

What is a Probability Distribution?

Reference: <https://freedium.cfd/https://news.lunartech.ai/fundamentals-of-statistics-for-data-scientists-and-data-analysts-69d93a05aae7>
(<https://freedium.cfd/https://news.lunartech.ai/fundamentals-of-statistics-for-data-scientists-and-data-analysts-69d93a05aae7>).

A probability distribution describes how probabilities are assigned to different values of a random variable. It tells us the likelihood of different outcomes in an experiment.

There are two main types of probability distributions:

Discrete Probability Distributions (for countable outcomes).

Continuous Probability Distributions (for uncountable outcomes like real numbers).

Three key related concepts associated with probability distributions include:

1. Probability Mass Function (PMF): associated with discrete variables, the PMF gives the probabilities for individual outcomes, for example, the probability of a family having one child, having two children, and so on. Assuming X is a random variable like the number of children in a family, and x is a specific value for X , then the PMF is denoted by $F(X)=P[X=x]$.
2. Probability Density Function (PDF): associated with continuous variables, the PDF describes the likelihood of a value falling within a range, for instance probability of a person's height falling between 5.5 feet and 6 feet. Following a similar notation, the PDF is denoted by $F(X)=P[x \leq X \leq x']$, with $x < x'$.
3. Cumulative Distribution Function (CDF): it indicates the cumulated probability up to a certain value, for example, the probability of a newborn baby weighing up to 2.5kg. It is denoted by $F(x)=P[X \leq x]$.

1. Discrete Probability Distributions

A discrete probability distribution applies to a discrete random variable, which takes finite or countable values (e.g., number of defective products, number of heads in coin flips)

1. Binomial Distribution

Use Case: It models the number of successes in a fixed number of independent trials, like predicting the probability of defective items in a production batch. Example: Estimating the likelihood of defective products in a factory.

Reference:

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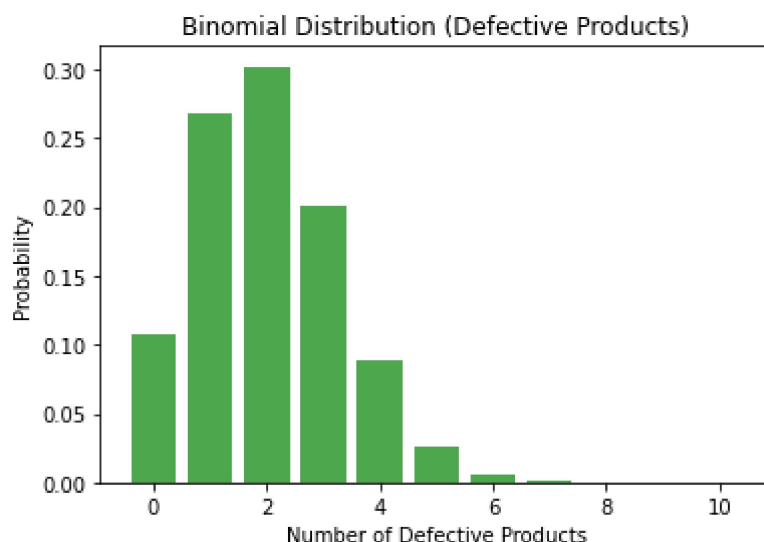
In [5]: 1 from scipy.stats import binom
2 import matplotlib.pyplot as plt
3
4 # Parameters
5 n = 10 # number of trials (products inspected)
6 p = 0.2 # probability of a defective product
7
8 # Generate binomial distribution
9 x = np.arange(0, n + 1)
10 probabilities = binom.pmf(x, n, p)
11 print(probabilities)
12 # Plot the probabilities
13 plt.bar(x, probabilities, color='green', alpha=0.7)
14 plt.title("Binomial Distribution (Defective Products)")
15 plt.xlabel("Number of Defective Products")
16 plt.ylabel("Probability")
17 plt.show()

```

```

[1.07374182e-01 2.68435456e-01 3.01989888e-01 2.01326592e-01
 8.80803840e-02 2.64241152e-02 5.50502400e-03 7.86432000e-04
 7.37280000e-05 4.09600000e-06 1.02400000e-07]

```

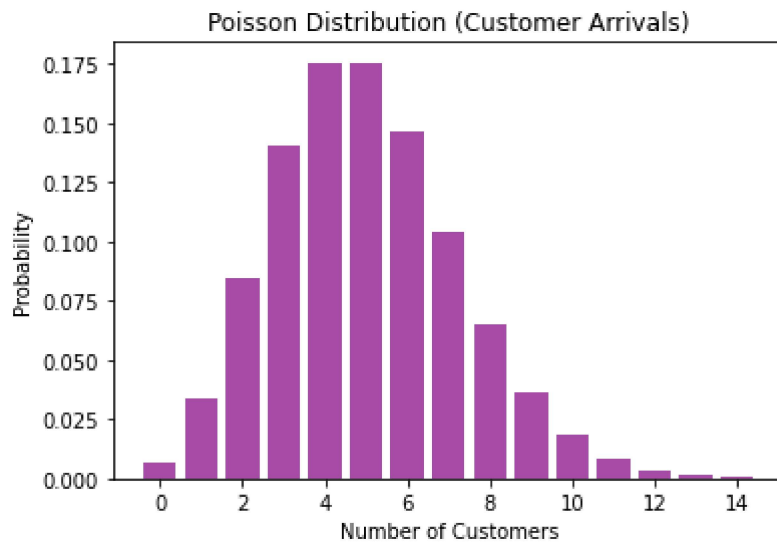


2. Poisson Distribution

Use Case: Used to model the number of events occurring within a fixed time, such as customer arrivals or call-center requests. Example: Modeling the number of customer arrivals at a coffee shop in an hour.

Reference: <https://www.statology.org/poisson-distribution/> (<https://www.statology.org/poisson-distribution/>)

```
In [30]: 1 from scipy.stats import poisson
2
3 # Parameters
4 lambda_val = 5 # average arrivals per hour
5
6 # Generate Poisson distribution
7 x = np.arange(0,15)
8 probabilities = poisson.pmf(x, lambda_val)
9
10 # Plot the distribution
11 plt.bar(x, probabilities, color='purple',alpha=0.7)
12 plt.title("Poisson Distribution (Customer Arrivals)")
13 plt.xlabel("Number of Customers")
14 plt.ylabel("Probability")
15 plt.show()
```



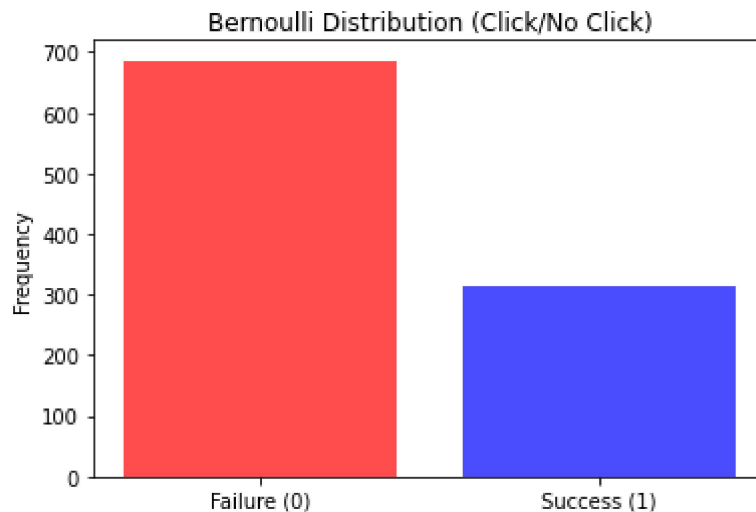
3. Bernoulli Distribution

Use Case: The Bernoulli distribution models a single experiment (trial) that has exactly two possible outcomes: success (1) or failure (0). It's commonly used in scenarios where we are dealing with binary outcomes, such as:

Example: Imagine you're running an A/B test to determine whether users click on a new feature button. The outcome for each user is binary: clicked (1) or not clicked (0).

```
In [5]: 1 import numpy as np
2 import matplotlib.pyplot as plt
3
4 # Probability of success (e.g., probability a user clicks on the button)
5 p = 0.3
6
7 # Generate 1000 Bernoulli trials
8 bernoulli_trials = np.random.binomial(n=1, p=p, size=1000)
9
10 # Calculate success and failure counts
11 success_count = sum(bernoulli_trials)
12 failure_count = len(bernoulli_trials) - success_count
13
14 print(f"Successes (Clicks): {success_count}")
15 print(f"Failures (No Clicks): {failure_count}")
16
17 # Plot Bernoulli Distribution
18 plt.bar([0, 1], [failure_count, success_count], color=['red', 'blue'], alpha=0.5)
19 plt.xticks([0, 1], ['Failure (0)', 'Success (1)'])
20 plt.title("Bernoulli Distribution (Click/No Click)")
21 plt.ylabel("Frequency")
22 plt.show()
```

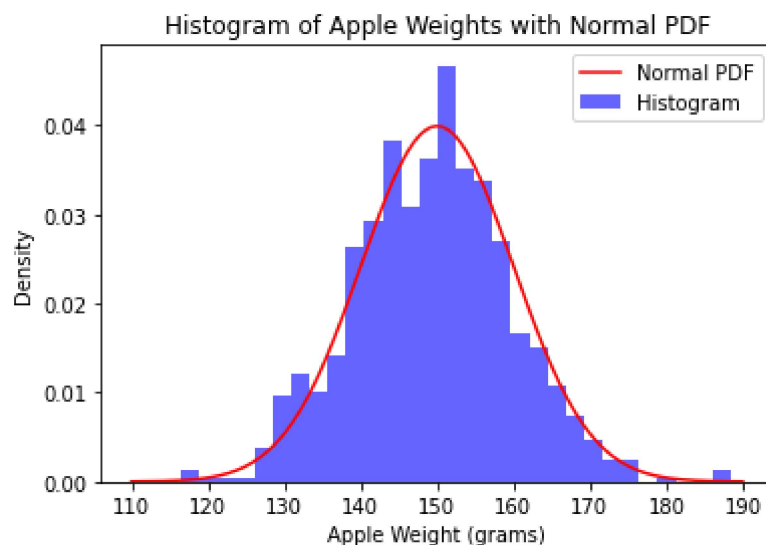
Successes (Clicks): 314
Failures (No Clicks): 686



2. Continuous Probability Distributions

A continuous probability distribution applies to a continuous random variable, which can take infinite values within a range (e.g., heights, weights, temperature).

```
In [24]: 1 import numpy as np
2 import matplotlib.pyplot as plt
3 from scipy.stats import norm
4
5 # Generate normal distribution
6 mean = 150 # Average weight in grams
7 std_dev = 10 # Standard deviation in grams
8 samples = np.random.normal(mean, std_dev, 1000) # Generate 1000 samples
9
10 # Plot histogram
11 plt.hist(samples, bins=30, density=True, alpha=0.6, color='b', label="Histogram")
12
13 # Overlay Normal PDF
14 x = np.linspace(mean - 4*std_dev, mean + 4*std_dev, 1000) # Generate x values
15 y = norm.pdf(x, mean, std_dev) # Compute PDF values
16
17 plt.plot(x, y, 'r-', label="Normal PDF") # Plot PDF
18 plt.xlabel("Apple Weight (grams)")
19 plt.ylabel("Density")
20 plt.title("Histogram of Apple Weights with Normal PDF")
21 plt.legend()
22 plt.show()
23
24
```



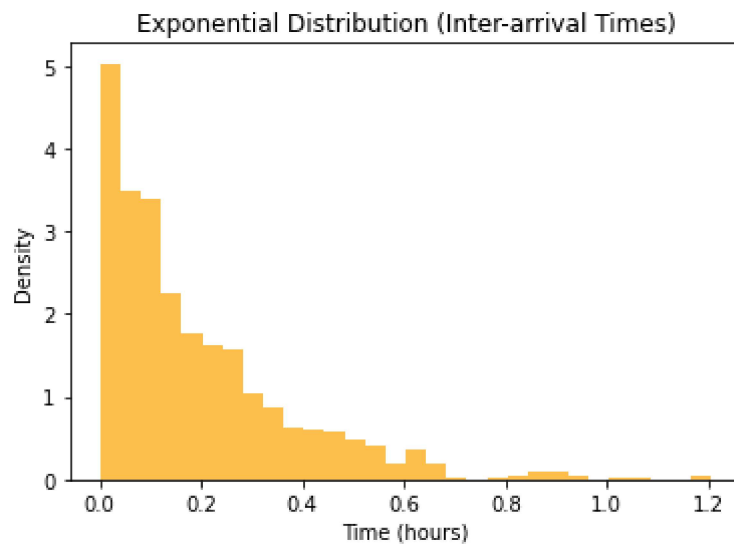
2. Exponential Distribution

Use Case: Used to model the time between events in a Poisson process, such as wait times in queues or failure rates of machines.

Example: Predicting the time until the next customer arrives at a service counter.

Reference: <https://www.statology.org/exponential-distribution/>
(<https://www.statology.org/exponential-distribution/>)

```
In [7]: 1 # Generate exponential distribution
2 scale = 1 / 5 # mean time between arrivals (e.g., 5 customers per hour)
3 arrival_times = np.random.exponential(scale, 1000)
4
5 # Plot the distribution
6 plt.hist(arrival_times, bins=30, density=True, alpha=0.7, color='orange')
7 plt.title("Exponential Distribution (Inter-arrival Times)")
8 plt.xlabel("Time (hours)")
9 plt.ylabel("Density")
10 plt.show()
```



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In [ ]:
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1
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In [ ]:
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1
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In [ ]:
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1
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