CS/ECE-4501/6501 **Due by Sep 14** 11 : 59 : 59**PM**

Problem Set I

1. (20%) Prove the properties of convolution. For all continuous function f, g, and h, the following axioms hold. **Please make sure to lay out all key steps of your proof for full credits.**

   - Associativity: $(f * g) * h = f * (g * h)$
   - Distributivity: $f * (g + h) = f * g + f * h$
   - Differentiation rule: $(f * g)' = f' * g = f * g'$
   - Convolution theorem: $\mathcal{F}(g * h) = \mathcal{F}(g)\mathcal{F}(h)$, where $\mathcal{F}$ denotes Fourier transform

2. (25%) Frequency smoothing:

   (a) Compute Fourier transform of the given image `lenaNoise.PNG` by using `fft2` function in Matlab (or `numpy.fft.fft2` in Python), and then center the low frequencies (e.g., by using `fftshift`).

   (b) Keep different number of low frequencies (e.g., $10^2, 20^2, 40^2$ and the full dimension), but set all other high frequencies to 0.

   (c) Reconstruct the original image (`ifft2`) by using the new generated frequencies in step (b).

   Submit the code and include the restored images with different number of low frequencies in your report.

3. (55%) Implement gradient descent algorithm for ROF model with total variation minimization. **All codes and a two-page report including problem description, a concrete optimization algorithm, and experimental results (a denoised image and a convergence graph that generated by your best-tuned parameters) with discussions should be submitted.**

   NOTE that

   - Test your program with different Gaussian noises ($\sigma = 0.01, 0.05, 0.1$) and include all results in your report. A matlab/python function of 'GenerateNoiseImage' is given for your reference.
   - The forward / backward difference for computing image gradient is given in `Dx / Dxt`. Feel free to use it or write your own.
   - A detailed class note of deriving total variation, computing gradient term, and gradient descent algorithm can be downloaded from Collab.