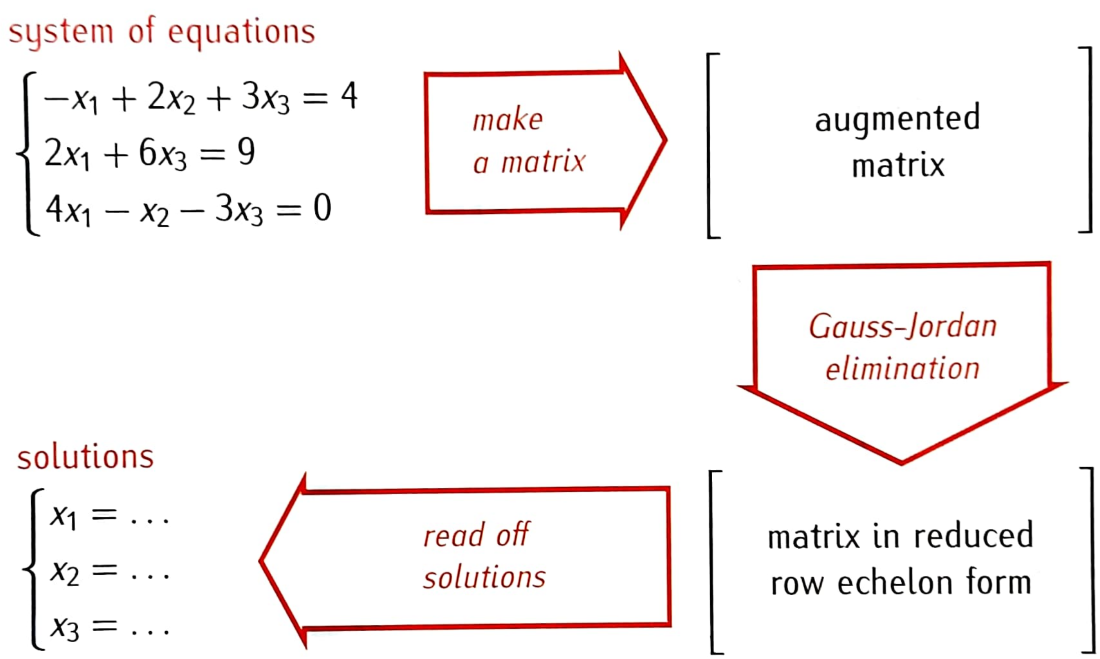
Next:

How to solve a system of linear equations



Matrices

matrix = rectangular array of numbers

Q: $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

Example.

$\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \text{. row}$$

columns

$$B = \begin{bmatrix} 1 & 2 & 0 \\ 7 & -5 & 1 \\ 8 & 10 & 7 \\ 6 & 4 & 3 \end{bmatrix}$$

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Note

Every system of linear equations can be represented by a matrix.

Example.

$$\begin{cases} -x_1 + 2x_2 + 3x_3 = 4 \\ 2x_1 + 6x_3 = 9 \\ 4x_1 - x_2 - 3x_3 = 0 \end{cases}$$

$\begin{bmatrix} -1 & 2 & 3 & 4 \\ 2 & 0 & 6 & 9 \\ 4 & -1 & 3 & 0 \end{bmatrix}$

$\left[\begin{array}{ccc|c} -1 & 2 & 3 & 4 \\ 2 & 0 & 6 & 9 \\ 4 & -1 & 3 & 0 \end{array}\right]$

coefficient matrix.

column of constants

Augmented matrix.

Elementary row operations:

1) Interchange of two rows.

1 $\to \begin{cases} x_1 + 2x_2 + 3x_3 = 4 \\ \quad\; x_2 + 5x_3 = 1 \\ 4x_1 + 3x_2 \quad = 7. \end{cases}$

Example. R_{12} $\quad R_1 \leftrightarrow R_2,$

interchang rows 1 2.

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 1 \\ 4 & 3 & 0 & 7 \end{array}\right] \longrightarrow \begin{bmatrix} 0 & 1 & 5 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 3 & 0 & 7 \end{bmatrix} \begin{cases} x_2 + 5x_3 = 1 \\ x_1 + 2x_2 + 3x_3 = 4 \\ 4x_1 + 3x_2 \quad = 7. \end{cases}$$

2) Multiplication of a row by a non-zero number.

Example.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 1 \\ 4 & 3 & 0 & 7 \end{bmatrix} \xrightarrow[3 \cdot R_1]{\times 3\, R_1} \begin{bmatrix} 3 & 6 & 9 & 12 \\ 0 & 1 & 5 & 1 \\ 4 & 3 & 0 & 7 \end{bmatrix} \to \begin{array}{l} 3x_1 + 6x_2 + 9x_3 = 12 \\ \quad x_2 + 5x_3 = 1 \\ 4x_1 + 3x_2 \quad = 7 \end{array}$$

3) Addition of a multiple of one row to another row.

Example.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 1 \\ 4 & 3 & 0 & 7 \end{bmatrix} \xrightarrow{R_3 - 4 \cdot R_1} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 1 \\ 0 & -5 & -12 & -9 \end{bmatrix} \to$$

$$\begin{array}{l} x_1 + 2x_2 + 3x_3 = 4 \\ \quad x_2 + 5x_3 = 1 \\ -5x_2 - 12x_3 = -9 \end{array}$$

Proposition

Elementary row operations do not change solutions of the system of equations represented by a matrix.