

Assignment-1

Q.no.1, ans Proposition is a declarative statement that is either 'TRUE' or 'FALSE'. It uses 'T' for 'True' and 'F' for 'FALSE'.

Example:

- i) Paris is in France (T).
- ii) Delhi is in Nepal (F).
- iii) $2 < 4$ (T).
- iv) $9 = 5$ (F)

Q.no.2.ans

A logical operator is an operator that connects two or more expressions. AND, OR and NOT are the common logical operators.

Types of logical Operator.

I) Negation operator (NOT) :

- If 'p' is the proposition then,
negation of 'p' is denoted by $\neg p$.

$\neg p$ means "it is not case that p" which is simply a "not p".

Example: p: "Today is Friday".
 $\neg p$: "Today is not Friday".

Truth table:

P	$\neg P$
T	F
F	T

2. Conjunction (AND): " \wedge "

↳ if 'p' and 'q' are two proposition, then

conjunction of 'p' and 'q' is ' $p \wedge q$ '

↳ $p \wedge q$ is True only when both 'p' & 'q' are
"True" else "False".

Examples:

p: "Today is Friday"

q: "It's raining today".

$p \wedge q$: "Today is Friday & it's raining."

Truth Table :

	p	q	$p \wedge q$
	T	T	T
	T	F	F
	F	T	F
	F	F	F

3. Disjunction (OR): " \vee "

↳ if "p" and "q" are two proposition, then the
disjunction of 'p & q' is " $p \vee q$ ".

↳ $p \vee q$ is False when both 'p' & 'q' are False
else TRUE.

Example: "Today is Friday": p

"It's raining today": q

↳ $p \vee q$: Today is Friday or its raining
today".

Truth Table

P	q	$P \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

4. Exclusive (XOR):

- ↳ if 'p' and 'q' are two proposition, then
Exclusive of 'P' and 'q' is ' $p \oplus q$ ' which
- ↳ ' $p \oplus q$ ' means either p or q but not both.

Examples:

- ↳ $p \oplus q$: "Either today is Friday or it's raining today"

Truth table:

P	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

5. Implication ($\text{if } p \rightarrow \text{then}$)

- ↳ if 'p' and 'q' are proposition then,

Implication of 'p' and 'q' is ' $p \rightarrow q$ '.

Example:

$p \rightarrow q$: If 'p' today is holiday, then 'q' college is closed

Truth Table

P	q	$P \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

6. Biconditional (if and only if)

→ if ' p ' & ' q ' are proposition, then

$p \leftrightarrow q$ is the bicondition

→ $(p \rightarrow q) \wedge (q \rightarrow p) = "p \leftrightarrow q"$

Examples: p : I'm breathing.

q : I'm alive.

$p \leftrightarrow q$: "I am breathing if and only if I am alive"

Truth table:

P	q	$q \leftrightarrow p$
T	T	T
T	F	F
F	T	F
F	F	T

Q.no.3 ans.

Given: p: "I bought a lottery ticket this week."

q: "I won million dollar jackpot!"

a) $\neg p$: I didn't buy a lottery ticket this week.

b) $p \vee q$: I bought a lottery ticket this week or I won million dollar jackpot.

c) $p \rightarrow q$: If I bought a lottery ticket this week, then I won million dollar ~~or~~ jackpot.

d) $p \wedge q$: I bought a lottery ticket this week and won a million dollar jackpot.

e) $p \leftrightarrow q$: I bought a lottery ticket this week if and only if I won million dollar jackpot.

f) $\neg p \rightarrow \neg q$: If I didn't buy a lottery ticket this week, then I won't win the million dollar jackpot.

g) $\neg p \wedge \neg q$: If I didn't buy a lottery ticket this week, then I didn't win the million dollar jackpot.

h) $\neg p \vee (p \wedge q)$: I didn't buy a lottery ticket this week or I bought a lottery ticket this week and I won million dollar jackpot.

Q.no.4. ans

p: "It is below freezing".

q: It is snowing."

a) $p \wedge q$

b) $p \wedge \neg q$

c) $\neg p \wedge \neg q$

d) $p \vee q$

e) $\cancel{p} \rightarrow p \rightarrow q$

f) $(p \vee q) \wedge (p \rightarrow \neg q)$

g) $p \leftrightarrow q$

Q.no.5) ans

p: "you have flu".

q: "you miss final examination."

r: you pass course.

a) $p \rightarrow q$: If you have flu, then you will miss final examination.

b) $\neg q \leftrightarrow r$: You wont miss final examination if and only if you pass the course.

c) $q \rightarrow \neg r$: If you miss the final examination then, you wont pass the course.

d) $p \vee q \vee r$: ~~If~~ you have flu or you miss the final examination or you pass the course.

e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$

\Rightarrow if you have flu, then you won't pass the course or if you miss final examination you won't pass the course..

f) $(p \wedge q) \vee (\neg q \wedge r)$

: you have flu and you miss the examination or you ~~won't~~ didn't miss the examination and pass the course.

Q. no. 6, ans a) $r \wedge \neg q$

b) $p \wedge q \wedge r$

c) $r \rightarrow p$

d) $p \wedge q \wedge r$

e) $(p \wedge q) \rightarrow r$

f) $r \leftrightarrow q \vee p$

- Q. no. 7 ans a) If it snows today, I will ski tomorrow
b) I come to class whenever there is going to be a quiz.

Converse: $(p \rightarrow q) \Rightarrow (q \rightarrow p)$

a) If I will ski tomorrow, then it will snow today.

b) There is going to be a quiz whenever I come to class.

Contrapositive: $(p \rightarrow q) \Rightarrow (\neg q \rightarrow \neg p)$

a) If I won't ski tomorrow, it doesn't snow today.

b) There is not going to be a quiz unless I come to class.

Inverse $(p \rightarrow q) \Rightarrow (\neg p \rightarrow \neg q)$

a) If it doesn't snow today, I won't ski tomorrow.

b) I won't come to class unless there is going to be ~~a~~ quiz.

Q.no.8 ans

a) If it rains tonight, then I will watch movie.

b) I go to beach whenever it is a sunny summer day.

Converse:

a) If I will watch movie, it rains tonight.

b) It is a sunny summer day, ^{whenever} I go to the beach

Contrapositive:

a) If I won't watch movie, then it doesn't rain tonight.

b) ~~If~~ It is never sunny summer day when I don't watch movie.

Q.no.9

Truth table

P	q	$\neg P$	$p \wedge \neg p$
T	F	F	F
F	T	T	F

b) $P \vee \neg P$

P	$\neg P$	$P \vee \neg P$
T	F	T
F	T	T

c) $(P \vee \neg q) \rightarrow q$

P	q	$\neg q$	$(P \vee \neg q)$	$(P \vee \neg q) \rightarrow q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

d) $(P \vee q) \rightarrow (P \wedge q)$

P	q	$(P \vee q)$	$(P \wedge q)$	$(P \vee q) \rightarrow (P \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

e) $(P \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg P)$

P	q	$\neg P$	$\neg q$	$P \rightarrow q$	$\neg q \rightarrow \neg P$	$(P \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg P)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	F	T	T

$$f) (p \rightarrow q) \rightarrow (q \rightarrow p)$$

p	q	$(p \rightarrow q)$	$(q \rightarrow p)$	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	F
F	F	T	T	T

Q.no.10

a) $(p \vee q) \rightarrow (p \oplus q)$

Here,

p	q	$p \vee q$	$p \oplus q$	$(p \vee q) \rightarrow (p \oplus q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

b) $(p \oplus q) \rightarrow (p \wedge q)$

p	q	$p \oplus q$	$p \wedge q$	$(p \oplus q) \rightarrow (p \wedge q)$
T	T	F	T	T
F	T	T	F	F
T	F	T	F	F
F	F	F	F	T

c) $(p \vee q) \oplus (p \wedge q)$

p	q	$p \vee q$	$(p \wedge q)$	$(p \vee q) \oplus (p \wedge q)$
T	T	T	T	F
T	F	T	F	T
F	T	T	F	T
F	F	F	F	F

Q.no
a) (

d) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$

p	q	$\neg p$	$p \leftrightarrow q$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
T	T	F	T	F	T
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	T	F	T

b)

e) $(p \oplus q) \rightarrow (p \oplus \neg q)$

p	q	$\neg q$	$(p \oplus q)$	$(p \oplus \neg q)$	$(p \oplus q) \rightarrow (p \oplus \neg q)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

Q.no. II ans

a) $(P \vee q) \vee r$

P	q	r	$P \vee q$	$(P \vee q) \vee r$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

b) $(P \vee q) \wedge r$

P	q	r	$P \vee q$	$(P \vee q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

c) $(p \wedge q) \vee r$

P	q	r	$(p \wedge q)$	$(p \wedge q) \vee r$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	T
T	F	F	F	F
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

d) $(p \wedge q) \wedge r$

P	q	r	$p \wedge q$	$(p \wedge q) \wedge r$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

$$e) (p \vee q) \wedge \neg r$$

p	q	r	$\neg r$	$p \vee q$
T	T	T	F	T
T	T	F	T	T
T	F	T	F	T
T	F	F	T	T
F	T	T	F	T
F	T	F	T	T
F	F	F	F	F
F	F	F	T	F

$$(p \vee q) \wedge \neg r$$

T
F
T
F
T
F
T
F

$$f) (p \wedge q) \vee \neg r$$

p	q	r	$\neg r$	$p \wedge q$
T	T	T	F	T
T	T	F	T	T
T	F	T	F	F
T	F	F	T	F
F	T	T	F	F
F	T	F	T	F
F	F	T	F	F
F	F	F	T	F

$$(p \wedge q) \vee \neg r$$

T
F
T
F
T
F
T
T

$$g) (p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$$

p	q	r	$p \leftrightarrow q$	$\neg p$	$\neg r$	$\neg p \leftrightarrow \neg r$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$
T	T	T	T	F	F	T	F
T	T	F	T	F	T	F	T
T	F	T	F	F	F	T	T
T	F	F	F	F	T	F	F
F	T	T	F	T	F	T	F
F	T	F	F	T	T	F	T
F	F	T	T	T	F	T	T
F	F	F	F	T	T	F	F

iii) p
q
r

iv) A

re

Q.no.12 i) $p =$ "you can have voting right."
 $q =$ "you are mentally unfit."
 $r =$ "you are over 18 years."
 \Rightarrow relation: $(q \wedge \neg r) \rightarrow \neg p$

ii) $p =$ "leaders will make correct decision."
 $q =$ "you choose a good leader".
 $r =$ "you raise your voice against incorrect decision."
 relation: $p \rightarrow (q \vee r)$

Q.no

iii) p = "You can see the movie."

q = "You are over 18 years old."

r = "You have permission of a parent."

relation: $p \rightarrow (q \vee r)$

iv) p = "You can graduate."

q = "You have completed the requirement of your major."

r = "You ~~do not~~ owe money to university."

s = "You ~~do not~~ have an overdue library book."

relation: $p \rightarrow (q \wedge r \wedge s)$

Q.no.3 ans.

a) $q \rightarrow p$

b) ~~$\neg q \rightarrow \neg p$~~

c) $q \rightarrow p$

d) $\neg q \rightarrow \neg p$