EE621 Lecture 1 Notes

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Random process $\{X(t), t \in T\}$ ← family of random variables (time index; index set)

eg $T = \mathbb{N}$ (discrete time process)

$T = \mathbb{R}, \mathbb{R}^+$ (continuous time process)

Def: A Bernoulli random variable $X \in \{0,1\}$ with $P(X=1) = p$.

Def: A Bernoulli process $\{X_n, n \in \mathbb{N}\}$ is a sequence of iid (independent, identically distributed) Bernoulli RVs with parameter $p$. We denote this as $BP(p)$.

A sample path of a Bernoulli process looks like $(1,0,1,1,0,0\ldots)$

Think of 1 as an arrival in a queue (or as a success in a coin toss)

① Let $A_1$ denote the time of the first arrival

$(A_1 = \min\{n \geq 1 \mid X_n = 1\})$

Distribution of $A_1$? Geometric RV!

$$P(A_1 = k) = (1-p)^{k-1} p \quad \{A_1 \sim \text{Geometric}(p)\}$$

$$\mathbb{E}[A_1] = \frac{1}{p}$$

② $N(t)$ = # of arrivals until time $t$

Distribution of $N(t)$? Binomial RV!

$$P(N(t) = k) = \binom{t}{k} p^k (1-p)^{t-k}$$

③ Let $A_k$ denote the time of the $k^{th}$ arrival

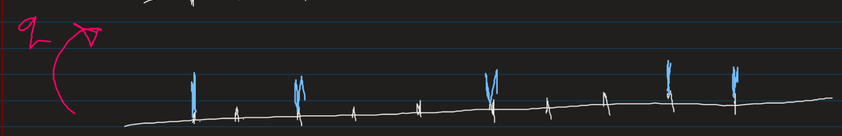
$T_k = A_k - A_{k-1}$ $(k \geq 2)$ inter-arrival times

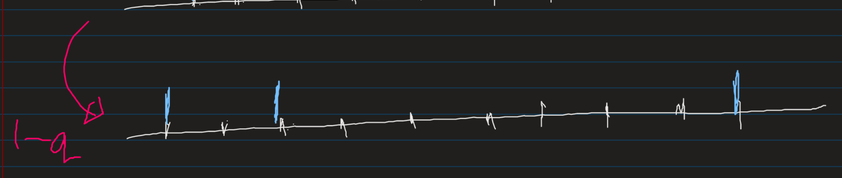
Distribution of $T_k$? $T_k \sim$ Geometric$(p)$

Def: A Bernoulli process $BP(p)$ is a $\{0,1\}$ random process with i.i.d Geometric$(p)$ interarrival times. (alternate definition)

## Properties of a Bernoulli process

① Splitting: Consider a $BP(p)$. Each "arrival" is made part of stream A with probability $q$ and stream B with probability $1-q$.


arrival → split into A: $p \times q$; split into B: $p \times (1-q)$

To show: Stream $A = BP(pq)$

Stream $B = BP(p(1-q))$

Streams A and B are not independent!

Clearly, because arrival in $A \Rightarrow$ no arrival in B

② Merging: Consider a $BP(p)$ and $BP(q)$, mutually independent.

We "merge" the two process as follows – an arrival occurs in the merged process if one occurs in either/both the original processes.

To show: Merged process $\sim BP(p+q-pq)$ $P(p \cup q) = P(p) + P(q) - P(p \cap q)$

(independent) $\Rightarrow pq$

③ Suppose $\{X_i\}_{i \geq 1}$ is a sequence of iid Geometric$(p)$ random variables.

$N \sim$ Geometric$(q)$, independent of $\{X_n\}$.

Define the random sum $Y = \sum_{i=1}^{N} X_i$. Distribution of $Y$?

Consider $BP(p)$ with interarrival time $\{X_i\}$. Thus $Y$ is the time of $N^{th}$ arrival in the process. Since $N$ is geometric, each arrival is picked with probability $q$ and not picked with probability $1-q$.

$Y$ = time of first "picked" arrival from $BP(p)$.

$\Rightarrow Y$ is the time of the first arrival in a split $BP(pq)$

i.e. $Y \sim$ Geometric$(pq)$

Note: A Bernoulli process is — independent, stationary — simplest possible stochastic process