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<u>Assignment 8 NUMERICAL DIFFERENTIATION</u>

1. Determine FIRST DIVIDED-DIFFERENCES of $f(x) = e^x$ at x = 2 with h = 0.25 by using

1.1 forward divided-difference which has O(h) and calculate true error

1.2 backward divided-difference which has O(h) and calculate true error

1.3 central divided-difference which has O(h^2) and calculate true error

2. Determine SECOND DIVIDED-DIFFERENCES of function $f(x) = e^{x/3} + x^2$ at x = -2.5 with h = 0.1 by using

2.1 forward divided-difference which has O(h^2) and calculate true error

2.2 backward divided-difference which has O(h^2) and calculate true error

2.3 central divided-difference which has O(h^4) and calculate true error

$e^x = e^x$

$e^{2x} = e^{2x} \, d(2x)$

$= 2e^{2x}$

1.

$f(x) = e^x \; ; \; x = 2 \; ; \; h = 0.25$ $\qquad\qquad$ $0.004,566282 \times 10^{-3}$

Diffก่อน $\quad f'(x) = e^x$

ดังนั้น ที่ $x=2$ หา root จะได้

$$f'(x=2) = e^2 = 7.389056$$

ถ้าใช้ interval $h = 0.25$ จะได้

$x_{i-1} = 2 - 0.25 = 1.75$ $\quad ; f(x_{i-1}) = 5.754603$

$x_i = 2$ $\quad ; f(x_i) = 7.389056$

$x_{i+1} = 2 + 0.25 = 2.25$ $\quad ; f(x_{i+1}) = 9.487736$

1.1 Forward : $f'(x_i) = [f(x_{i+1}) - f(x_i)]/h$

$$f'(2) = \frac{9.487736 - 7.389056}{0.25} = 8.39472$$

หา True error (ε_t) $= \left| \dfrac{7.389056 - 8.39472}{7.389056} \right| \times 100\% = 13.610182\%$

1.2 Backward : $f'(x_i) = [f(x) - f(x_{i-1})]/h$

$$f'(2) = \frac{7.389056 - 5.754603}{0.25} = 6.537812$$

หา True error (ε_t) $= \left| \dfrac{7.389056 - 6.537812}{7.389056} \right| \times 100\% = 11.520335\%$

1.3 Central : $f'(x_i) = [f(x_{i+1}) - f(x_{i-1})]/2h$

$$f'(2) = \frac{9.487736 - 5.754603}{0.5} = 7.466266$$

หา True error (ε_t) $= \left| \dfrac{7.389056 - 7.466266}{7.389056} \right| \times 100\% = 1.044924\%$

(2) $f(x) = e^{\frac{x}{3}} + x^2$, $x = -2.5$, $h = 0.1$

$x^2 = (x)(x)$

$-2.8^2 = (-2.8)(-2.8)$

Diff ก่อน $f'(x) = \frac{1}{3}e^{\frac{x}{3}} + 2x$

Diff ครั้งที่ 2 $f''(x) = \frac{1}{9}e^{\frac{x}{3}} + 2$

ดังนั้นที่ $x = -2.5$ จะได้ root เท่ากับ

root $\rightarrow$ $f''(-2.5) = \frac{1}{9}e^{\frac{-2.5}{3}} + 2 = 2.048289$

ถ้าใช้ interval $h = 0.1$ จะได้

$x_{i-3} = -2.5 - 0.1 - 0.1 - 0.1 = -2.8$; $f(x_{i-3}) = e^{\frac{-2.8}{3}} + (-2.8)^2 = 0.393241 + 7.84$
$= 8.233241$

$x_{i-2} = -2.5 - 0.1 - 0.1 = -2.7$; $f(x_{i-2}) = e^{\frac{-2.7}{3}} + (-2.7)^2 = 0.406569 + 7.29$
$= 7.696569$

$x_{i-1} = -2.5 - 0.1 = -2.6$; $f(x_{i-1}) = 0.420350 + 6.76 = 7.18035$

$x_i = -2.5$; $f(x_i) = 0.434598 + 6.25 = 6.684598$

$x_{i+1} = -2.5 + 0.1 = -2.4$; $f(x_{i+1}) = 0.449328 + 5.76 = 6.209328$

$x_{i+2} = -2.5 + 0.1 + 0.1 = -2.3$; $f(x_{i+2}) = 0.464559 + 5.29 = 5.754599$

$x_{i+3} = -2.5 + 0.1 + 0.1 + 0.1 = -2.2$; $f(x_{i+3}) = 0.480305 + 4.84 = 5.320305$

(2.1) Forward : $f''(x_i) = [-f(x_{i+3}) + 4f(x_{i+2}) - 5f(x_{i+1}) + 2f(x_i)] / h^2$

$= \dfrac{[-5.320305 + 23.018396 - 31.04664 + 13.369196]}{0.01} = 2.0647$

หา True error $(\varepsilon_t) = \left| \dfrac{2.048289 - 2.0647}{2.048289} \right| \times 100\% = 0.801205\%$

2.2 Backward : $f''(x_i) = [2f(x_i) - 5f(x_{i-1}) + 4f(x_{i-2}) - f(x_{i-3})]/h^2$

$$= \frac{[13.369196 - 35.90175 + 30.786276 - 8.233241]}{0.01}$$

$$= 2.0481$$

หา true error (ε_t) $= \left| \dfrac{2.048289 - 2.0481}{2.048289} \right| \times 100\% = 0.009227\%$

2.3 Central : $f''(x_i) = [-f(x_{i+2}) + 16f(x_{i+1}) - 30f(x_i) + 16f(x_{i-1}) - f(x_{i-2})]/12h^2$

$$= \frac{[-5.754599 + 99.349248 - 200.53794 + 114.8856 - 7.696569]}{0.12}$$

$$= 2.047833$$

หา True error (ε_t) $= \left| \dfrac{2.048289 - 2.047833}{2.048289} \right| \times 100\% = 0.022262\%$ #