

Lecture 7: Bipolar Junction Transistors

ECE3110J, Electronic Circuits

Xuyang Lu 2024 Summer



Recap of Last Lecture

Recap

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Op-amps



Topic to Be Covered

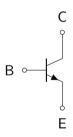
- Op-amps
- BJT

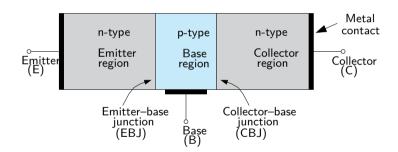
Bipolar Junction Transistors (BJTs)



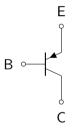
- These three—terminal devices are very useful for amplification, since an input signal can be applied across one pair of terminals and the output signal can be taken from another pair of terminals.
- bipolar junction transistor (BJTs) are still widely used in many analog circuits, including amplifiers, filters, mixers, etc.
- These are very fast devices, but also consume more power. This is the main reason CMOS has become the mainstream technology for digital circuits.

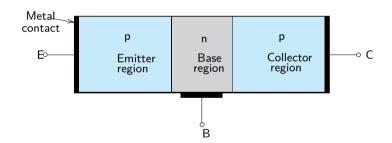












npn & pnp BJT Transistors



- There are three terminals: emitter, collector, and base
- There are two pn junctions, the base-emitter junction, and the base-collector junction, like two diodes that are connected back to back.
- However, the operation of the transistor is very different than just two back to back connected diodes.

BJT (Before Contact) $N_{d1} \gg N_{a}$



Emitter n - type

$$\begin{array}{c}
E_c \\
E_f \\
E_i
\end{array}
\qquad \downarrow q\phi_{n1}$$

$$E_v$$

$$n \cong N_{d1} = n_i e^{\frac{q\phi_{n1}}{kT}} \quad (1)$$

$$p \cong \frac{n_i^2}{N_{d1}} = n_i e^{\frac{-q\phi_{n1}}{kT}}$$
 (2)

Base p - type

$$E_c$$
 ————

$$E_{i} - q\phi_{p\updownarrow}$$

$$E_{v} - q\phi_{p\updownarrow}$$

$$p \cong N_a = n_i e^{\frac{q\phi_p}{kT}} \qquad (3)$$

$$n \cong rac{n_i^2}{N_a} = n_i e^{rac{-q\phi_p}{kT}} \quad (4)$$

Collector n - type

$$E_c$$
 E_f
 E_i $\uparrow q\phi_{n2}$

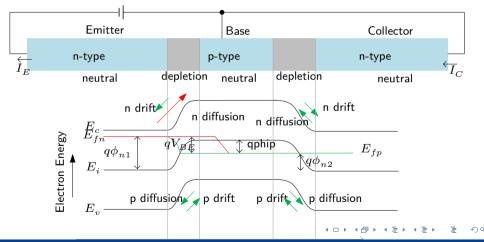
$$E_v$$

$$(3) n \cong N_{d2} = n_i e^{\frac{q\phi_{n2}}{kT}} (5)$$

$$n \cong \frac{n_i^2}{N_a} = n_i e^{\frac{-q\phi_p}{kT}}$$
 (4) $p \cong \frac{n_i^2}{N_{d2}} = n_i e^{\frac{-q\phi_{n2}}{kT}}$ (6)

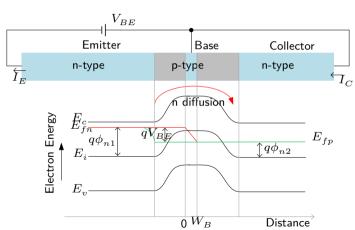
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$V_{BE} > 0$ and $V_{CB} = 0$ (W_B long)



$V_{BE}>0$ and $V_{CB}=0$ ($N_{d1}\gg N_a$, W_B very short)





$$I_B \cong 0$$
 (7)

$$I_C \cong I_E$$
 (8)

$$I_E = I_C + I_B \quad (9)$$

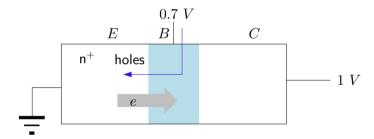
$$\frac{I_C}{I_E} = \alpha \qquad (10)$$

$$\frac{I_C}{I_B} = \beta \qquad (11)$$



- The n (electron) diffusion is much larger than the p (hole) diffusion at the Base-Emitter junction.
- Nearly all the n (electron) diffusion from the Base-Emitter junction pass through the Base, enter into the depletion region of the Base-Collector junction, and are swept to the Collector side by the built-in electric field.





- Because the BE junction is forward biased, we will turn on this junction.
- Not that because the base is p-doped, it means it has a lot more h⁺ than e⁻.
 Because the BE junction is forward biased, the concentration of electrons to increase in the base.



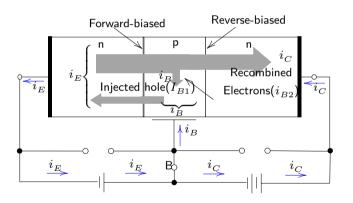
- Holes that are injected into the emitter do not change things too much in the emitter since electrons are majority charge carriers in the emitter and dominate the current in the emitter.
- The BC junction is reverse biased, therefore there are no carriers injected across this junction.

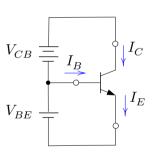


- Once electrons are injected into the base, their concentration becomes much higher at the edge of the base-emitter junction, compared to the base-collector junction.
- Because of this concentration gradient, e⁻ will start diffusing from the emitter side of the base, to the collector side of the base.
 If the base is short, then most of these electrons make it through the base, without recombining with any holes that are in the base.
- When electrons get to the edge of the base-collector junction, they see a very large electric field that pulls then from the base to the collector across this junction.
- Note that e⁻ are minority charge carriers in the base.











- The base current consists of h⁺ injected into emitter and h⁺ recombined with e⁻ that do not make across the base.
- The emitter current is h⁺ injected from the base, and e⁻ injected into the base.
- \bullet The collector current is mainly e^- collected at the edge of the base .

Collector Current



 \bullet I_C is mainly due to electrons that are injected from the BE junction and crossing the base.

$$i_C = I_S e^{v_{BE}/V_T} \tag{12}$$

 Note that the collector current is an exponential function of the base - emitter voltage, that is the current through the collector-emitter terminal changes when the voltage across the base-emitter terminal is modulated.

Base Current

Recap



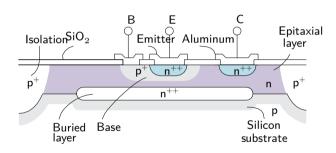
• i_B caused (mostly) by holes that are injected from the base into the emitter (although there is also a portion of this current that is due to those recombined with electrons in the base.

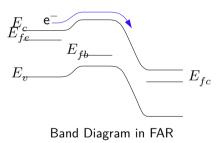
$$i_B = \frac{I_S}{\beta} e^{v_{BE}/V_T} \tag{13}$$

- But $i_B << i_C$ since most of the electrons from the emitter side make it to the collector side of the base.
- Where for a reasonable transistor β is 50-200, so the collector current is much larger than the base current, and the higher the β the better the transistor.
- \bullet Clearly, we can see that $i_E=i_B+i_C$

npn Device







Lateral View of a BJT

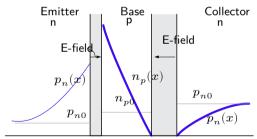


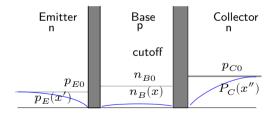
Minority Carrier Distribution

Recap









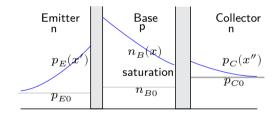
Cutoff Mode

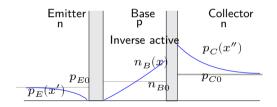
Forward Active Mode



Minority Carrier Distribution







Saturation Mode

Inverse Active Mode

BJT Operation Summary



$v_{BE} = v_B - v_E \approx 0.7V \tag{14}$

Ideal Case:

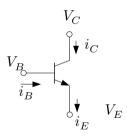
$$v_{BC} = v_B - v_C < 0$$
 (15)

Practice:

$$v_{BC} < -0.4$$
 (16)

$$i_C = I_S e^{v_{BE}/V_T} \tag{17}$$

Forward Active Region (FAR)



$$i_B = \frac{i_C}{\beta} \tag{18}$$

BJT Operation Summary

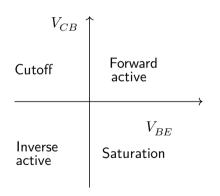
$$i_E = i_C + i_B \tag{20}$$

$$i_E = i_C + \frac{i_C}{\beta} = \frac{i_C(\beta + 1)}{\beta} \qquad (21)$$

$$i_C = \frac{\beta}{\beta + 1} i_E = \alpha i_E \qquad (22)$$

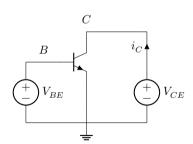
$$\beta \approx 50 - 200 \ (100)$$
 (23)

 α is less than 1, but ≈ 1



Summary in Forward - Active Mode





$$V_{CE} \ge V_{BE} \tag{24}$$

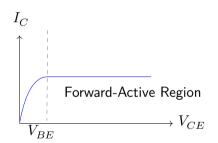
$$I_C = I_S \left(e^{\frac{qV}{BT}} - 1 \right) \tag{25}$$

$$\alpha = \frac{I_C}{I_E} \cong 1 \tag{26}$$

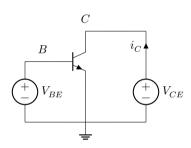
$$\beta = \frac{I_C}{I_B} = \frac{\alpha}{1 - \alpha} \tag{27}$$

 ${\cal I}_s$ is a constant in the spice model

I_C vs V_{CE} (Without Early Effect)



At given V_{RE} , DC sweep V_{CE}

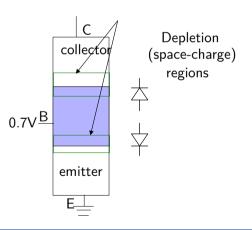


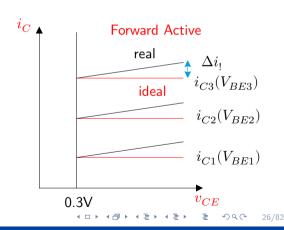
For
$$V_{CE} \geq V_{BE}$$
,

$$I_C = I_S \left(e^{\frac{qV_{BE}}{kT}} - 1 \right) \tag{28}$$

Early Effect, Named After Jim Early









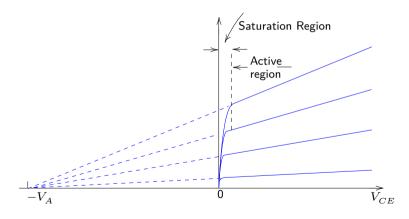
• As the collector voltage changes the width of the depletion region at the BC junction changes, and this causes the effective distance between the edge of the BE junction and BC junction to reduce. Thus smaller base width \rightarrow a higher collector current as there is less recombination.

$$i_C \approx I_s e^{v_{EB}/V_T} (1 + \frac{v_{CE}}{V_A}) \tag{29}$$

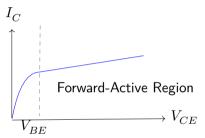
ullet V_A is called the Early voltage. Typically is 50-100 V.

Early Voltage

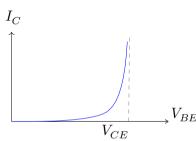




I_{C} vs V_{CE} (with Early Effect)



At given V_{BE} , DC sweep V_{CE}



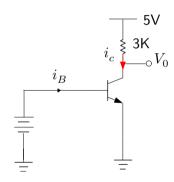
At given ${\cal V}_{CE}$, DC sweep ${\cal V}_{BE}$

$$\bullet \ \ {\rm For} \ V_{CE} \geq V_{BE} {\rm ,}$$

$$I_C = I_S \left(e^{\frac{qV_{BE}}{kT}} - 1 \right) \left(1 + \frac{V_{CE}}{V_A} \right) \tag{30}$$

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Example: Find i_C and i_E



given: $i_B = 10\mu A, \beta = 100$

$$i_C=\beta i_B=100\times 10\mu A=1mA$$
 (31)

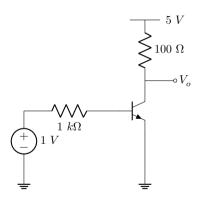
$$i_B = (\frac{I_S}{\beta})e^{\frac{V_{BE}}{V_T}} \tag{33}$$

$$v_{BE} = 0.71V$$
 (34)

$$v_{BC} = 0.71 - 2 = -1.29V (35)$$

Models

Example 2: Find V_{α}



$$\beta=100, v_{BE}=0.6V$$



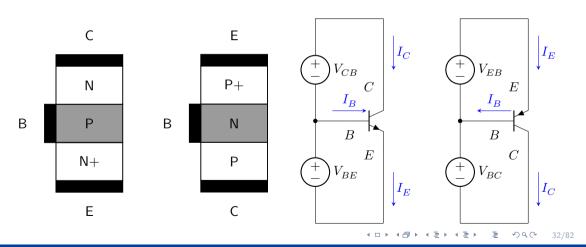
$$i_B = \frac{1 - 0.6V}{R_B} = 0.4 \ mA \tag{36}$$

$$i_C = \beta i_B = 100 \cdot 0.4 \ mA = 40 \ mA$$
 (37)

$$V_o = 5 \ V - 100 \ \Omega \cdot 40 mA = 1V$$
 (38)

pnp Transistor





pnp Transistor

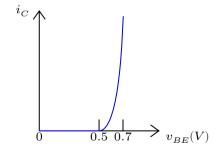


- For pnp, the basics of the operation are the same, except now instead of ecrossing the base, h+ cross the base.
- Because holes move a lot slower than e-, there would naturally be more recombination and fewer of them make it to the collector.
- So, the current gain is much lower for pnp transistors than for npn transistors.
- Note that all polarities are now different, for example for FAR operation, the BE junction has to be forward biased which means that the base-emitter voltage should be -0.7 V, or the emitter-base junction should be +0.7V so the BE junction is forward biased.



• We can now easily establish large signal models for the transistor that describe the current-voltage relation ship

$$i_C pprox I_S e^{v_{BE}/V_T}$$
 (39)

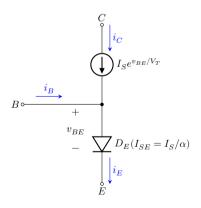


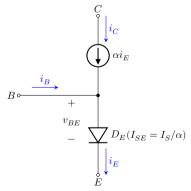


Large Signal Model: T-Model



• The base is shared between the input port (B-E) and the output port (B-C)



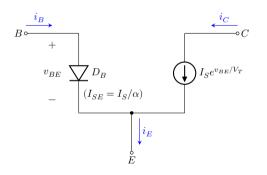


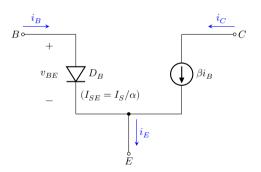
Hybrid- π Model



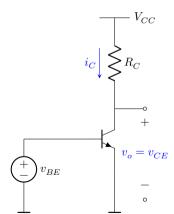
- In hybrid- π model, where the collector current is shown as a voltage controlled current source (VCCS).
- The hybrid- π model is more useful for analyzing the common-emitter amplifier. Notice that the emitter in this model is shared between the input port (B-E), and the output port (C-E).

Hybrid- π Model





Small Signal Gain



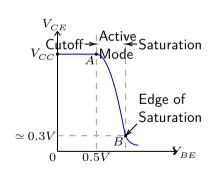
$$A_v = \frac{dv_o}{dv_{RE}} \tag{40}$$

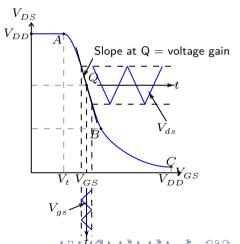
$$A_v = -\frac{I_C}{V_T} R_C \tag{41}$$

Small Signal Gain



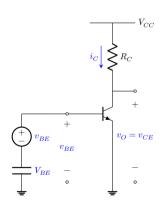
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Small Signal Gain

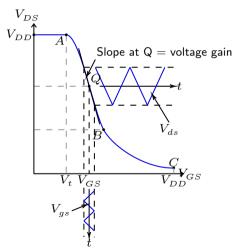




$$v_{CE}(t) = V_{CE} + v_{ce}(t)$$
 (42)

$$v_{BE}(t) = V_{BE} + v_{be}(t)$$
 (43)





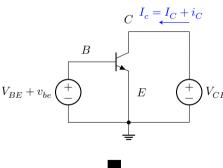
- Operating point for circuit
- Often called "bias" point

$$A_v = \frac{V_o}{v_{be}} \tag{44}$$

$$V_o = V_{CC} - R_C I_S e^{v_{BE}/V_T}$$
 (45)

Hybrid- π Model (g_m and r_π)





$$V_{CE} \geq V_{BE} \\ \Rightarrow \mathsf{Forward-Active}$$

$$I_C = I_S \left(e^{\frac{qV_{BE}}{kT}} - 1 \right) \left(1 + \frac{V_{CE}}{V_A} \right) \tag{46}$$

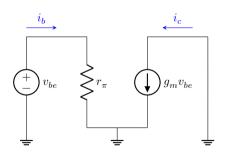
constant



Hybrid- π Model (Derivation of g_m and r_{π})



Small-signal circuit:



$$r_{\pi} = \frac{dV_{BE}}{dI_B} = \frac{1}{\frac{dI_C}{\beta dV_{BE}}} \tag{47}$$

$$=\frac{1}{\frac{g_m}{\beta}} = \frac{\beta}{g_m} \tag{48}$$

$$g_m = \frac{dI_C}{dV_{BE}} \cong \frac{I_C}{kT/q}$$
 (49)



• As we see from this relationship, the collector current should change when the collector-emitter voltage changes:

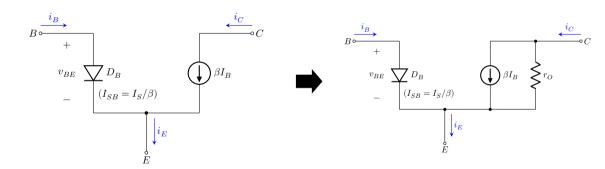
$$i_C \approx I_S e^{v_{EB}/V_T} \left(1 + \frac{v_{CE}}{V_A} \right)$$
 (50)

• So, this effect is included in the model shown below where:

$$r_o = \frac{V_A}{I_C} = \frac{\Delta v_{CE}}{\Delta i_C} \tag{51}$$

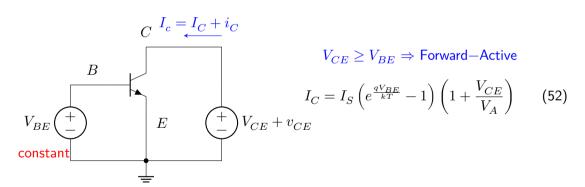
Models with the Early Effect Included





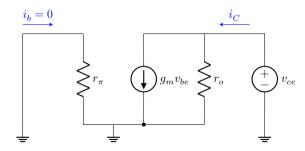
Hybrid- π Model (how to get r_o)





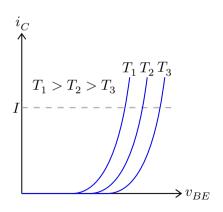
Hybrid- π Model (Derivation of r_o)





$$r_o = \frac{1}{\frac{dI_C}{dV_{CE}}} \cong \frac{V_A}{I_C}$$
 (53)





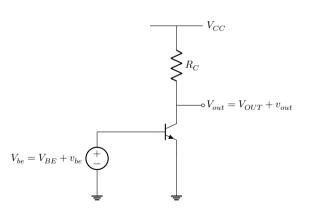
$$i_C = I_S e^{v_{BE}/V_T} \tag{54}$$

 $\bullet \ \, \text{Both} \,\, I_S \,\, \text{and} \,\, V_T \,\, \text{are temperature} \\ \, \text{dependent} \\$

Common-Emitter Amplifier



• Sedra 7.1.7.2.2. 7.2.3



Common-Emitter Amplifier

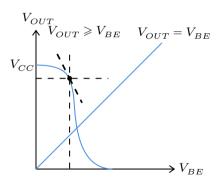


$$V_{OUT} = V_{CC} - I_C R_C \tag{55}$$

$$=V_{CC} - \frac{AqD_n n_i^2}{N_a W_B} \left(e^{\frac{qV_{BE}}{kT}} - 1 \right) R_C \quad (56)$$

$$A_V = \frac{dV_{OUT}}{dV_{BE}} \tag{57}$$

$$\cong -\frac{I_C}{kT/q}R_C = -g_m R_C \tag{58}$$



Common-Emitter Amplifier

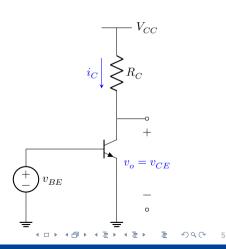


Gain varies a lot in the FAR region.

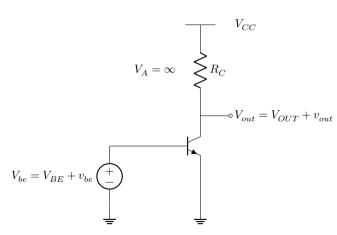
$$A_v = \frac{dv_o}{dv_{BE}} \tag{59}$$

$$A_V = \frac{dV_{OUT}}{dV_{BE}} \tag{60}$$

$$\cong -\frac{I_C}{kT/q}R_C = -g_m R_C \qquad (61)$$

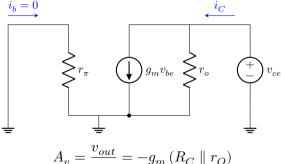


Common-Emitter Amplifier ($V_A = \infty$)



Common-Emitter Amplifier ($V_A = \infty$)





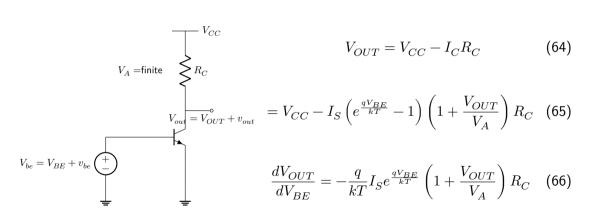
$$A_v = \frac{v_{out}}{v_{be}} = -g_m \left(R_C \parallel r_O \right)$$

$$=-g_m R_C \quad (\text{since } r_o = \infty)$$



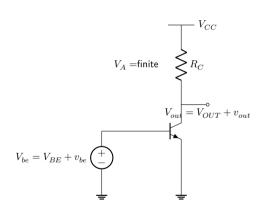
Models

Common-Emitter Amplifier (V_A = finite)



Common-Emitter Amplifier (V_A = finite)





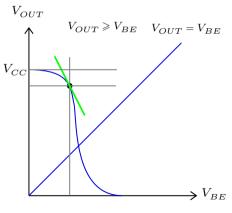
$$-I_S \left(e^{\frac{qV_{BE}}{kT}} - 1 \right) \frac{1}{V_A} \frac{dV_{OUT}}{dV_{BE}} R_C \tag{67}$$

$$\cong -g_m R_C - \frac{1}{r_o} \frac{dV_{OUT}}{dV_{BE}} R_C \tag{68}$$



$$A_V = \frac{dV_{OUT}}{dV_{BE}} \tag{69}$$

$$\cong g_m(R_C \parallel r_O) \tag{70}$$

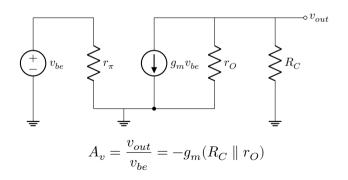


Common-Emitter Amplifier (V_A = finite)



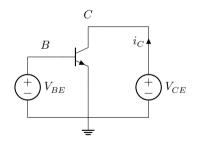
(71)

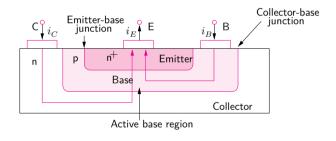
Small-Signal Analysis



NPN BJT Cross Section







NPN Pspice Model ($I_S = 10^{-18}$ **,** $B_E = 100$ **,** $V_{AE} = 100$ **)**



$$I_C = IS \left(e^{\frac{qV_{BE}}{kT}} - 1 \right) \left(1 + \frac{V_{CE}}{VAF} \right) \tag{72}$$

$$BF = \frac{I_C}{I_B} \tag{73}$$

$$I_E = I_C + I_B \tag{74}$$

$$gm \cong \frac{I_C}{kT/q}$$
 (75)

$$r_{\pi} = \frac{BF}{qm} \tag{76}$$

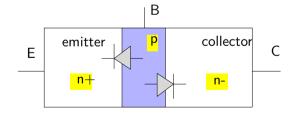
$$r_o \cong \frac{V_{AF}}{I_C}$$
 (77)



- Forward Active Region, where the BE junction is forward biased and the BC junction is reverse biased.
- Notice that since the emitter is more highly doped (it is n+) compared to the collector (which is more lightly doped, n-), we would expect the turn- on voltage for the BE junction to be higher than that of the BC junction.

Typically:

$$V_{BE}$$
 (ON) 0.7 V V_{BC} (ON) 0.5V



Forward Active Region (FAR)



- In FAR, the BE junction is forward biased (meaning V_{BE} (ON) \sim 0.7 V) and the BC junction is reverse biased, meaning V_{BC} (ON) < 0.4V.
- Remember that we indicated that: $i_C pprox I_S e^{v_{BE}/V_T}$
- This shows that the collector current depends on the voltage applied across the base-emitter junction.
- It also shows that the collector current is independent of the collector-base voltage, which as we will shortly see is in fact not quite the case.
- But for now let's assume this relationship is correct.

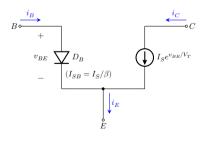
Recap

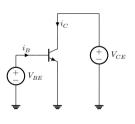
Forward Active Current

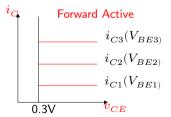
Recap



• Forward active model: the collector current does not change as a function of CE voltage (V_{CE}) .







Models for pnp Transistors

Recap



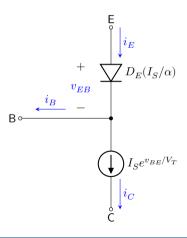
For pnp transistor, the relationship is the same except all polarities are reversed.
 The BE junction is forward biased when the emitter voltage is larger than the base voltage by about 0.7V and the collector current flows out of the collector and emitter current flows into the emitter.

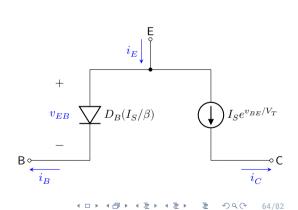
$$i_C \approx I_S e^{v_{EB}/V_T} \tag{78}$$



Models for pnp Transistors

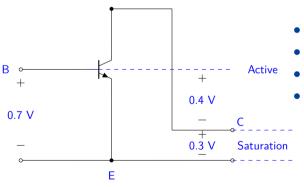






Graphical Depiction of FAR





- Forward Active
- $V_{CB} \ge -0.4V$
- $V_{CE} \ge 0.3V$
- Otherwise get "Saturation"

$$V_{BE}$$
 (ON) 0.7 V V_{BC} (ON) 0.5 V

Saturation Region

Recap



- What happens if the collector base junction begins to be forward biased?
- This means the collector voltage gets smaller until the base voltage become larger than it by about 0.4V.
- With BC junction forward-biased, most of the base current will simply be injected into the collector, as shown by the model below.
- As the BC voltage goes beyond 0.4V, the collector current will become smaller since the current coming from the diode DC will simply go down the voltage controlled current source.

Saturation Region

Recap

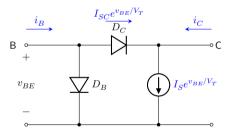


- This means the current gain Beta of the transistor will begin to go down as collector current decreases.
- When the base collector voltage becomes equal to the BE voltage, collector current becomes zero as the model shows.
- With BC junction forward-biased, most of the base current will simply be injected into the collector, as shown by the model below.
- The IV relationships for the transistor in Saturation can then be shown graphically.

Saturation Region

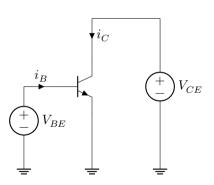


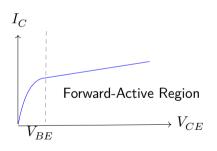
Mode	EBJ	CBJ
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward



Saturation Current

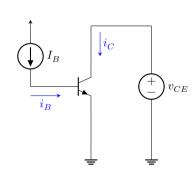


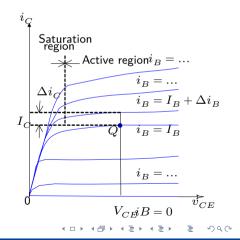




Saturation Region: Closer Look







Simplified Models for Two Regions



Active

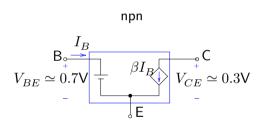
EBJ : Forward

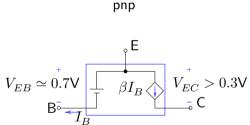
Biased

CBJ:

Reverse

Biased







Saturation

EBJ:

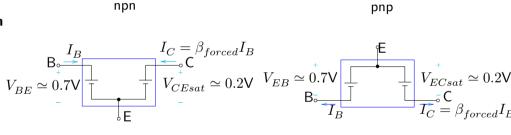
Recap

Reverse Biased

CBJ:

Forward

Biased





$$v_{BE} = v_B - v_E \approx 0.7 \text{ V}$$

$$i_B = \frac{i_C}{\beta} = \left(\frac{I_S}{\beta}\right) e^{v_{BE}/V_T}$$

$$v_{BC} = v_B - v_C < 0$$
(80)

$$i_C = I_S e^{v_{BE}/v_T}$$
 (81) $i_E = \frac{i_C}{\alpha} = \left(\frac{I_S}{\alpha}\right) e^{v_{BE}/V_T}$ (83)

Note: For the pnp transistor, replace \boldsymbol{v}_{BE} with \boldsymbol{v}_{EB}



$$i_C = \alpha i_E$$
 (84)
 $i_C = \beta i_B$ (85)

$$i_E = (\beta + 1)i_B \tag{87}$$

$$i_C = \beta i_B \tag{85}$$

$$\beta = \frac{\alpha}{1 - \alpha} \tag{88}$$

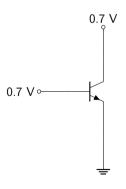
$$i_B = (1 - \alpha)i_E = \frac{i_E}{\beta + 1}$$
 (86)

$$\alpha = \frac{\beta}{\beta + 1} \tag{89}$$

 $V_T = \text{thermal voltage} = \frac{kT}{q} \simeq 25 \text{ mV}$ at room temperature

Condition for Forward Active

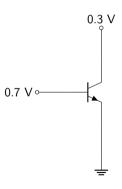




- Is this Forward Active?
- Yes



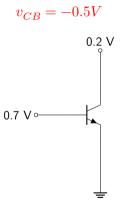
Not really on yet



- Is this Forward Active?
- Yes
- $V_{BC} = 0.4V$

Forward

How Far Can We Go?



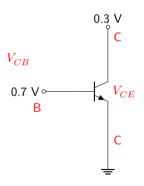
Is this Forward Active?

No

Forward

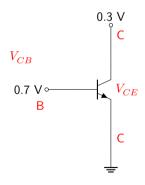


Forward Active Constraints



- $V_{BE} \approx 0.7V$
- $V_{BC} \leq 0.4V$ $V_{CB} \geq -0.4V$

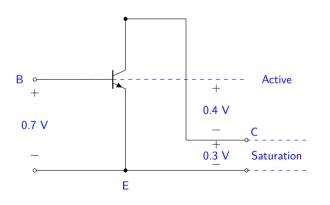




- Forward Active
- $V_{CB} \ge -0.4V$
- $V_{CE} \ge 0.3V$

If we are not in forward active, we are in saturation region.



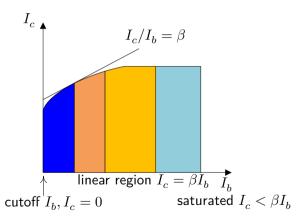


- Forward Active
- $V_{CB} \ge -0.4V$
- $V_{CE} \ge 0.3V$

Otherwise get "Saturation"

Saturation

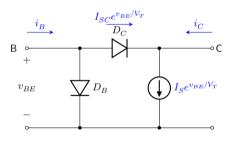




 I_c and I_b for the Different Operating Modes of a BJT







- BC diode starts conducting
- Less current for i_c (D_C is forward biased)
- Then Collect current falls

$$i_C = I_S e^{v_{BE}/V_T} - I_{SC} e^{V_{BC}/V_T}$$
 (90)

Base current rises

$$i_B = I_S/\beta e^{vBE/V_T} + I_S c e^{vBC/V_T}$$
 (91)