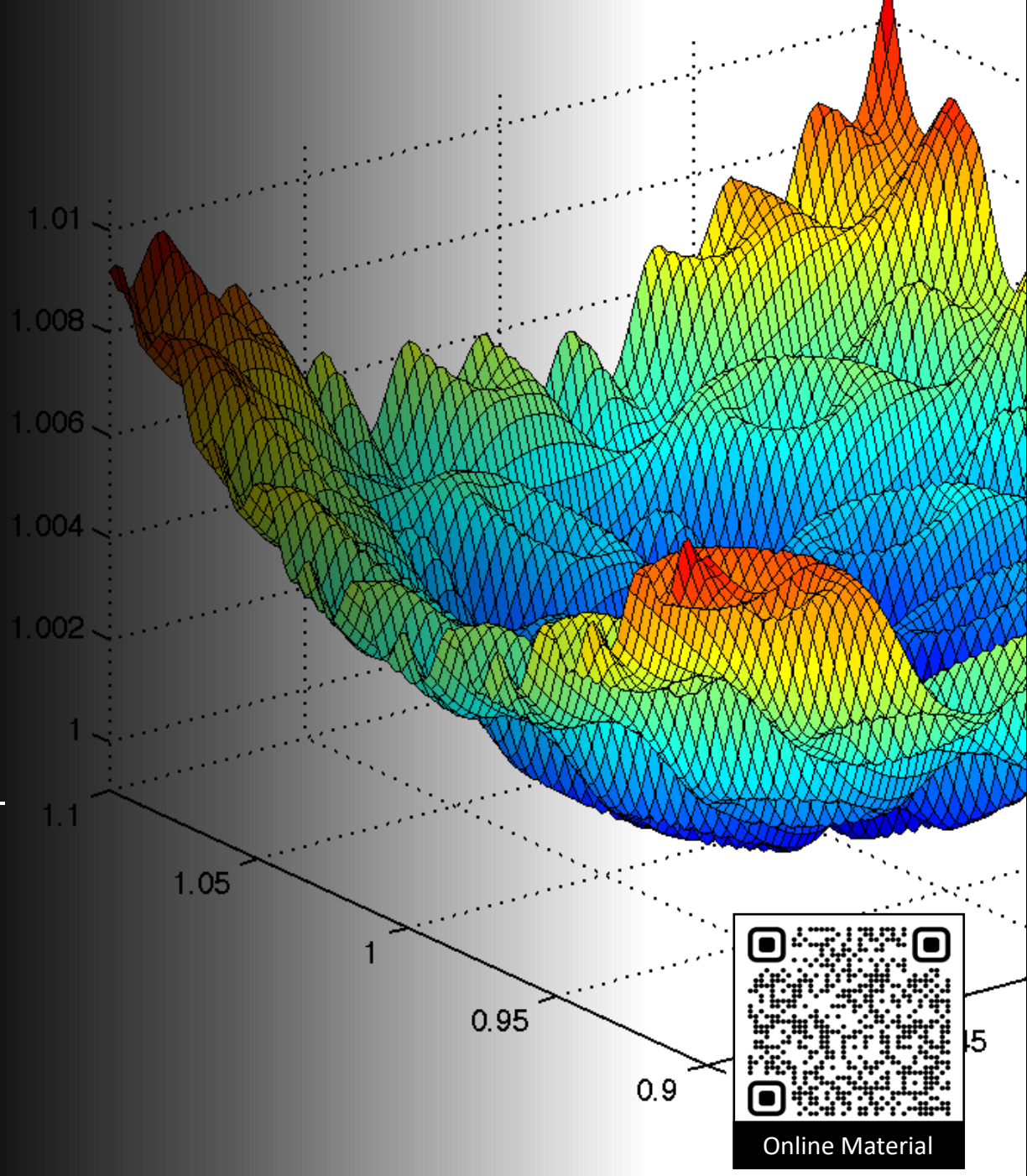
Discussion

# CS 5/7320
## Artificial Intelligence

# Local Search
## AIMA Chapters 4.1 & 4.2




Slides by Michael Hahsler
based on slides by Svetlana Lazepnik
with figures from the AIMA textbook.

This work is licensed under a Creative Commons Attribution-ShareAlike 4.0 International License.

# Recap: Uninformed and Informed Search

Tries to **plan** the

**best path**

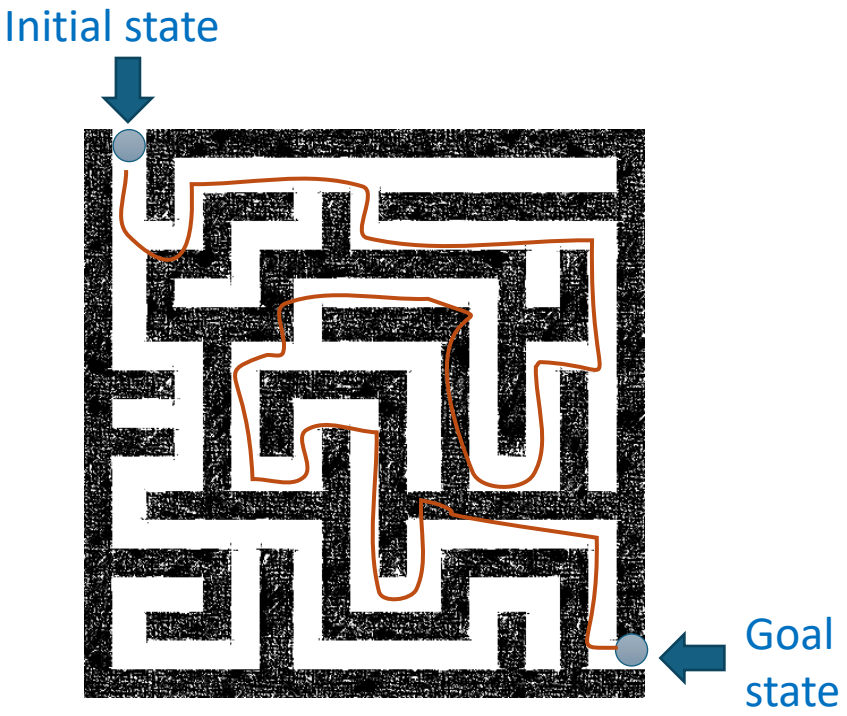
from a

**given initial state**

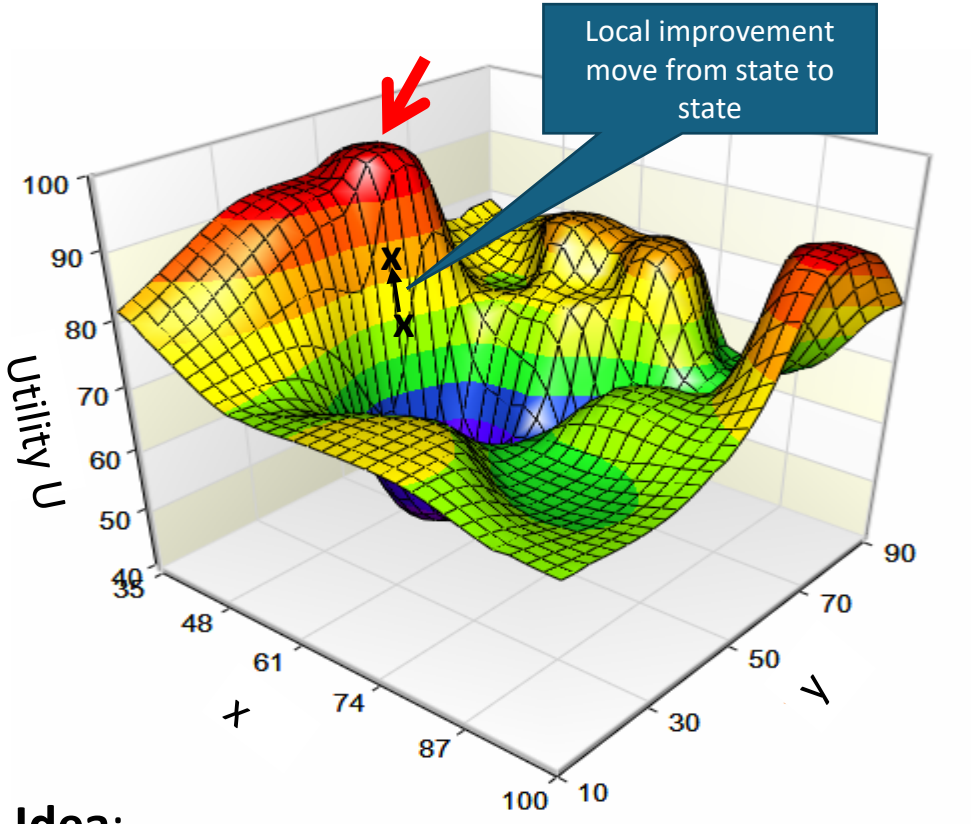
to a

**given goal state.**

- Often comes with completeness and optimality guarantees (BFS, A* Search, IDS).
- **Issue**: Typically have to search a large portion of the search space and therefore need a lot of time and memory.



# Local Search




- Assume we know the utility of each possible state given by a utility function
$$U = u(s)$$
- We use a factored state description. Here $s = (x, y)$
- How can we identify the best or at least a "good" state?
- This is the **optimization problem**:
$$s^* = \underset{s \in S}{\operatorname{argmax}}\, u(s)$$
- We need a fast and memory-efficient way to find the best/a good state.

**Idea**:

Start with a current solution (a state) and improve the solution by moving from the current state to a "neighboring" better state (a.k.a. performing a series of **local moves**).

# Hill-Climbing Search (Greedy Local Search)

Maximization

```
function HILL-CLIMBING(problem) returns a state that is a local maximum
    current ← problem.INITIAL
    while true do
        neighbor ← a highest-valued successor state of current
        if VALUE(neighbor) ≤ VALUE(current) then return current
        current ← neighbor
```

We often start with a random state

Use ≥ for minimization

**Variants**:

- **Steepest ascent hill climbing:** Check all possible successors and choose the highest-valued successor.
- **Stochastic hill climbing:** Choose randomly among **all** uphill (improvement) moves.
- **First-choice stochastic hill climbing**: Generate randomly **one** new successor at a time and only move to better ones. This is what people often mean by "stochastic hill climbing." It is equivalent to a, but computationally much cheaper.

Minimization


$h = 17$

This is what people often mean by "stochastic hill climbing."

# Stochastic Hill Climbing

- **First-choice stochastic hill climbing**: Generate randomly **one** new successor at a time and only move to better ones.
- **Steps:**

Minimization


$h = 17$

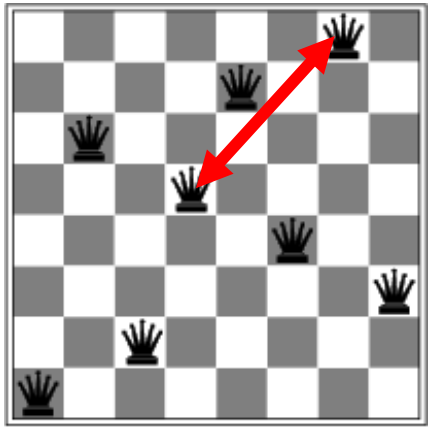
- Why is the result equivalent to stochastic hill climbing that calculates the heuristic for all moves?

# Local Optima

Hill-climbing search is like greedy best-first search with the objective function as a (maybe not admissible) heuristic. It only stores the current state (has no frontier data structure) and just stops at a dead end.

Is it complete/optimal?

- No – can get stuck in local optima.



$h = 1$

Example: local optimum for the 8-queens problem. No single queen can be moved within its column to improve the objective function.

Simple approach that can help with local optima:

**Random-restart hill climbing:** Restart hill-climbing many times with random initial states and return the best solution. This strategy can be used for any stochastic (i.e., randomized) algorithm.

# Simulated Annealing Algorithm

- Use first-choice stochastic hill climbing + escape local minima by allowing some "bad" moves but gradually decreasing their frequency as we get closer to the solution.
- Annealing tries to reach a low energy state: A negative $\Delta E$ means the solution gets better.
- The probability of accepting "bad" moves follows the annealing schedule, which reduces the temperature $T$ over time $t$.

Maximization

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
    current ← problem.INITIAL
    for t = 1 to ∞ do
        T ← schedule(t)
        if T = 0 then return current
        next ← a randomly selected successor of current
        ΔE ← VALUE(current) – VALUE(next)
        if ΔE ≤ 0 then current ← next
        else current ← next only with probability e^(−ΔE/T)
```

Typically, we start with a random state

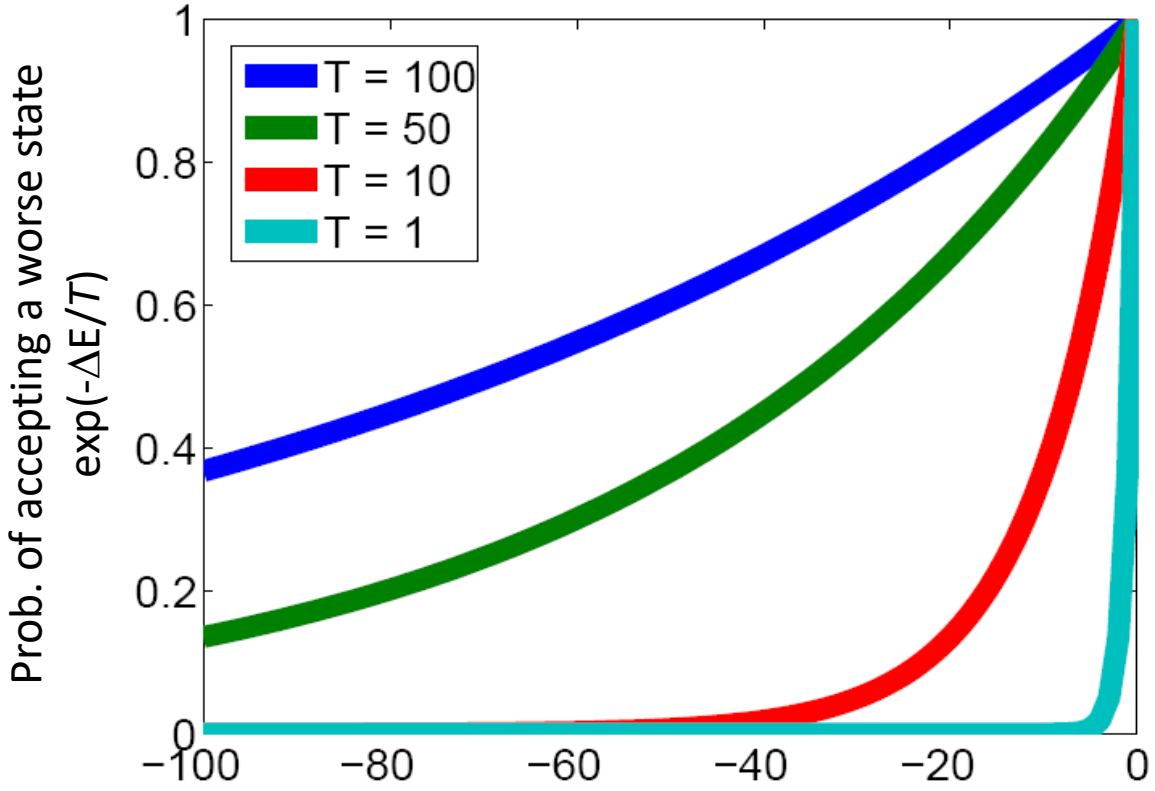
Always accept good moves that reduce the energy.

Accept "bad" moves with a probability inspired by the acceptance criterion in the Metropolis–Hastings MCMC algorithm.

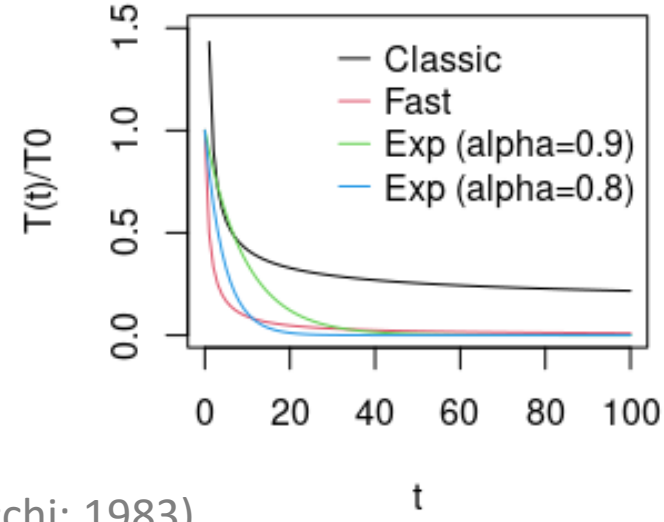
Note: Use $VALUE(next) - VALUE(current)$ for minimization

# The Effect of Temperature

Convert the changes due to “bad” moves into an acceptance probability depending on the temperature. The criterion uses the negative part of the exponential function.



# Cooling Schedule




The cooling schedule is very important.
Popular schedules for the temperature at time $t$:

- **Classic simulated annealing:** $T_t = T_0 \frac{1}{\log(1+t)}$
- **Exponential cooling** (Kirkpatrick, Gelatt and Vecchi; 1983)

$$T_t = T_0 \alpha^t \quad \text{for} \quad 0.8 < \alpha < 1$$

- **Fast simulated annealing** (Szy and Hartley; 1987)

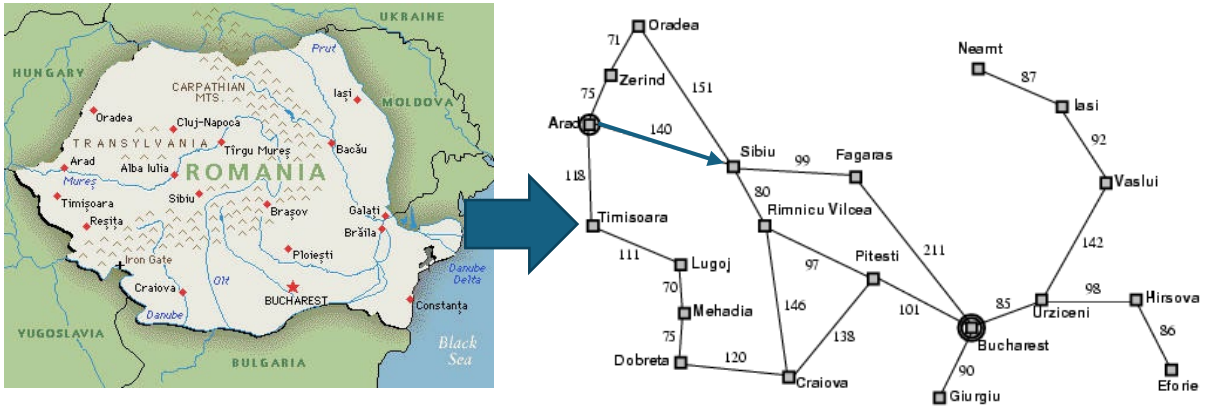
$$T_t = T_0 \frac{1}{1+t}$$

Notes:

- Choose $T_0$ to provide a high probability $p_0 = e^{-\frac{\Delta E}{T_0}}$ that any move will be accepted at time $t = 0$. $\Delta E$ is determined by the worst possible move.
- $T_t$ will not become 0 but very small. Stop when $T < \epsilon$ ($\epsilon$ is a very small constant).
- The best schedule (cooling rate) is typically determined by trial-and-error. The goal is to have a low chance of getting stuck in a local optima.

# Methods: Discretization of Continuous Space

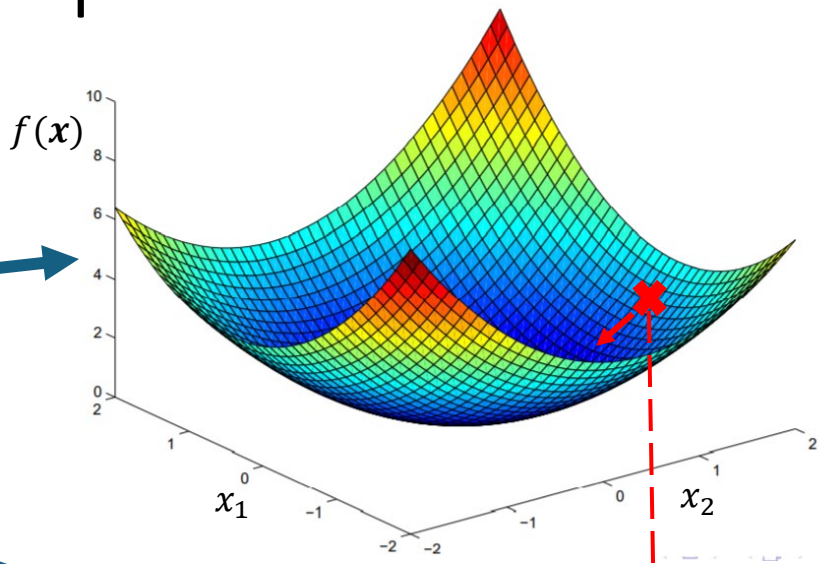
- Use atomic states and create a graph as the transition function.




- Use a grid with spacing of size $\delta$

  Note: You probably need a way finer grid!

# Search in Continuous Spaces: Gradient Descent




**State representation**: $\boldsymbol{x} = (x_1, x_2, \dots, x_k)$

**State space size**: infinite

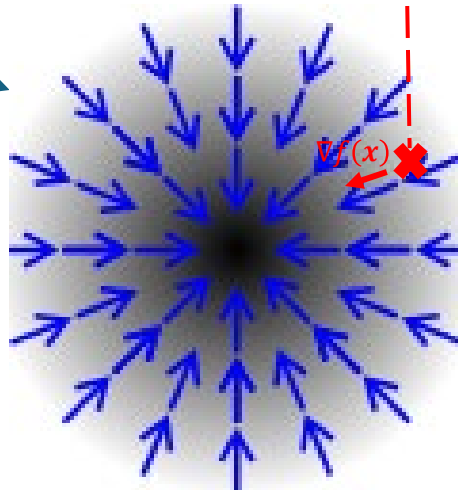
**Objective function**: min $f(\boldsymbol{x})$

**Local neighborhood**: small changes in $x_1, x_2, \dots, x_k$

Gradient at point $\boldsymbol{x}$: $\nabla f(\boldsymbol{x}) = \left(\frac{\partial f(\boldsymbol{x})}{\partial x_1}, \frac{\partial f(\boldsymbol{x})}{\partial x_2}, \dots, \frac{\partial f(\boldsymbol{x})}{\partial x_k}\right)$

(=evaluation of the Jacobian matrix at $\boldsymbol{x}$)

Find optimum by solving: $\nabla f(\boldsymbol{x}) = 0$




- **Gradient descent (= Steepest-ascend hill climbing for minimization)** with step size $\alpha$ (typically reduced over time)

  Repeat: $\boldsymbol{x} \leftarrow \boldsymbol{x} - \alpha \nabla f(\boldsymbol{x})$

- **Newton-Raphson method**
  uses the inverse of the Hessian matrix (second-order partial derivative of $f(\boldsymbol{x})$)
  $H_{ij} = \frac{\partial^2 f}{\partial x_i \partial x_j}$ as the optimal step size

  Repeat: $\boldsymbol{x} \leftarrow \boldsymbol{x} - \boldsymbol{H}_f^{-1}(\boldsymbol{x}) \nabla f(\boldsymbol{x})$

Note: May get stuck in a local optimum if the search space is non-convex! Use simulated annealing, momentum or other methods to escape local optima.