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Modeling Collaborative Similarity with the Signed Resistance Distance Kernel

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Information Retrieval
& Machine Learning

Outline

Abstract: *We want to apply the resistance distance to collaborative filtering. However, collaborative datasets contain negative edges, which are not supported by the resistance distance. Therefore, we introduce the* ***signed resistance distance****.*

1. Collaborative Filtering
2. Resistance Distance
3. Signed Resistance Distance
4. Evaluation
5. Conclusion

1. Collaborative Filtering – Definition

Users rate items

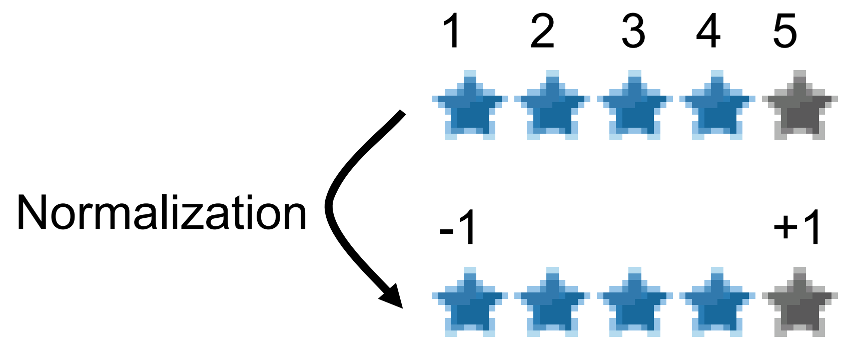
- Task: Recommend items to users
- Task: Find similar users
- Task: Find similar items

…using only ratings, not the content

Examples: Amazon, Netflix, …

1. Collaborative Filtering – Rating Matrix

Ratings as a matrix



A =

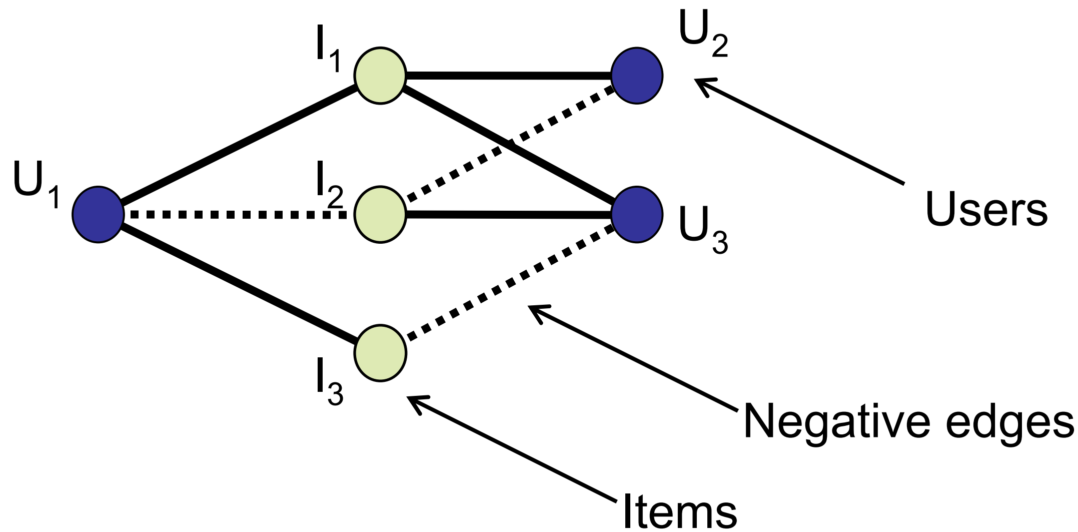
Items\users	U_1	U_2	U_3	U_4	U_5
I_1	+1	+1	+1		+1
I_2	-1	-½		-1	+1
I_3		+1	-1	+½	

- Rating matrix is sparse
- Ratings are signed: positive and negative values

1. Collaborative Filtering – Bipartite Rating Graph

Ratings as a graph

I\U	U_1	U_2	U_3
I_1	+1	+1	+1
I_2	-1	-1	+1
I_3	+1		-1



- Rating graph is bipartite
- Rating graph has edges weighted by signed values

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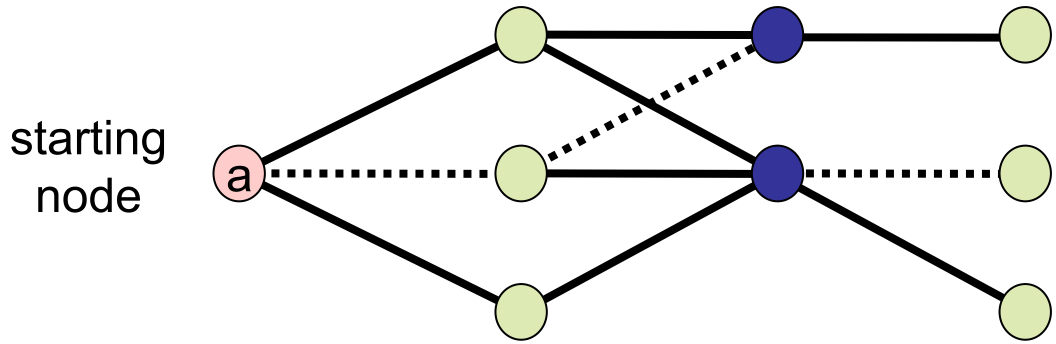
1. Collaborative Filtering – Motivation

We want to recommend users or items



•Recommendation algorithm: Find nearest nodes to *a*

•We need a distance function that takes into account negative edges

Can we define a "signed distance" ?

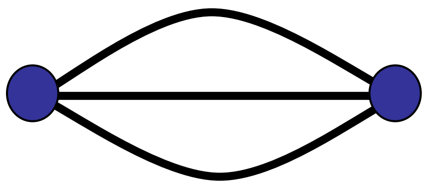
1. Collaborative Filtering – Three Requirements

Three requirements for a “signed” distance function:

1. Long paths separate more

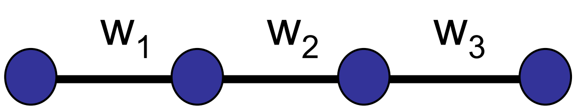
$d > 1$

2. Parallel paths separate less



$d < 1$

3. Sign multiplication rule



$\mathrm{sgn}(d) = \mathrm{sgn}(w_1)\ \mathrm{sgn}(w_2)\ \mathrm{sgn}(w_3)$

“The foe of a foe is a friend” corresponds to $-1 \times -1 = +1$

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1. Collaborative Filtering – Example

- Long paths:

$$\mathrm{dist}(c, d) < \mathrm{dist}(c, e)$$

- Parallel paths:

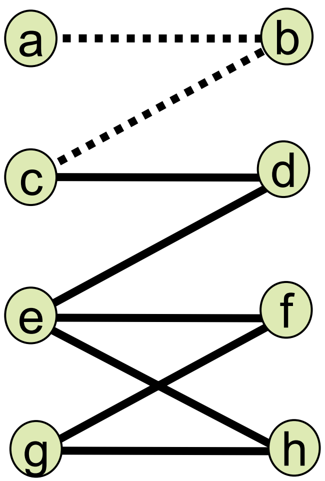
$$\mathrm{dist}(c, e) > \mathrm{dist}(e, g)$$

- Sign rule:

$$\mathrm{dist}(a, b) < 0$$

$$\mathrm{dist}(a, c) > 0$$

$$\mathrm{dist}(a, d) > 0$$



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1. Collaborative Filtering – Scalar Product

The (inverted) scalar product between rating vectors fulfills two requirements:

- Parallel paths:

$$a \cdot b > b \cdot c$$

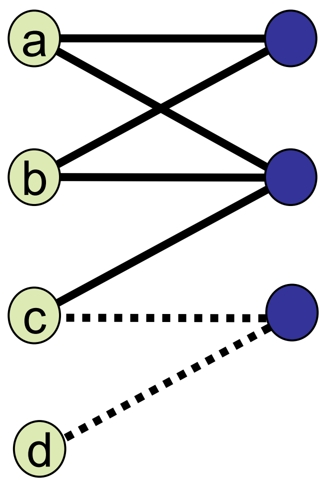
- Sign rule:

$$b \cdot c > 0,\ c \cdot d < 0$$

- Long paths do not work, however:

$$b \cdot d = 0$$

The scalar product is the basis for Pearson correlation and cosine distance.

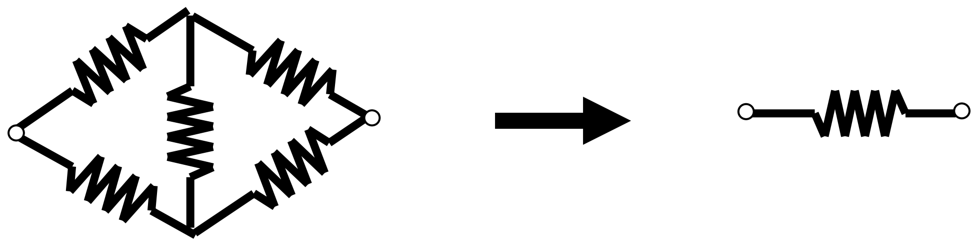


I\U	a	b	c	d
I_1	+1	+1		
I_2	+1	+1	+1	
I_3			-1	-1

column vector

2. Resistance Distance – Definition

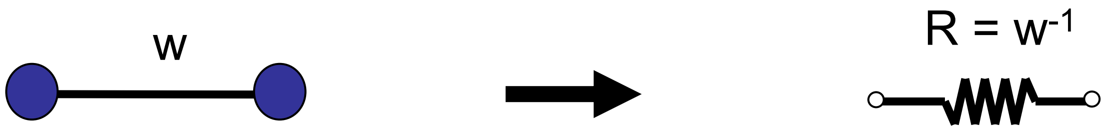
The total electrical resistance induced by a network of resistors measured at two nodes



Taken over all node pairs, defines a distance function called the **resistance distance** (or electric metric)

2. Resistance Distance – Collaborative Filtering

- Large edge weights should lead to small distances
- Large resistances correspond to large distances



We model the resistance as the inverted rating.

(Unknown ratings map to infinite resistances, which are equivalent to missing resistances.)

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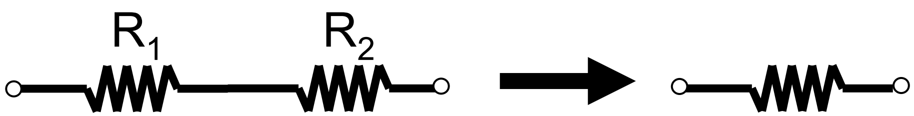
2. Resistance Distance – Properties

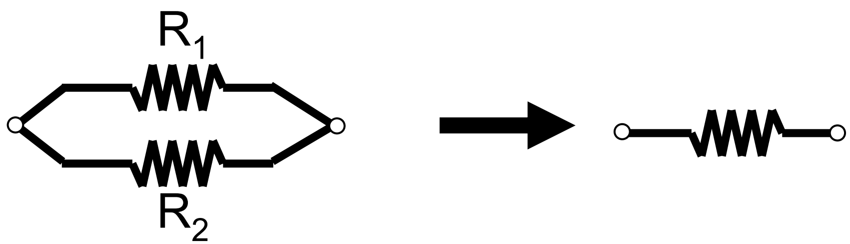
Fulfills the first two requirements:

- Series rule:



$R_{eq} = R_1 + R_2$

- Parallel rule:



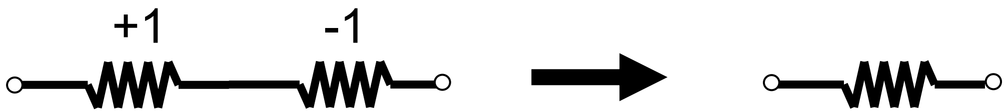
$R_{eq}^{-1} = R_1^{-1} + R_2^{-1}$

What about negative edges?

2. Resistance Distance – Negative Resistance?

Electrical resistances are non-negative.

We try to use negative resistance values anyway:



$(+1) + (-1) = 0$

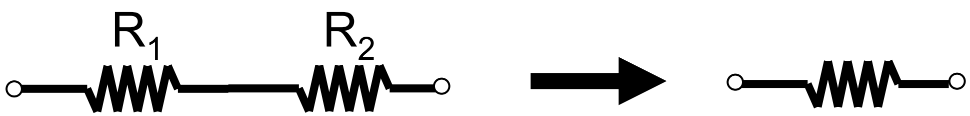
According to the long path and multiplication requirements, we want $R_{eq} < -1$.

Negative resistances don't work!

Really?

3. Signed Resistance Distance – Idea

We separate sign from magnitude in the series rule:

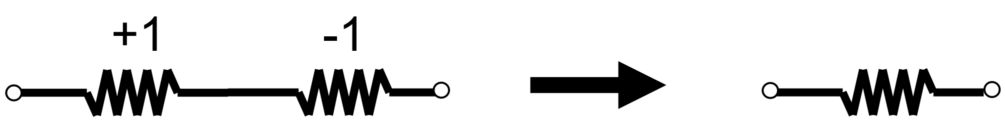


$|R_{eq}| = |R_1| + |R_2|$
$\mathrm{sng}(R_{eq}) = \mathrm{sgn}(R_1)\ \mathrm{sgn}(R_2)$

The signed series rule: $R_{eq}^{-1} = (R_1^{-1}\ R_2^{-1}) / (|R_1^{-1}| + |R_2^{-1}|)$

Note that the series rule can be written as:

$$R_{eq}^{-1} = (R_1^{-1}\ R_2^{-1}) / (R_1^{-1} + R_2^{-1}) = 1 / (R_1 + R_2)$$

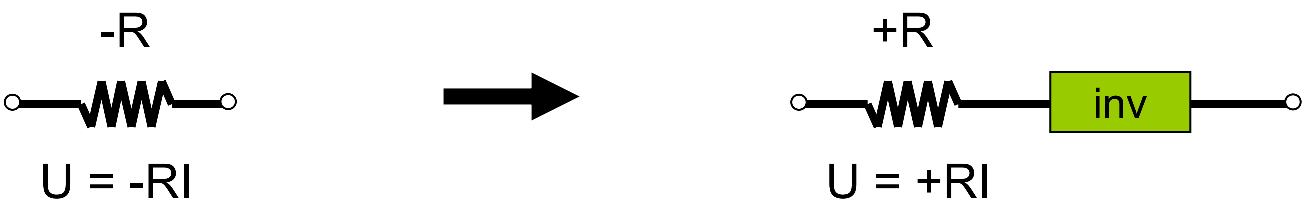


$R_{eq} = -2$

Can we construct a signed resistor that behaves like that?

3. Signed Resistance Distance – Construction

We model a negative resistor as a positive resistor in series with a "voltage inverter"



The voltage inverter inverts the voltage…

Which is impossible physically.

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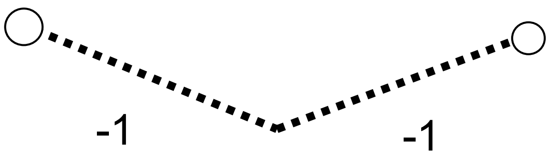
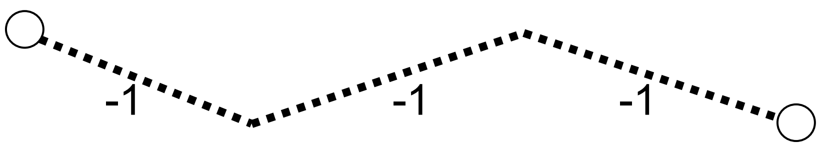
3. Signed Resistance Distance – Examples

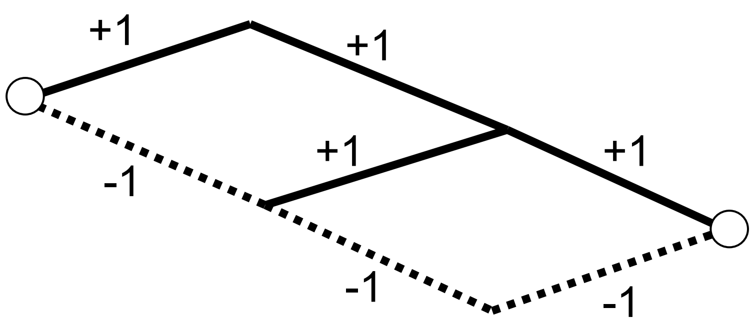


= +2

Voltage inverters in series annihilate each other to fulfill the sign requirement



= -3



= +2.75

How do we compute this?

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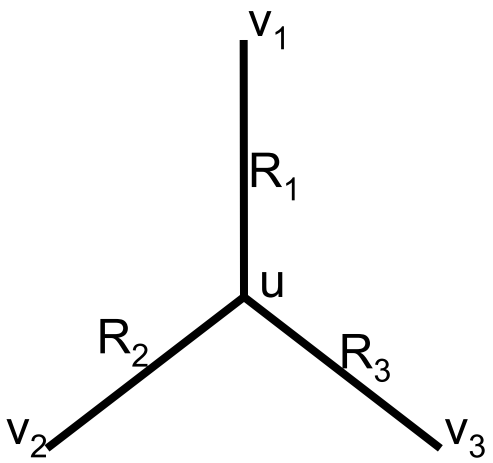
3. Signed Resistance Distance – Node Equation

In resistor networks, the voltage at each node is the weighted mean of the neighbors' voltages:

$$u = (\Sigma_i R_i^{-1} v_i) / (\Sigma_i R_i^{-1})$$



In "signed" resistor networks:

$$u = (\Sigma_i R_i^{-1} v_i) / (\Sigma_i |R_i^{-1}|)$$

Generalizing the signed series rule.

How do we compute this quickly?

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3. Signed Resistance Distance – Algebraic Formulation

The resistance distance between *(i, j)* is given by

$$d(i, j) = K_{ii} + K_{jj} - K_{ij} - K_{ji}$$

Resistance distance: $K = (D - A)^{+}$

Signed resistance distance: $K = (D^{\mathrm{abs}} - A)^{+}$

Moore-Penrose pseudoinverse

(with $D^{\mathrm{abs}}_{ii} = \Sigma_j |A_{ij}|$, $D_{ii} = \Sigma_j A_{ij}$)

$\mathrm{L}^{(abs)} = (D^{(abs)} - A)$ is the (signed) Laplacian matrix

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3. Signed Resistance Distance – Computation

Computing the pseudoinverse of the Laplacian:

•Approximate with dimensionality reduction of *k*:

$$L_k = U_k \Delta_k U_k^{\mathrm{T}}$$

$$K_k = U_k \Delta_k^{+} U_k^{\mathrm{T}}$$

Find *k* the smallest eigenvalues of *L*, giving the *k* largest eigenvalues of L^+.

What does *K* represent?

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3. Signed Resistance Distance – Kernel

A kernel is a similarity matrix which is positive semi-definite

For the resistance distance, K is a kernel:

- K is positive semi-definite and of rank (n - 1)
- The square root of the resistance distance is thus a Euclidean metric:

$$d(i, j) = (G_i - G_j)^2 \text{ for } K = GG^T$$

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3. Signed Resistance Distance – "Pseudo-Kernel"

For the signed resistance distance:

- K has full rank: we "lose" one degree of freedom due to voltage inversion, which fixes the zero potential
- K is not positive semi-definite in general: not a kernel!
- K is positive semi-definite in practice: use it as a kernel!
- The square root of the signed resistance distance is a pseudo-Euclidean metric

4. Evaluation – Application to Collaborative Filtering

Two ways to use it for collaborative filtering:

- Recommendation of users/items by finding nearest nodes
- Rating prediction using inverted distance in weighted mean of known ratings:

$$\tilde{A}_{ui} = \Sigma_{v\sim u}\, d(u, v)^{-1} A_{vi}$$

(traditionally implemented with the Pearson correlation)

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4. Evaluation – Netflix Prize

- Netflix Prize: 400,000 users rated 17,000 items
- Root mean squared error of rating predictions

lower is better

	User-based weighted mean	Item-based weighted mean
Pearson correlation	0.98	0.99
Resistance distance	1.30	1.29
Signed resistance distance	**0.97**	**0.98**

(evaluated on a subset of the full corpus)

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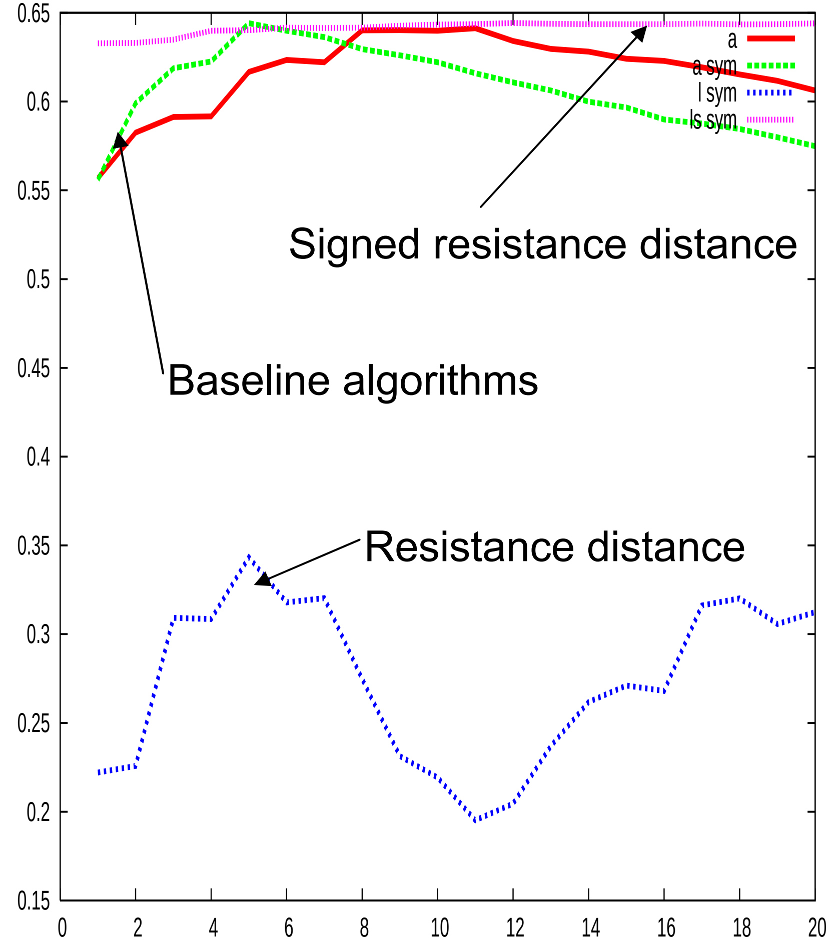
4. Evaluation – Slashdot Zoo (work in progress)

- Slashdot Zoo: 70,000 users tagged each other as “friends” and “foes”
- Graph is unipartite, directed, weighted
- Matrix is quadratic, assymmetric
- We ignore edge direction for evaluation and use sign of distance as prediction

higher is better



5. Conclusion

The signed resistance distance

- Applies to networks with negative edges
- Fulfills the sign multiplication condition
- Is better than ignoring/misrepreseting negative relations

Future work

- Full analysis of the unipartite case (Slashdot Zoo)
- Signed, directed Laplacian
- Signed centrality: Laplacian methods for trust, PageRank, etc.

Thank You!