# Parameters, Memories, and Computations in Transformers

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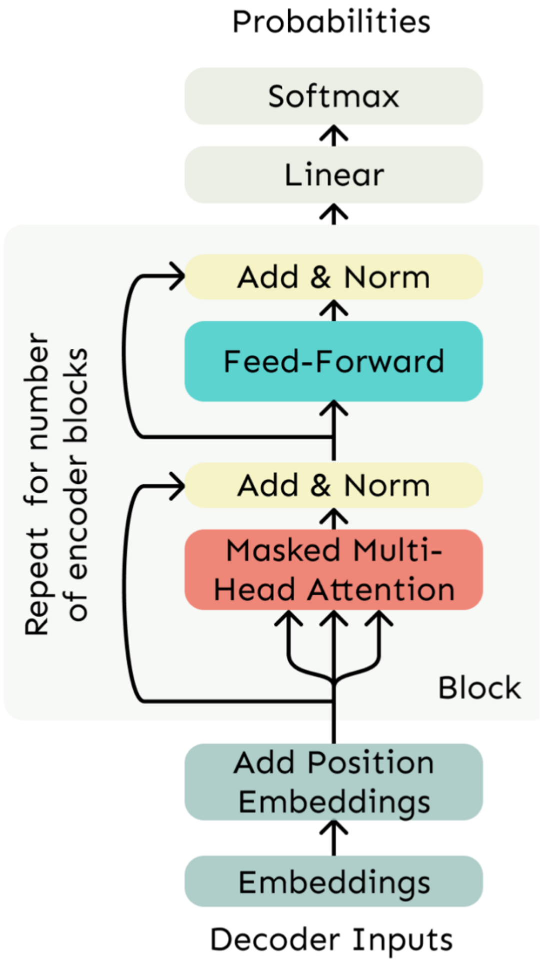
# PART 01

## Settings and Basics

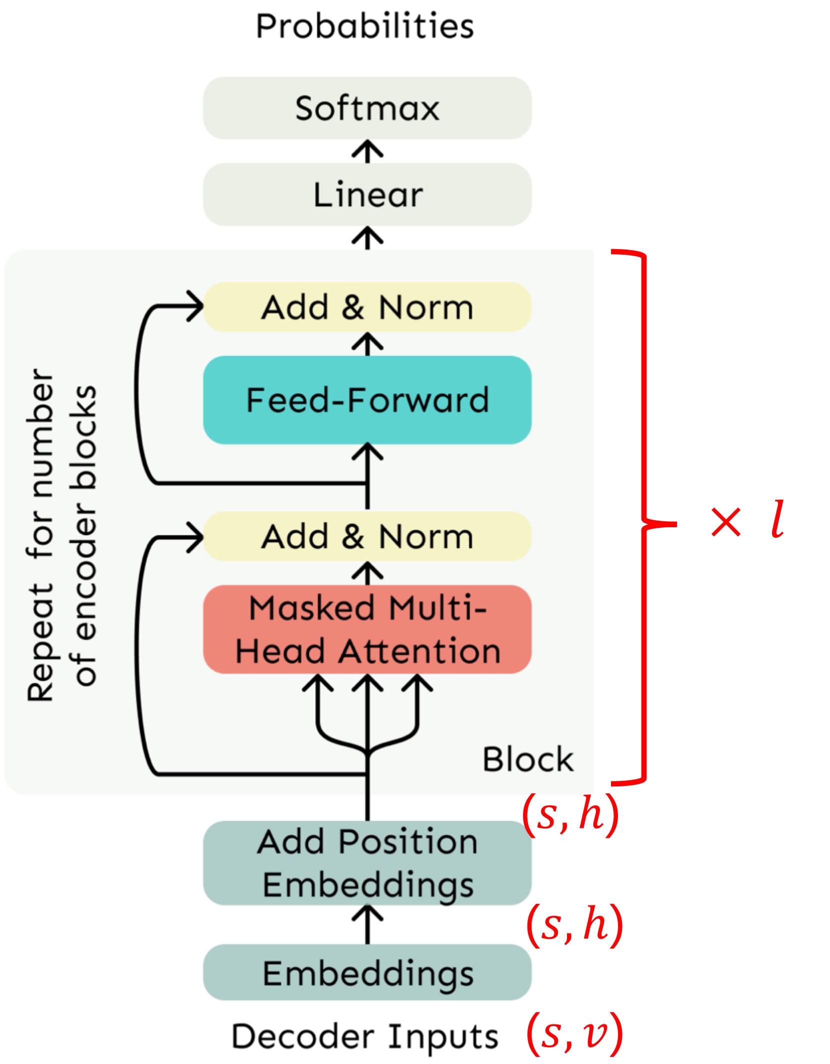
# Decoder-only Transformer

- GPT is based on the **decoder-only** transformer
- We will analyze the parameters, memories, and computation costs for decoder-only transformer

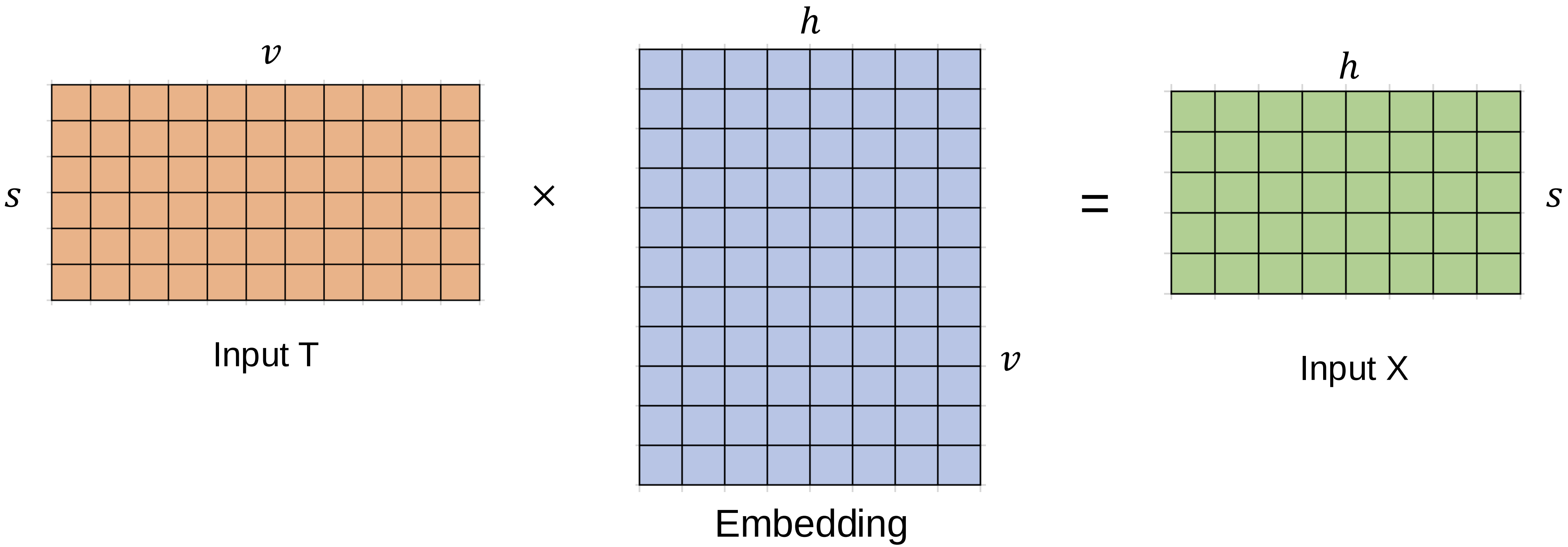


# Notations

- Number of the transformer layers: $l$
- Sequence length: $s$
- Vocabulary size: $v$
- Embedding representation dims: $h$



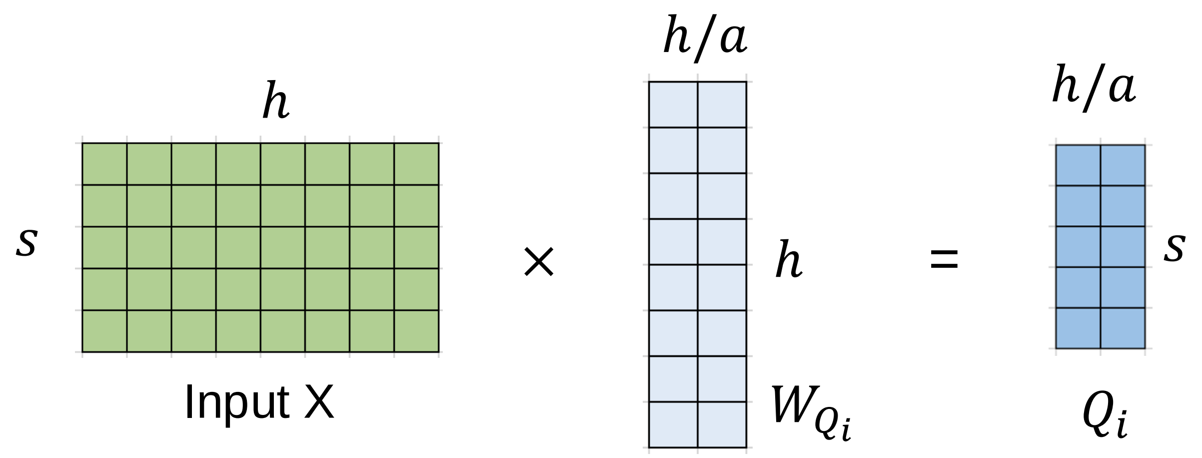
# Embedding

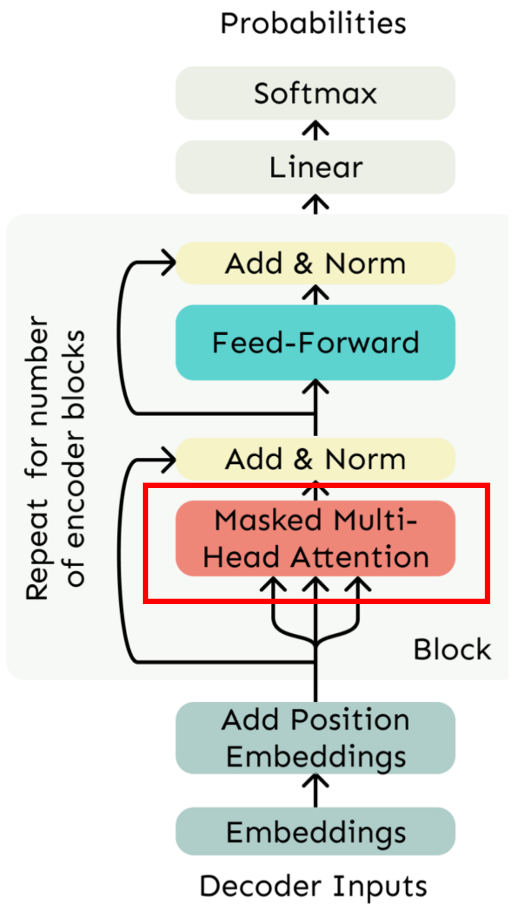


# Self-attention



- Number of heads: $a$
- Dims of each $W_{Q_i}$, $W_{K_i}$ and $W_{V_i}$: $h \times \frac{h}{a}$



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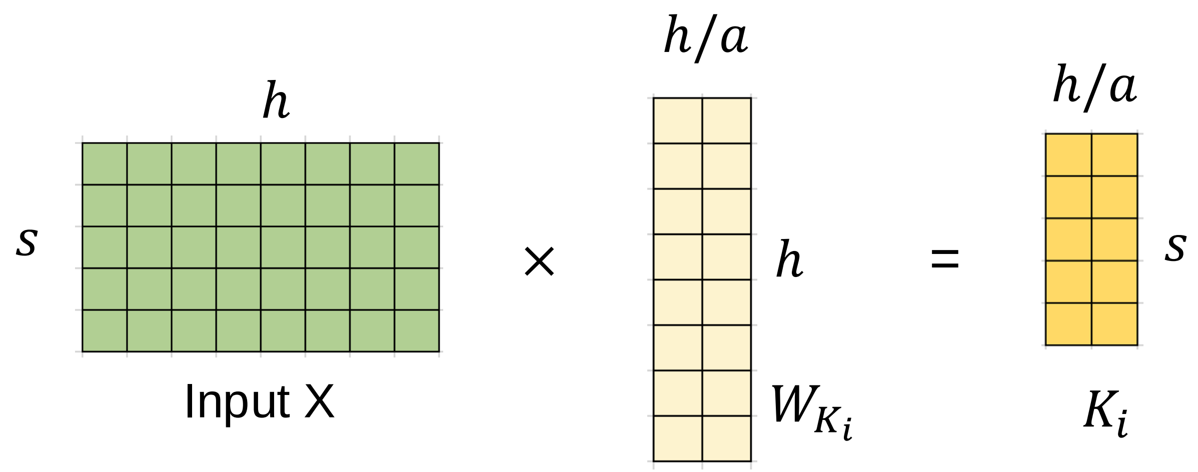

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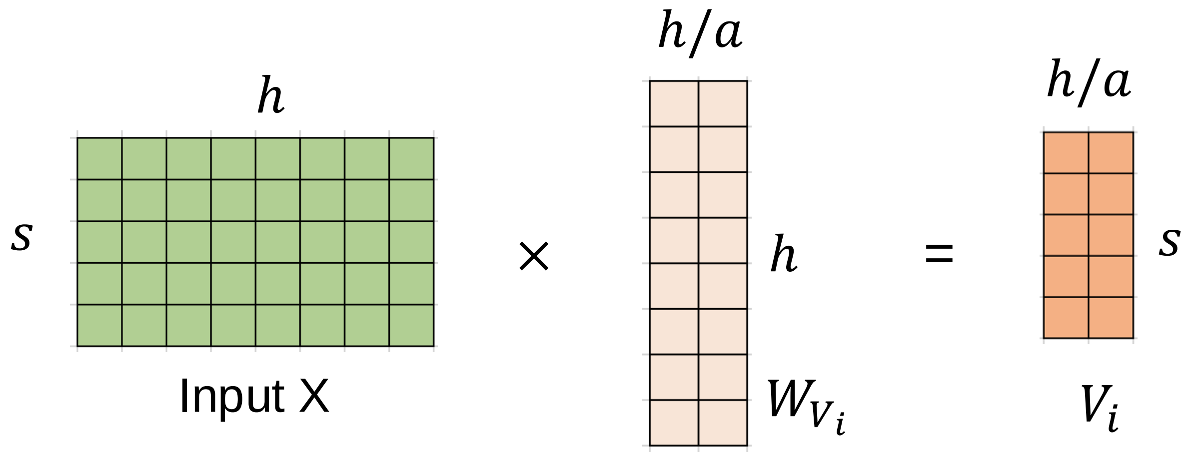
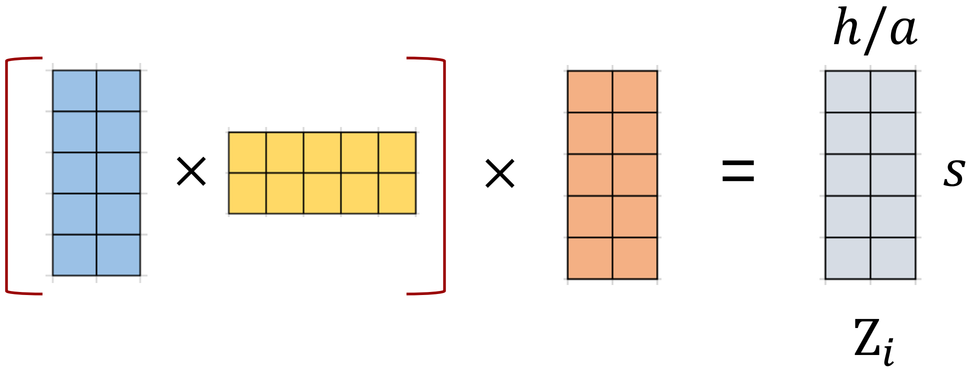


# Self-attention

- Number of heads: $a$
- Dims of each $W_{Q_i}$, $W_{K_i}$ and $W_{V_i}$: $h \times \frac{h}{a}$

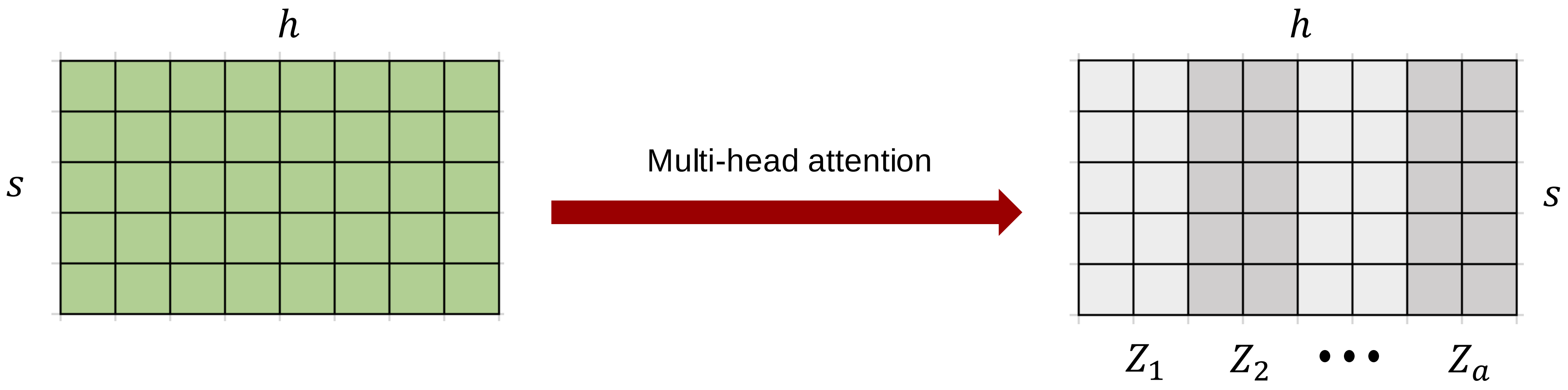
$$\text{softmax}\left(\frac{Q_i K_i^T}{\sqrt{h/a}}\right)V_i \quad = \quad \text{softmax}$$




One-head attention

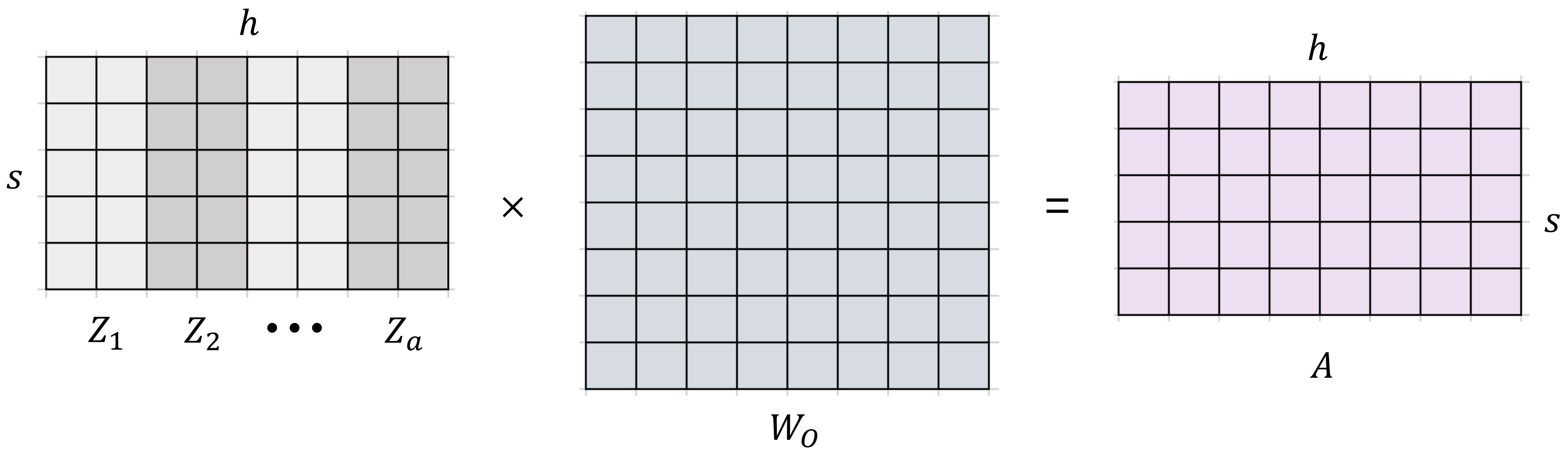
# Multi-head attentions

- Number of heads: $a$
- Dims of each $W_{Q_i}$, $W_{K_i}$ and $W_{V_i}$: $h \times \frac{h}{a}$



# Multi-head attentions

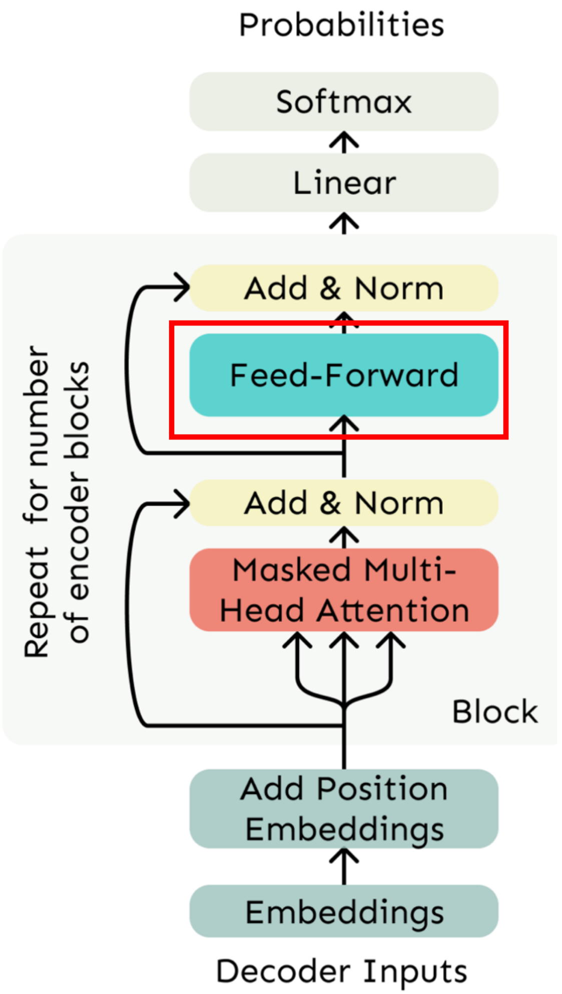
- Number of heads: $a$
- Dims of each $W_O$: $h \times h$
- Dims of each $W_{Q_i}$, $W_{K_i}$ and $W_{V_i}$: $h \times \frac{h}{a}$



# Feed-forward Layer

$$X' = \mathrm{ReLU}(A \cdot W_1 + b_1) \cdot W_2 + b_2$$

- Dims of $W_1$: $h \times 4h$
- Dims of each $W_2$: $4h \times h$

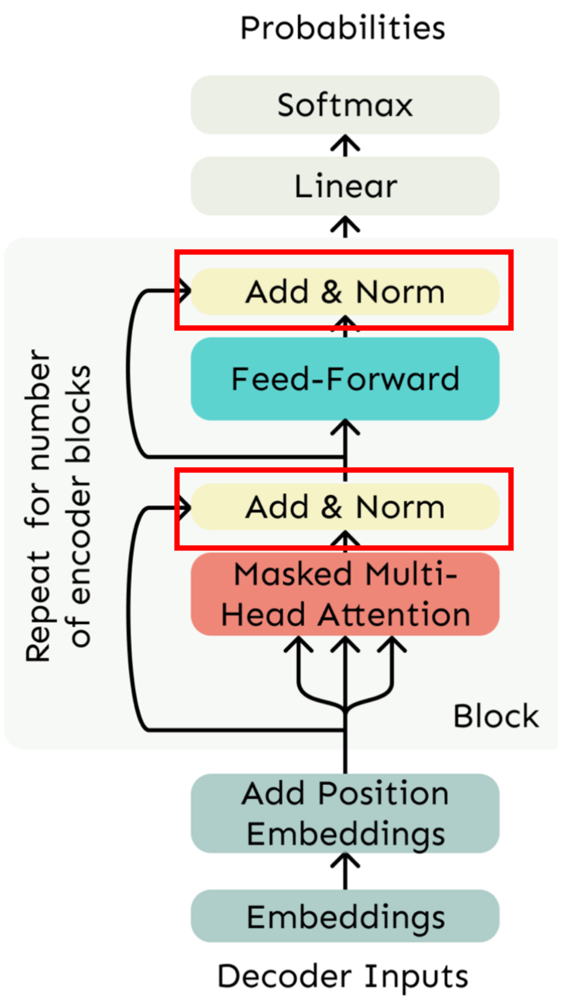


# Layer normalization

- Then layer normalization computes:

$$\text{output} = \frac{x \ - \mu}{\sqrt{\sigma} + \epsilon} * \gamma + \beta$$

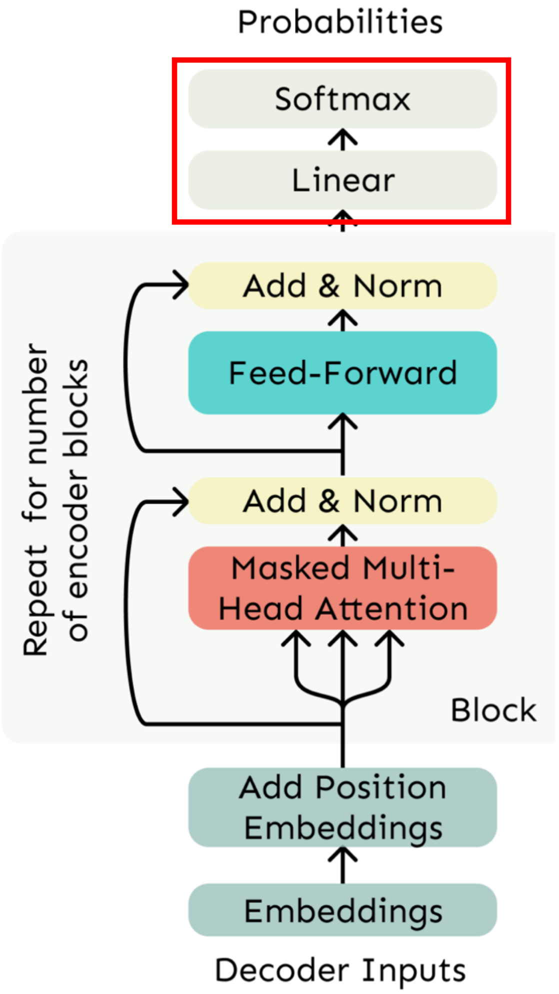
- Dims of $\gamma$ and $\beta$: $h$



# Probability prediction

$$p = \mathrm{Softmax}(X \cdot W_v + b_v)$$

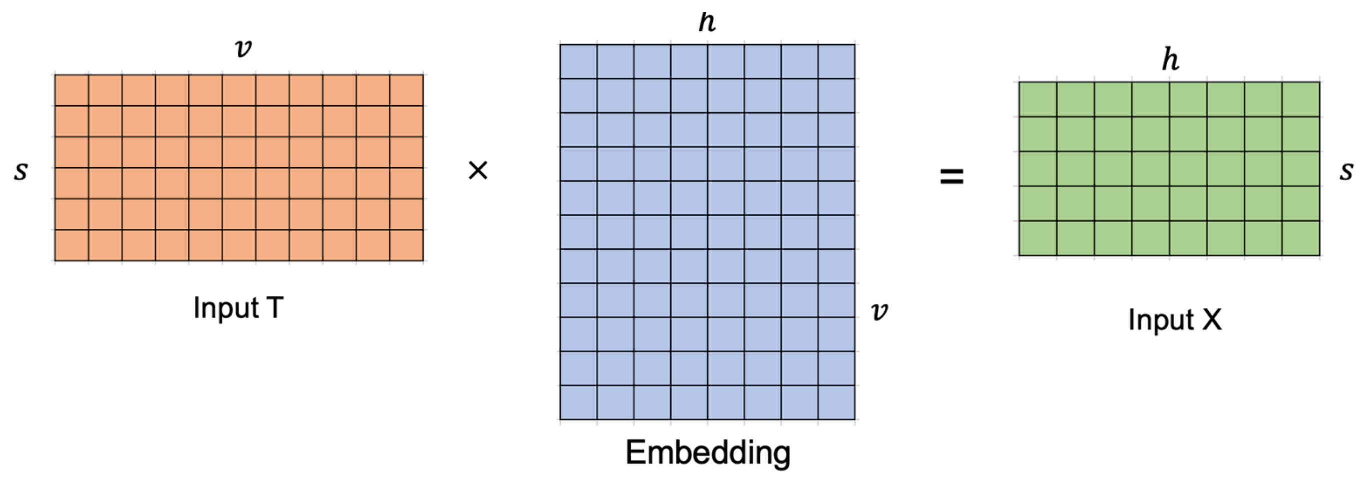
- Dims of $W_v$: $h \times \mathrm{v}$

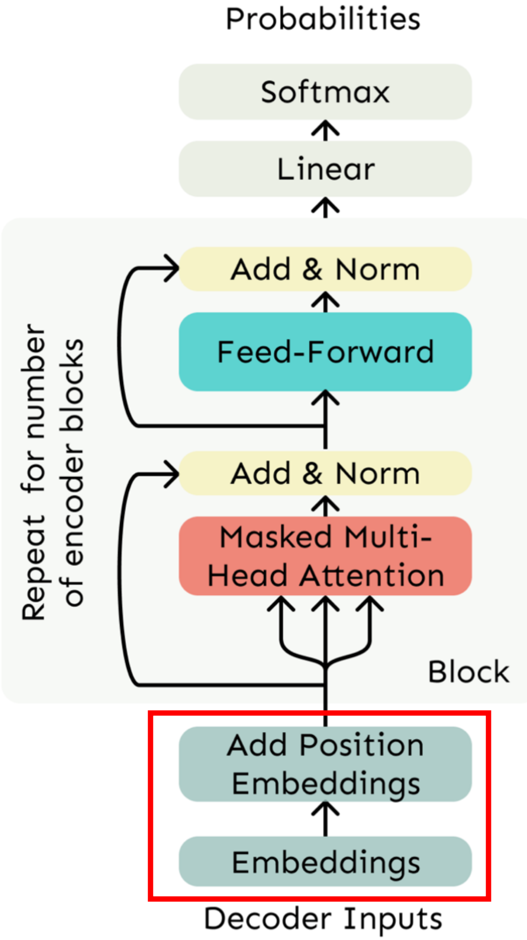


PART 02

# Parameters analysis

# Embedding



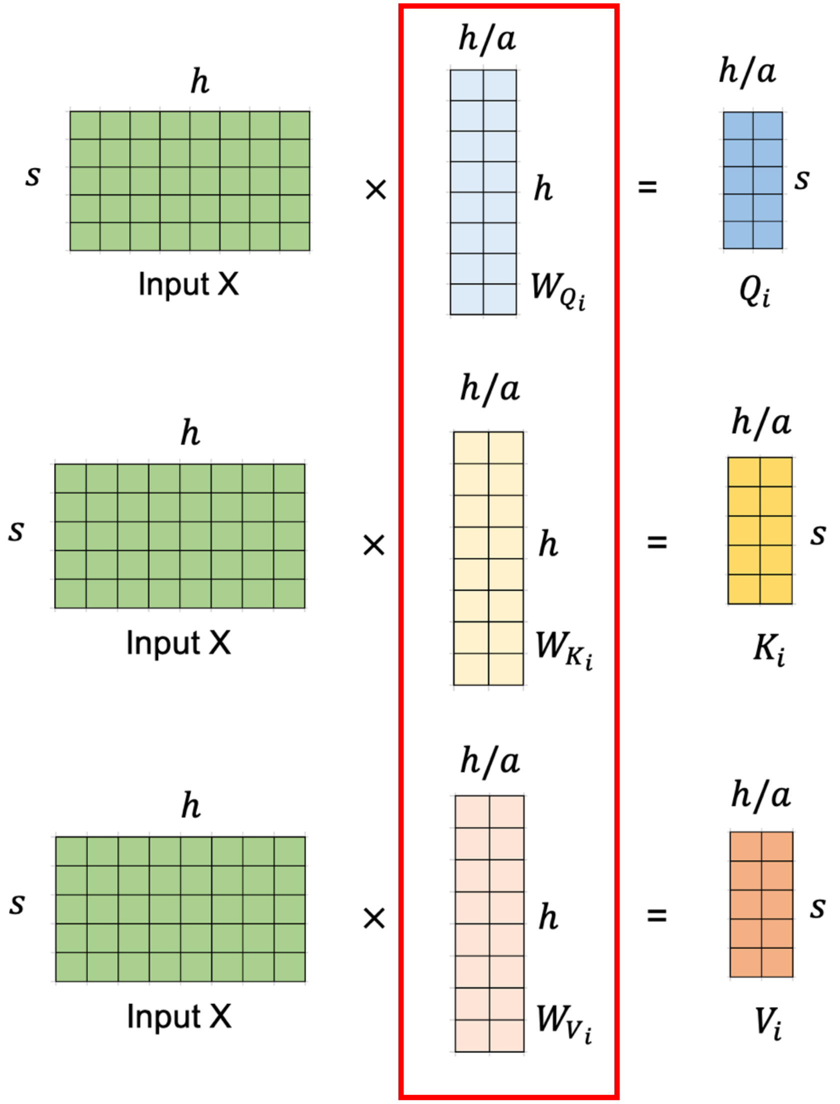



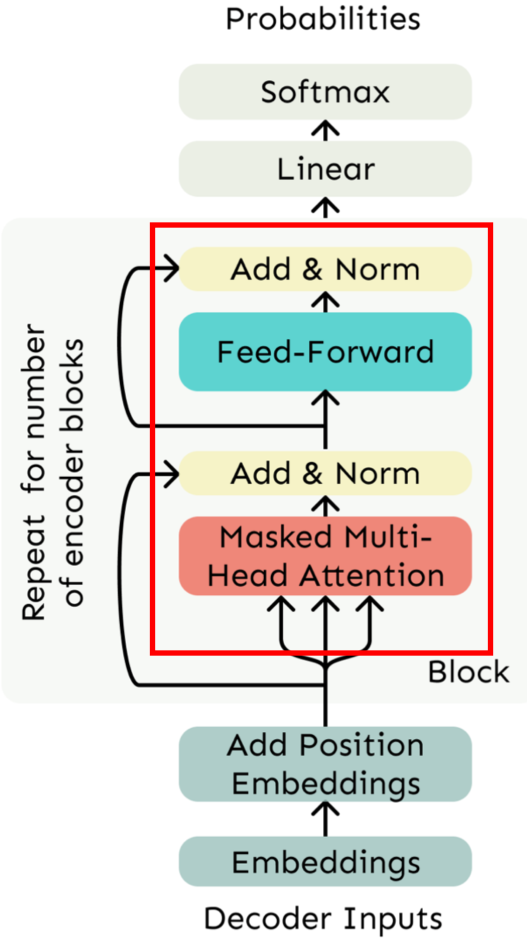

- We need to store the embedding with parameters $vh$
- Position embedding can be ignored when using RoPE and ALiBi

# Multi-head attentions

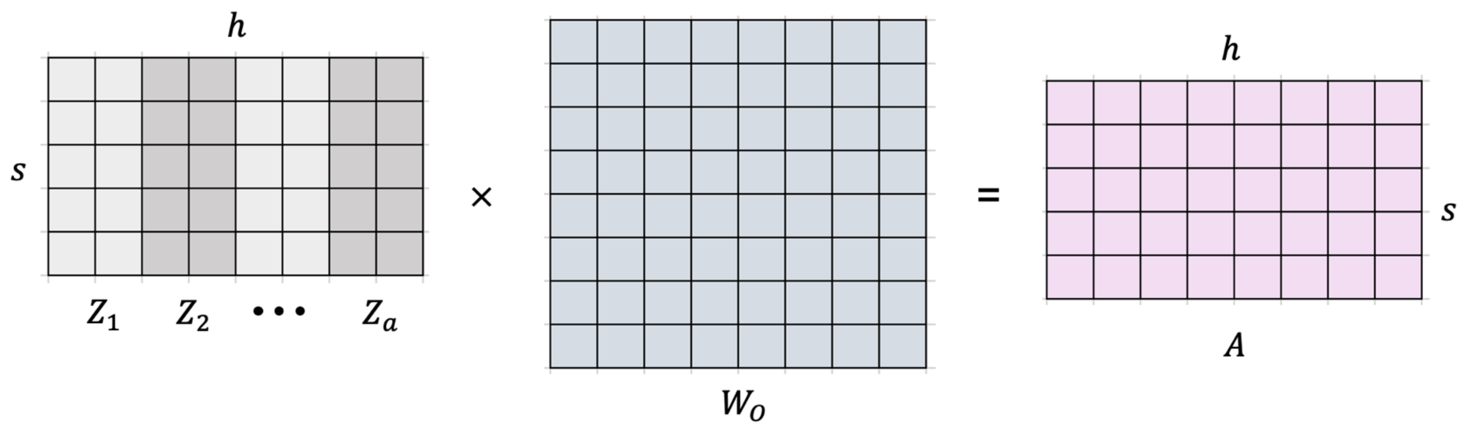
- We need to store $W_Q$, $W_K$ and $W_V$

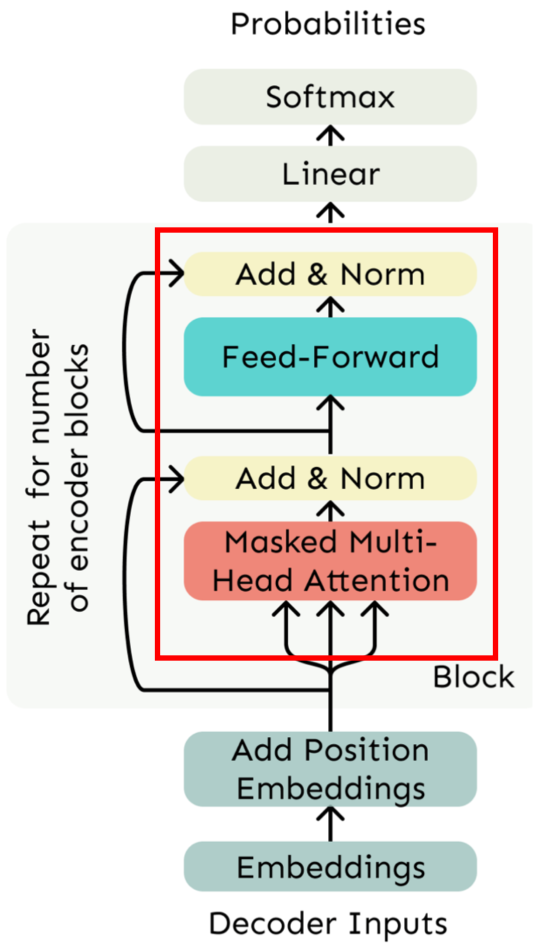
$$3(h^2/a) \times a = 3h^2$$



# Multi-head attentions




- We need to store $W_O$: $h^2$

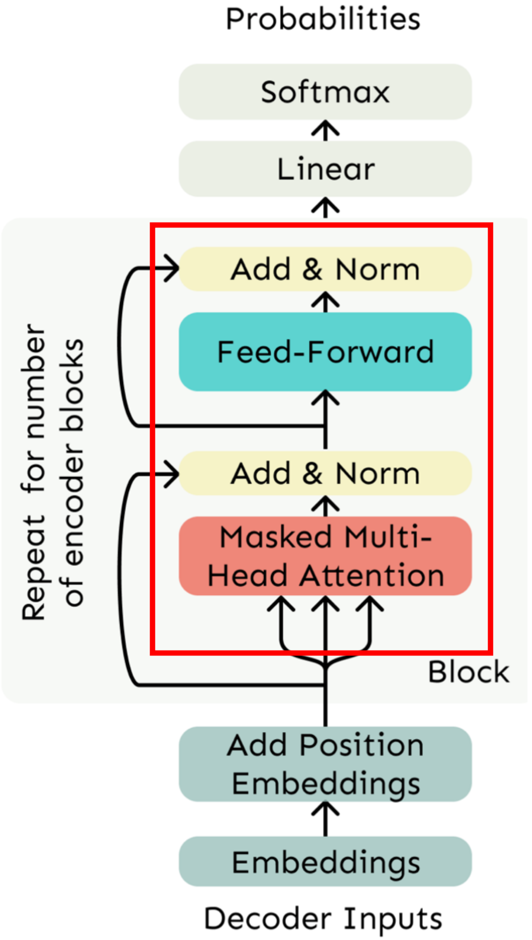
# Layer normalization

- Then layer normalization computes:

$$\text{output} = \frac{x - \mu}{\sqrt{\sigma + \epsilon}} * \gamma + \beta$$

- Dims of γ and β: $h$

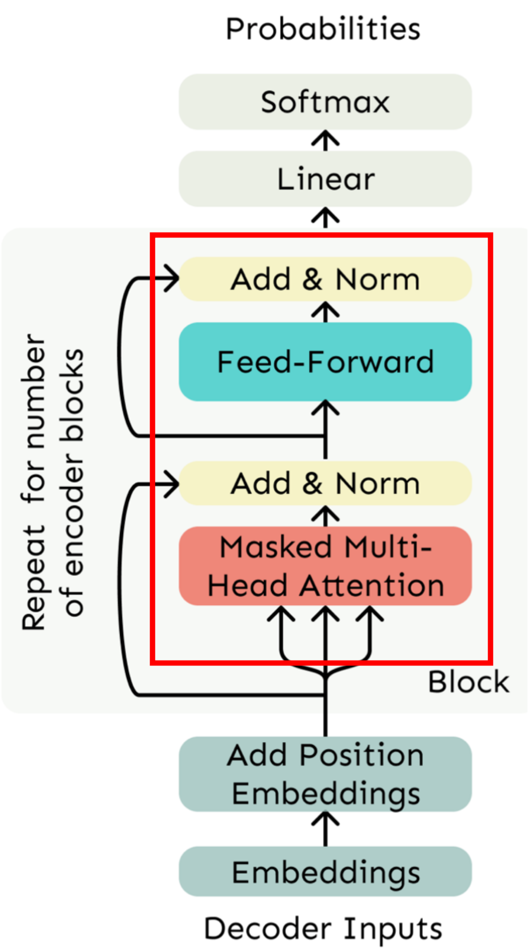
- We should store γ and β. Since their parameters are much smaller than $h^2$ and $vh$, we can ignore them



# Feed-forward layers

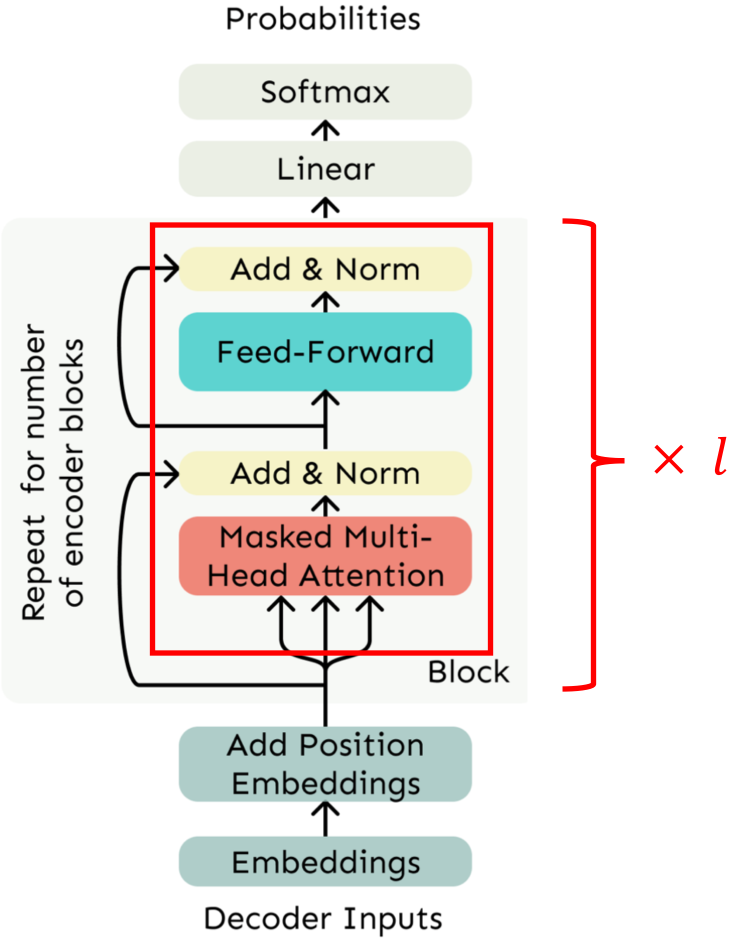
$$X' = \mathrm{ReLU}(A \cdot W_1 + b_1) \cdot W_2 + b_2$$

- Dims of $W_1$: $h \times 4h$
- Dims of each $W_2$: $4h \times h$
- We need to store $W_1$ and $W_2$ : $8h^2$
- The storage of $b_1$ and $b_2$ can be ignored



# Transformer block

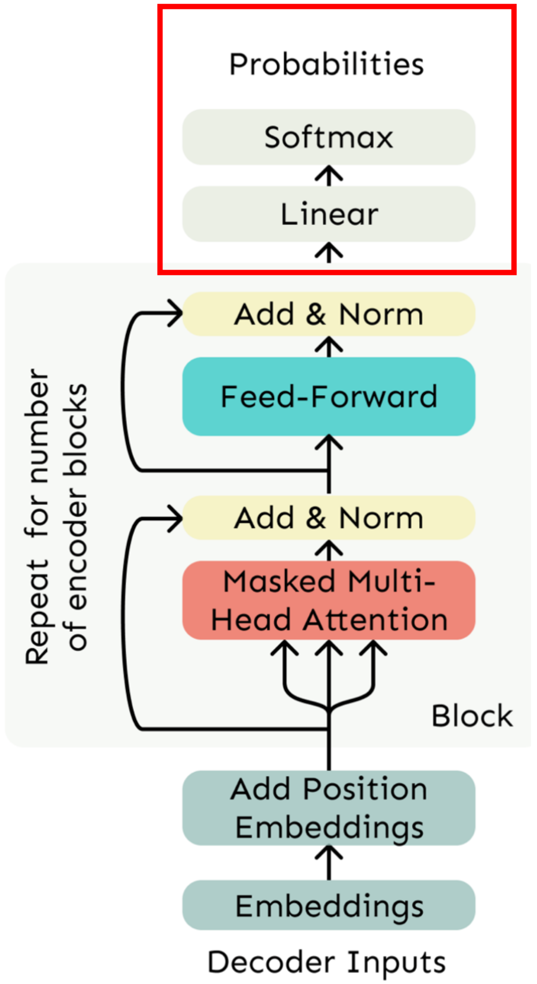
- Multi-head attentions: $4h^2$
- Feed-forward layers : $8h^2$
- $l$ layers of attentions : $(4h^2 + 8h^2) \times l = 12lh^2$



# Probability predictions

$$p = \mathrm{Softmax}(X \cdot W_v + b_v)$$

- Dims of $W_v$: $h \times \mathrm{v}$
- We need to store $W_v$: $hv$ parameters
- $b_v$ can be ignored

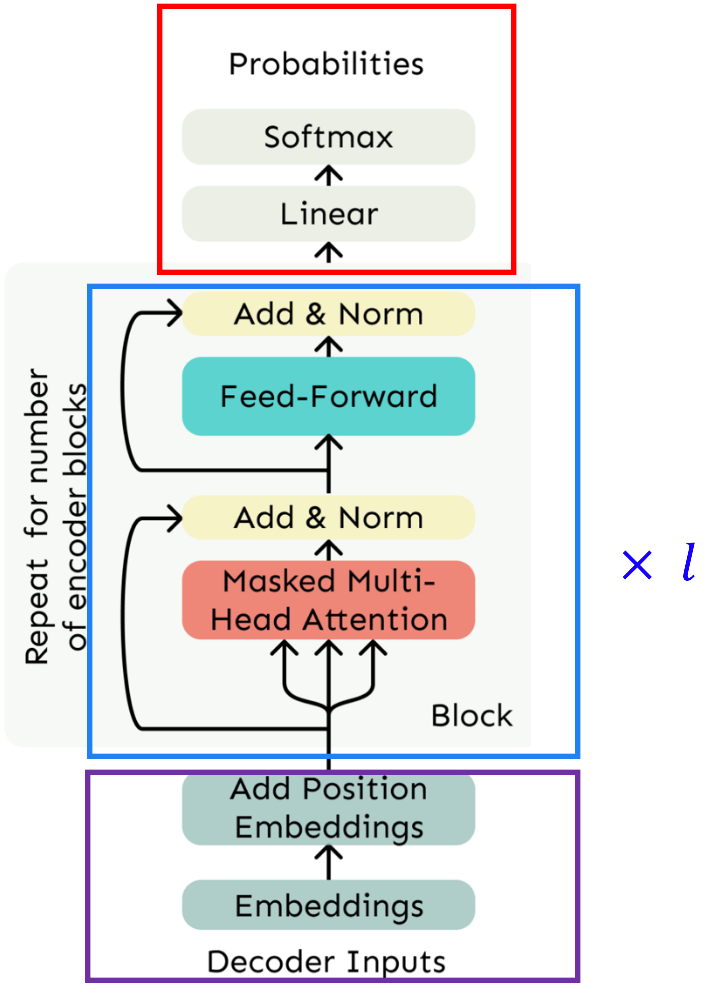


# Total parameters

- Embeddings: $vh$
- Attention blocks: $12lh^2$
- Probability predictions: $vh$

Total parameters:

$$12\ell h^2 + 2vh$$



# Example: LLaMA parameters

- Now we compare our theoretical evaluations with LLaMA model
- $12\ell h^2 + 2vh$ is a very accurate estimation

| 实际参数量 | Embedding h | Attention层数l | Vocab大小v | 预估参数量 |
|---|---|---|---|---|
| 6.7B | 4096 | 32 | 32000 | 6,704,594,944 |
| 13.0B | 5120 | 40 | 32000 | 12,910,592,000 |
| 32.5B | 6656 | 60 | 32000 | 32,323,665,920 |
| 65.2B | 8192 | 80 | 32000 | 64,948,797,440 |

PART 03

# Computations analysis

# Flops

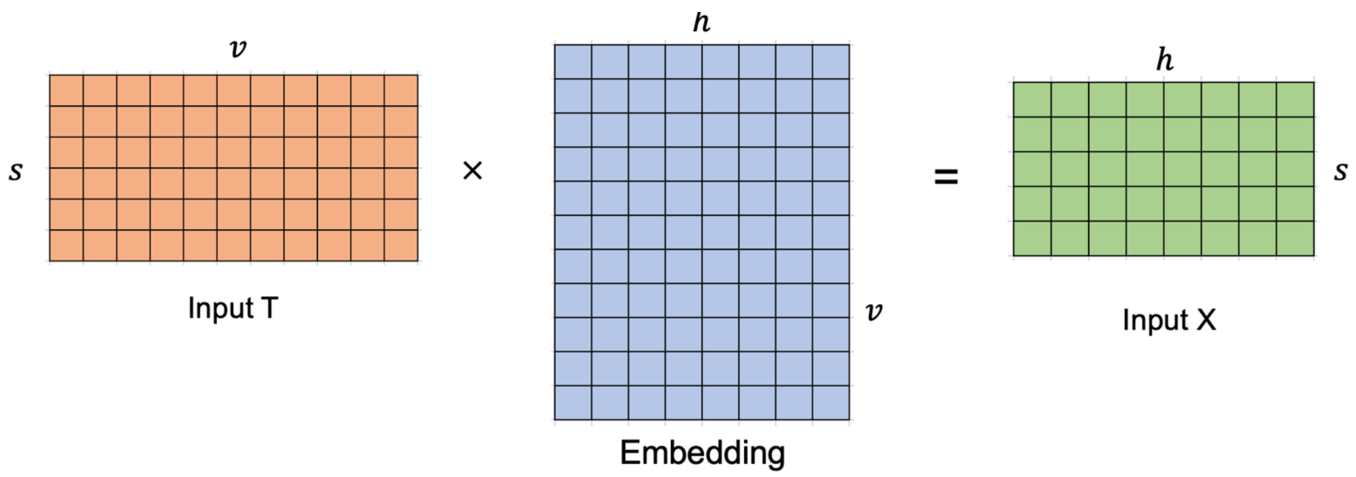
- FLOPs: Floating point operations; gauges the total amount of computations

- Given matrices $A \in \mathbb{R}^{m \times n}, B \in \mathbb{R}^{n \times p}$, to compute $AB$ , we need

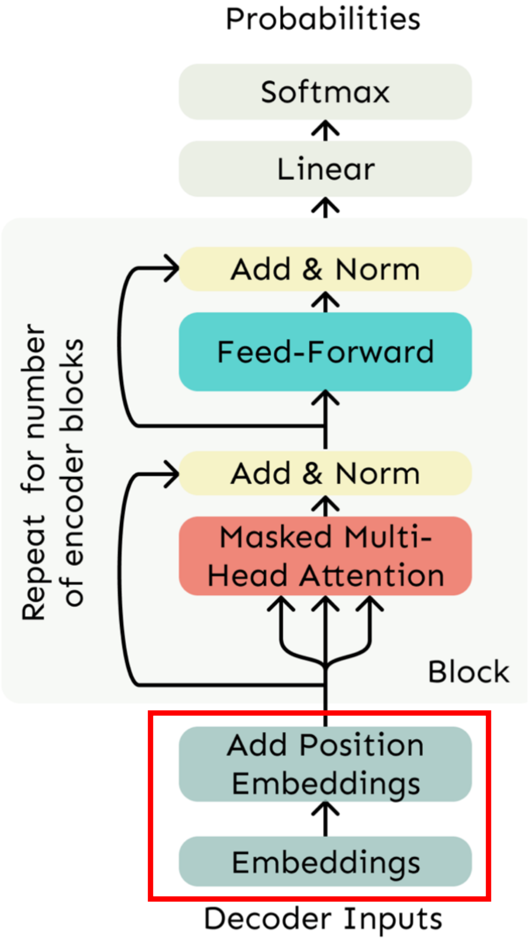
$$\left.\begin{array}{l} mnp \text{ additions} \\ mnp \text{ multiplications} \end{array}\right\} \quad 2mnp \text{ FLOPs}$$

- In transformers, we only count computations raised by matrix operations and ignore vector operations since the later is trivial

# Embedding




- Word embedding: $2svh$

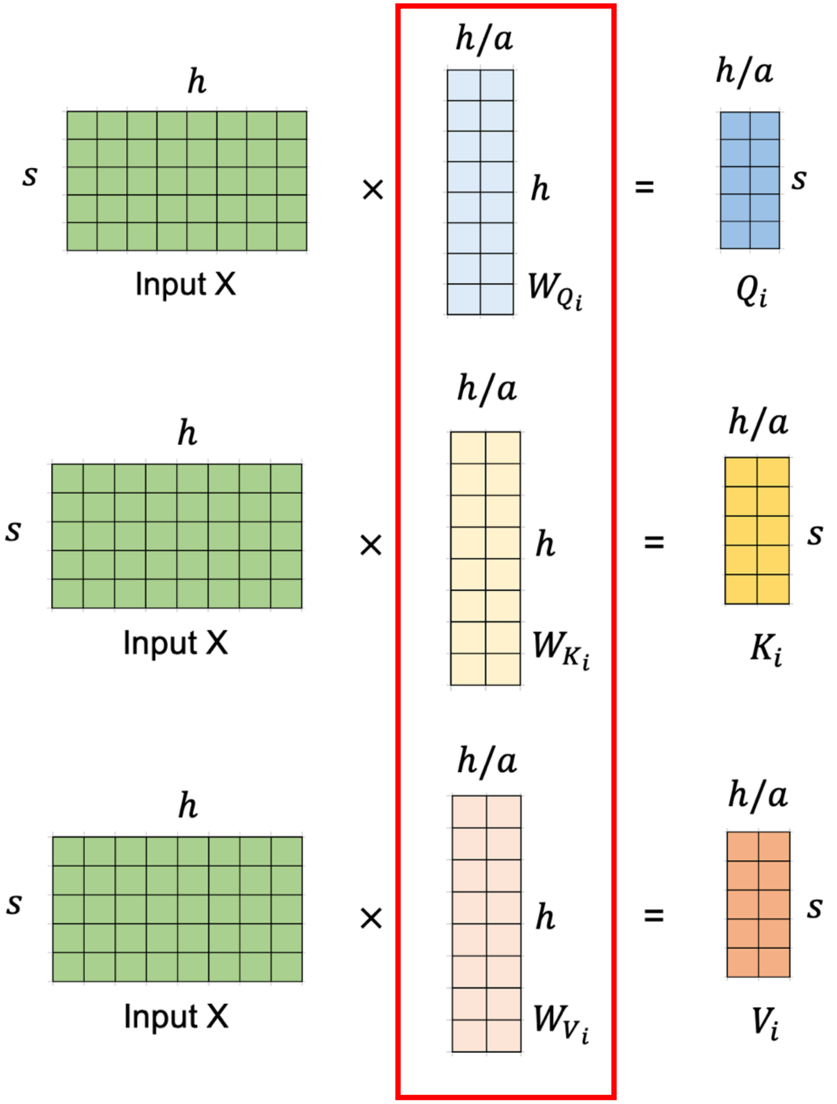


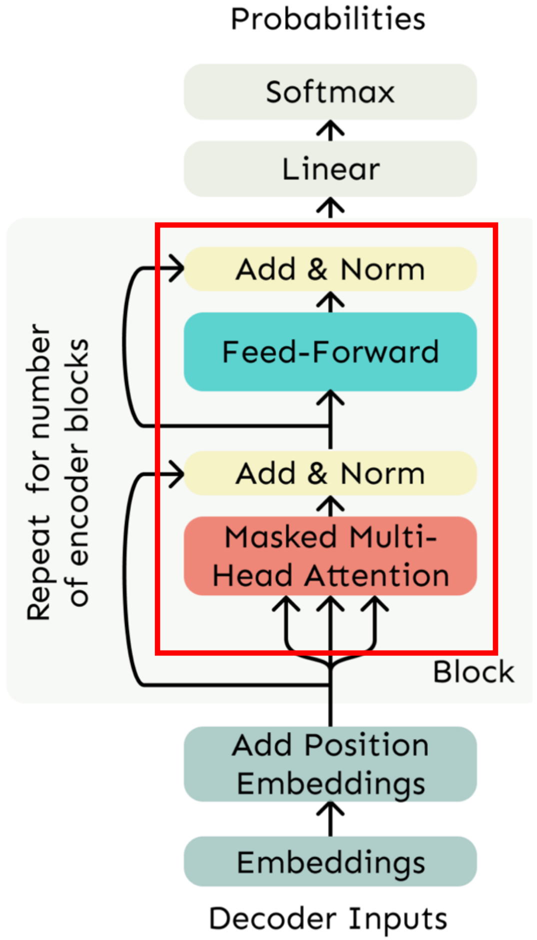
# Multi-head attentions



- Multi-head attentions

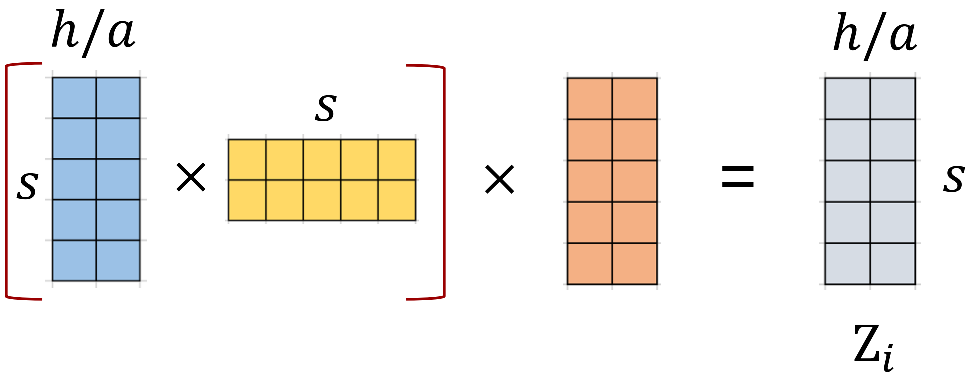
$$6(sh^2/a) \times a = 6sh^2$$



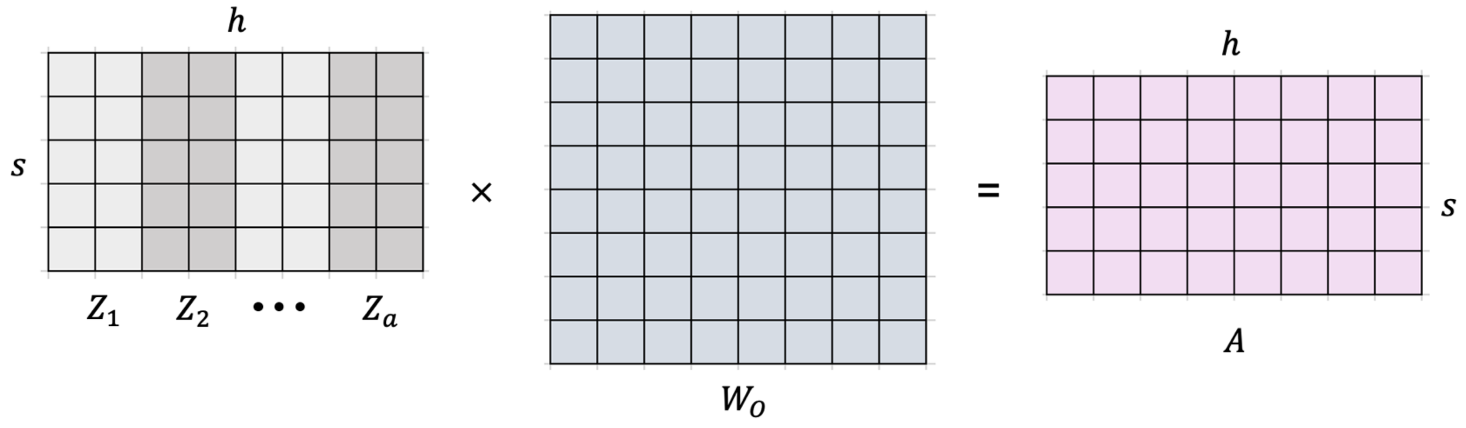
# Multi-head attentions

$$\mathrm{softmax}\left(\frac{Q_i K_i^T}{\sqrt{h/a}}\right)V_i \; = \; \text{softmax}$$

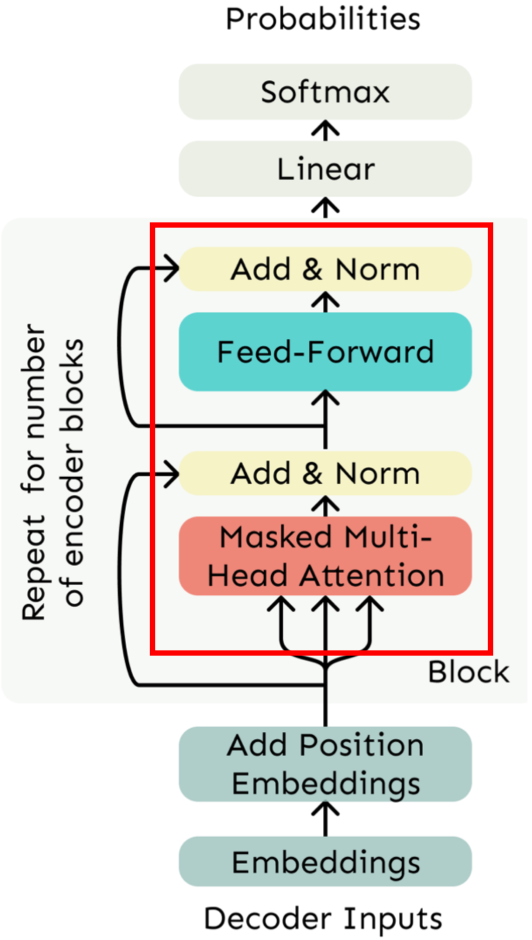



$$(2s^2h/a + 2s^2h/a) \times a = 4s^2h$$

# Multi-head attentions




$2sh^2$ Flops

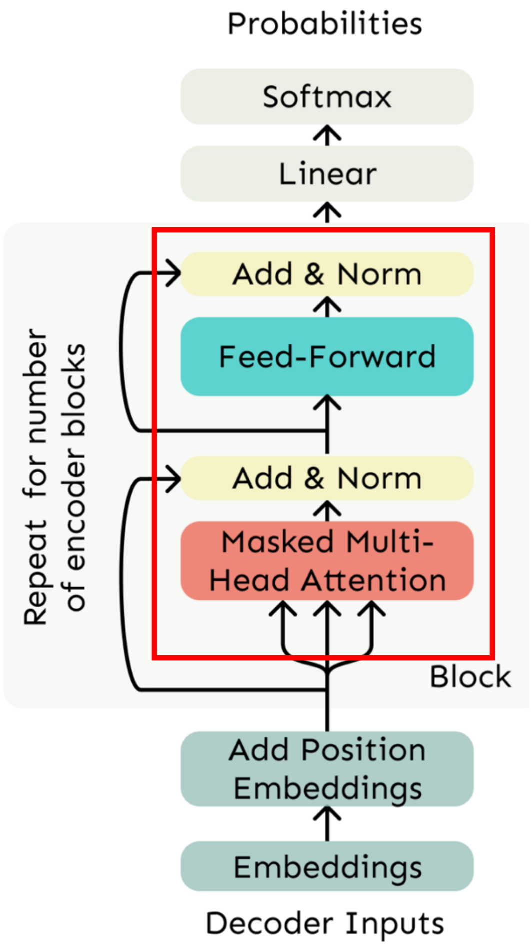


# Feed-forward layers

$$X' = \mathrm{ReLU}(A \cdot W_1 + b_1) \cdot W_2 + b_2$$

- Dims of $W_1$: $h \times 4h$
- Dims of each $W_2$: $4h \times h$
- $AW_1 + b_1$ needs: $8sh^2$
- $A'W_2 + b_2$ needs: $8sh^2$

$\left.\begin{array}{l} AW_1 + b_1 \text{ needs: } 8sh^2 \\ A'W_2 + b_2 \text{ needs: } 8sh^2 \end{array}\right\} 16sh^2$



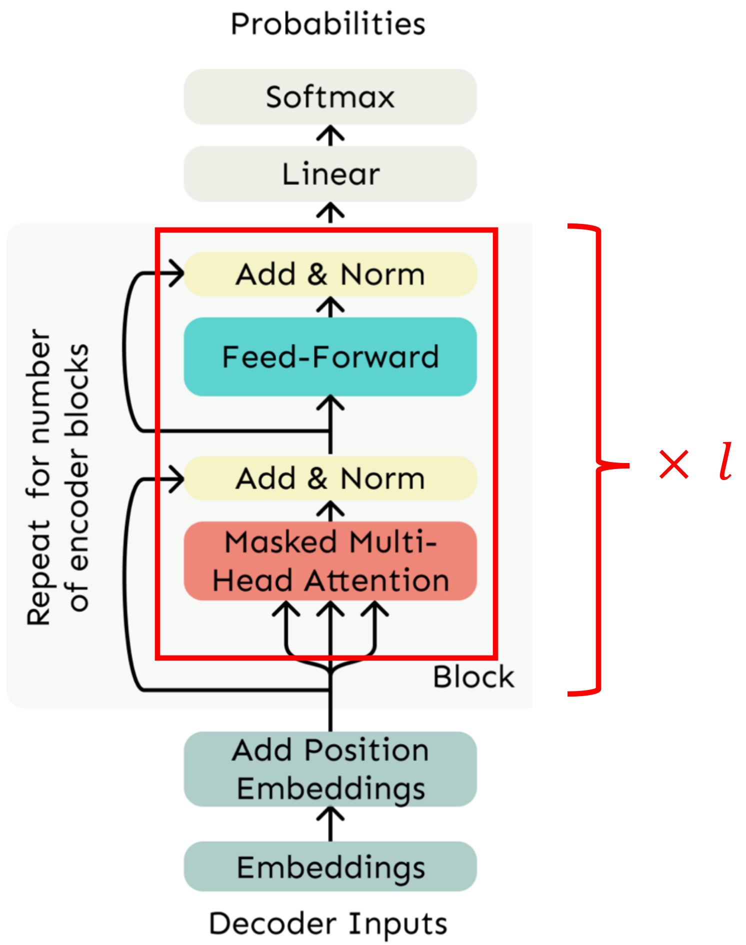
# Transformer block

- Multi-head attentions: $8sh^2 + 4s^2\mathrm{h}$

- Feed-forward layers : $16sh^2$

- $l$ layers of attentions :

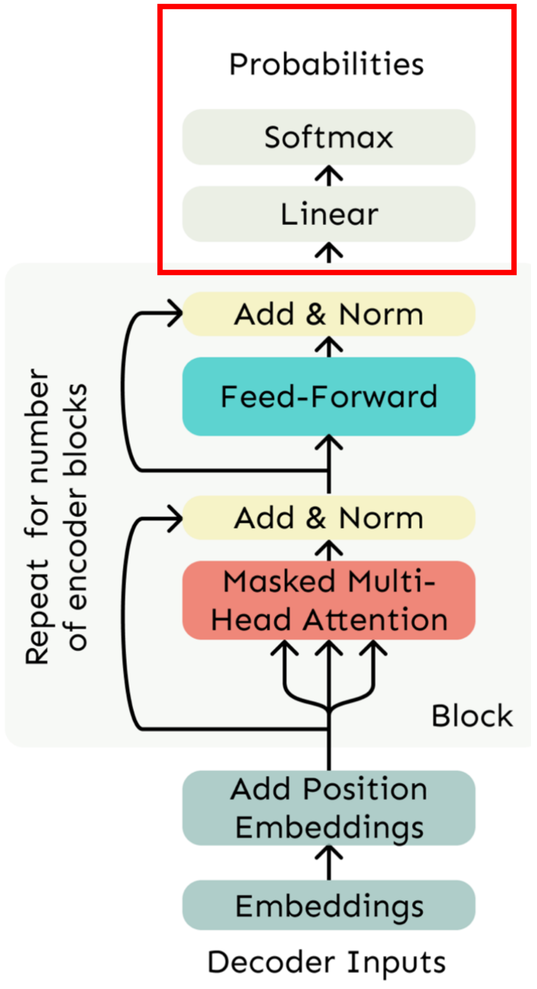
$$(8sh^2 + 16sh^2 + 4s^2h\,) \times l = 24slh^2 + 4s^2lh$$



# Probability predictions

$$p = \mathrm{Softmax}(X \cdot W_v + b_v)$$

- Dims of $W_v$: $h \times \mathrm{v}$
- We need to : $2shv$ FLOPs



# Total forward FLOPs

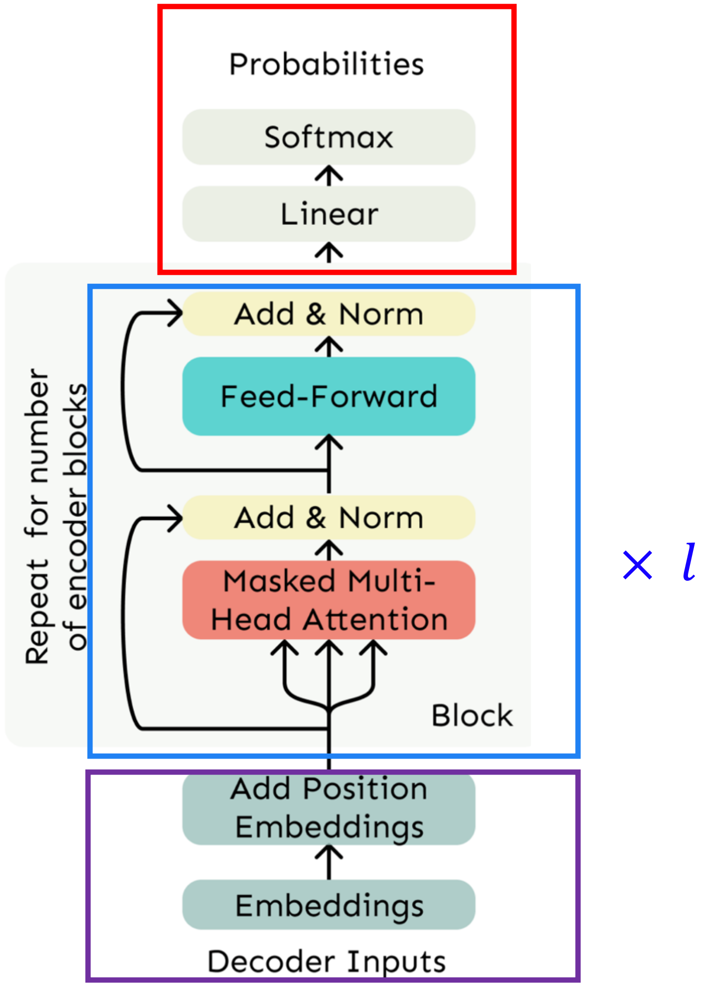
- Embeddings: $2svh$
- Attention blocks: $24lsh^2 + 4s^2lh$
- Probability predictions: $2svh$

Total forward FLOPs:

$$\ell(24sh^2 + 4s^2h) + 4svh$$

When using batch-size b, the total forward FLOPs:

$$b\ell(24sh^2 + 4s^2h) + 4bsvh$$



# Total forward-backward FLOPs

The backward computations are **twice amount** of the forward computations

Total forward-backward FLOPs

$$\Big(b\ell(24sh^2 + 4s^2h) + 4bsvh\Big) \times 3 = 3\Big(b\ell(24sh^2 + 4s^2h) + 4bsvh\Big)$$

When $h^2$ dominates, the above FLOPs can be simplified as $72bs\ell h^2$

When $h^2$ dominates, the parameters can be simplified as $P = 12\ell h^2$

Since $T = bs$ is the number of tokens, we thus have $\mathrm{FLOPs} = 6TP$

# Example: GPT3-175B

GPT-175B: 175B parameters, 300B tokens

$$6 \times 174600 \times 10^{6} \times 300 \times 10^{9} = 3.1428 \times 10^{23}\, flops$$

| Model | Total train compute (PF-days) | Total train compute (flops) | Params (M) | Training tokens (billions) | Flops per param per token | Mult for bwd pass | flops per active param per token | params active for each token |
|---|---|---|---|---|---|---|---|---|
| T5-Small | 2.08E+00 | 1.80E+20 | 60 | 1,000 | 3 | 3 | 1 | 0.5 |
| T5-Base | 7.64E+00 | 6.60E+20 | 220 | 1,000 | 3 | 3 | 1 | 0.5 |
| T5-Large | 2.67E+01 | 2.31E+21 | 770 | 1,000 | 3 | 3 | 1 | 0.5 |
| T5-3B | 1.04E+02 | 9.00E+21 | 3,000 | 1,000 | 3 | 3 | 1 | 0.5 |
| T5-11B | 3.82E+02 | 3.30E+22 | 11,000 | 1,000 | 3 | 3 | 1 | 0.5 |
| BERT-Base | 1.89E+00 | 1.64E+20 | 109 | 250 | 6 | 3 | 2 | 1.0 |
| BERT-Large | 6.16E+00 | 5.33E+20 | 355 | 250 | 6 | 3 | 2 | 1.0 |
| RoBERTa-Base | 1.74E+01 | 1.50E+21 | 125 | 2,000 | 6 | 3 | 2 | 1.0 |
| RoBERTa-Large | 4.93E+01 | 4.26E+21 | 355 | 2,000 | 6 | 3 | 2 | 1.0 |
| GPT-3 Small | 2.60E+00 | 2.25E+20 | 125 | 300 | 6 | 3 | 2 | 1.0 |
| GPT-3 Medium | 7.42E+00 | 6.41E+20 | 356 | 300 | 6 | 3 | 2 | 1.0 |
| GPT-3 Large | 1.58E+01 | 1.37E+21 | 760 | 300 | 6 | 3 | 2 | 1.0 |
| GPT-3 XL | 2.75E+01 | 2.38E+21 | 1,320 | 300 | 6 | 3 | 2 | 1.0 |
| GPT-3 2.7B | 5.52E+01 | 4.77E+21 | 2,650 | 300 | 6 | 3 | 2 | 1.0 |
| GPT-3 6.7B | 1.39E+02 | 1.20E+22 | 6,660 | 300 | 6 | 3 | 2 | 1.0 |
| GPT-3 13B | 2.68E+02 | 2.31E+22 | 12,850 | 300 | 6 | 3 | 2 | 1.0 |
| GPT-3 175B | 3.64E+03 | 3.14E+23 | 174,600 | 300 | 6 | 3 | 2 | 1.0 |