Vector PDE - Finite Difference Method

print("vector diffusion equation solver")

A sample vector PDE,

$$\frac{\partial \mathbf{u}}{\partial t} = \alpha \left(\frac{\partial^2 \mathbf{u}}{\partial x^2} + \hat{z} \times \frac{\partial^2 \mathbf{u}}{\partial y^2} \right) \tag{1}$$

,

i.e., each component of dependent variable depends on the other component,

$$\frac{\partial u_x}{\partial t} = \alpha \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} \right) \tag{2}$$

,

and,

$$\frac{\partial u_y}{\partial t} = \alpha \left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} \right) \tag{3}$$

,

the maximum value allowed for the solution to be stable is $\Delta t < \Delta t_{max} = \frac{\Delta x^2}{4\alpha}$