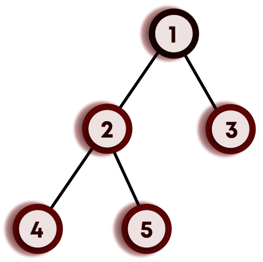
Graph traversal algorithms

Traversing a graph means examining the nodes and edges of the graph. There are two standard methods of graph traversal which we will discuss in this section. These two methods are -

a) Depth-first search
b) Breadth-first search.

While breadth-first search uses a **queue** as an auxiliary data structure to store nodes for further processing, the depth-first search scheme uses a **stack**. But both these algorithms make use of a bool variable **VISITED**. During the execution of the algorithm, every node in the graph will have the variable VISITED set to **false** or **true**, depending on its **current state,** whether the node has been processed/visited or not.

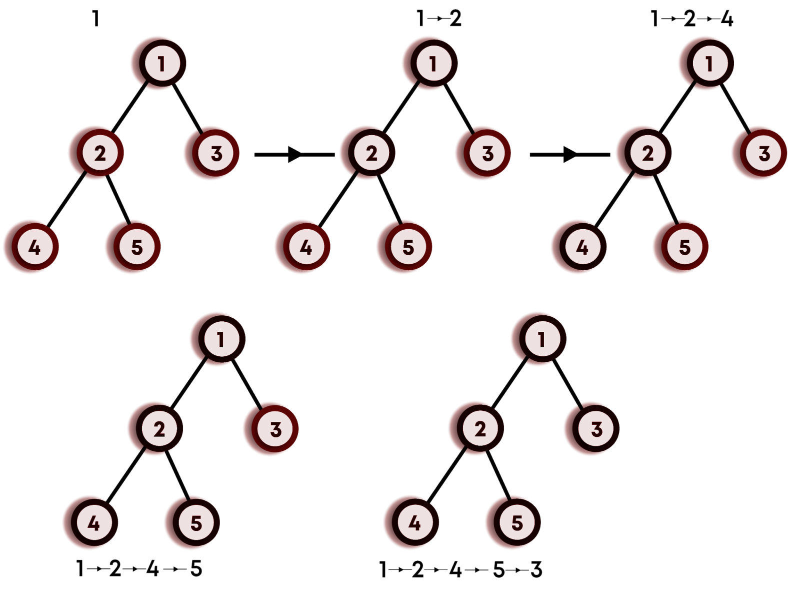
- **Depth-first search (DFS)**
  The **depth-first search(DFS)** algorithm, as the name suggests, first goes into the depth and then recursively does the same in other directions, it progresses by expanding the starting node of G and then going deeper and deeper until the goal node is found, or until a node that has no children is encountered. When a dead-end is reached, the algorithm backtracks, returning to the most recent node that has not been completely explored.
  In other words, the depth-first search begins at a starting node A which becomes the current node. Then, it examines each node along with a path P which begins at A. That is, we process a neighbor of A, then a neighbor of the processed node, and so on. During the execution of the algorithm, if we reach a path that has a node that has already been processed, then we backtrack to the current node. Otherwise, the unvisited (unprocessed) node becomes the current node.
  For example: DFS for the below graph is:



  DFS Traversal : 1->2->4->5->3
  Other possible DFS traversals for the above graph can be:
  1. 1->3->2->4->5
  2. 1->2->5->4->3
  3. 1->3->2->5->4

  We can clearly observe that there can be more than one DFS traversals for the same graph.

DFS



Implementation of DFS (Iterative) :

```
function DFS_iterative(graph,source)

        /*
                Let St be a stack, pushing source vertex in the stack.
                St represents the vertices that have been processed/visited
                so far.
        */

        St.push(source)

        //   Mark source vertex as visited.
        visited[source] = true

        //   Iterate through the vertices present in the stack

        while St is not empty
                //   Pop a vertex from the stack to visit its neighbors
                cur = St.top()
                St.pop()

                /*
                        Push all the neighbors of the cur vertex that have not been
                        visited yet, push them into the stack and mark them as
                        visited.
                */
```

```
            for all neighbors v of cur in graph:
                if visited[v] is false
                    St.push(v)
                    visited[v] = true

    return
```

Implementation of DFS (Recursive)

```
function DFS_recursive(graph,cur)

    //   Mark the cur vertex as visited.
    visited[cur] = true

    /*
        Recur for all the neighbors of the cur vertex that have not been
        visited yet.
    */

    for all neighbors v of cur in graph:
        if visited[v] is false
            DFS_recursive(graph,v)
    return
```

Features of Depth-First Search Algorithm

- **Time Complexity:** The time complexity of a depth-first search is proportional to the number of vertices plus the number of edges in the graphs that are traversed. The time complexity can be given as **(O(|V| + |E|))**, where **|V|** is the number of vertices and **|E|** is the number of edges in the graph considering the graph is represented by adjacency list.
- **Completeness:** Depth-first search is said to be a **complete** algorithm in the case of a finite graph. If there is a solution, a depth-first search will find it regardless of the kind of graph. But in the case of an infinite graph, where there is no possible solution, it will diverge.

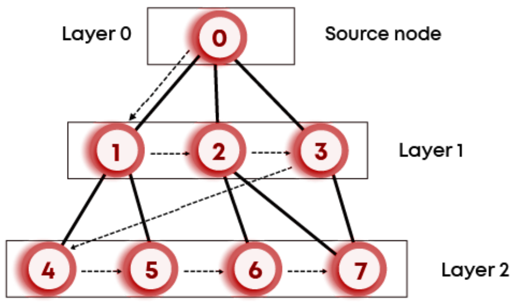
Applications of Depth-First Search Algorithm

- Finding a path between two specified nodes, u and v, of an unweighted graph.
- Finding a path between two specified nodes, u and v, of a weighted graph.
- Finding whether a graph is connected or not.
- Computing the spanning tree of a connected graph

- **Breadth-first search (BFS)**
  Breadth-first search(BFS) is a graph search algorithm where we start from the selected node and traverse the graph level-wise or layer-wise, thus exploring the neighbor nodes (which are directly connected to the starting node), and then moving on to the next level neighbor nodes.

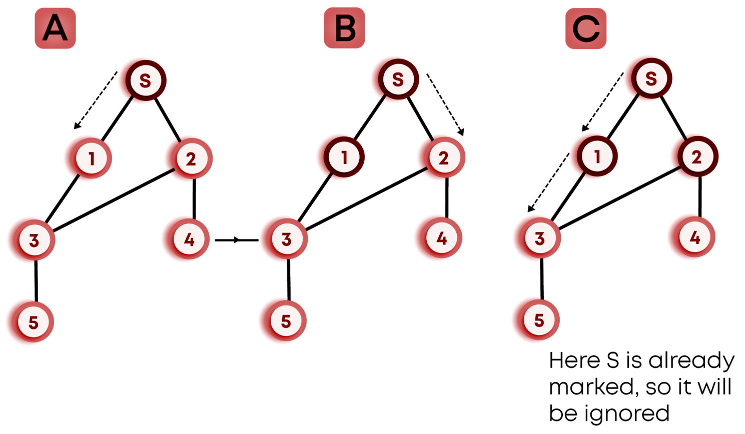
  As the name suggests:

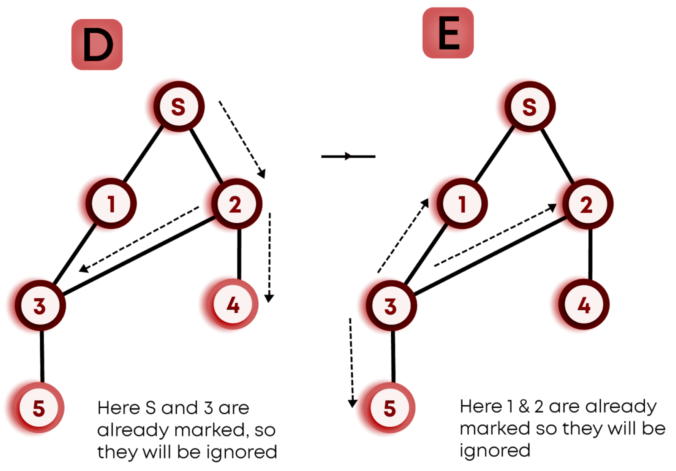
  - We first move horizontally and visit all the nodes of the current layer.
  - Then move to the next layer.



BFS Traversal: 0->1->2->3->4->5->6->7
Other possible BFS traversals for the above graph can be:
1. 0->3->2->1->7->6->5->4
2. 0->1->2->3->5->4->7->6

We can clearly observe that there can be more than one BFS traversals for the same graph, as shown for the above graph.
That is, we start examining say node A and then all the neighbors of A are examined. In the next step, we examine the neighbors of A, so on, and so forth. This means that we need to track the neighbors of the node and guarantee that every node in the graph is processed and no node is processed more than once. This is accomplished by using a queue that will hold the nodes that are waiting for further processing and a variable VISITED to represent the current state of the node.
For example: BFS for the below graph is:




D
E
S
1
2
3
4
5
Here S and 3 are already marked, so they will be ignored
Here 1 & 2 are already marked so they will be ignored

Implementation of BFS

```
function BFS(graph,source)

        /*
                Let Q be a queue, pushing source vertex in the queue.
               Q represents the vertices that have not been processed
               /visited so far, and their neighbors have not been processed.
        */

        Q.enqueue(source)
        visited[source] = true

        //  Iterate through the vertices in the queue.
        while Q is not empty
                //  Pop a vertex from the queue to visit its neighbors
                cur = Q.front()
```

```
            Q.dequeue()

            /*
                Push all the neighbors of the cur vertex that have not been
                visited yet, push them into the queue and mark them as
                visited.
            */

            for all neighbors v of cur in graph:
                if visited[v] is false
                    Q.enqueue(v)
                    visited[v] = true

        return
```

Features of Breadth-First Search Algorithm

- **Time Complexity:** The time complexity of a breadth-first search is proportional to the number of vertices plus the number of edges in the graphs that are traversed. The time complexity can be given as **$(O(|V| + |E|))$**, where **$|V|$** is the number of vertices in the graph and **$|E|$** is the number of edges in the graph, considering the graph is represented by adjacency list.
- **Completeness:** Breadth-first search is said to be a **complete** algorithm in the case of a finite graph because if there is a solution, the breadth-first search will find it regardless of the kind of graph. But in the case of an infinite graph where there is no possible solution, it will diverge.

Applications of Breadth-First Search algorithm

- Finding all connected components in a graph G.
- Finding all nodes within an individual connected component
- Finding the shortest path between two nodes, u and v, of an unweighted graph. (The shortest path is in terms of the minimum number of **moves** required to visit v from u or vice-versa).

- **NOTE**
  - The DFS and BFS algorithms work for both **directed** and **undirected** graphs, **weighted** and **unweighted** graphs, **connected** and **disconnected** graphs.
  - For disconnected graphs, for traversing the whole graph, we need to call DFS/BFS for each **unvisited** vertex.

- For getting the number of connected components in a graph, the number of times we need to call DFS/BFS traversal for the graph on an **unvisited** vertex gives the number of connected components in the graph.