Lecture 17: Differential Amplifiers

VE 311 Analog Circuits

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Recap of Last Lecture

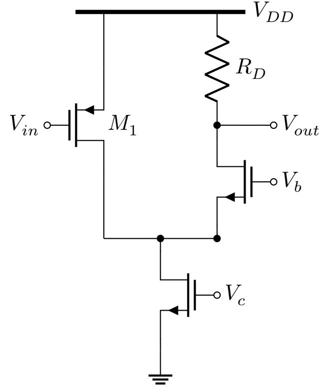
- Common Gate
- Cascode (Telescopic, Folded)
- Differential Amplifier (Current Equation, Superposition)

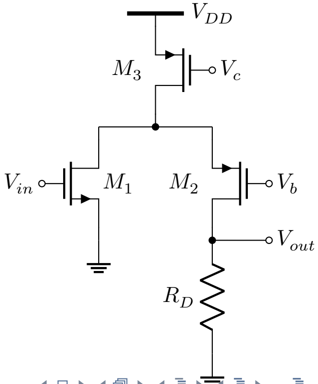
Topics to be Covered

- Differential Amplifier (Half circuit method)
- More on differential amplifier topologies
- Current mirror

Folded Cascode ($\lambda \neq 0$, $\gamma \neq 0$)

We don't care about M_3's non-ideality.





Folded Cascode ($\lambda \neq 0$, $\gamma \neq 0$)

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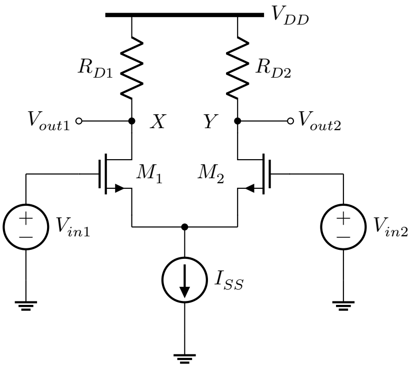
$$G_m = -g_{m1} \frac{(r_{o1} \parallel r_{o3})}{(r_{o1} \parallel r_{o3}) + \left(r_{o2} \parallel \frac{1}{g_{m2}+g_{mb2}}\right)} \tag{1}$$

$$R_{out} = [(r_{o1} \parallel r_{o3}) + r_{o2} + (g_{m2} + g_{mb2}) r_{o2} (r_{o1} \parallel r_{o3})] \parallel R_D \tag{2}$$

$$A_v = G_m R_{out} \tag{3}$$

Differential-Mode (Superposition)

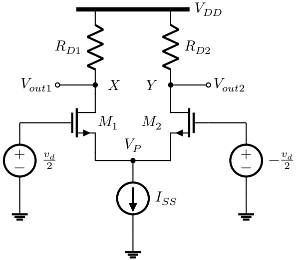


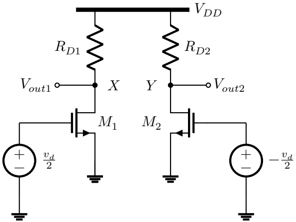
$$A_{DM} = \frac{V_{out1} - V_{out2}}{V_{in1} - V_{in2}} \tag{4}$$

$$= -g_m R_D \tag{5}$$

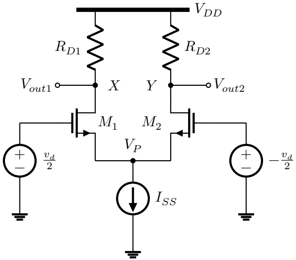
Small-Signal, Half-circuit ($\lambda \neq 0$, $\gamma \neq 0$)

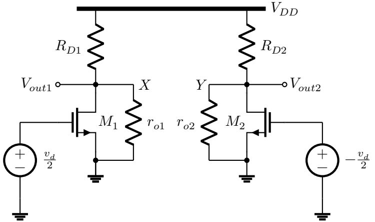
- Assume the circuit is fully symmetric.
- For $i_{d1} + i_{d2} = 0$ and $g_{m1}\frac{v_d}{2} + g_{m2}(-\frac{v_d}{2}) = 0$, V_P must be a constant voltage in DC analysis and a virtual ground in small-signal analysis.





Small-Signal, Half-circuit ($\lambda \neq 0$, $\gamma \neq 0$)





$$V_{out1} = -g_m(R_D \parallel r_o)\frac{v_d}{2} \tag{6}$$

$$V_{out2} = -g_m(R_D \parallel r_o)(-\frac{v_d}{2}) \tag{7}$$

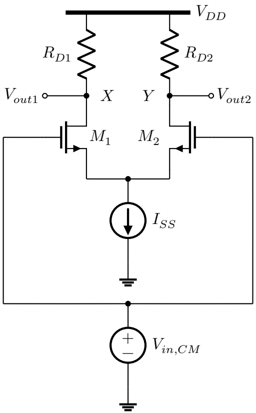
$$A_{DM} = \frac{V_{out1} - V_{out2}}{v_d} = -g_m(R_D \parallel r_o) \tag{8}$$

Common-Mode Response ($\lambda \neq 0$, $\gamma \neq 0$)

If the circuit is fully symmetric,

$$A_{CM-DM} = \frac{V_{out1} - V_{out2}}{v_{in,CM}} = 0 \qquad (9)$$

$$CMRR = \left| \frac{A_{DM}}{A_{CM-DM}} \right| = \infty \qquad (10)$$



Common-Mode Response ($\lambda \neq 0$, $\gamma \neq 0$)

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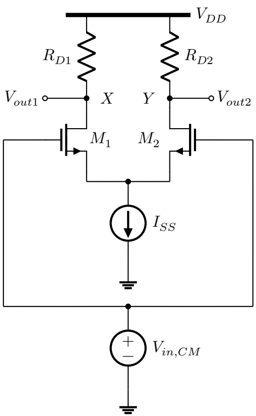
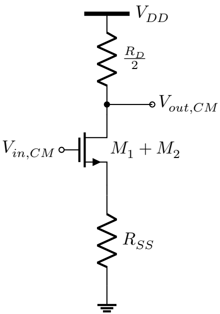




Perturbing biasing condition $\rightarrow$ Altering transconductance (g_m)

Common-Mode Response ($\lambda \neq 0$, $\gamma \neq 0$)

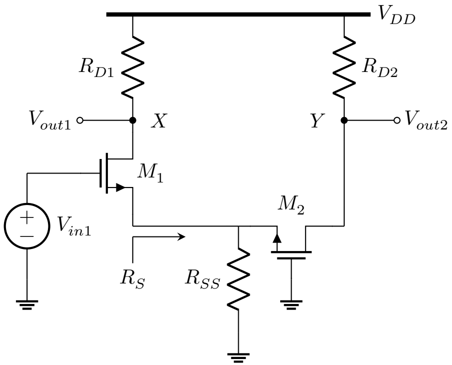
If the circuit is fully symmetric,

$$A_{CM} = \frac{V_{out,CM}}{V_{in,CM}} \tag{11}$$

$$= \frac{-2g_m \frac{r_o}{2}}{R_{SS} + \frac{r_o}{2} + (2g_m + 2g_{mb2})\frac{r_o}{2} R_{SS}} \cdot \frac{\left[R_{SS} + \frac{r_o}{2} + (2g_m + 2g_{mb})\frac{r_o}{2} R_{SS}\right] \frac{R_D}{2}}{\left[R_{SS} + \frac{r_o}{2} + (2g_m + 2g_{mb})\frac{r_o}{2} R_{SS}\right] + \frac{R_D}{2}} \tag{12}$$

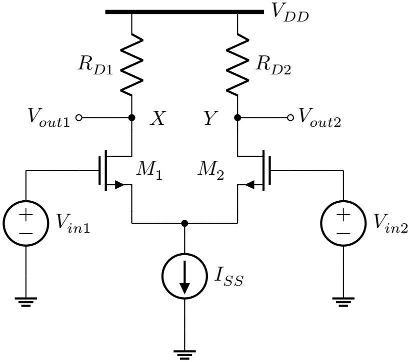
$$= 0 \quad \text{if } R_{SS} = \infty \tag{13}$$

Nonzero R_{SS} ($\lambda = 0$, $\gamma = 0$)



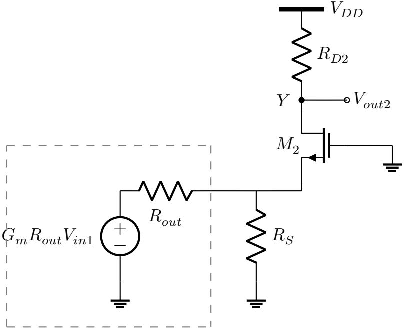
$$R_S = \frac{1}{g_{m2}} \parallel R_{SS} \tag{14}$$

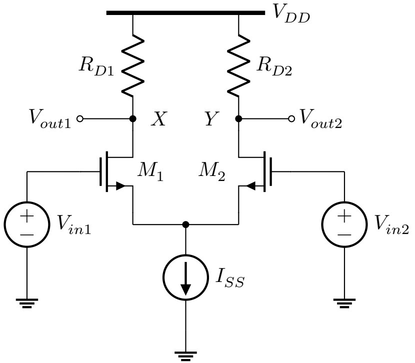
Nonzero R_{SS} ($\lambda = 0$, $\gamma = 0$)



$$V_{out1} = -\frac{R_D}{\frac{1}{g_{m1}} + \left(\frac{1}{g_{m2}} \parallel R_{SS}\right)} V_{in1} \quad (15)$$

Nonzero R_{SS} ($\lambda = 0$, $\gamma = 0$)



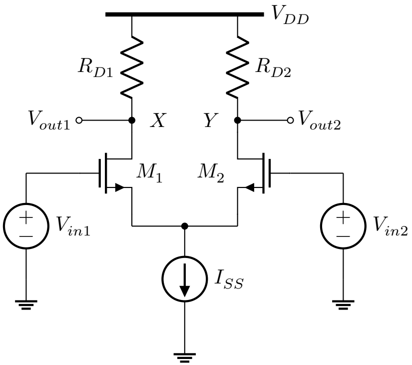


$$G_m = g_{m1} \tag{16}$$

$$R_{out} = \frac{1}{g_{m1}} \tag{17}$$

$$V_{out2} = -\frac{\frac{R_{SS}}{R_{SS}+\frac{1}{g_{m2}}} R_D}{\frac{1}{g_{m1}} + \left(\frac{1}{g_{m2}} \parallel R_{SS}\right)} V_{in1} \tag{18}$$

Nonzero R_{ss}



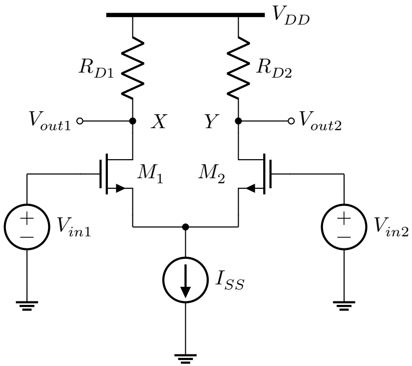
$$V_{out1} - V_{out2} = -\frac{(g_{m1} + 2R_{SS}g_{m1}g_{m2})R_D}{1 + (g_{m1} + g_{m2})R_{SS}} V_{in1} \quad (19)$$

$$= -g_m R_D V_{in1} \quad (20)$$

$$V_{out1} - V_{out2} = -\frac{(g_{m1} + 2R_{SS}g_{m1}g_{m2})R_D}{1 + (g_{m1} + g_{m2})R_{SS}} V_{in1} \quad (21)$$

$$= -g_m R_D V_{in2} \quad (22)$$

Nonzero R_{ss}

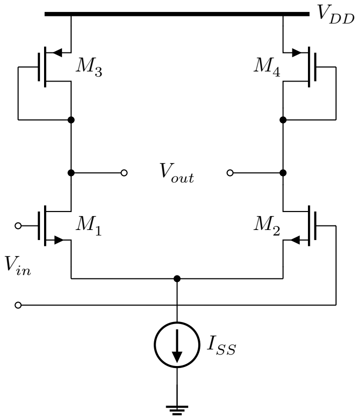


$$A_{DM} = \frac{V_{out1} - V_{out2}}{V_{in1} - V_{in2}} = -g_m R_D \qquad (23)$$

A_{DM} with MOS Loads ($\lambda \neq 0$, $\gamma \neq 0$)

Higher A_{DM}

- $\rightarrow$ Smaller $(W/L)_P$
- $\rightarrow$ Larger $(V_{SGP} – V_{THP})$
- $\rightarrow$ Smaller $V_{in,CM}$ headroom



A_{DM} with MOS Loads ($\lambda \neq 0$, $\gamma \neq 0$)

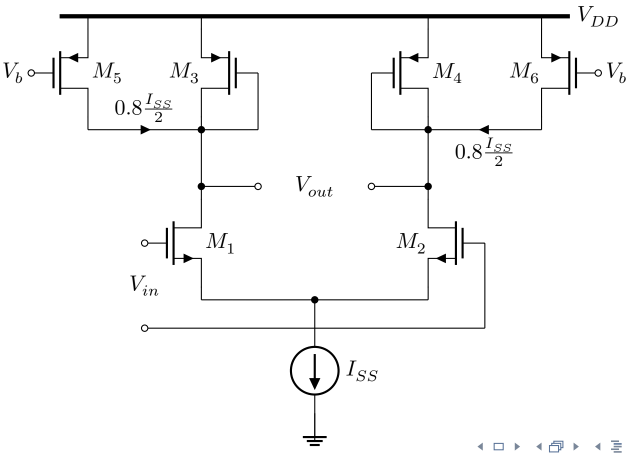
$$V_{out1} = -g_{mN}(r_{oN} \parallel r_{oP} \parallel \frac{1}{g_{mP}})\frac{v_d}{2} \tag{24}$$

$$V_{out2} = -g_{mN}(r_{oN} \parallel r_{oP} \parallel \frac{1}{g_{mP}})(-\frac{v_d}{2}) \tag{25}$$

$$A_{DM} = \frac{V_{out1} - V_{out2}}{v_d} = -g_{mN}\left(r_{oN} \parallel r_{oP} \parallel \frac{1}{g_{mP}}\right) \tag{26}$$

$$\approx -\frac{g_{mN}}{g_{mP}} \approx -\sqrt{\frac{\mu_n (W/L)_N}{\mu_p (W/L)_P}} \tag{27}$$

A_{DM} with MOS Loads ($\lambda \neq 0$, $\gamma \neq 0$)



A_{DM} with MOS Loads

$$V_{out1} = -g_{m1,2}\left(r_{o1,2} \parallel r_{o3,4} \parallel \frac{1}{g_{m3,4}} \parallel r_{o5,6}\right)\frac{v_d}{2} \tag{28}$$

$$V_{out2} = -g_{m1,2}\left(r_{o1,2} \parallel r_{o3,4} \parallel \frac{1}{g_{m3,4}} \parallel r_{o5,6}\right)(-\frac{v_d}{2}) \tag{29}$$

$$A_{DM} = \frac{V_{out1} - V_{out2}}{v_d} \approx -\frac{g_{m1,2}}{g_{m3,4}} \approx -\sqrt{\frac{5\mu_n(W/L)_{1,2}}{\mu_p(W/L)_{3,4}}} \tag{30}$$

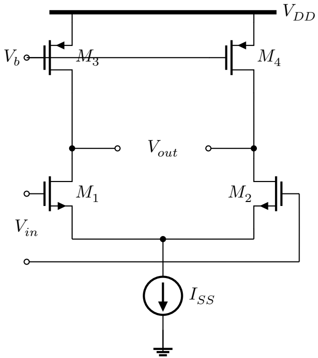
A_{DM} with MOS Loads ($\lambda \neq 0$, $\gamma \neq 0$)

$$V_{out1} = -g_{m1,2}(r_{o1,2} \parallel r_{o3,4})\frac{v_d}{2} \quad (31)$$

$$V_{out2} = -g_{m1,2}(r_{o1,2} \parallel r_{o3,4})(-\frac{v_d}{2}) \quad (32)$$

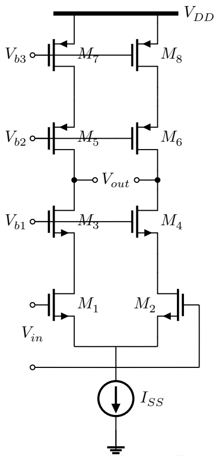
$$A_{DM} = \frac{V_{out1} - V_{out2}}{v_d} \quad (33)$$

$$= -g_{m1,2}(r_{o1,2} \parallel r_{o3,4}) \quad (34)$$



A_{DM} with MOS Loads ($\lambda \neq 0$, $\gamma \neq 0$)

Higher R_{out}
$\rightarrow$ High A_{DM}
$\rightarrow$ Small $V_{in,CM}$ headroom
Telescopic cascode



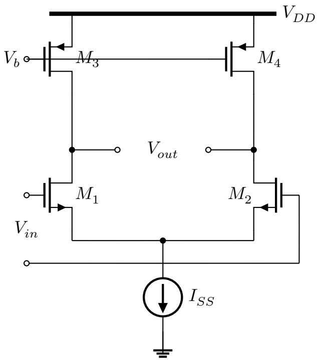
A_{DM} with MOS Loads ($\lambda \neq 0$, $\gamma \neq 0$)

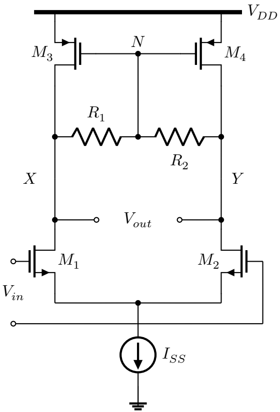
$$
\begin{aligned}
V_{out1} \cong & -g_{m1,2}\left\{[r_{o1,2} + r_{o3,4} + (g_{m3,4} + g_{mb3,4})r_{o3,4}r_{o1,2}] \parallel \right.\\
& \left.[r_{o7,8} + r_{o5,6} + (g_{m5,6} + g_{mb5,6})r_{o5,6}r_{o7,8}]\right\} \frac{v_d}{2}
\end{aligned} \tag{35}
$$

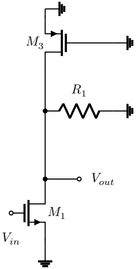
$$
\begin{aligned}
V_{out2} \cong & -g_{m1,2}\left\{[r_{o1,2} + r_{o3,4} + (g_{m3,4} + g_{mb3,4})r_{o3,4}r_{o1,2}] \parallel \right.\\
& \left.[r_{o7,8} + r_{o5,6} + (g_{m5,6} + g_{mb5,6})r_{o5,6}r_{o7,8}]\right\} (-\frac{v_d}{2})
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
A_{DM} = & \frac{V_{out1} - V_{out2}}{v_d} \cong -g_{m1,2}[(g_{m3,4} + gmb_{3,4})r_{o3,4}r_{o1,2} \parallel \\
& (g_{m5,6} + g_{mb5,6})r_{o5,6}r_{o7,8}]
\end{aligned} \tag{37}
$$

A_{DM} with MOS Loads



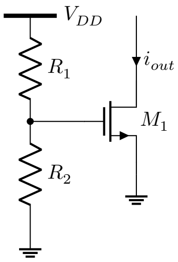




$$A_v = g_{mN} \| (r_{ON} r_{OP}) \qquad (38)$$

$$|A_v| = g_{m1}(r_{O1} =\| R_1 \| r_{O3}) \qquad (39)$$

Current Defined by Resistors



- I_{out} loosely defined.

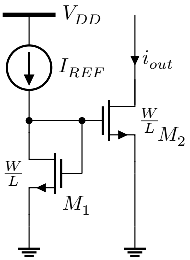
  If $\lambda = 0$

$$I_{out} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(\frac{R_2}{R_1 + R_2}V_{DD} - V_{TH1}\right)^2 \tag{40}$$

Variations by temperature: μ_n, V_{TH1}

Variations by process: $\frac{R_2}{R_1+R_2}, V_{TH1}$

Current Mirror



$$I_{out} = \frac{(W/L)_2}{(W/L)_1} I_{REF} \quad (41)$$

- μ_n and V_{TH} still vary with temperature, but are canceled out in ratio
- I_{out} precisely defined.

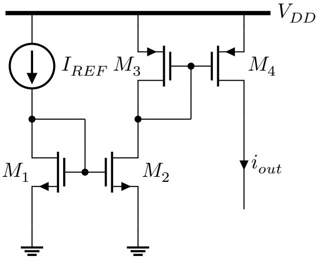
If $\lambda = 0$

$$I_{REF} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_A - V_{TH1})^2$$
$$I_{out} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_A - V_{TH2})^2 \quad (42)$$

$TH1, TH2:$ Almost identical if two transistors are drawn close to each other in layout.

Current Mirror Examples

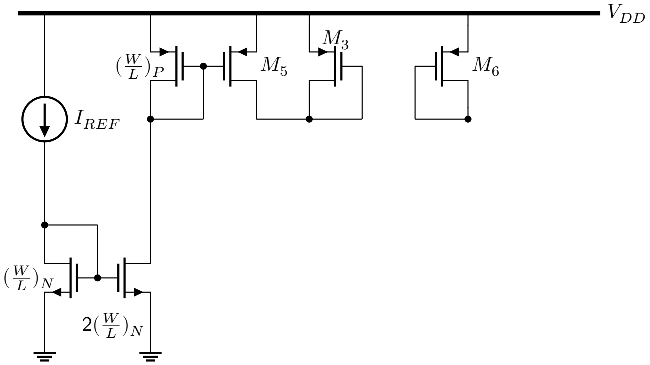
Calculate I_{out} in terms of I_{REF}



$$I_{out} = \frac{(W/L)_2}{(W/L)_1}\frac{(W/L)_4}{(W/L)_3} I_{REF} \tag{43}$$

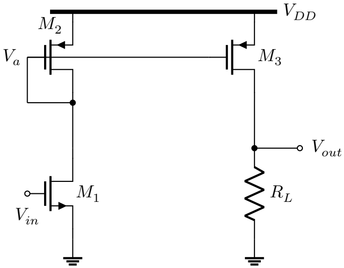
- DC or small-signal current amplification or reduction can be achieved if transistors are properly ratioed.

DC Amplification by Current Mirror



$$A_{\text{DM}} = \frac{v_{\text{out }1} - v_{\text{out }2}}{v_{\text{d}}} \approx -\frac{gm_{1,2}}{gm_{3,4}} \approx -\sqrt{\frac{5\mu_n (W/L)_{1,2}}{\mu_p (W/L)_{3,4}}} \tag{44}$$

Amplification by Current Mirror



$$v_a = -v_{\text{in}}\, gm_1 (r_{o1} \parallel r_{o2} \parallel \frac{1}{gm_2}) \qquad (45)$$

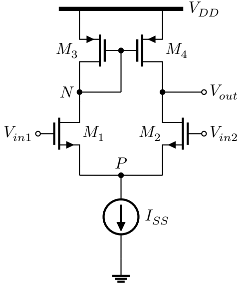
$$v_{\text{out}} = -v_a gm_3 (R_L \parallel r_{o3}) \qquad (46)$$

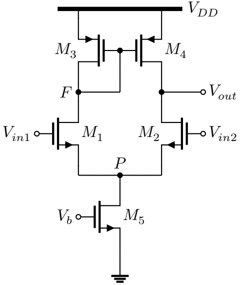
$$\frac{v_{\text{out}}}{v_{\text{in}}} = gm_1 (r_{o1} \parallel r_{o2} \parallel \frac{1}{gm_2}) gm_3 (R_L \parallel r_{o3}) \qquad (47)$$

If $\lambda = \gamma = 0$

$$\frac{v_{\text{out}}}{v_{\text{in}}} = gm_1 \frac{gm_3}{gm_2} R_L = gm_1 \frac{(W/L)_3}{(W/L)_2} R_L \qquad (48)$$

Differential Pair with Current Mirror

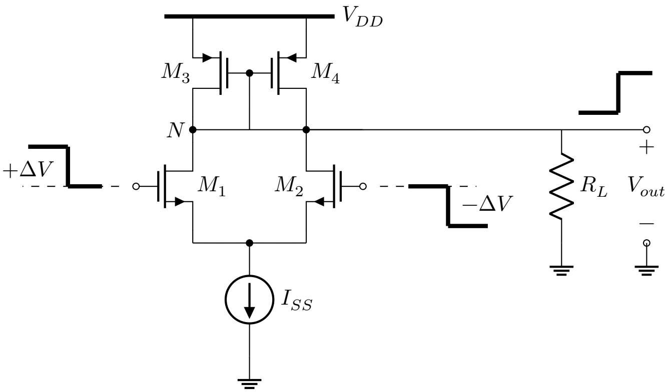




- V_{in} increases
- I_{D1}, I_{D3}, I_{D4} increases
- I_{D2} decreases
- V_{out} increases

- $V_{in1} > V_F + V_{TH1} \rightarrow M_1$ enters into triode
- For M_2 in saturation, $V_{\text{out}} \geq V_{in,CM} - V_{TH2}$

Differential Pair with Active Load



Asymmetric Differential Pair

Why N is not on the right branch?
Caveat: Because of the vastly different resistance magnitude at the drains of M_1 and M_2, the voltage swings at these two nodes are different and therefore node P cannot be viewed as a virtual ground when $V_{in2} = -V_{in1}$.