CS/ECE 4501/6501 **Due by Oct 6** $11:59:59$**PM**

Problem Set II

1. (20%) **Euler's method:** `https://en.wikipedia.org/wiki/Euler_method`
   For $\frac{dy}{dt} + 2y = 2 - e^{-4t}, y(0) = 1$,
   (a) Derive its closed-form solution on your own.
   (b) Use Euler's Method to find the approximation to the solution at $t = \{1, 2, 3, 4, 5\}$, and compare to the exact solution in (a) by plotting them on a same figure.
   (c) Use different step size $h = \{0.1, 0.05, 0.01, 0.005, 0.001\}$ and plot out your approximated function value.

2. (CS/ECE 4501 40%) (CS/ECE 6501 30%) **Principal Component Analysis.** Write your own program of PCA (no packages allowed) and test your implementation on MNIST handwritten digits database (provided in "mnist.mat") . Results listed below are required to be included in your report:
   (a) A plot of all eigenvalues.
   (b) How many numbers of principal components would you choose to achieve at least 90% of the data variance? Include a plot of (number of principal components) vs. (accumulated data variance).
   (c) Plots of the first 10 eigenvectors (same dimension as digit images).

3. (CS/ECE 4501 40%) (CS/ECE 6501 30%) **Geodesic shooting for diffeomorphic image registration.** Implement geodesic shooting by the following strategy and compute the final transformation ϕ_1 at time point $t = 1$. Deform a given source image (included in the data folder) by using the transformation ϕ_1. Note that the initial condition for v_0 is also given in the data folder, and the initial condition for ϕ_0 is an image coordinate, which can be easily generated from Matlab or Python.

$$\frac{dv_t}{dt} = -K[(Dv_t)^T \cdot v_t + \text{div}(v_t v_t^T)],$$
$$\frac{d\phi_t}{dt} = v_t \circ \phi_t,$$

   where K is a smoothing kernel, D is a Jacobian matrix, div is a divergence operator, and $\circ$ denotes an interpolation.

   Note: Use your code of frequency smoothing in PS1 to implement the smoothing operator K (set the truncated number of frequency as 16^2). The div is a divergence operator.

   * Use Euler integration to solve the above ordinary differential equations.

4. (**CS/ECE 6501 ONLY.** 20%) Compute the final transformation ϕ_1 at time point $t = 1$ by the following strategy. Compute the deformed image and compare the differences between the final transformations ϕ_1 and above.

$$\frac{dv_t}{dt} = K[(Dv_t)^T \cdot v_t + \text{div}(v_t v_t^T)],$$
$$\frac{d\phi_t}{dt} = -D\phi_t \cdot v_t.$$

IMPORTANT NOTES:

* Interpolation function: a Matlab function *interp2* with the option ‘spline’, or a Python function *scipy.ndimage.map_coordinates* with the option ‘order=3’.

* All results should be clearly reported and discussed in the report.