Introduction to Large Language Models

Mixed Precision Training

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Full precision training and low precision training

- Large language model is difficult to train
  - take massive resource to **compute**
  - take massive resource to **store**
- Full precision training (e.g. FP32)
  - used in training most DNNs; very precise
  - takes a lot of computations and memories
- Low precision training (e.g. FP16)
  - able to train larger models due to computational and memory efficiency
  - not precise enough; overflow and underflow occur occasionally

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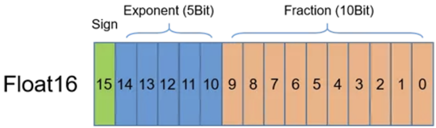

- **Sign**: 1 bit; 0 for positive and 1 for negative
- **Exponent**: 5 bits; range: 00001(1)-11110(30); value range: $2^{-14} \sim 2^{15}$

$$\text{Example:} \quad 00111(7) \quad \longrightarrow \quad 2^{7-15} = 2^{-8}$$

  where -15 is the offset
- **Fraction**: 10 bits;

$$\text{Example:} \quad 1001000000 \quad \longrightarrow \quad 1.1001000000$$
$$\longrightarrow \quad 1 + 576/1024 = 1.5625$$

  where binary 1001000000 translates into decimal 576

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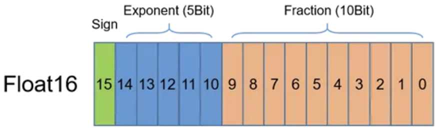

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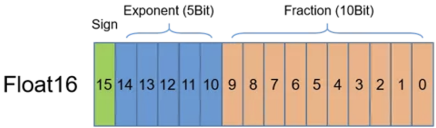

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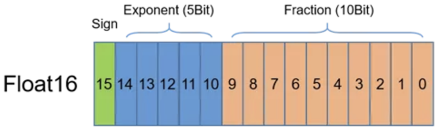

- **Translation law**

$$(-1)^{\text{sign}} \times 2^{\text{exponent}-15} \times (1 + \frac{\text{fraction}}{1024})$$

- **Largest positive number**:

$$(-1)^0 \times 2^{15} \times (1 + \frac{1023}{1024}) = 65504$$

The range of FP16 is $[-65504, +65504]$.

- **Smallest positive number**:

$$(-1)^0 \times 2^{-14} \times (1 + \frac{1}{1024}) \approx 6.1 \times 10^{-5}$$

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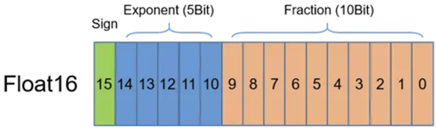

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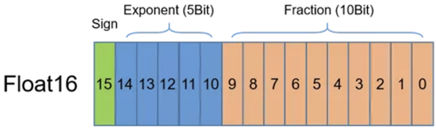

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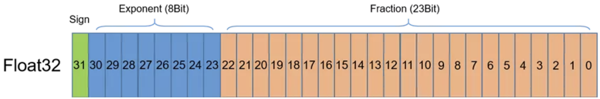
Binary	Hex	Value	Notes
0 00000 0000000000	0000	0	
0 00000 0000000001	0001	$2^{-14} \times (0 + \frac{1}{1024}) \approx 0.000000059604645$	smallest positive subnormal number
0 00000 1111111111	03ff	$2^{-14} \times (0 + \frac{1023}{1024}) \approx 0.000060975552$	largest subnormal number
0 00001 0000000000	0400	$2^{-14} \times (1 + \frac{0}{1024}) \approx 0.00006103515625$	smallest positive normal number
0 01101 0101010101	3555	$2^{-2} \times (1 + \frac{341}{1024}) \approx 0.33325195$	nearest value to 1/3
0 01110 1111111111	3bff	$2^{-1} \times (1 + \frac{1023}{1024}) \approx 0.99951172$	largest number less than one
0 01111 0000000000	3c00	$2^{0} \times (1 + \frac{0}{1024}) = 1$	one
0 01111 0000000001	3c01	$2^{0} \times (1 + \frac{1}{1024}) \approx 1.00097656$	smallest number larger than one
0 11110 1111111111	7bff	$2^{15} \times (1 + \frac{1023}{1024}) = 65504$	largest normal number
0 11111 0000000000	7c00	∞	infinity

Float 32 (FP32)



- **Translation law**

$$(-1)^{\text{sign}} \times 2^{\text{exponent}-127} \times (1 + \frac{\text{fraction}}{2^{23}})$$

- **Range**: $[-3.40282 \times 10^{38}, +3.40282 \times 10^{38}]$
- **Smallest positive number**: 1.17549×10^{-38}
- FP 32 is much more powerful than FP 16; but takes too much memory

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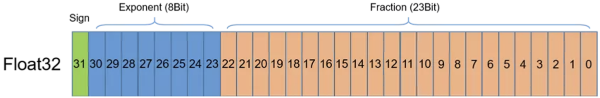

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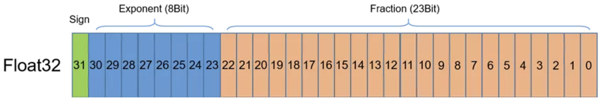

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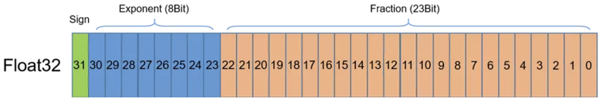

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Mixed precision training

- When both FP32 and FP16 are used in training, we get **Mixed precision training** (Micikevicius et al., 2017)
- Save memory and computations without hurting performance
- Three key techniques:
  - FP32 weight copies
  - Loss scaling
  - Arithmetic precisioin

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FP32 weight copies

- An FP32 weight copy is maintained and updated with gradient

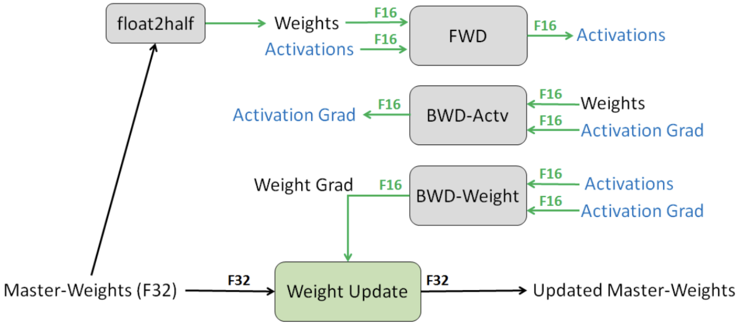

Figure 1: Mixed precision training iteration for a layer.

FP32 weight copies

- Reason I: **maintain small values in the weight update**
- Weight update = learning rate × gradient; typically very small in late phase
- Values less than $2^{-24} \approx 5.96 \times 10^{-8}$ become 0 when using FP16
- About 5% values are less than 2^{-24}

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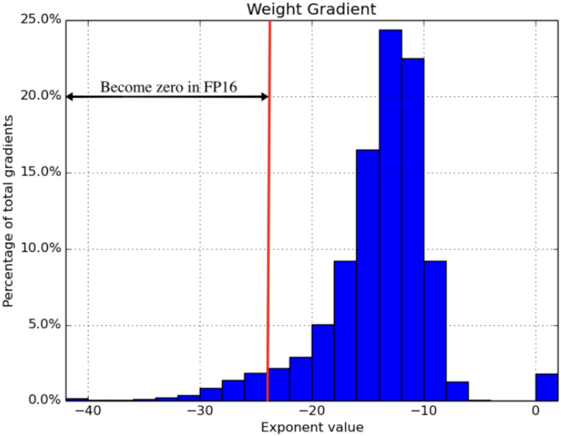

FP32 weight copies

- Reason II: **big value-to-update ratio**

[1] This figure is from wikipedia

FP32 weight copies

- Reason II: **big value-to-update ratio**
- The resolution in each period is shown as follows[1]

Min	Max	interval
0	2^{-13}	2^{-24}
2^{-13}	2^{-12}	2^{-23}
2^{-12}	2^{-11}	2^{-22}
2^{-11}	2^{-10}	2^{-21}
2^{-10}	2^{-9}	2^{-20}
2^{-9}	2^{-8}	2^{-19}
2^{-8}	2^{-7}	2^{-18}
2^{-7}	2^{-6}	2^{-17}
2^{-6}	2^{-5}	2^{-16}
2^{-5}	2^{-4}	2^{-15}
2^{-4}	$\frac{1}{8}$	2^{-14}
$\frac{1}{8}$	$\frac{1}{4}$	2^{-13}
$\frac{1}{4}$	$\frac{1}{2}$	2^{-12}
$\frac{1}{2}$	1	2^{-11}
1	2	2^{-10}
2	4	2^{-9}
4	8	2^{-8}
8	16	2^{-7}
16	32	2^{-6}
32	64	2^{-5}
64	128	2^{-4}
128	256	$\frac{1}{8}$
256	512	$\frac{1}{4}$
512	1024	$\frac{1}{2}$
1024	2048	1
2048	4096	2
4096	8192	4
8192	16384	8
16384	32768	16
32768	65519	32
65519	∞	∞

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FP32 weight copies

- If the value-to-update ratio is bigger than $2^{11} = 2048$, it holds that

$$\text{value} + \text{update} = \text{value}$$

- The update has no influence on the value
- For reasons I and II, we maintain FP32 copies for both the weight and weight decay

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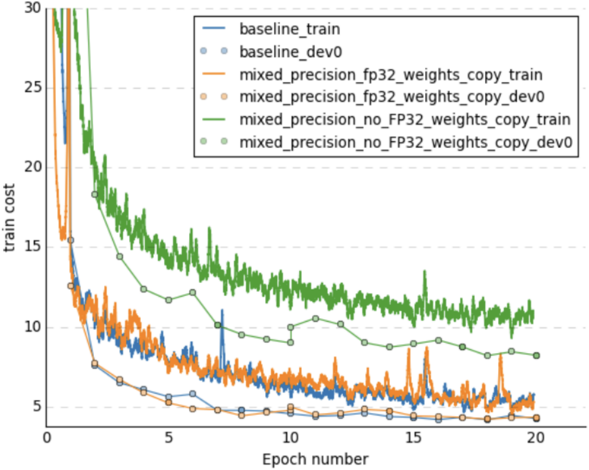
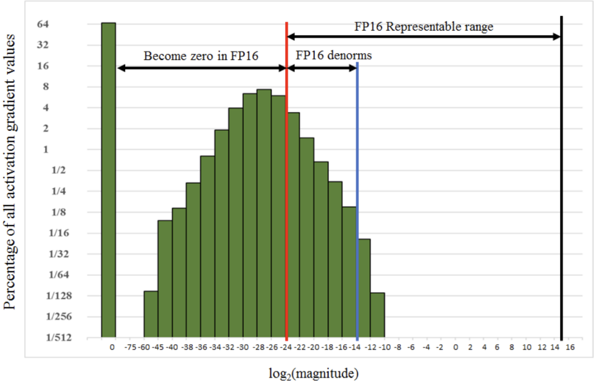


Loss scaling

- FP16 representation range $[2^{-24}, 2^{15}]$
- The activation gradient is typically very small; most values are smaller than 2^{-24}



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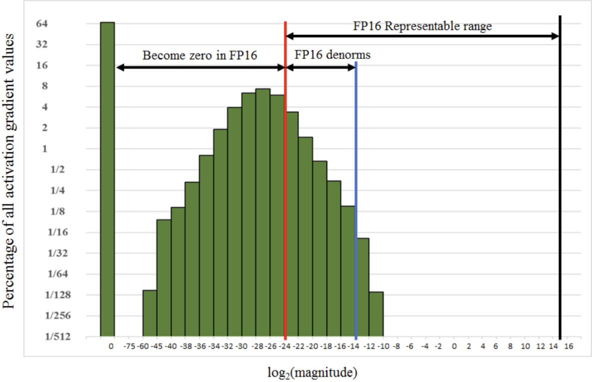

Loss scaling

- Scale-up the loss value before the back-propagation
- Unscale the gradient after back-propagation but before the update

$$g = \frac{\partial L}{\partial x} = \frac{1}{c}\frac{\partial (c \cdot L)}{\partial x}$$

- Effectively shift the graient value to the FP representation range
- Tricky to choose the scale-up coefficient
- $c = 8$ typically works

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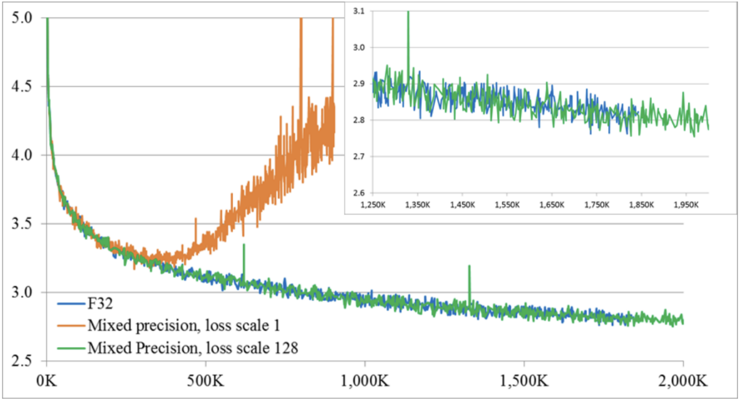

Figure 5: bigLSTM training perplexity

Arithmetic precision

- Not as important as the above two techniques
- Three key computation steps: vector dot-products; reductions; point-wise operations
- It is suggested in (Micikevicius et al., 2017) that vector dot-products and reductions are read and written in FP16 but carried out in FP32
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Numerical studies

Table 1: ILSVRC12 classification top-1 accuracy.

Model	Baseline	Mixed Precision	Reference
AlexNet	56.77%	56.93%	(Krizhevsky et al., 2012)
VGG-D	65.40%	65.43%	(Simonyan and Zisserman, 2014)
GoogLeNet (Inception v1)	68.33%	68.43%	(Szegedy et al., 2015)
Inception v2	70.03%	70.02%	(Ioffe and Szegedy, 2015)
Inception v3	73.85%	74.13%	(Szegedy et al., 2016)
Resnet50	75.92%	76.04%	(He et al., 2016b)

Numerical studies

Table 2: Detection network average mean precision.

Model	Baseline	MP without loss-scale	MP with loss-scale
Faster R-CNN	69.1%	68.6%	69.7%
Multibox SSD	76.9%	diverges	77.1%

Nvidia AMP[2]

AMP FOR PYTORCH

As simple as two lines of code

Wrap the model and optimizer

```
model, optimizer = amp.initialize(model, optimizer)
```

Apply automatic loss scaling and backpropagate with scaled loss

```
with amp.scaled_loss(loss, optimizer) as scaled_loss:
    scaled_loss.backward()
```

[2] https://nvlabs.github.io/iccv2019-mixed-precision-tutorial/

Nvidia AMP[3]: An example

```
import torch
import amp
model = ...
optimizer = ...
model, optimizer = amp.initialize(model, optimizer, opt_level="O1")
for data, label in data_iter:
    out = model(data)
    loss = criterion(out, label)
    optimizer.zero_grad()
    with amp.scaled_loss(loss, optimizer) as scaled_loss:
        scaled_loss.backward()
optimizer.step()
```

allows AMP to perform automatic casting

replaces `loss.backward()`

[3] https://nvlabs.github.io/iccv2019-mixed-precision-tutorial/

References I

P. Micikevicius, S. Narang, J. Alben, G. Diamos, E. Elsen, D. Garcia, B. Ginsburg, M. Houston, O. Kuchaiev, G. Venkatesh *et al.*, "Mixed precision training," *arXiv preprint arXiv:1710.03740*, 2017.

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