The Problem of Action at a Distance in Networks and the Emergence of Preferential Attachment from Triadic Closure

Jérôme KUNEGIS[*1], Fariba KARIMI[1,2], SUN Jun[1]

[1]University of Koblenz–Landau, Germany
[2]GESIS – Leibniz-Institut für die Sozialwissenschaften, Germany

*Corresponding author: kunegis@uni-koblenz.de

Abstract
In this paper, we characterise the notion of preferential attachment in networks as action at a distance, and argue that it can only be an emergent phenomenon – the actual mechanism by which networks grow always being the closing of triangles. After a review of the concepts of triangle closing and preferential attachment, we present our argument, as well as a simplified model in which preferential attachment can be derived mathematically from triangle closing.

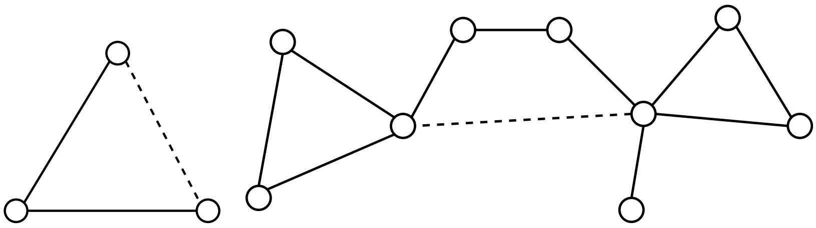
Keywords
Networks, preferential attachment, triangle closing, action at a distance

---

INTRODUCTION

Many natural and man-made phenomena are networks – i.e., ensembles of interconnected entities. To understand such structures is to understand their creation, their evolution and their decay. In fact, many models have been described for the evolution of networks, for the simple reason that such a large amount of systems consist of interconnected parts. Rules for the evolution of networks can be broadly classified into two classes: those postulating local growth, and those postulating global growth. An example for a mechanism of local growth is triangle closing: When two people become friends because they have a common friend, then a new triangle is formed, consisting of three persons. This tendency of networks to form triangles is a natural model not only for social networks, but for almost all types of networked data. For instance, if Alice likes a movie and Bob is friends with Alice, Bob might also come to like that movie. In this case, the triangle consists of two persons and one movie. In general, networks can contain any type of object being connected by many different types of connections, and thus many different types of such triangle closings are possible. We call this type of growth *local* because it only depends on the immediate neighbourhood of the two connected nodes; the rest of the network does not play a role.

On the other hand, there is preferential attachment. When, for instance, two people become friends with each other, not because they have a common friend, or go to the same class, but because they are both popular. Given two very popular persons, i.e. with many friends, it is more likely that they will become friends, than that two unpopular people will become friends, all else being equal. This phenomenon is referred to as preferential attachment. Preferential attachment is an often-used strategy to predict new connections, not only in social networks: a frequent movie-goer is much more likely to watch a popular film, than someone who almost never goes out to the movies watching an obscure film almost nobody knows or has seen. These types of



(a) Triangle closing (b) Preferential attachment

Figure 1: The two network growth mechanisms considered in this article: triangle closing and preferential attachment. In both models, new edges appear (shown as dashed lines), based on the network environment of the current graph. (a) Triangle closing: an edge is more likely to appear between nodes that have common neighbours, (2) Preferential attachment: An edge is more likely to appear between nodes that have high degree.

statements seem obviously true and indeed they are used widely in application systems: recommender systems give a big preference to popular movies, search engines give higher weight to well-connected web pages, and Facebook or Twitter will make a point to show you pictures that already have many likes. In that sense, preferential attachment is true empirically, and has been verified many times in experiments. Then, what is problematic with preferential attachment? Is it not always correct? No, we are not claiming that preferential attachment is wrong. What we argue is that preferential attachment is never a primitive phenomenon, but always a derived phenomenon, emerging as a result of more basic network evolution rules.

So, if preferential attachment is not a fundamental network evolution mechanism, what *makes* a fundamental network mechanism, and which fundamental network evolution rules are then fundamental? We will present in this paper arguments for the thesis that only the principle of triangle closing is fundamental, all forms of preferential attachment being derived from it. To give an argument in favour of our thesis, we will first review basic notions of networks and network evolution models, and then review preferential attachment, proposing various mechanisms by which it can arise from triangle closing, a fundamental notion in the evolution of networks.

NETWORKS

The statement *everything is a network* has become a cliché because it is true. Social networks, knowledge networks, information networks, communication networks – many papers in the field of network science motivate their use by enumerating fields in which they play a central role. Biological networks, molecules, lexical networks, Feynman diagrams – hardly a scientific field exists in which networks do not play a fundamental role. Instead of giving a hopelessly incomplete enumeration of examples, we will simply refer the reader to the introductory section of our Handbook of Network Analysis (Kunegis, 2016), in case she wishes to convince herself of this fact. In case this is not enough, we may point to the existence of entire fields of research incorporating the word *network* that have emerged in the last decade: network science (Börner et al., 2007; Newman, 2010), web science (Hendler et al., 2008) and others (Tiropanis et al., 2015). Then, why *is* everything a network? To find an answer, it is instructive to consider the field of machine learning. Most classical machine learning algorithms deal with datasets consisting of data points, each consisting of the same features. Mathematically, we may model such a dataset as a set of points in a space whose dimensions are the individual features (Salton et al., 1975). This formalism is very powerful, and still constitutes the backbone of many machine learning and data mining methods to this day. The standard formulation of classification, clustering and other learning problems all rely on the set-of-points-in-a-space model. However, not all do. While the set of words contained in text documents are well represented by the *bag of words* model, a social network is not. We may try to represent a social network as a bag of friends, but this representation is very unsatisfactory: each person has a set of friends, but the model does not consider the fact that a person contained in one of these bags is the same person as one *having* a bag of friends. Thus, the vector space model cannot find connections such as "the friend of my friend" – it can only find "a person that has the same friend as me". In other words, the vector space model disconnects the role of *having friends* and that of *being a friend*. Instead, the natural way to represent friendships is as a network. Using a network model, the symmetry of the friend relationship is included automatically in the model, and relationships such as *the friend of my friend* arise as the natural way to create new edges in the network, i.e., triangle closing. In fact, we will argue that this is the only way new edges can be created in a network, and that other models are merely consequences of it, such as preferential attachment.

PREFERENTIAL ATTACHMENT

Preferential attachment, also referred to by the phrase "the rich get richer", or as the Matthew effect, is observed in many social networks. In other words, who has many friends, will get more new friends than who has few. Movies that have been seen by many people will be seen by more people than movies that have not. Websites that have been linked to many times will receive more new links because of this. These statements seem true, and indeed, they are true empirically for many different network types (Kunegis et al., 2013).

In fact, preferential attachment is the basis for a whole class of network models. The most basic of these, the model of Albert-László Barabási and Réka Albert (1999), describes the growth of a network as follow: Start with a small graph, and at each step, add a node, and connect that node to k existing nodes with a probability proportional to the number of neighbours for each existing node. In the limit where many nodes have been added in that way, the network tends to become *scale-free*, i.e. tends to have a distribution of neighbour counts that follow a power law. Since power law degree distributions are observed in many natural networks, the usual conclusion is that preferential attachment is correct.

Preferential attachment is thus undeniably real. Why then, are we arguing against it? The reason is that preferential attachment cannot be a fundamental driving force for tie creation. How are two nodes, completely unconnected from each other, be supposed to choose to connect with each other? How can two completely disconnected nodes even *know* of each others existence? This is a fundamental problem with all nonlocal interactions. For instance, the classical theory of gravitation as defined and used by Isaac Newton (1687) includes nonlocal interactions. In that theory, two masses exert a force on each other, regardless of their position. While the force decreases with distance, it is always nonzero, and instantaneous. The conceptual problem with this type of interaction has been identified even by Newton himself (Hesse, 1955). In modern physics, Newton's formalism is replaced by more precise theories that do not include any action at a distance. The theory of general relativity as defined by Albert Einstein in 1916 for instance, only includes local interaction in the from of the Einstein field equations (Einstein, 1916). Einstein's general relativity is thus free from any problematic *action at a distance*, and has been verified at many experimental scales. This is also true for other types of physical interactions – instead of a force that acts at a distance between matter particles, quantum field theory models *bosons* that connect particles. In fact, such interactions can be represented by Feynman diagrams: graph-like representations of particles in which edges are particles and nodes are interactions – any interacting particles must be connected in one diagram, directly or indirectly. In this light, we may interpret preferential attachment as a theory that is true superficially, but must be explained by an underlying phenomenon. Specifically, an underlying phenomenon that does not rely on action at a distance. As this phenomenon, we propose the known mechanism of triangle closing.

TRIANGLE CLOSING

How do we make new friends? By meeting the friends of our friends. This represents a triangle formed by us, our previous friend and our new friend. What if we meet our new friend in another way – maybe at a party, or a concert, or at work ... in any case there is always *some* element in common. If we meet our new friend at a party, then we are both connected to the party, and by modelling the party as a node in our network, that new friendship is indeed created by the closing of a person–person-party triangle. Of course, we may continue to ask how our connection to the party arose. After all, we did not come to a random party or to a random party with many guests. No – we came to the party because a friend invited us, or for any other reason, as long as there is some connection. This game of connections can be played to any desired degree of precision. Maybe we *really* went from door to door until we found a party with many people. But then, how did we get from door to door? We surely have started somewhere, likely near to our home, and have then gone on to the next door, and to the next door, and so on. In doing this, we have only followed links: We are connected to our home by living there; our home is connected to the neighbouring house, which itself is connected to the next house, and so on. This example is of course exaggerated, but serves to illustrate the principle: in order for a new edge to appear, a path has to exist from one node to another; this can go over node representing any type of entity, and these nodes may be visible or hidden. All in all, there is no escaping the principle of triangle closing. However we arrived at the party, it must have been by a series of triangle closings.

Thus, triangles fulfil our expected as a fundamental mechanism of network growth, as it is purely local. However, we cannot deny the existence of preferential attachment, for which we must now find suitable explanations.

EXPLANATIONS

In recommender systems, such as that used on web sites that recommend movies to watch, preferential attachment is often taken as a solution to the cold start problem. The cold start problem in recommender systems refers to the situation in which a user has not yet entered any information about herself, and thus triangle closing cannot be used to recommend her anything. If the user has watched only a single movie, then we can find similar movies and recommend them. If a user has added only a single friend, then we can take movies liked by that friend and recommend them. But if the user is completely new, as has no friends and no ratings yet, then this strategy will not work. How then, do recommender systems give recommendations to new users? The solution is simple: they recommend the most popular items. If you subscribe to Twitter, you will be recommended popular accounts to follow. If you subscribe to Last.fm, you will be recommended popular music. For these sites, this strategy is better than not recommending anything, and in fact is a form of preferential attachment: Create, or rather recommend, links to nodes with many neighbours. How can we interpret this in terms of triangle closing? If a node has no connections yet, then surely it cannot acquire new nodes by triangle closing. How then will a node ever acquire new edges, if it starts without neighbours? The answer is that a node does not start without any neighbours. Everything is connected. A child when it is born does not start without connections; it is already connected to its parents and to its birthplace. Likewise a user on the Web never starts from scratch: every page has a referrer, and thus the user can be connected to another website. Even if the referring web page is not known, there has to be a referrer. If a user types in a URL by hand, she has to have taken it somewhere: maybe a friend gave it to her, maybe she read it in a magazine, on a billboard, or on a truck ... in all cases, the newly created connection is not created *ex nihilo* – it is created by triangle closing.

The explanation for preferential attachment thus lies in hidden nodes: Nodes that make indirect connections between things, but do not appear in the modelled system. On Facebook for instance, many new friendships are created between people who do not have common friends. These new friendships seemingly appear without the help of triangle closing. However, that is always due to the fact that Facebook does not know everything. Some people are simply not on Facebook, which means that if I meet a new friend through a friend of mine that is not on Facebook and then connect with my new friend via Facebook, then from the point of view of Facebook a new edge was created without triangle closing. But that is only true because Facebook does not know my initial friend. If it did, it could correctly infer the new friendship via triangle closing. Thus, any two nodes in a network can potentially be linked, even if they do not share common neighbours *in the network at hand*, because they may share a hidden common neighbour. The same argumentation applies to hidden nodes that represent non-actors, such as classes, hometowns, parties, etc.

If any edge can be explained by hidden nodes, how can it then be that edges connecting nodes with high degree are observed more often? Imagine a network, for instance a social network. Call this the known network. Then imagine a certain number of nodes outside of that network, that are connected at random to the nodes in the known network. Call these the unknown nodes. How many common neighbours do two members of the known network have outside of the known network? Without knowing the distribution of hidden edges, this question cannot be answered. But consider that triangle closing acts not only on known–unknown–known triangles, but also on known–known–unknown triangles. Starting with an equal probability for all known–unknown edges, performing triangle closing will lead to the creation of known–known–unknown triangles. The newly created known–unknown edges can then be combined with other unknown–known edges to perform, again, triangle closing, leading to new known–known edges. The result are new edges in the observed social network, with a probability proportional to the number of the initial known node's neighbours. Thus, preferential attachment emerges as a necessary consequence of iterated triangle closing, if hidden nodes are admitted. The next section will make this heuristic argument precise.

DERIVATION

This section gives an exemplary derivation of a simplified model that we introduce to illustrate our explanation, in which preferential attachment arises as a consequence of triangle closing in the presence of hidden nodes. The given scenario is very general and may be generalised easily for instance by considering multiple node or edge types. In this model, we distinguish two types of nodes: visible nodes in the set V, and hidden nodes in the set W. We will assume that there is a given, fixed number of visible nodes $|V|$, and a possibly very large number of hidden nodes $|W|$. In particular, we will consider the limit $|W| \to \infty$.

Let $G = (V \cup W, E)$ be a bipartite network in which only the nodes V and their degree are visible, the edges E and the nodes W are not visible. Assuming that two nodes in V connect with a probability proportional the number of common nodes they have. Edge between nodes in V will not be considered, except for their effect on the degree of nodes in V. Likewise, edges between nodes in W need not be considered. Thus, the considered network G is bipartite. We will use the convention that $n = |W|$, and the degree of a node x is denoted by $d(x)$. Seeing only nodes in V and their degree, preferential attachment can be observed in the following way.

In order to make our derivation, we need to make two assumptions:

- The edges of the graph are randomly distributed between possible node pairs.
- The typical degree of nodes are significantly smaller than the number of nodes, i.e., $d(x) \ll n$. This is precise when n goes to infinity.

Let $u, v \in V$ be two nodes of the network. Under the assumption that the edges are distributed randomly in the graph, the probability p that u and v are connected can be derived combinatorically by considering the number of configurations in which the two nodes do not share a common neighbor. Given that u and v have degree $d(u)$ and $d(v)$ respectively, the total number of configurations for the edges connected to the nodes is

$$\binom{n}{d(u)}\binom{n}{d(v)}.$$

Out of those, the number of configurations in which the neighbours of the two nodes are disjoint is given by

$$\binom{n}{d(u)}\binom{n-d(u)}{d(v)}.$$

Thus, the probability that the two nodes share a common neighbour is given by

$$p = 1 - \frac{\binom{n}{d(u)}\binom{n-d(u)}{d(v)}}{\binom{n}{d(u)}\binom{n}{d(v)}} = 1 - \frac{\binom{n-d(u)}{d(v)}}{\binom{n}{d(v)}}.$$

We now use the falling factorial to express binomial coefficients, i.e.,

$$n^{\underline{a}} = n(n-1)(n-2)\cdots(n-a+1).$$

The falling factorial has the property that in the limit where a is constant and n goes to infinity, we have

$$\lim_{n\to\infty} \frac{n^{\underline{a}}}{n^a} = 1$$

and also,

$$\binom{n}{a} = \frac{n^{\underline{a}}}{a!},$$

and thus

$$p = 1 - \frac{(n-d(u))^{\underline{d(v)}} d(v)!}{d(v)!\, n^{\underline{d(v)}}} = \frac{(n-d(u))^{\underline{d(v)}}}{n^{\underline{d(v)}}}$$

with the limit

$$p = 1 - \frac{(n-d(u))^{d(v)}}{n^{d(v)}} = 1 - \left(1 - \frac{d(u)}{n}\right)^{d(v)}$$

and using the limit $n \to \infty$ again:

$$p = \frac{d(u)d(v)}{n}.$$

It thus follows that $p \sim d(u)d(v)$, i.e., the probability of the nodes u and v being connected is proportional to both u and v. Thus, preferential attachment is a consequence of the triangle closing model. Preferential attachment itself then leads to a scale-free degree distribution, as per Barabási and Albert (1999).

RELATED WORK

The debate over the nature of preferential attachment mechanism dates back to the 1960s, when the economist H. Simon defended the role of randomness and the mathematician B. Mandelbrot defended the role of optimization (Barabási, 2012). The concept of preferential attachment is also able to explain the nature of scale-free degree distribution in biological networks such as metabolic networks (Jeong et al., 2000) and protein networks (Jeong et al., 2001). There are various suggestions to explain the nature of preferential attachment for instance by introducing hidden variable models in which nodes possess an intrinsic fitness to other nodes in unipartite (Boguñá and Pastor-Satorras, 2003) or bipartite networks (Kitsak and Krioukov, 2011). In a recent *Nature* paper, Papadopoulos et al. proposed a model based on geometric optimization of homophily space (Papadopoulos et al., 2012). However, in these models, triadic closure is not defined as the main principle for the formation of edges.

Triadic closure, a tendency to connect to friend of a friend (Rapoport, 1953), has been observed undeniably in many social networks such as friendship at university (Kossinets and Watts, 2006), scientific collaboration (Newman, 2001) and World Wide Web (Adamic, 1999). The concept of triadic closure was first suggested by German sociologist Georg Simmel 1950 and later on popularized by Fritz Heider and Mark Granovetter as the theory of cognitive Balance in which if two individuals feel the same way about the an object/person, they seek closure by closing the triad between themselves (Heider, 2013). Since the classic preferential attachment model lacks to explain the number of clusters in many social networks, many attempts have been make to include triadic closure to the model (Holme and Kim, 2002; Vázquez, 2003), in which nodes with certain probability connect based on the principle of triadic closure. These works have shown that the scaling law for the degree distribution and clustering coefficient can be reproduced based on these models (Klimek and Thurner, 2013).

Hence,the scale-free nature of networks and the abundance of triangles beg for a more fundamental explanation. Moreover, the observable part of the systems is not necessarily completely representative for the entire system. Networks are generally multi-layered or multiplex, in which some layers can be hidden or simply not possible to observe (Kivelä et al., 2014). For instance, the creation of a new Facebook tie can be caused by attending the same class, sharing the same hobby or living in a same neighborhood, which is hidden from the observable data. Consequently, these "focal" points contribute to the tie creation known as "focal closure" and need to be considered in modeling realistic networks, as argued by Kossinets and Watts (2006).

DISCUSSION

The status of a mechanism as *fundamental* is not clear cut. When a phenomenon is explained by another, more fundamental phenomenon, then we can consider it as derived. But how can we be sure that a phenomenon is not explained by a more basic phenomenon? What does it mean for a phenomenon to be fundamental? Just as physics cannot declare one theory to be final, network science cannot declare one network growth mechanism to be final. Thus, individual instances of triangle closing can for instance be explained by several layers of triangle closing, just as in physics a direct interaction can be explained by a new mediating particle. In the end however, this applies only to specific instances of triangle closing, as it replaces them with other, more detailed instances of triangle closing. Thus triangle closing *does* play a fundamental role in network models, only that we cannot state which triangle closing is the fundamental one. In the end, the only judge of the validity of a model remains the experiment, and in practice, used models do not have to be fundamental – recommenders and information retrieval systems have had enough success by applying preferential attachment directly.

References

Adamic L. A. (1999). The small world web. In *Int. Conf. on Theory and Practice of Digital Libraries*, pp. 443–452. Springer.

Barabási A.-L. (2012). Network science: Luck or reason. *Nature 489*(7417), 507–508.

Barabási A.-L., Albert R. (1999). Emergence of scaling in random networks. *Science 286*(5439), 509–512.

Boguñá M., Pastor-Satorras R. (2003). Class of correlated random networks with hidden variables. *Phys. Rev. E 68*(3), 036112.

Börner K., Sanyal S., Vespignani A. (2007). Network science. *Annual Rev. of Information Science and Technology 41*(1), 537–607.

Einstein A. (1916). Die Grundlagen der allgemeinen Relativitätstheorie. *Ann. Phys. 49*, 769–822.

Heider F. (2013). *The psychology of interpersonal relations*. Psychology Press.

Hendler J., Shadbolt N., Hall W., Berners-Lee T., Weitzner D. (2008, July). Web science: An interdisciplinary approach to understanding the web. *Commun. ACM 51*(7), 60–69.

Hesse M. B. (1955). Action at a distance in classical physics. *Isis 46*(4), 337–353.

Holme P., Kim B. J. (2002). Growing scale-free networks with tunable clustering. *Phys. Rev. E 65*(2), 026107.

Jeong H., Mason S. P., Barabási A.-L., Oltvai Z. N. (2001). Lethality and centrality in protein networks. *Nature 411*(6833), 41–42.

Jeong H., Tombor B., Albert R., Oltvai Z. N., Barabási A.-L. (2000). The large-scale organization of metabolic networks. *Nature 407*(6804), 651–654.

Kitsak M., Krioukov D. (2011). Hidden variables in bipartite networks. *Phys. Rev. E 84*, 026114.

Kivelä M., Arenas A., Barthelemy M., Gleeson J. P., Moreno Y., Porter M. A. (2014). Multilayer networks. *J. of Complex Networks 2*(3), 203–271.

Klimek P., Thurner S. (2013). Triadic closure dynamics drives scaling laws in social multiplex networks. *New J. of Phys. 15*(6), 063008.

Kossinets G., Watts D. J. (2006). Empirical analysis of an evolving social network. *Science 311*(5757), 88–90.

Kunegis J. (2016). *Handbook of Network Analysis [KONECT – the Koblenz Network Collection].*

Kunegis J., Blattner M., Moser C. (2013). Preferential attachment in online networks: Measurement and explanations. In *Proc. Web Science Conf.*, pp. 205–214.

Newman M. (2010). *Networks: an introduction*. Oxford university press.

Newman M. E. (2001). Clustering and preferential attachment in growing networks. *Phys. Rev. E 64*(2), 025102.

Newton I. (1687). *Philosophiæ Naturalis Principia Mathematica*. First edition.

Papadopoulos F., Kitsak M., Serrano M. A., Boguñá M., Krioukov D. (2012). Popularity versus similarity in growing networks. *Nature 489*(7417), 537–540.

Rapoport A. (1953). Spread of information through a population with socio-structural bias: Ii. various models with partial transitivity. *The Bull. of Math. Biophys. 15*(4), 535–546.

Salton G., Wong A., Yang C. S. (1975, November). A vector space model for automatic indexing. *Commun. ACM 18*(11), 613–620.

Simmel G., Wolff K. H. (1950). *The sociology of georg simmel*, Volume 92892. Simon and Schuster.

Tiropanis T., Hall W., Crowcroft J., Contractor N., Tassiulas L. (2015, July). Network science, web science, and internet science. *Commun. ACM 58*(8), 76–82.

Vázquez A. (2003). Growing network with local rules: Preferential attachment, clustering hierarchy, and degree correlations. *Phys. Rev. E 67*(5), 056104.