

## Assignment 2 Roots of Equations

1. Use Bisection method to find the root of  $\sqrt[4]{13}$  in the interval of 1.5 and 2.0

1.1 show your work for 4 iterations

**Iteration 0:**  $X_L=1.5, X_R=2.0$

$$\text{Step1: } X_M = \frac{1.5+2.0}{2} = 1.75$$

$$f(x): x = \sqrt[4]{13} \rightarrow x^4 = 13 \rightarrow x^4 - 13 = 0$$

$$\text{find } f(X_M) = (1.75)^4 - 13 = -3.621093$$

$$\text{find } f(X_R) = (2.0)^4 - 13 = 16 - 13 = 3$$

$$\text{Step2: } f(X_M) \cdot f(X_R) = -3.621093 \cdot 3 = -10.863279 < 0$$

$$\text{Step3: } f(X_M) \cdot f(X_R) < 0 \rightarrow X_L = X_M \rightarrow X_L = 1.75$$

**Iteration 1:**  $X_L=1.75, X_R=2$

$$\text{Step1: } X_M = \frac{1.75+2}{2} = 1.875$$

$$\text{find } f(X_M) = (1.875)^4 - 13 = -0.64$$

$$\text{find } f(X_R) = (2.0)^4 - 13 = 16 - 13 = 3$$

$$\text{Step2: } f(X_M) \cdot f(X_R) < 0$$

$$\text{Step3: } f(X_M) \cdot f(X_R) < 0 \rightarrow X_L = X_M \rightarrow X_L = 1.875$$

$$\text{Step4: } \left| \frac{1.875 - 1.75}{1.875} \right| \times 100\% = 6.67\%$$

**Iteration 2:**  $X_L=1.875, X_R=2$

$$\text{Step1: } X_M = \frac{1.875+2}{2} = 1.9375$$

$$\text{find } f(X_M) = (1.9375)^4 - 13 = 1.09$$

$$\text{find } f(X_R) = (2.0)^4 - 13 = 16 - 13 = 3$$

$$\text{Step2: } f(X_M) \cdot f(X_R) > 0$$

$$\text{Step3: } f(X_M) \cdot f(X_R) > 0 \rightarrow X_R = X_M \rightarrow X_R = 1.9375$$

$$\text{Step4: } \left| \frac{1.9375 - 1.875}{1.9375} \right| \times 100\% = 3\%$$

**Iteration 3:**  $X_L = 1.875$ ,  $X_R = 1.9375$

$$\text{Step1: } X_M = \frac{1.875 + 1.9375}{2} = 1.90625$$

$$\text{find } f(X_M) = (1.90625)^4 - 13 = 0.204$$

$$\text{find } f(X_R) = (1.9375)^4 - 13 = 1.09$$

$$\text{Step2: } f(X_M) \cdot f(X_R) > 0$$

$$\text{Step3: } f(X_M) \cdot f(X_R) > 0 \rightarrow X_R = X_M \rightarrow X_R = 1.90625$$

$$\text{Step4: } \left| \frac{1.90625 - 1.9375}{1.90625} \right| \times 100\% = 1\%$$

**Iteration 4:**  $X_L = 1.875$ ,  $X_R = 1.90625$

$$\text{Step1: } X_M = \frac{1.875 + 1.90625}{2} = 1.890625$$

$$\text{find } f(X_M) = (1.890625)^4 - 13 = -0.223$$

$$\text{find } f(X_R) = (1.90625)^4 - 13 = 0.204$$

$$\text{Step2: } f(X_M) \cdot f(X_R) < 0$$

$$\text{Step3: } f(X_M) \cdot f(X_R) < 0 \rightarrow X_L = X_M \rightarrow X_L = 1.890625$$

$$\text{Step4: } \left| \frac{1.890625 - 1.90625}{1.890625} \right| \times 100\% = 0.8\%$$

1.2 write a program to iteratively find the answer till there is no changes in the answer for six significant figures

```
f=function(x){  
    return Math.pow(x,4)-13;  
}  
var l=1.5;  
var r=2.0;  
var t=1.898828;  
var m;  
for(i=0;i<5;i++){  
    m=(l+r)/2;  
    if(f(m)*f(r)>0){  
        r=m;  
    }  
    else{  
        l=m;  
    }  
    console.log(m);  
}
```

```
[nodemon] starting node home1.js  
1.75  
1.875  
1.9375  
1.90625  
1.890625  
[nodemon] clean exit - waiting for changes before restart  
█
```

```
f = function(x) {
```

```
  return Math.pow(x, 4) - 13;
```

```
var prev m = 0;
```

```
var error = Math.abs(m - prev);
```

```
error = Math.abs((m - prev) / m) * 100;
```

```
error = Math.abs((m - prev) / m) * 100;
```

```
  {  
    var r = 1.5;  
    var l = 1.0;  
    var m = 0;  
    var f = 180 * Math.PI / 180;
```

```
    for (i = 0; i < 5; i++) {
```

```
      m = (l + r) / 2;
```

```
      if (f(m) * f(r) > 0) {
```

```
        r = m;
```

```
      } else {
```

```
        l = m;
```

```
      console.log(m);
```

```
      console.log(error);
```

2. Use False-Position method to find the value of  $\frac{1}{43}$  in the interval of 0.02 and 0.03

2.1 show your work for 4 iterations

$$f(x) : x = \frac{1}{43} \rightarrow 43x = 1 \rightarrow 43x - 1 = 0$$

Iteration0:  $X_L=0.02$  ,  $X_R=0.03$

$$\begin{aligned}\text{Step1: } X_1 &= \frac{X_L f(X_R) - X_R f(X_L)}{f(X_R) - f(X_L)} \\ &= \frac{(0.02)(0.29) - (0.03)(-0.14)}{(0.29) - (-0.14)} = \frac{(0.0058) - (-0.0042)}{0.43} = 0.02325\end{aligned}$$

$$f(X_L) = -0.14$$

$$f(X_R) = 0.29$$

$$f(X_1) = 43(0.02325) - 1 = -0.00025$$

$$\text{Step2: } f(X_1) \cdot f(X_R) < 0$$

$$\text{Step3: } X_L = X_1 \rightarrow X_L = 0.02325$$

$$\text{Step4: } \left| \frac{X_1^{\text{new}} - X_1^{\text{old}}}{X_1^{\text{new}}} \right| \times 100\% = \left| \frac{0.02325 - 0.03}{0.02325} \right| \times 100\% = 29\%$$

Iteration1:  $X_L=0.02325$  ,  $X_R=0.03$

$$\begin{aligned}\text{Step1: } X_1 &= \frac{X_L f(X_R) - X_R f(X_L)}{f(X_R) - f(X_L)} = \\ &= \frac{(0.0067425) - (-0.0000075)}{0.29} = 0.02327\end{aligned}$$

$$f(X_L) = 43(0.02325) - 1 = -0.00025$$

$$f(X_R) = 43(0.03) - 1 = 0.29$$

$$f(X_1) = 43(0.02327) - 1 = 0.00061$$

$$\text{Step2: } f(X_1) \cdot f(X_R) > 0$$

$$\text{Step3: } X_R = X_1 \rightarrow X_R = 0.02327$$

$$\begin{aligned} \text{Step4: } & \left| \frac{X_1^{\text{new}} - X_1^{\text{old}}}{X_1^{\text{new}}} \right| \times 100\% \\ &= \left| \frac{0.02327 - 0.02325}{0.02327} \right| \times 100\% = 0.08594\% \end{aligned}$$

$$\text{Iteration2: } X_L = 0.02325, X_R = 0.02327$$

$$\begin{aligned} \text{Step1: } X_1 &= \frac{X_L f(X_R) - X_R f(X_L)}{f(X_R) - f(X_L)} = \\ &= \frac{(0.000014) - (-0.0000058)}{0.00086} = 0.02302 \end{aligned}$$

$$f(X_L) = 43(0.02325) - 1 = -0.00025$$

$$f(X_R) = 43(0.02327) - 1 = 0.00061$$

$$f(X_1) = 43(0.02302) - 1 = -0.01014$$

$$\text{Step2: } f(X_1) \cdot f(X_R) < 0$$

$$\text{Step3: } X_L = X_1 \rightarrow X_L = 0.02302$$

$$\text{Step4: } \left| \frac{X_1^{\text{new}} - X_1^{\text{old}}}{X_1^{\text{new}}} \right| \times 100\% = \left| \frac{0.02302 - 0.02327}{0.02302} \right| \times 100\% = 1.08\%$$

$$\text{Iteration3: } X_L = 0.02302, X_R = 0.02327$$

$$\text{Step1: } X_1 = \frac{X_L f(X_R) - X_R f(X_L)}{f(X_R) - f(X_L)} = \frac{(0.000014) - (-0.00023)}{0.01061} = 0.0229$$

$$f(X_L) = -0.01$$

$$f(X_R) = 0.00061$$

$$f(X_1) = -0.0153$$

Step2:  $f(X_1) \cdot f(X_R) < 0$

Step3:  $X_L = X_1 \rightarrow X_L = 0.0229$

Step4:  $\left| \frac{X_1^{new} - X_1^{old}}{X_1^{new}} \right| \times 100\% = \left| \frac{0.0229 - 0.02302}{0.0229} \right| \times 100\% = 0.524\%$

Iteration4:  $X_L = 0.0229, X_R = 0.02327$

Step1:  $X_1 = \frac{X_L f(X_R) - X_R f(X_L)}{f(X_R) - f(X_L)} = \frac{(0.000013) - (-0.00035)}{0.01591} = 0.0228$

$f(X_L) = -0.0153$

$f(X_R) = 0.00061$

$f(X_1) = -0.0196$

Step2:  $f(X_1) \cdot f(X_R) < 0$

Step3:  $X_L = X_1 \rightarrow X_L = 0.0228$

Step4:  $\left| \frac{X_1^{new} - X_1^{old}}{X_1^{new}} \right| \times 100\% = \left| \frac{0.0228 - 0.0229}{0.0228} \right| \times 100\% = 0.4385\%$

2.2 write a program provide that the error is lesser than 0.000001%

```
f=function(x){
    return (43*x)-1;
}
var l=0.02;
var r=0.03;
var t =0.023255;
var x1;
for(i=0;i<5;i++){
    x1=(l*f(r))-(r*f(l))/f(r)-f(l);
    if((f(x1)*f(r))>0){
        r=x1;
    }
}
```

```
else{  
    l=x1;  
}  
console.log(x1);  
}  
  
[nodemon] starting `node home2.js`  
0.16028275862068966  
0.2616515536288036  
0.34859375895297784  
0.42327917881581506  
0.4874651881877338  
[nodemon] clean exit - waiting for changes before  
[ ]
```

3. Use ONE-POINT ITERATION method to find the value of  $\frac{1}{2}$   
with an initial guess of 0.00

3.1 show your work for 4 iterations

$$F(x): x = \frac{1}{2} \rightarrow 2x - 1 = 0$$

$$\text{Iteration1: } X_{i+1} = 2x_i - 1 \rightarrow \text{using } 0.00$$

$$\text{Iteration2: } X_2 = 2x_1 - 1 = 2(0.00) - 1 = -1$$

$$\left| \frac{X_{\text{new}} - X_{\text{old}}}{X_{\text{new}}} \right| \times 100\% = \left| \frac{-1 - 0.00}{-1} \right| \times 100\% = 100\%$$

$$\text{Iteration3: } X_3 = 2x_2 - 1 = 2(-1) - 1 = -3$$

$$\left| \frac{X_{\text{new}} - X_{\text{old}}}{X_{\text{new}}} \right| \times 100\% = \left| \frac{-3 - (-1)}{-3} \right| \times 100\% = 66.67\%$$

$$\text{Iteration4: } X_4 = 2x_3 - 1 = 2(-3) - 1 = -7$$



```
f = function(x){  
  return (43 * x) - 1;  
}
```

```
↳
```

```
var l = 1.5;
```

```
var r = 2.0;
```

```
var x1;
```

```
var prev_x1 = 0;
```

```
var error;
```

```
for (i = 0; i < 10; i++) {
```

```
  x1 = (l * f(r) - r * f(l)) / (f(r) - f(l));
```

```
  if (f(x1) * f(r) > 0) {
```

```
    r = x1;
```

```
  }  
  else {
```

```
    l = x1;
```

```
  }  
  console.log(x1);  
  console.log(error);  
}
```

$error = \left( \text{Math.abs}(\text{prev\_x1} - \text{new\_x1}) \right) * 100;$   
 $\text{prev\_x1} = \text{error};$

$$\left| \frac{X_{new} - X_{old}}{X_{new}} \right| \times 100\% = \left| \frac{-7 - (-3)}{-7} \right| \times 100\% = 57.14\%$$

3.2 write a program with error lesser than 0.000001%

```
f=function(x){  
    return (2*news)-1;  
}  
var x1=0.00;  
  
var news=0.00;  
for(i=0;i<5;i++){  
    news=f(x1);  
  
    console.log(news);  
}
```

```
[nodemon] starting `node home3.js`  
-1  
-3  
-7  
-15  
-31  
[nodemon] clean exit - waiting for changes before restart
```

$$x - \frac{1}{2}$$

```
f = function(x) {  
  return news - c1(2);  
}
```

↳

```
var x1 = 0.00;  
var news = 0.00;
```

```
var error =
```

```
for (i = 0; i < 6; i++) {
```

```
  news = f(x1);
```

```
  error = Math.abs(news - x1) / news * 100;
```

```
  console.log(news);
```

```
  console.log(error);
```

↳