Systems of linear equations

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{cases}$$

$y = mx + c.$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b, \\ a x_1^2 + a x_2 + \ldots + a_n x_n = b, \\ a_1 x + b_1 y + 0.z + 0.w = c, \quad a_1 x + b_1 y = c. \\ 0.x + 0.y + a_2 z + b_2 w = d, \quad a_2 z + b_2 w = d. \end{cases}$$

$x_1 + 2x_2 = 3$ — (1)
$3x_1 + 5x_2 = 5$ — (2)

System of li. equations in x_1 & x_2.

(2) $- 3 \times$ (1)

$$\begin{array}{r} 3x_1 + 5x_2 = 5 \\ -\; 3x_1 + 6x_2 = 9 \\ \hline -x_2 = -4 \end{array}$$

$x_2 = 4.$

Question: How many solutions a system of linear equations can have?

Example: Systems of equations in 2 variables.

$$\begin{cases} x_1 + x_2 = 1 \\ x_1 - x_2 = 1 \end{cases} \qquad \frac{x_1}{1} + \frac{x_2}{1} = 1$$

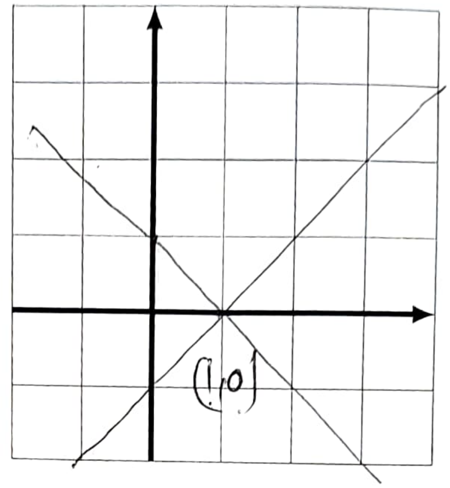
$$a x_1 + b x_2 = c. \qquad 3x_1 + x_2 = \cancel{2}\,1$$

$$\frac{x_1}{d_1} + \frac{x_2}{d_2} = 1 \Rightarrow \frac{x_1}{1/3} + \frac{x_2}{1} = 1$$

$(d_1, 0)$

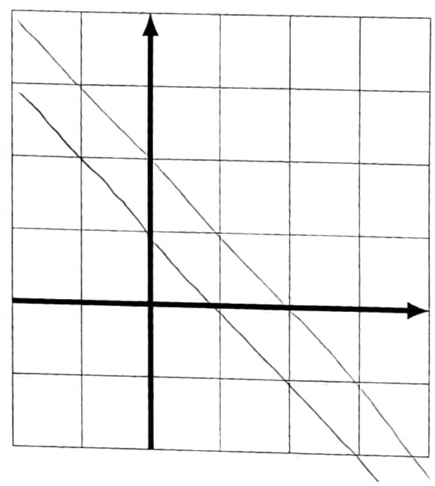
$(0, d_2)$

$\rightarrow$ One solution



$$\begin{cases} x_1 + x_2 = 1 \\ x_1 + x_2 = 2 \end{cases}$$

No solution

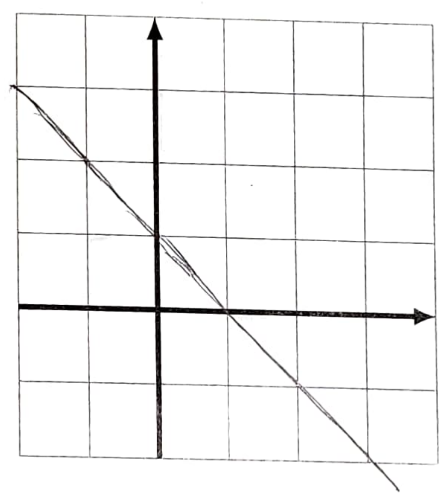


$$\begin{cases} x_1 + x_2 = 1 \\ 2x_1 + 2x_2 = 2 \end{cases}$$

$\downarrow$

$x_1 + x_2 = 1$

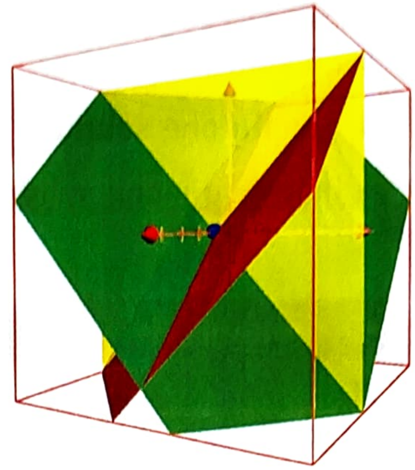
infinitely many solutions.



Example: Systems of equations in 3 variables.

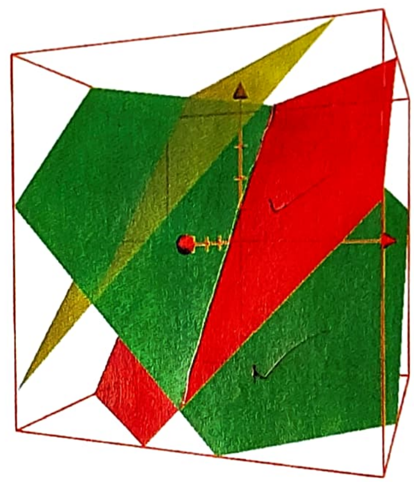
$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 - x_2 + x_3 = 1 \\ x_1 = 1 \end{cases}$$

one sol^n.



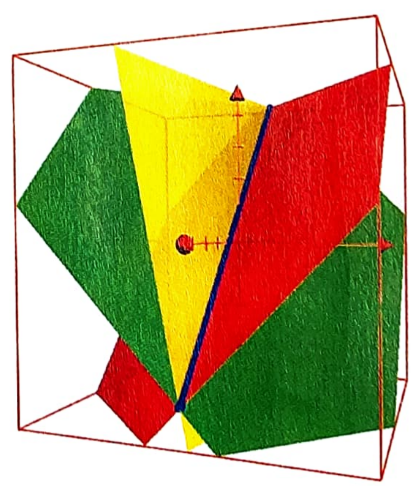
$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 - x_2 + x_3 = 1 \\ x_1 - x_2 + x_3 = 6 \end{cases}$$

no ~~so~~ solution.



$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 - x_2 + x_3 = 1 \\ x_1 + 5x_2 + x_3 = 1 \end{cases}$$

many solutions.



In general:

A system of linear equations can have either

- no solutions
- exactly one solution
- infinitely many solutions

Definition

If as system of linear equations which has no solutions is called an *inconsistent system*. Otherwise the system is *consistent*.