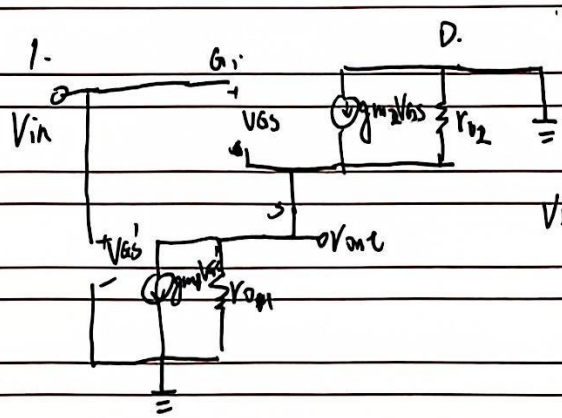
上海交通大學

年　　月　　日

Homework 7

Question 1.

1.

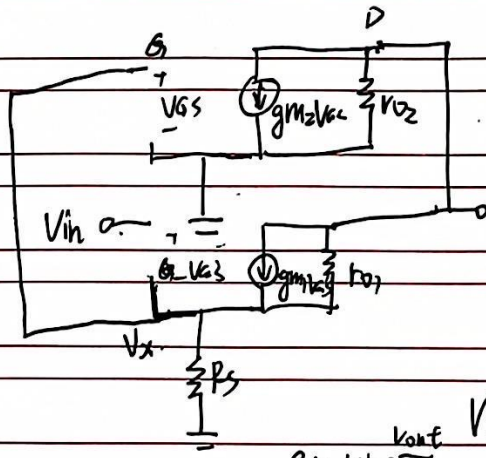


$$Vin \cdot g_{m_1} + \frac{Vout - 0}{r_{O_1}} = g_{m_2} \cdot \frac{(Vin - Vout)}{1} + \frac{-Vout}{r_{O_2}}$$

$$Vout\left(\frac{1}{r_{O_2}} + \frac{1}{r_{O_1}} + g_{m_2}\right) = Vin\left(g_{m_2} - g_{m_1}\right)$$

$$\Rightarrow A = \frac{-g_{m_1} + g_{m_2}}{g_{m_2} + \frac{1}{r_{O_1}} + \frac{1}{r_{O_2}}}$$

2.



$$\begin{cases} \frac{Vx}{Rs} = g_{m_1}(Vin - Vx) + \frac{Vout - Vx}{r_{O_1}} \\ g_{m_2} Vx + \frac{Vout}{r_{O_2}} + g_{m_1}(Vin - Vx) + \frac{Vout - Vx}{r_{O_1}} = 0 \end{cases}$$

$$\Downarrow$$

$$g_{m_2} Vx + \frac{Vx}{Rs} + \frac{Vout}{r_{O_2}} = 0$$

$$Vx\left(\frac{1}{Rs} + g_{m_1} + \frac{1}{r_{O_1}}\right) = g_{m_1} Vin + \frac{Vout}{r_{O_1}}$$

$$\left(g_{m_2} + \frac{1}{Rs}\right) \cdot \frac{g_{m_1} Vin + \frac{Vout}{r_{O_1}}}{\frac{1}{Rs} + g_{m_1} + \frac{1}{r_{O_1}}} + \frac{Vout}{r_{O_2}} = 0$$

$$A = \frac{r_{O_2} g_{m_1} g_{m_2} Rs\, r_{O_1} + g_{m_1} r_{O_1} r_{O_2}}{r_{O_2} g_{m_2} Rs + r_{O_2} + r_{O_1} + g_{m_1} r_{O_1} Rs + Rs}$$

$$A = \frac{g_{m_1} g_{m_2} Rs\, r_{O_1} r_{O_2} + g_{m_1} r_{O_1} r_{O_2}}{g_{m_2} Rs\, r_{O_2} + r_{O_2} + r_{O_1} + g_{m_1} r_{O_1} Rs + Rs}$$

Question 2.

1) If M_1 is at the edge of triode region:

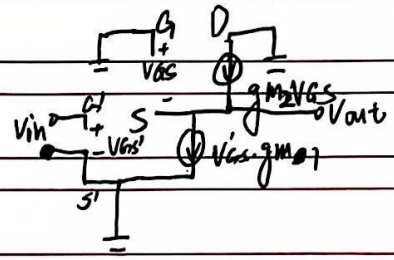
$V_{GS}-V_{TH}=V_{DS}$

$V_{in}-0.7=V_{out}$.

M_2: $V_{GS}=V_{DS}>V_{GS}-V_{TH}$, in saturation region: (Assume $V_{th2}=0.7V$)

$100\cdot(V_{out})^2=20\cdot(2.3-V_{out})^2$

$\Rightarrow V_{out}=0.71V$, $V_{in}=1.41V$.



$g_{m1}\cdot(-V_{out})=V_{in}\cdot g_{m2}$,

since $g_{m1}=5g_{m2}$,

$A=\frac{V_{out}}{V_{in}}=-\sqrt{\frac{g_{m1}}{g_{m2}}}=-2.236$

2) $V_{GS}-V_{TH}=V_{DS}+50mV$.

$\Rightarrow V_{in}-0.7=V_{out}$., Also,

$\frac{1}{2}\cdot20\cdot(2.3-V_{out})^2=100\cdot[(V_{out}+0.05)\cdot V_{out}-\frac{1}{2}V_{out}^2]$

$V_{out}=0.68V$, $V_{in}=1.43V$.

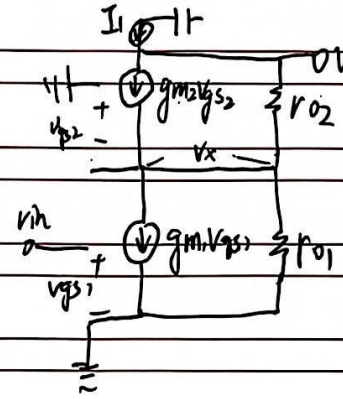
$A=-\frac{1}{5}\cdot\frac{V_{DS}}{V_{GS}-V_{TH}}=-\frac{1}{5}\cdot\frac{V_{out}}{2.3-V_{out}-0.7}$ $A=-\sqrt{\frac{g_{m1}}{g_{m2}}}=-\sqrt{\frac{20\cdot(V_{GS}-V_{TH})}{100\cdot V_{DS}}}=-\frac{1}{5}\cdot\sqrt{5\cdot\frac{V_{out}}{(2.3-V_{out})-0.7}}$

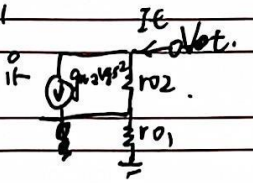
$=-2.10$.

$=-1.90$.

Question 3.

① When $I_1 = 0.01\,mA$, $g_m = \sqrt{2K_n \frac{W}{L} I_1}$





$R_{out} = \frac{V_t}{I_t}$,

$I_t r_{O1} = g_{m2} \cdot (V_t - I_t r_{O1}) + \frac{1}{r_{O2}}(V_t - I_t r_{O1})$

$I_t (r_{O1} + g_{m2} r_{O1} r_{O2} + r_{O1}) = \frac{1}{r_{O2}} V_t$

$\Rightarrow R_{out} = \frac{V_t}{I_t} = r_{O1} + r_{O2} + g_{m2} r_{O1} r_{O2}$.

$$\begin{cases} g_{m2} \cdot (-V_x) + \frac{V_{out} - V_x}{r_{O2}} = g_{m1} V_{in} + \frac{V_x}{r_{O1}} \\ g_{m2}(-V_x) + \frac{V_{out} - V_x}{r_{O2}} = I_1 \end{cases}$$

$V_x \cdot (g_{m2} + \frac{1}{r_{O2}}) = \frac{V_{out}}{r_{O2}} - I_1$

$I_1 = g_{m1} V_{in} + \frac{1}{r_{O1}} \cdot \frac{V_{out} - I_1 r_{O2}}{g_{m2} r_{O2} + 1}$

$V_{out} = (I_1 - g_{m1} V_{in}) \cdot r_{O1} \cdot (g_{m2} r_{O2} + 1) + I_1 r_{O2}$

$= V_{in} \cdot (-g_{m1} r_{O1} r_{O2} g_{m1} g_{m2}) + (r_{O1} + r_{O2} + r_{O1} r_{O2} g_{m2}) I_1$,

$A = \frac{\partial V_{out}}{\partial V_{in}} = -r_{O1} g_{m1} - r_{O1} r_{O2} g_{m1} g_{m2}$

$R_{out} = 1.32 \times 10^9\,\Omega$, $A_V = -2.76 \times 10^{-5}$.

② When $I = 0.01\,mA$, $R_{out} = 4.20 \times 10^7\,\Omega$, $A_V = -2.77 \times 10^4$.

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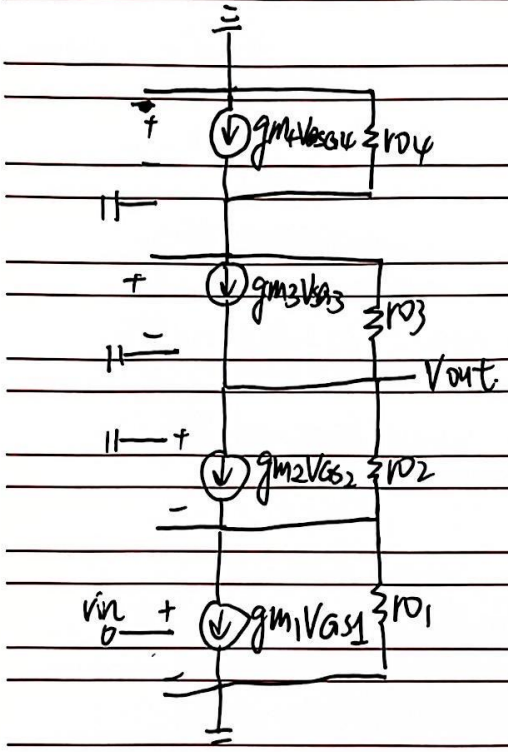
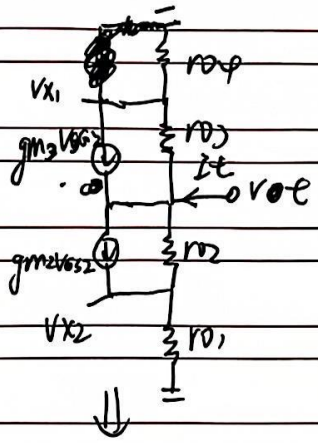
Question 4

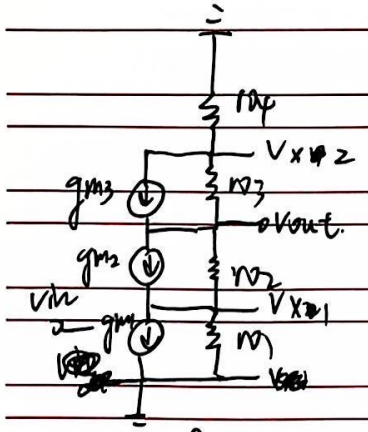


Rout:



$$\begin{cases} \frac{Vx1}{ro4} + gm_3 \cdot Vx_1 + \frac{Vx_1 - Vt}{ro_3} = 0 \\ gm_3 Vx_1 + \frac{Vx_1 - Vt}{ro_3} + It = gm_2(-Vx_2) + \frac{Vt - Vx_2}{ro_2} \\ gm_2(-Vx_2) + \frac{Vt - Vx_2}{ro_2} = \frac{Vx_2}{ro_1} \end{cases}$$

$$\Rightarrow Rout = \frac{Vt}{It} = \frac{(ro_1 + ro_2 + gm_2 ro_1 ro_2)(ro_3 + ro_4 + gm_3 ro_3 ro_4)}{ro_4 + ro_1 + ro_2 + ro_3 + gm_2 ro_1 ro_2 + gm_3 ro_3 ro_4}$$



$$\Rightarrow gm_1 Vin + \frac{Vx_1}{ro_1} = gm_2 \cdot (-Vx_1) + \frac{Vout - Vx}{ro_1} =$$

$$gm_3 Vx_2 + \frac{Vx_2 - Vout}{ro_3} = \frac{-Vx_2}{ro_4}$$

$$\Rightarrow Av = \frac{gm_1}{(\frac{1}{ro_1} + \frac{1}{ro_2} + gm_2)\left(\frac{gm_3 + \frac{1}{ro_3}}{(gm_2 + \frac{1}{ro_2})(gm_3 + (\frac{1}{ro_3} + \frac{1}{ro_4}))ro_3} - \frac{\frac{1}{ro_2} + \frac{1}{ro_3}}{\frac{1}{ro_2} + gm_2}\right) + \frac{1}{ro_2}}$$

If $I = 0.01mA$, $Rout = 3.98 \times 10^8 \Omega$, $Av = -8.3 \times 10^4$

If $I = 0.1mA$, $Rout = 1.297 \times 10^7 \Omega$, $Av = -8.4 \times 10^3$

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