

Lecture 20: Miller Effect

ECE3110J, Electronic Circuits

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Recap of Last Lecture

- Review of 216 Frequency Response
- Bode plot Amplitude and Phase Estimation

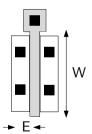
Topic to be covered

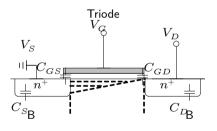


- Source of capacitance in Transistors
- Bandwidth of MOSFET amplifiers



NMOS







Review

$$C_{GS} = W_{ov} + 1/2 (WLC_{ox})$$
 (1)

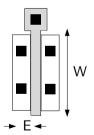
$$C_{SB} = WEC_j + 2(W + E)C_{jsw}$$
 (3)

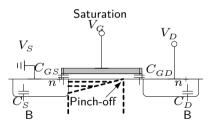
$$C_{GD} = WC_{ov} + 1/2 (WLC_{ox})$$
 (2)

$$C_{DB} = WEC_j + 2(W+E)C_{jsw}$$
 (4)



NMOS







$$C_{GS} = WC_{ov} + 2/3 (WLC_{ox})$$
 (5)

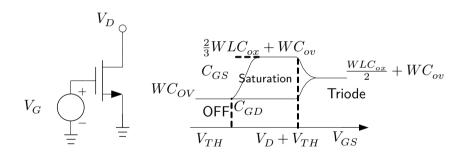
$$C_{GD} = WC_{ov} \tag{6}$$

$$C_{SB} = WC_j + 2(W+E)C_{jsw}$$
 (7)

Frequency Domain

$$C_{DB} = WEC_j + 2(W+E)C_{jsw}$$
 (8)

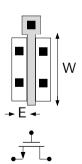


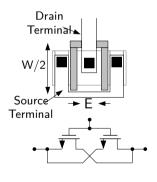




Calculate C_{SB} and C_{DB} of the two structures below.

NMOS







Solution:

Left:
$$C_{DB} = \frac{W}{2}EC_j + 2\left(\frac{W}{2} + E\right)C_{jsw}$$
 (9)

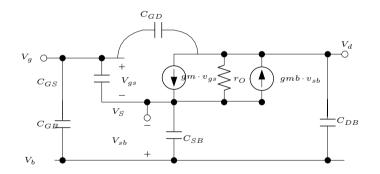
Right:
$$C_{DB} = C_{SB} = WEC_j + 2(W + E)C_{jsw}$$
 (10)

$$C_{SB} = 2\left[\frac{W}{2}EC_j + 2\left(\frac{W}{2} + E\right)C_{jsw}\right]$$
$$= WEC_j + 2(W + 2E)C_{jsw}$$
(11)

Drain junction capacitance is greatly reduced

Complete Small-Signal Model





Only when MOSFET is off should we need to consider C_{GB} , which includes the gate oxide capacitance and the depletion region capacitance in series.



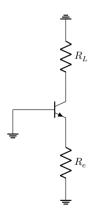
Spice Model



NMOS Model			
LEVEL = 1	VTO = 0.7	$GAMMA\ = 0.45$	PHI = 0.9
$NSUB = 9\mathrm{e} + 14$	$LD \ = 0.08\mathrm{e} - 6$	UO = 350	$LAMBDA\ = 0.1$
TOX = 9e - 9	PB = 0.9	CJ = 0.56e - 3	$CJSW\ = 0.35\mathrm{e} - 11$
$MJ\ = 0.45$	MJSW = 0.2	$CGDO \ = 0.4\mathrm{e} - 9$	JS = 1.0e - 8
PMOS Model			
LEVEL = 1	VTO = -0.8	GAMMA = 0.4	PHI = 0.8
$NSUB = 5\mathrm{e} + 14$	$LD \ = 0.09\mathrm{e} - 6$	UO = 100	$LAMBDA\ = 0.2$
TOX = 9e - 9	PB = 0.9	CJ = 0.94e - 3	$CJSW\ = 0.32\mathrm{e} - 11$
MJ = 0.5	MJSW = 0.3	$CGDO \ = 0.3\mathrm{e} - 9$	JS = 0.5e - 8

Impedances



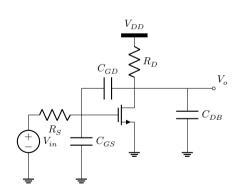


$$R_{c} = r_{o} + \left(R_{e} \parallel r_{\pi}\right) + g_{m}r_{o}\left(R_{e} \parallel r_{\pi}\right) \tag{12} \label{eq:eq:energy_constraints}$$

$$= r_e + (g_m r_o) (R_e \parallel r_{\pi})$$
 (13)

$$R_{in} = r_{\pi} \parallel \left[\frac{r_o + R_L}{g_m r_o + 1} \right] = \frac{1}{g_m} + \frac{R_L}{g_m r_o}$$
 (14)





$$\frac{V_x - V_{in}}{R_S} + \frac{V_x}{\frac{1}{SC_{GS}}} + \frac{V_x - V_{out}}{\frac{1}{SC_{GD}}} = 0$$
 (15)

$$\frac{V_{out} - V_x}{\frac{1}{SC_{GD}}} + \frac{V_{out}}{\frac{1}{SC_{DB}}} + \frac{V_{out}}{R_D} + V_x g_{m1} = 0$$
(16)

$$\gamma \neq 0, \lambda = 0$$





$$\frac{v_{out}}{v_{in}} = \frac{\left(C_{GD}S - g_{m1}\right)R_{D}}{R_{S}R_{D}\xi S^{2} + \left[R_{S}\left(1 + g_{m1}R_{D}\right)C_{GD} + R_{S}C_{GS} + R_{D}\left(C_{DB} + C_{GD}\right)\right]S + 1} \tag{17}$$

$$\xi = C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB} \tag{18}$$



$$= \frac{-g_{m1}R_D\left(1 - \frac{S}{\omega_z}\right)}{\left(1 + \frac{S}{\omega_{p1}}\right)\left(1 + \frac{S}{\omega_{p2}}\right)} \tag{19}$$

$$= \frac{-g_{m1}R_D\left(1 - \frac{S}{\omega_z}\right)}{\frac{S^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)S + 1}$$

$$(20)$$

If $\omega_{p1}\ll\omega_{p2}$, $\frac{1}{\omega_{p2}}$ can be ignored.



$$\omega_{p1} = \frac{1}{R_S (1 + g_{m1} R_D) C_{GD} + R_S C_{GS} + R_D (C_{DB} + C_{GD})}$$
(21)

$$\approx \frac{1}{R_S \left(1 + g m_1 R_D\right) C_{GD} + R_S C_{GS}} \tag{22}$$

$$\omega_{p2} = \frac{R_S \left(1 + g_{m1} R_D\right) C_{GD} + R_S C_{GS} + R_D \left(C_{DB} + C_{GD}\right)}{R_S R_D \left(C_{GS} C_{GD} + C_{GS} C_{DB} + C_{GD} C_{DB}\right)} \tag{23}$$

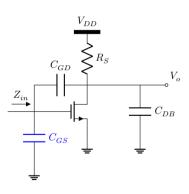
$$\approx \frac{R_S C_{GS}}{R_S R_D \left(C_{GS} C_{GD} + C_{GS} C_{DB} \right)} = \frac{1}{R_D \left(C_{GD} + C_{DB} \right)} \tag{24}$$

If C_{GS} dominates.



Input Impedance





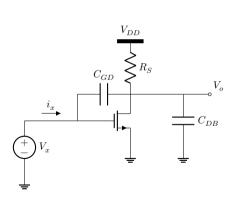
 Z_{in1} is the input impedance excluding C_{GS}

$$Z_{in} = Z_{in1} \parallel \frac{1}{SC_{GS}} \tag{25}$$

$$\gamma \neq 0, \lambda = 0$$

Input Impedance





$$\dot{v}_x = \frac{v_x - v_{out}}{\frac{1}{SC_{CR}}} \tag{26}$$

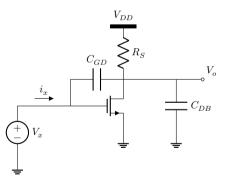
$$= \frac{v_{out}}{R_D} + \frac{v_{out}}{\frac{1}{SC_{DB}}} + v_x g_{m1}$$
 (27)

$$Z_{in1} = \frac{v_x}{i_x} \tag{28}$$

$$= \frac{1 + R_D \left(C_{GD} + C_{DB} \right) S}{C_{GD} S \left(1 + g_{m1} R_D + R_D C_{DB} S \right)} \tag{29}$$

Input Impedance



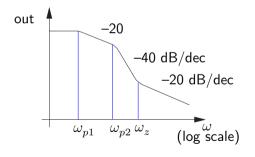


We set the output cap small and look at relatively low frequency to determine the low cut-off frequency.

$$\approx \frac{1}{C_{GD}S\left(1 + g_{m1}R_D\right)} \tag{30}$$

Effect of C_{GD}





Miller Effect

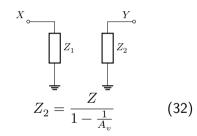


$$X \stackrel{Z}{\circ} Y \stackrel{\circ}{\circ}$$

$$Z_1 = \frac{Z}{1 - A_v}$$
 (31)

Proof:

$$\frac{X-Y}{Z} = \frac{X}{Z_1} \to Z_1 = \frac{Z}{1-\frac{Y}{X}}$$
 (33)

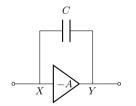


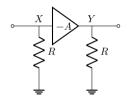
$$\frac{X - Y}{Z} = \frac{-Y}{Z_2} \to Z_2 = \frac{Z}{1 - \frac{X}{Y}}$$
 (34)

Calculate the Input Capacitance



(35)





$$Z_1 = \frac{\frac{1}{SC_F}}{1+A} = \frac{1}{S(1+A)C_F}$$

$$Z_2 = \frac{\frac{1}{SC_F}}{1 + \frac{1}{A}} = \frac{1}{S\left(1 + \frac{1}{A}\right)C_F}$$
 (36)

Miller Effect Example



The amount of charge flowing into C_F from X is $C_F(1+A)\Delta V$, as if the input capacitance increased to $(1+A)C_F$.

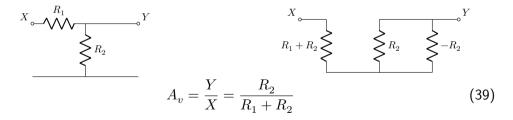
$$R_{in} = \frac{1}{S(1+A)C_F} \tag{37}$$

$$R_{out} = 0 (38)$$

Limitation on Miller Effect



If the impedance Z forms the ONLY signal path between X and Y, then Miller conversion is often invalid.



Limitation on Miller Effect

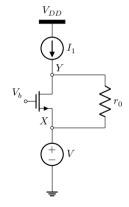


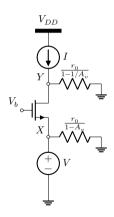
$$Z_1 = \frac{R_1}{1 - \frac{R_2}{R_1 + R_2}} = R_1 + R_2 \tag{40}$$

$$Z_2 = \frac{R_1}{1 - \frac{R_1 + R_2}{R_2}} = -R_2 \tag{41}$$

Miller Effect Example







Miller Effect Example



$$A_v = \frac{Y}{X} = 1 + (g_{m1} + g_{mb1}) r_{o1}$$
 (42)

$$Z_1 = \frac{r_{01}}{1 - [1 + (g_{m1} + g_{mb1}) r_{01}]} = \frac{-1}{g_{m1} + g_{mb1}}$$
(43)

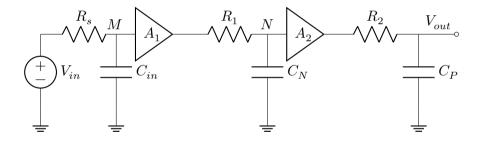
$$R_{in} = \frac{-1}{g_{m1} + g_{mb1}} \parallel \frac{1}{g_{m1} + g_{mb1}} = \infty$$
 (44)

- $\bullet\ r_o$ in parallel with the main signal path. Miller conversion is valid.
- The result is the same as direct small-signal analysis.



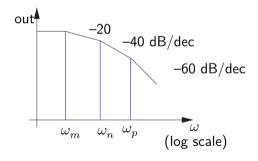
Association of Poles with Nodes





Association of Poles with Nodes





Association of Poles with Nodes



$$v_{out} = v_{in} \frac{\frac{1}{sC_{in}}}{R_S + \frac{1}{SC_{in}}} A_1 \frac{\frac{1}{SC_N}}{R_1 + \frac{1}{sC_N}} A_2 \frac{\frac{1}{SC_P}}{R_2 + \frac{1}{sC_P}}$$
(45)

$$\frac{v_{out}}{v_{in}}(\omega) = \frac{A_1}{1 + SR_SC_{in}} \frac{A_2}{1 + SR_1C_N} \frac{1}{1 + SR_2C_P}$$
(46)

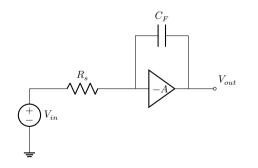
$$=A_1 A_2 \frac{1}{\left(1 + \frac{s}{\omega_M}\right) \left(1 + \frac{s}{\omega_N}\right) \left(1 + \frac{s}{\omega_P}\right)} \tag{47}$$

$$\omega_{M} = \frac{1}{R_{S}C_{in}}$$
 (48) $\omega_{N} = \frac{1}{R_{1}C_{N}}$ (49) $\omega_{P} = \frac{1}{R_{2}C_{P}} (rad/s)$ (50)

Miller conversion often discards the zeros in the transfer function.



Simplification through Miller effect and then association of poles with nodes.

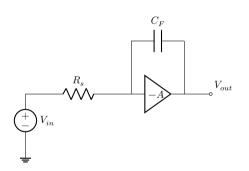


$$\frac{v_{out}}{v_{in}}(\omega \text{ close to } DC) = -A \tag{51} \label{eq:51}$$

$$\frac{v_{out}}{v_{in}} = \frac{-A}{1 + SR_S C_F (1+A)}$$
 (52)

Method 2 (Derivation of transfer function using KCL)



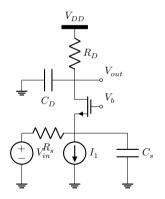


$$v_x - \frac{v_{in} - v_x}{R_S} \frac{1}{SC_F} = v_{out}$$
 (53)

$$v_{out} = v_x(-A) \tag{54}$$

$$\frac{v_{out}}{v_{in}} = \frac{-A}{1 + SR_S C_F (1+A)}$$
 (55)





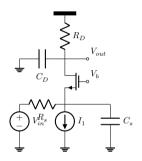
$$\frac{v_{out}}{v_{\text{in}}} = \frac{(g_{m1} + g_{mb1}) R_D}{1 + (g_{m1} + g_{mb1}) R_s} \frac{1}{\left(1 + \frac{s}{\omega_X}\right) \left(1 + \frac{s}{\omega_Y}\right)}$$
(56)

$$\omega_X = \frac{1}{\left(\frac{1}{g_{m1} + g_{mb1}} \parallel R_S\right) C_s} (rad/s) \tag{57}$$

$$\omega_Y = \frac{1}{R_D C_D} \quad (rad/s) \tag{58}$$

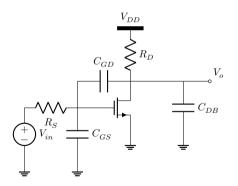
Method 2 (Derivation of transfer function using KCL)





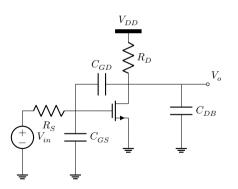
$$\frac{v_{in}}{R_S + \left(\frac{1}{g_{m1} + g_{mb1}} \parallel \frac{1}{SC_S}\right)} \frac{\frac{1}{SC_S}}{\frac{1}{SC_S} + \frac{1}{g_{m1} + g_{mb1}}} \left(R_D \parallel \frac{1}{SC_D}\right) = v_{out}$$
 (59)





- C_{GD} in parallel with the main signal path. So, Miller conversion is valid.
- Zeros are neglected in the transfer function.
- $A_v = -g_{m1}R_D$ is the low-frequency gain. In reality, it changes with frequency.



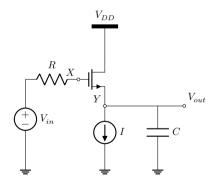


$$\omega_{in} = \frac{1}{R_S \left[C_{GS} + (1 + q_{m1} R_D) C_{GD} \right]} \tag{60}$$

$$\omega_{out} \approx \frac{1}{R_D \left(C_{DB} + C_{GD} \right)} \tag{61}$$

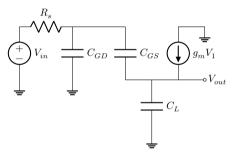
$$\frac{v_{out}}{v_{in}} = \frac{-g_{m1}R_D}{\left(1 + \frac{S}{\omega_{in}}\right)\left(1 + \frac{S}{\omega_{out}}\right)} \tag{62}$$





- C_{GS} forms the only signal path between V_{in} and V_{out} . Miller conversion may not be applied here.
- Strong interaction between V_{in} and V_{out} through C_{GS} . Association of poles with nodes method may not be applied here.





$$= C_{GS} \underbrace{\downarrow}_{g_m V_1} \underbrace{v_{\text{out}}}_{\circ V_{out}} + \frac{v_{\text{out}} - v_a}{\frac{1}{SC_{GS}}} + (v_{\text{out}} - v_a) g_{m1} = 0$$
 (63)

$$\frac{v_a - v_{\text{out}}}{\frac{1}{SC_{GS}}} + \frac{v_a}{\frac{1}{SC_{GD}}} + \frac{v_a - v_{\text{in}}}{R_S} = 0$$
 (64)

$$\lambda = \gamma = 0$$





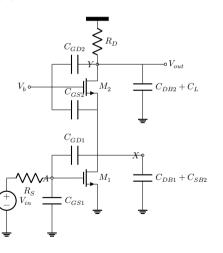
$$\frac{v_{out}}{v_{in}} = \frac{1 + \frac{C_{GS}S}{g_{m1}}}{\frac{R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)}{g_{m1}} S^2 + \frac{g_{m1}R_SC_{GD} + C_L + C_{GS}}{g_{m1}} + 1}$$
(65)

$$= \frac{1 + \frac{C_{GS}S}{g_{m1}}}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{S}{\omega_{p2}}\right)} = \frac{1 + \frac{C_{GS}S}{g_{m1}}}{\frac{S^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)S + 1}$$
(66)

$$\omega_{p1} = \frac{g_{m1}}{g_{m1}R_SC_{GD} + C_L + C_{GS}} = \frac{1}{R_SC_{GD} + \frac{C_L + C_{GS}}{g_{m1}}} \quad \text{If } \omega_{p1} \ll \omega_{p2}$$
 (67)

Cascode





- No Miller multiplication of capacitance at the CG stage.
- Miller multiplication of capacitance suppressed at the CS stage.

$$\frac{X}{A} = -\frac{g_{m1}}{g_{m2} + g_{mb2}} \tag{68}$$

Cascode



$$\omega_{p,A} = \frac{1}{R_S \left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2} + g_{mb2}} \right) C_{GD1} \right]}$$
 (69)

$$\omega_{p,X} = \frac{1}{\frac{1}{g_{m2} + g_{mb2}} \left[C_{DB1} + C_{SB2} + C_{GS2} + \left(1 + \frac{g_{m2} + g_{mb2}}{g_{m1}} \right) C_{GD1} \right]}$$
(70)

$$\omega_{p,Y} = \frac{1}{R_D \left[C_{GD2} + C_{DB2} + C_L \right]} \tag{71}$$

Cascode



$$\frac{v_{out}}{v_{in}} = -g_{m1}R_D \frac{1}{\left(1 + \frac{s}{\omega_{p,A}}\right)\left(1 + \frac{s}{\omega_{p,X}}\right)\left(1 + \frac{s}{\omega_{p,Y}}\right)}$$
(72)

Recall for CS stages

$$\omega_{in} = \frac{1}{R_S \left[C_{GS} + (1 + g_{m1} R_D) C_{GD} \right]} \tag{73}$$