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# Assignment\_1.1

**1.** What is Proposition? Explain with Examples.

**2.** What are Logical Operators? Explain each of them with examples and Truth tables.

**3.** Let p and q be the propositions,

p: "*I bought a lottery ticket this week.*"

q: "*I won the million dollar jackpot.*"

Express each of these propositions as an English sentence.

- |                           |                               |                      |                 |                          |                                |
|---------------------------|-------------------------------|----------------------|-----------------|--------------------------|--------------------------------|
| a) $\neg p$               | b) $p \vee q$                 | c) $p \rightarrow q$ | d) $p \wedge q$ | e) $p \leftrightarrow q$ | f) $\neg p \rightarrow \neg q$ |
| g) $\neg p \wedge \neg q$ | h) $\neg p \vee (p \wedge q)$ |                      |                 |                          |                                |

**4.** Let p and q be the propositions,

p: "*It is below freezing.*"

q: "*It is snowing.*"

Write these propositions using p and q and logical connectives (including negations).

- a) It is below freezing and snowing.
- b) It is below freezing but not snowing.
- c) It is not below freezing and it is not snowing
- d) It is either snowing or below freezing (or both).
- e) If it is below freezing, it is also snowing.
- f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
- g) That it is below freezing is necessary and sufficient for it to be snowing

**5.** Let p, q, and r be the propositions,

p : "*You have the flu.*"

q : "*You miss the final examination.*"

r : "*You pass the course.*"

Express each of these propositions as an English sentence.

- |                      |                               |                           |                      |   |  |
|----------------------|-------------------------------|---------------------------|----------------------|---|--|
| a) $p \rightarrow q$ | b) $\neg q \leftrightarrow r$ | c) $q \rightarrow \neg r$ | d) $p \vee q \vee r$ | e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$ | f) $(p \wedge q) \vee (\neg q \wedge r)$ |
|----------------------|-------------------------------|---------------------------|----------------------|---|--|

**6.** Let p, q, and r be the propositions,

p : "*You get an A on the final exam.*"

q : "*You do every exercise in this book.*"

r : "*You get an A in this class.*"

Write these propositions using p, q, and r and logical connectives (including negations).

- a) You get an A in this class, but you do not do every exercise in this book.
- b) You get an A on the final, you do every exercise in this book, and you get an A in this class.
- c) To get an A in this class, it is necessary for you to get an A on the final.
- d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.
- e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
- f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final

**7.** State the converse, contrapositive, and inverse of each of these conditional statements.

a) *If it snows today, I will ski tomorrow.*

b) *I come to class whenever there is going to be a quiz.*

**8.** State the converse, contrapositive, and inverse of each of these conditional statements.

- a) *If it rains tonight, then I will watch movie.*
- b) *I go to the beach whenever it is a sunny summer day.*

**9.** Construct a truth table for each of these compound propositions.

- a)  $p \wedge \neg p$
- b)  $p \vee \neg p$
- c)  $(p \vee \neg q) \rightarrow q$
- d)  $(p \vee q) \rightarrow (p \wedge q)$
- e)  $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
- f)  $(p \rightarrow q) \rightarrow (q \rightarrow p)$

**10.** Construct a truth table for each of these compound propositions.

- a)  $(p \vee q) \rightarrow (p \oplus q)$
- b)  $(p \oplus q) \rightarrow (p \wedge q)$
- c)  $(p \vee q) \oplus (p \wedge q)$
- d)  $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg q)$
- e)  $(p \oplus q) \rightarrow (p \oplus \neg q)$

**11.** Construct a truth table for each of these compound propositions.

- a)  $(p \vee q) \vee r$
- b)  $(p \vee q) \wedge r$
- c)  $(p \wedge q) \vee r$
- d)  $(p \wedge q) \wedge r$
- e)  $(p \vee q) \wedge \neg r$
- f)  $(p \wedge q) \vee \neg r$
- g)  $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$

**12.** Translate into mathematical expression

- i) *You can't have voting right if you are mentally unfit and you are not over 18 years.*
- ii) *Leaders will make correct decision only if you choose a good leader or you raise your voice against incorrect decision.*
- iii) *You can see the movie only if you are over 18 years old or you have the permission of a parent.*
- iv) *You can graduate only if you have completed the requirements of your major and you do not owe money to the university and you do not have an overdue library book.*

**13.** Express these system specifications using the propositions:

- p: “*The message is scanned for viruses*”  
 q : “*The message was sent from an unknown system*”

- a) “The message is scanned for viruses whenever the message was sent from an unknown system.”
- b) “The message was sent from an unknown system but it was not scanned for viruses.”
- c) “It is necessary to scan the message for viruses whenever it was sent from an unknown system.”
- d) “When a message is not sent from an unknown system it is not scanned for viruses.”

Assignment-1.

Q.no.1, ans Proposition is a declarative statement that is either 'TRUE' or 'FALSE'. It uses 'T' for 'True' and 'F' for 'FALSE'.

Example:

- i) Paris is in France (T).
- ii) Delhi is in Nepal (F).
- iii)  $2 < 4$  (T).
- iv)  $9 = 5$  (F)

Q.no.2.ans

A logical operator is an operator that connects two or more expressions. AND, OR and NOT are the common logical operators.

Types of Logical Operator.

## I) Negation Operator (NOT) :

- If 'p' is the proposition then,  
negation of 'p' is denoted by  $\neg p$ .

$\neg p$  means "it is not case that p" which is simply a "not p".

Example: p: "Today is Friday".  
 $\neg p$ : "Today is not Friday".

Truth table:

P	$\neg P$
T	F
F	T

## 2. Conjunction (AND): " $\wedge$ "

↳ if 'p' and 'q' are two proposition, then

conjunction of 'p' and 'q' is ' $p \wedge q$ '

↳  $p \wedge q$  is True only when both 'p' & 'q' are "True" else "False".

Examples:

$p$ : "Today is Friday"

$q$ : "It's raining today".

$p \wedge q$ : "Today is Friday & it's raining."

Truth Table :

	$p$	$q$	$p \wedge q$
	T	T	T
	T	F	F
	F	T	F
	F	F	F

## 3. Disjunction (OR): " $\vee$ "

↳ if "p" and "q" are two proposition, then the disjunction of 'p & q' is " $p \vee q$ ".

↳  $p \vee q$  is False when both 'p' & 'q' are False  
else TRUE.

Example: "Today is Friday": p

"It's raining today": q

↳  $p \vee q$ : Today is Friday or its raining today".

## Truth Table

$P$	$q$	$P \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

## 4. Exclusive (XOR):

- ↳ if 'p' and 'q' are two proposition, then  
Exclusive of 'P' and 'q' is ' $p \oplus q$ ' which
- ↳ ' $p \oplus q$ ' means either  $p$  or  $q$  but not both.

### Examples:

- ↳  $p \oplus q$ : "Either today is Friday or it's raining today"

### Truth table:

$P$	$q$	$P \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

## 5. Implication ( $\text{if } p \rightarrow \text{then}$ )

- ↳ if 'p' and 'q' are proposition then,

Implication of 'p' and 'q' is ' $p \rightarrow q$ '.

### Example:

$p \rightarrow q$ : If 'p' today is holiday, then 'q' college is closed

## Truth Table

$P$	$q$	$P \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

6. Biconditional (if and only if)

→ if 'p' & q are proposition, then

$p \leftrightarrow q$  is the bicondition

→  $(P \rightarrow q) \wedge (q \rightarrow P) = "p \leftrightarrow q"$

Examples:  $p$ : I'm breathing.  
 $q$ : I'm alive.  
 $p \leftrightarrow q$  : "I am breathing if and only if I am alive"

Truth table:

$P$	$q$	$q \leftrightarrow p$
T	T	T
T	F	F
F	T	F
F	F	T

Q.no.3ans.

Given: p: "I bought a lottery ticket this week."  
 q: "I won million dollar jackpot!"

- a)  $\neg p$ : I didn't buy a lottery ticket this week.
- b)  $p \vee q$ : I bought a lottery ticket this week or I won million dollar jackpot.
- c)  $p \rightarrow q$ : If I bought a lottery ticket this week, then I won million dollar ~~or~~ jackpot.
- d)  $p \wedge q$ : I bought a lottery ticket this week and won a million dollar jackpot.
- e)  $p \leftrightarrow q$ : I bought a lottery ticket this week if and only if I won million dollar jackpot.
- f)  $\neg p \rightarrow \neg q$ : If I didn't buy a lottery ticket this week, then I won't win the million dollar jackpot.
- g)  $\neg p \wedge \neg q$ : If I didn't buy a lottery ticket this week, then I didn't win the million dollar jackpot.
- h)  $\neg p \vee (p \wedge q)$ : I didn't buy a lottery ticket this week or I bought a lottery ticket this week and I won million dollar jackpot.

Q.no.4. ans

p: "It is below freezing".

q: It is snowing."

a)  $p \wedge q$

b)  $p \wedge \neg q$

c)  $\neg p \wedge \neg q$

d)  $p \vee q$

e)  $\rightarrow p \rightarrow q$

f)  $(p \vee q) \wedge (p \rightarrow \neg q)$

g)  $p \leftrightarrow q$

Q.no.5) ans

p: "you have flu".

q: "you miss final examination."

r: you pass course.

a)  $p \rightarrow q$ : If you have flu, then you will miss final examination.

- b)  $\neg q \leftrightarrow r$ : You wont miss final examination if and only if you pass the course.
- c)  $q \rightarrow \neg r$ : If you miss the final examination then, you wont pass the course.
- d)  $p \vee q \vee r$ : ~~If~~ you ~~as~~ have flu or you miss the final examination or you pass the course.
- e)  $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$   
⇒ if you have flu, then you won't pass the course or if you miss final examination you won't pass the course.
- f)  $(p \wedge q) \vee (\neg q \wedge r)$   
: you have flu and you miss the examination or you ~~won't~~ didn't miss the examination and pass the course.

Q. no. 6, ans a)  $r \wedge \neg q$

b)  $p \wedge q \wedge r$

c)  $r \rightarrow p$

d)  $p \wedge q \wedge r$

e)  $(p \wedge q) \rightarrow r$

f)  $r \leftrightarrow q \vee p$

- Q. no. 7 ans a) If it snows today, I will ski tomorrow.  
 b) I come to class whenever there is going to be a quiz.

Converse:  $(p \rightarrow q) \Rightarrow (q \rightarrow p)$

a) If I will ski tomorrow, then it will snow today.

b) There is going to be a quiz whenever I come to class.

Contrapositive:  $(p \rightarrow q) \Rightarrow (\neg q \rightarrow \neg p)$

a) If I won't ski tomorrow, it doesn't snow today.

b) There is not going to be a quiz unless I come to class.

Inverse  $(p \rightarrow q) \Rightarrow (\neg p \rightarrow \neg q)$

- a) If it doesn't snow today, I won't ski tomorrow.
- b) I won't come to class unless there is going to be ~~a~~ quiz.

Q.no.8 ans

- a) If it rains tonight, then I will watch movie.
- b) I go to beach whenever it is a sunny summer day.

Converse:

- a) If I will watch movie, it rains tonight.
- b) It is a sunny summer day, <sup>whenever</sup> I go to the beach

Contrapositive:

- a) If I won't watch movie, then it doesn't rain tonight.
- b) ~~If~~ It is never sunny summer day when I don't watch movie.

Q.no.9

Truth table

P	q	$\neg P$	$P \wedge \neg P$
T	F	F	F
F	T	T	F

b)  $P \vee \neg P$

$P$	$\neg P$	$P \vee \neg P$
T	F	T
F	T	T

c)  $(P \vee \neg q) \rightarrow q$

$P$	$q$	$\neg q$	$(P \vee \neg q)$	$(P \vee \neg q) \rightarrow q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

d)  $(P \vee q) \rightarrow (P \wedge q)$

$P$	$q$	$(P \vee q)$	$(P \wedge q)$	$(P \vee q) \rightarrow (P \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

e)  $(P \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg P)$

$P$	$q$	$\neg P$	$\neg q$	$P \rightarrow q$	$\neg q \rightarrow \neg P$	$(P \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg P)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

$$f) (p \rightarrow q) \rightarrow (q \rightarrow p)$$

$p$	$q$	$(p \rightarrow q)$	$(q \rightarrow p)$	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	F
F	F	T	T	T

Q.no.10

a)  $(p \vee q) \rightarrow (p \oplus q)$

Here,

$p$	$q$	$p \vee q$	$p \oplus q$	$(p \vee q) \rightarrow (p \oplus q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

b)  $(p \oplus q) \rightarrow (p \wedge q)$

$p$	$q$	$p \oplus q$	$p \wedge q$	$(p \oplus q) \rightarrow (p \wedge q)$
T	T	F	T	T
F	T	T	F	F
T	F	T	F	F
F	F	F	F	T

c)  $(p \vee q) \oplus (p \wedge q)$

$p$	$q$	$p \vee q$	$(p \wedge q)$	$(p \vee q) \oplus (p \wedge q)$
T	T	T	T	F
T	F	T	F	T
F	T	T	F	T
F	F	F	F	F

d)  $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$

$p$	$q$	$\neg p$	$p \leftrightarrow q$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
T	T	F	T	F	T
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	T	F	T

e)  $(p \oplus q) \rightarrow (p \oplus \neg q)$

$p$	$q$	$\neg q$	$(p \oplus q)$	$(p \oplus \neg q)$	$(p \oplus q) \rightarrow (p \oplus \neg q)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

Q.no. II ans

a)  $(P \vee q) \vee r$

P	q	r	$P \vee q$	$(P \vee q) \vee r$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

b)  $(P \vee q) \wedge r$

P	q	r	$P \vee q$	$(P \vee q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

c)  $(p \wedge q) \vee r$

$p$	$q$	$r$	$(p \wedge q)$	$(p \wedge q) \vee r$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	T
T	F	F	F	F
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

d)  $(p \wedge q) \wedge r$

$p$	$q$	$r$	$p \wedge q$	$(p \wedge q) \wedge r$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

$$e) (P \vee q) \wedge \neg r$$

P	q	r	$\neg r$	$P \vee q$
T	T	T	F	T
T	T	F	T	T
T	F	T	F	T
T	F	F	T	T
F	T	T	F	T
F	T	F	T	T
F	F	F	F	F
F	F	F	T	F

$$(P \vee q) \wedge \neg r$$

P	q	$\neg r$
T	F	T
F	F	T
T	T	F
F	T	F
F	F	F

$$f) (P \wedge q) \vee \neg r$$

P	q	r	$\neg r$	$P \wedge q$	$(P \wedge q) \vee \neg r$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	F	F
F	T	T	F	F	F
F	T	F	T	F	T
F	F	T	F	F	F
F	F	F	T	F	T

$$(P \wedge q) \vee \neg r$$

P	q	$\neg r$
T	F	T
F	F	T
T	T	F
F	T	F
F	F	F

$$g) (p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$$

$p$	$q$	$r$	$p \leftrightarrow q$	$\neg p$	$\neg r$	$\neg p \leftrightarrow \neg r$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$
T	T	T	T	F	F	T	F
T	T	F	T	F	T	F	T
T	F	T	F	F	F	T	T
T	F	F	F	F	T	F	F
F	T	T	F	T	F	T	F
F	T	F	F	T	T	F	T
F	F	T	T	T	F	T	T
F	F	F	F	T	T	F	F

iii) p

q

r

iv) A

C

re

Q.no.12 i)  $p =$  "you can have voting right."  
 $q =$  "you are mentally unfit."  
 $r =$  "you are over 18 years."  
 $\Rightarrow$  relation:  $(q \wedge \neg r) \rightarrow \neg p$

ii)  $p =$  "leaders will make correct decision."  
 $q =$  "you choose a good leader".  
 $r =$  "you raise your voice against incorrect decision."

relation:  $p \rightarrow (q \vee r)$

Q.no

iii)  $p = \text{"You can see the movie."}$

$q = \text{"You are over 18 years old."}$

$r = \text{"You have permission of a parent."}$

relation:  $p \rightarrow (q \vee r)$

iv)  $p = \text{"You can graduate."}$

$q = \text{"You have completed the requirement of your major."}$

$r = \text{"You } \cancel{\text{do not}} \text{ owe money to university."}$

$s = \text{"You } \cancel{\text{do not}} \text{ have an overdue library book."}$

relation:  $p \rightarrow (q \wedge r \wedge s)$

Q.no. B ans.

a)  $q \rightarrow p$

b)  ~~$\neg q \rightarrow q \rightarrow \neg p$~~

c)  $q \rightarrow p$

d)  $\neg q \rightarrow \neg p$