Pizza Assortment Optimization

Kunlei Lian

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Let $\mathcal{S} = \{1, 2, \cdots, 10\}$ denote the set of stores under consideration. Let $\mathcal{T} = \{A, B, C\}$ indicate all the available types of pizzas to display. The price and cost of a certain pizza type is dependent on the store location and type, and are represented by p_{st} and c_{st}, respectively. The demand of pizza is related to its display *quantity*:

$$d_{st} = \alpha \cdot quantity^{\beta}, \tag{1}$$

where both α and β are given parameters.

1 Model 1

In this problem, we need to decide the number of pizzas to display for each type in each store in order to maximize the total profit across the chain. To model this problem, we define the following decision variables:

- x_{st}: a nonnegative integer variable that represents the number of pizzas to display for store s and pizza type t.

The problem could then be formulated as,

$$\begin{aligned}
\text{max.} \quad & \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} (p_{st} - c_{st}) \alpha_{st} x_{st}^{\beta_{st}} && (2) \\
\text{s.t.} \quad & \sum_{t \in \mathcal{T}} x_{st} \leq 20, \ \forall s \in \mathcal{S} && (3) \\
& \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} c_{st} x_{st} \leq 100000 && (4) \\
& x_{st} \in N^0 = \{0, 1, 2, \cdots\}, \ \forall s \in \mathcal{S}, t \in \mathcal{T} && (5)
\end{aligned}$$

In this formulation, the objective function (2) tries to maximize the total profit across all stores. The constraints (3) make sure that there are at most 20 pizzas on display in each store. Constriants (4) is the budget constraint, making sure the total cost does not exeed the given budget limit. The last constraints (5) define variable types.

2 Model 2

In this problem, the stores are divided into 3 groups $\mathcal{G}$ for each pizza type. To model this problem, we define the following decision variables:

- x_{st}: a nonnegative integer variable that represents the number of pizzas to display for store s and type t.
- y_{stg}: a binary variable that equals 1 if store s is assigned to group g for pizza type t, 0 otherwise.
- v_{stg}: a nonnegative integer variable that inidicates the number of type t pizzas to display for group g in store s. Note that v_{stg} can take on positive values only when $y_{stg} = 1$.
- z_{tg}: a nonnegative integer variable that represents the number of type t pizzas to display for group g.

$$
\begin{aligned}
\text{max.} \quad & \sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}(p_{st} - c_{st})\alpha_{st}x_{st}^{\beta_{st}} && (6)\\
\text{s.t.} \quad & \sum_{t\in\mathcal{T}} x_{st} \leq 20,\ \forall s \in \mathcal{S} && (7)\\
& \sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}} c_{st}x_{st} \leq 100000 && (8)\\
& x_{st} = \sum_{g\in\mathcal{G}} v_{stg},\ \forall s \in \mathcal{S}, t \in \mathcal{T} && (9)\\
& \sum_{g\in\mathcal{G}} y_{stg} = 1,\ \forall s \in \mathcal{S}, t \in \mathcal{T} && (10)\\
& \sum_{s\in\mathcal{S}} y_{stg} \geq 2,\ \forall t \in \mathcal{T}, g \in \mathcal{G} && (11)\\
& v_{stg} \geq z_{tg} - (1 - y_{stg})M,\ \forall s \in \mathcal{S}, t \in \mathcal{T}, g \in \mathcal{G} && (12)\\
& v_{stg} \leq y_{stg}M,\ \forall s \in \mathcal{S}, t \in \mathcal{T}, g \in \mathcal{G} && (13)\\
& v_{stg} \geq 0,\ \forall s \in \mathcal{S}, t \in \mathcal{T}, g \in \mathcal{G} && (14)\\
& v_{stg} \leq z_{tg},\ \forall s \in \mathcal{S}, t \in \mathcal{T}, g \in \mathcal{G} && (15)\\
& x_{st}, v_{stg}, z_{tg} \in N^0,\ \forall s \in \mathcal{S}, t \in \mathcal{T}, g \in \mathcal{G} && (16)\\
& y_{stg} \in \{0, 1\},\ \forall s \in \mathcal{S}, t \in \mathcal{T}, g \in \mathcal{G} && (17)
\end{aligned}
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In this formulation, the objective function (6) aims to maximize the total profit across all stores. The constraints (7) make sure that there are at most 20 pizzas on display in each store. Constriants (8) is the budget constraint, making sure the total cost does not exeed the given budget limit. Constraints (9) derive the number of type t pizzas to display for store s. Constraints (10) ensure that a store can only to assigned to one and only one group for a certain pizza type t. Constraints (11) require that there are at least two stores in every group of type t. Constraints (12) - (15) are the linearization of $v_{stg} = y_{stg} * z_{tg}$, in which M is a large number. In our case, M could take the value of 20 as we can have at most 20 pizzas in a store. These contraints together make sure that all the stores assgigned to the same pizza group have the same number of pizzas to display for that group. Also it is guaranteed that v_{stg} can only take on positive values when y_{stg} is equal to 1. The last constraints (17) define variable types.