Lecture 3: Diode

ECE3110J,Electronic Circuits

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2024 Summer

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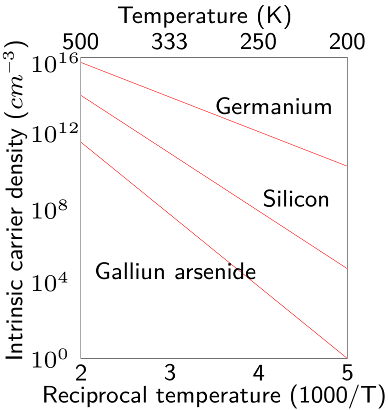
Recap of Last Lecture

- Scaling
- Review of Thevenin and Norton Equivalent
- Semiconductor Basics
- Diodes

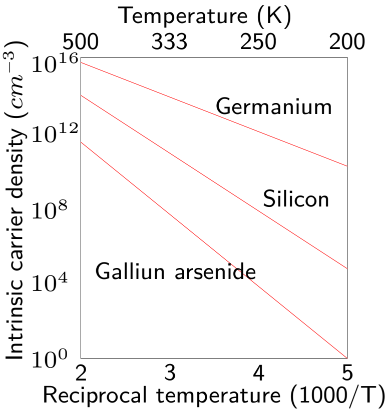
Intrinsic Si



$$np = n_i^2 = BT^3 \exp(-\frac{E_G}{kT}) \tag{1}$$

- n Free electrons in the conduction band.
- p Free holes in the valence band.
- n_i Intrinsic carrier density.

Intrinsic Si

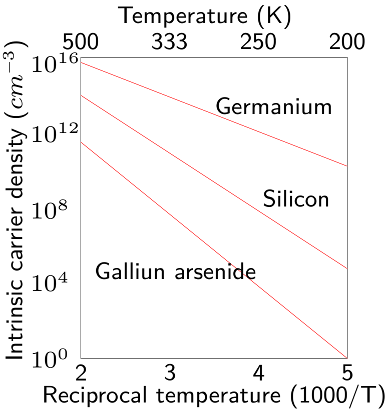


k (Boltzmann's Constant)

$$= 1.38 \times 10^{-23}\ \mathrm{J/K} = 8.62 \times 10^{-5}\ e\mathrm{V/K} \quad (2)$$

	B ($\mathrm{K}^{-3}\cdot\mathrm{cm}^{-6}$)	E_G (eV)
Si	1.08×10^{31}	1.12
Ge	2.31×10^{30}	0.66
GaAs	1.27×10^{29}	1.42

Intrinsic Si



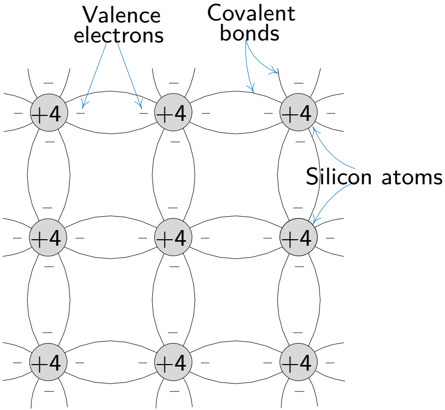
At 300K

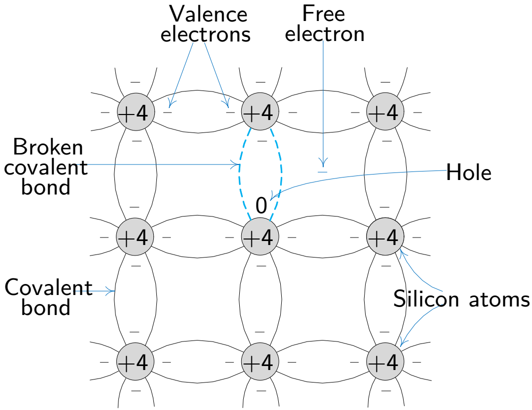
$$n_i^2 = (1.08 \cdot 10^{31}) \cdot 300^3 \cdot e^{\frac{-1.12}{(8.62 \cdot 10^{-5}) \cdot 300}}$$

$$= 4.52 \times 10^{19} (1/\mathrm{cm}^6) \quad (3)$$

$$n_i = 6.73 \times 10^9 (1/\mathrm{cm}^3) \approx 10^{10} (1/\mathrm{cm}^3) \quad (4)$$

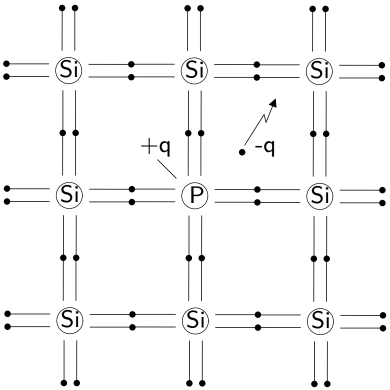
Electrons and Holes





Doped Si (N-type)

- At room temperature, nearly all phosphorus dopants are ionized.
- Each dopant donates one electron away, which creates an electron in the conduction band.
- P: $1s^2 2s^2 2p^6 3s^2 3p^3$
- B: $1s^2 2s^2 2p^1$



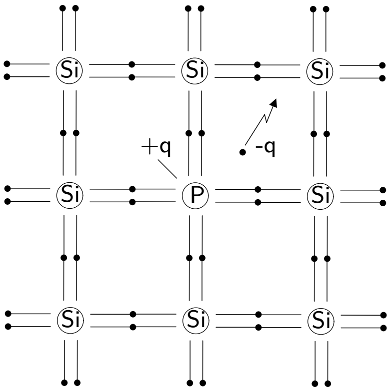
Doped Si (N-type)

- If the n-type dopant concentration $N_d \gg n_i$, $n = N_d$ and $p = n_i^2/N_d$
- charge neutrality

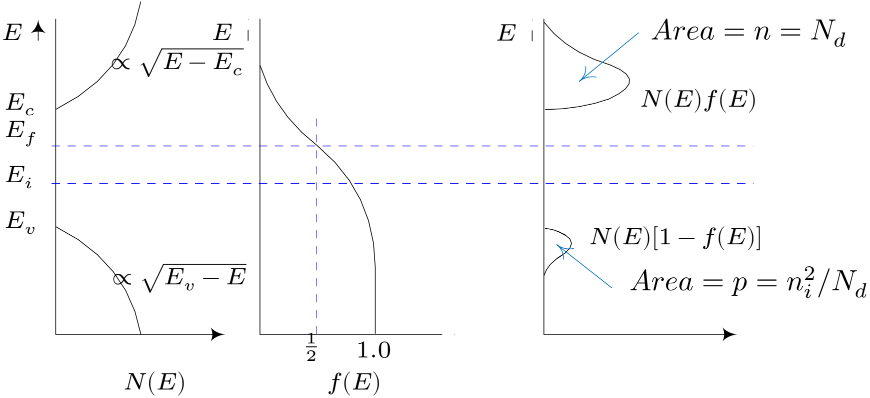
$$q\left(N_D + p - N_A - n\right) = 0 \quad (5)$$

$$n^2 - \left(N_D - N_A\right)n - n_i^2 = 0 \quad (6)$$

$$n = \frac{\left(N_D - N_A\right) + \sqrt{\left(N_D - N_A\right)^2 + 4n_i^2}}{2} \quad (7)$$

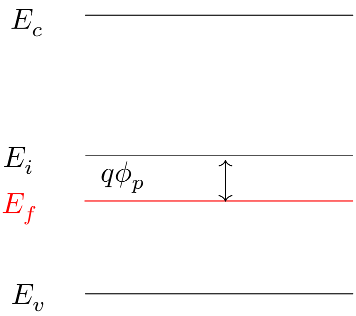


Density of States (N-type)



- Notice that the fermi level is shifted.

P-Type



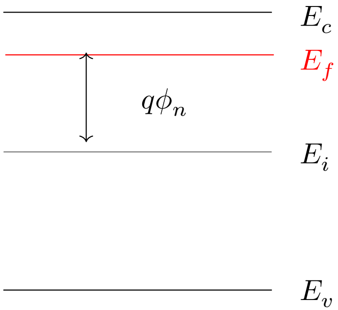
$$np = n_i^2 \tag{8}$$

$$p = N_a = n_i e^{\frac{E_i - E_f}{kT}} = n_i e^{\frac{q\phi_p}{kT}} \tag{9}$$

$$n = \frac{n_i^2}{N_a} = n_i e^{\frac{E_f - E_i}{kT}} = n_i e^{\frac{-q\phi_p}{kT}} \tag{10}$$

- Hu, Chenming. Modern semiconductor devices for integrated circuits.

N-Type



$$np = n_i^2 \tag{11}$$

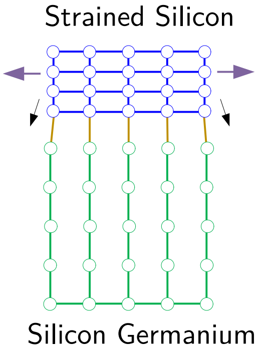
$$n = N_d = n_i e^{\frac{E_f - E_i}{kT}} = n_i e^{\frac{q\phi_n}{kT}} \tag{12}$$

$$p = \frac{n_i^2}{N_d} = n_i e^{\frac{E_i - E_f}{kT}} = n_i e^{\frac{-q\phi_n}{kT}} \tag{13}$$

Carrier Concentration Practice

- By ion implantation on crystalline Si with phosphorous concentration equal to 10^{17} cm^{-3}
- What is the value of n, p, and E$_f$?

Strained Silicon for Higher Mobility

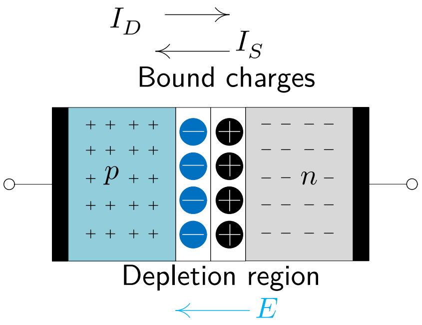


Johnson's figure of merit

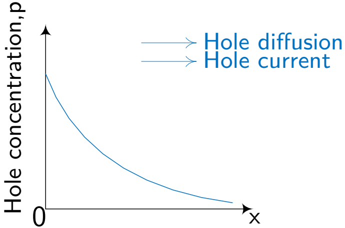
Material	Saturation velocity $\times 10^{-5}$ m/s	$V_{breakdown}$ MV/cm	JFM
Silicon	1.0	0.3	1.0
GaAs	1.5	0.4	2.7
SiC	2.0	3.5	20
InP	0.67	0.5	0.33
GaN	2.5	3.3	27.5

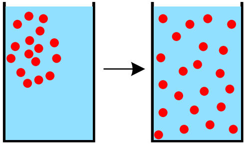
PN Junction

- Diffusion current
- space-charge (Depletion) region
- Drift current



Diffusion

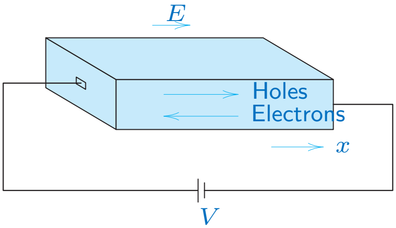




- Magnitude of the current at any point is proportional to the slope of the concentration profile, or the **concentration gradient** , at that point

$$J_{D,n} = qD_n \frac{dn(x)}{dx} \tag{14}$$

Drift Current



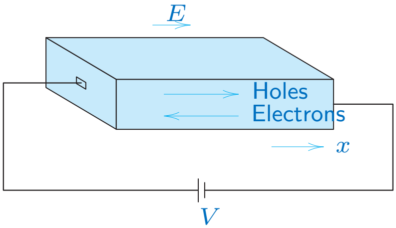
$$I_p = Aqpv_{p\text{-drift}} \tag{15}$$

$$I_p = Aqp\mu_p E \tag{16}$$

- For intrinsic silicon $\mu_p = 480$ cm^2/V · s.
- For the electron mobility, $\mu_n = 1350$ cm^2/V

$$J_S = J_{S,p} + J_{S,n} = q\left(p\mu_p + n\mu_n\right)E \tag{17}$$

Drift Current

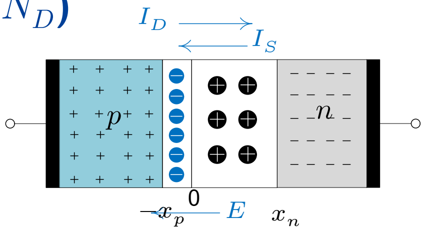


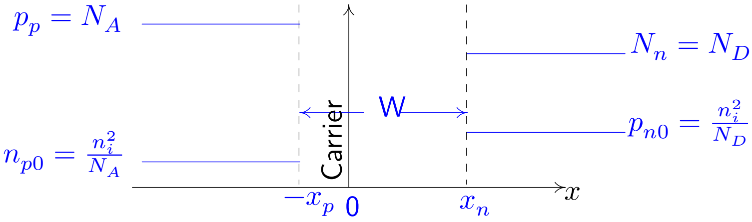
- The Junction Built-in Voltage

$$V_0 = V_T \ln(\frac{N_A N_D}{n_i^2}) \tag{18}$$

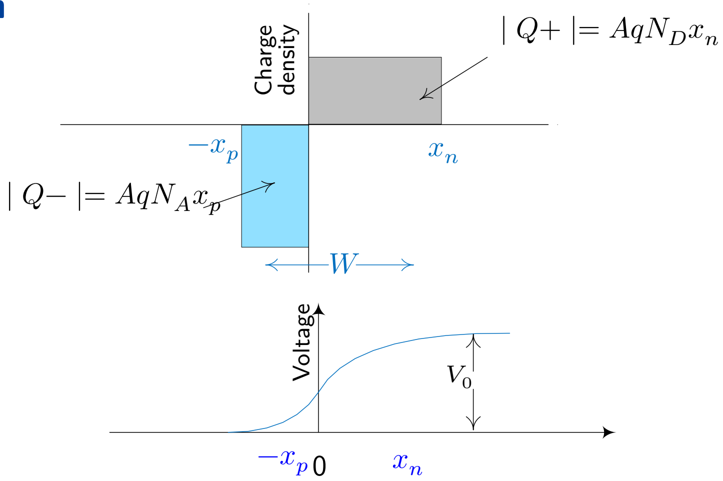
- Derivation: see Streetman and Bannerjee

PN Junction $(N_A > N_D)$

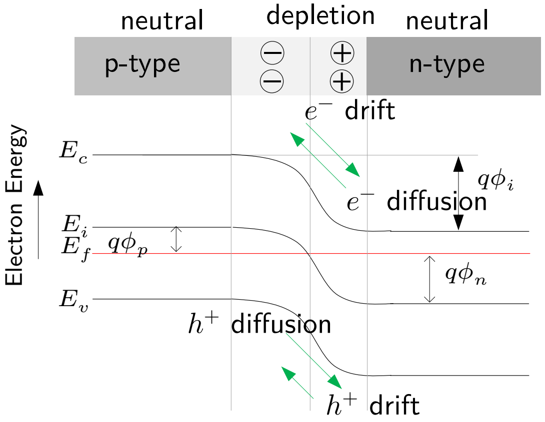




PN Junction

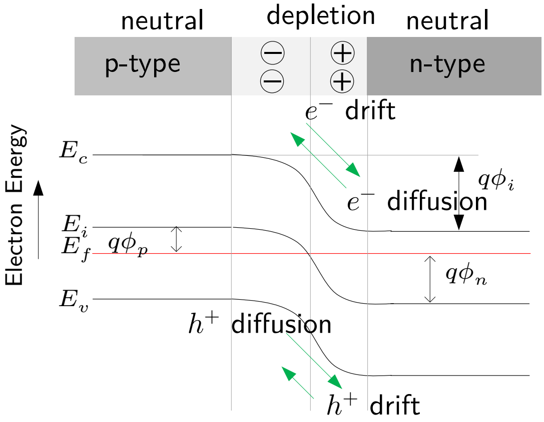


PN Junction



1. Electrons/holes near the junction diffuse to the opposite sides.
2. Ionized dopants, fixed in the lattice, are left behind.
3. Formation of built-in electric field and energy barrier $(q\phi_i)$ for diffusion.

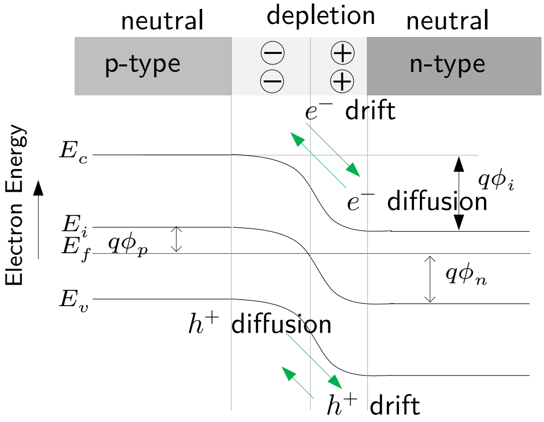
PN Junction



Note:

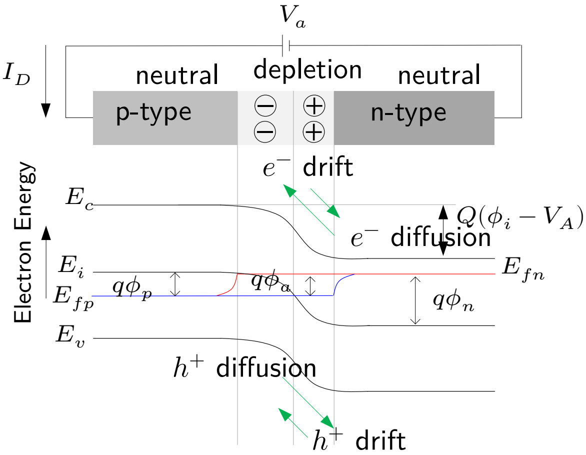
- E$_C$, E$_i$ and E$_V$ are parallel to each other.
- E$_C$, E$_i$ and E$_V$ bending means there is electric field.
- E$_f$ bending means there is current.

PN Junction



4. Some electrons/holes in the neutral regions with sufficient energy continuously diffuse to the opposite sides. $\rightarrow$ Formation of diffusion current.
5. Some electrons/holes wandering into the in the depletion region get swept by the built-in electric field. $\rightarrow$ Formation of drift current.
6. Diffusion current cancels drift current. No net current flowing.

Forward Bias (When $V_a > 0$)



$$I_D = I_S\left(e^{\frac{qV_a}{kT}} - 1\right) > 0 \tag{19}$$

1. $e^{\frac{qV_a}{kT}}$ is the diffusion current.
2. -1 stands for the drift current.
3. The energy barrier formed by the built-in electric field becomes smaller, $q(\phi_i - V_a)$.
4. Neamen, Donald A. Semiconductor Physics and Devices Basic Principles, chapter 8

Forward Bias (When $V_a > 0$)

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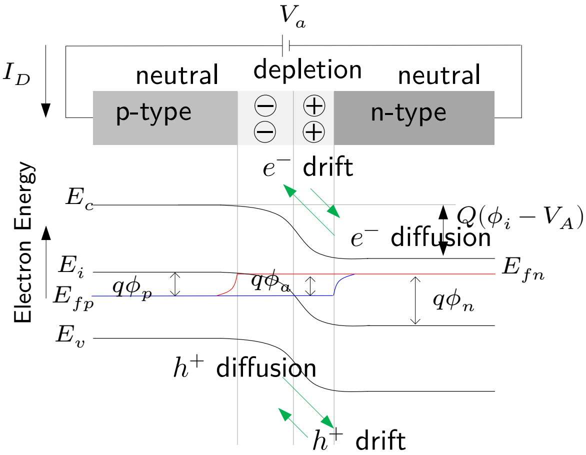
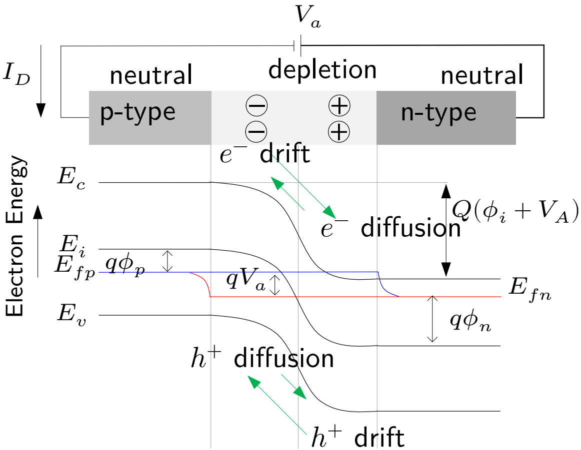


1. More electrons/holes diffuse to the opposite sides. $\rightarrow$ Diffusion current increases.
2. The drift current is limited by the number of minority carriers on either side of the p-n junction and is unchanged by the increased E-field.
3. There is $(+)$ net current flowing.
4. The depletion width becomes narrower.

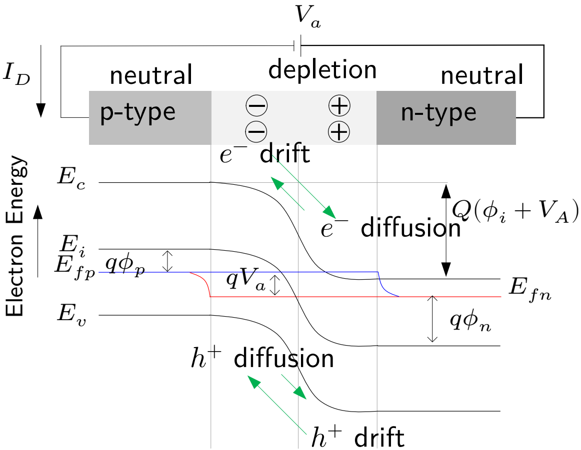
Reverse Bias ($V_a < 0$)



$$I_D = I_S \left(e^{\frac{qV_a}{kT}} - 1 \right) < 0 \qquad (20)$$

1. The energy barrier formed by the built-in electric field becomes larger, $q(\phi_i + V_a)$.

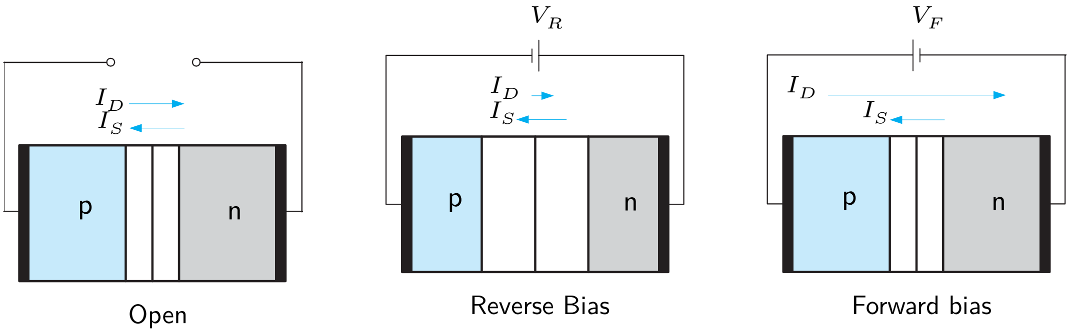
Reverse Bias ($V_a < 0$)



$$I_D = I_S\left(e^{\frac{qV_a}{kT}} - 1\right) < 0 \qquad (21)$$

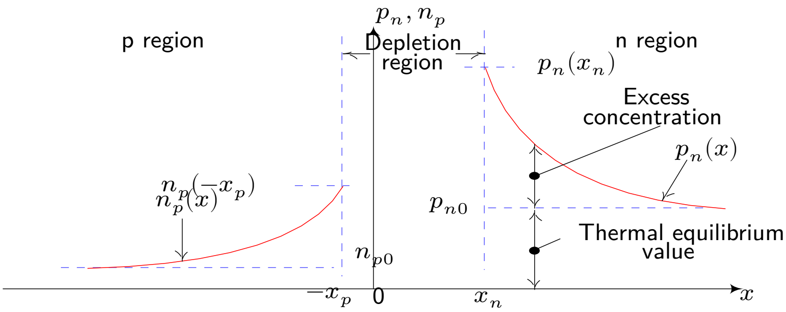
1. Less electrons/holes diffuse to the opposite sides. → Diffusion current decreases, while drift current remains the same.
2. There is (-) net current flowing.
3. The depletion width becomes wider.

Operating Conditions of Diodes



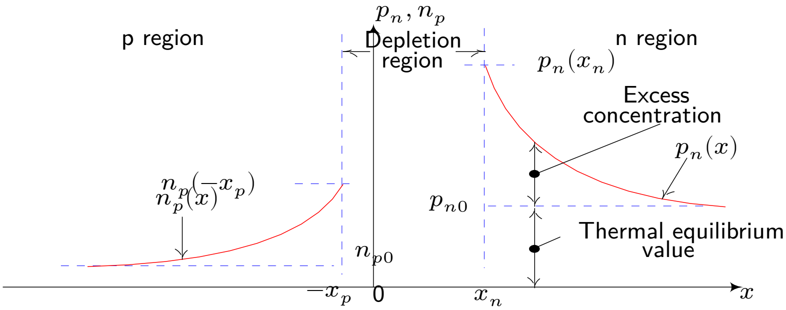
Diode Current (Forward Bias, $N_A \gg N_D$)

Sedra: 3.5.2



- a greater number of holes (electrons) to overcome the barrier and diffuse into the n (p) region

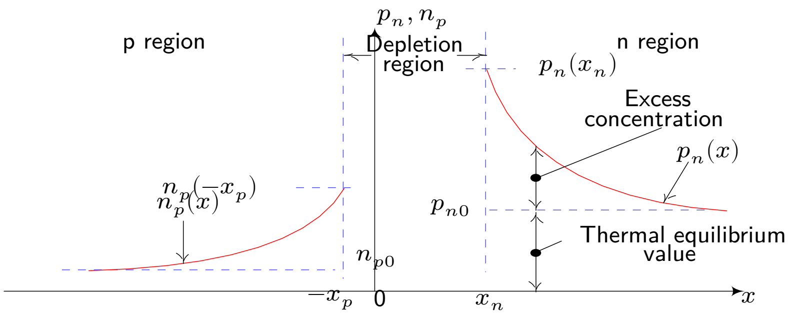
Diode Current



the steady-state concentration at the edge of the depletion region will be

$$p_n(x_n) = p_{n0} e^{V/V_T} \tag{22}$$

Location-Dependent Recombination



$$p_n(x) = Pn_0 + pn_0(e^{V/V_T} - 1)e^{-(x-x_n)/L_p} \tag{23}$$

L_p is the diffusion length

Diode Current

the value of the hole–diffusion current density, D is diffusivity.

$$J_p(x) = -qD_p \frac{dp_n(x)}{dx} \tag{24}$$

$$p_n(x) = p_{n0} + p_{n0}\left(e^{V/V_T} - 1\right) e^{-(x-x_n)/L_p} \tag{25}$$

$$J_p(x) = q\left(\frac{D_p}{L_p}\right) p_{n0}\left(e^{V/V_T} - 1\right) e^{-(x-x_n)/L_p} \tag{26}$$

$J_p(x)$ is highest at $x = x_n$,

$$J_p\left(x_n\right) = q\left(\frac{D_p}{L_p}\right) p_{n0}\left(e^{V/V_T} - 1\right) \tag{27}$$

Derivation of PN Junction Current Equation

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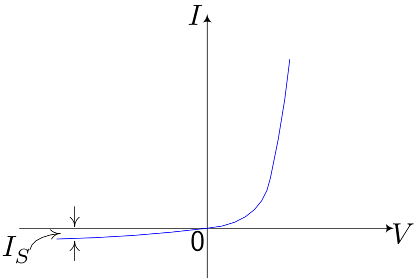
$$I = A\left(J_p + J_n\right) \tag{28}$$

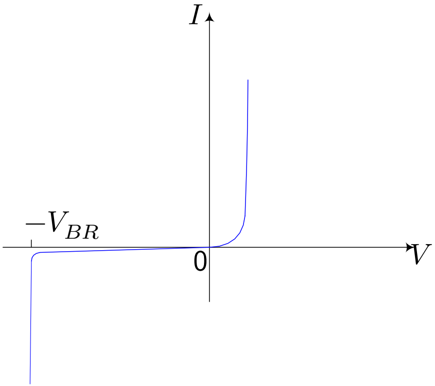
$$I = Aq\left(\frac{D_p}{L_p}p_{n0} + \frac{D_n}{L_n}n_{p0}\right)\left(e^{V/V_T} - 1\right) \tag{29}$$

Remember the following equation

$$I = I_S\left(e^{V/V_T} - 1\right) \tag{30}$$

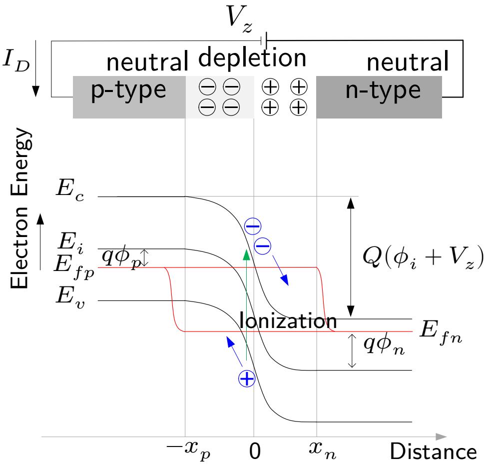


Reverse Breakdown



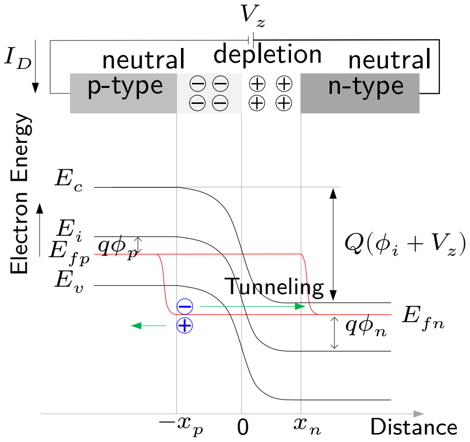
- the reverse-breakdown current is limited by the external circuit to a "safe" value.
- Two possible mechanisms for pn junction breakdown: the zener effect and the avalanche effect.

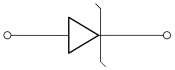
Avalanche Breakdown



- Collision of electrons with atoms
- Si diode with breakdown voltages greater than about 5.6 V enter breakdown through an avalanche mechanism.
- Carriers accelerated by electric field gain sufficient energy to break covalent bonds upon impact, thereby creating electron-hole pairs.

Zener Breakdown





- Si diode with very heavy doping (i.e. very narrow depletion region) easily enter into Zener breakdown under reverse bias.
- Electrons tunnel directly between valence and conduction bands.

Diode

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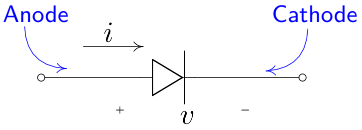
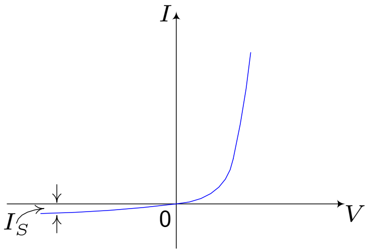
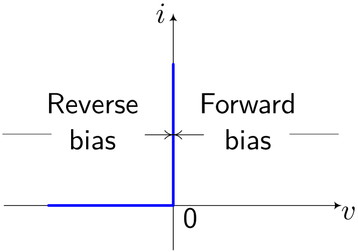
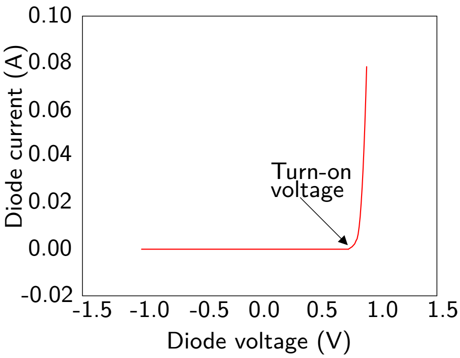
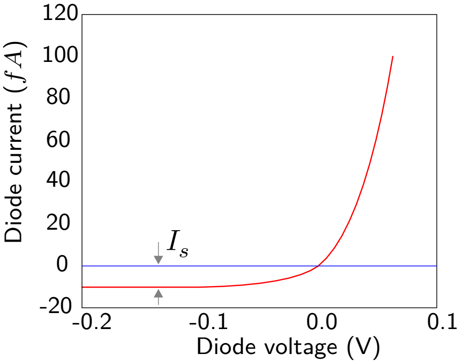






Diode


0.10
0.08
0.06
0.04
0.02
0.00
-0.02
Diode current (A)
-1.5 -1.0 -0.5 0.0 0.5 1.0 1.5
Diode voltage (V)
Turn-on voltage


120
100
80
60
40
20
0
-20
Diode current (fA)
-0.2 -0.1 0.0 0.1
Diode voltage (V)
I_s

Diode

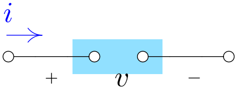
$$I_D = I_S\left(e^{\frac{qV_a}{kT}} - 1\right) \tag{31}$$

- Turn-on voltage typically 0.5 to 0.7 V
- Saturation current (I_S) typically 10^{-18} to 10^{-9} A
- $kT/q = 0.025875$ V at 300 K

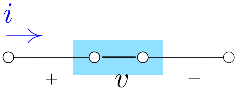
Diode Models

- 1. Ideal
- 2. Constant voltage drop
- 3. Large signal model

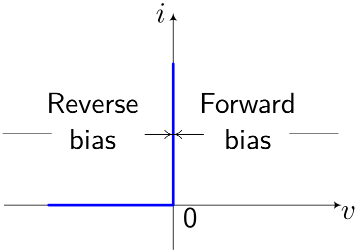
Ideal Model



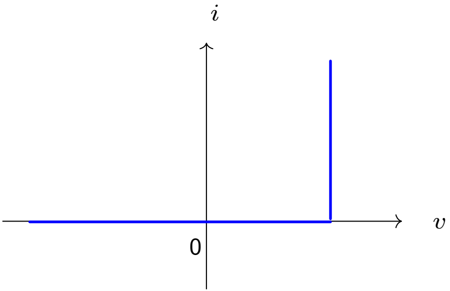
$v < 0 \Rightarrow i = 0$



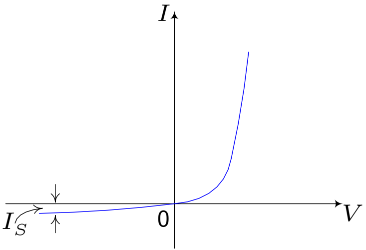
$i > 0 \Rightarrow v = 0$



Different Diode Models



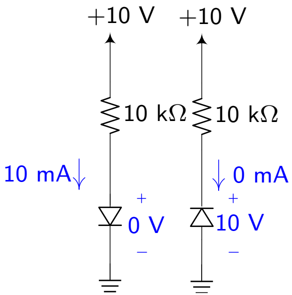
Constant Voltage Drop Model

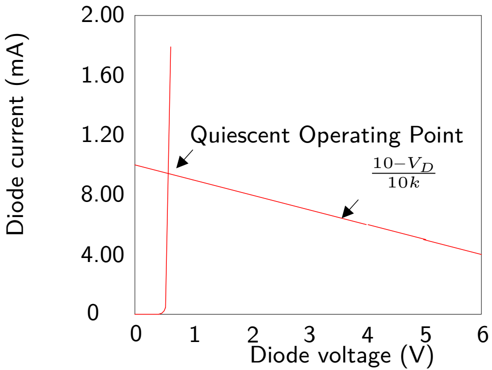


Large signal model

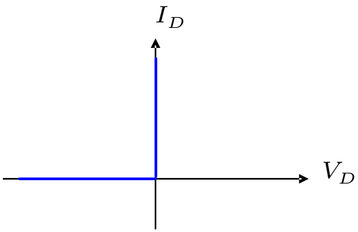
$$I_D = I_S\left(e^{\frac{qV_a}{kT}} - 1\right) \qquad (32)$$

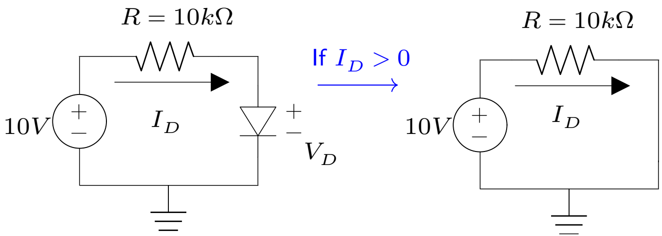
Simple Diode Circuits with Load-Line Method





Simplified Analysis (Ideal Diode)

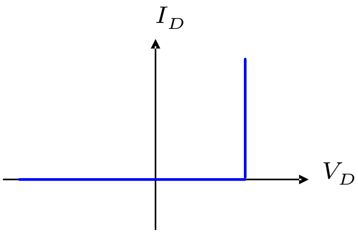


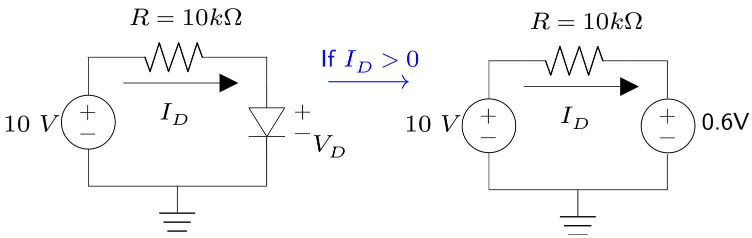


$$V_D = 0 \tag{33}$$

$$I_D = \frac{(10-0)V}{10k\Omega} = 1 \text{ mA} \tag{34}$$

Constant Voltage Drop





$$I_D = \frac{(10 - 0.6)\ V}{10\ k\Omega} = 0.94\ \text{mA} \tag{35}$$

Comparison of Different Diode Models

	V_D	I_D
Graphical Analysis	0.6 V	0.95 mA
Mathematical Analysis	0.5742 V	0.944 mA
Ideal Diode Model	0 V	1 mA
Constant Voltage Drop Model	0.6 V	0.94 mA

More Diode Circuit Examples (Constant V-drop)

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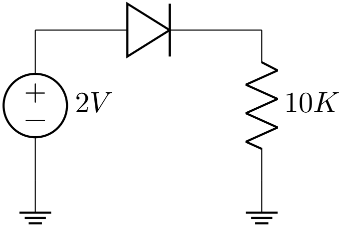
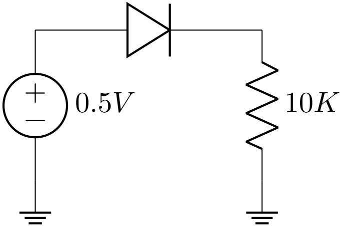
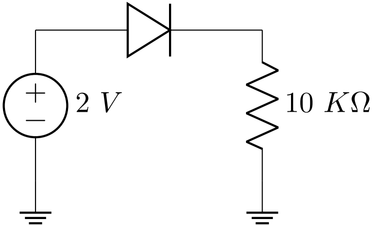
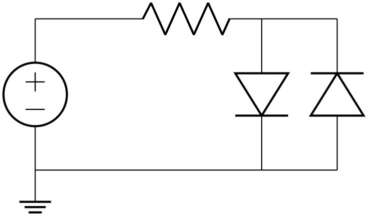




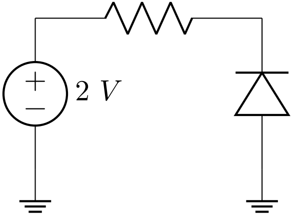
Examples With Diode Resistance of 100 Ω

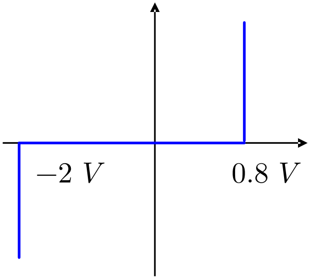


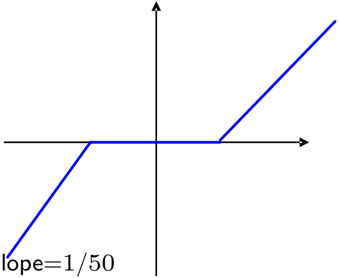
Diodes as a Clipper Circuit



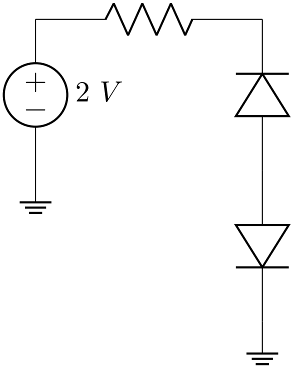
Zener Diode Examples







Examples



Examples with Constant V-Drop Model ($V_{on} = 0.6V$)

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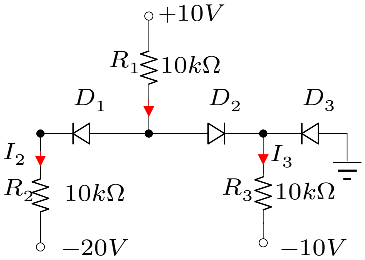


Assume no current flowing through D_3

$$\frac{10 - V_B}{10k} = \frac{V_B - 0.6 + 20}{10k} + \frac{V_B - 0.6 + 10}{10k} \quad (36)$$

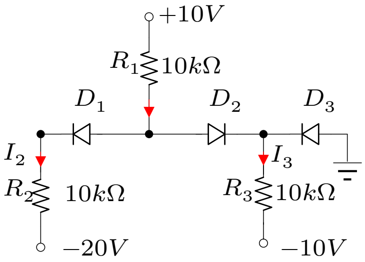
$$V_B = -6.27V \quad (37)$$

$$V_C = -6.87\ V \Rightarrow D_3 \text{ in forward bias} \quad (38)$$

Assumption NOT valid

Examples with Constant V-Drop Model ($V_{on} = 0.6V$)

Calculate V_D and I_D of each diode.



Assume no current flowing through D_2

$$\frac{10 - V_B}{10k} = \frac{V_B - 0.6 + 20}{10k} \tag{39}$$

$V_B = -4.7\ V$

$V_C = -0.6\ V \Rightarrow D_2$ indeed in reverse bias

Assumption valid

$V_{D1} = 0.6\ V \quad V_{D2} = -4.1\ V \quad V_{D3} = 0.6\ V$

$I_{D1} = 1.47\ mA \quad I_{D2} = 0\ mA \quad I_{D3} = 0.94\ mA$

Examples with Ideal Model($V_{on} = 0\ V$)

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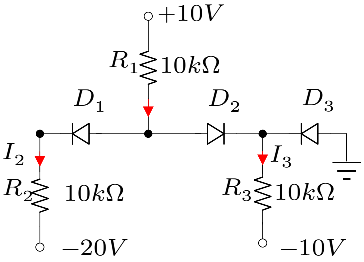


Assume D_3 in reverse bias

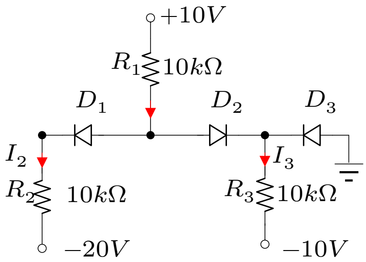
$$\frac{10 - V_B}{10k} = \frac{V_B + 20}{10k} + \frac{V_B + 10}{10k} \tag{40}$$

$$V_B = -6.67\ V \tag{41}$$

$$V_C = -6.67\ V \Rightarrow D_3 \text{ in forward bias} \tag{42}$$

Assumption NOT valid

Examples with Ideal Model ($V_{on} = 0\ V$)



Assume D_2 in reverse bias

$$\frac{10 - V_B}{10k} = \frac{V_B + 20}{10k} \tag{43}$$

$V_B = -5\ V$

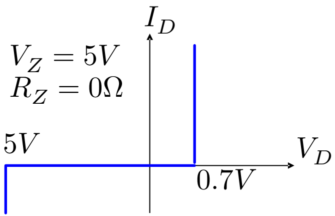
$V_C = 0\ V \Rightarrow D_2$ indeed in reverse bias

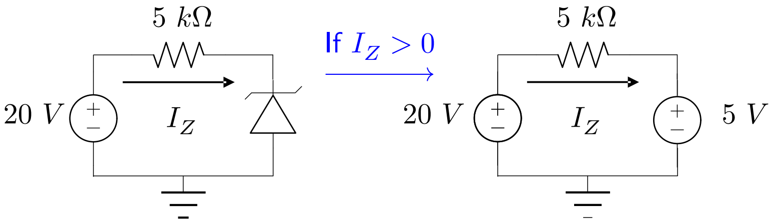
Assumption valid

$V_{D1} = 0\ V \quad V_{D2} = -5\ V \quad V_{D3} = 0\ V$

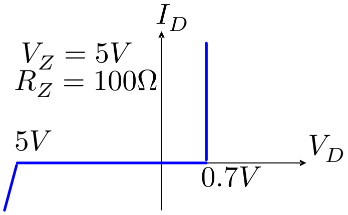
$I_{D1} = 1.5\ mA \quad I_{D2} = 0\ mA \quad I_{D3} = 1\ mA$

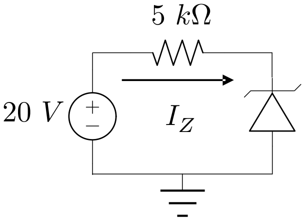
Zener Diode ($R_Z = 0$)





Zener Diode ($R_Z \neq 0$)





If $I_Z > 0$

Zener Diode ($R_Z \neq$ 0, If $I_Z > 0$)

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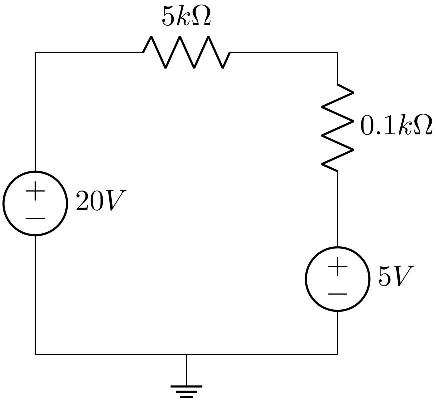

Regulator Using Zener (R_Z=0)

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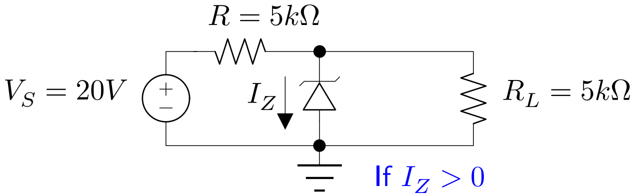


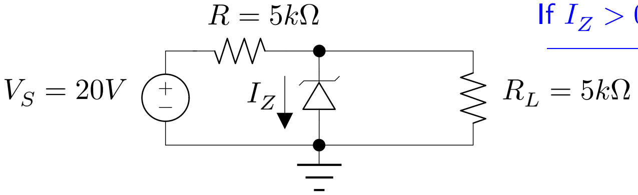
If $I_Z > 0$

What is the smallest R_L for reverse breakdown to happen?

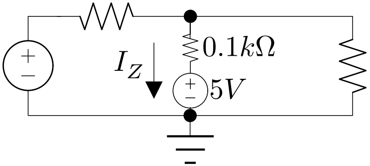
$$R_{L,min} = 1.67\ k\Omega \tag{44}$$

As long as the zener diode operates in reverse breakdown ($I_Z > 0$), a constant voltage ($5V$) appears across R_L.

Zener Diode $(R_Z \neq 0)$



If $I_Z > 0$



$$\frac{20 - V_L}{5k} = \frac{V_L - 5}{0.1k} + \frac{V_L}{5k} \tag{45}$$

$$I_Z = \frac{5.19 - 5}{0.1k} = 1.9\ mA > 0 \tag{46}$$

$$V_L = 5.1923\ V \tag{47}$$