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Assignment 5 INTERPOLATION AND EXTRAPOLATION

$\downarrow y = f(x)$

1. A below table shows a relationship between x and y

Point No.	x	y
1	0	9.81
2	20,000	9.7487
3	40,000	9.6879
4	60,000	9.6879
5	80,000	9.5682

Determine y at x = 42,000 using Newton's divided-differences to do

1.1 LINEAR INTERPOLATION (use point no. 1 and 5) x_0 = point 1 x_1 = point 5

1.2 QUADRATIC INTERPOLATION (use point no. 1, 3 and 5)

1.3 POLYNOMIAL INTERPOLATION (use point no. 1, 2, 3, 4 and 5)

2. Given a relationship between x and y in a below table

$\downarrow y = f(x)$

Point No.	x	Y
1	-100	215
2	0	202
3	100	206
4	200	215
5	300	228
6	400	249

Determine y at x = 250 using LAGRANGE INTERPOLATION to do

2.1 LINEAR INTERPOLATION (use point no. 1 and 6)

2.2 QUADRATIC INTERPOLATION (use point no. 1, 3 and 6)

2.3 POLYNOMIAL INTERPOLATION (use point no. 1, 2, 3, 4, 5 and 6)

1.1 หา $f(42{,}000)$ หา point ที่ 1 กับ 5 linear interpolation

จากสูตร $f(x) = f(x_0) + (x - x_0)\dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$$= 9.81 + (42{,}000 - 0)\dfrac{(9.5682 - 9.81)}{80{,}000 - 0}$$

$$f(42{,}000) = 9.81 + 42{,}000(-3.0225 \times 10^{-5})$$

$$= 9.81 - 126.945 \times 10^{-5}$$

$$= 9.81 - 0.126945 = 9.683055 \quad \#$$

1.2 QUADRATIC interpolation หา point 1, 3, 5

จากสูตร $f(x) = c_0 + c_1(x - x_0) + c_2(x - x_0)(x - x_1)$

หา $c_0 = 9.81$

หา $c_1 = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \dfrac{9.6879 - 9.81}{40{,}000 - 0} = \dfrac{-0.1221}{40{,}000} = -3.0525 \times 10^{-5}$

หา $c_2 = \dfrac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0} = \dfrac{\left(\dfrac{9.5682 - 9.6879}{80{,}000 - 40{,}000}\right) - \left(\dfrac{9.6879 - 9.81}{40{,}000 - 0}\right)}{80{,}000 - 0}$

$$= \frac{-2.9925 \times 10^{-6} - (-3.0525 \times 10^{-5})}{80{,}000} = \frac{-0.29925 \times 10^{-5} + 3.0525 \times 10^{-5}}{80{,}000}$$

$$= \frac{2.75325 \times 10^{-5}}{80{,}000} = 3.4415625 \times 10^{-10}$$

$$\therefore f(42{,}000) = 9.81 - 3.0525 \times 10^{-5}(42{,}000 - 0) + 3.4415625 \times 10^{-10}(42{,}000 - 0)(42{,}000 - 40{,}000)$$

$$= 9.81 - 128205 \times 10^{-5} + 0.028909125 = 9.81 - 1.28205 + 0.028909125$$

$$= 8.556859125$$

1.3 Polynomial interpolation

x	$f(x)$	first	second	third
0	9.81			
		$\frac{f_2-f_1}{x_2-x_1}=\frac{9.7487-9.81}{20{,}000-0}=-3.065\times10^{-6}$		
20,000	9.7487		$\frac{f_2-f_1}{x_2-x_1}=\frac{-3.04\times10^{-6}-(-3.065\times10^{-6})}{40{,}000-0}=6.25\times10^{-7}\times10^{-6}=6.25\times10^{-13}$	
		$\frac{-0.0608}{40{,}000-20{,}000}=-3.04\times10^{-6}$		$=\frac{7.6\times10^{-11}-0.0625\times10^{-11}}{60{,}000-0}=1.25625\times10^{-15}$
40,000	9.6879		$=\frac{0-(-3.04\times10^{-6})}{60{,}000-20{,}000}=7.6\times10^{-5}\times10^{-6}=7.6\times10^{-11}$	
		$\frac{0}{20{,}000}=0$		$=\frac{-14.9625\times10^{-11}-7.6\times10^{-11}}{80{,}000-20{,}000}=\frac{-22.5625\times10^{-11}}{60{,}000}=-3.76041667\times10^{-15}$
60,000	9.6879		$=\frac{-5.985\times10^{-6}-0}{80{,}000-40{,}000}=-1.49625\times10^{-4}\times10^{-6}=-1.49625\times10^{-10}$	
		$\frac{-0.1197}{80{,}000-60{,}000}=-5.985\times10^{-6}$		
80,000	9.5682			

four $=\frac{-3.76041667\times10^{-15}-1.25625\times10^{-15}}{80{,}000-0}=6.270833338\times10^{-20}$

$$f(42{,}000)=f(x_0)+(x-x_0)f[x_1,x_0]+(x-x_0)(x-x_1)f[x_2,x_1,x_0]+(x-x_0)(x-x_1)(x-x_2)f[x_3,x_2,x_1,x_0]$$

$$+(x-x_0)(x-x_1)(x-x_2)(x-x_3)f[x_4,x_3,x_2,x_1,x_0]$$

$$=9.81+(-128730\times10^{-6})+(42{,}000)(22{,}000)(6.25\times10^{-13})+(42{,}000)(22{,}000)(2{,}000)(1.25625\times10^{-15})$$
$$+(42{,}000)(22{,}000)(2{,}000)(-18{,}000)(6.270833338\times10^{-20})$$

$$=9.81-0.12873+5.775\times10^{-4}+2.32155\times10^{-3}-2.085930002\times10^{-3}$$
$$=9.68208312$$

\#

2.1 linear interpolation point 1 กับ 6

จากสูตร $f(x) = L_0(x)f(x_0) + L_1(x)f(x_1)$

หา $L_0(250) = \dfrac{x_1 - x}{x_1 - x_0} = \dfrac{400-250}{400-(-100)} = \dfrac{150}{500} = 0.3$

หา $L_1(250) = \dfrac{x_0 - x}{x_0 - x_1} = \dfrac{-100-250}{-100-400} = \dfrac{-350}{-500} = 0.7$

$\therefore f(250) = 0.3(215) + 0.7(249) = 64.5 + 174.3 = 238.8$ #

2.2 quadratic interpolation point 1, 3, 6

จากสูตร $f(x) = L_0(x)f(x_0) + L_1(x)f(x_1) + L_2(x)f(x_2)$

$L_0(250) = \dfrac{(x_2 - x)(x_1 - x)}{(x_2 - x_0)(x_1 - x_0)} = \dfrac{(400-250)(100-250)}{(400-(-100))(100-(-100))} = \dfrac{150(-150)}{500(200)}$

$= \dfrac{-22500}{100000} = -0.225$

$L_1(250) = \dfrac{(x_2 - x)(x_0 - x)}{(x_2 - x_1)(x_0 - x_1)} = \dfrac{150(-350)}{300(-200)} = \dfrac{-52500}{-60000} = 0.875$

$L_2(250) = \dfrac{(x_1 - x)(x_0 - x)}{(x_1 - x_2)(x_0 - x_2)} = \dfrac{(-150)(-200)}{(-300)(-500)} = \dfrac{30,000}{150,000} = 0.2$

$\therefore f(250) = (-0.225)(215) + 0.875(206) + 0.2(249) = -48.375 + 180.25 + 49.8$

$= 181.675$ #

(2.3) polynomial interpolation point 1-6

สูตร $f(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$ * Σ คือผลรวมของ point ทั้งหมด

หา $L_i(x)$ โดยเริ่มจาก $L_0(x) = \prod_{\substack{j=0 \\ j\neq 1}}^{6} \frac{x-x_j}{x_0-x_j} = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)(x_0-x_5)}$

$$= \frac{250(150)(50)(-50)(-150)}{(-100)(-200)(-300)(-400)(-500)} = \frac{1.40625\times10^{10}}{-1.2\times10^{12}} = -1.171875\times10^{-2}$$

$$= 0.01171875$$

หา $L_1(x) = \prod_{\substack{j=1 \\ j\neq 2}}^{6} \frac{x-x_j}{x_1-x_j} = \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)}$

$$= \frac{350(150)(50)(-50)(-150)}{100(-100)(-200)(-300)(-400)} = \frac{1.96875\times10^{10}}{2.4\times10^{11}} = 8.203125\times10^{-1}$$

$$= 0.8203125$$

หา $L_2(x) = \prod_{\substack{j=2 \\ j\neq 3}}^{6} \frac{x-x_j}{x_2-x_j} = \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)(x-x_5)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)(x_2-x_5)} = \frac{150(250)(50)(-50)(-150)}{(200)(100)(-100)(-200)(-300)}$

$$= \frac{1.40625\times10^{10}}{-1.2\times10^{11}} = -1.171875\times10^{-1} = -0.1171875$$

หา $L_3(x) = \prod_{\substack{j=3 \\ j\neq 4}}^{6} \frac{x-x_j}{x_3-x_j} = \frac{350(250)(150)(-50)(-150)}{300(200)(100)(-100)(-200)} = \frac{9.84375\times10^{10}}{1.2\times10^{11}}$

$$= 8.203125\times10^{-1}$$

$$= 0.8203125 \quad \#$$

จาก $L_4(x) = \frac{350(250)(150)(50)(-150)}{400(300)(200)(100)(-100)} = \frac{-9.84375 \times 10^{10}}{-2.4 \times 10^{11}} = 4.1015625 \times 10^{-1}$

$= 0.41015625$

จาก $L_5(x) = \frac{(350)(250)(150)(50)(-50)}{(500)(400)(300)(200)(100)} = \frac{-3.28125 \times 10^{10}}{1.2 \times 10^{12}} = -2.734375 \times 10^{-2}$

$= -0.02734375$ #

$\therefore f(250) = L_0(x)f(x_0) + L_1(x)f(x_1) + L_2(x)f(x_2) + L_3(x)f(x_3) + L_4(x)f(x_4) + L_5(x)f(x_5)$

$= 2.51953125 + 165.703125 - 24.140625 + 176.3671875$

$+ 93.515625 - 6.08859375$

$\therefore f(250) = 410.15625$ #