# RC4

ECE 3110J Analog Circuits

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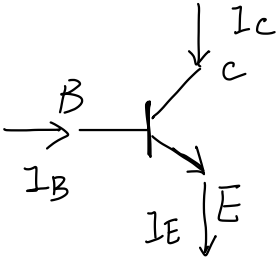
# About BJT Exercise:



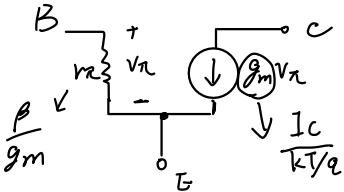

① Use

$$I_C = I_S\left(e^{\frac{qV_{BE}}{kT}} - 1\right)$$
$$\alpha = \frac{I_C}{I_E}$$
$$\beta = \frac{I_C}{I_B} = \frac{\alpha}{1-\alpha}$$

+ KCL, KVL you learned in VE215

⇒ get the DC value ($I_{xx}$, $V_{xx}$)

② Use to get the parameter of the small signal




$r_o = \frac{V_A}{I_C}$ ⇒ again use the KVL, KCL to some the circuit

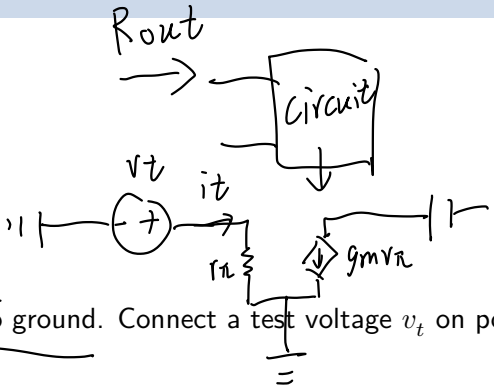
1 BJT

$R_{out}$ and $R_{in}$

Common-Emitter Amplifier

Exercise

# $R_{out}$




How to calculate $R_{out}$?

1. Draw small signal model.
2. Connect port with $V_{in}$ to ground. Connect a test voltage $v_t$ on port with Vout.
3. Derive $i_t$
4. Calculate $R_{out} = \frac{v_t}{i_t}$

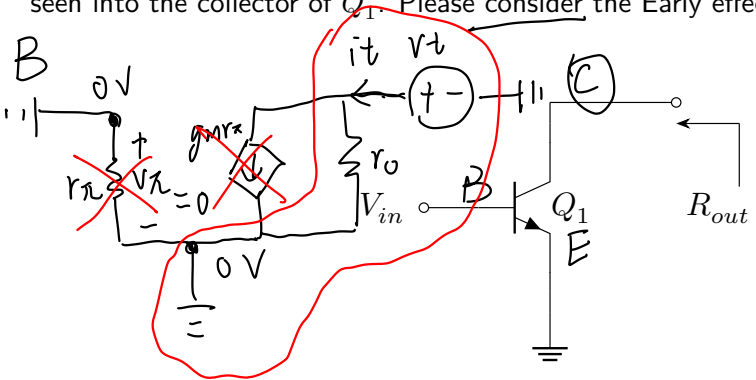
# $R_{in}$

How to calculate $R_{in}$?

1. Draw small signal model.
2. Connect a test voltage $v_t$ on port with $V_{in}$. Do nothing on output port.
3. Derive $i_t$
4. Calculate $R_{in} = \frac{v_t}{i_t}$

# Exercise 1

Assuming that the BJT operates in the forward active region, determine the impedance seen into the collector of $Q_1$. Please consider the Early effect.

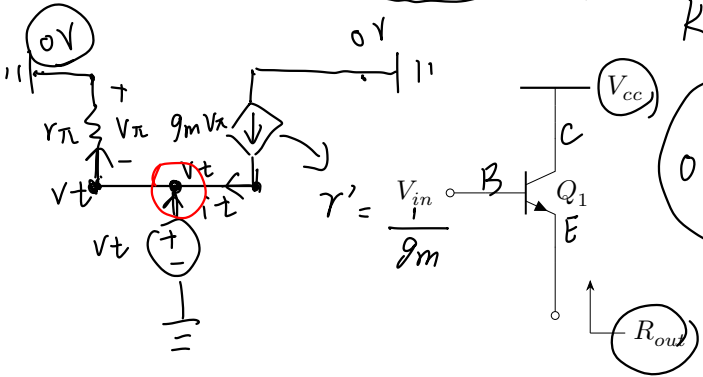



$v_t = i_t \cdot r_o$

$R_{out} = r_o$

# Exercise 2

Assuming that the BJT operates in the forward active region, determine the impedance seen at the emitter. Please neglect the Early effect for simplicity.




KCL

$$i_t = \frac{v_t}{r_\pi} - g_m v_\pi$$

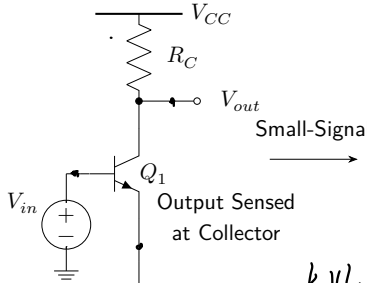
$$0 - v_\pi = v_t$$

$$v_\pi = -v_t$$

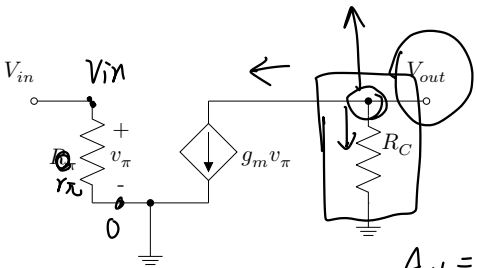
$$R_{out} = \frac{v_t}{i_t} = r_\pi \parallel \frac{1}{g_m} = \frac{1}{\frac{1}{r_\pi} + g_m}$$

# CE Amplifier (Without Early Effect)

$A_v = \frac{V_{out}}{V_{in}}$ $\quad i_b = 0$




Small-Signal




KVL: $\begin{cases} V_{in} = V_\pi \\ \frac{V_{out}}{R_C} + g_m V_\pi = 0 \end{cases}$

$A_v = \frac{V_{out}}{V_{in}} = -g_m R_C$

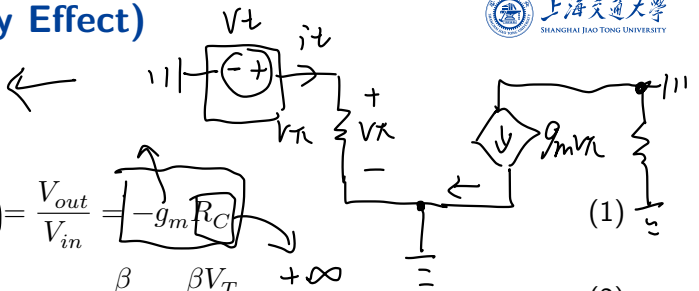
# CE Amplifier (Without Early Effect)

$v_t = i_t r_\pi$

$R_{in} = r_\pi$

$v_t = i_t \cdot R_C$

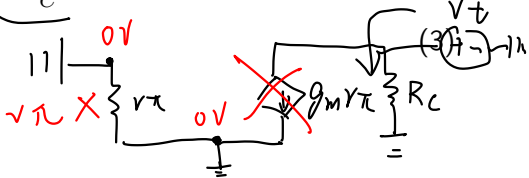
$R_{out} = R_C$



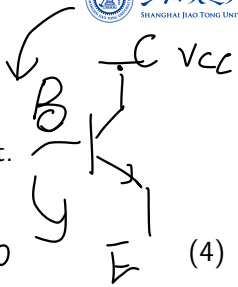

$$\boxed{A_v} = \frac{V_{out}}{V_{in}} = \boxed{-g_m \boxed{R_C}} \quad (1)$$

$$R_{in} = r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C} \quad (2)$$

$$R_{out} = R_C \quad (3)$$



# CE Amplifier (Without Early Effect)



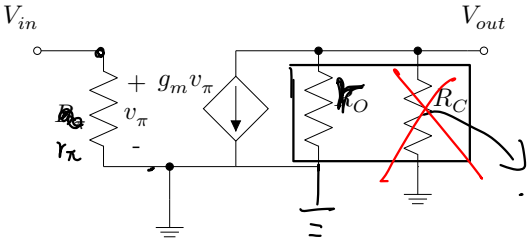

Important properties of the CE stage:

1. The small-signal gain is negative, i.e. $V_{BE}$ ↑and $I_C$ ↑lower Vout.
2. The voltage gain of the stage is limited by the supply voltage.

$$| A_v | = g_m R_C = \frac{I_C R_C}{V_T} \to +\infty \tag{4}$$

$I_C R_C$ indicates the voltage drops at the collector resistor $R_C$.
Because $I_C R_C < V_{CC}$, $| A_v | < \frac{V_{CC}}{V_T}$.
Finally, as the BJT is in the active region, we can say that, $| A_v | < \boxed{\frac{V_{CC} - V_{BE}}{V_T}}$.
However, when $R_C \to \infty$, $| A_v | \to \infty$. Contradiction!

# CE Amplifier (With Early Effect)




$R_C \to \infty$

$$A_v = \frac{V_{out}}{V_{in}} = -g_m(R_C \parallel r_O) \quad (5)$$

$R_{in}$

$$R_{in} = r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C} \quad (6)$$

$$R_{out} = R_C \parallel r_O \quad (7)$$

$$A_V = -g_m r_o$$

# CE Amplifier (With Early Effect)

If $R_C \to \infty$, $A_v = -g_m r_O \to$ intrinsic gain
Because

$$g_m = \frac{I_C}{V_T},\ r_O = \frac{V_A}{I_C} \tag{8}$$

we can get:

$$A_v = -\frac{V_A}{V_T} \tag{9}$$

The Early effect limits the voltage gain even if $R_C$ approaches infinity.

# CE Stage With Emitter Degeneration (Without Early Effect)

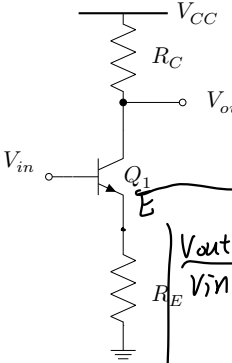
KCL:

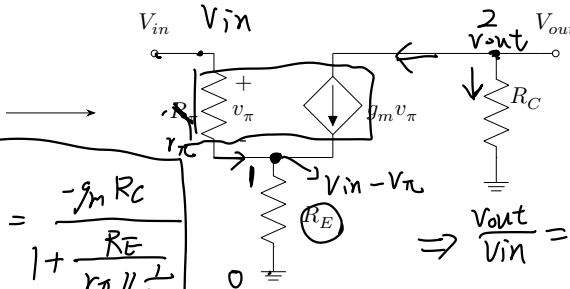
$$\frac{v_\pi}{r_\pi} + g_m v_\pi = \frac{v_{in} - v_\pi}{R_E} \quad (1)$$

$$\frac{v_{out}}{R_C} + g_m v_\pi = 0 \quad (2)$$

$$\Rightarrow \quad v_\pi = -\frac{v_{out}}{g_m R_C}$$

$$-\frac{v_{out}}{g_m R_C r_\pi} + -\frac{v_{out}}{R_C} = \frac{v_{in} + \frac{v_{out}}{g_m R_C}}{R_E}$$

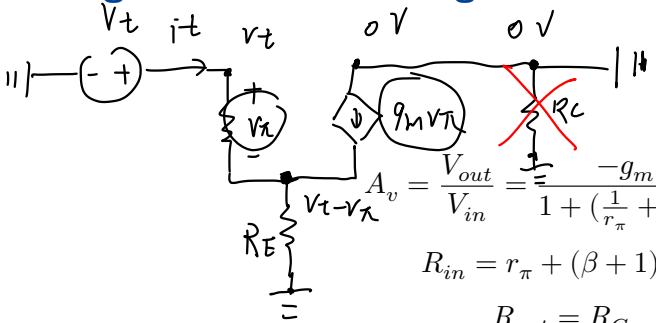



$$\boxed{\frac{v_{out}}{v_{in}} = \frac{-g_m R_C}{1 + \frac{R_E}{r_\pi \parallel \frac{1}{g_m}}}}$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = \frac{-g_m R_C}{1 + \left(\frac{1}{r_\pi} + g_m\right) R_E}$$

# CE Stage With Emitter Degeneration (Without Early Effect)




$$A_v = \frac{V_{out}}{V_{in}} = \frac{-g_m R_C}{1 + (\frac{1}{r_\pi} + g_m)R_E} \quad (10)$$

$$R_{in} = r_\pi + (\beta + 1)R_E \quad (11)$$

$$R_{out} = R_C \quad (12)$$

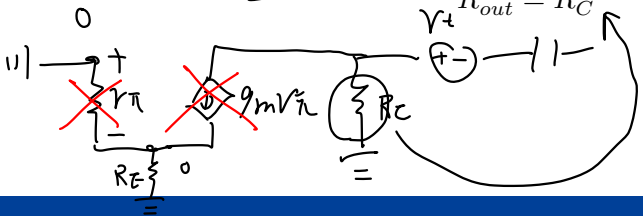
$$v_t = i_t \cdot r_\pi + (i_t + g_m v_\pi) R_E$$

$$\frac{v_t}{i_t} = r_\pi + \left(1 + \frac{g_m v_\pi}{i_t}\right) R_E$$

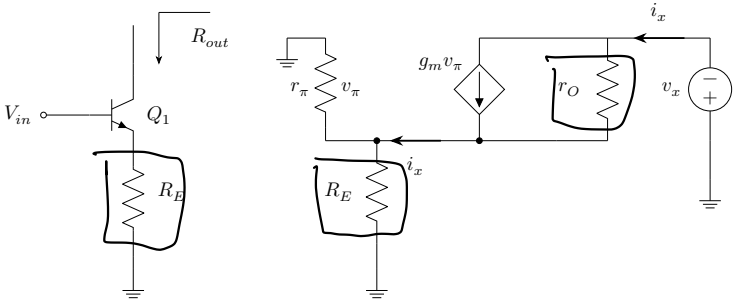
$$v_\pi = i_t \cdot r_\pi$$

$$R_{in} = r_\pi + (1 + g_m r_\pi) R_E$$

$$= r_\pi + (1 + \beta) R_E$$



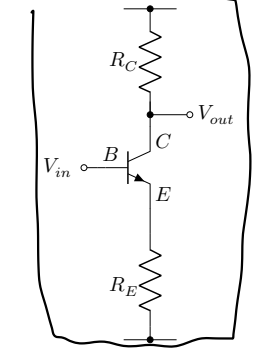
# CE Stage With Emitter Degeneration (With Early Effec



# CE Stage With Emitter Degeneration (With Early Effec

$$R_{out} = r_O + (1 + g_m r_O)(R_E \parallel r_\pi) \approx r_O[1 + g_m(R_E \parallel r_\pi)] \tag{13}$$

# Summary




| | Definition | Expression | |
|---|---|---|---|
| | | With emitter degeneration | Without emitter degeneration; i.e., $R_E = 0$ |
| Current gain | $A_i$ | $\beta$ | $\beta$ |
| Voltage gain | $A_v$ | $-\frac{\beta R_C}{r_\pi + (\beta+1)R_E}$ | $-g_m R_C$ |
| Input impedance | $r_{in}$ | $r_\pi + (\beta+1)R_E$ | $r_\pi$ |
| Output impedance | $r_{out}$ | $R_C$ | $R_C$ |

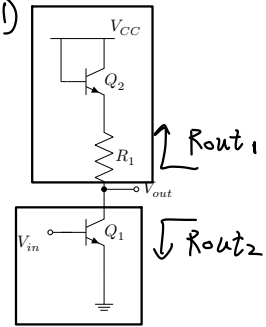
# Exercise 3

(23FA Mid/5) Using a small-signal model, determine output impedance of the circuit shown below with and without Early effect.[10 points]

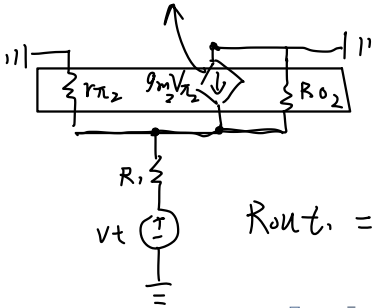
$R_{out} = R_{out_1} // R_{out_2}$

$r = \frac{1}{g_{m2}}$

(1)



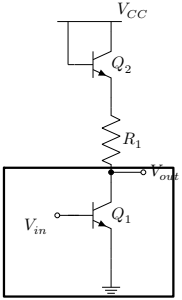

$R_{out_1}$ :

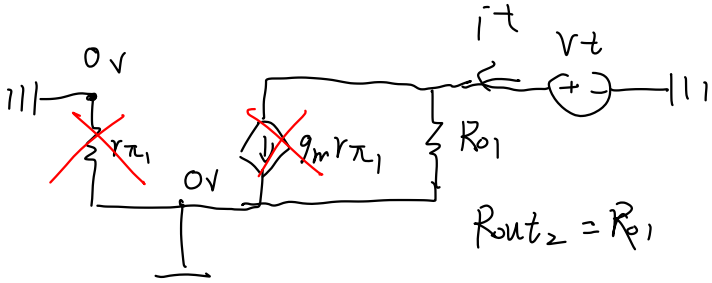



$R_{out_1} = R_1 + \left(R_{o2} // \frac{1}{g_{m2}} // r_{\pi_2}\right)$

# Exercise 3

(23FA Mid/5) Using a small-signal model, determine output impedance of the circuit shown below with and without Early effect.[10 points]



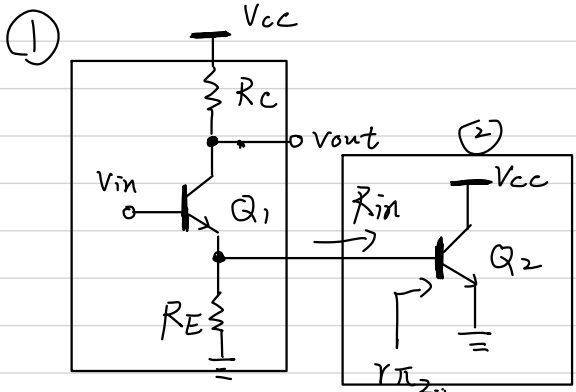



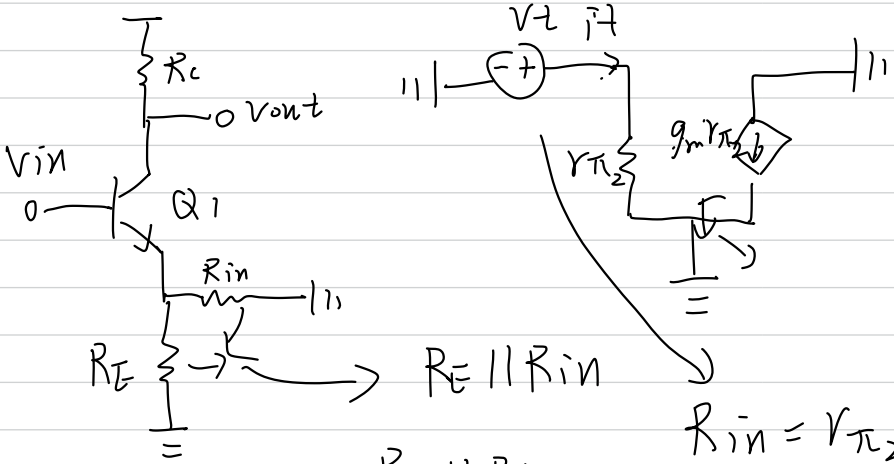

$$R_{out_2} = R_{o1}$$

$$R_{out} = R_{o1} \parallel \left( R_1 + \left( R_{o2} \parallel \frac{1}{g_{m2}} \parallel \frac{1}{r_{\pi_2}} \right) \right)$$

Exercise 4.

Determine the voltage gain of the stage shown below. No early effect.



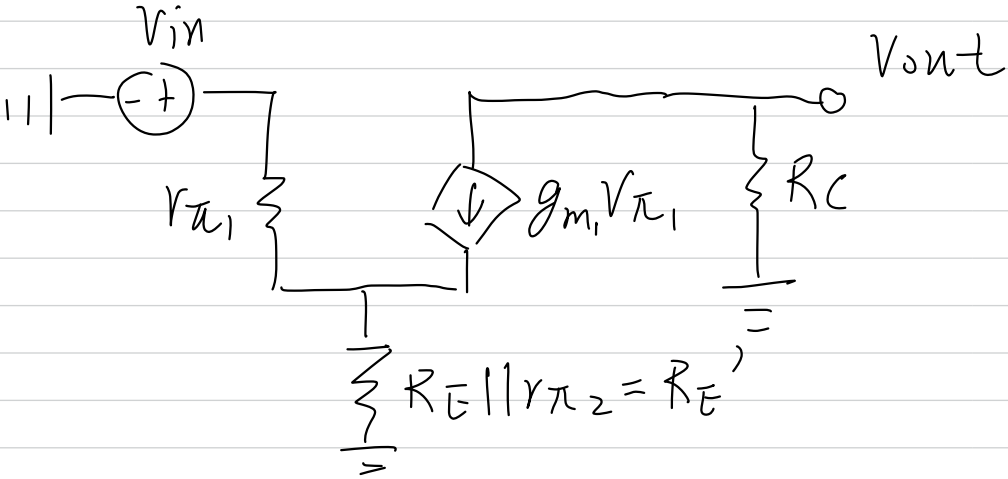




$R_E \| R_{in}$

$R_{in} = r_{\pi_2}$

$= R_E \| R_{in}$

$= R_E \| r_{\pi_2}$




$$A_V = \frac{V_{out}}{V_{in}} = \frac{-g_{m_1} R_C}{1 + R_E'\left(\frac{1}{r_{\pi_1}} + g_{m_1}\right)}$$

use the formular given in the table

$$= \frac{-g_{m_1} R_C}{1 + (R_E \| r_{\pi_2})\left(\frac{1}{r_{\pi_1}} + g_{m_1}\right)}$$