

Lecture 16: Cascode Amplifiers

ECE3110J, Electronic Circuits

Xuyang Lu 2024 Summer



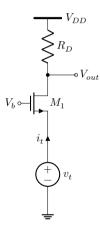


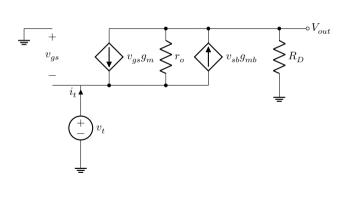
- Source Follower
- Emitter Follower
- Common Gate

Topics to be Covered



- Common Gate
- Cascode
- Differential amplifier





CG Input Impedance ($\lambda \neq 0$, $\gamma \neq 0$)



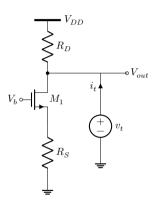
$$i_t = v_t(g_m + g_{mb}) + \frac{v_t - v_{out}}{r_o}$$
 (1)

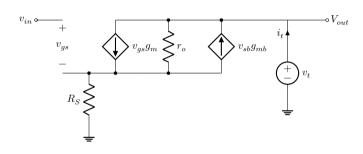
$$V_{out} = R_D i_t \tag{2}$$

$$R_{in} = \frac{R_D + r_o}{1 + (g_m + g_{mb})r_o} \begin{cases} \text{If } R_D = 0 & R_{in} = r_o \parallel \frac{1}{g_m} \parallel \frac{1}{g_{mb}} \\ \text{If } R_D = \infty & R_{in} = \infty \end{cases}$$
 (3)

CG Output Impedance ($\lambda \neq 0$, $\gamma \neq 0$)







Same as CS with source degradation

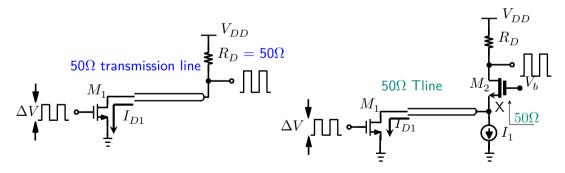
$$R_{out} = [R_S + r_{o1} + (g_{m1} + g_{mb1})r_{o1}R_S] \parallel R_D$$
(4)

Applications of CG

Recap of Last Lecture

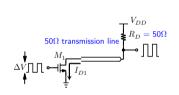


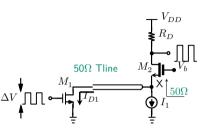
Calculate the small-signal voltage gain at low frequencies of the circuits below. To minimize wave reflection at point X, the input impedance must be equal to 50Ω .



Applications of CG

Recap of Last Lecture





$$A_v = -g_{m1}R_D \tag{6}$$

 $A_v = -g_{m1}R_D$ (5) R_D can be much larger than 50Ω , so as to achieve a much higher gain.

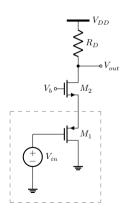
$$R_{in} = \frac{R_D + r_{o2}}{1 + (g_{m2} + g_{mb2}) r_{o2}} = 50\Omega \tag{7}$$

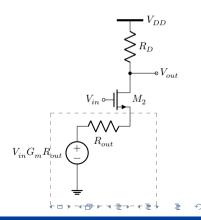
CG Example



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Calculate the small-signal voltage gain of the circuit below. $(\lambda \neq 0, \gamma \neq 0)$



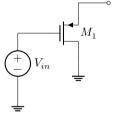


CG Example

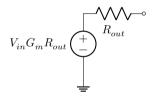












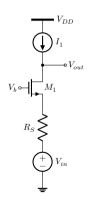
$$\begin{cases} G_m = & g_{m1} \\ R_{out} = & r_{o1} \parallel \frac{1}{g_{m1}} \parallel \frac{1}{g_{mb1}} \end{cases}$$

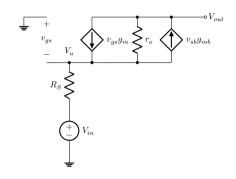


Example



Calculate the small-signal voltage gain of the circuit below. $(\lambda \neq 0, \gamma \neq 0)$





Since no current flowing in R_S , $V_a = V_{in}$

Example



$$V_{out} - V_{in}(g_m + g_{mb})r_o = V_{in}$$

$$\tag{9}$$

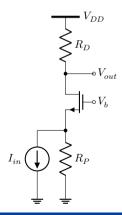
$$A_v = \frac{V_{out}}{V_{in}} = 1 + (g_m + g_{mb})r_o$$
 (10)

Example

Recap of Last Lecture



Calculate the small-signal trans-impedance gain of the circuit below. $(\lambda \neq 0, \gamma \neq 0)$



$$R_{in} = \frac{R_D + r_o}{1 + (q_m + q_{mb})r_o} \tag{11}$$

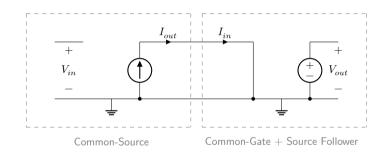
$$-i_{in}\frac{R_P}{R_{in} + R_P}R_D = V_{out} \tag{12}$$

$$\frac{V_{out}}{i_{in}} = -\frac{R_P}{R_{in} + R_P} R_D \tag{13}$$

$$=\frac{-R_P R_D [1+(g_m+g_{mb})r_o]}{R_D+r_o+R_P+(g_m+g_{mb})r_o R_P}$$
 (14)

Recap of Last Lecture

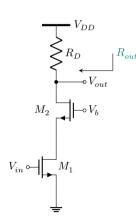




Cascode 000000000

Cascode is a common gate on top of a common source.





$$G_m = -g_{m1} \frac{r_{o1}}{r_{o1} + \left(r_{o2} \parallel \frac{1}{g_{m2} + g_{mb2}}\right)}$$
(15)

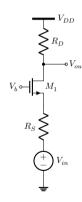
$$R_{out} = [r_{o1} + r_{o2} + (g_{m2} + g_{mb2})r_{o2}r_{o1}] \parallel R_D$$
 (16)

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Cascode

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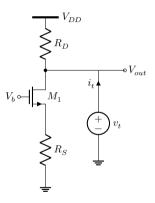


$$G_m = \frac{(g_m + g_{mb})r_{o1} + 1}{r_{o1} + R_S + (g_m + g_{mb})r_{o1}R_S}$$

Cascode

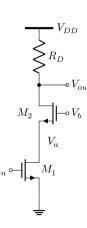
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$$R_{out} = [R_S + r_{o1} + (g_{m1} + g_{mb1})r_{o1}R_S] \parallel R_D$$
 (18)





$$A_v = G_m R_{out} \tag{19}$$

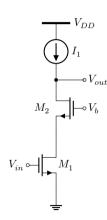
$$V_a \geqq V_{in} - V_{TH1} \tag{20}$$

$$V_b - V_{GS2} \ge V_{in} - V_{TH1} \tag{21}$$

$$V_b \ge V_{in} - V_{TH1} + V_{GS2} \tag{22}$$

$$V_{out} \geqq V_b - V_{TH2} \geqq (V_{in} - V_{TH1}) + (V_{GS2} - V_{TH2}) ~~ \mbox{(23)} \label{eq:vout}$$





$$V_{DD} \geqq V_{out} \geqq V_{ov1} + V_{ov2} \tag{24}$$

$$G_m = -g_{m1} \frac{r_{o1}}{r_{o1} + \left(r_{o2} \parallel \frac{1}{q_{m2} + q_{mb2}}\right)}$$
(25)

$$R_{out} = r_{o1} + r_{o2} + (g_{m2} + g_{mb2})r_{o2}r_{o1}$$
 (26)

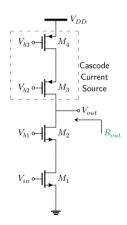
$$A_v = G_m R_{out} (27)$$

CS + CG with Ideal Current Source ($\lambda \neq 0$, $\gamma \neq 0$)

Cascode

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$$G_m = -g_{m1} \frac{r_{o1}}{r_{o1} + (r_{o2} \parallel \frac{1}{g_{m2}g_{mb2}})}$$
 (28)

$$R_{out} = [r_{o1} + r_{o2} + (g_{m2} + g_{mb2})r_{o2}r_{o1}]$$

$$\parallel [r_{o3} + r_{o4} + (g_{m3} + g_{mb3})r_{o3}r_{o4}]$$
(29)

$$A_v = G_m R_{out} \tag{30}$$

$$V_{DD} - V_{ov3} - V_{ov4} \ge V_{out} \ge V_{ov1} + V_{ov2}$$
 (31)

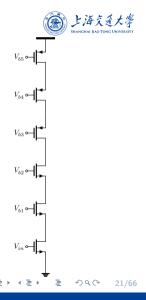
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CS + CG with Ideal Current Source

Can we cascode more to get higher gain? Problems:

Headroom

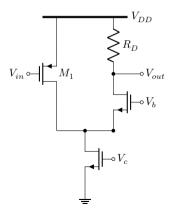
- 2 nA of current makes it sensitive towards leakage. PVT. noise
- 3 Because how output impedance scales, you will only see triple cascode in MOSFET but not in BJT.

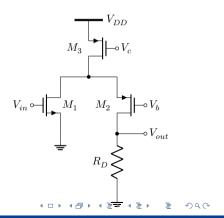


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Folded Cascode ($\lambda \neq 0$, $\gamma \neq 0$)

We don't care about M_3 's non-ideality.





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Folded Cascode ($\lambda \neq 0$, $\gamma \neq 0$)



$$G_m = -g_{m1} \frac{(r_{o1} \parallel r_{o3})}{(r_{o1} \parallel r_{o3}) + (r_{o2} \parallel \frac{1}{q_{m2} + q_{mb2}})}$$
(32)

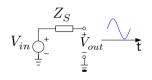
$$R_{out} = [(r_{o1} \parallel r_{o3}) + r_{o2} + (g_{m2} + g_{mb2})r_{o2}(r_{o1} \parallel r_{o3})] \parallel R_D$$
 (33)

$$A_v = G_m R_{out} (34)$$

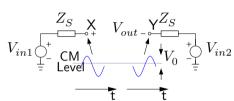
Single-Ended vs Differential Signals



Single-ended



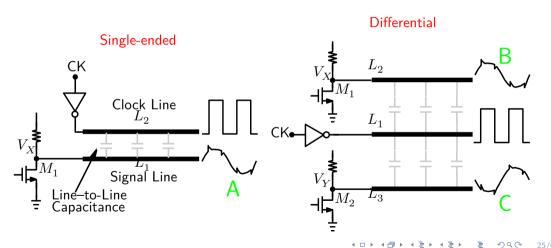
Differential



- B-C=A (matters)
- (B+C)/2 = common-mode level (doesn't matter)
- Single-ended signal: a voltage signal measured with respect to ground
- Differential signal: a voltage signal measured between two nodes, each having equal amplitude and opposite phase around a common-mode (CM) level

Common-Mode Noise Rejection





Common-Mode Noise Rejection



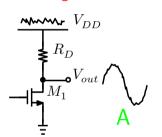
- A corrupted; B corrupted; C corrupted
- $(B+C)/2 = \mathsf{CM}$ corrupted
- (B-C) not corrupted

Increased Output Swing

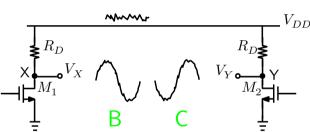
Recap of Last Lecture







Differential

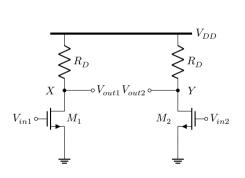


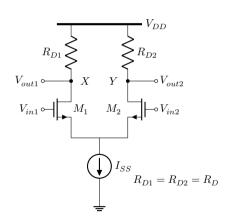
- $(V_{GS1} V_{TH1}) \le A \le V_{DD}$
- $(V_{GS1,2} V_{TH1,2}) V_{DD} \le (B C) \le V_{DD} (V_{GS1,2} V_{TH1,2})$



$V_{in,CM}$ and $V_{out,CM}$



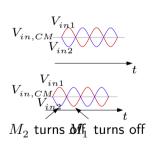


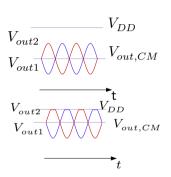




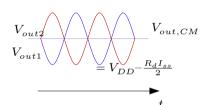
$V_{in,CM}$ and $V_{out,CM}$

Recap of Last Lecture



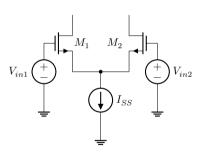


ullet $V_{out,CM}$ dependent on $\dot{V}_{in,CM}$

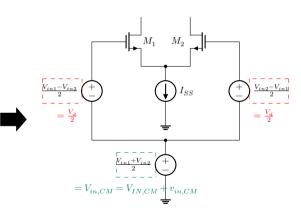


- $V_{out,CM}$ independent from $V_{in,CM}$
- Better design

Common-Mode + Differential-Mode



Not necessarily fully differential





Common-Mode + Differential-Mode

$$A_{DM} = \frac{V_{out1} - V_{out2}}{V_d} \tag{35}$$

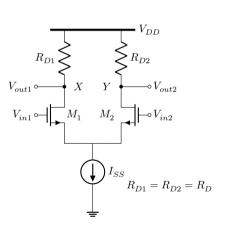
$$A_{CM} = \frac{V_{out,CM}}{V_{in,CM}} \tag{36}$$

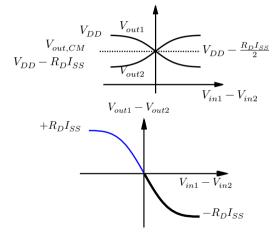
$$A_{CM-DM} = \frac{V_{out1} - V_{out2}}{V_{in,CM}}$$
 (37)

$$CMRR = \left| \frac{A_{DM}}{A_{CM-DM}} \right| \tag{38}$$

Differential-Mode (Qualitative Analysis)







Differential-Mode (DC Analysis) ($\lambda = 0$, $\gamma = 0$)

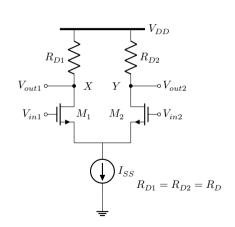
Cascode

$$V_{in1} - V_{GS1} = V_{in2} - V_{GS2} (39)$$

$$V_{in1} - V_{in2} = V_{GS1} - V_{GS2} (40)$$

$$= (V_{GS1} - V_{TH}) - (V_{GS2} - V_{TH}) \quad \mbox{(41)}$$

$$= \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} \frac{W}{L}}}$$
 (42)



Differential-Mode (DC Analysis)

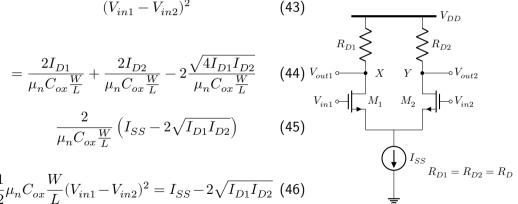
$$(V_{in1} - V_{in2})^2 (43)$$

Cascode

$$= \frac{2I_{D1}}{\mu_n C_{ox} \frac{W}{L}} + \frac{2I_{D2}}{\mu_n C_{ox} \frac{W}{L}} - 2\frac{\sqrt{4I_{D1}I_{D2}}}{\mu_n C_{ox} \frac{W}{L}}$$

$$rac{2}{\mu_n C_{ox} rac{W}{L}} \left(I_{SS} - 2\sqrt{I_{D1}I_{D2}}
ight)$$

$$\frac{1}{2}\mu_{n}C_{ox}\frac{W}{L}(V_{in1}-V_{in2})^{2}=I_{SS}-2\sqrt{I_{D1}I_{D2}} \ \ \mbox{(46)}$$



Differential-Mode (DC Analysis)



$$\frac{1}{2}\mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2})^2 - I_{SS} = -2\sqrt{I_{D1}I_{D2}}$$
(47)

$$\frac{1}{4}(\mu_n C_{ox} \frac{W}{L})^2 (V_{in1} - V_{in2})^4 + I_{SS}^2 - \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2})^2 I_{SS} = 4I_{D1}I_{D2}$$
 (48)

Cascode

$$\frac{1}{4}(\mu_{n}C_{ox}\frac{W}{L})^{2}(V_{in1}-V_{in2})^{4}+I_{SS}^{2}-\mu_{n}C_{ox}\frac{W}{L}(V_{in1}-V_{in2})^{2}I_{SS}=I_{SS}^{2}-(I_{D1}-I_{D2})^{2}$$

$$\Delta I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \Delta V_{in} \sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - \Delta V_{in}^2}$$
 (50)

Differential-Mode (DC Analysis)



$$\Delta I_{D} = \frac{1}{2} \mu_{n} C_{ox} \frac{W}{L} \Delta V_{in} \sqrt{\frac{4I_{SS}}{\mu_{n} C_{ox} \frac{W}{L}} - \Delta V_{in}^{2}} - \sqrt{\frac{2I_{SS}}{\mu_{n} C_{ox} \frac{W}{L}}}} \sqrt{\frac{2I_{SS}}{\mu_{n} C_{ox} \frac{W}{L}}} V_{in}}$$

$$(51)$$

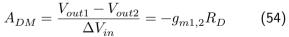
Differential-Mode (DC Analysis)

$$G_m = \frac{\partial \Delta I_D}{\partial \Delta V_{in}} \tag{52}$$

Cascode

$$= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \frac{\frac{4I_{ss}}{\mu_n C_{ox} \frac{W}{L}} - 2\Delta V_{in}^2}{\sqrt{\frac{4I_{ss}}{\mu_n C_{ox} \frac{W}{L}} - \Delta V_{in}^2}}$$
(53)

At $\Delta V_{in} = 0$.

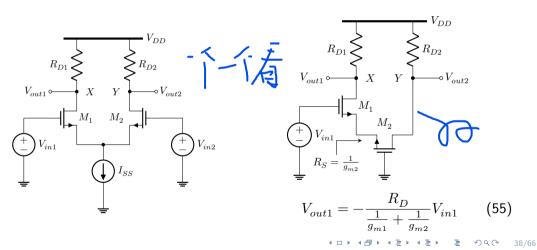


- Larger I_{SS} leads to higher G_m and wider input range.
- Smaller W/L leads to lower G_m but wider input range.





Differential-Mode (Superposition)

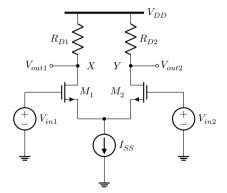




Small-signal Analysis

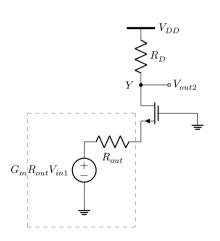
$$\lambda \neq 0$$

$$\gamma \neq 0$$



Differential-Mode (Superposition)



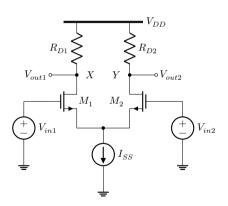


$$G_m = g_{m1} \tag{56}$$

$$R_{out} = \frac{1}{g_{m1}} \tag{57}$$

$$V_{out2} = \frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} V_{in1}$$
 (58)

Differential-Mode (Superposition)



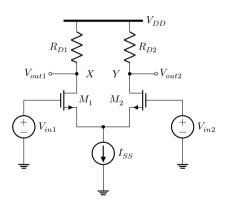
$$V_{out1} - V_{out2} = -\frac{2R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} V_{in1}$$
 (59)

$$=-g_m R_D V_{in1} (60)$$

$$-V_{out1} + V_{out2} = -\frac{2R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} V_{in2}$$
 (61)

$$= -g_m R_D V_{in2} \tag{62}$$



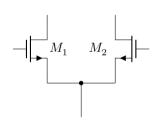


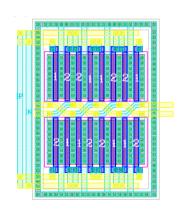
$$A_{DM} = \frac{V_{out1} - V_{out2}}{V_{in1} - V_{in2}}$$
 (63)

$$= -g_m R_D \tag{64}$$

Differential pair







Example

Recap of Last Lecture



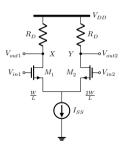
Calculate the A_{DM} of the differential pair below if the biasing conditions of M_1 and M_2 are the same.

Cascode

$$V_{out1} - V_{out2} = -\frac{2R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} V_{in1} = -\frac{4}{3} g_m R_D V_{in1}$$
 (65)

$$V_{out1} - V_{out2} = -\frac{2R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} V_{in1} = -\frac{1}{3} g_m R_D V_{in1}$$
 (65)
$$-V_{out1} - V_{out2} = -\frac{2R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} V_{in2} = -\frac{4}{3} g_m R_D V_{in2}$$
 (66)
$$V_{out1} - V_{out2} = -\frac{2R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} V_{in2} = -\frac{4}{3} g_m R_D V_{in2}$$
 (66)

$$A_{DM} = \frac{V_{out1} - V_{out2}}{V_{in1} - V_{in2}} = -\frac{4}{3}g_m R_D \tag{67}$$

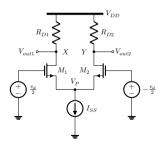


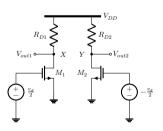
Small-Signal, Half-circuit ($\lambda \neq 0$, $\gamma \neq 0$)



- Assume the circuit is fully symmetric.
- For $i_{d1}+i_{d2}=0$ and $g_{m1}\frac{v_d}{2}+g_{m2}(-\frac{v_d}{2})=0$, V_P must be a constant voltage in DC analysis and a virtual ground in small-signal analysis.

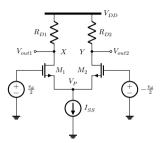
Cascode

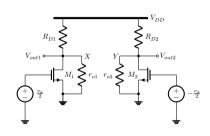




Small-Signal, Half-circuit ($\lambda \neq 0$, $\gamma \neq 0$)







$$V_{out1} = -g_m(R_D \parallel r_o) \frac{v_d}{2} \tag{68}$$

$$V_{out2} = -g_m(R_D \parallel r_o)(-\frac{v_d}{2}) \tag{69}$$

$$A_{DM} = \frac{V_{out1} - V_{out2}}{v_d} = -g_m(R_D \parallel r_o)$$
 (70)

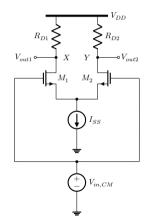
Common-Mode Response ($\lambda \neq 0$, $\gamma \neq 0$)



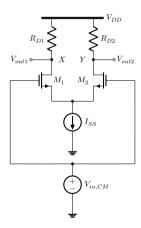
If the circuit is fully symmetric,

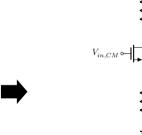
$$A_{CM-DM} = \frac{V_{out1} - V_{out2}}{v_{in,CM}} = 0$$
 (71)

$$CMRR = \left| \frac{A_{DM}}{A_{CM-DM}} \right| = \infty \tag{72}$$



Common-Mode Response ($\lambda \neq 0$, $\gamma \neq 0$)





Perturbing biasing condition \rightarrow Altering transconductance (g_m)

Common-Mode Response ($\lambda \neq 0$, $\gamma \neq 0$)



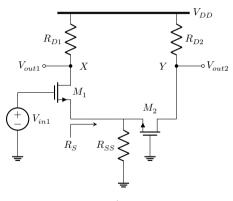
If the circuit is fully symmetric.

$$A_{CM} = \frac{V_{out,CM}}{V_{in,CM}} \tag{73}$$

$$= \frac{-2g_m \frac{r_o}{2}}{R_{SS} + \frac{r_o}{2} + (2g_m + 2g_{mb2})\frac{r_o}{2}R_{SS}} \cdot \frac{\left[R_{SS} + \frac{r_o}{2} + (2g_m + 2g_{mb})\frac{r_o}{2}R_{SS}\right]\frac{R_D}{2}}{\left[R_{SS} + \frac{r_o}{2} + (2g_m + 2g_{mb})\frac{r_o}{2}R_{SS}\right] + \frac{R_D}{2}}$$
(74)

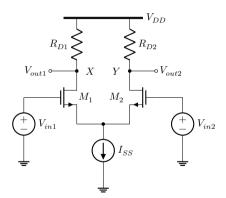
$$=0 \quad \text{if } R_{SS} = \infty \tag{75}$$

Nonzero R_{SS} ($\lambda=0$, $\gamma=0$)



$$R_S = \frac{1}{g_{m2}} \parallel R_{SS} \tag{76}$$

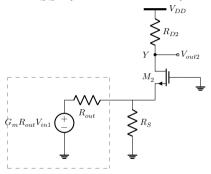
Nonzero R_{SS} ($\lambda=0$, $\gamma=0$)



$$V_{out1} = -\frac{R_D}{\frac{1}{g_{m1}} + \left(\frac{1}{g_{m2}} \parallel R_{SS}\right)} V_{in1} \quad (77)$$



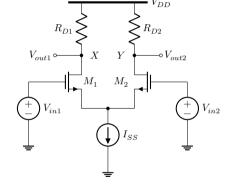
Nonzero R_{SS} ($\lambda=0$, $\gamma=0$)



$$G_m = g_{m1} \tag{78}$$

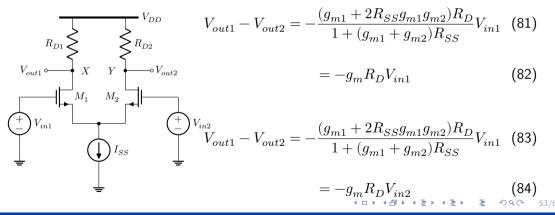
(79)

$$R_{out} = \frac{1}{g_{m1}}$$

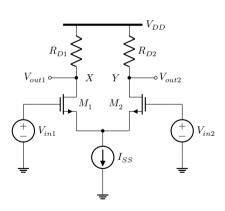


$$V_{out2} = -rac{rac{R_{SS}}{R_{SS} + rac{1}{g_{m2}}} R_D}{rac{1}{g_{m2}} + \left(rac{1}{g_{m2}} \parallel R_{SS}
ight)} V_{in1} \quad (80)$$

Nonzero R_{ss}





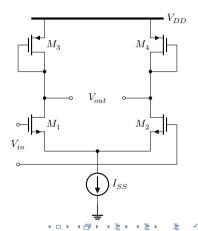


$$A_{DM} = \frac{V_{out1} - V_{out2}}{V_{in1} - V_{in2}} = -g_m R_D$$
 (85)



Higher A_{DM}

- \rightarrow Smaller $(W/L)_P$
- \rightarrow Larger $(V_{SGP} V_{THP})$
- ullet \to Smaller $V_{in.CM}$ headroom



Cascode



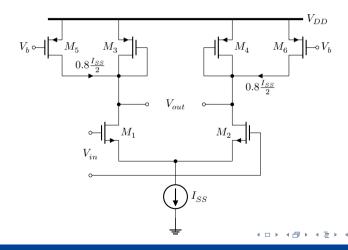
$$V_{out1} = -g_{mN}(r_{oN} \parallel r_{oP} \parallel \frac{1}{g_{mP}}) \frac{v_d}{2}$$
 (86)

$$V_{out2} = -g_{mN}(r_{oN} \parallel r_{oP} \parallel \frac{1}{g_{mP}})(-\frac{v_d}{2}) \tag{87}$$

$$A_{DM} = \frac{V_{out1} - V_{out2}}{v_d} = -g_{mN} \left(r_{oN} \parallel r_{oP} \parallel \frac{1}{g_{mP}} \right)$$
 (88)

$$\approx -\frac{g_{mN}}{g_{mP}} \approx -\sqrt{\frac{\mu_n(W/L)_N}{\mu_p(W/L)_P}} \tag{89}$$





Cascode

A_{DM} with MOS Loads

$$V_{out1} = -g_{m1,2} \left(r_{o1,2} \parallel r_{o3,4} \parallel \frac{1}{g_{m3,4}} \parallel r_{o5,6} \right) \frac{v_d}{2}$$
 (90)

$$V_{out2} = -g_{m1,2} \left(r_{o1,2} \parallel r_{o3,4} \parallel \frac{1}{g_{m3,4}} \parallel r_{o5,6} \right) \left(-\frac{v_d}{2} \right)$$
 (91)

$$A_{DM} = \frac{V_{out1} - V_{out2}}{v_d} \approx -\frac{g_{m1,2}}{g_{m3,4}} \approx -\sqrt{\frac{5\mu_n(W/L)_{1,2}}{\mu_p(W/L)_{3,4}}}$$
(92)

A_{DM} with MOS Loads ($\lambda \neq 0$, $\gamma \neq 0$)

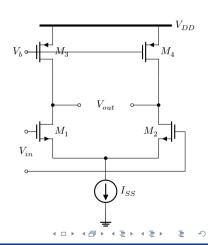


$$V_{out1} = -g_{m1,2}(r_{o1,2} \parallel r_{o3,4}) \frac{v_d}{2}$$
 (93)

$$V_{out2} = -g_{m1,2}(r_{o1,2} \parallel r_{o3,4})(-\frac{v_d}{2})$$
 (94)

$$A_{DM} = \frac{V_{out1} - V_{out2}}{v_d} \tag{95}$$

$$= -g_{m1,2}(r_{o1,2} \parallel r_{o3,4}) \tag{96}$$



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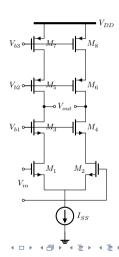


Higher R_{out}

Recap of Last Lecture

- \rightarrow High A_{DM}
- \rightarrow Small $V_{in,CM}$ headroom

Telescopic cascode



A_{DM} with MOS Loads ($\lambda \neq 0$, $\gamma \neq 0$)

$$V_{out1} \cong -g_{m1,2} \left\{ [r_{o1,2} + r_{o3,4} + (g_{m3,4} + g_{mb3,4})r_{o3,4}r_{o1,2}] \parallel \\ [r_{o7,8} + r_{o5,6} + (g_{m5,6} + g_{mb5,6})r_{o5,6}r_{o7,8}] \right\} \frac{v_d}{2}$$

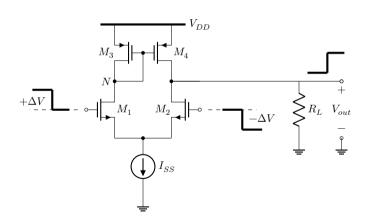
$$(97)$$

$$V_{out2} \cong -g_{m1,2} \left\{ [r_{o1,2} + r_{o3,4} + (g_{m3,4} + g_{mb3,4})r_{o3,4}r_{o1,2}] \parallel \\ [r_{o7,8} + r_{o5,6} + (g_{m5,6} + g_{mb5,6})r_{o5,6}r_{o7,8}] \right\} \left(-\frac{v_d}{2} \right)$$
(98)

$$A_{DM} = \frac{V_{out1} - V_{out2}}{v_d} \cong -g_{m1,2}[(g_{m3,4} + gmb_{3,4})r_{o3,4}r_{o1,2} \parallel (g_{m5,6} + g_{mb5,6})r_{o5,6}r_{o7,8}]$$

$$(99)$$

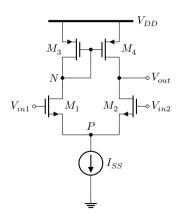
Differential Pair with Active Load



Asymmetric Differential Pair



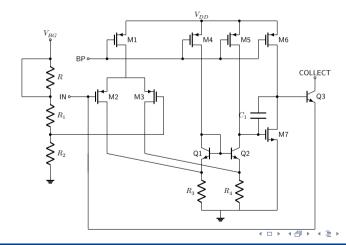
Why N is not on the right branch? Caveat: Because of the vastly different resistance magnitude at the drains of M_1 and M_2 , the voltage swings at these two nodes are different and therefore node P cannot be viewed as a virtual ground when $V_{in2}=-V_{in1}$.



Recap of Last Lecture

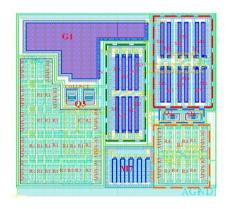


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OPA Layout





OPA Layout

Lecture 16: Cascode Amplifiers



