
HOMWORK 4. ZERO-ORDER OPTIMIZATION

Kun Yuan

October 18, 2023

Attention: Turn in your homework at the beginning of our lecture on Oct. 24, 2023

1 Convergence Analysis of ZO-GD

ZO-GD with forward difference iterates as follows

$$g_k = \sum_{i=1}^d \frac{1}{\tau_k} (f(x_k + \tau_k e_i) - f(x_k)) e_i \quad (1a)$$

$$x_{k+1} = x_k - \gamma g_k \quad (1b)$$

Prove the following theorem.

Theorem 1.1. Assume $f(x)$ to be L -smooth. If we set $\gamma = 1/L$ and $\sum_{k=0}^K \tau_k^2 \leq R^2$, ZO-GD (1) converges as follows

$$\frac{1}{K+1} \sum_{k=0}^K \|\nabla f(x_k)\|^2 \leq \frac{2L(f(x_0) - f(x^*))}{K+1} + \frac{dL^2 R^2}{4(K+1)}. \quad (2)$$

Hint: The proof argument in Theorem 3.3 is helpful.