Lecture 5: Operational Amplifiers

ECE 3110J, Electronic Circuits

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2024 Summer

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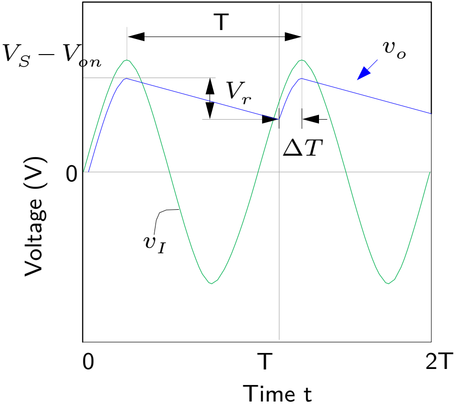
Recap of Last Lecture

- Diode Circuits
  Look at Rectifiers again
- Op-amps

Half-Wave Rectifier with RC Load

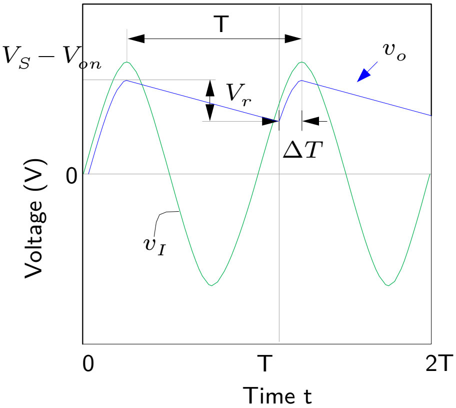


Here “DC” means we approximate the discharging period with a constant voltage.

$$V_{dc} = V_s - V_{on} \tag{1}$$

$$I_{dc} = \frac{V_{dc}}{R} \tag{2}$$

Calculating Ripple Voltage

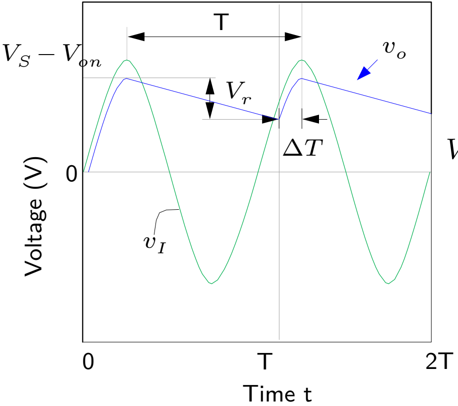


$$V_r = (V_s - V_{on})\left(1 - e^{-\frac{T-\Delta T}{RC}}\right) \tag{3}$$

$$\cong (V_s - V_{on})\left(\frac{T-\Delta T}{RC}\right) \text{ if } (T-\Delta T) \ll RC \tag{4}$$

$$\cong (V_s - V_{\text{on}})\left(\frac{T}{RC}\right) \text{ if } \Delta T \ll T \tag{5}$$

Calculating Conduction Angle and Interval

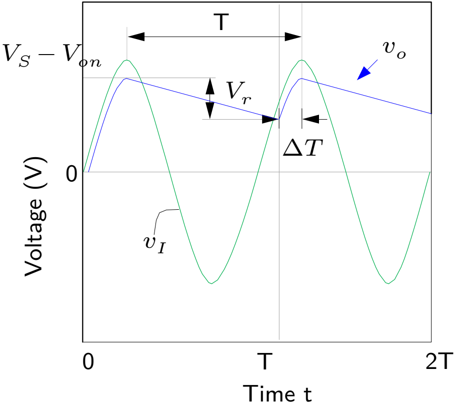


$$V_s \sin\left[\omega\left(\frac{5T}{4} - \Delta T\right)\right] - V_{on} = (V_s - V_{on}) - V_r \quad (6)$$

$$V_s \sin\left(\frac{5\pi}{2} - \theta_c\right) - V_{on} = (V_s - V_{on}) - V_r \quad (7)$$

$$V_s \cos\theta_c = V_s - V_r \quad (8)$$

Calculating Conduction Angle and Interval



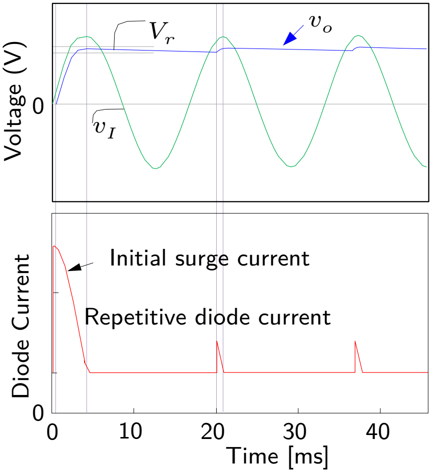
$$\cos\theta_c = \frac{V_s - V_r}{V_s} \tag{9}$$

$$\cong 1 - \frac{\theta_c^2}{2} \text{ if } \theta_c \text{ very small} \tag{10}$$

$$\theta_c = \sqrt{\frac{2V_r}{V_s}} \tag{11}$$

$$\Delta T = \frac{\theta_c}{\omega} = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_s}} \tag{12}$$

Derive Peak Current



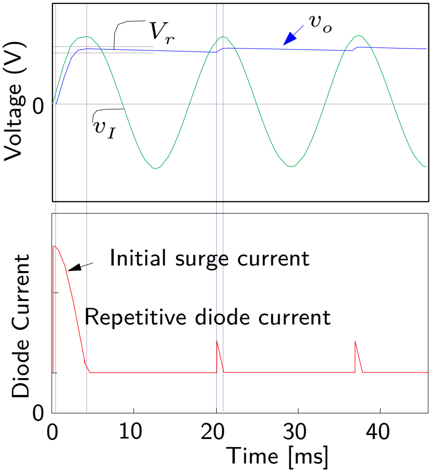
The charge filled on C during ΔT is discharged during $T - \Delta T$

$$Q \cong \frac{I_{\text{peak}}\,\Delta T}{2} \tag{13}$$

$$= I_{dc}(T - \Delta T) \cong I_{dc}T \tag{14}$$

$$I_{\text{peak}} = \frac{2I_{\text{dc}}\,T}{\Delta T} \tag{15}$$

Derive Surge Current



During charging period (ΔT), almost all diode current goes to C.

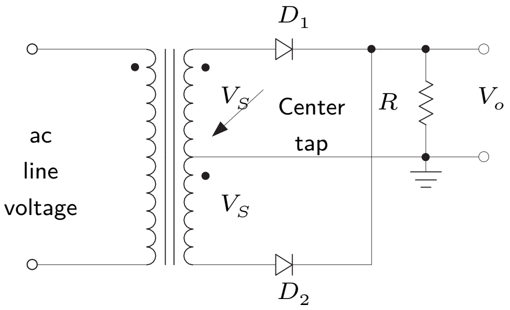
$$\left|\frac{1}{SC}\right| = \frac{1}{2\pi\frac{C}{T}} \tag{16}$$

$$= \frac{T}{2\pi C} \ll R \quad \text{if } RC \gg T \tag{17}$$

$$I_{\text{surge}} = C\frac{d\left(V_s \sin\omega t - V_{\text{on}}\right)}{dt} \tag{18}$$

$$= \omega C V_s \text{ if } t = 0 \tag{19}$$

Transformer-Based Full Wave Rectifier



What does the ground symbol mean?

Diode-Based Full Wave Rectifier ($V > 2V_D$)

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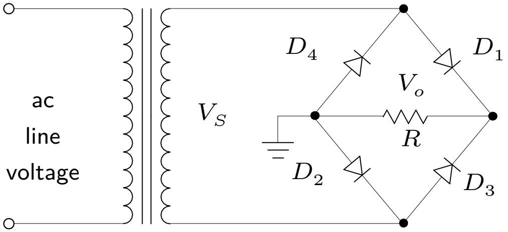
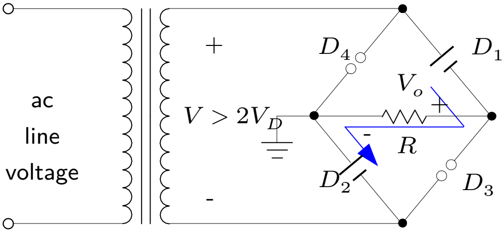
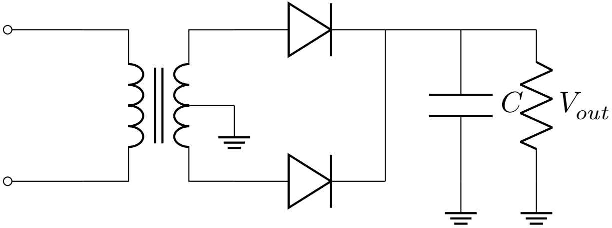
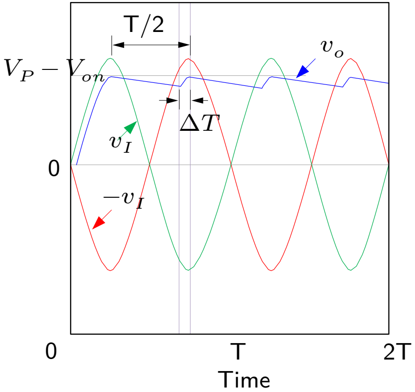


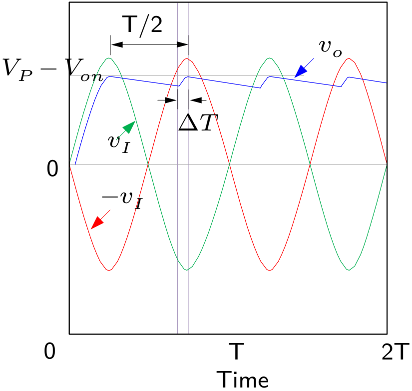


Diode-Based Full Wave Rectifier





Ripple Voltage Derivation

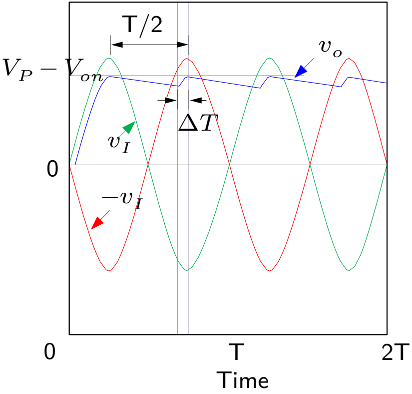


$$V_{dc} = V_s - V_{on} \tag{20}$$

$$I_{dc} = \frac{V_{dc}}{R} \tag{21}$$

$$V_r = (V_s - V_{on})\left(1 - e^{-\frac{T/2-\Delta T}{RC}}\right) \tag{22}$$

Ripple Voltage Derivation



$$V_r = (V_s - V_{on})\left(1 - e^{-\frac{T/2-\Delta T}{RC}}\right) \tag{23}$$

if $\left(\frac{T}{2} - \Delta T\right) \ll RC$

$$\cong (V_s - V_{on})\left(\frac{T/2 - \Delta T}{RC}\right) \tag{24}$$

if $\Delta T \ll \frac{T}{2}$

$$\cong (V_s - V_{on})\left(\frac{T}{2RC}\right) \tag{25}$$

Conduction Angle Derivation

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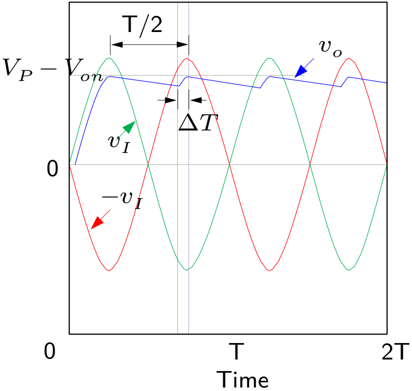


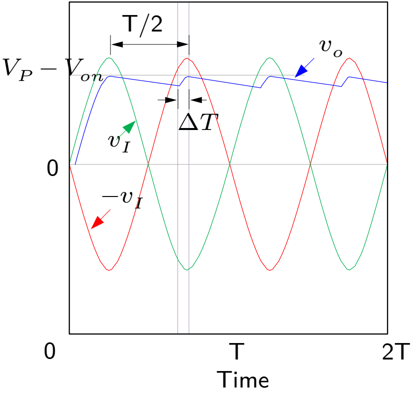
$$-V_s \sin\left[\omega\left(\frac{3T}{4} - \Delta T\right)\right] - V_{on} = (V_s - V_{on}) - V_r \quad (26)$$

$$-V_s \sin\left(\frac{3\pi}{2} - \theta_c\right) - V_{on} = (V_s - V_{on}) - V_r \quad (27)$$

$$V_s \cos\theta_c = V_s - V_r \quad (28)$$

$$\theta_c = \sqrt{\frac{2V_r}{V_s}} \quad (29)$$

Conduction Angle Derivation



$$\cos\theta_c = \frac{V_s - V_r}{V_s} \cong 1 - \frac{\theta_c^2}{2} \text{ if } \theta_c \text{ very small} \tag{30}$$

$$\Delta T = \frac{\theta_c}{\omega} = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_s}} \tag{31}$$

Surge and Peak Current Estimation

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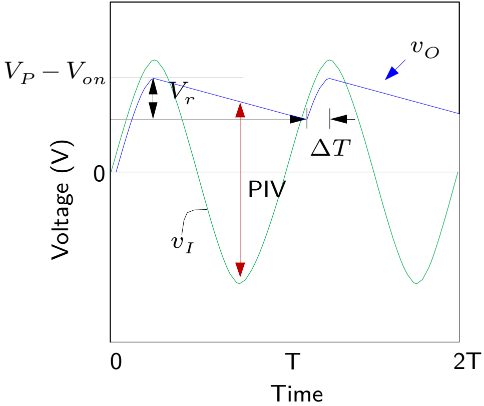

$$Q \cong \frac{I_{\text{peak}}\,\Delta T}{2} \tag{32}$$

$$= I_{dc}\left(\frac{T}{2} - \Delta T\right) \cong I_{dc}\frac{T}{2} \tag{33}$$

$$I_{\text{peak}} = \frac{I_{dc}T}{\Delta T} \tag{34}$$

$$I_{\text{surge}} = \omega C V_s \tag{35}$$

$$\text{PIV} = 2V_s - V_{on} \tag{36}$$

Full-wave Bridge Rectifier Example

Design a full-wave bridge rectifier to provide a dc output voltage 15 V with no more than 1 percent ripple at a load current of 2A. $(V_{on} = 1\ V, T = 1/60\ \mathrm{sec})$.

$$V_{dc} = 15\ V \tag{37}$$

$$V_r < 0.15 \tag{38}$$

Given that load current $I_{dc} = 2\ A$, the load resistance is

$$15/2 = 7.5\ \Omega \tag{39}$$

Full-wave Bridge Rectifier Ripple Calculation

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Assume turn-on is 1 V, The required transformer voltage

$$V_s = 15 + 2 = 17\ V \text{ or } \frac{17}{\sqrt{2}}\ (V_{rms}) \tag{40}$$

$$V_r \cong (V_s - 2V_{on})\left(\frac{T}{2RC}\right) = 15\left(\frac{1}{2 \cdot 60 \cdot 7.5 \cdot C}\right) = 0.15 \Rightarrow C = 0.111\ F \tag{41}$$

$$\Delta T = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_s}} = \frac{1}{2\pi \cdot 60}\sqrt{\frac{2 \cdot 0.15}{17}} = 0.352 \cdot 10^{-3}\ (\text{sec}) \tag{42}$$

Full-wave Bridge Rectifier Surge and Peak Current

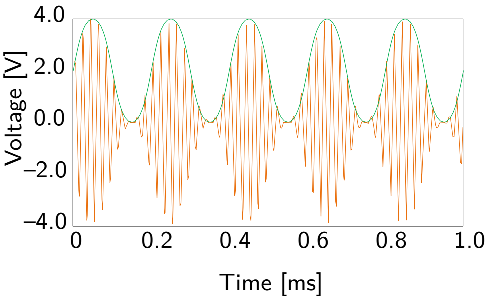
$$I_{peak} = \frac{I_{dc}T}{\Delta T} = \frac{2 \cdot \frac{1}{60}}{0.352 \cdot 10^{-3}} = 94.7 \ A \tag{43}$$

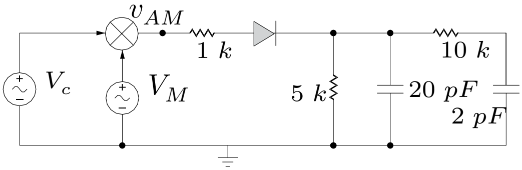
$$I_{surge} = \omega C V_s = 2\pi \times 60 \cdot 0.111 \cdot 17 = 711 \ A \tag{44}$$

- Make sure the diodes can handle these large currents

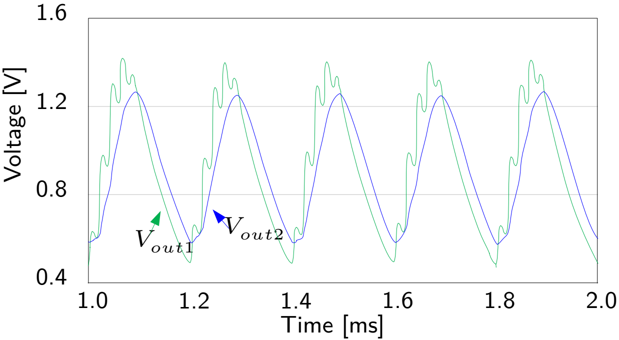
amplitude modulated Demodulator

An amplitude modulated (AM) signal is shown below. The envelope of the AM signal contains the information being transmitted, and the envelope can be recovered using a single half-wave rectifier.





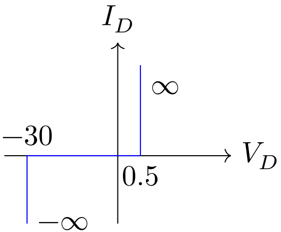
Amplitude Modulated (AM) signal

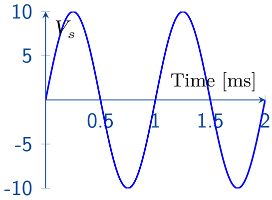


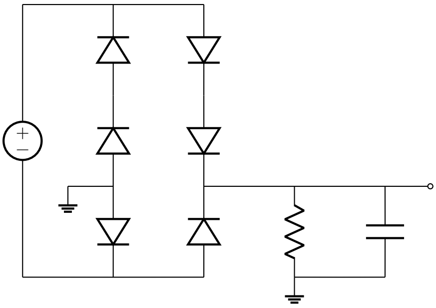
- The envelope of the AM signal contains the information being transmitted, and the envelope can be recovered using a single half-wave rectifier.

Full-Wave Bridge Rectifier Example

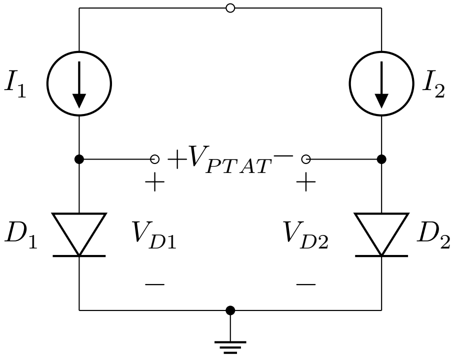
A diode has the I-V characteristic as shown. $R = 1\ k\Omega$. Estimate the minimum value of C, so that the ripple voltage (V_r) is smaller than 0.1V.







Diode Temperature Dependence



For a fixed $I_D \gg I_s$:

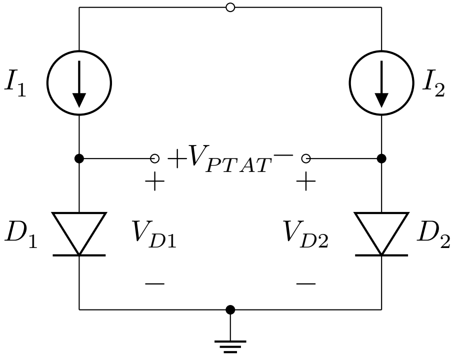
$$I_D = I_S\left(e^{\frac{qV_a}{kT}} - 1\right) \tag{45}$$

$$V_a = \frac{kT}{q}\ln\left(\frac{I_D}{I_S} + 1\right) \cong \frac{kT}{q}\ln\frac{I_D}{I_S} \tag{46}$$

$$V_{D1} = \frac{kT}{q}\ln\frac{I_1}{I_S} \tag{47}$$

$$V_{D2} = \frac{kT}{q}\ln\frac{I_2}{I_S} \tag{48}$$

Diode Temperature Dependence



$$V_{PTAT} = V_{D1} - V_{D2} \tag{49}$$

$$= \frac{kT}{q} \ln \frac{I_1}{I_2} = T \times \text{constant} \tag{50}$$

Diode Spice Model

$$I_D = IS\left[\exp\left(\frac{qV_a}{NkT}\right) - 1\right] \tag{51}$$

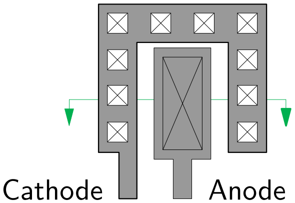
$$C_D = TT\frac{I_D}{N(kT/q)} \tag{52}$$

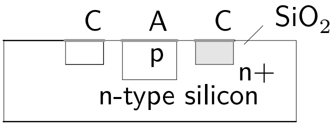
$$C_j = \frac{CJO}{\left(1 - \frac{V_a}{VJ}\right)M} RAREA \tag{53}$$

Diode Spice Parameter Equivalences

Parameter	SPICE	Typical Default Values
Saturation current	IS	$10\ fA$
Ohmic series resistance	RS	$0\ \Omega$
Ideality factor	N	1
Transit time	TT	$0\ sec$
Zero-bias junction capacitance for a unit area diode RAREA $=1$	CJO	$0\ F$
Built-in potential	VJ	$1\ V$
Junction grading coefficient	M	0.5
Relative junction area	RAREA	1

Diode Layout





- For forming an ohmic contact between metal and n^- type silicon

Short Circuit Current

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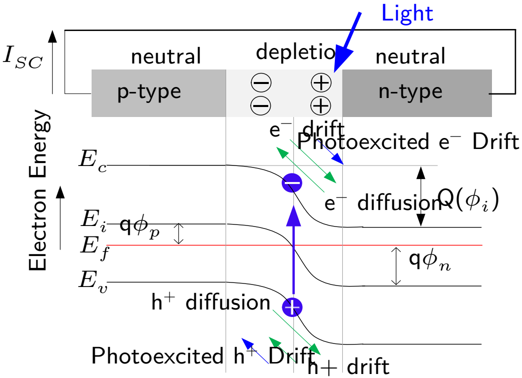
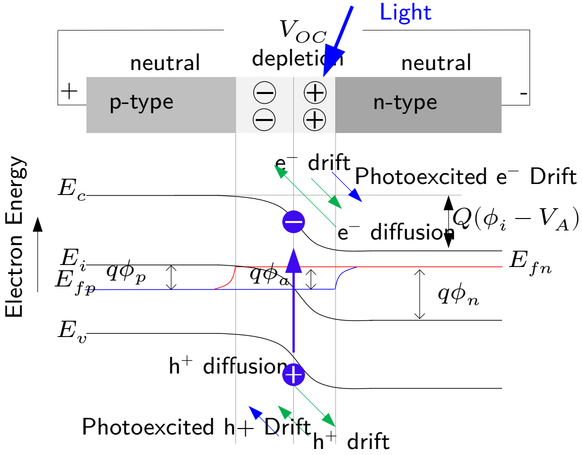


$$EQE = \frac{\text{electrons /sec}}{\text{photons /sec}} = \frac{(\text{ current })/(\text{ charge of one electron })}{(\text{ total power of photons })/(\text{ energy of one photon })} \tag{54}$$

Open Circuit Voltage



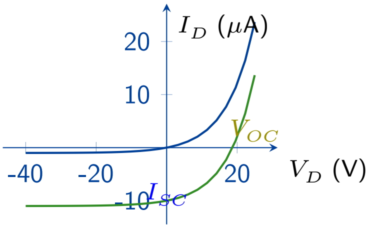
- e^- drift + photoexcited e^- drift = e^- diffusion
- h^+ drift + photoexcited h^+ drift = h^+ diffusion

$$I = I_s \left[\exp\left(\frac{qV}{nkT}\right) - 1\right] - I_L \qquad (55)$$

where I_L is light generated current.
$I_L = I_{SC}$ when source resistance is small.

$$V_{OC} = \frac{nkT}{q} \ln\left(\frac{I_L}{I_s} + 1\right) \qquad (56)$$

I-V Curve of Solar Cells



IV of Diodes vs. that of solar Cells

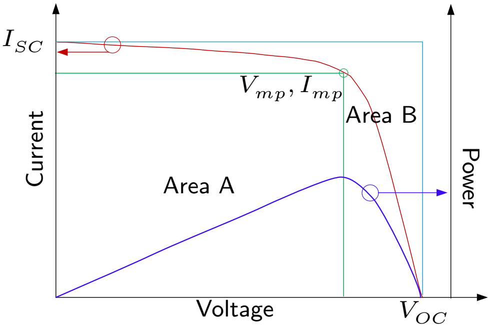
$$P_{\max} = V_{OC} I_{SC} FF \tag{57}$$

$$\eta = \frac{V_{OC} I_{SC} FF}{P_{\text{in}}} \tag{58}$$

To find V_{MP}, we will set

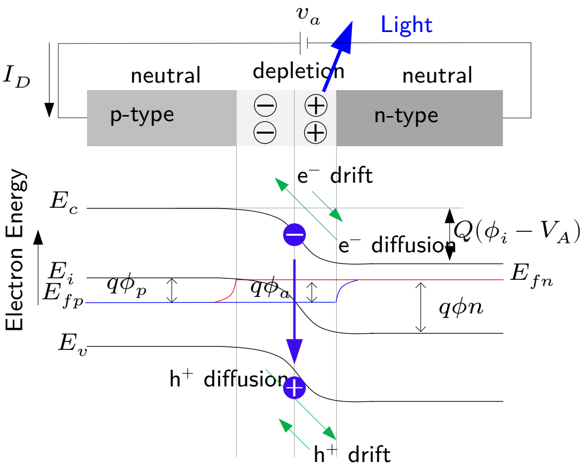
$$\frac{d(IV)}{dV} = 0 \tag{59}$$

Derivation of V_{MP}



$$V_{MP} = V_{OC} - \frac{nkT}{q} \ln\left(\frac{qV_{MP}}{nkT} + 1\right) \quad (60)$$

Charge Injection



- Direct bandgap semiconductor is required. Note that silicon is indirect bandgap semiconductor.
- Under forward bias, charge carriers are injected and recombined in the depletion region to emit photons.

Review: Signal Types

Sedra & Smith 1.4, 1.5

- discrete time and amplitude
- discrete time but continuous amplitude
- continuous time and continuous amplitude

Linear vs. Nonlinear

Linear two-port circuits consist only of R, L, and C elements and transformers, and where the output is linearly related to the input.

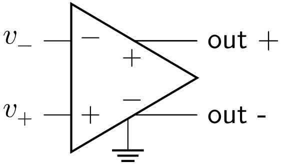
- Non-linear circuits typically include diodes, transistors
- Therefore, all active circuits, and amplifiers are active circuits, are nonlinear over a large enough signal range. But amplifiers by definition need to be linear (meaning that the output and the input should only be related through a constant factor so no distortion is introduced into the amplified output

Amplifiers

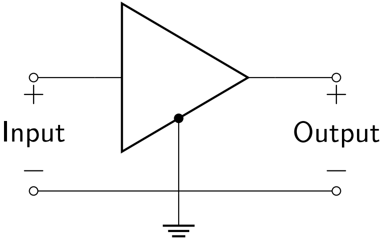
An amplifier is a two-port circuit which takes an input voltage or current and increases one or both in magnitude. Note that a linear amplifier is not supposed to add any new frequency components into the signal.

- Amplifiers can be differential or single-ended.
- Single-ended amplifiers share one of their lines (terminals) and the input and output signals are both referenced to the common line, which usually is the ground in a circuit.

Amplifiers



Differential Amplifier



single-ended Amplifier

Classification of Amplifiers

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An amplifier receives a signal from a source and delivers it to a load. Gains are dimensionless, and are usually expressed in terms of decibels (dB):

$$\text{Voltage Gain} = A_v = \frac{v_o}{v_i} \tag{61}$$

$$\text{Current Gain} = A_i = \frac{i_o}{i_i} \tag{62}$$

$$\text{Power Gain} = A_p = \frac{v_o i_o}{v_i i_i} = A_v A_i \tag{63}$$

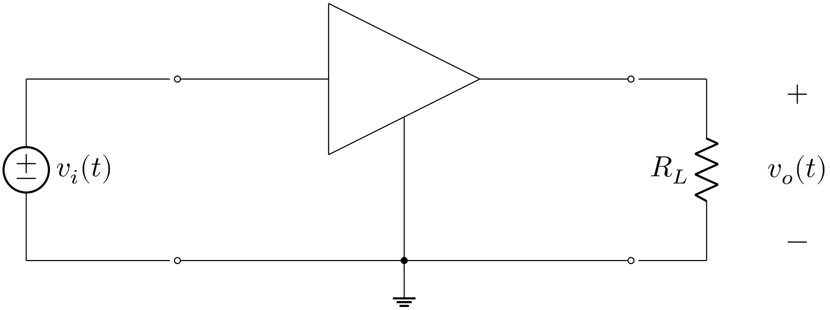
$$\text{Voltage Gain} = 20 \log A_v \tag{64}$$

$$\text{Current Gain} = 20 \log A_i \tag{65}$$

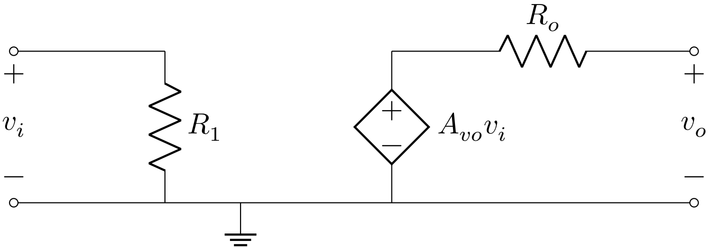
$$\text{Power Gain} = 10 \log A_p \tag{66}$$

Voltage Amplifier

As mentioned all active circuits are nonlinear. But amplifiers need to be linear. Therefore we need to use an amplifier circuit over only a limited range where the relationship between the input and out is linear.



Generic Model for Voltage Amplifier



There are several models for amplifiers. For a voltage amplifier the model is shown. This is a voltage-controlled voltage source (VCVS) model. It means that the output voltage is controlled by the input voltage.

Generic Model for Voltage Amplifier

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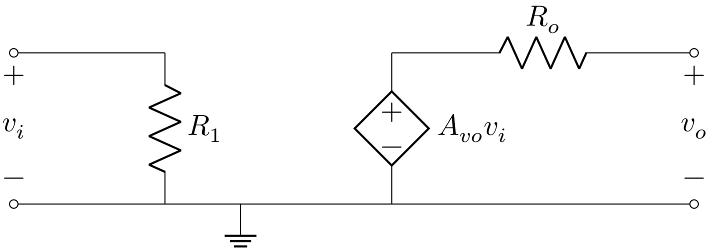

In order for the voltage gain to be maximum, as you can see we need the output resistance R_o to be as small as possible so there is no voltage dropped across the output resistance. The input resistance R_i needs to be as large as possible so all of the input voltage is applied to the input.

Current Amplifier

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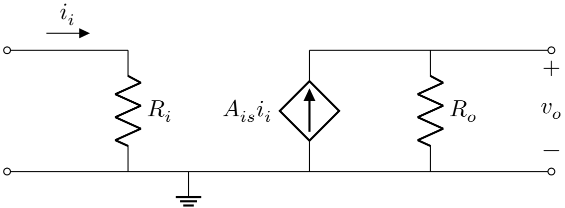


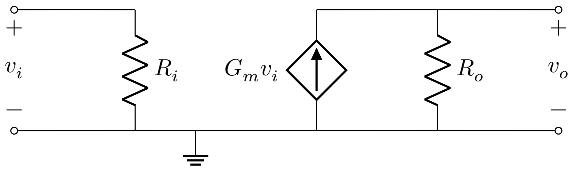
Short-Circuit Current Gain

$$A_{is} \equiv \frac{i_o}{i_i}\Big|_{v_o=0} \quad (A/A) \tag{67}$$

$$R_i = 0 \tag{68}$$

$$R_o = \infty \tag{69}$$

Transconductance Amplifier



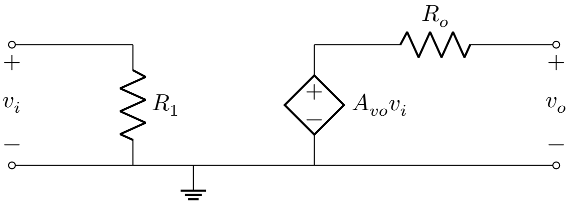
Short-Circuit Transconductance

$$G_m \equiv \frac{i_o}{v_i}\Big|_{v_o=0} \quad (A/V) \tag{70}$$

$$R_i = \infty \tag{71}$$

$$R_o = \infty \tag{72}$$

Transresistance Amplifier



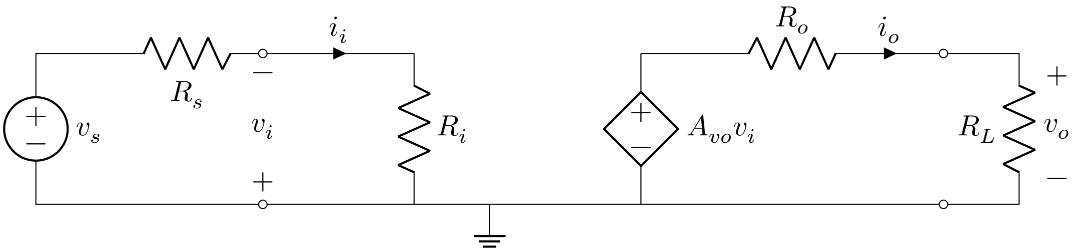
Open-Circuit Transresistance

$$R_m \equiv \frac{v_o}{i_i}\Big|_{i_o=0} \quad (V/A) \tag{73}$$

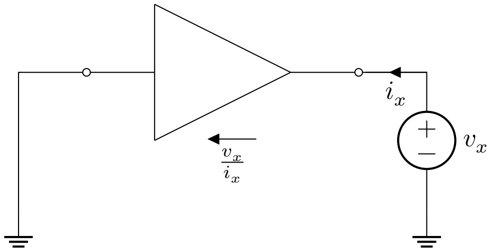
$$R_i = 0 \tag{74}$$

$$R_o = 0 \tag{75}$$

Voltage Amplifier in a Circuit



Determining R_o

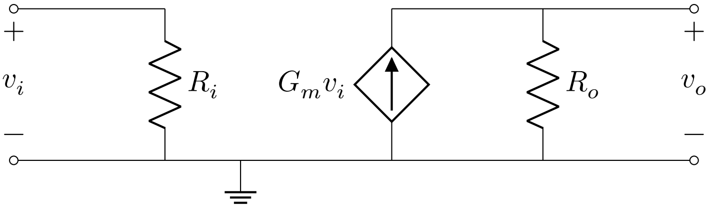


To find the output resistance, we follow the same procedure as we did for finding the Thevenin equivalent resistance.

- Short the input terminal since it is an independent voltage source
- Apply a test source v_x to find i_x
- Find the ratio of $R_o = \frac{v_x}{i_x}$

Norton Model of Transconductance Amplifiers

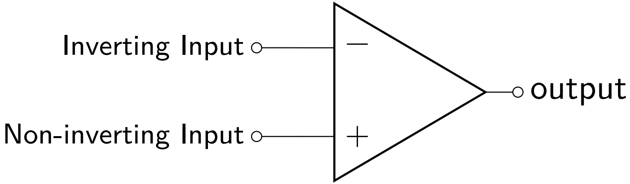
上海交通大學 SHANGHAI JIAO TONG UNIVERSITY



The input is a voltage and the output is a current.

- Therefore, we need the input resistance to be as large as possible and the output resistance to be also as large as possible so that all of the output current is delivered to the load (not shown)

Ideal Op-amps



Operational Amplifier (op-amp) is a circuit building block that is used extensively to implement a variety of functions.

- Op-amps were originally used in analog computers to perform operations such as addition and subtraction, thus the name operational amplifier.

Ideal Op-amps

- Today, op-amps are all integrated circuits, are low-cost, and are used to implement a variety of functions, including signal amplification, filtering, buffering, etc., mostly analog functions.
- The op-amp is used in a circuit that almost always employs feedback.
- Let's first consider an ideal op-amp and see how basic analog functions can be implemented using an ideal op-amp.
- Non-ideal behavior is then discussed.