

# An Adaptive Ant Colony Optimization for Solving Assembly Line Balancing Problem

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## Abstract

An adaptive ant colony optimization was proposed to solve the assembly line balancing problem (ALBP). According to the characteristics of the ALBP, a method of solution constructing strategy was developed, and a better differentiation of objective function was proposed to appraisal solution quality. However, general ant algorithm often falls into local optimal and consume excessive time, in order to overcome these shortcoming, an improved ACO was presented by adaptive adjustment of the parameters in the algorithm, which has a good ability of searching better solution at higher convergence speed. Finally, the proposed algorithm was tested and compared against best known algorithms reported in the literatures, and the experimental results indicate the feasibility and effectiveness of the proposed algorithm.

**Key words:** assembly line balancing; ant colony optimization (ACO); adaptive; artificial intelligence

## 1 Introduction

Assembly line is a widely used component of manufacturing factories and it faces many problems in the design and operation of assembly lines, assembly line balancing problem (ALBP) being one of most important ones. A small improvement towards ALBP often leads to significant efficiency enhancement and cost reduction (Salveso, 1955); On the other hand, ALBP is a classical NP-hard combinatorial optimization problem in which the complexity increases exponentially with more number of jobs and there yet exists polynomial-time algorithms to find optimization solutions for this problem (Kilinc, 2010). Currently, there exist three solution strategies to solve ALBP: exact algorithms, heuristic algorithms and artificial intelligence algorithms. Exact algorithms are able to find the optimal solutions, for only small-sized problems and

with tremendous computation times, therefore, they can hardly be applied in real-world production systems (Peeters and Degraeve, 2006; Scholl and Becker, 2006). Heuristic algorithms have received many attentions from researchers due to its theoretical simplicity; on the other hand, they generally take long time to identify the optimal solution and it is often hard to verify the solutions they found are optimal (Ponnambalam et al., 1999). In recent years, artificial intelligence algorithms, including genetic algorithms, simulated annealing and tabu search, witness significant advances in the fields and have been used to solve ALBP successfully (O. et al., 2011; Ozcan and Toklu, 2008).

Ant colony optimization (ACO) is another intelligent optimization algorithm proposed by Colnani (Colnani et al., 1991) in 1991 and has been applied to various combinatorial optimization problems. Bautista and Pereira (2002) made the first attempt to solve a simple assembly line balancing problem using ant colony algorithms based on ant system and the optimization results were not optimal. McMullen and Tarasewich (2003) obtained superior performance of ACO in solving the assembly balancing problem with multiple job types, stochastic processing times and parallel workstations. Bautista and Pereira (2007) studied an assembly balancing problem with timing and spatial constraints using ant colony algorithms.

The aforementioned researches in assembly balancing problems using ACO suffer from inferior performance when compared with other algorithms and the reasons are twofold. First, the way the pheromone is accumulated in some algorithms is too simplified, which prevents ACO finding optimal solutions. Second, the objective function considers only limited factors, which makes it difficult to differentiate good solutions from bad solutions. To address this, we propose an adaptive ant colony algorithm to solve the assembly line balancing problems. It tries to avoid local optima by utilizing both external and historical information to dynamically adjust global pheromone evaporation factor in the process of path construction of an ant. In addition, the algorithm incorporates balancing and smoothing as part of the objective function, which improves ACO's ability in differentiating solutions. The superiority of the proposed algorithm is validated on benchmark problem instances.

## 2 Assembly Line Balancing and Mathematical Model

ALBP refers to the assignment of finite job set to finite workstation set in order to maximize workstation utilization, minimize overall overload time and minimize balancing objective value, subject to processing constraints and workstation processing time satisfying cycling requirements. It involves the coordination of various processes within an assembly line and needs to address the inconsistency in process machining times. Assembly line productivity as well as product quality are both greatly affected by its balancing level.

The ALBP this paper aims to address can be mathematically described as follows:

$$\max. \quad \lambda L - I \tag{1}$$

$$\text{s.t.} \quad L = \frac{\sum_{i=1}^n t_i}{mC} \tag{2}$$

$$I = \sqrt{\frac{\sum_{k=1}^m (\max(T(S_k)) - T(S_k))^2}{m}} \tag{3}$$

$$S_i \cap S_j = \emptyset, \quad i, j = \{1, 2, \dots\}, i \neq j \tag{4}$$

$$\cup_k S_k = E, \quad \forall i \in S_x, j \in S_y, 1 \leq x, y \leq n, x \leq y \text{ if } P_{ij} = 1 \tag{5}$$

$$T(S_k) \leq C \tag{6}$$

In this model,  $L$  is the balancing rate of an assembly line;  $I$  is the smoothing factor;  $\lambda$  is an user-defined parameter and  $\lambda > 1$ ;  $E$  is the set of jobs within the assembly line;  $S_k$  is the set of jobs assigned to workstation  $k$ ;  $C$  is the assembly line cycle;  $t_i$  is the processing time of job  $i$ ;  $T(S_k)$  is the total processing time of workstation  $k$ ;  $P$  is the precedence matrix of ALBP and  $P = [P_{ij}]_{n \times n}$ ,  $P_{ij} = 1$  job  $i$  must be processed right before job  $j$ , 0 otherwise.

### 3 An Adaptive ACO for the ALBP

#### 3.1 Solution construction strategy

In order to use ACO to solve the ALBP, the processing jobs can be seen as nodes on a graph which will be traversed by ants, also the connections between jobs and workstations can be seen as arcs on the graph. The assignment of processing jobs to workstations can then be seen as an ant colony travels through the graph with the guidance of pheromone and heuristic information.

The solution construction is a key step in employing ACO to solve ALBP and this paper constructs a feasible solution by gradually assigning processing jobs to corresponding workstations. To this end, we define the following notations:

- no-assigned task: a task that hasn't been assigned to any workstation yet
- available task: a task that hasn't yet been assigned but satisfy precedence constraints between tasks, also all of its preceding tasks have been assigned
- assignable task: an available task that satisfies cycling constraints.

The feasible solution construction algorithm can then be described as follows:

1. open a workstation
2. identify the set of available tasks from all no-assigned tasks and the precedence constraints among tasks, if the resulting set is empty, go to step 7
3. identify the set of assignable tasks from available tasks and the cycling constraint
4. if the set of assignable tasks is empty, go to step 6
5. select a task from the set of assignable tasks according to defined rules and assign it to the current workstation, go to step 2
6. open a new workstation, go to step 3
7. stop

Using the way defined in the above algorithm to identify the set of assignable tasks, there always exists an optimal assignable task in the set.

One key characteristic of ACO is its utilization of pheromone feedback and heuristic information during its search for global optimality; therefore, the way of pheromone updating and heuristic information selection have a huge impact on the performance of ACO. In this paper, we define  $\tau_{ij}$  as the pheromone intensity on arc  $(i, j)$  traversed by ants and represents the expectation of assigning task  $i$  to workstation  $j$ . The corresponding heuristic information is computed by static precedence rules. In addition, the heuristic information of ACO consists of the maximal task completion time and maximal number of succeeding tasks, and the visibility  $\eta_i$  of task  $i$  can be computed as

$$\eta_i = \frac{t_i}{C} + \frac{U_i}{\max_{i=1,2,\dots,N} U_i} \quad (7)$$

where  $U_i$  is the number of succeeding tasks of task  $i$ .

In order to improve ACO's search capability and avoid stagnating into local optima, we propose a hybrid search strategy by which an ant chooses task  $i$  and assigns it to workstation  $j$ :

$$\alpha(x) = \begin{cases} I_1 & \text{argmax}_{i \in N_j} (\tau_{ij}(t))^\alpha (\eta_i)^\beta, 0 \leq r \leq r_1 \\ I_2 & p_{ij} = \frac{(\tau_{ij}(t))^\alpha (\eta_i)^\beta}{\sum_{z \in N_j} (\tau_{zj}(t))^\alpha (\eta_z)^\beta}, r_1 < r \leq r_1 + r_2 \\ I_3 & i \in N, r_1 + r_2 < r \leq r_1 + r_2 + r_3 \end{cases} \quad (8)$$

where  $r$  is a random number chosen from  $(0, 1)$ ;  $r_1, r_2, r_3$  is a user-defined parameter and  $0 \leq r_1, r_2, r_3 \leq 1, r_1 + r_2 + r_3 = 1$ ;  $N_j$  is the set of assignable tasks for ant  $i$  on the current workstation  $j$ ;  $z$  is an assignable

task; and  $\alpha, \beta$  are two key parameters deciding pheromone intensity and heuristic information.

This hybrid search strategy probabilistically chooses one of the three strategies: 1) utilization: choose with probability  $r_1$  from assignable tasks  $N_j$  the task with maximal  $(\tau_{ij}(t))^\alpha (\eta_i)^\beta$  to assign to workstation  $j$ ; 2) exploration: choose with probability  $r_2$  that is given by  $I_2$  in the equation a task; 3) random selection: choose with probability  $r_3$ ) a task from the set of assignable tasks.

Following the aforementioned strategy, save the assignable task chosen by ant  $k$  into table  $tabu_k$  and when all  $n$  tasks have been added to  $tabu_k$ , ant  $k$  has finished on traversal of the search graph and the resulting task sequence is a feasible solution to the underlying problem.

### 3.2 Objective function

The quality of a constructed solution following the strategy described in the previous section must be evaluated and this section defines the objective function for this purpose. The type 1 assembly line balancing problem (ALBP-1) aims to minimize the number of workstations given a fixed cycle time; However, many solutions tend to have the same objective value if we use the number of workstations  $m$  as objective function and it is therefore difficult to identify good solutions. In order to facilitate the pheromone acculation during ants' searching process, this paper uses the following objective function:

$$\max f(m) = \lambda L - I = \lambda \frac{\sum_{i=1}^n t_i}{mC} - \sqrt{\frac{\sum_{k=1}^m (\max(T(S_k)) - T(S_k))^2}{m}} \quad (9)$$

This function employs both assembly line balancing rate  $L$  and smoothing factor  $I$  to evaluate the quality of an assembly line balancing solution. Using this objective function, an assembly line with better balance has higher  $L$  value and smaller  $I$  value. We give  $L$  a bigger weight  $\lambda > 1$  considering the comparative importance of  $L$  and  $I$ . This objective function improves ACO's capability to identify better solutions from solutions with the same number of workstations, enhances the learning and upating of pheromones and speeds up the algorithm's convergence rate.

### 3.3 Pheromone update strategy

The pheromone update strategy in this paper utilizes a combination of local pheromone update and global pheromone update from ant colony system.

- local pheromone trail update: when the algorithm constructs a feasible solution, the pheromone on arc  $(i, j)$  is updated using the following equation after an ant assigns task  $i$  to workstation  $j$ :

$$\tau_{ij}(n) = (1 - \rho_1)\tau_{ij}(n-1) + \rho_1\tau_0 \quad (10)$$

where  $\rho_1$  is the evaporation factor of local pheromone and  $0 \leq \rho_1 \leq 1$ ;  $\tau_0$  is the initial pheromone level.

Local pheromone trail updating aims to reduce the impact of assigned tasks on following ants traversing the same arc and improve the search capacity of un-explored arcs, which increases the search space of the algorithm in order to explore those areas that contain the optimal solution.

- global pheromone trail update: After all the ants finish traversal, on the solution with the best objective value will be used to update the pheromone, which increases the search capability of ACO. In the current iteration, the best ant updates the global pheromone using the following equation:

$$\tau_{ij}(t) = (1 - \rho_2)\tau_{ij}(t-1) + \rho_2\Delta\tau_{ij}^{gb}(t) \quad (11)$$

where

$$\Delta\tau_{ij}^{gb}(t) = \begin{cases} f(m) & (i, j) \text{ belongs to the current best solution} \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

and  $\rho_2$  is the global pheromone evaporation factor,  $0 \leq \rho_2 \leq 1$  and  $f(m)$  is the objective value of the current optimal solution.

Using the aforementioned global pheromone trail updating strategy, the pheromone volume that belongs to the current best solution will be higher than other solutions after a few searching iterations. This can increase the convergence speed but might also suffer from local optima. In order to improve ACO's global searching capability, this paper proposes a strategy to adaptively modify the value of  $\rho_2$  in order to prevent the pheromone intensity of certain arcs from getting too low or too high. We set the initial value of  $\rho_2$  as  $\rho_2(t_0) = 1$  and decrease its value when the best objective value doesn't improve after  $N$  iterations:

$$\rho_2(t) = \begin{cases} 0.95\rho_2(t-1) & 0.95\rho_2(t-1) \geq \rho_{2min} \\ \rho_{2min} & \text{otherwise} \end{cases} \quad (13)$$

where  $\rho_{2min}$  is the minimal value of  $\rho_2$  and is used to prevent  $\rho_2$  from getting too small during the search process, which may lead to slow convergence speed of the overall algorithm.

### 3.4 Algorithm workflow

The overall algorithm works as follows

1. initialize algorithm parameters

2. open a workstation and create the set of initial available tasks and assignable tasks
3. conduct the following steps for every ant
  - (a) initialize the pheromone value for every arc
  - (b) choose a task from the set of assignable tasks using formula (8) and assign it to the current open workstation
  - (c) add the assigned task into table *tabu*
  - (d) update the pheromone according to formula (10)
  - (e) move to the next task and open a new workstation if the current open workstation is full
  - (f) create new set of available tasks and assignable tasks
  - (g) repeat the above steps until all tasks are assigned
4. compute the objective value for every ant using formula (9) and identify the best solution, update the global best solution if the current best solution is better
5. update the global pheromone trail evaporation factor using formula (13)
6. update the global pheromone trail using formula (11)
7. set  $N_c = N_c + 1$ ;
  - (a) if  $N_c > N_{C_{max}}$ , then output the best solution and stop
  - (b) clear the table *tabu* for all ants and go to step 2.

## 4 Computational Experiments

Based on the above descriptions, we validated the performance of ACO on benchmark instances. The algorithm was implemented in C++ on a Windows XP machine with Pentium IV 2.20GHz and 2G memory. Inasmuch as parameter settings have big impacts of ACO's efficiency, we identified the parameters using multiple trails and the final parameters are set as follows: the number of ants ( $N_{ANT\_COUNT}$ ) = number of tasks ( $N_{TASK\_COUNT}$ ), the maximum number of iterations  $N_{C_{max}} = 50$ ,  $\tau_0 = 1$ ,  $\alpha = 1.0$ ,  $\beta = 2.0$ ,  $\rho_1 = 0.1$ ,  $\rho_2(t_0) = 1$ ,  $\rho_{2_{min}} = 0.5$ ,  $r_1 = 0.6$ ,  $r_2 = 0.3$ ,  $r_3 = 0.1$ .

We first analyze the proposed ACO's performance based on the results for problem  $C = 57$  from Kilbridge benchmark set, then present the overall performance comparison for all 24 benchmark problems in the set. The task precedence graph and computational results for problem Kilbridge are given in Figure 1 and Table 1, respectively.

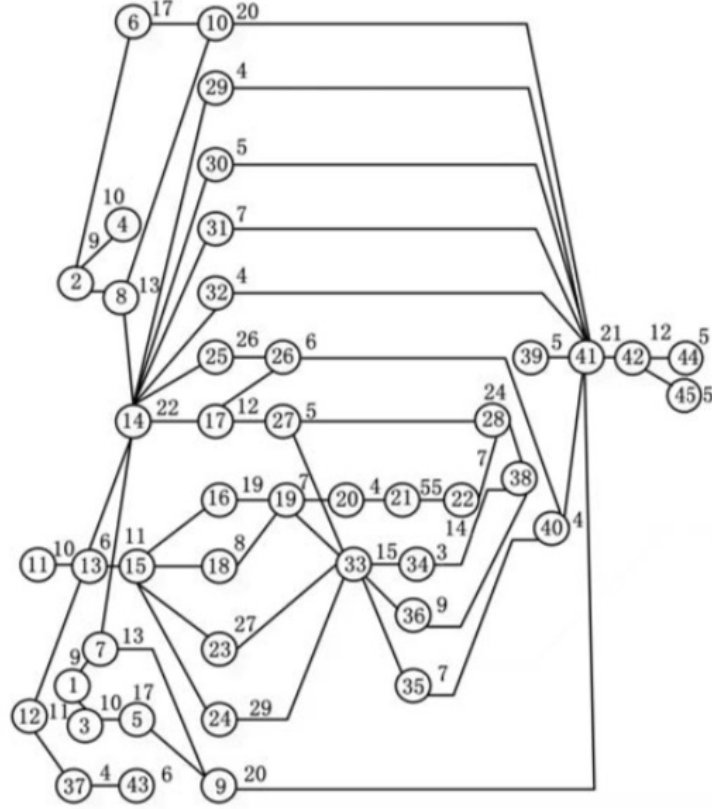


Figure 1: Task Precedence Graph of Problem Kilbridge

Table 1: Computational Result of Problem Kilbridge

Workstation No.	Tasks	Workstation Time(s)	Free Time (s)
1	1, 11, 12, 13, 15, 18, 39	55	2
2	2, 7, 8, 16	54	3
3	14, 17, 19, 20, 27, 31	57	0
4	21	55	2
5	23, 24	56	1
6	3, 4, 22, 30, 33, 34	57	0
7	5, 25, 29, 36	56	1
8	6, 26, 28, 35	54	3
9	9, 10, 32, 38, 40	55	2
10	37, 41, 42, 43, 44, 45	53	4

It can be seen from the computational results that our proposed algorithm was able to identify the solution with minimal number of workstations and balance rate of 96.8% for the problem Kilbridge. Figure 2 shows the workstation workload is well balanced and the solution was identified within 1s, which proved the efficiency of the proposed algorithm.

Table ?? shows the computational results of the proposed algorithm with weighted algorithm and genetic



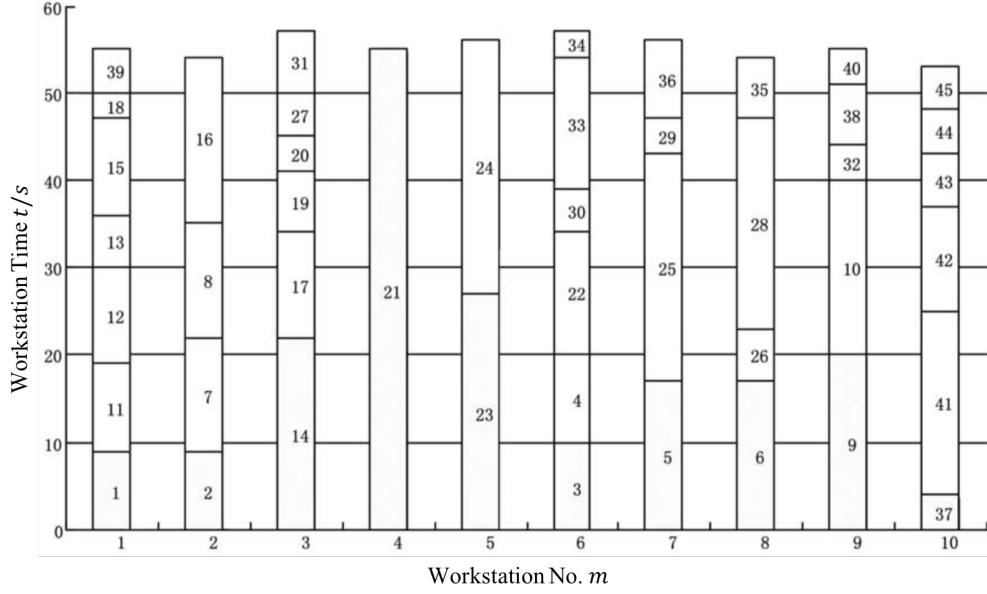


Figure 2: Task Precedence Graph of Problem Kilbridge

algorithm (GA). It can be seen from the table that both the proposed ACO and GA can identify the solution with minimal number of workstations for all the 24 benchmarking problems and their overall performance is superior than the weighted algorithm. It is worth noting that the weighted algorithm uses a static precedence rule and its selection criterion is unique and deterministic; the proposed algorithm is stochastic in nature and it can fully utilize the information between ants and previous experience, which helps the algorithm jump out of local optima and therefore find the optimal solutions.

## 5 Conclusions

This paper proposes an adaptive ACO algorithm for the assembly line balancing problem. It encompasses the following characteristics: comprehensive considerations of three rules of utilization, exploration and random search to construct feasible assignment plans; comprehensive considerations of assembly line balancing rate as well as smoothing factor to evaluate balancing performance to increase differentiation capability of different solutions; adaptive modifications of global pheromone evaporation factor in the construction phase to help the algorithm jump out of local optima and improve its global search capability. Computational results validated the superior performance of the proposed algorithm.

This paper only considers single deterministic assembly balancing problem and the proposed algorithm can be used to solve more complex assembly balancing problems, like dynamic problem, stochastic problem, multi-objective problem and multi-model assembly line balancing problems.

Table 2: Computational Result Comparisons on Benchmarking Problems

Problem	No. Tasks $n$	Cycle $C$	Time $t_{sum}$	Min. No. Workstation $m_{min}$	Adaptive ACO			Weight	GA
					m	L	I		
Jackson	11	10	46	5	5	92.0%	1.095	6	5
		13		4	4	88.5%	0.707	4	4
Heskiaoff	28	205	1024	5	5	99.9%	0.447	6	5
		256		4	4	100.0%	0	5	4
Buxey	29	30	324	12	12	90.0%	2.309	12	12
		41		8	8	98.8%	0.707	9	8
		54		7	7	85.7%	4.629	7	7
Sawyer	30	41	324	8	8	98.8%	0.707	9	8
		47		7	7	98.5%	2.390	8	7
Lutzi	32	1414	14140	11	11	90.9%	134.400	12	11
		1572		10	10	89.9%	134.270	11	10
		2020		8	8	87.5%	110.640	8	8
		2828		6	6	83.3%	449.620	6	6
Kilbridge	45	57	552	10	10	96.8%	2.240	11	10
		79		7	7	99.8%	0.377	8	7
		92		6	6	100.0%	0	7	6
		110		6	6	83.6%	18.876	6	6
		138		4	4	100.0%	0	5	4
		184		3	3	100.0%	0	4	3
Tonge	70	176	3510	21	21	95.0%	8.423	22	21
		364		10	10	96.4%	4.795	11	10
		410		9	9	95.1%	6.896	10	9
		468		8	8	93.8%	15.050	8	8
		527		7	7	95.1%	14.540	7	7

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