

ON MODELING 3D JET GEOMETRY

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ABSTRACT

Optical and radio images of the M87 jet show a huge variety of parsec-scale bends and helical distortion from HST-1 to knot C. The sinusoidal pattern in the outer jet is observed in both bands, suggesting a possible double helical structure. We developed a mathematical model that converts the observed 2D projection of the jet to a 3D configuration by using three inputs: the viewing angle (estimated from 20 years of HST monitoring of the jet), distances and relative angles between bends measured from the HST optical and VLA/VLBA radio images of the M87 jet. Our code is written in Python, combining Markov Chain Monte Carlo (MCMC) methods and computer graphics to model and demonstrate the jet geometry. By understanding the real geometry we will be able to disentangle details regarding jet kinematics and dynamics, as well as the magnetic field structure and particle acceleration mechanisms. This code is still in the testing phase as we try to understand its numerical behavior and instabilities, as well as how it responds to various line of sight angles, bend types and structural complexities. Our hope is to be able to plug in an image and from it constrain parameters for various wiggles and bends. We hope to extend the work to a variety of jets in the future.

Subject headings: galaxies: jets — galaxies: jets: individual(M87)

1. INTRODUCTION

2. MODEL

We consider part of an active galaxy jet with a viewing angle θ , which has a bend structure (Fig. 1). O, A, B are neighboring knots in the jet frame, while A' and B' are the projections of knot A and B in the sky frame. The distance between A and B in the jet frame is d , and the apparent distance between A' and B' is s . The projected bend angle relative to the x-axis (jet axis) is represented as η .

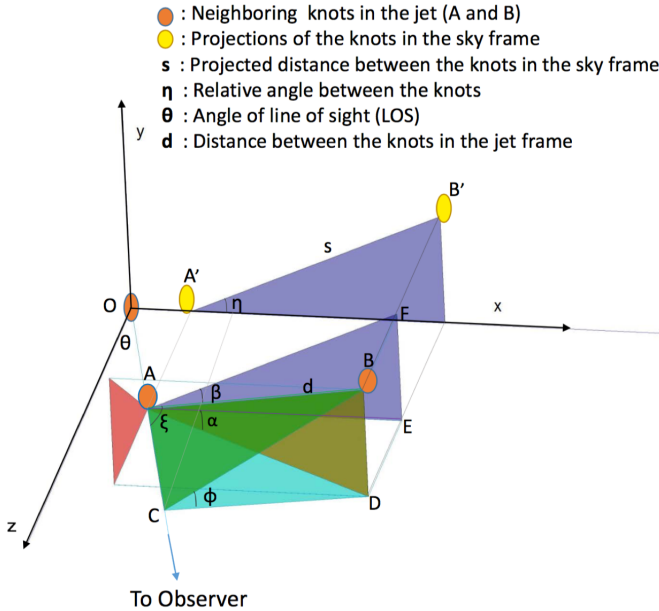


FIG. 1.— Local jet 3D structure model when $\xi < \pi/2 - \theta$.

Set ξ as angle BAC, when $\xi < \pi/2 - \theta$, the equation set that describes the local jet geometry is derived below: In $\triangle A'B'D'$,

$$\tan \eta = \frac{\sin(\xi) \sin(\phi)}{\cos(\xi) \sin(\theta) + \sin(\xi) \cos(\phi) \cos(\theta)} \quad (1)$$

$$\xi = \text{angle} BAC.$$

$$\phi = \text{angle} BCD.$$

$$\theta = \text{the viewing angle of the jet.}$$

$$\eta = \text{the angle } B'A'D'.$$

Because $\triangle AEF$ is the projection of $\triangle ABD$ onto the xy-plane (sky frame), and $\triangle ADE$ is the projection of $\triangle ABF$ onto the xz-plane. In right $\triangle AEF$,

$$AF = s = d \cos(\beta) \quad (2)$$

Because $\triangle AGH$ is the projection of $\triangle ABD$ onto the yz-plane, and $\triangle AHD$ is the projection of $\triangle ABG$ onto the xz-plane. In right triangle ABG:

$$BG = HD = A'D' = s \cos(\eta)$$

$$s \cos(\eta) = d \sin(\gamma)$$

(3)

$$\gamma = \text{angle} BAG.$$

In right triangle AGH:

$$\begin{aligned}
AG^2 &= AH^2 + GH^2 \\
AH &= DE = s \cos(\eta) \tan(\alpha) \\
GH &= BD = B'D' = s^* \sin(\eta)
\end{aligned}$$

$$(d \cos(\gamma))^2 = (s \sin(\eta))^2 + (s \cos(\eta) \tan(\alpha))^2 \quad (4)$$

Equation (4) can be simplified by using equation (2):

$$\cos^2(\gamma) = \cos^2(\beta) \sin^2(\eta) + \cos^2(\beta) \cos^2(\eta) \tan^2(\alpha) \quad (5)$$

Equation (3) and (5) can be combined into:

$$\left(\frac{\tan(\beta)}{\tan(\alpha)}\right)^2 = \cos^2(\eta) \quad (6)$$

In right triangle CID:

$$CI = DC \sin(\theta) = d \sin(\xi) \cos(\phi) \sin(\theta)$$

and $AC \cos(\theta) = AH + CI$, so:

$$d \cos(\xi) \cos(\theta) = s \cos(\eta) \tan(\alpha) + d \sin(\xi) \cos(\phi) \sin(\theta) \quad (7)$$

Also, in right triangle ABC, $AB^2 = AC^2 + BC^2$, so:

$$d^2 = s^2 \left[\frac{\sin(\eta)}{\sin(\phi)}\right]^2 + s^2 \left[\frac{\cos(\eta)}{\cos(\alpha)}\right]^2 \sin^2(\theta + \alpha) \quad (8)$$

Equation (1), (2), (6), (7), and (8) form an equation set of five equations and five unknowns: $\alpha, \beta, \phi, \xi, d$, while η and s can be measured from the image of jet, and θ , the viewing angle, can be estimated from other properties of the jet.

$$d^2 = s^2 \left[\frac{\sin(\eta)}{\sin(\phi)}\right]^2 + s^2 \left[\frac{\cos(\eta)}{\cos(\alpha)}\right]^2 \sin^2(\theta + \alpha)$$

$$\left(\frac{\tan(\beta)}{\tan(\alpha)}\right)^2 = \cos^2(\eta)$$

$$d \cos(\xi) \cos(\theta) = s \cos(\eta) \tan(\alpha) + d \sin(\xi) \cos(\phi) \sin(\theta)$$

$$\tan \eta = \frac{\sin(\xi) \sin(\phi)}{\cos(\xi) \sin(\theta) + \sin(\xi) \cos(\phi) \cos(\theta)}$$

$$s = d \cos(\beta)$$

When the local jet structure has large ξ , that is when $\xi \geq \pi/2 - \theta$ (Fig.2), the equation set that describes the local jet geometry is slightly different:

$$d^2 = s^2 \left[\frac{\sin(\eta)}{\sin(\phi)}\right]^2 + s^2 \left[\frac{\cos(\eta)}{\cos(\alpha)}\right]^2 \sin^2(\theta - \alpha)$$

$$\left(\frac{\tan(\beta)}{\tan(\alpha)}\right)^2 = \cos^2(\eta)$$

$$d \cos(\xi) \cos(\theta) + s \cos(\eta) \tan(\alpha) = d \sin(\xi) \cos(\phi) \sin(\theta)$$

$$\tan \eta = \frac{\sin(\xi) \sin(\phi)}{\cos(\xi) \sin(\theta) + \sin(\xi) \cos(\phi) \cos(\theta)}$$

$$s = d \cos(\beta)$$

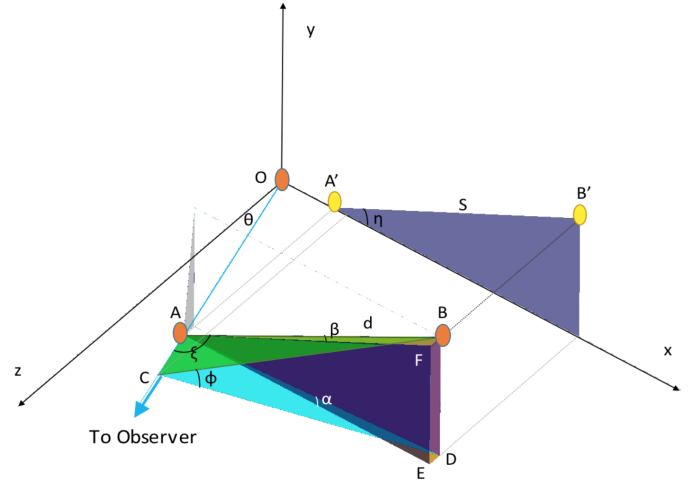


FIG. 2.— Local jet 3D structure model when $\xi \geq \pi/2 - \theta$.

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