

# Spectral Analysis of Signed Graphs for Clustering, Prediction and Visualization

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## Summary

The **Laplacian matrix** applies to **signed graphs**

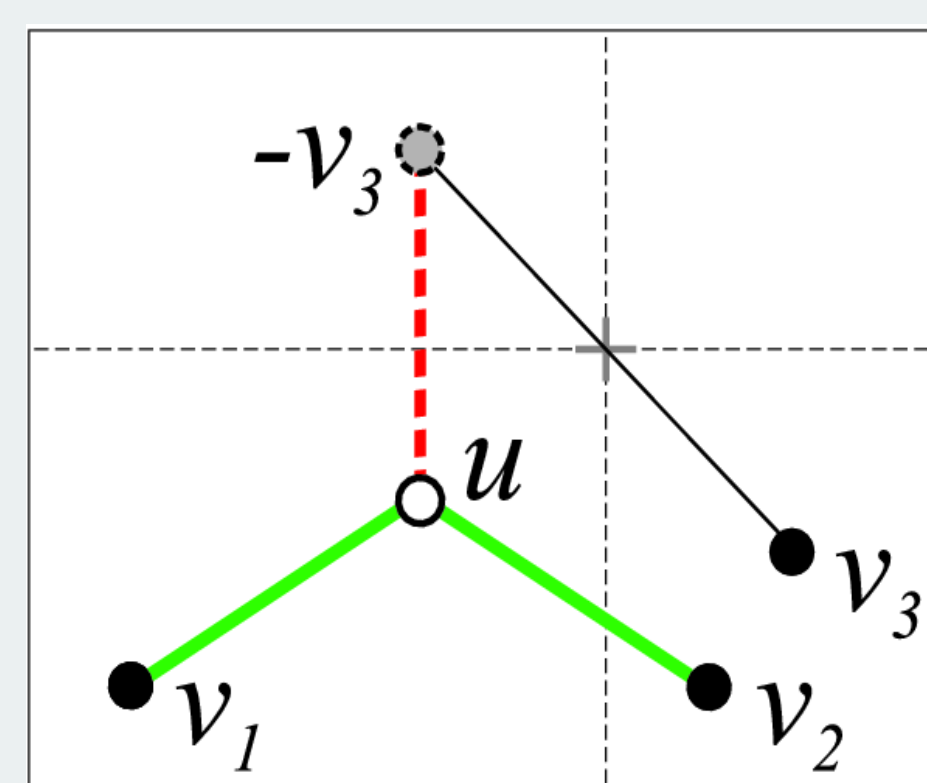
- The Laplacian spectrum denotes **graph balance**
- The Laplacian implements **antipodal proximity**
- The Laplacian implements **signed cuts**
- The Laplacian models negation as **inversion of electrical potential**

## Signed Graph Drawing

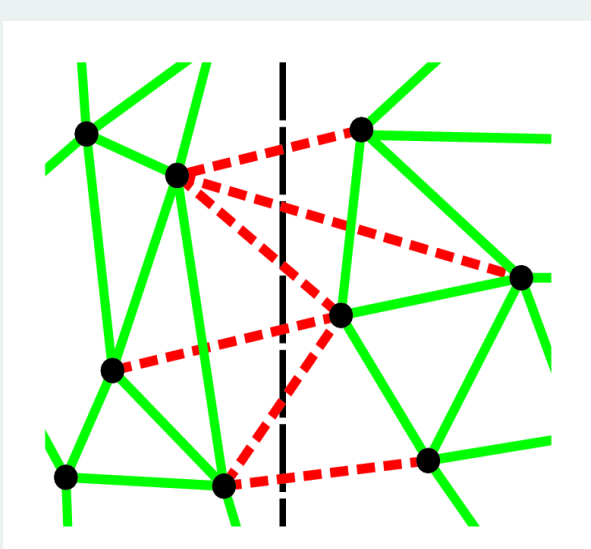
- Place node near positive neighbors
- Place node far from negative neighbors

$$u = (1/3) (v_1 + v_2 - v_3)$$

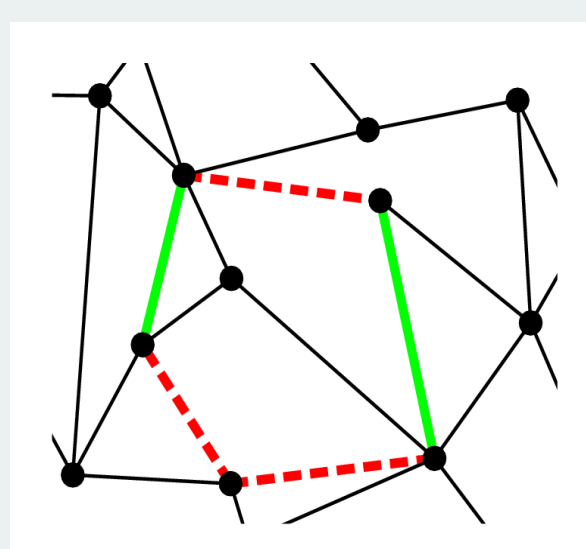
- Leads to lower eigenvectors of signed Laplacian  
 $L = D - A$



## Balance and Conflict



Balance

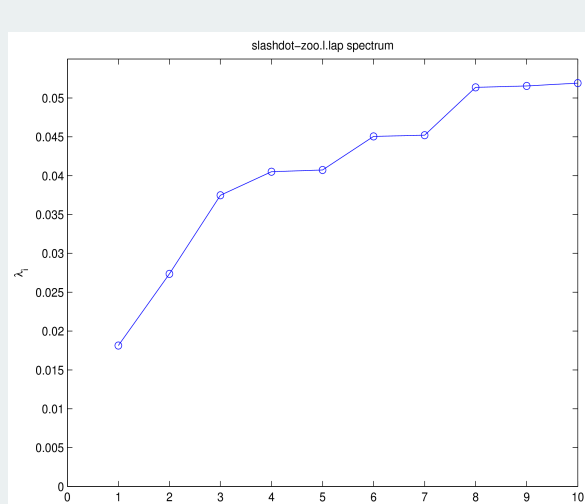


Conflict

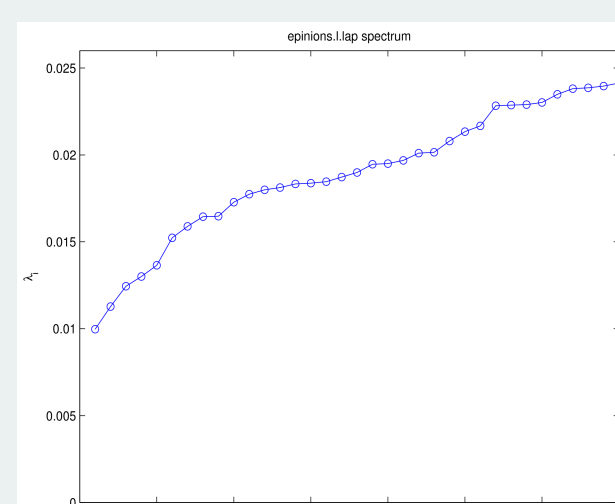
A network is balanced when:

- There is a 2-clustering consistent with edge signs
- All cycles have an odd number of negative edges

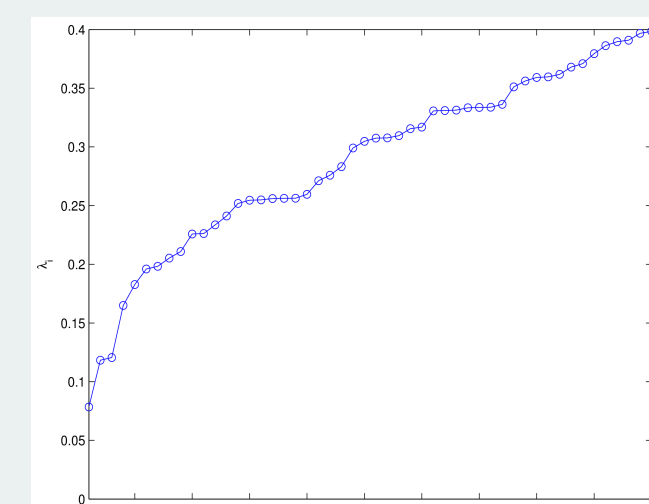
## Signed Spectrum of Large Networks



Slashdot Zoo

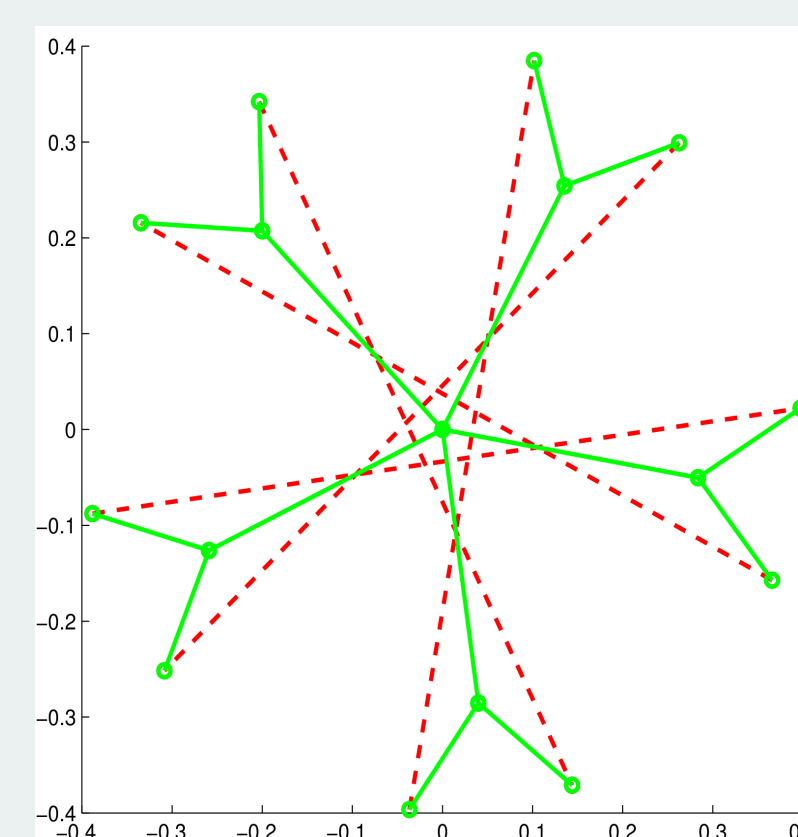


Epinions

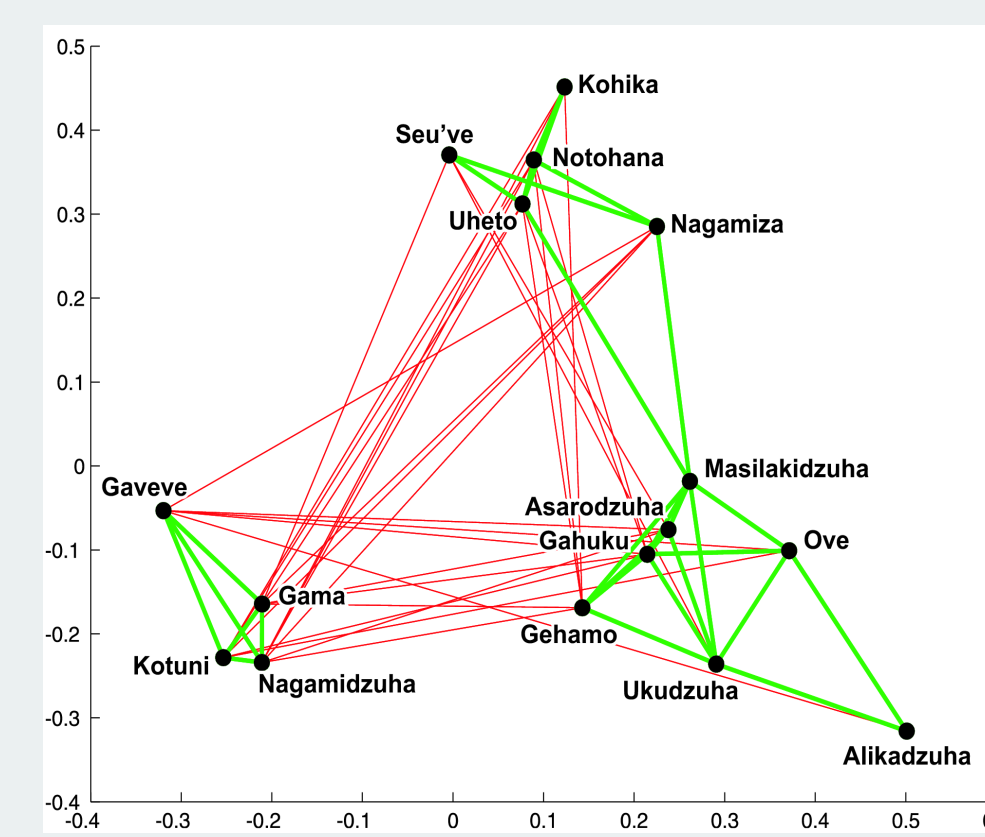


MovieLens

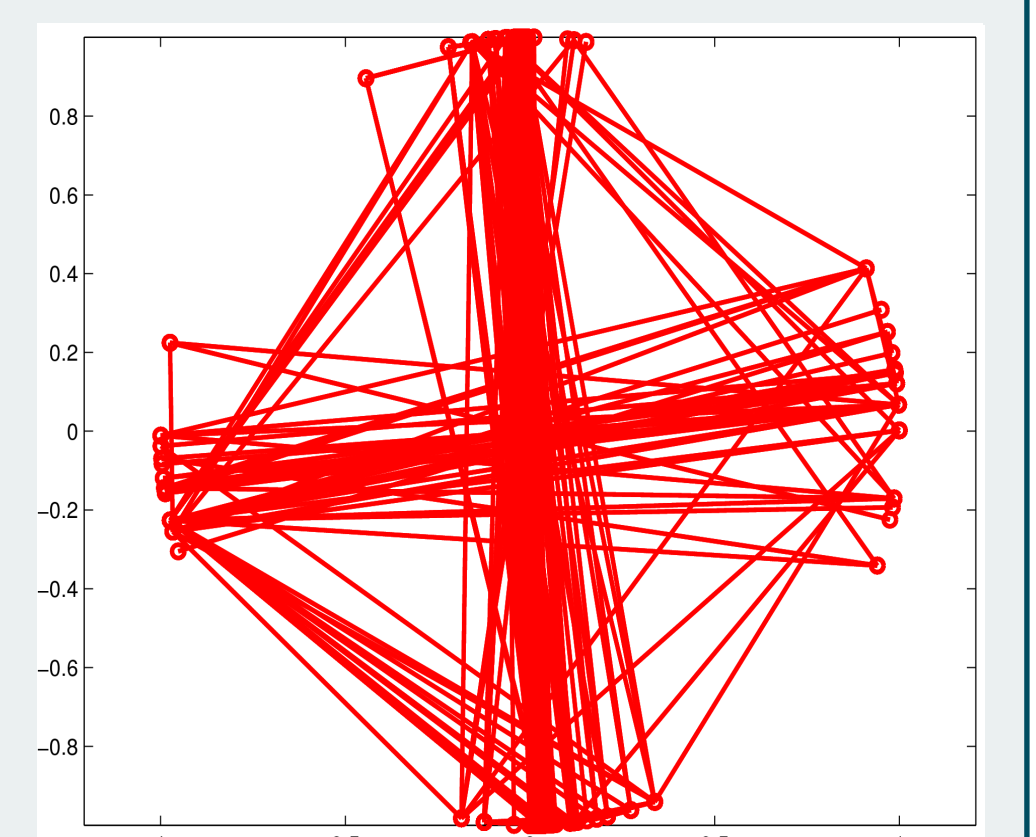
## Examples – Signed Graphs



Synthetic example



Tribal groups of the Eastern Central Highlands of New Guinea  
Friends ('Rova') and Foes ('Hina')



Wikipedia reverts on controversial article  
'Criticism of Prem Rawat'

## The Signed Graph Laplacian

$$A = \{0, +1, -1\}^{n \times n}$$

$$D_{ii} = \sum_j |A_{ij}|$$

$$L = D - A$$

Adjacency matrix

Degree matrix

Signed Laplacian

- Positive semidefinite:  $x^T L x = \sum_{ij} |A_{ij}| (x_i - \text{sgn}(A_{ij}) x_j)^2 \geq 0$
- Positive definite when the network is unbalanced
- Smallest eigenvalue denotes conflict: It is zero when the network is balanced and larger when there is conflict

## Signed Spectral Clustering

- Communities in signed graphs:
  - Positive edges inside communities
  - Negative edges between communities

- Minimize the signed ratio cut:

$$\min (2 \text{ pos}(X, Y) - \text{neg}(X, X) - \text{neg}(Y, Y)) (|X|^{-1} + |Y|^{-1})$$

- $\text{pos}(X, Y)$  counts positive edges between X and Y
- $\text{neg}(X, Y)$  counts negative edges between X and Y
- Relaxation gives the lower eigenvalues of L