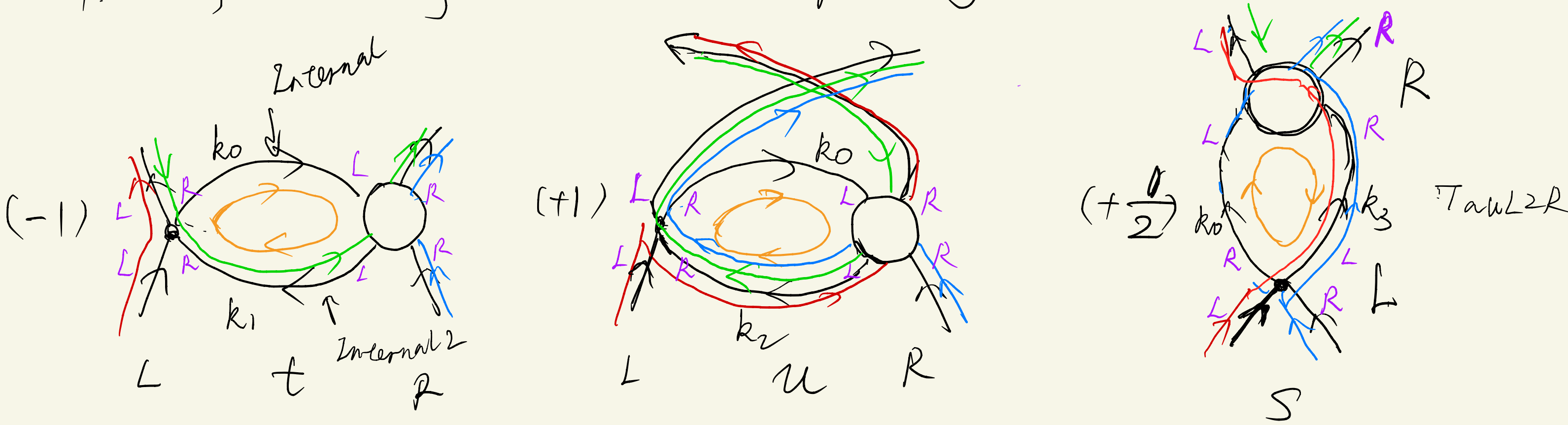
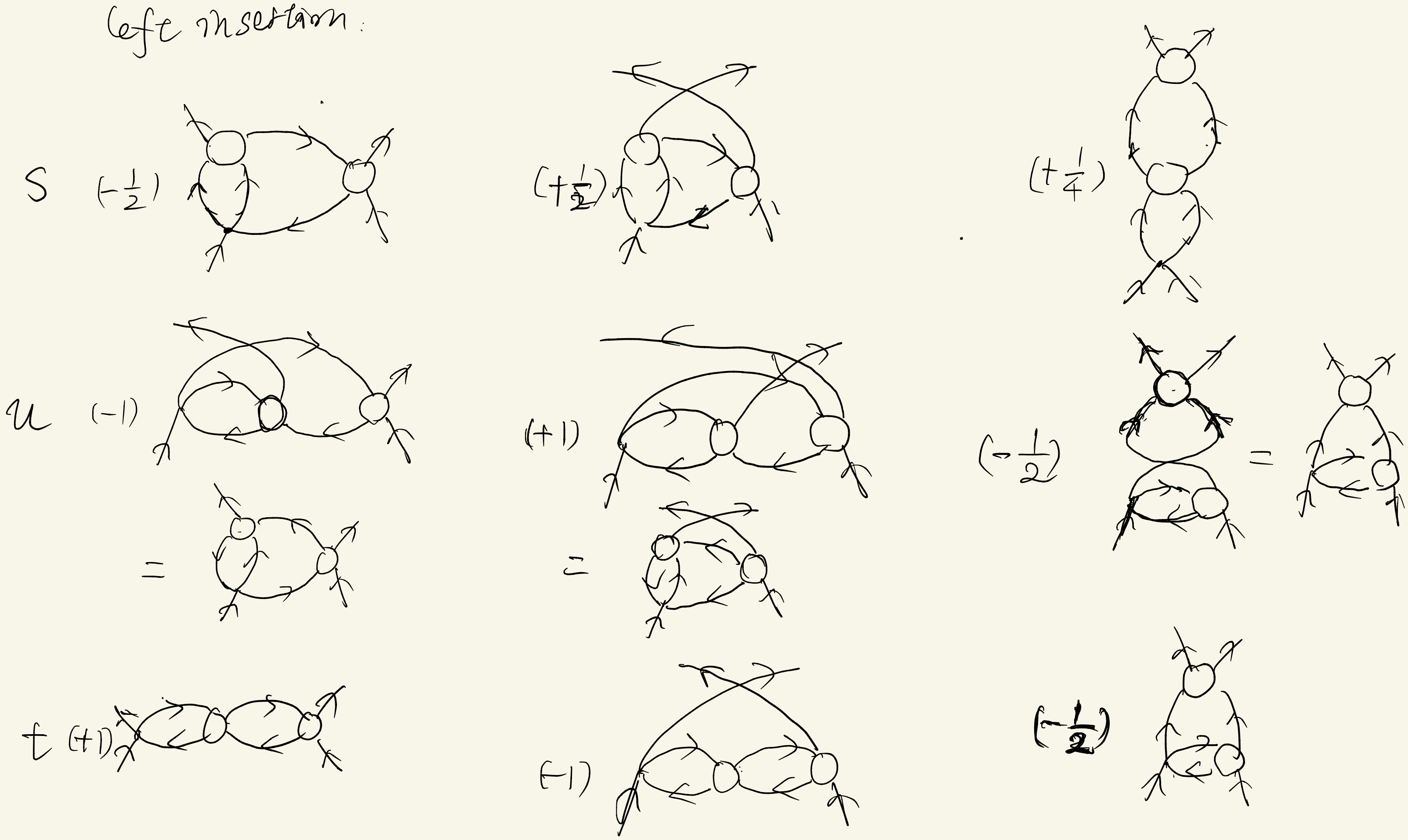
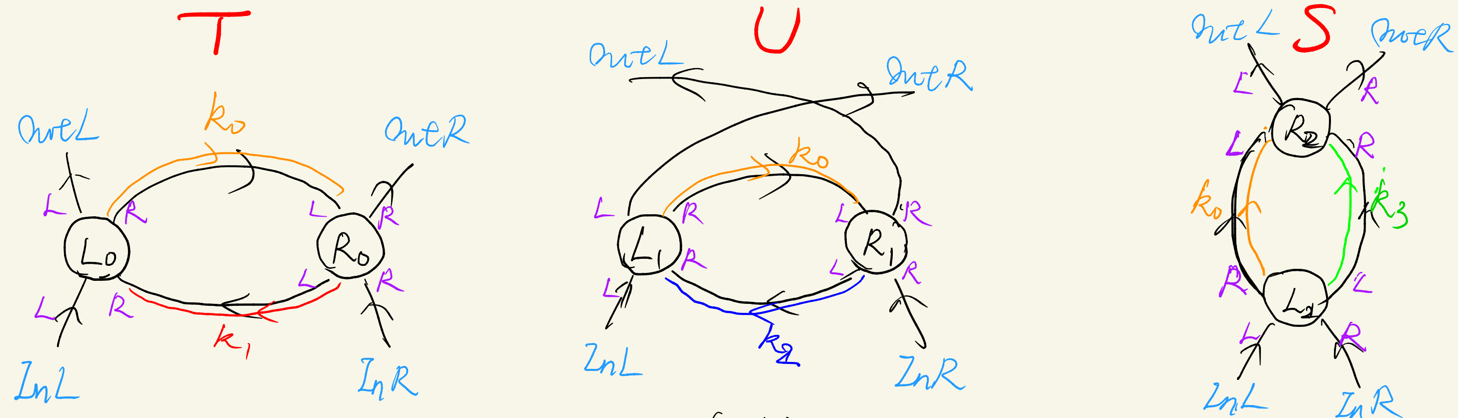
1. Definition of the one-loop diagram.



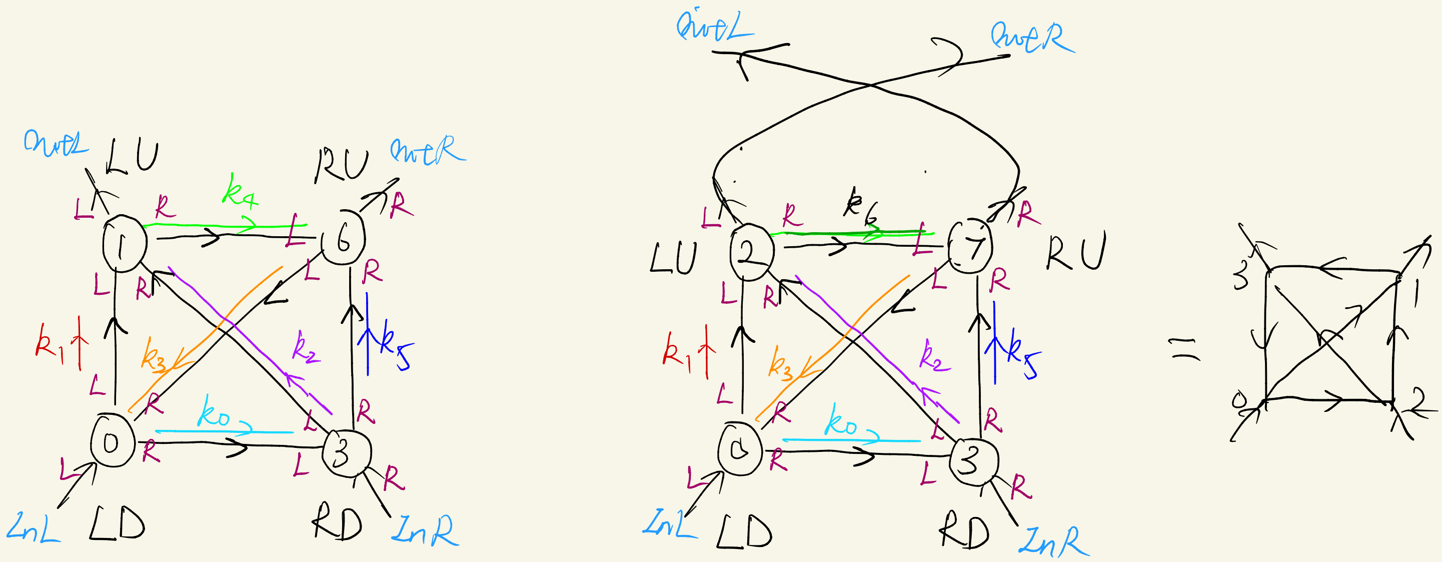
left insertion:



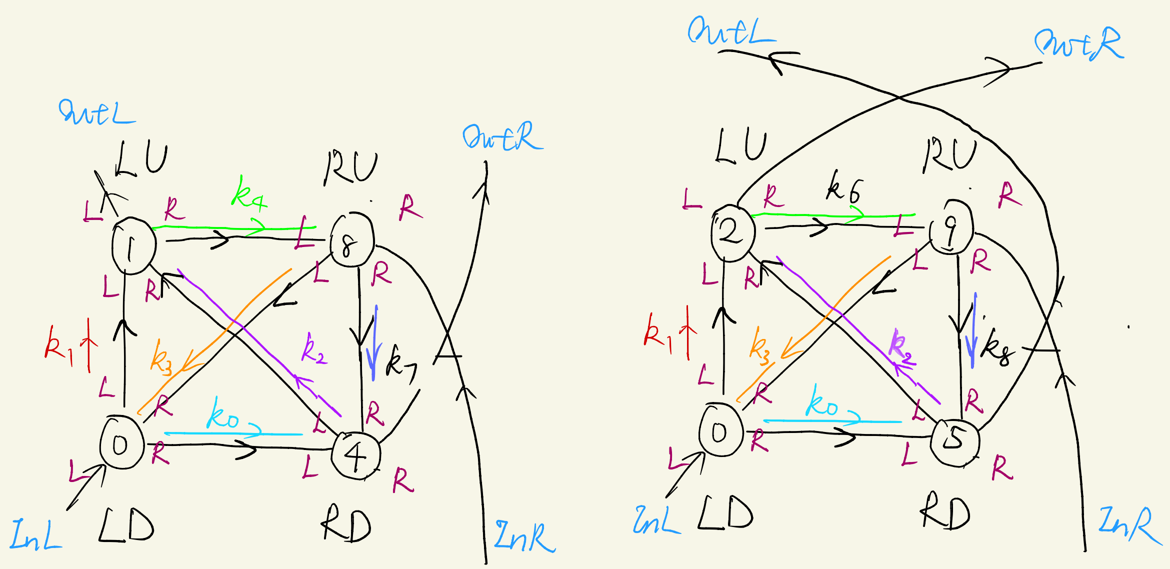
2.174



(+1) (−1) (−1/2)

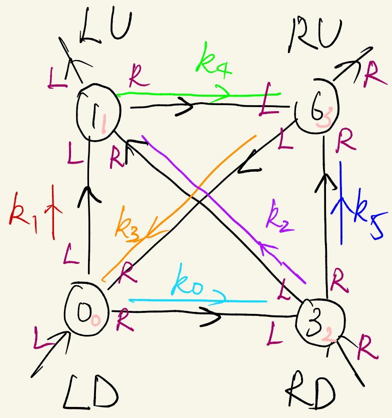


(1) (−1, 0) (2) (+1, 0)

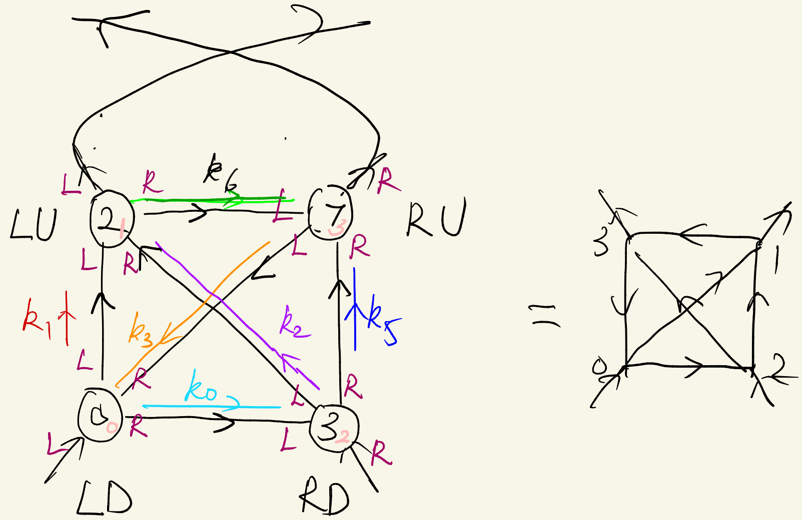


(3) (−1, 0) (4) (+1, 0)

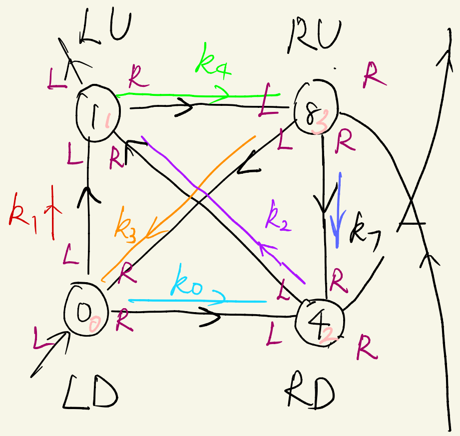
All LD, LU, RD, RU share the same set of Tau variables



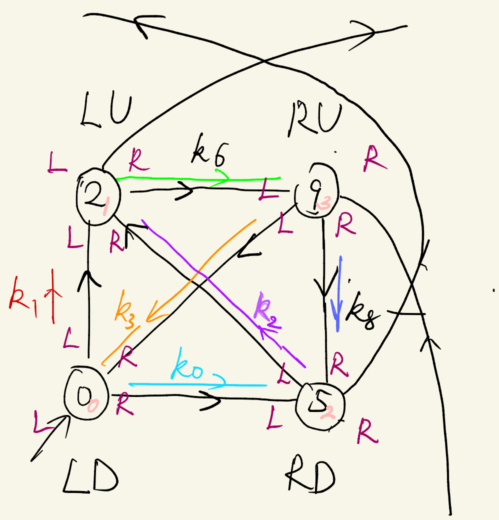
(1) (-1,0)



(2) (+1,0)



(3) (-1,0)



(4) (+1,0)

All LD, LU, RD, RU share the same set of Tau variables

Diagram 1 and 2

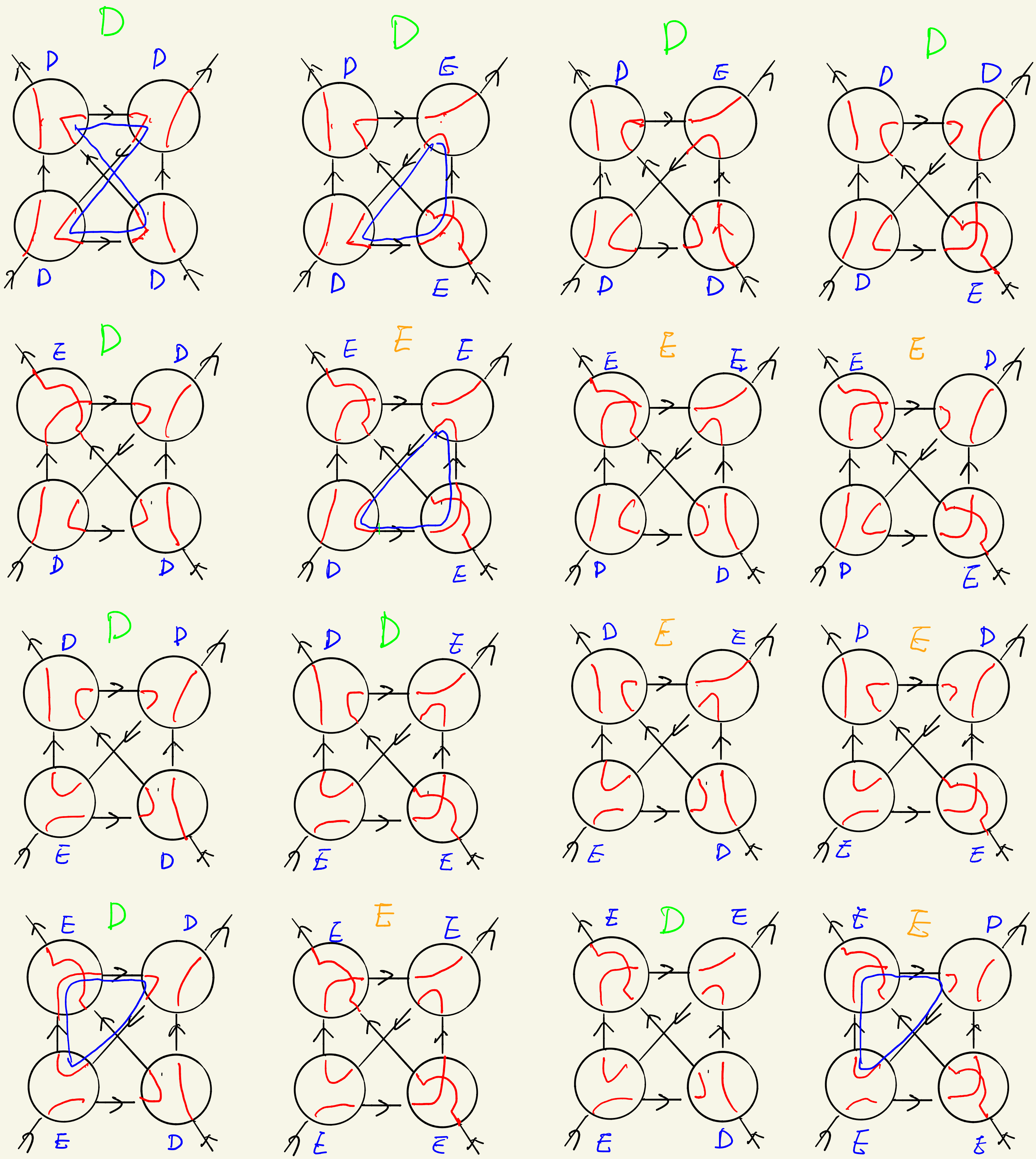
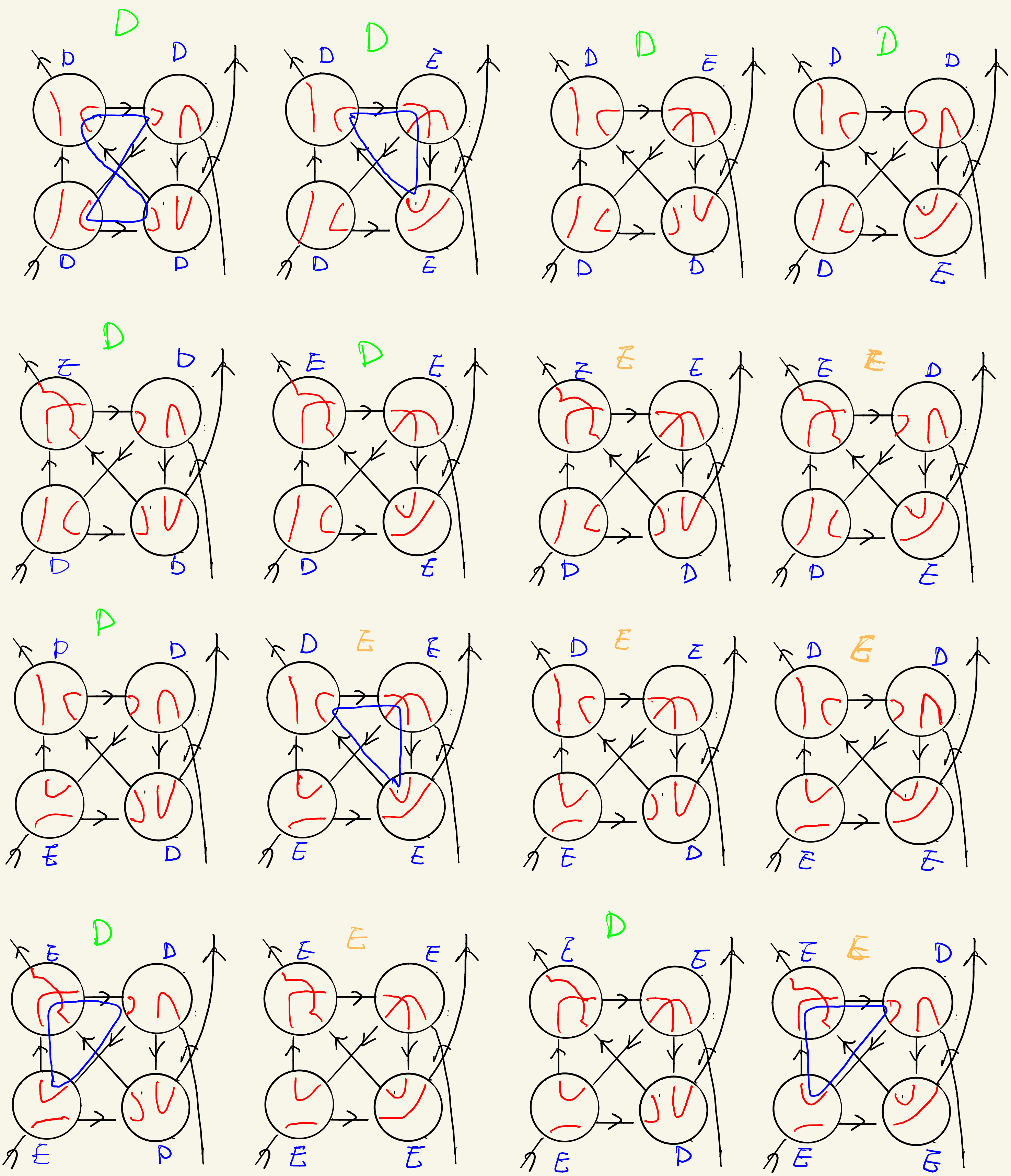
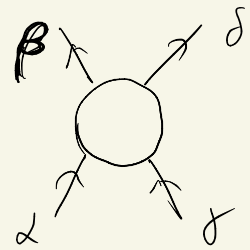


Diagram 3.4



Spin Index

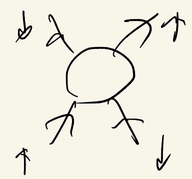


$$A = A_s \cdot \delta_{\alpha\beta}\delta_{\gamma\delta} + A_a \cdot \vec{\sigma}_{\alpha\beta}\cdot\vec{\sigma}_{\gamma\delta}$$

$$= \Gamma^{dir}\cdot\delta_{\alpha\beta}\delta_{\gamma\delta} + \Gamma^{ex}\delta_{\alpha\delta}\delta_{\gamma\beta}$$

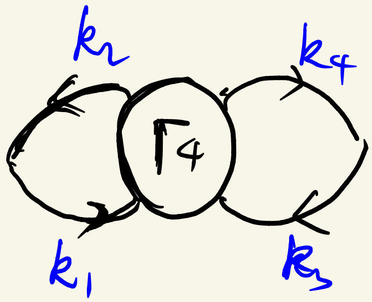
$$\Gamma_{\substack{\uparrow\uparrow\uparrow\uparrow \\ \alpha\beta\gamma\delta}} = A_s + A_a = \Gamma^{dir} + \Gamma^{ex}$$

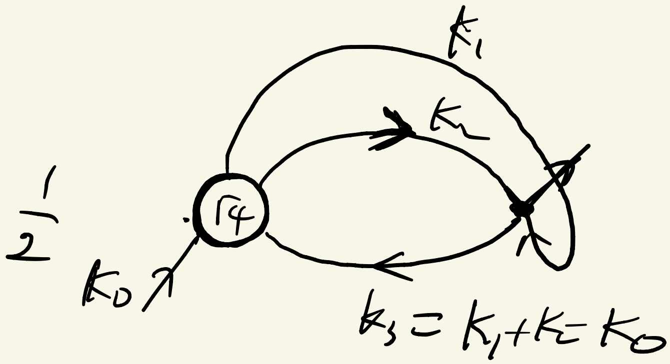
$$\Gamma_{\substack{\uparrow\uparrow\downarrow\downarrow \\ \alpha\beta\gamma\delta}} = A_s - A_a = \Gamma^{dir}$$

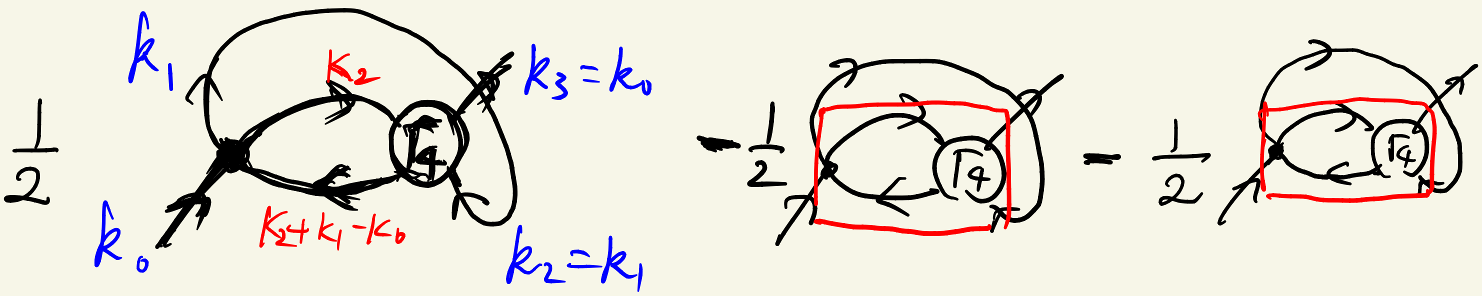


$$\Gamma_{\substack{\uparrow\downarrow\downarrow\uparrow \\ \alpha\beta\gamma\delta}} = +2A_a = \Gamma^{ex}$$

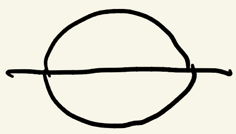
$$A_s = \Gamma^{dir} + \tfrac{1}{2}\Gamma^{ex}, \quad A_a = \tfrac{1}{2}\Gamma^{ex}$$







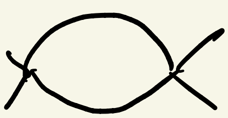
Note: k_1 and k_2 are symmetric



$$= \int \frac{d^Dp_1}{(2\pi)^D}\frac{d^Dp_2}{(2\pi)^D}\frac{1}{p_1^2+m^2}\frac{1}{p_2^2+m^2}\cdot\frac{1}{(q+p_1-p_2)^2+m^2}$$

$$= -\frac{1}{D-3}\left[3m^2A(q)+B(q)\right]$$

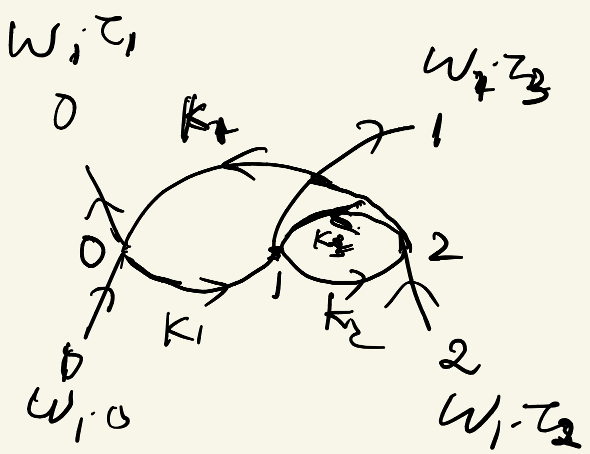
$$A(q) = \int\frac{d^Dp_1}{(2\pi)^D}\frac{d^Dp_2}{(2\pi)^D}\frac{1}{(p_1^2+m^2)(p_2^2+m^2)[(q+p_1+p_2)^2+m^2]^2}$$



$$= \int\frac{d^Dp}{(2\pi)^D}\frac{1}{(p^2+m^2)}\cdot\frac{1}{(p+k)^2+m^2}$$

$$= \frac{1}{(4\pi)^{D/2}}\frac{\Gamma(2-\frac{D}{2})}{\Gamma(2)}\int_0^1 dx\,\frac{1}{[k^2x(1-x)+m^2]^{2-D/2}}$$

$$\int\frac{d^3k}{(2\pi)^3}\frac{1}{(k^2+1)^2} = \frac{4\pi}{8\pi^3}\int_0^{+\infty}\frac{k^2dk}{(k^2+1)^2} = \frac{4\pi}{8\pi^3}\cdot\frac{\pi}{4} = \frac{1}{8\pi}$$



$$= F(\tau_1-\tau_0,\ \tau_2-\tau_0,\ \tau_3-\tau_0)$$

$$F(\tau_0,\tau_1,\tau_2,\tau_3)\,e^{-i\omega_0\tau_0}$$

$$\int d\tau_0 d\tau_1 d\tau_2\, F(\tau_0,\tau_1,\tau_2,\tau_3)\,e^{-i\omega_0\tau_0+i\omega_0\tau_1-i\omega_0\tau_2+i\omega_0\tau_3}$$

static source, thus τ needs to be integrated

$$F_0(\tau_0) = \int_0^\beta d\tau\, G(\tau)\, G(\tau-\tau_0) \qquad [0<\tau_0<\beta]$$

$$= \int_0^{\tau_0} d\tau\, G(\tau)\, G(\tau-\tau_0) + \int_{\tau_0}^\beta G(\tau)\, G(\tau-\tau_0)$$

$$= \int_0^{\tau_0} d\tau\, (1-n_k) e^{-\varepsilon_k \tau} (-n_k) e^{+\varepsilon_k(\tau-\tau_0)}$$

$$+ \int_{\tau_0}^\beta (1-n_k) e^{-\varepsilon_k \tau} (1-n_k) e^{-\varepsilon_k(\tau-\tau_0)}$$

$$\int_{\tau_0}^\beta e^{-2\varepsilon_k \tau} d\tau\, e^{\varepsilon_k \tau_0}$$

$$= \frac{1}{2\varepsilon_k} e^{-2\varepsilon_k \tau}\Big|_{\tau_0}^\beta e^{\varepsilon_k \tau_0}$$

$$= \frac{1}{2\varepsilon_k}\left(e^{-2\varepsilon_k \beta} - e^{-2\varepsilon_k \tau_0}\right) e^{\varepsilon_k \tau_0}$$

$$= -\tau_0 (1-n_k) n_k e^{-\varepsilon_k \tau_0} + (1-n_k)^2 \frac{e^{\varepsilon_k(\tau_0-2\beta)} - e^{-\varepsilon_k \tau_0}}{\varepsilon_k}$$

$$= -\tau_0 (1-n_k) n_k e^{-\varepsilon_k \tau_0} + \frac{1-n_k}{2\varepsilon_k} \frac{1}{1+e^{-\varepsilon_k \beta}} \left[e^{\varepsilon_k(\tau_0-2\beta)} - e^{-\varepsilon_k \tau_0}\right.$$

$$\underline{e^{-\varepsilon_k(\beta-\tau_0)}\left[e^{2\varepsilon_k(\beta-\tau_0)} - 1\right]}$$

$$= -\tau_0 (1-n_k) n_k e^{-\varepsilon_k \tau_0} + \frac{(1-n_k) n_k}{2\varepsilon_k} \left[e^{\varepsilon_k(\tau_0-\beta)} - e^{-\varepsilon_k(\tau_0-\beta)}\right.$$

$$= (1-n_k) n_k \left[-\tau_0 e^{-\varepsilon_k \tau_0} + \frac{1}{\varepsilon_k} \sinh \varepsilon_k(\tau_0-\beta)\right]$$

$$= -n_k (1-n_k) \left[\tau_0 e^{-\varepsilon_k \tau_0} + (\beta-\tau_0) \frac{\sinh \varepsilon_k(\beta-\tau_0)}{\varepsilon_k(\beta-\tau_0)}\right]$$

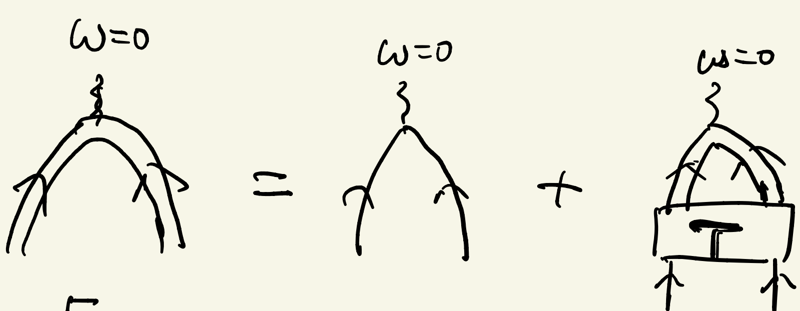
$$n_k(1-n_k)$$

$$= \frac{1}{1+e^{\beta\varepsilon_k}} \frac{1}{1+e^{-\beta\varepsilon_k}}$$

$$= \frac{1}{2+e^{\beta\varepsilon_k}+e^{-\beta\varepsilon_k}}$$

$$= \frac{1}{2(1+\cosh \beta\varepsilon_k)}$$

$$\frac{e^{-\beta\varepsilon_k}}{2e^{-\beta\varepsilon_k}+1+e^{-2\beta\varepsilon_k}}$$



F

$$e^{\varepsilon_k(\beta-\tau_0)} - e^{-\varepsilon_k(\beta-\tau_0)} \qquad e^{-\varepsilon_k\beta} e^{\varepsilon_k \tau_0} \cdot \left[e^{2\varepsilon_k(\beta-\tau_0)} - 1\right]$$

$$= e^{-\varepsilon_k(\beta-\tau_0)}\left[e^{2\varepsilon_k(\beta-\tau_0)} - 1\right]$$