Comparing POMP to Birth-Death-Immigration ODE

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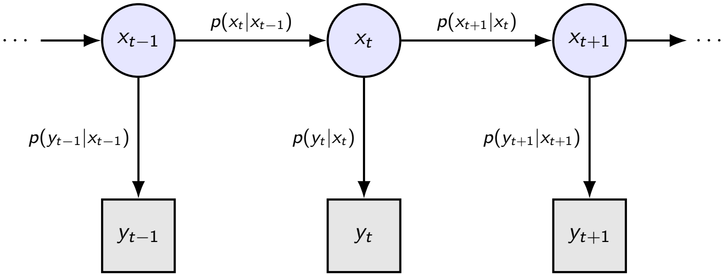
State Space Model Structure



Top Layer: Latent process evolving via transition density.

$$p(x_t \mid x_{t-1}) \sim \frac{1}{2N_t}\text{Binomial}(2N_t, x_{t-1}), \quad x_t \in [0,1] \tag{1}$$

Bottom Layer: Observation process conditioned on latent state.

$$p(y_t \mid x_t) \sim \text{Binomial}(n, x_t), \quad y_t \in \{0, \ldots, n\} \tag{2}$$

Approximate Derivation of the BDI

The ordinary differential equation for the birth-death-immigration process is

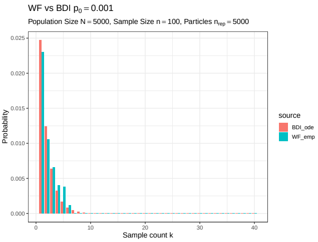
$$\begin{aligned} \frac{d}{dt}p_k(t) &= \big(f + (k-1)b(t)\big)p_{k-1}(t) \\ &\quad - \big(f + k(b(t)+d(t))\big)p_k(t) \\ &\quad + (k+1)d(t)p_{k+1}(t), \quad k \geq 0 \end{aligned} \tag{3}$$

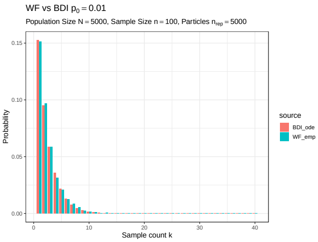
This is derived from

$$p_k(t) = \int_0^1 \underbrace{\binom{n}{k} x^k (1-x)^{n-k}}_{\text{Poisson approximation}} \underbrace{p_t(x)}_{\text{Diffusion}} \, dt \tag{4}$$

which makes the integration analytic, hence the recursion in equation (3).

Results


WF vs BDI $p_0 = 0.001$
Population Size $N = 5000$, Sample Size $n = 100$, Particles $n_{rep} = 5000$
Probability
Sample count k
source
BDI_ode
WF_emp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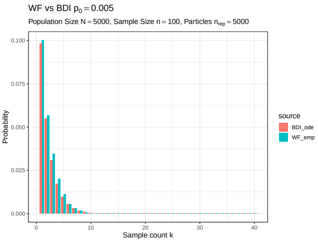
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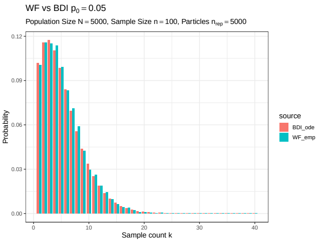
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Thank you for listening

Questions?