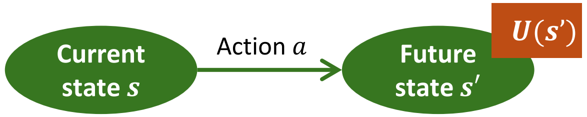
CS 5/7320
Artificial Intelligence

Reinforcement Learning
AIMA Chapter 17+22

Slides by Michael Hahsler
with figures from the AIMA textbook.

Online Material

Remember Chapter 16:
Making Simple Decisions



For a decision that we make frequently and making it once does not affect the future decisions (**episodic environment**), we can use the **Principle of Maximum Expected Utility (MEU).**

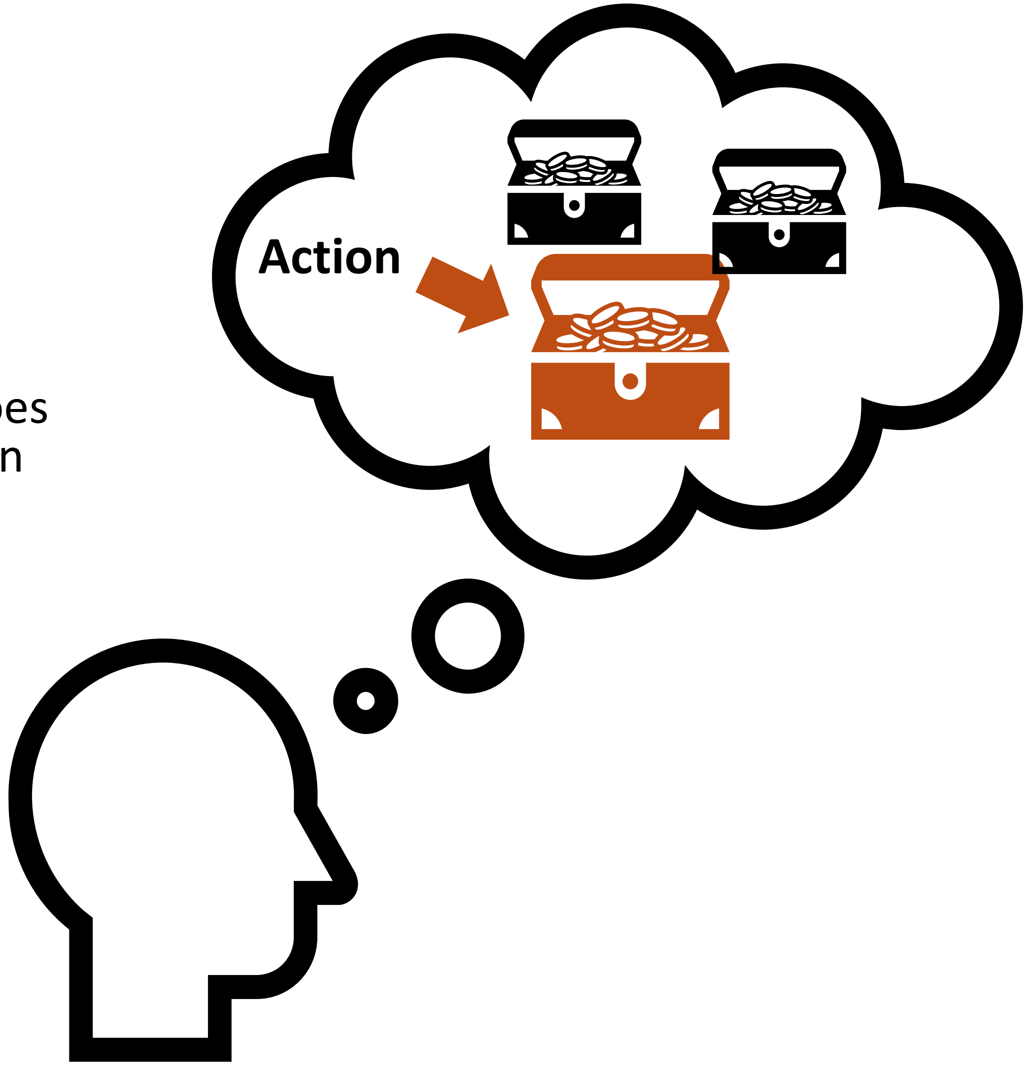
Given the expected utility of an action

$$EU(a) = \sum_{s'} \sum_{s} P(s)\, P(s'|s,a)\, U(s')$$

choose action that maximizes the expected utility:

$$a^* = \text{argmax}_a\, EU(a)$$

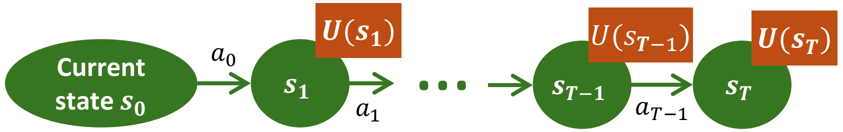
Now we will talk about **sequential decision making**.



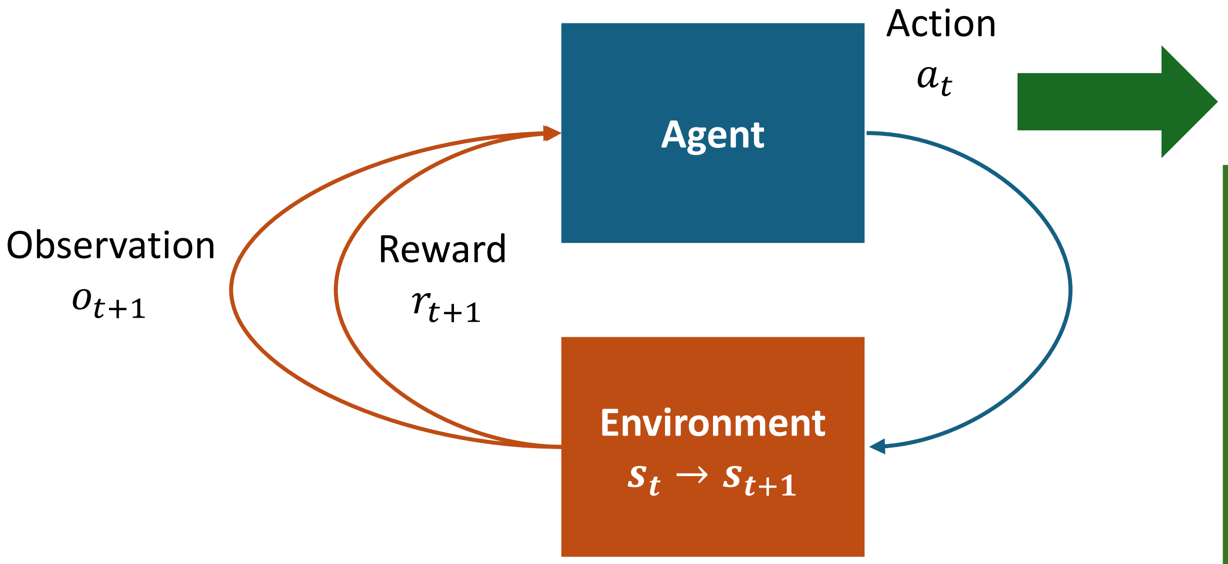
Making Complex Decisions: Sequential Decision Making

AIMA Chapter 17

Sequential Decision Problems



- **Utility-based agent**: The agent's utility depends on a sequence of decisions that depend on each other.
- Sequential decision problems incorporate utility (called immediate and long-term reward), uncertainty, and sensing.



Sequence: $\overbrace{(o_0, r_0)}^{s_0}, a_0, \overbrace{(o_1, r_1)}^{s_1}, a_1, \overbrace{(o_2, r_2)}^{s_2}, a_2, \ldots$

Goal: Observations and rewards depend on the state of the system, and the agent wants to maximize the expected discounted reward:

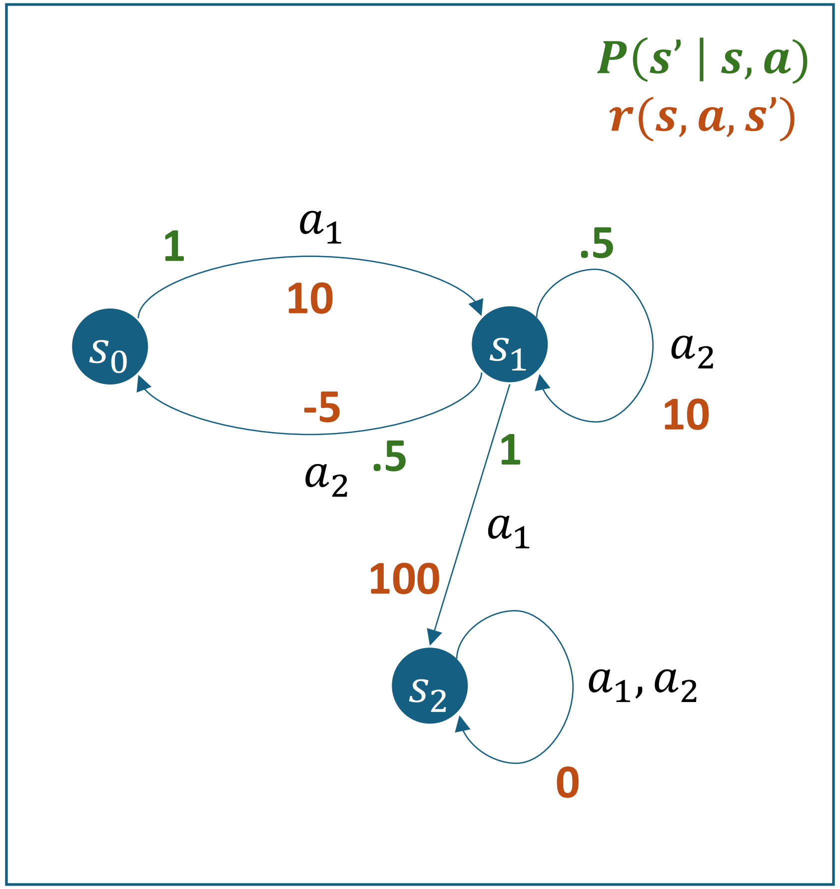
$$U = E\left[\sum_{t=0}^{T} \gamma^t R_t\right]$$

R_t... reward at time t as a random variable
γ ... discounting factor
T ... time horizon may be infinity

Definition: Markov Decision Process (MDP)

MDPs are discrete-time stochastic control processes defined by:

- a finite set of **states** $S = \{s_0, s_1, s_2, \dots\}$ (initial state s_0)
- a set of available **actions** $ACTIONS(s)$ in each state s
- a **transition model** $P(s' \mid s, a)$ where $a \in ACTIONS(s)$
- a **reward function** $r(s)$ where the immediate reward depends on the current state (often $r(s, a, s')$ is used to make modelling easier)



MDPs model sequential decision problems with

- a fully observable, stochastic, and known environment;
- a Markovian transition model (i.e., future states do not depend on past states given the current state);
- additive immediate rewards.

Time horizon

- **Infinite horizon**: non-episodic (continuous) tasks with no terminal state.
- **Finite horizon**: episodic tasks. Episode ends after a number of periods or when a terminal state is reached. Episodes contain a sequence of several actions that affect each other.

This is different from the previous definition of an **episodic** environment!

Example: 4x3 Grid World

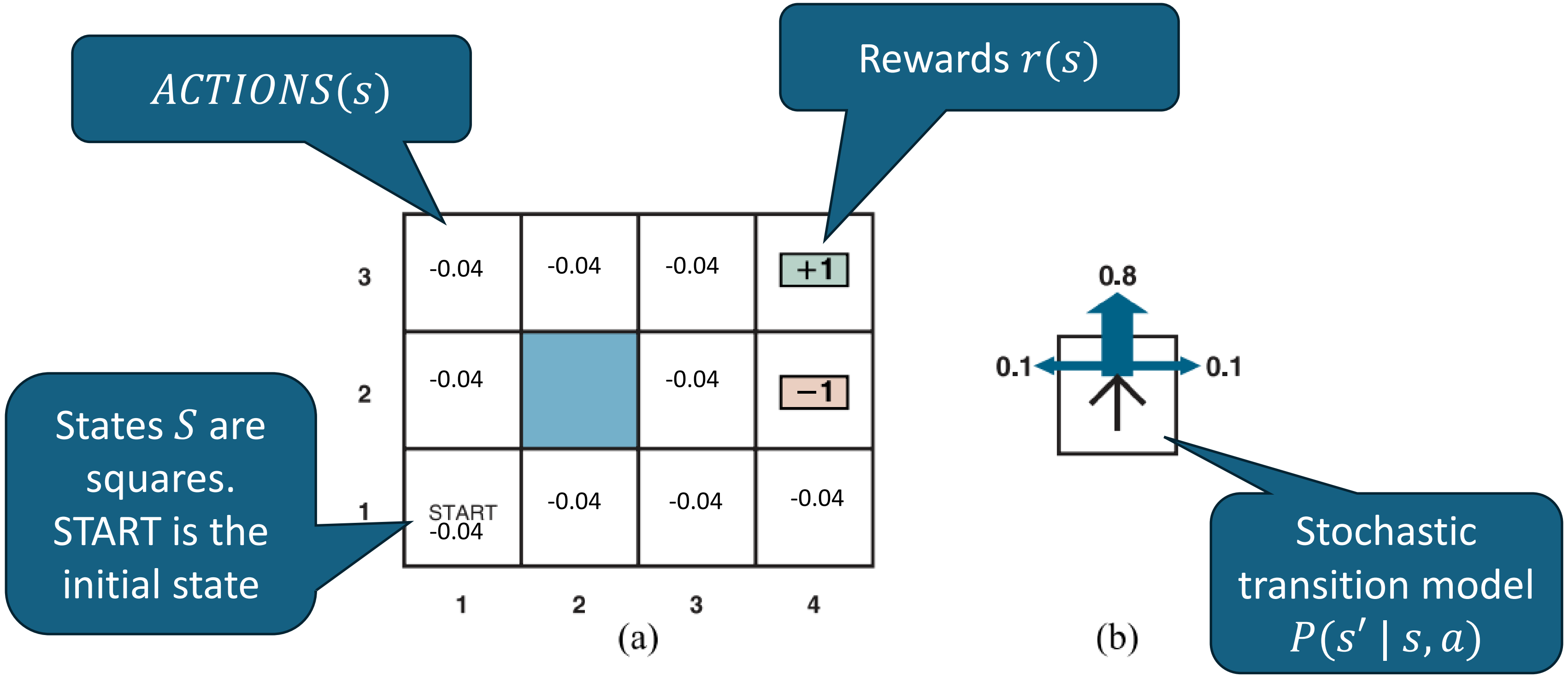


Figure 17.1 (a) A simple, stochastic 4×3 environment that presents the agent with a sequential decision problem. (b) Illustration of the transition model of the environment: the "intended" outcome occurs with probability 0.8, but with probability 0.2 the agent moves at right angles to the intended direction. A collision with a wall results in no movement. Transitions into the two terminal states have reward +1 and –1, respectively, and all other transitions have a reward of –0.04.

Since we know the complete MDP model, we can solve this as a **planning problem**.

For each square: specify what direction should we try to go to maximize the expected total utility.

This is called a **policy** written as the function

$$\pi: S \rightarrow ACTIONS(S)$$

Policy as a Table

s	**Action $\pi(s)$**
(1,1)	Up
…	…
…	…

Value Function

- A policy $\boldsymbol{\pi} = \{\pi(s_0), \pi(s_1), \dots\}$ defines for each state which action to take.
- The expected utility of being in state s under policy π (i.e., following the policy starting from s) can be calculated as the sum over the generated sequence of states:

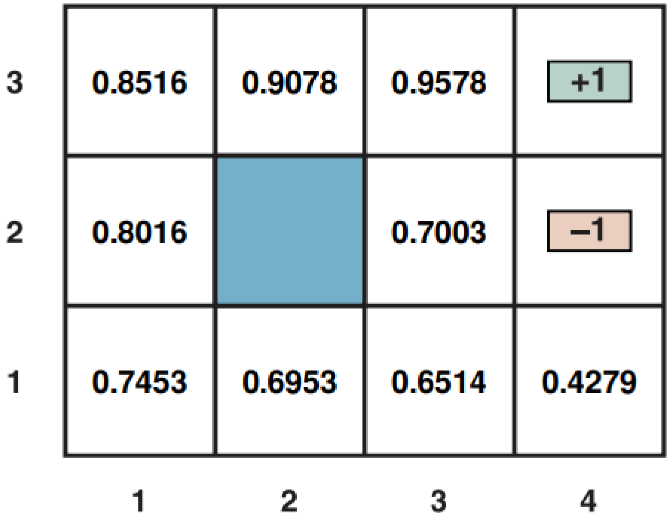
$$U^\pi(s) = E_\pi\left[\sum_{t=0}^{\infty} \gamma^t r(s_t) \,|s_0 = s\right]$$

- $U^\pi(s)$ (often also written as $V(s)$) is called **the value function**. It is often stored as a table.

γ is a discounting factor to give more weight to immediate rewards.

E_π is the expectation over sequences that can be created by following π.

Value Function



Value Function

s	State Value $V(s)$
(1,1)	0.7453
(1,2)	0.8016
…	…

Planning: Finding the Optimal Policy

- The goal of solving an MDP is to find an optimal policy $\boldsymbol{\pi}$ that maximizes the expected future utility for each state

$$\pi^*(s) = \underset{\pi}{\operatorname{argmax}}\, U^\pi(s) \quad \text{for all } s \in S$$

- **Issue**: π^* depends on U^π and vice versa!

- The problem can be formulated recursively using the **Bellman equation** which holds for the optimal value function U ("Bellman optimality condition"):

$$U^{\pi^*}(s) = r(s) + \gamma \max_{a \in A} \underbrace{\sum_{s'} P(s'|s,a)}_{\text{Expectation}}\ U^{\pi^*}(s')$$

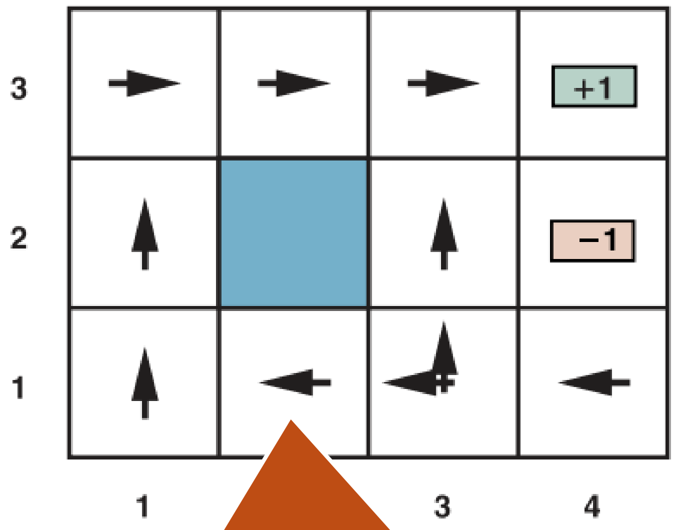
Immediate Reward

π^* uses the best action

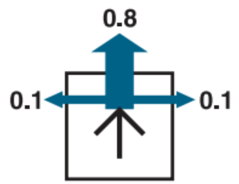
Utility of the next state

Example Solution: 4x3 Grid World

Optimal action in each state
(policy π^*)



Greedy policy:
Always pick the action leading to the state with the highest expected utility.



Value of being in a state $U^{\pi^*}(s)$
(given that we will follow π^*)

	1	2	3	4
3	0.8516	0.9078	0.9578	+1
2	0.8016		0.7003	−1
1	0.7453	0.6953	0.6514	0.4279

$\gamma = 1$

It is optimal to walk away from the +1 square!

How do we find the optimal value function/optimal policy?

Policy Iteration

Value Iteration

Q-Function

- $Q(s,a)$ is called the state-action value function. It gives the expected utility of taking action a in state s and then following the policy.

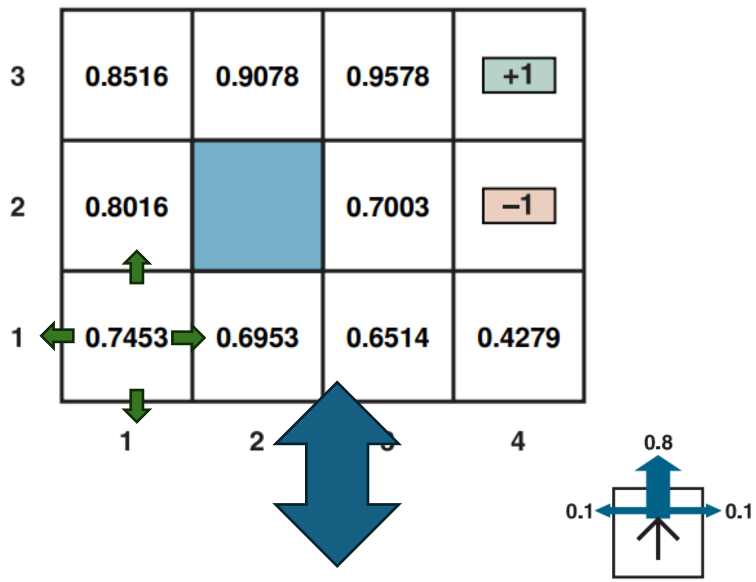
$$Q(s,a) = r(s) + \gamma \sum_{s'} P(s'|s,a)U(s')$$

Immediate Reward

Expected utility of the next state

- The Relationship with the state value function: $U(s) = \max_a Q(s,a)$
- The Q-function lets us compare the value of taking different action is a given state. It is used in algorithms to determine what action is the best.

Value Function

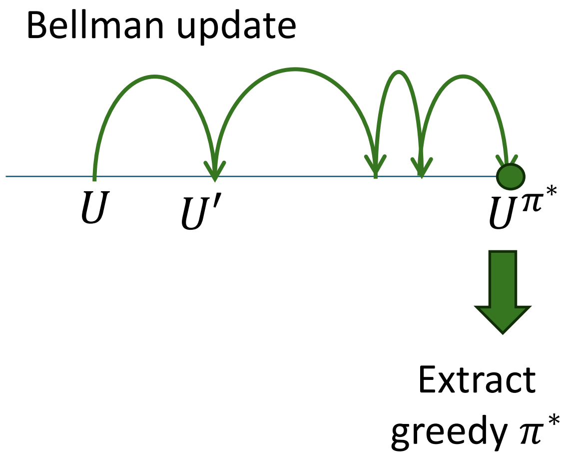


Q-Table

s	a	$Q(s,a)$
(1,1)	Up	0.7453
(1,1)	Right	0.6709
(1,1)	Down	0.7003
(1,1)	Left	0.7109
…	…	…

Value Iteration: Estimate the Optimal Value Function U^{π^*}

Algorithm: Start with a U vector of 0 for all states and then update (Bellman update) the vector iteratively until it converges to the unique optimal solution U^{π^*} (because it is a fixed point of the Bellam operator).



function VALUE-ITERATION(mdp, ϵ) **returns** a utility function
  inputs: mdp, an MDP with states S, actions $A(s)$, transition model $P(s' \mid s, a)$,
    rewards $R(s, a, s')$, discount γ
    ϵ, the maximum error allowed in the utility of any state
  local variables: U, U', vectors of utilities for states in S, initially zero
    δ, the maximum relative change in the utility of any state

  repeat
    $U \leftarrow U'$; $\delta \leftarrow 0$
    for each state s **in** S **do**
      $U'[s] \leftarrow \max_{a \in A(s)}$ Q-VALUE(mdp, s, a, U)
      if $|U'[s] - U[s]| > \delta$ **then** $\delta \leftarrow |U'[s] - U[s]|$
  until $\delta \leq \epsilon(1-\gamma)/\gamma$
  return U

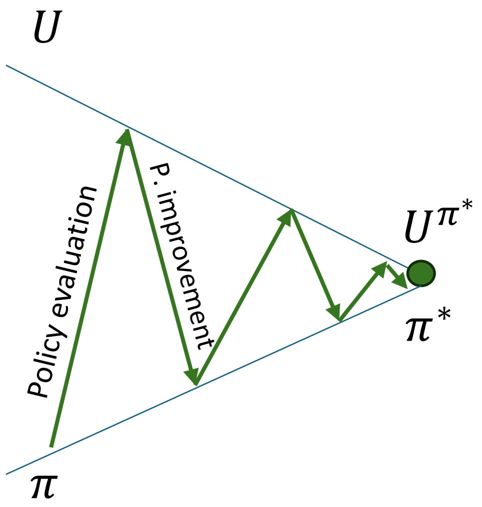
Update with the value of the best action in state s.

Uses a proxy for policy loss
$\|U^\pi - U\|_\infty$ as the stopping criterion

U converges to U^{π^*} and we can extract π^*

Policy Iteration: Find the Optimal Policy π^*

Policy iteration tries to directly find the optimal policy by iterating policy evaluation and improvement.



function POLICY-ITERATION(mdp) **returns** a policy
  inputs: mdp, an MDP with states S, actions $A(s)$, transition model $P(s' \mid s, a)$
  local variables: U, a vector of utilities for states in S, initially zero
    π, a policy vector indexed by state, initially random

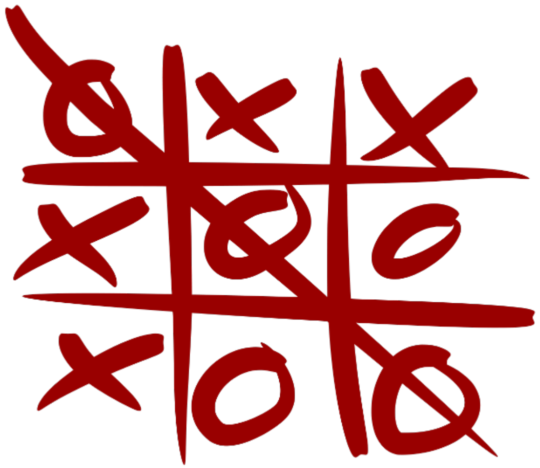
  repeat
    $U \leftarrow$ POLICY-EVALUATION(π, U, mdp)
    $unchanged? \leftarrow$ true
    for each state s **in** S **do**
      $a^* \leftarrow \operatorname{argmax}_{a \in A(s)}$ Q-VALUE(mdp, s, a, U)
      if Q-VALUE(mdp, s, a^*, U) $>$ Q-VALUE($mdp, s, \pi[s], U$) **then**
        $\pi[s] \leftarrow a^*$; $unchanged? \leftarrow$ false
  until $unchanged?$
  return π

Calculate U given current policy (either solve an LP or value iteration with fixed policy)

Greedy policy Improvement

π converges to π^*
(and U converges to U^{π^*})

Playing a Game as a Sequential Decision Problem: Tic-Tac-Toe



- Definitions from Chapter 5 on Games for a goal-based agent:

s_0	Empty board.
$Actions(s)$	Play empty squares.
$Result(s, a)$	Symbol (x/o) is placed on empty square.
$Terminal(s)$	Did a player win or is the game a draw?
$Utility(s)$	+1 if x wins, -1 if o wins and 0 for a draw.
	Utility is only defined for terminal states.

Stochastic transition model $P(s'|s, a)$

Reward function $r(s)$

- We can set up an MDP to find the optimal policy $\pi^*(s)$, but it will be hard to solve since:
  - There are many states, so the table U(s) has many entries.
  - The stochastic transition model $P(s'|s, a)$ needs to be known. The tables are very large.
  - All the reward is delayed. Immediate rewards are always 0 until the end of the game.
- This makes planning hard! A solution is to use online learning like model-free reinforcement learning.

(Model-Free) Reinforcement Learning

AIMA Chapter 22

Reinforcement Learning (RL)

- RL assumes that the problem can be modeled as a **Markov Decision Process (MDP)**.
- However, we do not know the transition or the reward model. This means we have an **unknown environment**.
- We cannot use offline planning in unknown environments. The agent needs to interact with the environment (try actions) and **use the reward signal to update its estimate of the utility of states and actions**. This is a learning process where the reward provides positive reinforcement.
- A popular algorithm is Q-Learning, which tries to learn the state-action value function of important states.

Q-Table

s	a	$Q(s,a)$

Q-Learning

Q-Learning learns the state-action value function as a table from interactions with the environment.

function Q-LEARNING-AGENT(*percept*) **returns** an action
  inputs: *percept*, a percept indicating the current state s' and reward signal r
  persistent: Q, a table of action values indexed by state and action, initially zero
    N_{sa}, a table of frequencies for state–action pairs, initially zero
    s, a, the previous state and action, initially null

A new episode starts with no previous state.

  if s is not null **then**
    increment $N_{sa}[s, a]$
    $Q[s,a] \leftarrow Q[s,a] + \alpha(N_{sa}[s,a])(r + \gamma \max_{a'} Q[s',a'] - Q[s,a])$
  $s, a \leftarrow s', \text{argmax}_{a'} f(Q[s',a'], N_{sa}[s',a'])$
  return a

Learning rate

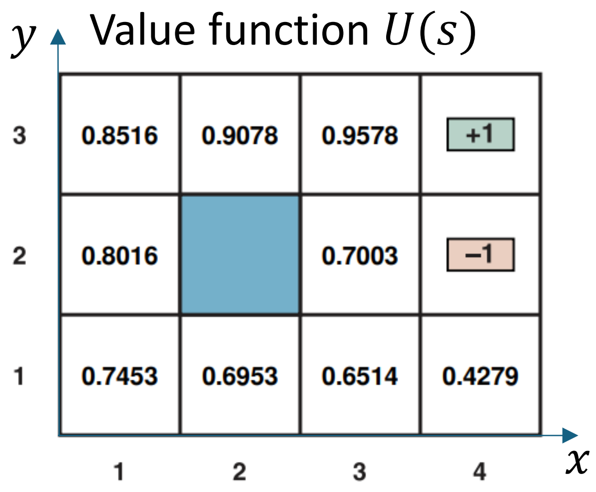
Make $Q[s,a]$ a little more similar to the received reward + the best Q-value of the successor state.

$f(\cdot)$ is the exploration function and decides on the next action. As N increases, it can exploit good actions more.

Value Function Approximation

- U (or Q) tables needs to store and estimate one entry for each state (state/action combination).
- Issues and solutions
  - Too many entries to store → lossy compression
  - Many combinations are rarely seen → generalize to unseen entries

- **Idea**: Estimate the state value by learning an approximation function $\hat{U}(s) = g_\theta(s)$ based on features of s.
- **Example**: 4x3 Grid World with a linear combination of state features (x, y) and learn $\boldsymbol{\theta}$ from observed data.



Learn $\boldsymbol{\theta}$ from observed interactions with the environment to approximate $U(s)$

$$\hat{U}_\theta(x, y) = \theta_0 + \theta_1 x + \theta_2 y$$

$\boldsymbol{\theta}$ can be updated iteratively after each new observed utility using gradient descent.

Traditional Q-Learning

s →

s	a	$Q(s,a)$
1	1	0.7
1	2	0.3
…	…	…

→ $Q(s,a)$

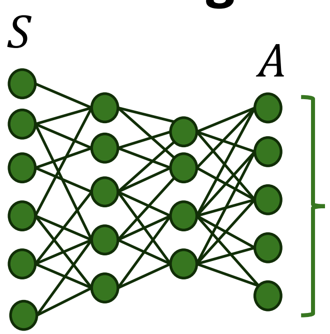
Q-Table

function Q-LEARNING-AGENT(*percept*) **returns** an action
  inputs: *percept*, a percept indicating the current state s' and reward signal r
  persistent: Q, a table of action values indexed by state and action, initially zero
    N_{sa}, a table of frequencies for state–action pairs, initially zero
    s, a, the previous state and action, initially null

  if s is not null **then**
    increment $N_{sa}[s,a]$
    $Q[s,a] \leftarrow Q[s,a] + \alpha(N_{sa}[s,a])(\overbrace{r + \gamma \max_{a'} Q[s',a']}^{\text{target}} - \overset{\text{prediction}}{\boxed{Q[s,a]}})$
  $s, a \leftarrow s', \operatorname{argmax}_{a'} f(Q[s',a'], N_{sa}[s',a'])$
  return a

Deep Q-Learning

s →



→ $Q(s,a)$

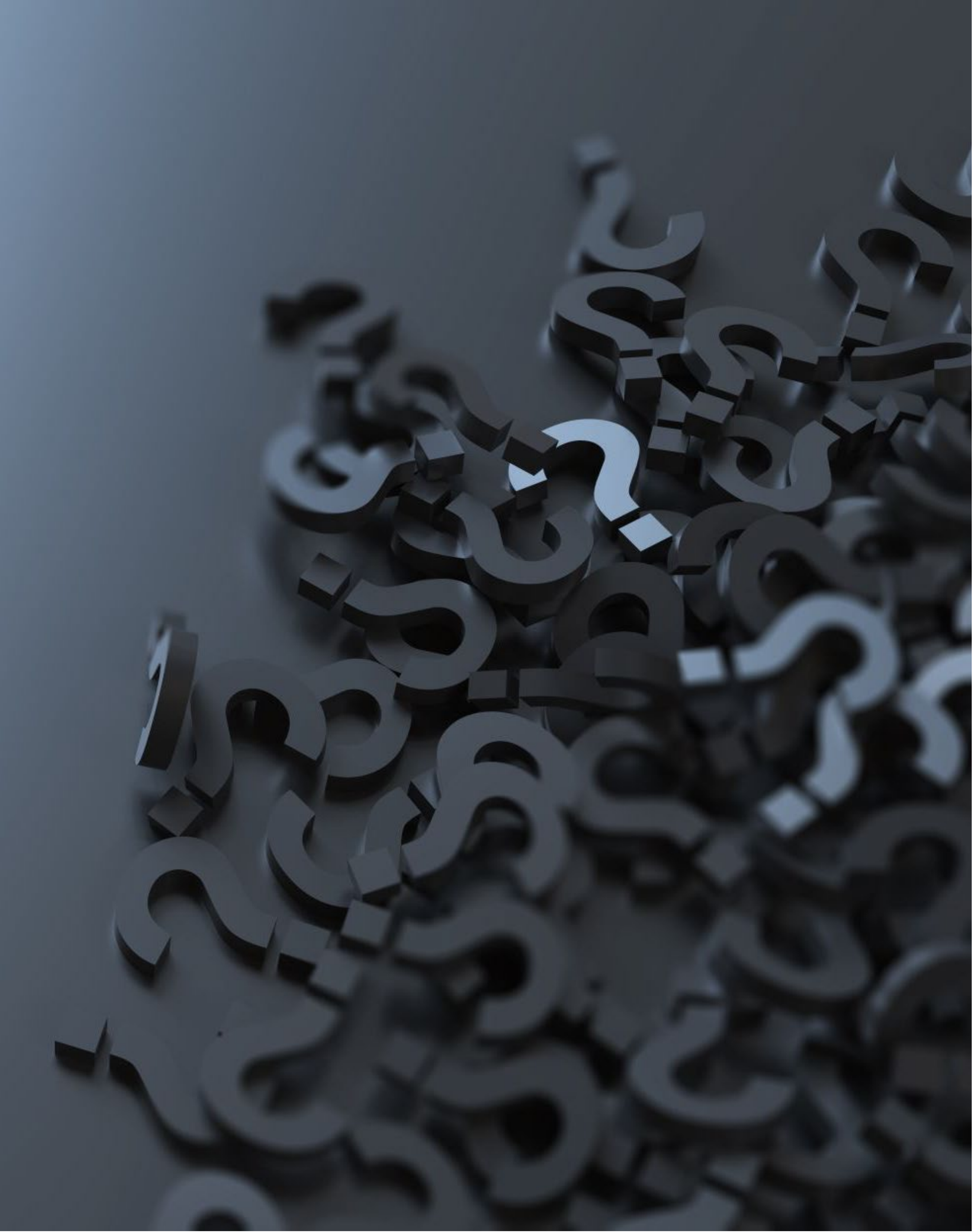
Deep Q-Network (DQN)

Target networks: It turns out that the Q-Network is unstable if the same network is used to estimate $Q(s,a)$ and also $Q(s',a')$. Deep Q-Learning uses a second target network for $Q(s',a')$ that is updated with the prediction network every C steps.

Experience replay: To reduce instability more, generate actions using the current network and store the experience $\langle s,a,r,s'\rangle$ in a table. Update the model parameters by sampling from the table.

Loss function: squared difference between prediction and target.

Volodymyr Mni et al., [Playing Atari with Deep Reinforcement Learning](), NIPS Deep Learning Workshop 2013.



Summary

- Agents can learn the value of being in a state from **reward signals**.
- Rewards can be delayed (e.g., at the end of a game).
- Not being able to fully **observe the state** makes the problem more difficult (POMDP).
- **Unknown transition models** lead to the need for exploration by trying actions (model-free methods like Q-Learning).
- All RL problems are computationally very expensive and often can only be solved by **approximation**. State-of-the-art is to use deep artificial neural networks for function approximation.