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The Link Prediction Problem in Bipartite Networks

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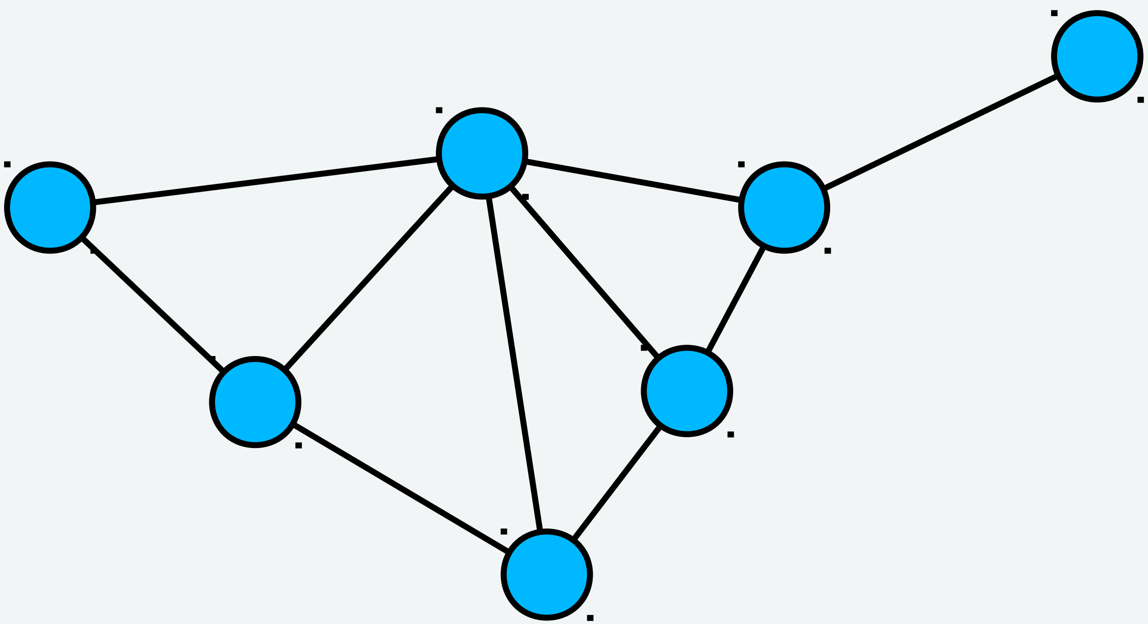
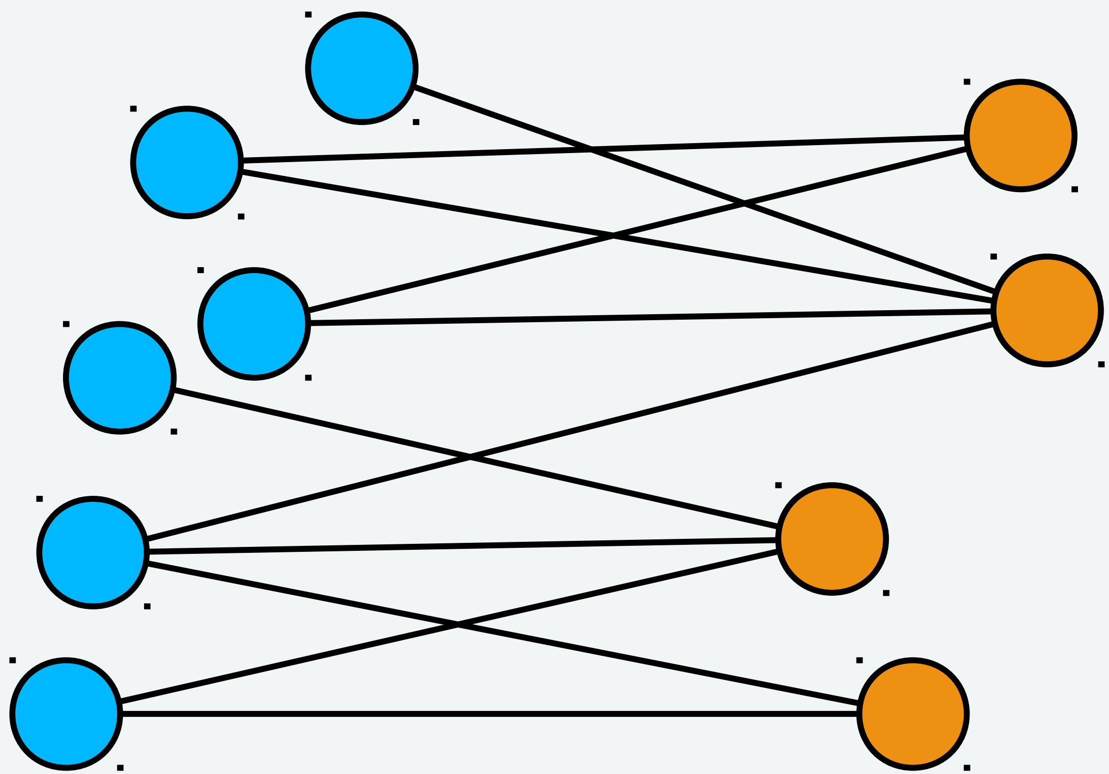
Introduction -- Networks

- Networks as a model of data
- E.g.: social networks, interactions, etc.



Bipartite Networks

Some networks are bipartite



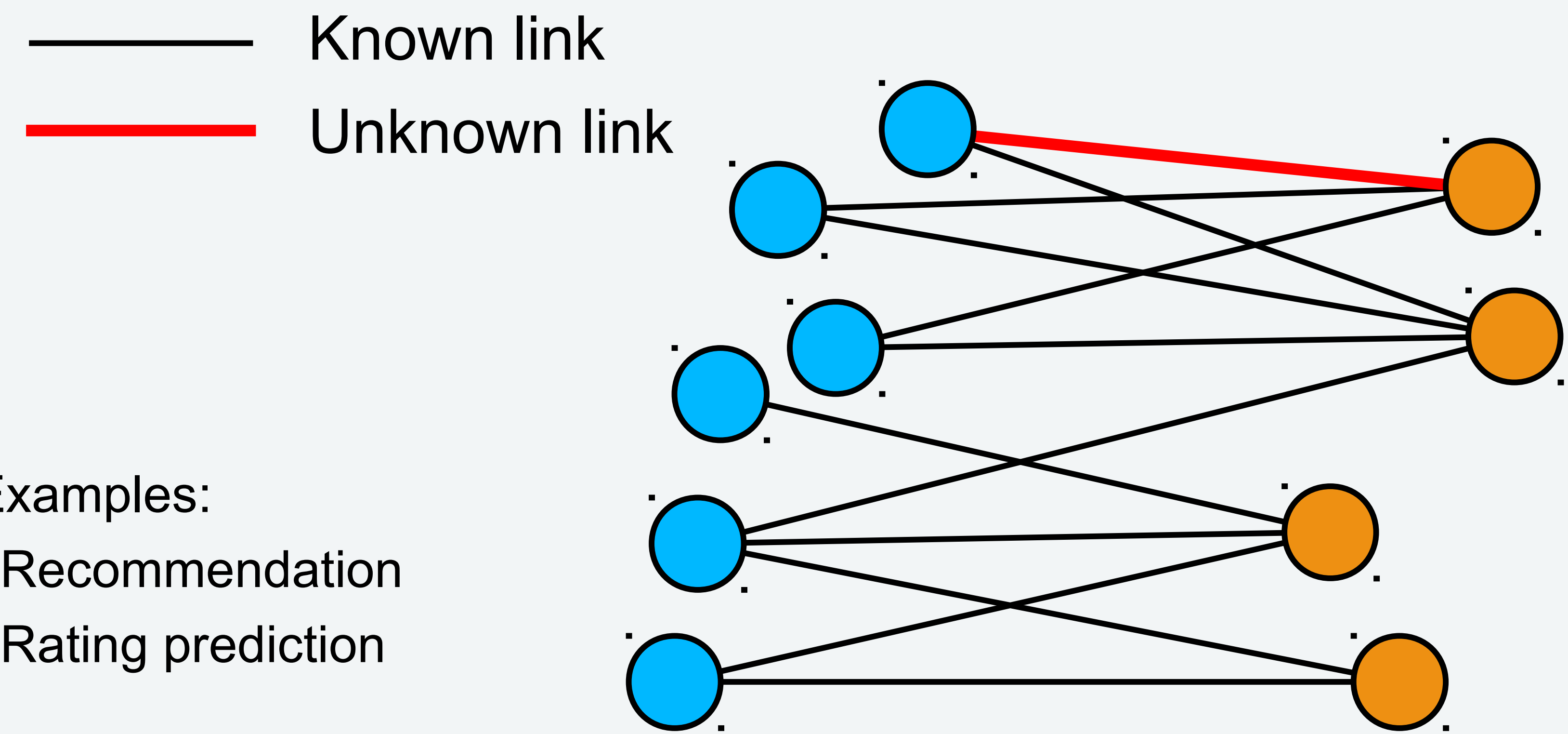
Bipartite Networks

Examples:

- Authorship (user-document)
- Ratings (user-item)
- Interaction (e.g. user-service)
- Membership (e.g. user-group)
- Taxonomies (document-category)
- Text (document-word)

Link Prediction in Bipartite Networks

Task: predict links in the network



Examples:

Recommendation

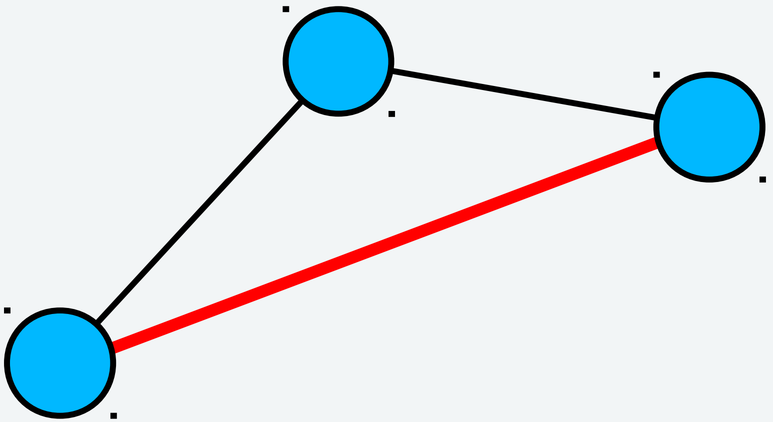
Rating prediction

Outline

- Introduction
- Local Link Prediction
- Graph Kernels
- Bipartite Pseudokernels
- Experiments

Local Link Prediction Methods

Triangle closing



Method: count common neighbors, Jaccard coeff., cosine, Pearson, Adamic/Adar, …

Does not work in bipartite networks!

We need more flexible link prediction methods.

Graph Kernels

Graph kernels are defined algebraically

$n \times n$ adjacency matrix $\mathbf{A}$: $\mathbf{A}_{ij} = 1$ when (i,j) is an edge, 0 otherwise

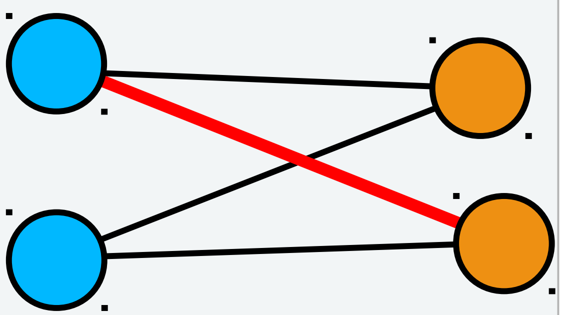
Graph kernel: function $F(\mathbf{A})$ that is positive semidefinite

Exponential and Hyperbolic Sine

Example: matrix exponential

$$\exp(\mathbf{A}) = \mathbf{I} + \mathbf{A} + \tfrac{1}{2}\,\mathbf{A}^2 + \tfrac{1}{6}\,\mathbf{A}^3 + \ldots$$

Each power $\mathbf{A}^n$ denotes paths of length n.



Use only paths of odd lengths:

$$\mathbf{A} + \tfrac{1}{6}\,\mathbf{A}^3 + {}^{1}/_{120}\,\mathbf{A}^5 + \ldots = \sinh(\mathbf{A})$$

Instead of the exponential we need the **hyperbolic sine**

The Bipartite von Neumann Pseudokernel

The Von Neumann kernel:

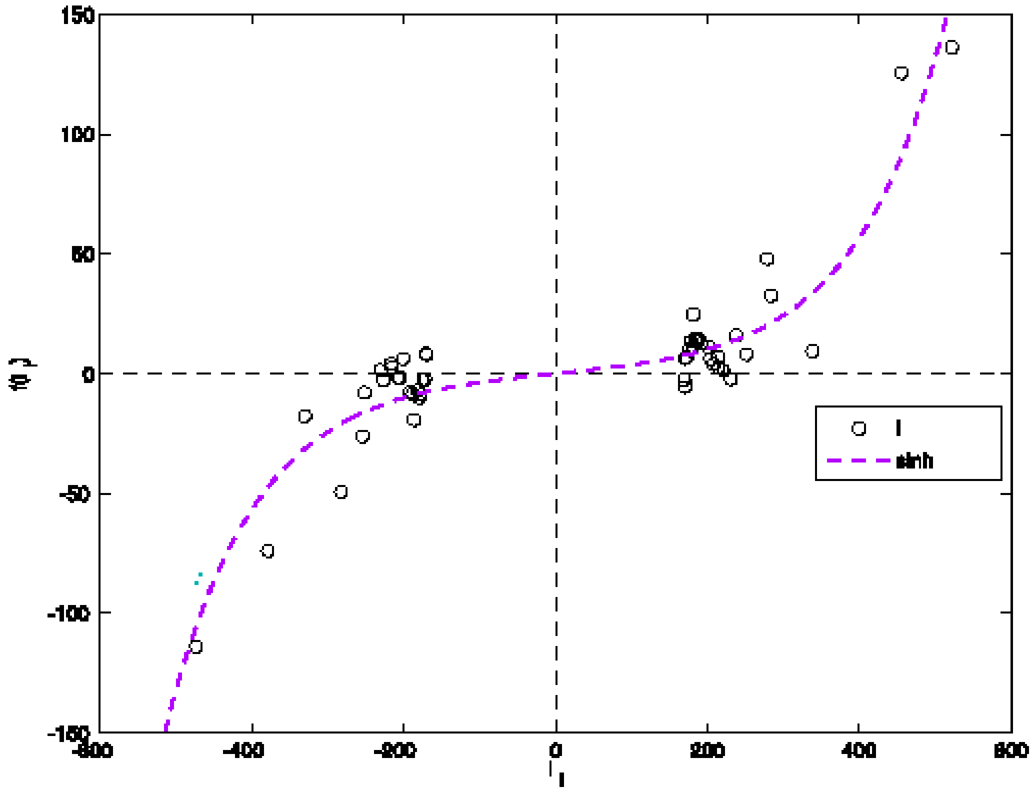
$$N(\mathbf{A}) = (\mathbf{I} - \alpha\mathbf{A})^{-1} = \mathbf{I} + \alpha\mathbf{A} + \alpha^2\mathbf{A}^2 + \ldots$$

The corresponding bipartite von Neumann kernel:

$$N_{bip}(\mathbf{A}) = \alpha\mathbf{A}(\mathbf{I} - \alpha^2\mathbf{A}^2)^{-1} = \alpha\mathbf{A} + \alpha^3\mathbf{A}^3 + \alpha^5\mathbf{A}^5 + \ldots$$

Note: We need $\alpha^{-1} > \lambda_1$, where λ_1 is the network's spectral radius.

Hyperbolic Sine -- Example



Computation – Eigenvalue Decomposition

Graph kernels can be computed with the eigenvalue decomposition:

$$\mathbf{A} = \mathbf{U}\, \mathbf{\Lambda}\, \mathbf{U}^{\mathrm{T}}$$

$$\exp(\alpha\mathbf{A}) = \mathbf{U} \exp(\alpha\mathbf{\Lambda})\, \mathbf{U}^{\mathrm{T}}$$

$$(\mathbf{I} - \alpha\mathbf{A})^{-1} = \mathbf{U}\, (\mathbf{I} - \alpha\mathbf{\Lambda})^{-1}\, \mathbf{U}^{\mathrm{T}}$$

Computation

Adjacency matrix has block structure $\mathbf{A} = [0\ \mathbf{B};\ \mathbf{B}^T\ 0]$, where $\mathbf{B}$ is the biadjacency matrix.

It is enough to compute the singular value decomposition of $\mathbf{B}$.

$$\mathbf{B} = \mathbf{V}\ \mathbf{\Sigma}\ \mathbf{W}^T$$

Where $F(\mathbf{B}) = \mathbf{V}\ F(\mathbf{\Sigma})\ \mathbf{W}^T$ when F is an odd function.

Note: $\mathbf{\Sigma}$ corresponds to $\mathbf{\Lambda}$ up to sign

Note: $[\mathbf{V}\ \mathbf{V};\mathbf{W}\ -\mathbf{W}]\cdot 2^{-1/2}$ equals $\mathbf{U}$

Experiments

- Task: Predict links in bipartite networks
- Networks: collection of large bipartite graphs, weighted and unweighted
- Retain a 30% test set of edges
- Performance measure: mean average precision (MAP)

  1 = perfect prediction
- Learn parameters by the method of (Kunegis 2009)
- Algorithms:
  - Odd polynomial
  - Odd nonnegative polynomial
  - Hyperblic sine
  - Von Neumann pseudokernel
  - Rank reduction
  - Preferential attachment

Experiments -- Results

Dataset	Nodes	Edges	Poly.	NN-poly.	Sinh	Red.	Odd Neu.	Pref.
BibSonomy tag-item	975,963	2,555,080	0.921	**0.925**	**0.925**	0.782	0.917	0.924
BibSonomy user-item	777,084	2,555,080	0.748	0.771	0.771	0.645	0.750	**0.821**
BibSonomy user-tag	210,467	2,555,080	0.801	0.820	0.820	0.777	0.295	**0.878**
CiteULike tag-item	885,046	2,411,819	0.593	0.608	0.608	0.510	0.635	**0.698**
CiteULike user-item	754,484	2,411,819	0.853	**0.856**	**0.856**	0.735	0.855	0.838
CiteULike user-tag	175,992	2,411,819	0.812	0.836	0.836	0.782	0.202	**0.881**
DBpedia artist-genre	47,293	94,861	0.824	**0.971**	0.833	0.736	0.841	0.961
DBpedia birthplace	191,652	273,695	0.952	0.977	**0.978**	0.733	0.813	0.968
DBpedia football club	41,846	131,084	**0.685**	0.678	0.674	0.505	0.159	0.680
DBpedia starring	83,252	141,942	0.908	0.916	**0.924**	0.731	0.570	0.897
DBpedia work-genre	156,145	222,517	0.879	0.941	0.908	0.746	0.867	**0.966**
Epinions	876,252	13,668,320	0.644	**0.690**	0.546	0.501	0.061	**0.690**
French Wikipedia	3,989,678	41,392,490	0.667	0.744	0.744	0.654	0.108	**0.803**
German Wikipedia	3,357,353	51,830,110	0.673	0.699	0.699	0.651	0.156	**0.799**
Japanese Wikipedia	1,892,869	18,270,562	0.740	0.752	0.755	0.618	0.076	**0.776**
Jester	25,038	616,912	0.575	0.571	**0.581**	0.461	0.579	0.501
MovieLens 100k	2,625	100,000	**0.822**	0.774	0.738	0.718	0.631	0.812
MovieLens 10M	136,700	10,000,054	**0.683**	0.682	0.663	0.500	0.298	0.680
MovieLens 1M	9,746	1,000,209	0.640	**0.662**	0.538	0.500	0.221	**0.662**
MovieLens tag-item	24,129	95,580	0.860	0.860	0.860	0.737	**0.865**	0.863
MovieLens user-item	11,610	95,580	0.755	0.741	0.728	0.659	0.674	**0.812**
MovieLens user-tag	20,537	95,580	0.782	0.798	0.798	0.672	0.663	**0.915**
Netflix	497,959	100,480,507	**0.674**	0.671	0.670	0.500	0.322	0.672
Spanish Wikipedia	2,684,231	23,392,353	0.634	0.750	0.750	0.655	0.094	**0.799**
Wikipedia categories	2,036,440	3,795,796	0.591	0.659	0.663	0.500	0.589	**0.675**

Conclusion

Summary:

- Simple link prediction does not work in bipartite networks
- Instead, use **algebraic methods**
- **Hyperbolic sine** and von **Neumann pseudokernels**

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Thank You

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