Lecture 4: Diode Circuits

ECE 3110J, Electronic Circuits

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2024 Summer

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Recap of Last Lecture

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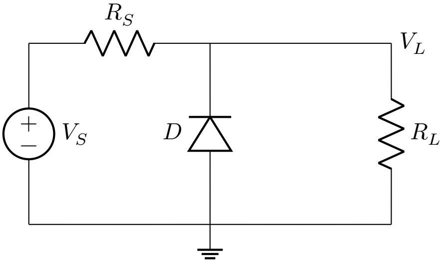
- Diodes
- Diode Circuits

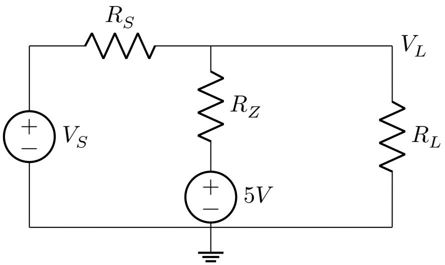
Line Regulation



- Line Regulation: how sensitive the output voltage (V_L) is to input voltage (V_S) changes, when $R_L = \infty$.

$$\text{Line Regulation} = \frac{\partial V_L}{\partial V_S} = \frac{R_Z}{R_S + R_Z} \tag{1}$$

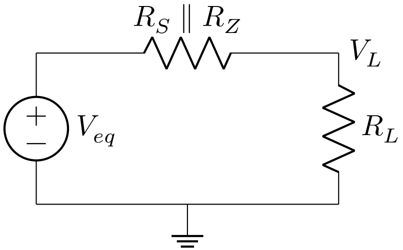
Load Regulation



- Load Regulation: output impedance of the voltage regulator.

$$\text{Load Regulation} = \frac{\partial V_L}{\partial I_L} = R_S \| R_Z \tag{2}$$

Thevenin Equivalent of a Voltage Regulator



$$V_{eq} = 5 + \frac{V_S - 5}{R_S + R_Z} R_Z \qquad (3)$$

$$V_L = V_{eq} \frac{R_L}{(R_S \| R_Z) + R_L} \qquad (4)$$

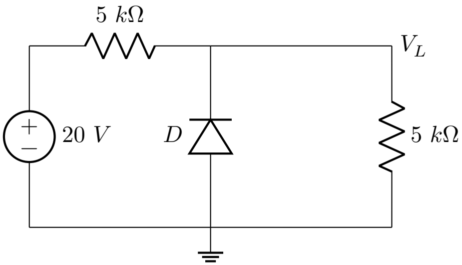
Thevenin Equivalent of a Voltage Regulator

Numerical Test: $V_S = 20\ V, R_S = 5\ k\Omega, R_Z = 0.1\ k\Omega, V_Z = \ 5\ V, R_L = 5\ k\Omega$

$$V_{eq} = 5 + \frac{20 - 5}{5\ k + 0.1\ k} 0.1\ k = 5.2941\ V \tag{5}$$

$$V_L = 5.2941 \times \frac{5\ k}{\frac{5\ k \times 0.1\ k}{5\ k + 0.1\ k} + 5\ k} = 5.1923\ V \tag{6}$$

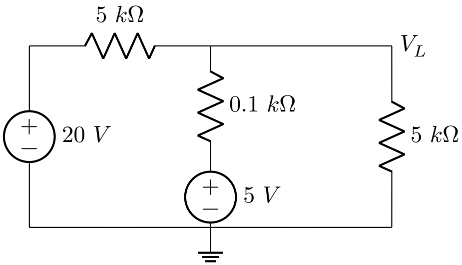
Line Regulation



- Line Regulation

$$\frac{R_Z}{R_Z + R_S} = \frac{0.1k}{5k + 0.1k} = 19.6\ mV/V \tag{7}$$

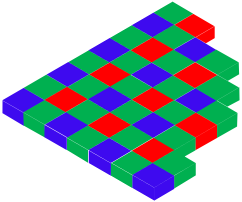
Load Regulation



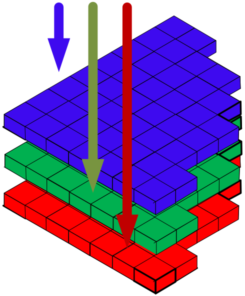
- Load Regulation

$$R_Z \parallel R_S = 5\ k \parallel 0.1\ k = 98\ \Omega \tag{8}$$

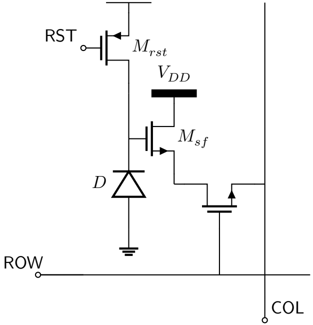
CMOS Image Sensors



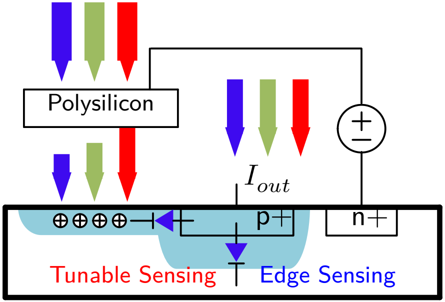
Bayer Filter

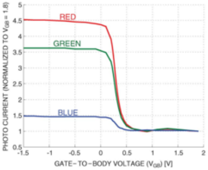


Foveon X3 sensor



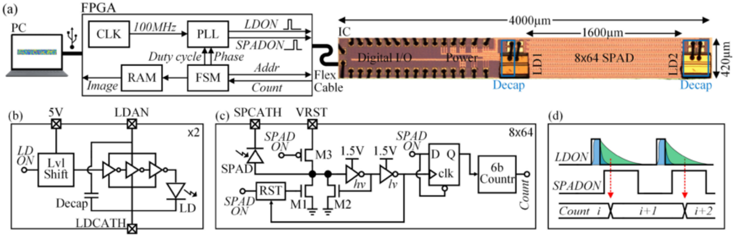
CIS Applications





D. Ho et al., "CMOS Tunable-Color Image Sensor With Dual-ADC Shot-Noise-Aware Dynamic Range Extension," in IEEE Transactions on Circuits and Systems I: Regular Papers.

CIS Applications



Jaebin Choi et al. "Fully Integrated Time-Gated 3D Fluorescence Imager for Deep Neural Imaging", in IEEE Transactions on Biomedical Circuits and Systems.

Sensitivity of Diode current to its Voltage

- Calculate V_a for a silicon diode with $I_S = 0.1\ fA$ and I_D increasing from 300 μA to 10 mA at 300 K.

$$300 \times 10^{-6} = \left(0.1 \times 10^{-15}\right)\left(e^{0.025875} - 1\right) \tag{9}$$

$$V_a = 0.743\ V \tag{10}$$

$$10 \times 10^{-3} = \left(0.1 \times 10^{-15}\right)\left(e^{\frac{V_a}{0.025875}} - 1\right) \tag{11}$$

$$V_a = 0.834\ V \tag{12}$$

Example

- Calculate I_S for a silicon diode with $I_D = 2.5\ mA$ and $V_a = 0.736\ V$ at 50 $^{\circ}C$.

$$2.5 \times 10^{-3} = I_S \left(\frac{1.6 \times 10^{-19} \cdot 0.736}{e^{1.38 \times 10^{-23} \cdot 323}} - 1 \right) \tag{13}$$

$$I_S = 8.4\ fA \tag{14}$$

Example

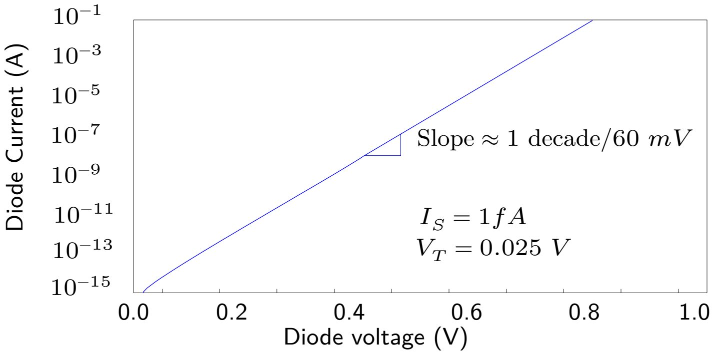
- Calculate the required V_a for I_D of a silicon diode to increase by a factor 10 at 300 K. Assume $I_D \gg I_S$.

$$\begin{cases} I_{D1} = I_S\left(e^{\frac{qV_{a1}}{kT}} - 1\right) \approx I_S e^{\frac{qV_{a1}}{kT}} \\ I_{D2} = I_S\left(e^{\frac{qV_{a2}}{kT}} - 1\right) \approx I_S e^{\frac{qV_{a2}}{kT}} \end{cases} \tag{15}$$

$$\frac{I_{D2}}{I_{D1}} = 10 = \frac{I_S e^{\frac{qVa2}{kT}}}{I_S e^{\frac{qV_{a1}}{kT}}} = e^{\frac{V_{a2}-V_{a1}}{0.025875}} \tag{16}$$

$$V_{a2} - V_{a1} = 0.025875 \times \ln 10 = 0.05958\ V \approx 60\ mV \tag{17}$$

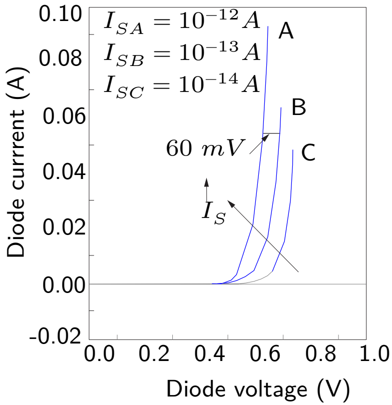
Sensitivity of Diode Current



Diode $i - v$ characteristic on semilog scale

- The diode voltage changes by about 60 mV per decade change in diode current.

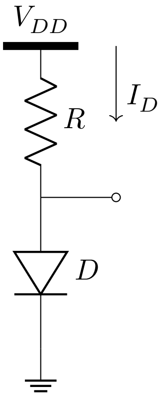
I_D and I_S versus V_a



$$I_D = I_S(e^{\frac{qV_a}{kT}} - 1) \tag{18}$$

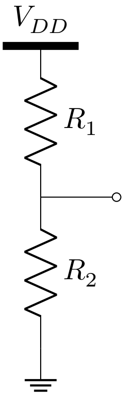
- At the same I_D, when I_S increases by 10, V_a decreases by 60 mV.
- At the same I_S, when I_D increases by 10, V_a increases by 60 mV.

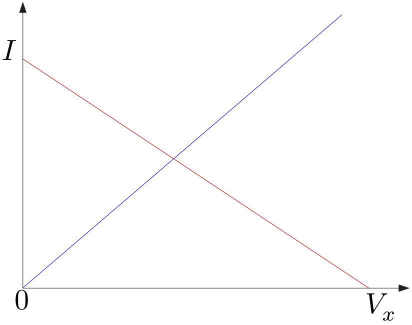
Large Signal Model



$$I_D = \frac{V_{DD} - V_D}{R} \tag{19}$$

Graphical "Load Line" Method





Graphical "Load Line" Method

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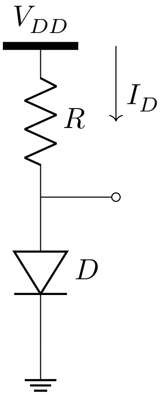
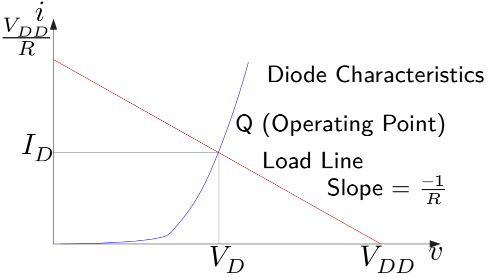
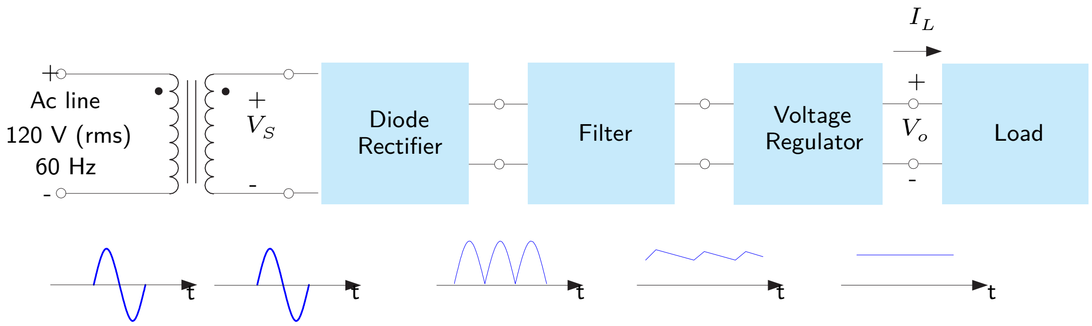


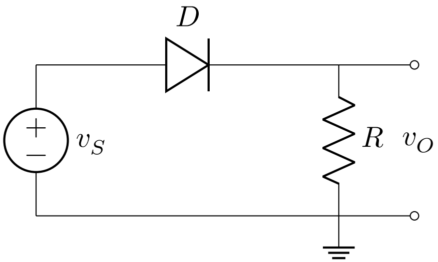


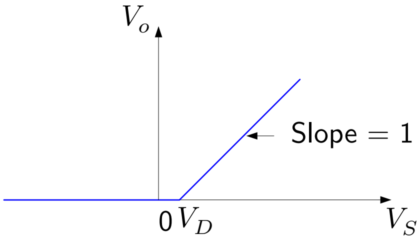
- Can be used for non-linear devices as well.
- It is very useful in future when we discuss transistors.

Block Diagram of a DC Power Supply

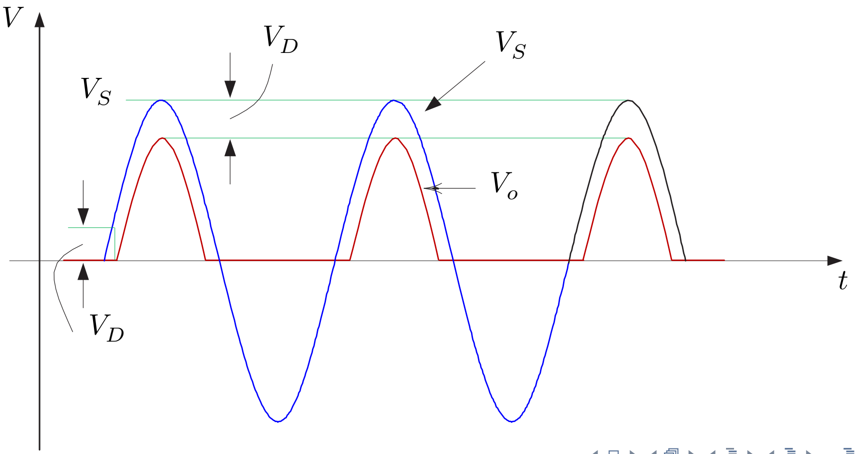


Half-Wave Rectifier with Resistive Load





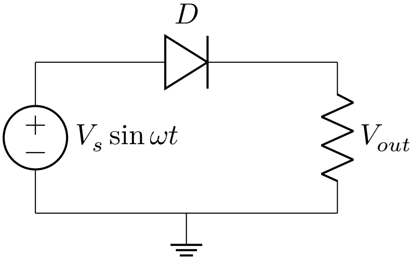
Half-Wave Rectifier with Resistive Load



Half-Wave Rectifier with Resistive Load

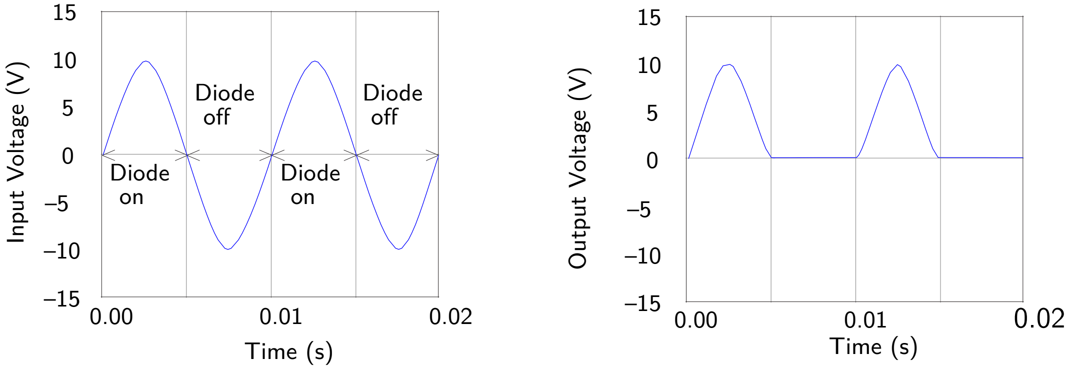
$$V_s = 10\text{V} \tag{20}$$

$$\omega = 2\pi f = 2\pi \frac{1}{0.01} = 200\pi(\text{rad/sec}) \tag{21}$$

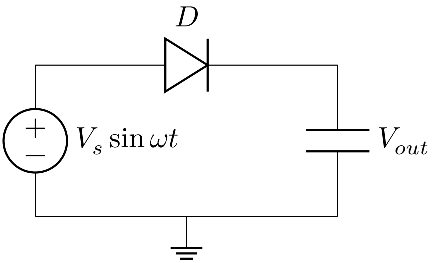


- V_{out} is not DC.

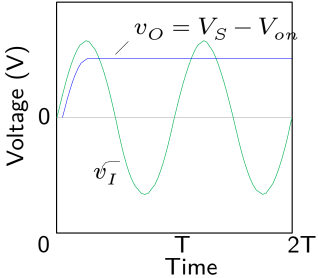
Half-Wave Rectifier with Resistive Load



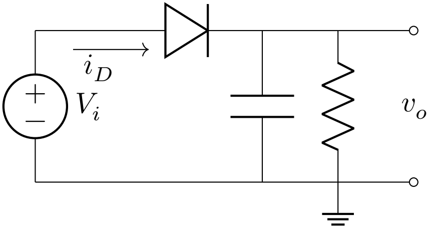
Half-Wave Rectifier with Capacitive Load (Peak Rectifier)

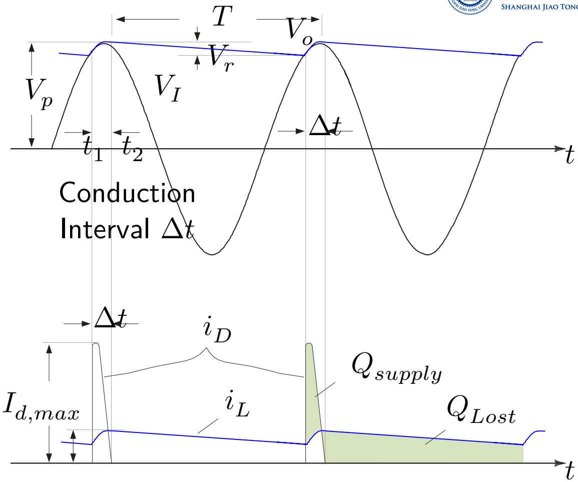


- V_{out} is DC



Peak Rectifier with Load



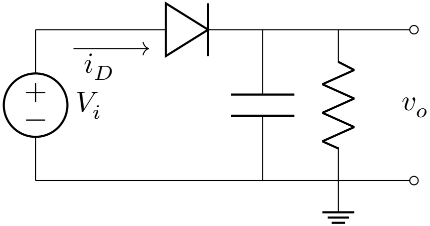


Peak Rectifier with Load

$$i_L = V_o/R \tag{22}$$

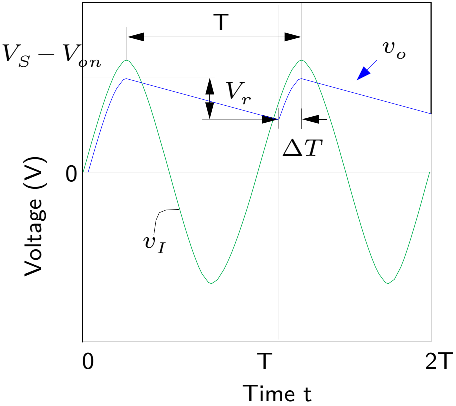
$$i_D = i_C + i_L = C\frac{dv_I}{dt} + i_L \tag{23}$$

$$V_p - V_r \simeq V_p e^{-T/CR} \tag{24}$$



- A very large capacitor is required to reduce the ripple

Half-Wave Rectifier with RC Load (I)

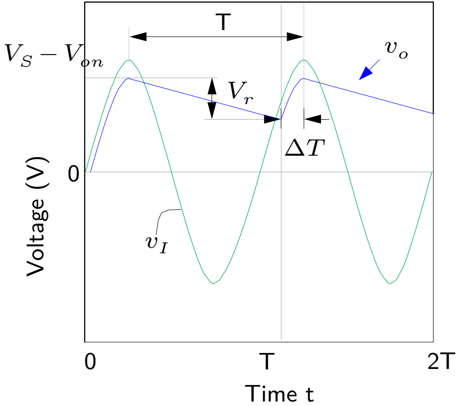


Here “DC” means we approximate the discharging period with a constant voltage.

$$V_{dc} = V_s - V_{on} \tag{25}$$

$$I_{dc} = \frac{V_{dc}}{R} \tag{26}$$

Calculating Ripple Voltage

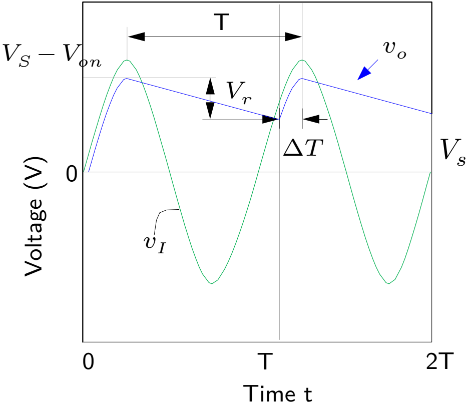


$$V_r = (V_s - V_{on})\left(1 - e^{-\frac{T-\Delta T}{RC}}\right) \quad (27)$$

$$\cong (V_s - V_{on})\left(\frac{T-\Delta T}{RC}\right) \text{ if } (T-\Delta T) \ll RC \quad (28)$$

$$\cong (V_s - V_{\text{on}})\left(\frac{T}{RC}\right) \text{ if } \Delta T \ll T \quad (29)$$

Calculating Conduction Angle and Interval

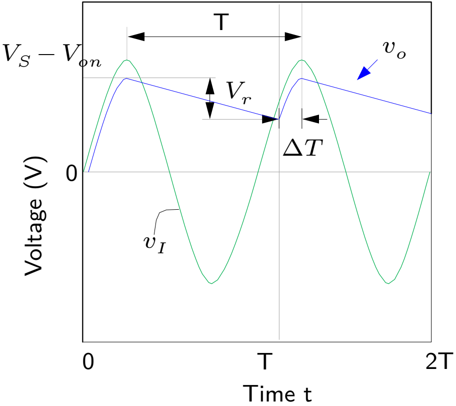


$$V_s \sin\left[\omega\left(\frac{5T}{4} - \Delta T\right)\right] - V_{on} = (V_s - V_{on}) - V_r \quad (30)$$

$$V_s \sin\left(\frac{5\pi}{2} - \theta_c\right) - V_{on} = (V_s - V_{on}) - V_r \quad (31)$$

$$V_s \cos\theta_c = V_s - V_r \quad (32)$$

Half-Wave Rectifier with RC Load

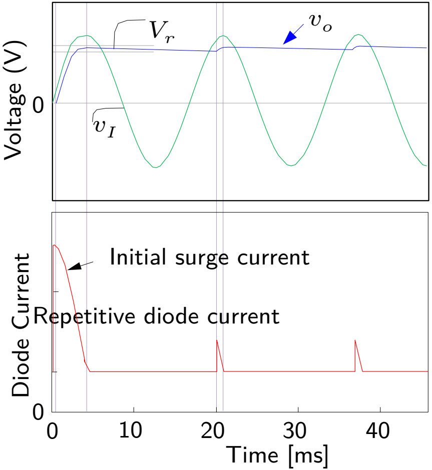


$$\cos\theta_c = \frac{V_s - V_r}{V_s} \simeq 1 - \frac{\theta_c^2}{2} \text{ if } \theta_c \text{ very small} \tag{33}$$

$$\theta_c = \sqrt{\frac{2V_r}{V_s}} \tag{34}$$

$$\Delta T = \frac{\theta_c}{\omega} = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_s}} \tag{35}$$

Derive Peak Current

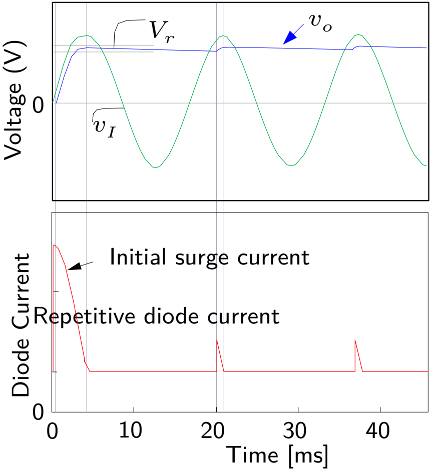


The charge filled on C during ΔT is discharged during $T - \Delta T$

$$Q \cong \frac{I_{\text{peak}}\,\Delta T}{2} = I_{dc}(T - \Delta T) \cong I_{dc}T \qquad (36)$$

$$I_{\text{peak}} = \frac{2I_{\text{dc}}\,T}{\Delta T} \qquad (37)$$

Derive Surge Current



During charging period (ΔT), almost all diode current goes to C.

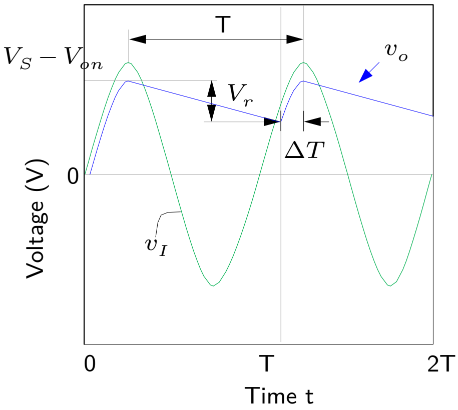
$$\left|\frac{1}{SC}\right| = \frac{1}{2\pi\frac{C}{T}} \tag{38}$$

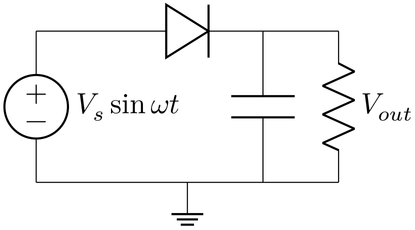
$$= \frac{T}{2\pi C} \ll R \quad \text{if } RC \gg T \tag{39}$$

$$I_{\text{surge}} = C\frac{d\left(V_s \sin\omega t - V_{\text{on}}\right)}{dt} \tag{40}$$

$$= \omega C V_s \text{ if } t = 0 \tag{41}$$

Half-Wave Rectifier with RC Load (IV)





- Peak-inverse-voltage $\cong 2V_s - V_{on}$
- If too large, the diode breaks down.

Example

Find the value of the dc output voltage, dc output current, ripple voltage, conduction interval, conduction angle and diode peak current for a half-wave rectifier driven from a transformer having a secondary voltage of 12.6 V_{rms} (60 Hz) with R=15 Ω and C=25,000 μF. Assume V_{on}=1 V.

$$V_{dc} = 12.6\sqrt{2} - 1 = 16.8\ V \tag{42}$$

$$I_{dc} = \frac{16.8}{15} = 1.12\ A \tag{43}$$

$$V_r \cong V_{dc}\frac{T}{RC} = 16.8\frac{\frac{1}{60}}{15 \times 25000 \times 10^{-6}} = 0.747\ V \tag{44}$$

Example

$$\theta_c \cong \sqrt{\frac{2\ V_r}{V_s}} = \sqrt{\frac{2 \times 0.747}{12.6 \times \sqrt{2}}} = 0.29\ (rad) \text{ or } 16.6^\circ \tag{45}$$

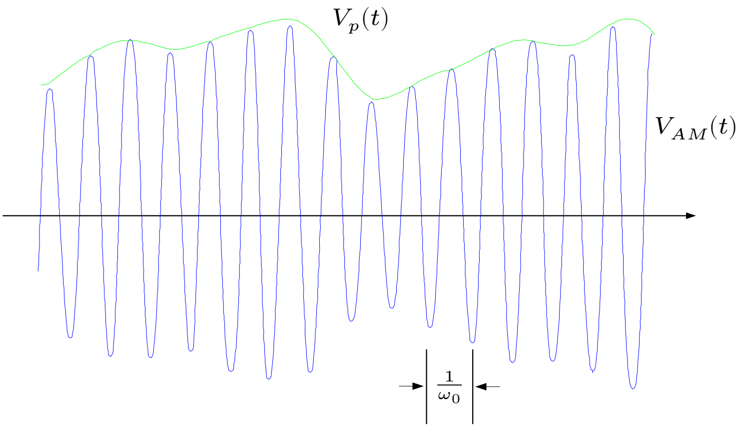
$$\Delta T \cong \frac{\theta_c}{\omega} = \frac{0.29}{2\pi \times 60} = 7.69 \times 10^{-4}\ (sec) \tag{46}$$

$$I_{\text{peak}} = \frac{2 \times 1.12 \times \frac{1}{60}}{7.69 \times 10^{-4}} = 48.6\ A \tag{47}$$

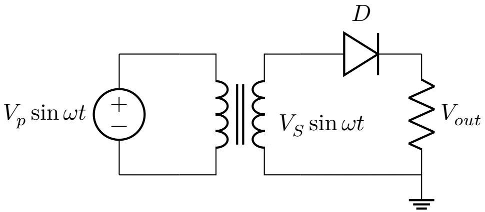
Example

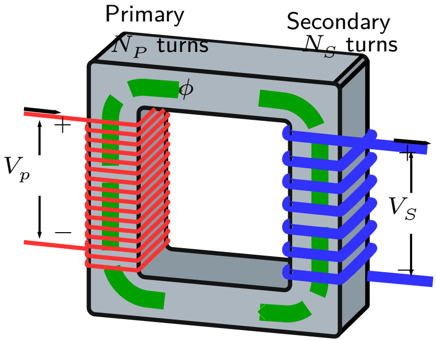
- Make sure all assumptions are valid.
- Since R is small (15 Ω), C needs to be large (25,000 μF) to maintain a low V_r.
- The diode must be able to handle these repetitively high peak currents.

Peak Rectifier in Communication



Transformer





- The output of an ideal transformer can be represented as an ideal voltage source.

Transformer

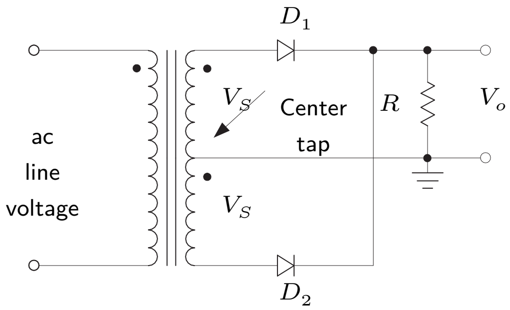
- By Faraday's law of induction

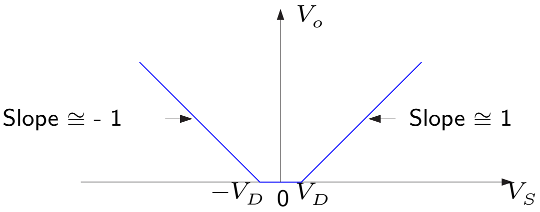
$$V_s = -N_s \frac{d\phi}{dt} \tag{48}$$

$$V_p = -N_p \frac{d\phi}{dt} \tag{49}$$

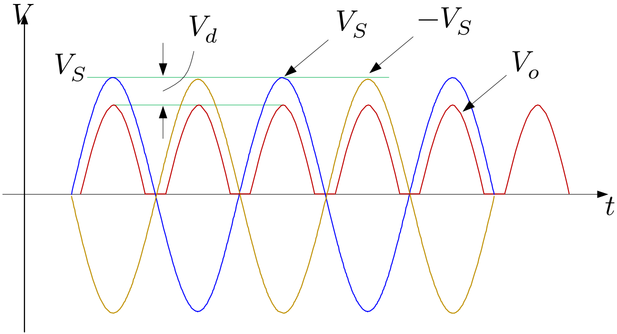
$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \tag{50}$$

Full Wave Rectifier

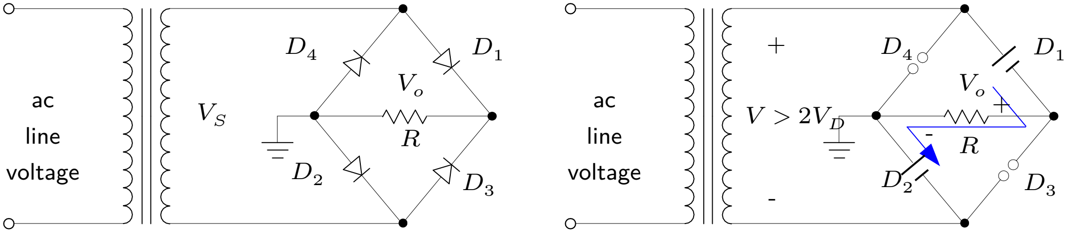




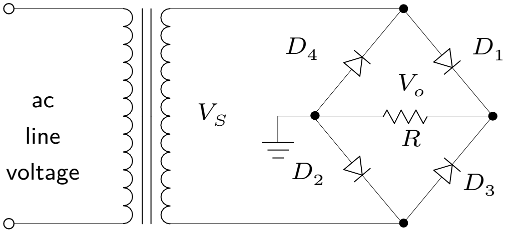
Full Wave Rectifier

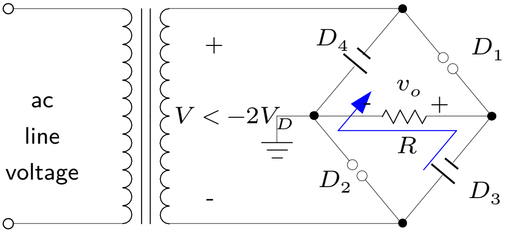


Full Wave Rectifier ($V > 2V_D$)

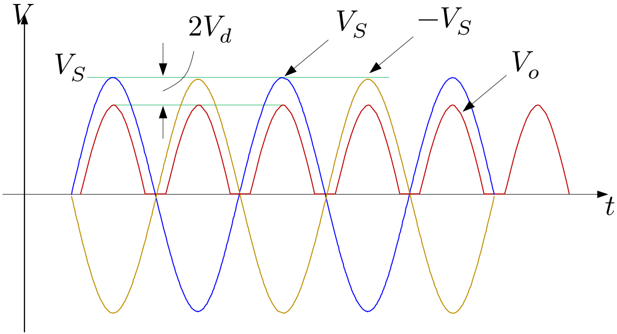


Full Wave Rectifier ($V < -2V_D$)

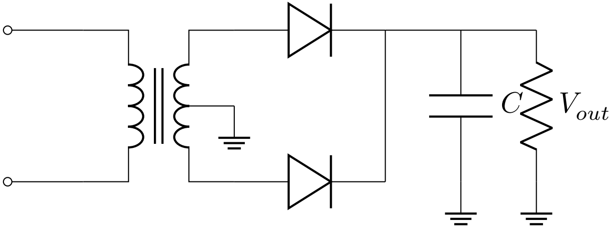


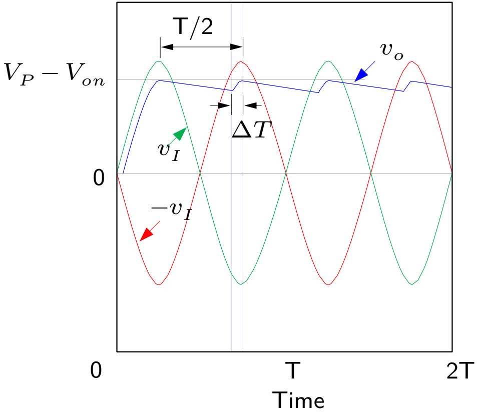


Full Wave Rectifier



Full Wave Rectifier (I)





Full Wave Rectifier (I)

$$V_{dc} = V_s - V_{on} \tag{51}$$

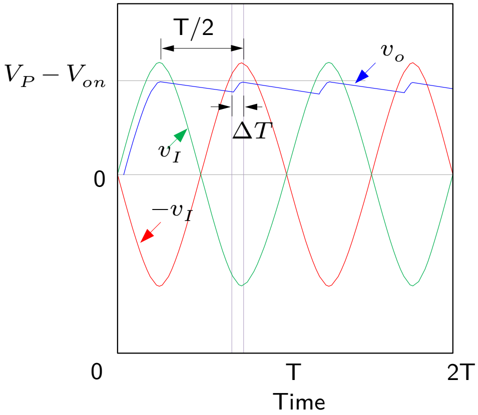
$$I_{dc} = \frac{V_{dc}}{R} \tag{52}$$

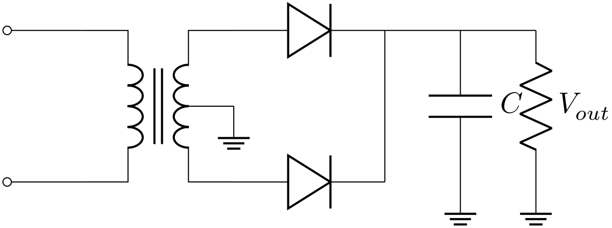
$$V_r = (V_s - V_{on}) \left(1 - e^{-\frac{T/2 - \Delta T}{RC}}\right) \tag{53}$$

$$\cong (V_s - V_{on}) \left(\frac{T/2 - \Delta T}{RC}\right) \text{ if } \left(\frac{T}{2} - \Delta T\right) \ll RC \tag{54}$$

$$\cong (V_s - V_{on}) \left(\frac{T}{2RC}\right) \text{ if } \Delta T \ll \frac{T}{2} \tag{55}$$

Full Wave Rectifier (II)





Full Wave Rectifier (II)

$$-V_s \sin\left[\omega\left(\frac{3T}{4} - \Delta T\right)\right] - V_{on} = (V_s - V_{on}) - V_r \tag{56}$$

$$-V_s \sin\left(\frac{3\pi}{2} - \theta_c\right) - V_{on} = (V_s - V_{on}) - V_r \tag{57}$$

$$V_s \cos\theta_c = V_s - V_r \tag{58}$$

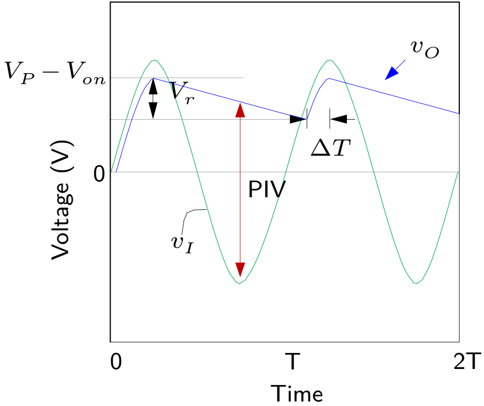
$$\theta_c = \sqrt{\frac{2V_r}{V_s}} \tag{59}$$

Full Wave Rectifier(II)

$$\cos\theta_c = \frac{V_s - V_r}{V_s} \cong 1 - \frac{\theta_c^2}{2} \text{ if } \theta_c \text{ very small} \quad (60)$$

$$\Delta T = \frac{\theta_c}{\omega} = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_s}} \quad (61)$$

Full Wave Rectifier (III)



$$Q \cong \frac{I_{\text{peak}}\,\Delta T}{2} = I_{dc}\left(\frac{T}{2} - \Delta T\right) \cong I_{dc}\frac{T}{2} \qquad (62)$$

$$I_{\text{peak}} = \frac{I_{dc}T}{\Delta T} \qquad (63)$$

$$I_{\text{surge}} = \omega C V_s \qquad (64)$$

$$\text{PIV} = 2V_s - V_{on} \qquad (65)$$