Assignment 3 ROOTS OF EQUATIONS 2

1. Use ONE-POINT ITERATION method to find the root of following equation

$$f(x) = e^{-\frac{x}{4}}(2 - x) - 1$$

with 2 initial guesses of 0 and the last digit of your student ID. For instance, if your student ID is 60-040626-1006-6, the initial guess will be 6 (there are 2 cases)

60-040626-6009-1

1.1 show your work for 4 iterations

1.2 write a program to find it with the tolerance below 0.000001

2. Use Taylor series to approximate value of the following function

$$f(x) = \ln x$$

b = 4, a = 2

when x = 4 with an initial guess of x_0 = 2 using n from 0 to 3 for Taylor series. Also, show error obtained from using each n^{th} term in the approxiamtion

3. Use Newton Raphson method to find the value of $\sqrt{7}$ using x = 2.0 as the initial guess

3.1 show your work for 4 iterations

3.2 write a program to find it providing the tolerance is below 0.000001

1.1 $f(x) = e^{-\frac{x}{4}}(2-x) - 1$

จัดสมการ $0 = 2e^{-\frac{x}{4}} - xe^{-\frac{x}{4}} - 1$

$xe^{-\frac{x}{4}} = 2e^{-\frac{x}{4}} - 1$

$x = \frac{2e^{-\frac{x}{4}} - 1}{e^{-\frac{x}{4}}} = \frac{2e^{-\frac{x}{4}}}{e^{-\frac{x}{4}}} - \frac{1}{e^{-\frac{x}{4}}} = 2 - e^{\frac{x}{4}}$

$x_{i+1} = 2 - e^{\frac{x_i}{4}}$

กรณีแรก initial guess = 1 (6004062600 9[1]) ใช้ระบบเลขทศนิยมตำแหน่ง

iteration 1 : $x_i = 1$, $x_{i+1} = 2 - e^{\frac{1}{4}} = 2 - 1.284 = 0.716$

error : $\left|\frac{x_{new} - x_{old}}{x_{new}}\right| \times 100\% = \left|\frac{0.716 - 1}{0.716}\right| \times 100\% = 39.66\%$

iteration 2 : $x_i = 0.716$ เป็น initial guess ในตัวถัดไป , $x_{i+1} = 2 - e^{\frac{0.716}{4}} = 2 - e^{0.179} \simeq 2 - 1.196 = 0.804$

error : $\left|\frac{x_{new} - x_{old}}{x_{new}}\right| \times 100\% = \left|\frac{0.804 - 0.716}{0.804}\right| \times 100\% = 10.945\%$

iteration 3 : $x_i = 0.804$ เป็น initial guess ในตัวถัดไป , $x_{i+1} = 2 - e^{\frac{0.804}{4}} = 2 - 1.22 = 0.78$

error : $\left|\frac{x_{new} - x_{old}}{x_{new}}\right| \times 100\% = \left|\frac{0.78 - 0.804}{0.78}\right| \times 100\% = 3.077\%$

iteration 4 : $x_i = 0.78$, $x_{i+1} = 2 - e^{\frac{0.78}{4}} = 0.7846$

error : $\left|\frac{x_{new} - x_{old}}{x_{new}}\right| \times 100\% = \left|\frac{0.7846 - 0.78}{0.7846}\right| \times 100\% = 0.5862\%$

กรณีสอง : ให้ initial guess = 0 ← โดยเดาก้านนอกมา

iteration 1: $x_i = 0$, $x_{i+1} = 2 - e^{\frac{0}{4}} = 2 - 1 = 1$

error: $\left| \frac{x_{new} - x_{old}}{x_{new}} \right| \times 100\% = \left| \frac{1-0}{1} \right| \times 100\% = 100\%$

iteration 2: $x_i = 1$, $x_{i+1} = 2 - e^{\frac{1}{4}} = 2 - 1.28 = 0.716$

error: $\left| \frac{x_{new} - x_{old}}{x_{new}} \right| \times 100\% = \left| \frac{0.716 - 1}{0.716} \right| \times 100\% = 39.66\%$

iteration 3: $x_i = 0.716$, $x_{i+1} = 2 - e^{\frac{0.716}{4}} = 0.804$

error: $\left| \frac{x_{new} - x_{old}}{x_{new}} \right| \times 100\% = \left| \frac{0.804 - 0.716}{0.804} \right| \times 100\% = 10.945\%$

iteration 4: $x_i = 0.804$, $x_{i+1} = 2 - e^{\frac{0.804}{4}} = 0.777$

error: $\left| \frac{x_{new} - x_{old}}{x_{new}} \right| \times 100\% = \left| \frac{0.777 - 0.804}{0.777} \right| \times 100\% = 3.474\%$

1.2 กรณีแรก

```
JS lab3.3.js > ...
1     var xi=1;
2     for(i=0;i<14;i++){
3         var e=2.71828;
4         var xi1=(2-(Math.pow(e,xi/4)));
5         var error=Math.abs(((xi1-xi)/xi1)*100);
6         xi=xi1;
7         console.log(error+"%");
8     }
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL

```
[nodemon] clean exit - waiting for changes before restart
[nodemon] restarting due to changes...
[nodemon] starting `node lab3.3.js`
39.669720367844 18%
10.946960939496755%
3.4227236454913 83%
1.0319405605175496%
0.3146310837045551%
0.09560379085611898%
0.029080187687575303%
0.008842659960940923%
0.002689119610905673 5%
0.000817757813979501 9%
0.000248681295353260 9%
0.000075624126764495 55%
0.000022997359778246607%
0.000006993514477301 3616%
```

กรณีสอง

JS lab3.3.1.js > ...

```
1    var xi=0;
2    for(i=0;i<15;i++){
3        var e=2.71828;
4        var xi1=2-Math.pow(e,xi/4);
5        var error=Math.abs(((xi1-xi)/xi1)*100);
6        xi=xi1;
7        console.log(error+"%");
8    }
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL

```
[nodemon] restarting due to changes...
[nodemon] starting `node lab3.3.1.js`
100%
39.669720367844 18%
10.946960939496755%
3.4227236454913 83%
1.0319405605175496%
0.3146310837045551%
0.09560379085611898%
0.029080187687575303%
0.008842659960940923%
0.002689119610905 6735%
0.000817757813979 5019%
0.000248681295353 2609%
0.000075624126764 49555%
0.000022997359778246607%
0.000006993514477 3013616%
```

2. $f(x) = \ln x$

สูตร $f(b) = f(a) + (b-a)f'(a) + \frac{(b-a)^2}{2!}f''(a) + \frac{(b-a)^3}{3!}f'''(a)$

Term0: $f(a)$ ใช้ $x_0 = 2$

$f(x_0) = \ln 2 = 0.693$

P_{term0} ใช้ $f(b) = f(a) = 0.693$

$Error = |f(x) - P_{term0}(x)|$ หรือ $|f(a) - f(b)| = |0.693 - 0.693| = 0$ #

Term1: $f'(a) = \frac{1}{x} = \frac{1}{2} = 0.5$

$(b-a) = 4-2 = 2$

P_{term1} ใช้ $f(b) = f(a) + (b-a)f'(a) = 0.693 + 2(0.5) = 0.693 + 1 = 1.693$

$Error = |f(x) - P_{term1}(x)| = |0.693 - 1.693| = 1$ #

Term2: $f''(a) = \frac{1}{x} = x^{-1} = -x^{-2} = \frac{-1}{x^2} = \frac{-1}{2^2} = \frac{-1}{4} = -0.25$

$\frac{(b-a)^2}{2!} = \frac{(4-2)^2}{2!} = \frac{4}{2} = 2$

P_{term2} ใช้ $f(b) = f(a) + (b-a)f'(a) + \frac{(b-a)^2}{2!}f''(a)$

$= 1.693 + 2(-0.25) = 1.693 - 0.5 = 1.193$

$Error = |f(x) - P_{term2}(x)| = |0.693 - 1.193| = 0.5$ #

Term3: $f'''(a) = \frac{-1}{x^2} = -x^{-2} = -(-2)x^{-3} = 2x^{-3} = \frac{2}{x^3} = \frac{2}{8} = 0.25$

$\frac{(b-a)^3}{3!} = \frac{(4-2)^3}{3!} = \frac{2^3}{6} = \frac{8}{6} = 1.33$

$P_{term_3}(b) = f(a) + (b-a)f'(a) + \frac{(b-a)^2}{2!}f''(a) + \frac{(b-a)^3}{3!}f'''(a)$

$= 1.193 + 1.33(0.25) = 1.193 + 0.3325 = 1.5255$

$Error = |f(x) - P_{term_3}(x)| = |0.693 - 1.52551| = 0.8325$ ##

3. $f(x) = \sqrt{7} \rightarrow x = \sqrt{7} \rightarrow x^2 = 7 \rightarrow x^2 - 7 = 0$

iteration 1: ใช้ $\Delta x_{k+1} = -\frac{f(x_k)}{f'(x_k)}$

ให้ $x_k = 2$ เป็น initial guess

$f(x_k) = 2^2 - 7 = 4 - 7 = -3$

$f'(x_k) = 2x - 0 \rightarrow f'(2) = 2(2) = 4$

$\Delta x_{k+1} = \frac{-(-3)}{4} = \frac{3}{4} = 0.75$ ✓

ได้ $x_{k+1} = x_k + \Delta x_{k+1} = 2 + 0.75 = 2.75$ ✓

ได้ error: $\left|\frac{\Delta x_{k+1}}{x_{k+1}}\right| \times 100\% = \left|\frac{0.75}{2.75}\right| \times 100\% = 27.27\%$ #

iteration 2: ใช้ $\Delta x_{k+1} = \frac{-f(x_k)}{f'(x_k)}$

ให้ $x_k = 2.75$ เป็น initial guess ตัวต่อไป

$f(x_k) = (2.75)^2 - 7 = 0.5625$

$f'(x_k) = 2x - 0 = 2(2.75) = 5.5$

$\Delta x_{k+1} = \frac{-0.5625}{5.5} = -0.102$ ✓ → ได้ $x_{k+1} = x_k + \Delta x_{k+1} = 2.75 + (-0.102) = 2.648$

ได้ error: $\left|\frac{\Delta x_{k+1}}{x_{k+1}}\right| \times 100\% = \left|\frac{-0.102}{2.648}\right| \times 100\% = 3.8519\%$

iteration 3: ใช้ $\Delta x_{k+1} = \frac{-f(x_k)}{f'(x_k)}$

ให้ $x_k = 2.648$ เป็น initial guess ตัวต่อไป

$f(x_k) = (2.648)^2 - 7 = 0.0119$

$f'(x_k) = 2x - 0 = 2(2.648) = 5.296$

$\Delta x_{k+1} = \frac{-0.0119}{5.296} = -0.00225$

ดังนั้น $x_{k+1} = x_k + \Delta x_{k+1} = 2.648 + (-0.00225) = 2.64575$

ดังนั้น error: $\left|\frac{\Delta x_{k+1}}{x_{k+1}}\right| \times 100\% = \left|\frac{-0.00225}{2.64575}\right| \times 100\% = 0.00085 \times 100\% = 0.085\%$ #

iteration 4: ดังนั้น $\Delta x_{k+1} = \frac{-f(x_k)}{f'(x_k)}$

ให้ $x_k = 2.64575$ ได้จาก initial guess ครั้งที่ 3

$f(x_k) = (2.64575)^2 - 7 = 0.00000693375$

$f'(x_k) = 2x - 0 = 2(2.64575) = 5.2915$

$\Delta x_{k+1} = \frac{-0.00000693375}{5.2915} = -0.00000131$

ดังนั้น $x_{k+1} = x_k + \Delta x_{k+1} = 2.64575 + (-0.00000131) = 2.64574$

ดังนั้น error: $\left|\frac{\Delta x_{k+1}}{x_{k+1}}\right| \times 100\% = \left|\frac{-0.00000131}{2.64574}\right| \times 100\% = 0.00004955\%$ #

3.2

JS lab3.2.js > ...

```
1   var x=2.00;
2   for(i=0;i<5;i++){
3       var fk=Math.pow(x,2)-7;
4       var ffk=(2*x);
5       var xxk1 = -(fk/ffk);
6       var xk1= x + xxk1;
7       var error = Math.abs((xxk1/xk1)*100);
8       x=xk1;
9       console.log(error+"%");
10  }
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL

```
3.8826085204032 54e-12%
6.3441315692866 08e-15%
[nodemon] clean exit - waiting for changes before res
[nodemon] restarting due to changes...
[nodemon] starting `node lab3.2.js`
[nodemon] restarting due to changes...
27.272727272727%
3.862660944206008%
0.07465644211361269%
0.00002786792951224296%
3.882608520403254e-12%
[nodemon] starting `node lab3.2.js`
27.272727272727%
3.862660944206008%
0.07465644211361269%
0.00002786792951224296%
3.882608520403254e-12%
```