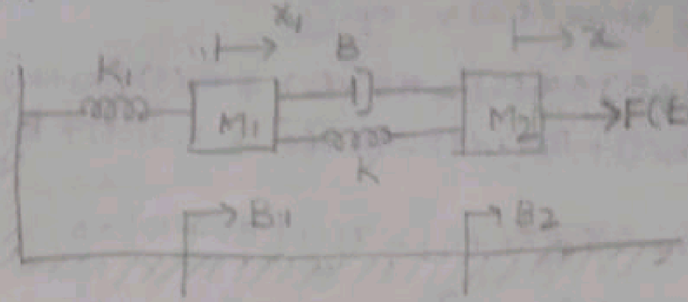
$F = F_K \propto x$

$F = F_K = Kx$

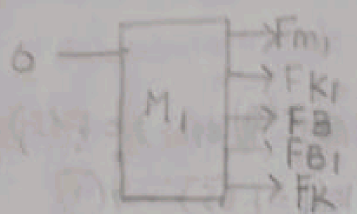


$F = F_K = K(x_2 - x_1)$.

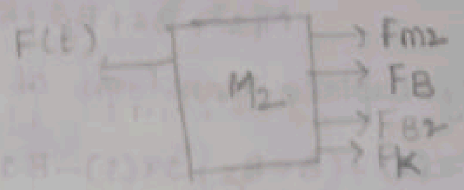
Write the differential equation & obtain the transfer function for the mechanical system shown in the diagram.



Free body diagram of M_1



Free body diagram of M_2



$$F_{m_1} + F_{K_1} + F_B + F_{B_1} + F_K = 0$$

$$F_{m_1} = M_1 \frac{d^2 x_1}{dt^2}, \quad F_{K_1} = K_1 x_1$$

$$F_B = B \frac{d}{dt}(x_1 - x), \quad F_{B_1} = B_1 \frac{dx_1}{dt}$$

$$F_K = K(x_1 - x)$$

$$F_B + F_K + F_{B_2} + F_{m_2} = F(t)$$

$$F_B = B\frac{d}{dt}(x - x_1), \quad F_{B_2} = B_2 \frac{dx}{dt}$$

$$F_K = K(x - x_1),$$

$$F_{m_2} = M_2 \frac{d^2 x}{dt^2}$$

Substitute in above equation

$$M_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + B\frac{d}{dt}(x_1 - x) + B_1 \frac{dx_1}{dt} + K(x_1 - x) = 0.$$

$$M_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + B\frac{d}{dt}x_1 - B\frac{d}{dt}x + B_1\frac{dx_1}{dt} + Kx_1 - Kx = 0$$

$$M_1 \frac{d^2 x_1}{dt^2} + \cancel{K_1 x_1} + (B + B_1)\frac{d}{dt}x_1 - B\frac{d}{dt}x + x_1(K_1 + K) - Kx = 0 \rightarrow (1)$$

Substitue F_B, F_K, F_{B2}, F_{m2} in M_2 equation

$$B\frac{d}{dt}(x - x_1) + K(x - x_1) + B_2\frac{dx}{dt} + M_2\frac{d^2 x}{dt^2} = F(t)$$

$$B\frac{d}{dt}x - B\frac{d}{dt}x_1 + Kx - Kx_1 + B_2\frac{dx}{dt} + M_2\frac{d^2 x}{dt^2} = F(t)$$

$$M_2\frac{d^2 x}{dt^2} + (B + B_2)\frac{dx}{dt} - B\frac{d}{dt}x_1 + Kx - Kx_1 = F(t) \rightarrow (2)$$

Obtain the transfer function of the system shown in the diagram.

Apply Laplace transform of equ ①

$$M_1 s^2 x_1(s) + (B+B_1) s\, x_1(s) - Bs\, x(s) + x_1(s)[K_1+K] - K x(s) = 0$$

$$M_1 s^2 x_1(s) + Bs\, x_1(s) + B_1 s\, x_1(s) - Bs\, x(s) + K_1 x_1(s) + K x_1(s) - K x(s) = 0$$

$$x_1(s)[M_1 s^2 + Bs + B_1 s + K_1 + K] - x(s)[Bs + K] = 0$$

$$x_1(s) = \frac{x(s)[Bs+K]}{M_1 s^2 + Bs + B_1 s + K_1 + K} \rightarrow ③$$

Apply Laplace transform of equ ②

$$M_2 s^2 x(s) + (B+B_2) s\, x(s) - Bs\, x_1(s) + K x(s) - K x_1(s) = F(s)$$

$$x(s)[M_2 s^2 + (B+B_2)s + K] - x_1(s)[Bs+K] = F(s) \rightarrow ④$$

$$F(s) = x(s)[M_2 s^2 + (B+B_2)s + K] - \frac{x(s)[Bs+K][Bs+K]}{M_1 s^2 + Bs + B_1 s + K_1 + K}$$

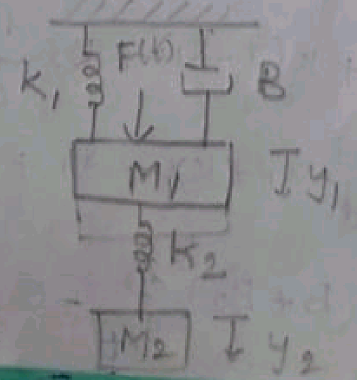
$$F(s) = x(s)\left[\frac{(M_2 s^2 + (B+B_2)s + K)(M_1 s^2 + (B+B_1)s + K + K_1) - (Bs+K)^2}{M_1 s^2 + (B+B_1)s + K_1 + K}\right]$$

$$\frac{x(s)}{F(s)} = \frac{M_1 s^2 + (B+B_1)s + K_1 + K}{[M_2 s^2 + (B+B_2)s + K][M_1 s^2 + (B+B_1)s + K + K_1] - Bs+K} \rightarrow ⑤$$

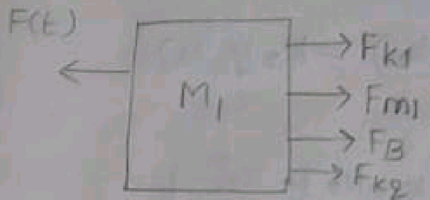
where equ ① & equ ② are differential equations

equ ⑤ is transfer function.

Obtain the transfer function of the system shown in the diagram.



Free body diagram of M_1.



$$F(t) = F_{k_1} + F_{m_1} + F_B + F_{k_2}$$

$$F_{m_1} = M_1 \frac{d^2 y_1}{dt^2}, \quad F_{k_1} = k_1 y_1$$

$$F_B = B \frac{dy_1}{dt}, \quad F_{k_2} = K_2 (y_1 - y_2)$$

Substitute in above eqn

$$F(t) = k_1 y_1 + M_1 \frac{d^2 y_1}{dt^2} + B \frac{dy_1}{dt} + k_2 (y_1 - y_2)$$

$$F(t) = k_1 y_1 + M_1 \frac{d^2 y_1}{dt^2} + B \frac{dy_1}{dt} + k_2 y_1 - k_2 y_2$$

$$F(t) = M_1 \frac{d^2 y_1}{dt^2} + B \frac{dy_1}{dt} + k_1 y_1 + k_2 y_1 - k_2 y_2 \rightarrow ①$$

Free body diagram of M_2

$$F_{k_2} = k_2 (y_2 - y_1)$$

$$F_{m_2} = M_2 \frac{d^2 y_2}{dt^2}$$

$$F_{k_2} + F_{m_2} = 0$$

$$k_2 (y_2 - y_1) + M_2 \frac{d^2 y_2}{dt^2} = 0$$

$$M_2 \frac{d^2 y_2}{dt^2} + k_2 y_2 - k_2 y_1 = 0 \rightarrow ②$$

Apply Laplace transform ~~to~~ eqn ①.

$$F(s) = M_1 s^2 y_1(s) + B s\, y_1(s) + k_1 y_1(s) + k_2 y_1(s) - k_2 y_2(s)$$

$$F(s) = M_1 s^2 y_1(s) + B s\, y_1(s) + (k_1 + k_2) y_1(s) - k_2 y_2(s) \rightarrow ③$$

Apply Laplace transfrom to eqn ②

$$M_2 s^2 y_2(s) + k_2 y_2(s) - k_2 y_1(s) = 0$$

$$y_2(s) \left[M_2 s^2 + k_2 \right] - k_2 y_1(s) = 0$$

$$k_2 y_1(s) = y_2(s) \left[M_2 s^2 + k_2 \right]$$

$$y_1(s) = \frac{y_2(s) \left[M_2 s^2 + k_2 \right]}{k_2} \rightarrow ④$$

consider eqn ③

$$F(s) = M_1 s^2 y_1(s) + B s\, y_1(s) + (k_1 + k_2) y_1(s) - k_2 y_2(s)$$

$$F(s) = y_1(s) \left[M_1 s^2 + Bs + k_1 + k_2 \right] - k_2 y_2(s) \rightarrow ⑤$$

Substitute eqⁿ 4 in eqⁿ 5

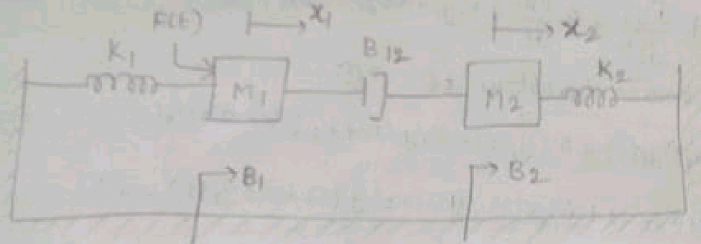
$$F(s) = \frac{y_2(s)\left[M_2 s^2 + k_2\right]}{k_2}(M_1 s^2 + Bs + k_1 + k_2) - k_2 y_2(s)$$

$$F(s) = y_2(s)\left[\frac{(M_2 s^2 + k_2)(M_1 s^2 + Bs + k_1 + k_2) - k_2^2}{k_2}\right]$$

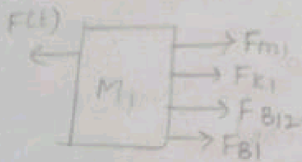
$$\boxed{\frac{y_2(s)}{F(s)} = \frac{k_2}{(M_2 s^2 + k_2)(M_1 s^2 + Bs + k_1 + k_2) - k_2^2}}$$

which is the required transfer function.

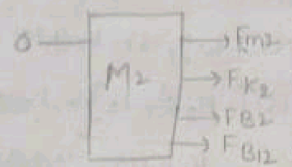
Obtain the transfer function $\frac{x_1(s)}{F(s)}$ & $\frac{x_2(s)}{F(s)}$ for the given diagram.



Free body diagram of M_1



Free body diagram of M_2



$F_{K_2} = K_2 x_2$

$F_{B_2} = B_2 \frac{dx_2}{dt}$

$F_{m_2} = M_2 \frac{d^2 x_2}{dt^2}$

$F_{B_{12}} = B_{12} \frac{d(x_2 - x_1)}{dt}$

$F_{B_1} = B_1 \frac{dx_1}{dt}$, $F_{B_{12}} = B_{12} \frac{d(x_1 - x_2)}{dt}$

$F_{K_1} = k_1 x_1$, $F_{m_1} = M_1 \frac{d^2 x_1}{dt^2}$

$F_{k_2} + F_{B_2} + F_{m_2} + F_{B_{12}} = 0$

$F(t) = F_{B_1} + F_{B_{12}} + F_{m_1} + F_{K_1}$

$$F(t) = B_1 \frac{dx_1}{dt} + B_{12}\frac{d(x_1 - x_2)}{dt} + M_1 \frac{d^2 x_1}{dt^2} + k_1 x_1$$

$$= M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B_{12}\frac{dx_1}{dt} - B_{12}\frac{dx_2}{dt} + k_1 x_1$$

$$F(t) = M_1 \frac{d^2 x_1}{dt^2} + (B_1 + B_{12})\frac{dx_1}{dt} - B_{12}\frac{dx_2}{dt} + k_1 x_1 \longrightarrow 1$$

Substitute F_{K_2}, F_{B_2}, F_{m_2}, $F_{B_{12}}$ values in M_2 eqⁿ.

$$k_2 x_2 + B_2 \frac{dx_2}{dt} + M_2 \frac{d^2(x_2)}{dt^2} + B_{12}\frac{d(x_2 - x_1)}{dt} = 0$$

$$M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{dx_2}{dt} + B_{12}\frac{dx_2}{dt} - B_{12}\frac{dx_1}{dt} + k_2 x_2 = 0$$

$$M_2 \frac{d^2 x_2}{dt^2} + (B_2 + B_{12})\frac{dx_2}{dt} - B_{12}\frac{dx_1}{dt} + k_2 x_2 = 0 \longrightarrow 2$$

Apply Laplace transform to eqⁿ 1

$$F(s) = M_1 s^2 x_1(s) + (B_1 + B_{12})\, s\, x_1(s) - B_{12} s\, x_2(s) + k_1 x_1(s)$$

$$F(s) = x_1(s)\left[M_1 s^2 + (B_1 + B_{12})s + k_1\right] - B_{12} s\, x_2(s) \longrightarrow 3$$

Apply Laplace transform to eqⁿ 2

$$M_2 s^2 x_2(s) + (B_2 + B_{12})\, s\, x_2(s) - B_{12} s\, x_1(s) + k_2 x_2(s) = 0$$

$$x_2(s)\left[M_2 s^2 + (B_2 + B_{12})s + k_2\right] - B_{12} s\, x_1(s) = 0$$

$$x_2(s)\left[M_2 s^2 + (B_2 + B_{12})s + k_2\right] = B_{12} s\, x_1(s) \longrightarrow 4$$

From eqn 4

$$x_2(s) = \frac{B_{12} s\, x_1(s)}{M_2 s^2 + (B_2 + B_{12})s + k_2}$$

Substitute in eqⁿ 3

$$F(s) = x_1(s)\left[M_1 s^2 + (B_1 + B_{12})s + k_1\right] - B_{12}s \frac{B_{12}s\, x_1(s)}{M_2 s^2 + (B_2 + B_{12})s + k_2}$$

$$F(s) = x_1(s)\left[\frac{(M_1 s^2 + (B_1+B_{12})s + k_1)(M_2 s^2 + (B_2+B_{12})s + k_2) - (B_{12}s)^2}{M_2 s^2 + (B_2 + B_{12})s + k_2}\right]$$

$$\boxed{\frac{x_1(s)}{F(s)} = \frac{M_2 s^2 + (B_2 + B_{12})s + k_2}{\left[M_1 s^2 + (B_1+B_{12})s + k_1\right]\left[M_2 s^2 + (B_2+B_{12})s + k_2\right] - (B_{12}s)^2}}$$

From eq^n 4

$$x_1(s) = \frac{x_2(s)\left[M_2 s^2 + (B_2 + B_{12})s + k_2\right]}{B_{12}s}$$

Substitute in eq^n 3

$$F(s) = \frac{x_2(s)\left[M_2 s^2 + (B_2 + B_{12})s + k_2\right]}{B_{12}s}\left[M_1 s^2 + (B_1+B_{12})s + k_1\right] - B_{12}s\, x_2(s)$$

$$F(s) = x_2(s)\left[\frac{(M_2 s^2 + (B_2+B_{12})s + k_2)(M_1 s^2 + (B_1+B_{12})s + k_1) - (B_{12}s)^2}{B_{12}s}\right]$$

$$\boxed{\frac{x_2(s)}{F(s)} = \frac{B_{12}s}{\left[M_2 s^2 + (B_2+B_{12})s + k_2\right]\left[M_1 s^2 + (B_1+B_{12})s + k_1\right] - (B_{12}s)^2}}$$

Mechanical Rotational System

⇒ Symbols used in mechanical rotational system

θ – angular displacement (radians)

$\frac{d\theta}{dt} = V \rightarrow$ angular velocity (radians/sec)

$a = \frac{d^2\theta}{dt^2}$ = angular acceleration (radians/sec^2)

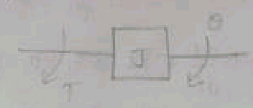
T = applying Torque (Newtons.m)

J = Moment of inertia (kg.m^2/rad)

V = Rotational frictional coefficient

K = Stiffness of the string (N.m/rad)

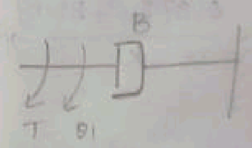
Mass: consider a mass with negligible friction & stiffness. The applied torque is directly proportional to the angular acceleration

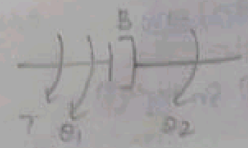


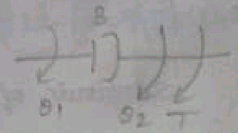
$$T = T_J \propto \frac{d^2\theta}{dt^2}$$

$$T = T_J = J\frac{d^2\theta}{dt^2}$$

Dash pot or Damper: consider a Dash pot with negligible moment of inertia & stiffness. The applied torque is directly proportional to the angular velocity.





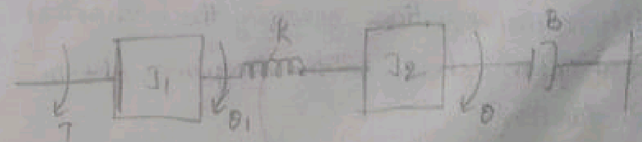


$T_B \propto V$ or $\frac{d\theta}{dt}$ $\quad T_B = B\frac{d}{dt}(\theta_1 - \theta_2) \quad T_B = B\frac{d}{dt}(\theta_2 - \theta_1)$

$T_B = B\frac{d\theta}{dt}$

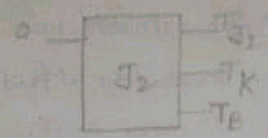
Spring: consider a spring with negligible moment of inertia & friction. The applied Torque is directly proportional to the angular displacement.

1. Write the differential equations governing the mechanical rotational systems shown in the diagram obtain the transfer function.



Free body diagram of J_1

Free body diagram of J_2



$T_{J_2} = J_2 \frac{d^2\theta}{dt^2}$

$T_k = K(\theta - \theta_1)$

$T_B = B\frac{d\theta}{dt}$

$T = T_k + T_{J_1}$

$T_k = K(\theta_1 - \theta)$, $T_{J_1} = J_1 \frac{d^2\theta_1}{dt^2}$

$T = K(\theta_1 - \theta) + J_1 \frac{d^2\theta_1}{dt^2}$

$T = K\theta_1 - K\theta + J_1 \frac{d^2\theta_1}{dt^2} \rightarrow ①$

Apply Laplace transform on B.S

$J_1 s^2 \theta_1(s) + K\theta_1(s) - K\theta(s) = T(s)$

$\theta_1(s)\left(J_1 s^2 + K\right) - K\theta(s) = T(s) \rightarrow ③$

$T_{J_2} + T_k + T_B = 0$

$J_2 \frac{d^2\theta}{dt^2} + K(\theta - \theta_1) + B\frac{d\theta}{dt} = 0$

$J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta - K\theta_1 = 0 \rightarrow ②$

Apply Laplace transform on B.S

$J_2 s^2\theta(s) + Bs\theta(s) + K\theta(s) - K\theta_1(s) = 0$

$\theta(s)\left[J_2 s^2 + Bs + K\right] = K\theta_1(s)$

$\theta_1(s) = \frac{\theta(s)\left[J_2 s^2 + Bs + K\right]}{K} \rightarrow ④$

Substitute eq^n ④ in eq^n ③

$$\frac{\theta(s)\left[J_2 s^2 + Bs + K\right]\left[J_1 s^2 + K\right]}{K} - K\theta(s) = T(s)$$

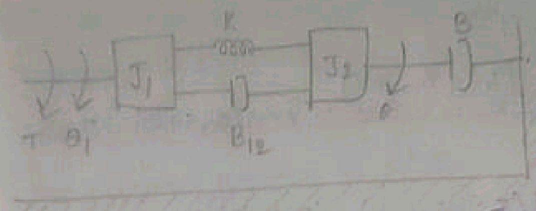
$$\theta(s)\left[\frac{(J_2 s^2 + Bs + K)(J_1 s^2 + K) - K^2}{K}\right] = T(s)$$

$$\boxed{\frac{\theta(s)}{T(s)} = \frac{K}{(J_2 s^2 + Bs + K)(J_1 s^2 + K) - K^2}} \rightarrow ⑤$$

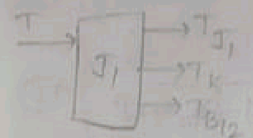
Here, eq^n ① & eq^n ② are differentiatial eq^ns

eq^n ⑤ is the transfer function for the given system.

2. Write the differential equations governing the mechanical ~~system~~ rotational system shown in the diagram obtain the transfer function.



Free body diagram of J_1



$T_{J_1} = J_1 \frac{d^2\theta_1}{dt^2}$, $T_K = K(\theta_1 - \theta)$

$T_{B_{12}} = B_{12}\frac{d(\theta_1 - \theta)}{dt}$

$T_{J_1} + T_K + T_{B_{12}} = T$

$J_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta) + B_{12}\frac{d(\theta_1 - \theta)}{dt} = T$

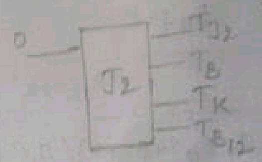
$J_1 \frac{d^2\theta_1}{dt^2} + B_{12}\frac{d\theta_1}{dt} - B_{12}\frac{d\theta}{dt} + K\theta_1 - K\theta = T \rightarrow ①$

Apply Laplace transform on B.S

$J_1 s^2\theta_1(s) + B_{12}s\theta_1(s) - B_{12}s\theta(s) + K\theta_1(s) - K\theta(s) = T(s)$

$\theta_1(s)\left[J_1 s^2 + B_{12}s + K\right] - \theta(s)\left[B_{12}s + K\right] = T(s) \rightarrow ③$

Free body diagram of J_2



$T_{J_2} = J_2 \frac{d^2\theta}{dt^2}$

$T_B = B\frac{d\theta}{dt}$

$T_{B_{12}} = B_{12}\frac{d(\theta - \theta_1)}{dt}$

$T_K = K(\theta - \theta_1)$

$T_{J_2} + T_{B_{12}} + T_B + T_K = 0$

$J_2 \frac{d^2\theta}{dt^2} + B_{12}\frac{d(\theta - \theta_1)}{dt} + B\frac{d\theta}{dt} + K(\theta - \theta_1) = 0$

$J_2 \frac{d^2\theta}{dt^2} + B_{12}\frac{d\theta}{dt} - B_{12}\frac{d\theta_1}{dt} + B\frac{d\theta}{dt} + K\theta - K\theta_1 = 0 \rightarrow ②$

Apply Laplace transform to eq^n ②

$J_2 s^2\theta(s) + B_{12}s\theta(s) - B_{12}s\theta_1(s) + Bs\theta(s) + K\theta(s) - K\theta_1(s) = 0$

$\theta(s)\left[J_2 s^2 + B_{12}s + Bs + K\right] - \theta_1(s)\left[B_{12}s + K\right] = 0$

$\theta(s)\left[J_2 s^2 + B_{12}s + Bs + K\right] = \theta_1(s)\left[B_{12}s + K\right]$

$$\theta_1(s) = \frac{\theta(s)\left[J_2 s^2 + B_{12}s + Bs + K\right]}{(B_{12}s + K)} \rightarrow ④$$

Substitute eq^n (4) in eq^n (3)

$$\theta(s)\frac{(J_2s^2+B_{12}s+Bs+k)(J_1s^2+B_{12}s+k)}{B_{12}s+k} - \theta(s)(B_{12}s+k) = T(s)$$

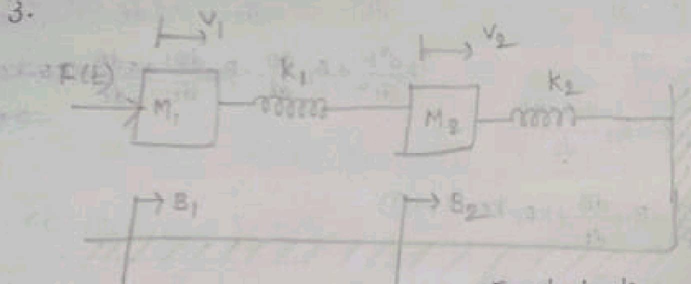
$$\theta(s)\left[\frac{(J_2s^2+B_{12}s+Bs+k)(J_1s^2+B_{12}s+k)-(B_{12}s+k)^2}{B_{12}s+k}\right] = T(s)$$

$$\boxed{\frac{\theta(s)}{T(s)} = \frac{B_{12}s+k}{(J_2s^2+B_{12}s+Bs+k)(J_1s^2+B_{12}s+k)-(B_{12}s+k)^2}} \rightarrow (5)$$

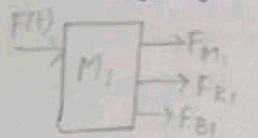
Here, eq^n (1) & eq^n (2) are differential eq^{n}s

eq^n (5) is the transfer function.

3.



Free body diagram of M_1



$F(t) = F_{M_1} + F_{K_1} + F_{B_1}$

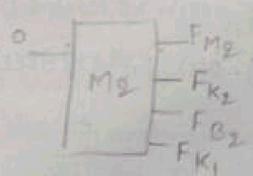
$F_{M_1} = M_1\frac{d^2v_1}{dt^2}$, $F_{K_1} = k_1(v_1-v_2)$

$F_{B_1} = B_1\frac{dv_1}{dt}$

$$F(t) = M_1\frac{d^2v_1}{dt^2} + k_1(v_1-v_2) + B_1\frac{dv_1}{dt}$$

$$F(t) = M_1\frac{d^2v_1}{dt^2} + B_1\frac{dv_1}{dt} + k_1v_1 - k_1v_2 \rightarrow (1)$$

Free body diagram of M_2



$F_{M_2} = M_2\frac{d^2v_2}{dt^2}$

$F_{K_2} = k_2v_2$

$F_{B_2} = B_2\frac{dv_2}{dt}$

$F_{K_1} = k_1(v_2-v_1)$

$F_{M_2} + F_{K_2} + F_{B_2} + F_{K_1} = 0$

$$M_2\frac{d^2v_2}{dt^2} + k_2v_2 + B_2\frac{dv_2}{dt} + k_1(v_2-v_1) = 0$$

$$M_2\frac{d^2v_2}{dt^2} + k_2v_2 + B_2\frac{dv_2}{dt} + k_1v_2 - k_1v_1 = 0 \rightarrow (2)$$

Apply Laplace transform to eq^n (1)

$M_1s^2V_1(s) + B_1sV_1(s) + k_1V_1(s) - k_1V_2(s) = F(s)$

$V_1(s)\left[M_1s^2+B_1s+k_1\right] - k_1V_2(s) = F(s) \rightarrow (3)$

Apply Laplace transform to eq^n (2)

$M_2s^2V_2(s) + B_2sV_2(s) + K_2V_2(s) + K_1V_2(s) - K_1V_1(s) = 0$

$V_2(s)\left[M_2s^2+B_2s+K_2+k_1\right] - K_1V_1(s) = 0$

$$V_1(s) = \frac{V_2(s)\left[M_2s^2+B_2s+k_1+k_2\right]}{K_1} \rightarrow (4)$$

Substitute eq^n (4) in eq^n (3)

$$\frac{V_2(s)(M_2s^2+B_2s+k_1+k_2)\left[M_1s^2+B_1s+k_1\right]}{K_1} - k_1V_2(s) = F(s)$$

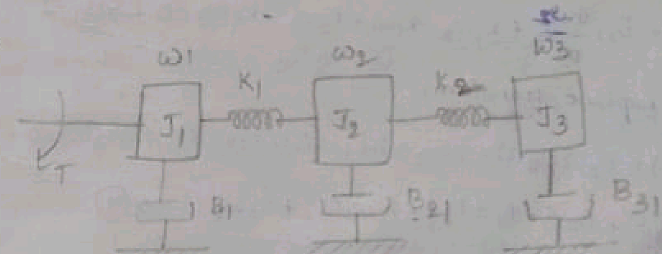
$$V_2(s)\left[\frac{(M_2s^2+B_2s+k_1+k_2)(M_1s^2+B_1s+k_1)-k_1^2}{k}\right] = F(s)$$

$$\boxed{\frac{V_2(s)}{F(s)} = \frac{k}{(M_2s^2+B_2s+k_1+k_2)(M_1s^2+B_1s+k_1)-k_1^2}} \rightarrow (5)$$

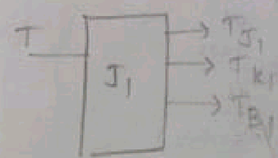
Here, eq^n (1) & eq^n (2) are differential equations

eq^n (5) is the transfer function.

4.



Free body diagram of J_1



$T_{J_1} = J_1\frac{d^2\omega_1}{dt^2}$, $T_{K_1} = k_1(\omega_1-\omega_2)$

$T_{B_1} = B_1\frac{d\omega_1}{dt}$

$T_{J_1} + T_{K_1} + T_{B_1} = T$

$$J_1 \frac{d^2\omega_1}{dt^2} + B_1 \frac{d\omega_1}{dt} + k_1(\omega_1 - \omega_2) = T$$

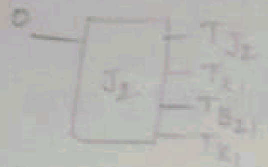
$$J_1 \frac{d^2\omega_1}{dt^2} + B_1 \frac{d\omega_1}{dt} + k_1\omega_1 - k_1\omega_2 = T \rightarrow (1)$$

Apply Laplace transform

$$J_1 s^2 \omega_1(s) + B_1 s\omega_1(s) + k_1\omega_1(s) - k_1\omega_2(s) = T(s)$$

$$\omega_1(s)\left[J_1 s^2 + B_1 s + k_1\right] - k_1\omega_2(s) = T(s) \rightarrow (2)$$

Free body diagram of J_2



$$T_{k_1} = k_1(\omega_2 - \omega_1),\quad T_{B_{21}} = B_{21}\frac{d\omega_2}{dt},\quad T_{k_3} = k_3(\omega_2 - \omega_3)$$

$$T_{J_2} = J_2 \frac{d^2\omega_2}{dt^2}$$

$$T_{J_2} + T_{k_1} + T_{B_{21}} + T_{k_3} = 0$$

$$J_2 \frac{d^2\omega_2}{dt^2} + k_1(\omega_2 - \omega_1) + B_{21}\frac{d\omega_2}{dt} + k_3(\omega_2 - \omega_3) = 0$$

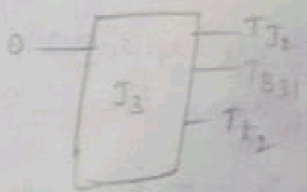
$$J_2 \frac{d^2\omega_2}{dt^2} + B_{21}\frac{d\omega_2}{dt} + k_1\omega_2 - k_1\omega_1 + k_3\omega_2 - k_3\omega_3 = 0 \rightarrow (3)$$

Apply Laplace transform

$$J_2 s^2\omega_2(s) + B_{21} s\omega_2(s) + k_1\omega_2(s) - k_1\omega_1(s) + k_3\omega_2(s) - k_3\omega_3(s) = 0$$

$$\omega_2(s)\left[J_2 s^2 + B_{21}s + k_1 + k_3\right] = k_1\omega_1(s) + k_3\omega_3(s) \rightarrow (4)$$

Free body diagram of J_3



$$T_{J_3} = J_3 \frac{d^2\omega_3}{dt^2}$$

$$T_{B_{31}} = B_{31}\frac{d\omega_3}{dt}$$

$$T_{k_2} = k_2(\omega_3 - \omega_2)$$

$$J_3 \frac{d^2\omega_3}{dt^2} + B_{31}\frac{d\omega_3}{dt} + k_2(\omega_3 - \omega_2) = 0$$

$$J_3 \frac{d^2\omega_3}{dt^2} + B_{31}\frac{d\omega_3}{dt} + k_2\omega_3 - k_2\omega_2 = 0 \rightarrow (5)$$

Apply Laplace Transform

$$J_3 s^2\omega_3(s) + B_{31} s\omega_3(s) + k_2\omega_3(s) - k_2\omega_2(s) = 0$$

$$\omega_3(s)\left[J_3 s^2 + B_{31}s + k_2\right] = k_2\omega_2(s)$$

$$\omega_2(s) = \frac{\omega_3(s)\left(J_3 s^2 + B_{31}s + k_2\right)}{k_2} \rightarrow (6)$$

substitute in eq^n (4)

$$\frac{\omega_3(s)\left(J_3 s^2 + B_{31}s + k_2\right)\left(J_2 s^2 + B_{21}s + k_1 + k_3\right)}{k_2} = k_1\omega_1(s) + k_3\omega_3(s)$$

$$\frac{\omega_3(s)\left(J_3 s^2 + B_{31}s + k_2\right)\left(J_2 s^2 + B_{21}s + k_1 + k_3\right)}{k_2} - k_3\omega_3(s) = k_1\omega_1(s)$$

$$\omega_3(s)\left[\frac{\left(J_3 s^2 + B_{31}s + k_2\right)\left(J_2 s^2 + B_{21}s + k_1 + k_3\right) - k_2k_3}{k_2}\right] = k_1\omega_1(s)$$

$$\omega_3(s)\left[\frac{\left(J_3 s^2 + B_{31}s + k_2\right)\left(J_2 s^2 + B_{21}s + k_1 + k_3\right) - k_1k_2k_3}{k_1k_2}\right] = \omega_1(s) \rightarrow (7)$$

substitute eq^n (6), eq^n (7) in eq^n (2)

$$\omega_3(s)\left[\frac{\left(J_3 s^2 + B_{31}s + k_2\right)\left(J_2 s^2 + B_{21}s + k_1 + k_3\right) - k_1k_2k_3}{k_1k_2}\right]\left[J_1 s^2 + B_1 s + k_1\right]$$

$$- k_1 \cdot \frac{\omega_3(s)\left[J_3 s^2 + B_{31}s + k_2\right]}{k_2} = T(s)$$

$$\omega_3(s)\left\{\frac{\left[\left(J_3 s^2 + B_{31}s + k_2\right)\left(J_2 s^2 + B_{21}s + k_1 + k_3\right) - k_1k_2k_3\right]\left[J_1 s^2 + B_1 s + k_1\right] - k_1^2\left(J_3 s^2 + B_{31}s + k_2\right)}{k_1k_2}\right\} = T(s)$$

$$\frac{\omega_3(s)}{T(s)} = \frac{\left[\left(J_3 s^2 + B_{31} s + K_2\right)\left(J_2 s^2 + B_{21} s + K_1 + K_3\right) - K_1 K_2 K_3\right]\left(J_1 s^2 + B_1 s + K_1\right) - K_1 s^2 \left(J_3 s^2 + B_{31} s + K_2\right)}{K_1 K_2}$$

$$\frac{\omega_3(s)}{T(s)} = \frac{\left(J_3 s^2 + B_{31} s + K_2\right)\left[\left[\left(J_2 s^2 + B_{21} s + K_1 + K_3\right] - K_1 K_2 K_3\right]\left(J_1 s^2 + B_1 s + K_1\right) - K_1^2\right.}{K_1 K_2}$$

Unit-II

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