

# Action at a Distance in Networks

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## ABSTRACT

In this short paper, we characterise the notion of preferential attachment in networks as action at a distance, and argue that it can only be an emergent phenomenon – the actual mechanism by which networks grow always being the closing of triangles.

## Categories and Subject Descriptors

H.4 [Information Systems Applications]: Miscellaneous

## Keywords

Networks, preferential attachment, triangle closing, action at a distance

## 1. INTRODUCTION

Many natural and man-made phenomena are networks – ensembles of interconnected entities. To understand such structures is to understand their creation, their evolution and their decay. In fact, many models have been described for the evolution of networks, for the simple reason that such a large amount of things are actually networks. Rules for the evolution of networks can be broadly classified into two classes: those postulating local growth, and those postulating global growth. An example for a mechanism of local growth is triangle closing: When two people become friends because they have a common friend, then a new triangle is formed, consisting of three persons. This tendency of networks to form triangles is a natural model not only for social networks, but for almost all types of networked data. For instance, if Alice likes a movie and Bob is friends with Alice, Bob might also come to like that movie. In this case, the triangle consists of two persons and one movie. In general, networks can contain any type of object being connected by many different types of connections, and thus many different types of such triangle closings are possible. We call this type of growth local because it only depends on the immediate neighbourhood of the two connected nodes; the rest of the network does not play a role.

On the other hand, there is preferential attachment. When, for instance, two people become friends with each other, not because they have a common friend, or go to the same class, but because they are both popular. Given two very popular persons, i.e. with many friends, it is more likely that they will become friends, than that two unpopular people will become friends, all else being equal. This phenomenon is referred to as preferential attachment. Preferential attachment is an often-used strategy to predict new connections, not only in social networks: a frequent movie-goer is much more likely to watch a popular film, than someone who almost never goes out to the movies watching an obscure film almost nobody knows or has seen. These types of statements seem obviously true and indeed they are used widely in application systems: recommender systems give a big preference to popular movies, search engines give higher weight to well-connected web pages, and Facebook will make a point to show you pictures that already have many likes. In that sense, preferential attachment is true empirically, and has been verified many times in experiments. Then, what is problematic with preferential attachment? Is it not always correct? No, we are not saying that preferential attachment is wrong. What we argue is that preferential attachment is never a primitive phenomenon, but always a derived phenomenon, emerging as a result of more basic network evolution rules.

So, if preferential attachment is not a fundamental network evolution mechanism, what *makes* a fundamental network mechanism, and which fundamental network evolution exist? We will present in this paper the thesis that only the principle of triangle closing is fundamental, all forms of preferential attachment being derived from it. To give an argument in favour of our thesis, we will first review basic notions of networks and network evolution models, and then review preferential attachment, proposing various mechanisms by which it can arise from triangle closing, a fundamental notion in the evolution of networks.

## 2. NETWORKS

The statement *everything is a network* has become a cliché because it is true. Social networks, knowledge networks, information networks, communication networks, many papers in the field of network science motivate their use by enumerating fields in which they play a central role. Biological networks, molecules, lexical networks, Feynman diagrams, hardly a scientific field exists in which networks do not play a fundamental role. Instead of giving a hopelessly incomplete

enumeration of examples, we will simply refer the reader to the introductory section of our Handbook of Network Analysis [4], in case she wishes to convince herself of this fact. In case this is not enough, we may just have a look at entire fields of research incorporating the word *network* or one of its many synonyms: network science and web science.

Then, why *is* everything a network? To find an answer, we must go back in history, to a time when computers were new, “artificial intelligence” was a developing field, and scientists were coming up with many basic algorithms that are now part of the machine learning canon. Most of these classical machine learning algorithms deal with datasets consisting of data points, each consisting of the same features. Mathematically, we may model such a dataset as a set of points in a space whose dimensions are the individual features. This formalism is very powerful, and still constitutes the backbone of many machine learning and data mining methods to this day. The standard formulation of classification, clustering and other learning problems all rely on the set-of-points-in-a-space model. However, not all do. While the set of words contained in text documents are well represented by the *bag of words* model, a social network is not. We may try to represent a social network as a *bag of friends*, but this representation is very unsatisfactory: each person has a set of friends, but the model does not consider the fact that a person contained in one of these bags is the same person as one *having* a bag of friends. Thus, the vector space model cannot find connections such as “the friend of my friend” – it can only find “a person that has the same friend as me”. In other words, the vector space model disconnects the role of *having friends* and that of *being a friend*. Instead, the natural way to represent friendships is as a network. Using a network model, the symmetry of the friend relationship is included automatically in the model, and relationships such as *the friend of my friend* arise as the natural way to create new edges in the network. In fact, we will argue that this is the only way new edges can be created in a network, and that other models are merely consequences of it, such as preferential attachment.

### 3. PREFERENTIAL ATTACHMENT

Preferential attachment is a powerful method. Preferential attachment is illustrated by the phrase “the rich get richer”. In other words, who has many friends, will get more new friends than who has few. Movies that have been seen by many people will be seen by more people than movies that have not. Websites that have been linked to many times will receive more new links because of this. These statements seem true, and indeed, they are true empirically for many different network types [5].

In fact, preferential attachment is the basis for a whole class of network models. The most basic of these, the model of Albert-László Barabási and Réka Albert [1], describes the growth of a network as follow: Start with a small graph, and at each step, add a node, and connect that node to  $k$  existing nodes with a probability proportional to the number of neighbours for each existing node. In the limit where many nodes have been added in that way, the network tends to become *scale-free*, i.e. tends to have a distribution of neighbour counts that follow a power law. Since power law degree distributions are observed on many natural networks, the usual

conclusion is that preferential attachment is correct.

Preferential attachment is thus undeniably real. Why then, are we arguing against it? The reason is that as a fundamental driving force, it is flawed. How are two nodes, completely unconnected from each other, be supposed to choose to connect with each other? How can two completely disconnected nodes even *know* of each others existence? This is a fundamental problem with all nonlocal interactions. For instance, the classical theory of gravitation as defined and used by Isaac Newton includes nonlocal interactions [6]. In that theory, two masses exert a force on each other, regardless of their position. While the force decreases with distance, it is always nonzero, and instantaneous. The conceptual problem with this type of interaction has been identified even by Isaac Newton himself [3]. In modern physics, Newton’s formalism is replaced by more precise theories that do not include any action at a distance. The theory of general relativity as defined by Albert Einstein in 1916 for instance, only includes local interaction in the form of the Einstein field equations [2]. Einstein’s general relativity is thus free from any problematic *action at a distance*, and has been verified at all experimental scales. This is also true for other types of physical interactions – instead of a force that acts at a distance between matter particles, quantum field theory models *bosons* that connect particles. In fact, such interactions can be represented by Feynman diagrams: graph-like representations of particles in which edges are particles and nodes are interactions – any interacting particles must be connected in one diagram, directly or indirectly. In this light, we may interpret preferential attachment as a theory that is true superficially, but must be explained by an underlying phenomenon. Specifically, an underlying phenomenon that does not rely on action at a distance. As this phenomenon, we propose the known mechanism of triangle closing.

### 4. TRIANGLE CLOSING

How do we make new friends? By meeting the friends of our friends. This represents a triangle made out of us, our previous friend and our new friend. What if we meet our new friend in another way – maybe at a party, or a concert, or at work ... in any case there is always *some* element in common. If we meet our new friend at a party, then we are both connected to the party, and by modelling the party as a node in our network, that new friendship is indeed created by triangle closing. Of course, we may continue to ask how our connection to the party arose. After all, we did not come to a random party or to a random party with many guests. No – we came to the party because a friend invited us, or for any other reason, as long as there is some connection. This game of connections can be played to any desired degree of precision. Maybe we *really* went from door to door until we found a party with many people. But then, how did we get from door to door? We surely have started somewhere, likely near to our home, and have then gone on to the next door, and to the next door, and so on. In doing this, we have only followed links: We are connected to our home by living there; our home is connected to the neighbouring house, which itself is connected to the next house, and so on. All in all, there is no escaping the principle of triangle closing. However we arrived at the party, it must have been by (a series of) triangle closing.

Although triangle closing is the fundamental mechanism by which networks grow, we cannot deny the existence of preferential attachment, for which we must now find suitable explanations.

## 5. EXPLANATIONS

In recommender systems, for instance in systems that recommend movies to watch, preferential attachment is often taken as a solution to the cold start problem. The cold start problem in recommender systems refers to the situation in which a user has not yet entered any information about herself, and thus triangle closing cannot be used to recommend her anything. If the user has watched only a single movie, then we can find similar movies and recommend them. If a user has added only a single friend, then we can take movies liked by that friend and recommend them. But if the user is completely new, as has no friends and no ratings yet, then this strategy will not work. How then, do recommender systems give recommendations to new users? The solution is simple: they recommend the most popular items. If you subscribe to Twitter, you will be recommended popular accounts to follow. If you subscribe to Last.fm, you will be recommended popular music. For these sites, this strategy is better than not recommending anything, and in fact is a form of preferential attachment: Create, or rather recommend, links to nodes with many neighbours. How can we interpret this in terms of triangle closing? If a node has no connections yet, then surely it cannot acquire new nodes by triangle closing. How then will a node ever acquire new edges, if it starts without neighbours? The answer is that a node does not start without any neighbours. Everything is connected. A child when it is born does not start without connections; it is already connected to its parents and to its birthplace. Likewise a user on the Web never starts from scratch: every page has a referrer, and thus the user can be connected to another website. Even if the referring web page is not known, there has to be a referrer. If a user types in a URL by hand, she has to have taken it somewhere: maybe a friend gave it to her, maybe she read it in a magazine, on a billboard or on a truck ... in all cases, the newly created connection is not created *ex nihilo* – it is created by triangle closing.

The explanation for preferential attachment thus lies in hidden nodes: Nodes that make indirect connections between things, but do not appear in the modelled system. On Facebook for instance, many new friendships are created between people who do not have common friends. These new friendships seemingly appear without the help of triangle closing. However, that is always due to the fact that Facebook does not know everything. Some people are simply not on Facebook, which means that if I meet a new friend through a friend of mine that is not on Facebook and then connect with my new friend via Facebook, then from the point of view of Facebook a new edge was created without triangle closing. But that is only true because Facebook does not know my initial friend. If it did, it could correctly infer the new friendship via triangle closing. Thus, any two nodes in a network can potentially be linked, even if they do not share common neighbours *in the network at hand*, because they may share hidden common neighbours.

If any edge can be explained by hidden nodes, how can it

then be that edges connecting nodes with high degree are observed more often? Imagine a network, for instance a social network. Call this the known network. Then imagine a certain number of nodes outside of that network, that are connected at random to the nodes in the known network. Call these the unknown nodes. How many common neighbours do two members of the known network have outside of the known network? Without knowing the distribution of hidden edges, this question cannot be answered. But consider that triangle closing acts not only on known–unknown–known triangles, but also on known–known–unknown triangles. Starting with an equal likelihood for all known–unknown edges, performing triangle closing will lead to the creation of known–known–unknown triangles. The newly created known–unknown edges can then be combined with other unknown–known edges to perform, again, triangle closing, leading to new known–known edges. The result are new edges in the observed social network, with a probability proportional to the number of the initial known node's neighbours. Thus, preferential attachment emerges as a necessary consequence of iterated triangle closing, if hidden nodes are admitted.

## 6. DISCUSSION

The status of a mechanism as *fundamental* is not clear cut. When a phenomenon is explained by another, more fundamental phenomenon, then we can consider it as derived. But how can we be sure that a phenomenon is not explained by a more basic phenomenon? What does it mean for a phenomenon to be fundamental? Just as physics cannot declare one theory to be final, network science cannot declare one network growth mechanism to be final. Thus, individual instances of triangle closing can for instance be explained by several layers of triangle closing, just as in physics a direct interaction can be explained by a new mediating particle. In the end however, this applies only to specific instances of triangle closing, as it replaces them with other, more detailed instances of triangle closing. Thus triangle closing *does* play a fundamental role in network models, only that we cannot state which triangle closing is the fundamental one. In the end, the only judge of the validity of a model remains the experiment, and in practice, used models do not have to be fundamental – recommenders and information retrieval systems have had enough success by applying preferential attachment directly.

## 7. REFERENCES

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