EE 621 Lecture 2 Notes 10 January 2021 20:25 Discrete line Markov chain (DTMCs) Reall: Two events A,B une independent it P(A|B) = P(A) = P(B) = assumes positive probability We say events A or B are independent given event C, if P(A/BC) = P(A/C) = P(B/C) 4 Let 5 denote a countable set Def: A random process { Xn3n20 taking values in 5 is a liscrete time Markor chain (DTMC) if State space ₩n, Lo, L, L2... L, L, J = 5, $P(X_{n+1} = J | X_0 = i_0, X_1 = i_1 - ... X_{n-1} = i_{n-1}, X_n = i_s)$ future = P(Xn+1=j/Xn=i) = Pij(n) present

probability For a Markov process, future and past are independent, given the present. eg Suppose {Yn}, is a BP(p). Define $X_0 = 0$, $X_N = X_{N-1} + X_N (N \ge 1)$ [Xn] defines a random walk over Zn. At each Time, the process "moves right with probability p and stays put with Claim: &Xn3 is a DTMC over Z+ . state spice P(Xn+1=1/Xo=io, X1=1, ... Xn-1=in-1, Xn=i) = $\begin{cases} P, j=i+1 \\ 1-P, j=i \end{cases}$ toes not depend on the his of therwise just

For instance, $X_8 = 5$, $X_{10} \in \{5,6,7\}$. does not depend on the history! Not true that X_8 and X_{10} are independent! They are independent given the value of the present i.e. X_9 . Py (n) is the probability that the process transitions from state i at time in to state just time int. Mostly, we will be interested in time-homogeneous DMC5 where Py(n) = Py (e. transition probabilities do not depend on time. The random walk example is time-homogeneous. For time-homogeneous ITMCs, we represent them pictorially using a transition probability diagram 19 This 2-state DTMC describes the evolution of the quality of a wireless channel. La Good & Bad & Good model "

Bad & model" The transition probability diagram is a weighted directed graph Nodes are states, edge i - jexists if Pij>0. Weight of ah edge is its transition probability. We will find it useful to "pack" the transition probabilities into a matrix P called the transition probability matrix (TPM). P=[Py]. Note that P is 15/X/5/ (Square, in a way). P is also stochastic which means vows sum up to 1 individually. $\left(\sum_{j \leq 5} P_{ij} = 1\right)$ god bad antinally For the G-E model, P = [-a a] (Good) B initially & bad [b 1-b] (Good) B