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<u>**Assignment 7** NUMERICAL INTEGRATION</u>

1. Evaluate the following integral

$$I = \int_{2}^{8} (4x^5 - 3x^4 + x^3 - 6x + 2)dx \qquad (1)$$

1.1 Using Single Trapezoidal Rule and calculate the true error.

1.2 Using Composite Trapezoidal Rule with n= 2, 4, 6 and calculate the true and approximation errors.

1.3 Write a program to do as in 1.1

1.4 Write a program to do as in 1.2

2. Evaluate the following integral

$$I = \int_{-1}^{2} (x^7 + 2x^3 - 1)dx \qquad (2)$$

2.1 Using SIMPSON'S RULE and calculate the true error.

2.2 Using COMPOSITE SIMPSON'S RULE with n= 2, 4, 6 and calculate the true and approximation errors.

2.3 Write a program to do as in 2.1

2.4 Write a program to do as in 2.2

3. The force on a sailboat mast can be represented by the following function

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$$F = \int_0^H 200\left(\frac{z}{5+z}\right)e^{-\frac{2z}{H}}dz \qquad (3)$$

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where z is the elevation above the deck and H is the height of the mast. This function has no close form solution. Compute F for a case where H equals to 30 using Romberg integration to a tolerance of ε_s = 0.5%.

4. Write a program to evaluate the function in equation (3) as in question 3.

1.1 Trapezoidal rule

$$I = \int_2^8 (4x^5 - 3x^4 + x^3 - 6x + 2)\,dx$$

$x_0 = a = 2$; $f(x_0) = 128 - 48 + 8 - 12 + 2 = 78$

$x_1 = b = 8$; $f(x_1) = 131072 - 12288 + 512 - 48 + 2 = 119250$

$$\therefore I = \frac{b-a}{2}[f(x_0) + f(x_1)] = \frac{8-2}{2}(78 + 119250) = 3(119328) = 357984$$

หา exact integral is

$$I = \int_2^8 (4x^5 - 3x^4 + x^3 - 6x + 2)\,dx = \left[\frac{4x^6}{6} - \frac{3x^5}{5} + \frac{x^4}{4} - \frac{6x^2}{2} + 2x\right]_2^8 = [174762.666667 - 19660.8 + 1024 - 192 + 16]$$

$$- [42.666667 - 19.2 + 4 - 12 + 4]$$

$$= 155949.866667 - (19.466667) = 155930.4$$

$\therefore$ True error is

$$\varepsilon_t = \left(\frac{155930.4 - 357984}{155930.4}\right) \times 100\% = -129.579351\%$$

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1.2 Composite trapezoidal rule

$\boxed{\text{เริ่ม } n = 2}$

$$h = \frac{b-a}{n} = \frac{8-2}{2} = \frac{6}{2} = 3$$

$f(x_0 = 2) = 78$

$f(x_1 = 5) = 12500 - 1875 + 625 - 30 + 2 = 11222$

$f(x_2 = 8) = 119250$

$$I = \frac{h}{2}\left[f(x_0) + f(x_2) + 2\sum_{i=1}^{2-1} f(x_i)\right] = \frac{3}{2}[78 + 119250 + 2(11222)]$$

$$= 1.5[119328 + 22444] = 212658$$

หา Exact solution

เอามา $I = 155930.4$
(จากข้อเดิม)

$\therefore$ True error is

$$\varepsilon_t = \left(\frac{155930.4 - 212658}{155930.4}\right) \times 100\% = -36.380077\% \quad \#$$

$\boxed{\text{เริ่ม } n=4}$ $\rightarrow h = \frac{b-a}{n} = \frac{8-2}{4} = \frac{6}{4} = 1.5$

$f(x_0 = 2) = 78$

$f(x_1 = 3.5) = 2100.875 - 450.1875 + 42.875 - 21 + 2 = 1674.5625$

$f(x_2 = 5) = 11222$

$f(x_3 = 6.5) = 46411.625 - 5355.1875 + 274.625 - 39 + 2 = 41294.0625$

$f(x_4 = 8) = 119250$

$$I = \frac{h}{2}\left[f(x_0) + f(x_4) + 2\sum_{i=1}^{4-1} f(x_i)\right] = \frac{1.5}{2}\left[78 + 119250 + 2(1674.5625 + 11222 + 41294.0625)\right]$$

$$= 0.75[119328 + 108381.25] = 170781.9375$$

หา Exact solution

$I = 155930.4$

$\therefore$ True error is

$$\varepsilon_t = \left(\frac{155930.4 - 170781.9375}{155930.4}\right) \times 100\% = -9.524466\% \quad \#$$

$\boxed{\text{เริ่ม } n=6}$ $\rightarrow h = \frac{b-a}{n} = \frac{8-2}{6} = \frac{6}{6} = 1$

$f(x_0 = 2) = 78$

$f(x_1 = 3) = 972 - 243 + 27 - 18 + 2 = 740$

$f(x_2 = 4) = 4096 - 768 + 64 - 24 + 2 = 3370$

$f(x_3 = 5) = 11222$

$f(x_4 = 6) = 31104 - 3888 + 216 - 36 + 2 = 27398$
$f(x_5 = 7) = 67228 - 7203 + 343 - 42 + 2 = 60328$
$f(x_6 = 8) = 119250$

$$I = \frac{h}{2}\left[f(x_0) + f(x_6) + 2\sum_{i=1}^{6-1} f(x_i) \right] = \frac{1}{2}\left[78 + 119250 + 2(740 + 3370 + 11222 + 27398 + 60328) \right]$$
$$= 0.5\,[119328 + 206116] = 162722$$

ឬ Exact solution
$I = 155930.4$

$\therefore$ True error is

$$\varepsilon_t = \left(\frac{155930.4 - 162722}{155930.4} \right) \times 100\% = -4.355533\% \quad \#$$

2.1 Simpson's Rule $\quad I = \int_{-1}^{2} x^7 + 2x^3 - 1 \, dx$

ឬ $h = \frac{b-a}{2} = \frac{2-(-1)}{2} = \frac{3}{2} = 1.5$

$f(x_0 = -1) = -1 - 2 - 1 = -4$
$f(x_1 = 0.5) = 0.0078125 + 0.25 - 1 = -0.7421875$
$f(x_2 = 2) = 128 + 16 + 1 = 145$

$$I = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2) \right] = \frac{1.5}{3}\left[-4 + 4(-0.7421875) + 145 \right]$$
$$= 0.5\,[138.03125] = 69.015625$$

ตอบ Exact solution

$$I = \left[\frac{x^8}{8} + \frac{2x^4}{4} - x \right]_{-1}^{2} = [32 + 16 - 2] - [\frac{1}{8} + \frac{2}{4} + 1]$$

$$I = 46 - [0.125 + 0.5 + 1] = 46 - 1.625 = 44.375$$

True error is

$$\varepsilon_t = \left(\frac{44.375 - 69.015625}{44.375} \right) \times 100\% = -55.528169\% \quad \#$$

2.2 Composite simpson' Rule $n = 2, 4, 6$

$$h = \frac{b-a}{n} = \frac{2-(-1)}{2} = \frac{3}{2} = 1.5$$

$f(x_0 = -1) = -4$

$f(x_1 = 0.5) = -0.7421875$

$f(x_2 = 2) = 145$

$$I = \frac{h}{3} \left[f(x_0) + f(x_n) + 4 \sum_{i=1}^{2-1} f(x_i) + 2 \sum_{i=2}^{2-2} f(x_i) \right]$$

$$= \frac{1.5}{3} [-4 + 145 + 4(-0.7421875) + 2(0)]$$

$$= 0.5 [141 - 2.96875]$$

$$= 69.015625$$

ตอบ Exact Solution จาก I เดิม จะได้

$I = 44.375$

หา True error is

$$\varepsilon_t = \left(\frac{44.375 - 69.015625}{44.375}\right) \times 100\% = -55.528169\%$$

เริ่ม $n = 4$

$$h = \frac{b-a}{n} = \frac{2-(-1)}{4} = \frac{3}{4} = 0.75$$

$f(x_0 = -1) = -4$
$f(x_1 = -0.25) = -0.968811$
$f(x_2 = 0.5) = -0.742188$
$f(x_3 = 1.25) = 7.674622$
$f(x_4 = 2) = 145$

$$\therefore I = \frac{0.75}{3}\left[-4 + 145 + 4\sum_{i=1,3}^{3} f(x_i) + 2\sum_{i=2,4}^{2} f(x_i)\right]$$

$$I = 0.25[141 + 4(-0.968811 + 7.674622) + 2(-0.742188 + 145)]$$

$$= 0.25[141 + 26.823244 + 288.515624]$$

$$I = 114.084717$$

หา Exact solution

$$I = 44.375$$

หา True error

$$\varepsilon_t = \left(\frac{44.375 - 114.084717}{44.375}\right) \times 100\% = -157.09232\%$$

กรณี $n = 6$

$$h = \frac{b-a}{n} = \frac{2-(-1)}{6} = \frac{3}{6} = 0.5$$

$$f(x_0 = -1) = -4$$
$$f(x_1 = -0.5) = -1.257813$$
$$f(x_2 = 0) = -1$$
$$f(x_3 = 0.5) = -0.742188$$
$$f(x_4 = 1) = 1+2-1 = 2$$
$$f(x_5 = 1.5) = 22.835938$$
$$f(x_6 = 2) = 145$$

$$\therefore I = \frac{0.5}{3}\left[-4+145+4\sum_{i=1,3,5}^{5} f(x_i) + 2\sum_{i=2,4}^{4} f(x_i)\right]$$

$$I = 0.166667\left[141+4(146)+2(22.09375)\right]$$
$$= 0.166667\left[141+584+44.1875\right]$$
$$I = 128.198173$$

หา Exact solution

$$I = 44.375$$

หา True error

$$\varepsilon_t = -188.897291\ \%$$

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3. $F = \int_0^{30} 200\left(\frac{z}{5+z}\right) e^{-\frac{2z}{30}}\, dz$ $\qquad x_0 = 0$

$\qquad x_1 = 30$

ใช้ $n = 1, 2, 4, 8$ ใช้ในการหา error

หา $h_1 = b-a = 30-0 = 30$

$I(h_1) = \frac{h}{2}[f(x_0) + f(x_1)]$

หา $f(x_0 = 0) = 200\left(\frac{0}{5+0}\right) e^{-\frac{2(0)}{30}}$

$f(x_0 = 0) = 0$

$f(x_1 = 30) = 200\left(\frac{30}{35}\right) e^{-\frac{60}{30}}$

$f(x_1 = 30) = 23.20033427$

$I(h_1) = 15[0 + 23.20033427] = 348.005014$

หา $h_2 = \frac{(b-a)}{n} = \frac{30-0}{2} = 15$

$f(x_0 = 0) = 0$

$f(x_1 = 15) = 200\left(\frac{15}{20}\right) e^{-\frac{30}{30}} = 55.18191618$

$f(x_2 = 30) = 200\left(\frac{30}{35}\right) e^{-\frac{60}{30}} = 23.20033427$

$I(h_2) = \frac{h}{2}\left[f(x_0) + f(x_n) + 2\left(\sum_{i=1}^{2-1} f(x_i)\right)\right] = \frac{15}{2}[0 + 23.20033427 + 2(55.18191618)]$

$I(h_2) = 1001.73125$

ul7 $h_4 = \frac{b-a}{n} = \frac{30-0}{4} = 7.5$

$f(x_0=0)=0$

$f(x_1=7.5) = 200\left(\frac{7.5}{12.5}\right)e^{\frac{-7.5}{30}} = 72.78367917$

$f(x_2=15) = 55.18191618$

$f(x_3=22.5) = 200\left(\frac{22.5}{27.5}\right)e^{-\frac{45}{30}} = 36.51220802$

$f(x_4=30) = 23.20033427$

$I(h_4) = \frac{7.5}{2}\left[f(x_0)+f(x_n)+2\left(\sum_{i=1}^{4-1} f(x_i)\right)\right]$

$= 3.75[0+23.20033427+2(164.47778034)]$

$I(h_4) = 1320.584779$

ul7 $h_8 = \frac{b-a}{n} = \frac{30-0}{8} = 3.75$

$f(x_0=0)=0$

$f(x_1=3.75) = 200\left(\frac{3.75}{8.75}\right)e^{\frac{-7.5}{30}} = 66.75435283$

$f(x_2=7.5) = 200\left(\frac{7.5}{12.5}\right)e^{\frac{-15}{30}} = 72.78367917$

$f(x_3=11.25) = 200\left(\frac{11.25}{16.25}\right)e^{\frac{-22.5}{30}} = 65.40459961$

$f(x_4=15) = 55.18191618$

$f(x_5=18.75) = 200\left(\frac{18.75}{23.75}\right)e^{-\frac{37.5}{30}} = 45.2375995$

$f(x_6=22.5) = 36.51220802$

$f(x_7 = 26.25) = 200\left(\frac{26.25}{31.25}\right)e^{-\frac{52.5}{30}} = 29.1940225$

$f(x_8 = 30) = 23.20033427$

$$I(h_8) = \frac{3.75}{2}\left[f(x_0) + f(x_n) + 2\sum_{i=1}^{8-1}(f(x_i))\right]$$

$$= 1.875\left[0 + 23.20033427 + 2(371.0683778)\right]$$

$I(h_8) = 1435.007044$

ใช้วิธี Romberg $k = 1, 2, 3$

$n = 1$ ที่ $k = 1$ ใช้ $I(h_1)$ กับ $I(h_2)$ * I_m คือ I_2 และ I_L คือ I_1

$$I = \frac{4^k I_m - I_L}{4^k - 1} = \frac{4^1(1001.73125) - 348.005014}{4^1 - 1}$$

$I = 1219.639995$

หา error $\varepsilon_a = \left|\frac{1219.639995 - 1001.73125}{1219.639995}\right| \times 100\% = 17.86664477\%$

$n = 1$ ที่ $k = 2$ ใช้ $I(h_2)$ กับ $I(h_4)$

$$I = \frac{4^k I_m - I_L}{4^k - 1} = 1341.841681$$

หา error(ε_a) $= \left|\frac{1341.841681 - 1320.584779}{1341.841681}\right| \times 100\% = 1.584157535\%$

n = 1 ที่ k = 3 ใช้ $I(h_4)$ กับ $I(h_8)$

$$I = \frac{4^k I_m - I_L}{4^k - 1} = 1436.82327 \quad \#$$

หา error (ε_a) = $\left| \frac{1436.82327 - 1435.007044}{1436.82327} \right| \times 100\% = 0.1264056942\% \quad \#$

$\therefore$ the approximated integral is 1436.82327 (F)