Parameters, Memories, and Computations in Transformers

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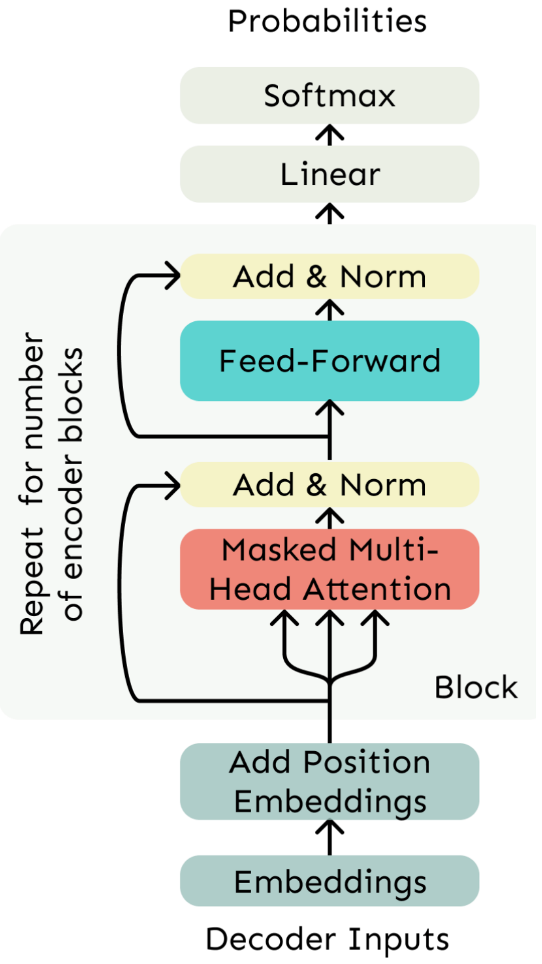
PART 01

Settings and Basics

Decoder-only Transformer

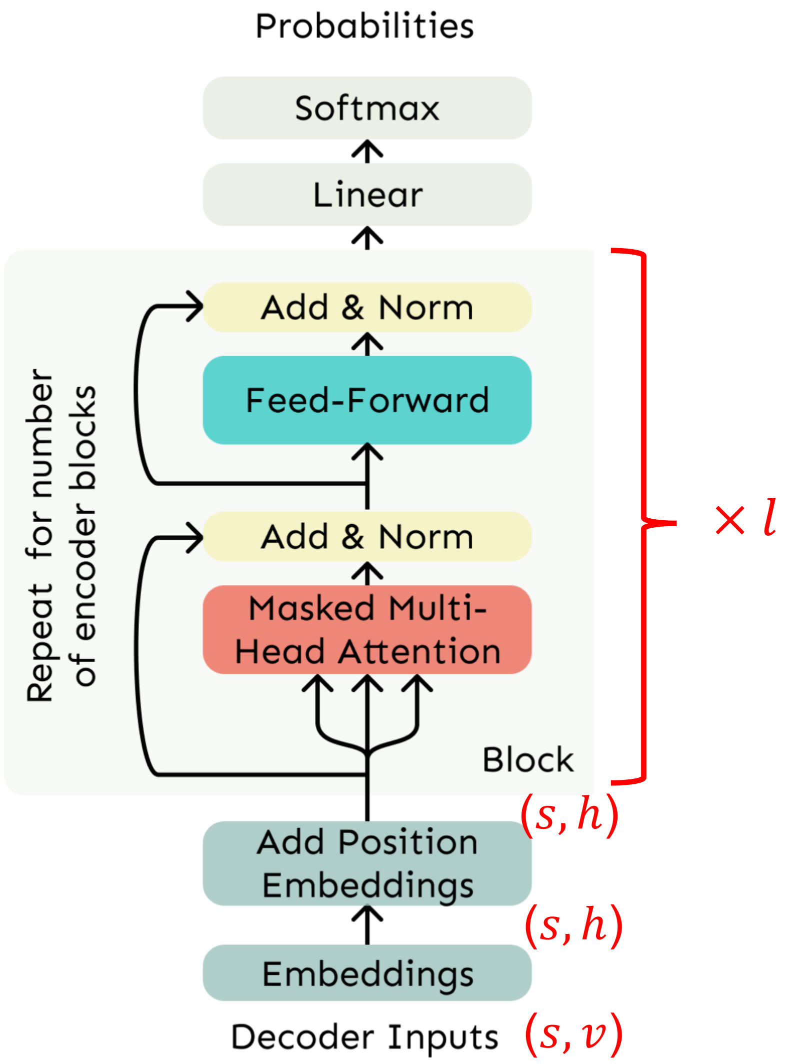
- GPT is based on the **decoder-only** transformer

- We will analyze the parameters, memories, and computation costs for decoder-only transformer

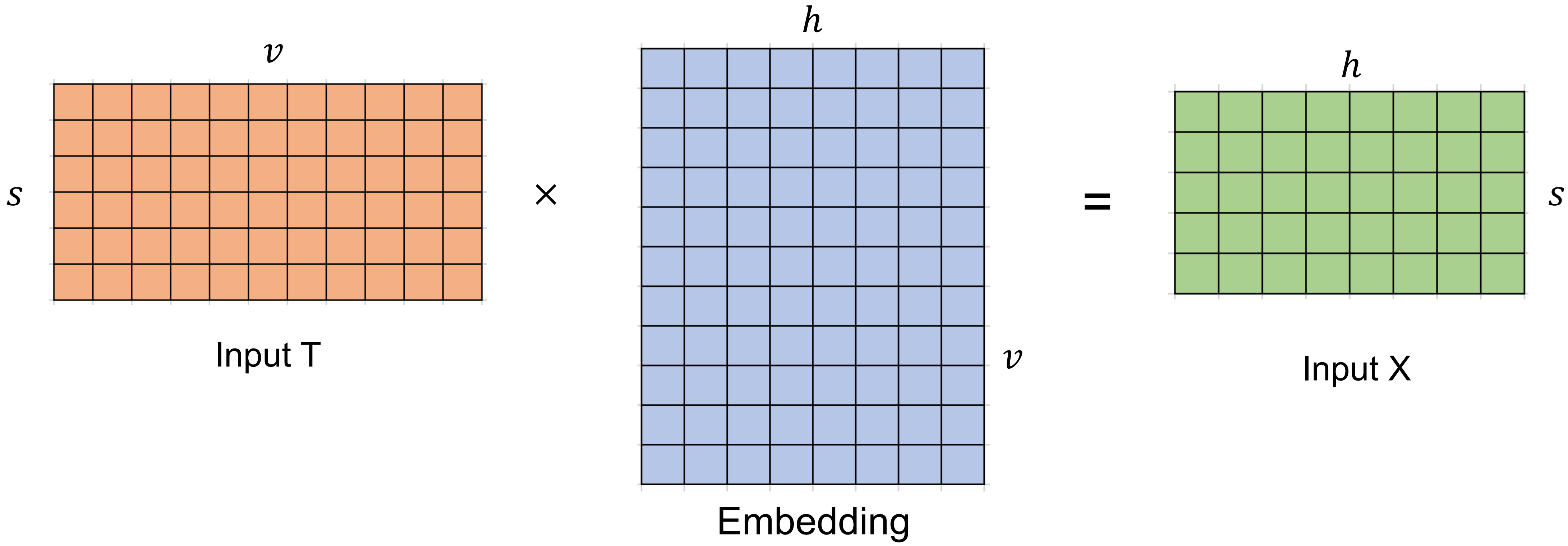


Notations

- Number of the transformer layers: l
- Sequence length: s
- Vocabulary size: v
- Embedding representation dims: h

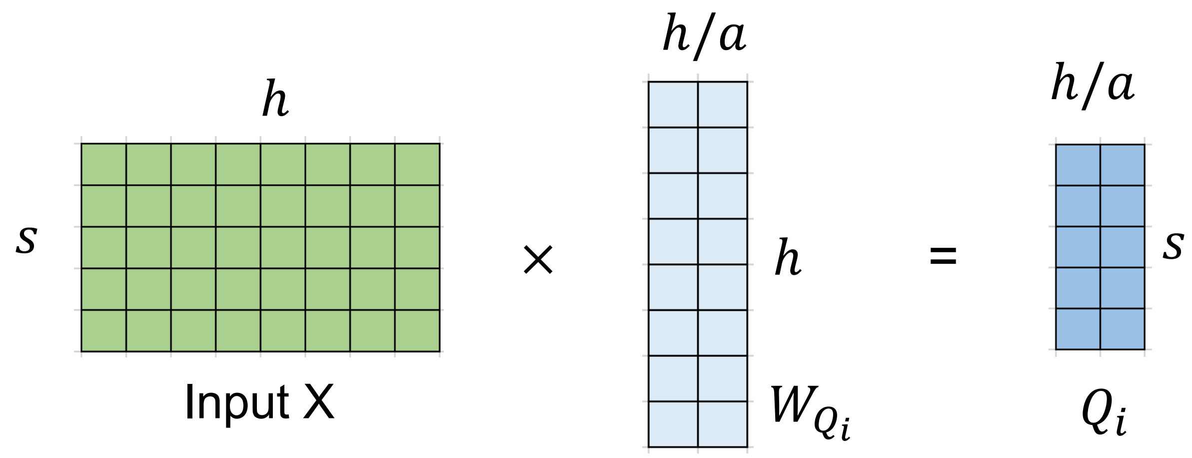


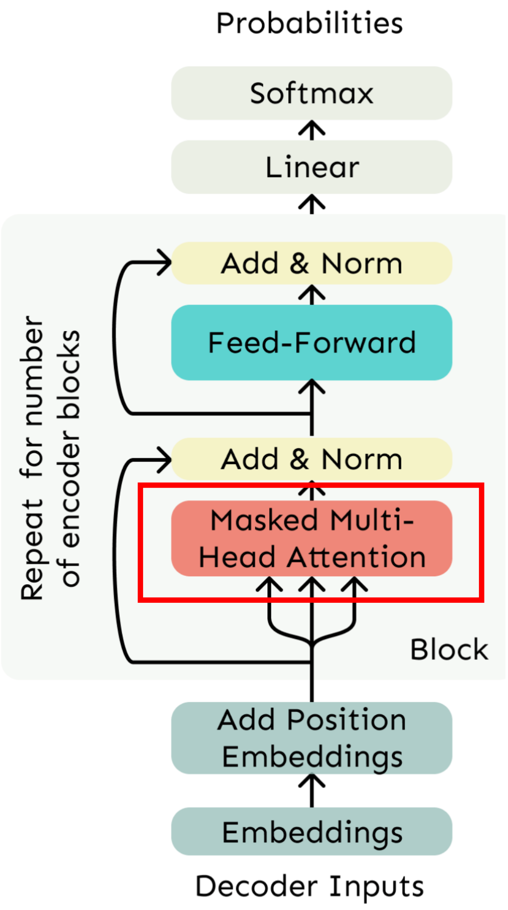
Embedding



Self-attention

- Number of heads: a
- Dims of each W_{Q_i}, W_{K_i} and W_{V_i}: $h \times \frac{h}{a}$





Self-attention

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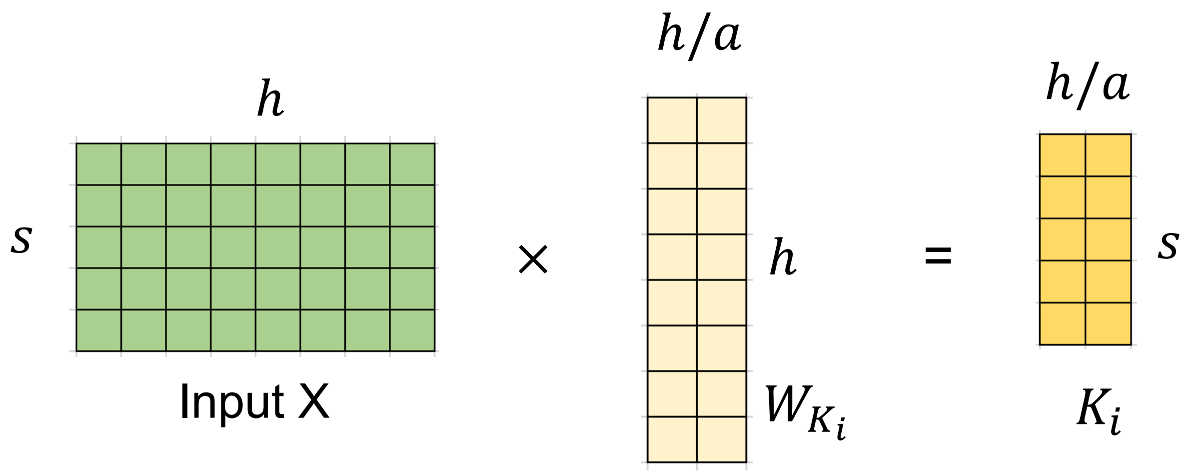

Self-attention

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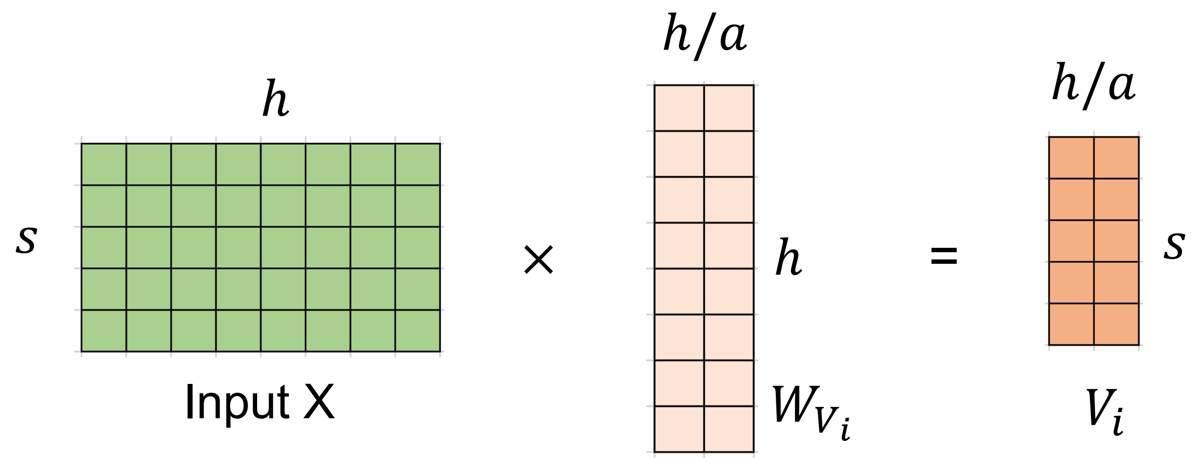
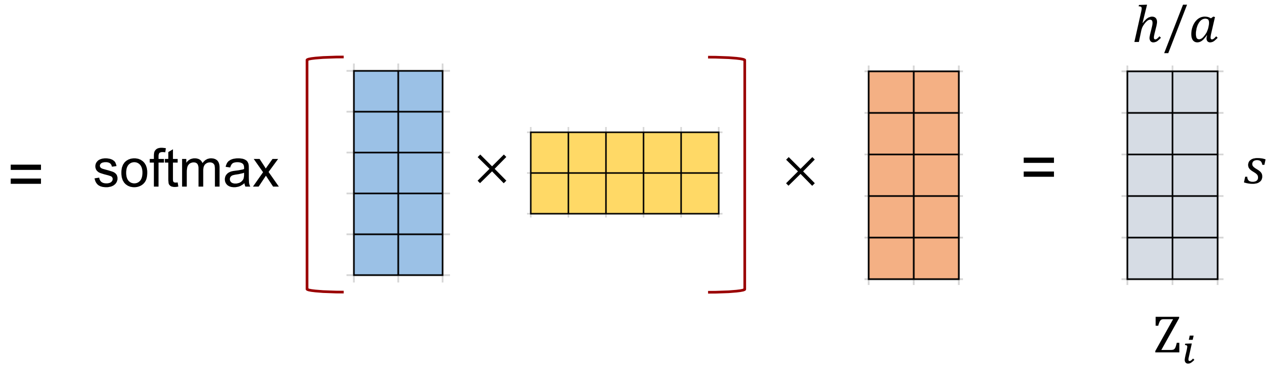


Self-attention

- Number of heads: a
- Dims of each W_{Q_i}, W_{K_i} and W_{V_i}: $h \times \frac{h}{a}$

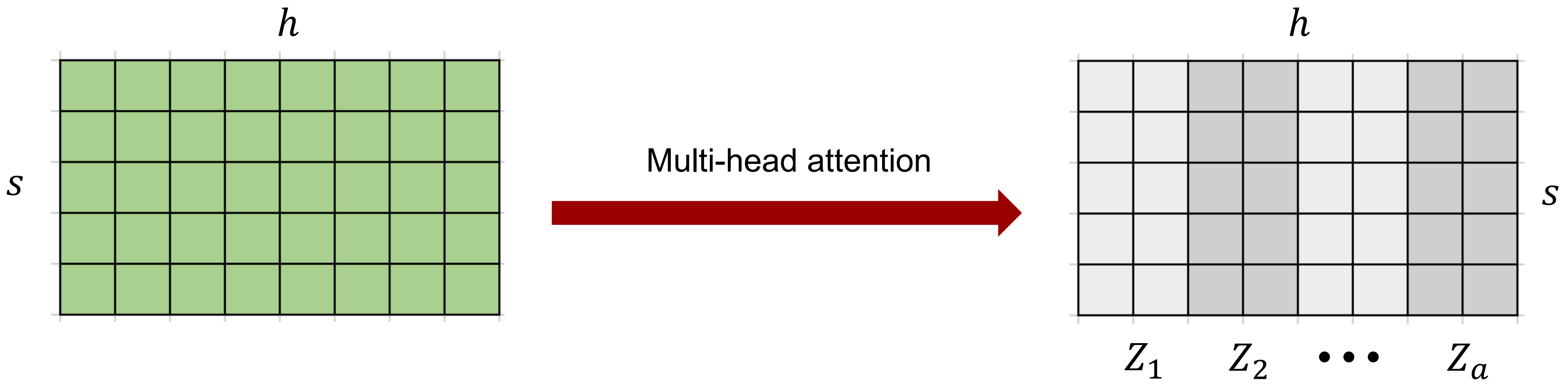
$$\mathrm{softmax}\left(\frac{Q_i K_i^T}{\sqrt{h/a}}\right)V_i$$



One-head attention

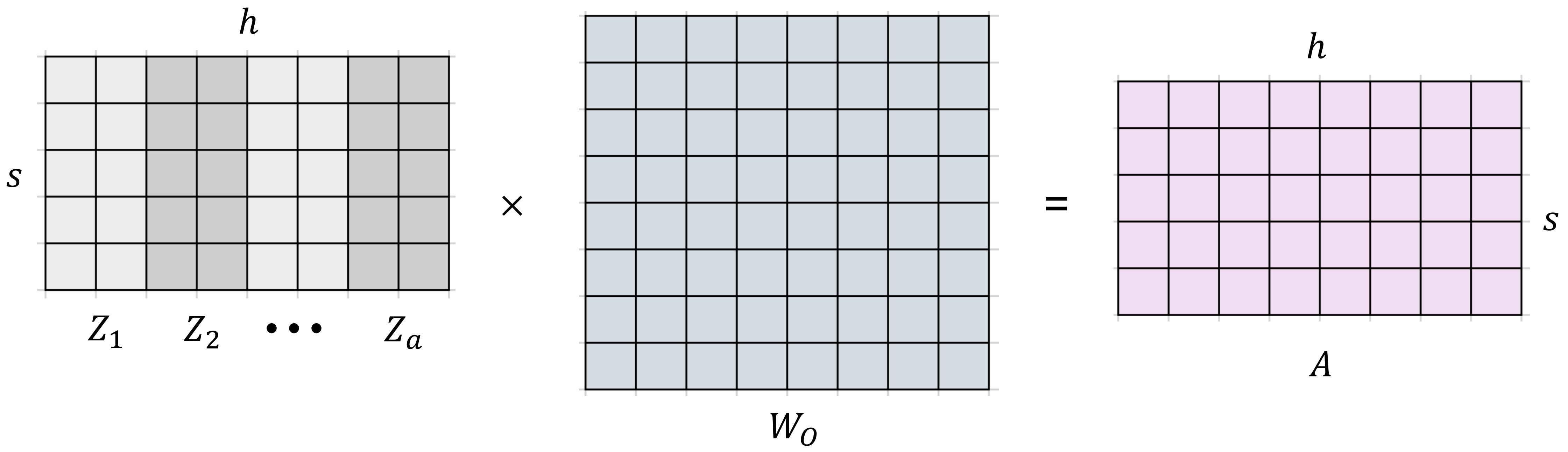
Multi-head attentions

- Number of heads: a
- Dims of each W_{Q_i}, W_{K_i} and W_{V_i}: $h \times \frac{h}{a}$



Multi-head attentions

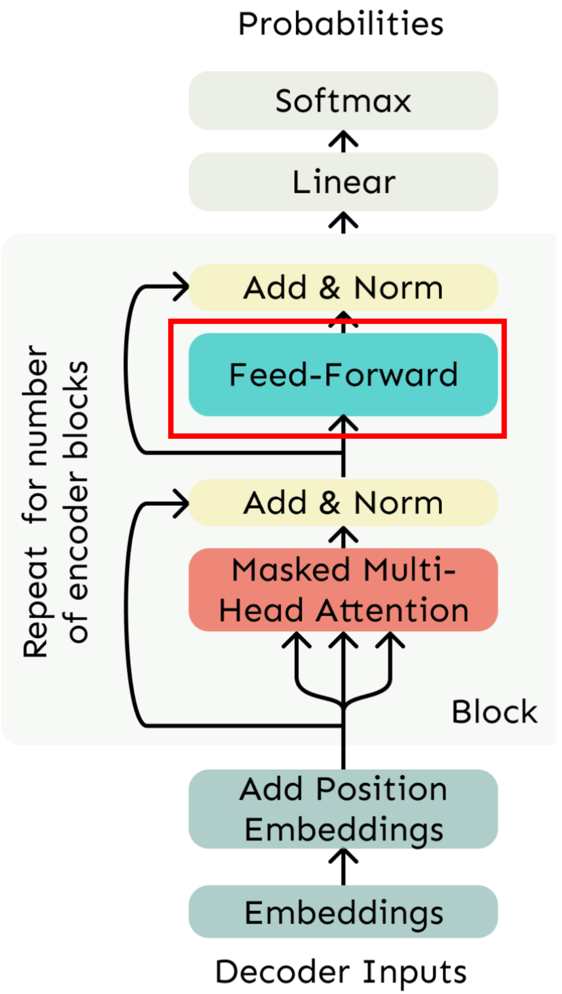
- Number of heads: a
- Dims of each W_O: $h \times h$
- Dims of each W_{Q_i}, W_{K_i} and W_{V_i}: $h \times \frac{h}{a}$



Feed-forward Layer

$$X' = \mathrm{ReLU}(A \cdot W_1 + b_1) \cdot W_2 + b_2$$

- Dims of W_1: $h \times 4h$
- Dims of each W_2: $4h \times h$

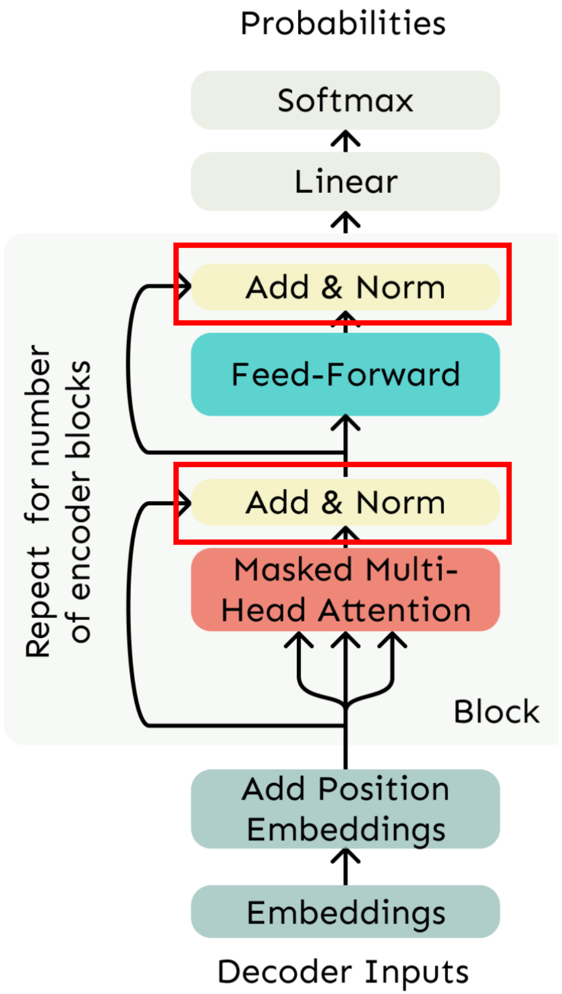


Layer normalization

- Then layer normalization computes:

$$\text{output} = \frac{x - \mu}{\sqrt{\sigma + \epsilon}} * \gamma + \beta$$

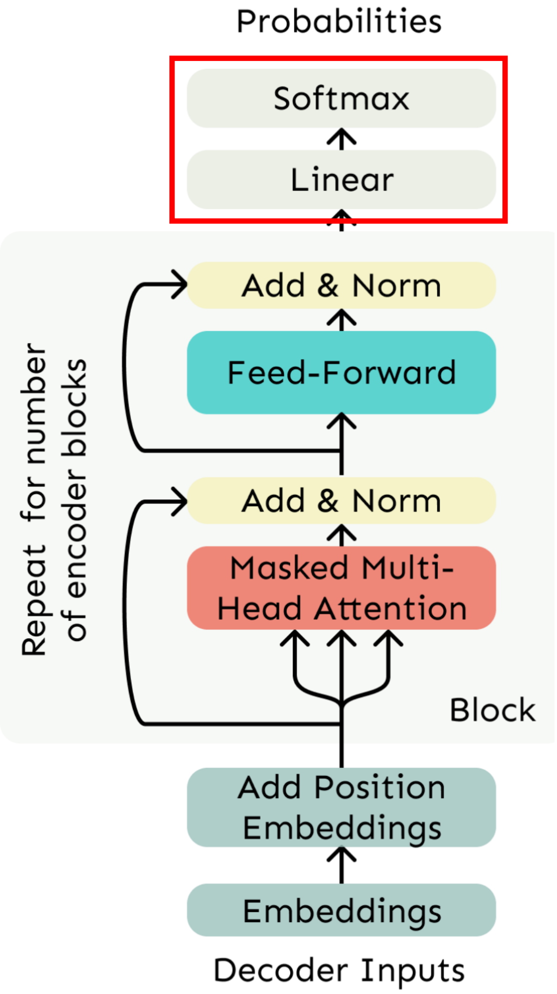
- Dims of γ and β: h



Probability prediction

$$p = \mathrm{Softmax}(X \cdot W_v + b_v)$$

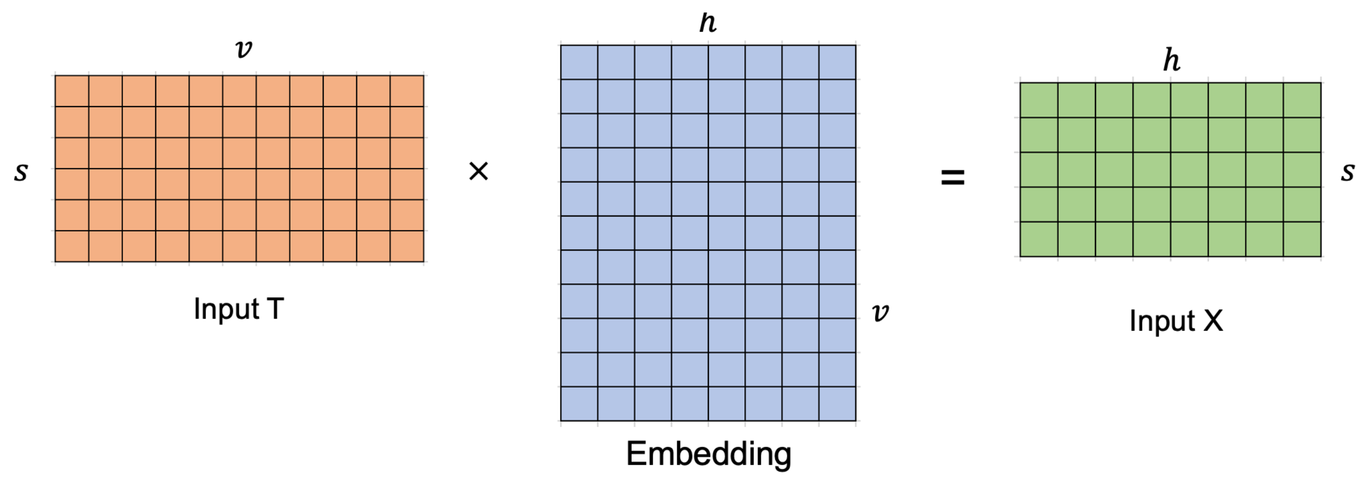
- Dims of W_v: $h \times \mathrm{v}$

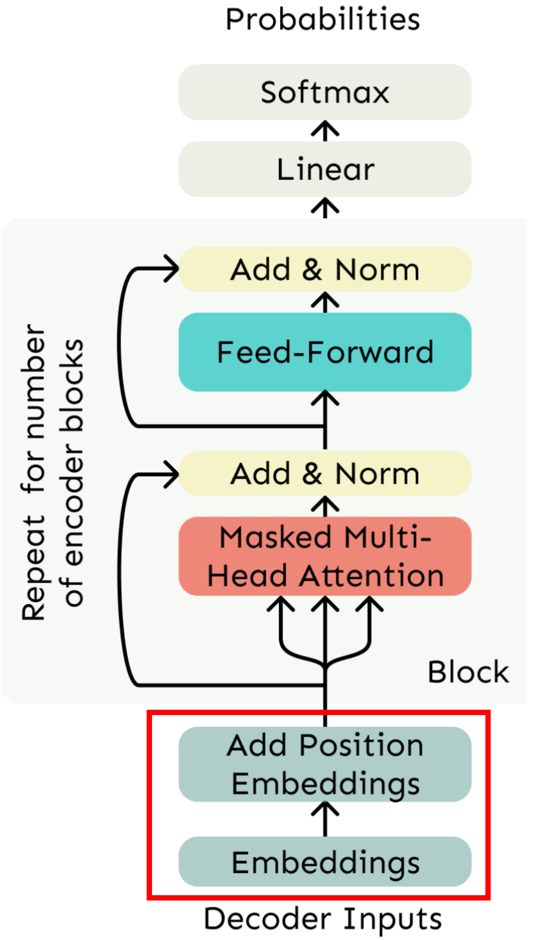


PART 02

Parameters analysis

Embedding



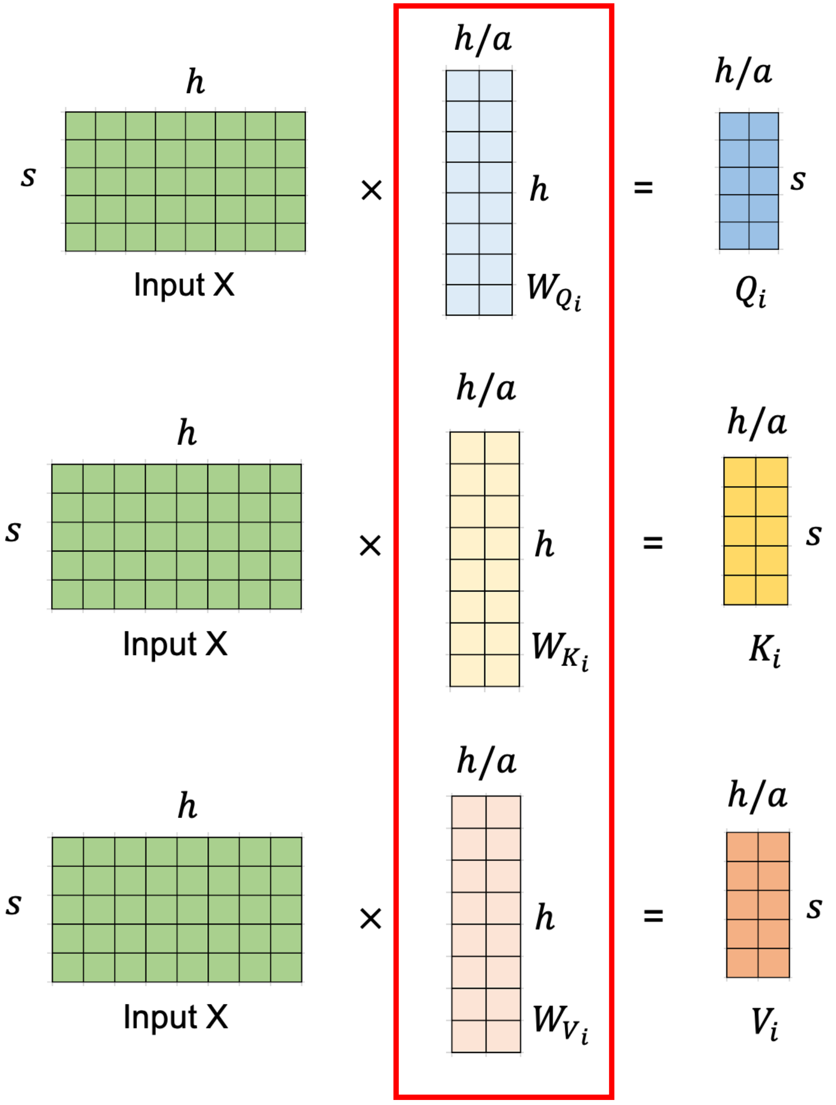


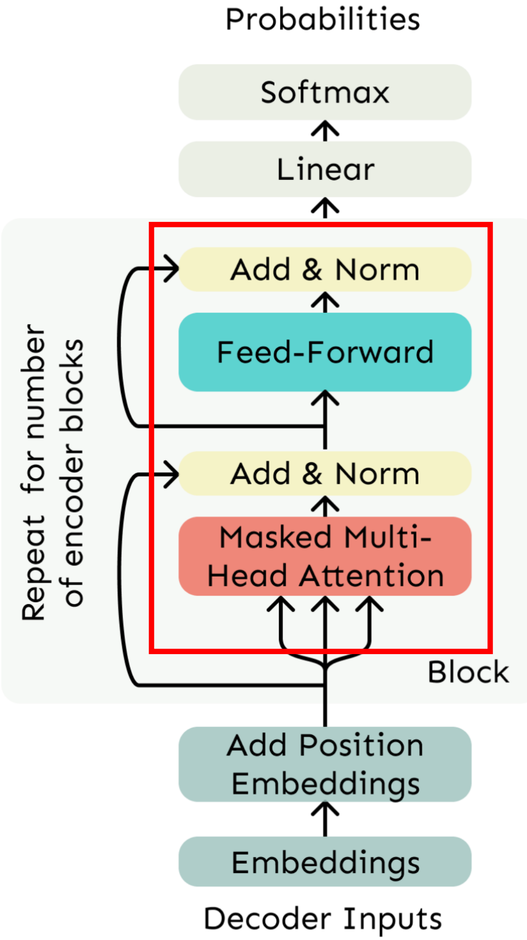
- We need to store the embedding with parameters vh
- Position embedding can be ignored when using RoPE and ALiBi

Multi-head attentions

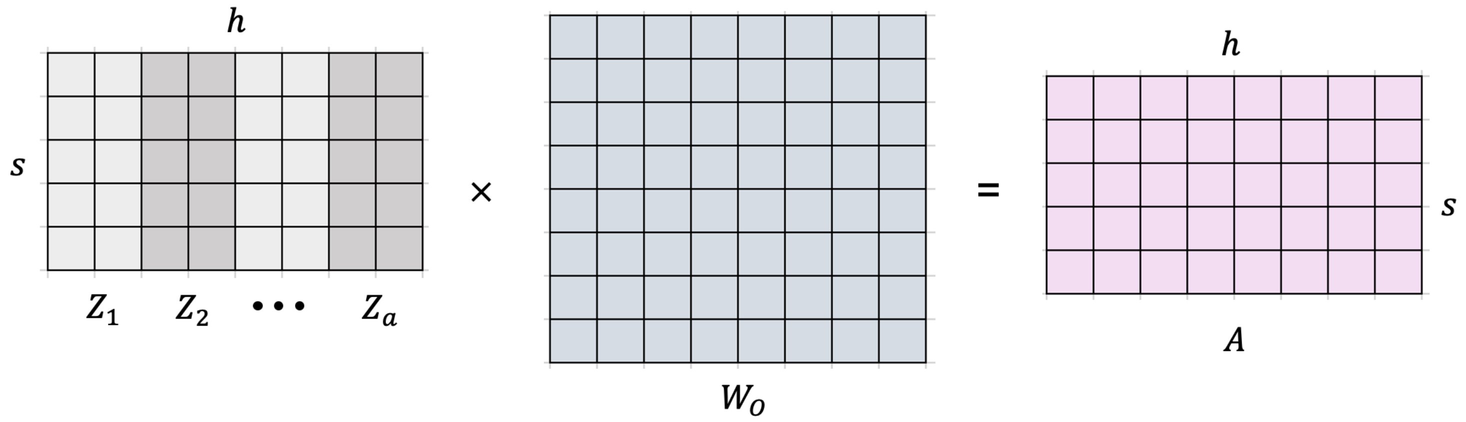
- We need to store W_Q, W_K and W_V

$$3(h^2/a) \times a = 3h^2$$

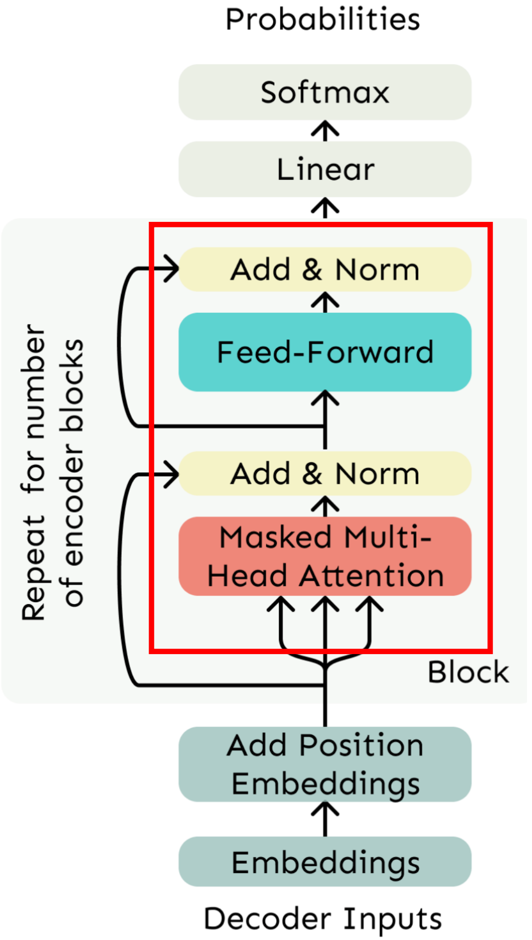




Multi-head attentions



- We need to store W_O: h^2



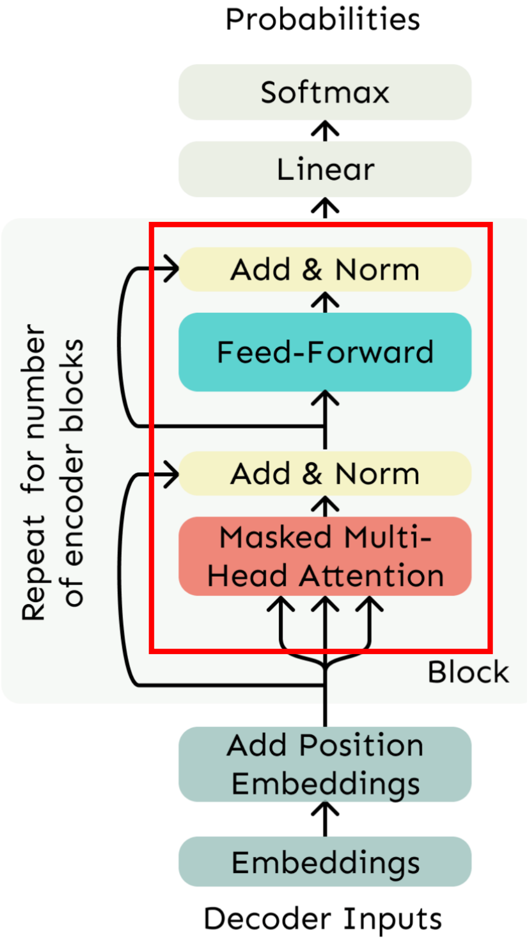
Layer normalization

- Then layer normalization computes:

$$\text{output} = \frac{x - \mu}{\sqrt{\sigma + \epsilon}} * \gamma + \beta$$

- Dims of γ and β: h

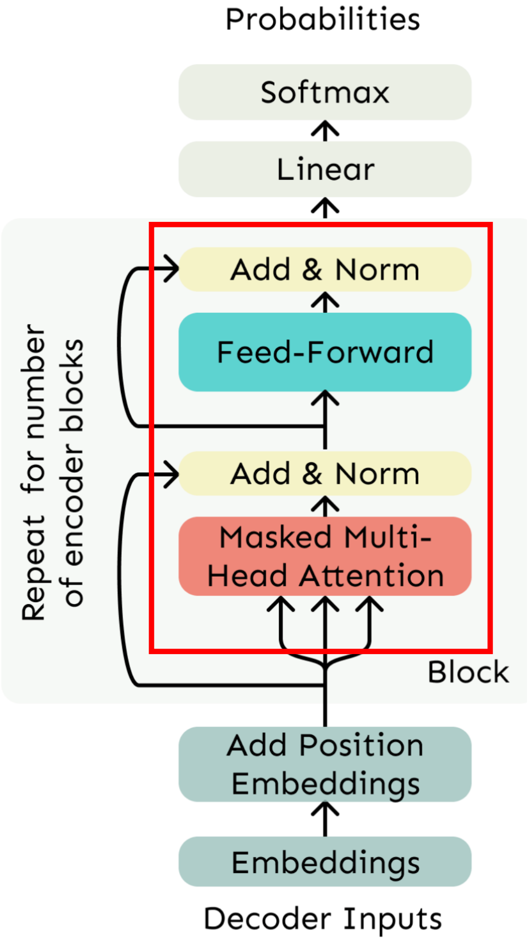
- We should store γ and β. Since their parameters are much smaller than h^2 and vh, we can ignore them



Feed-forward layers

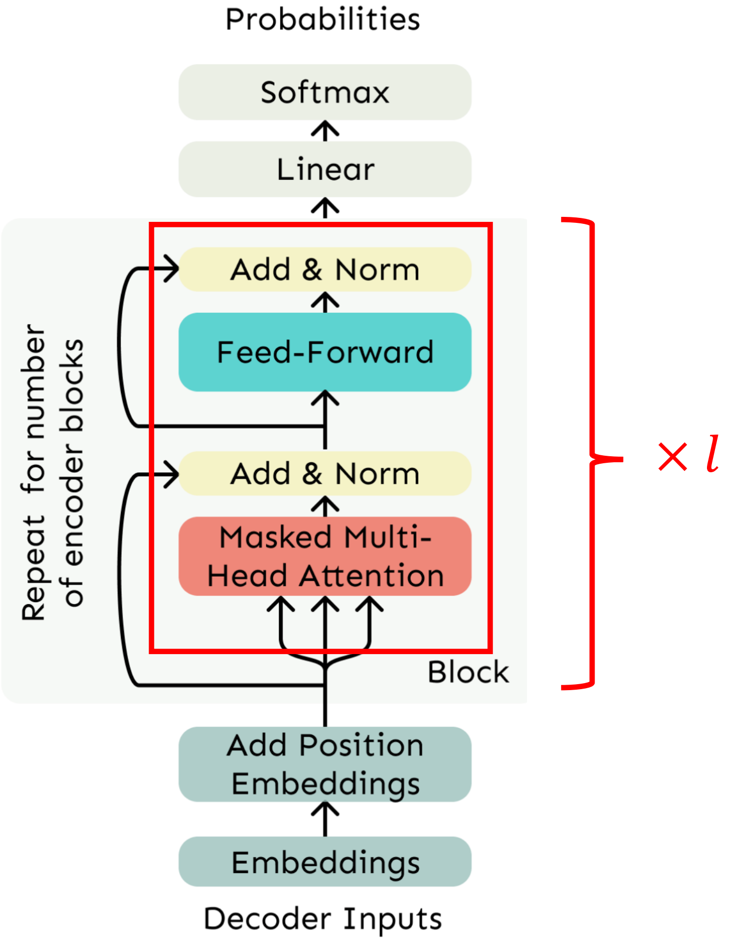
$$X' = \mathrm{ReLU}(A \cdot W_1 + b_1) \cdot W_2 + b_2$$

- Dims of W_1: $h \times 4h$
- Dims of each W_2: $4h \times h$
- We need to store W_1 and W_2 : $8h^2$
- The storage of b_1 and b_2 can be ignored



Transformer block

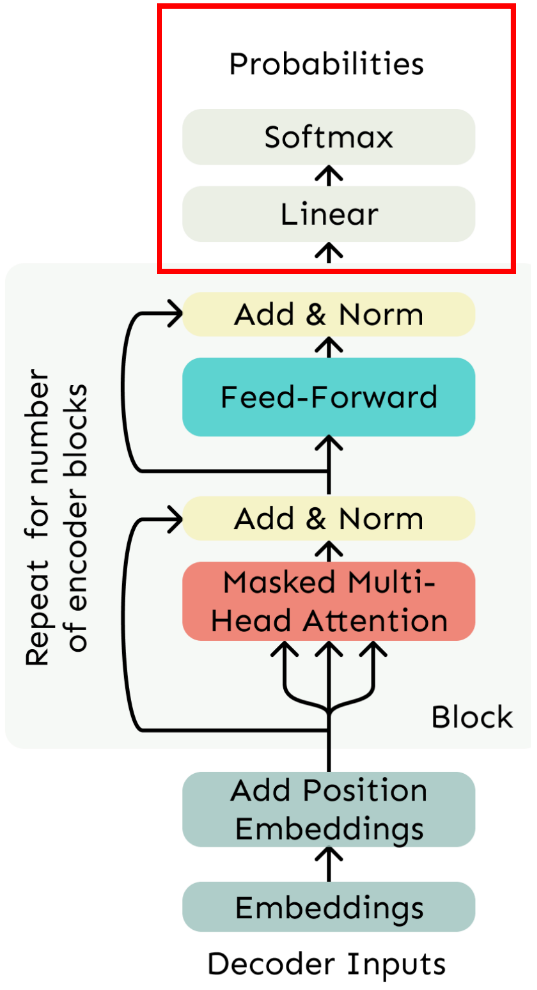
- Multi-head attentions: $4h^2$
- Feed-forward layers : $8h^2$
- l layers of attentions : $(4h^2 + 8h^2)\times l = 12lh^2$



Probability predictions

$$p = \mathrm{Softmax}(X \cdot W_v + b_v)$$

- Dims of W_v: $h \times \mathrm{v}$
- We need to store W_v: hv parameters
- b_v can be ignored

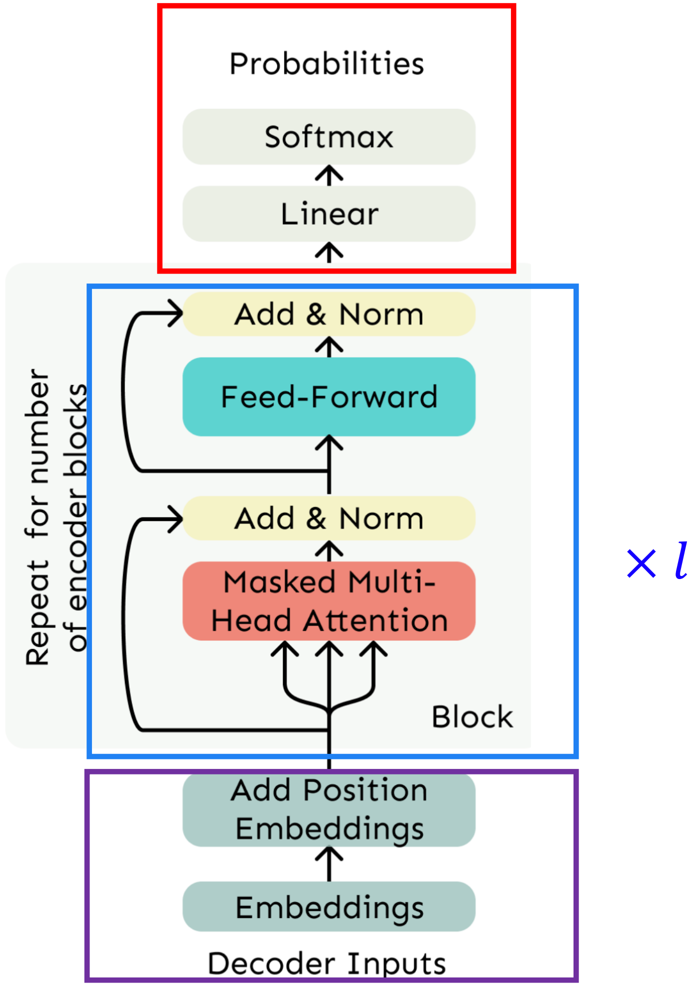


Total parameters

- Embeddings: vh
- Attention blocks: $12lh^2$
- Probability predictions: vh

Total parameters:

$$12\ell h^2 + 2vh$$



Example: LLaMA parameters

- Now we compare our theoretical evaluations with LLaMA model
- $12\ell h^2 + 2vh$ is a very accurate estimation

实际参数量	Embedding h	Attention层数l	Vocab大小v	预估参数量
6.7B	4096	32	32000	6,704,594,944
13.0B	5120	40	32000	12,910,592,000
32.5B	6656	60	32000	32,323,665,920
65.2B	8192	80	32000	64,948,797,440

PART 03

Computations analysis

Flops

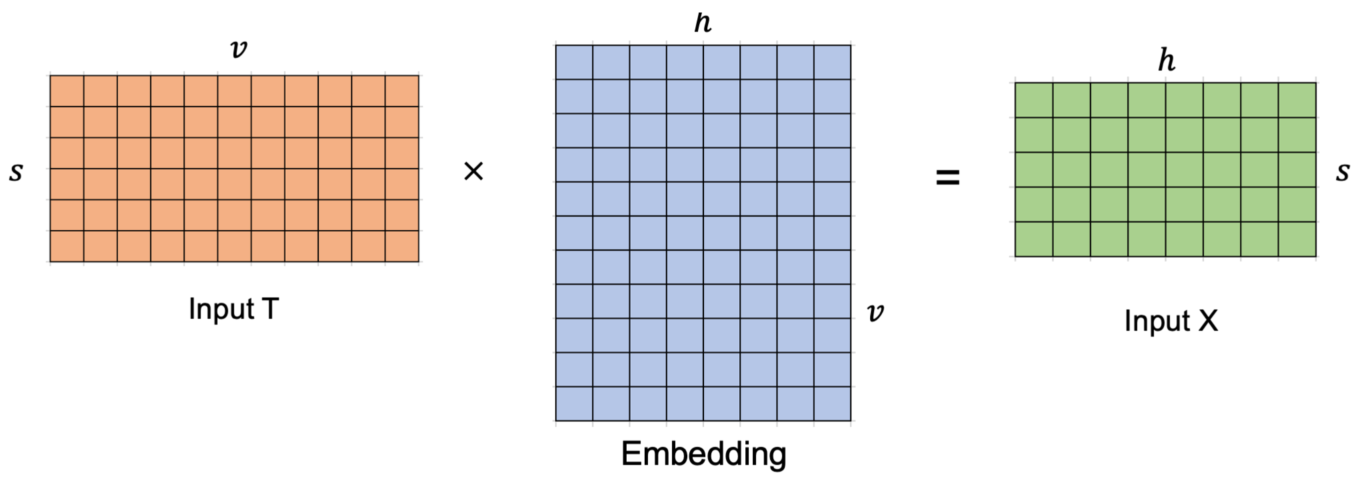
- FLOPs: Floating point operations; gauges the total amount of computations

- Given matrices $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{n\times p}$, to compute AB, we need

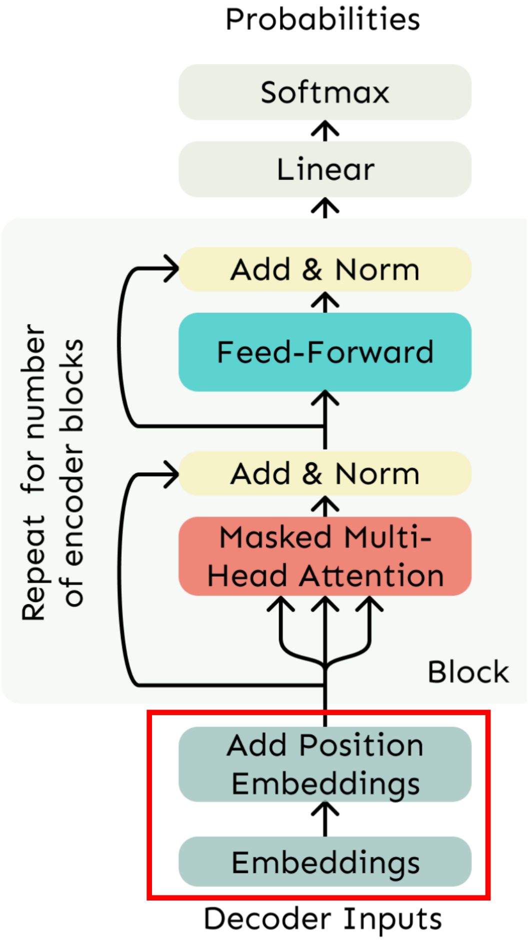
$$\left.\begin{array}{l} mnp \text{ additions} \\ \\ mnp \text{ multiplications} \end{array}\right\} \quad 2mnp \text{ FLOPs}$$

- In transformers, we only count computations raised by matrix operations and ignore vector operations since the later is trivial

Embedding



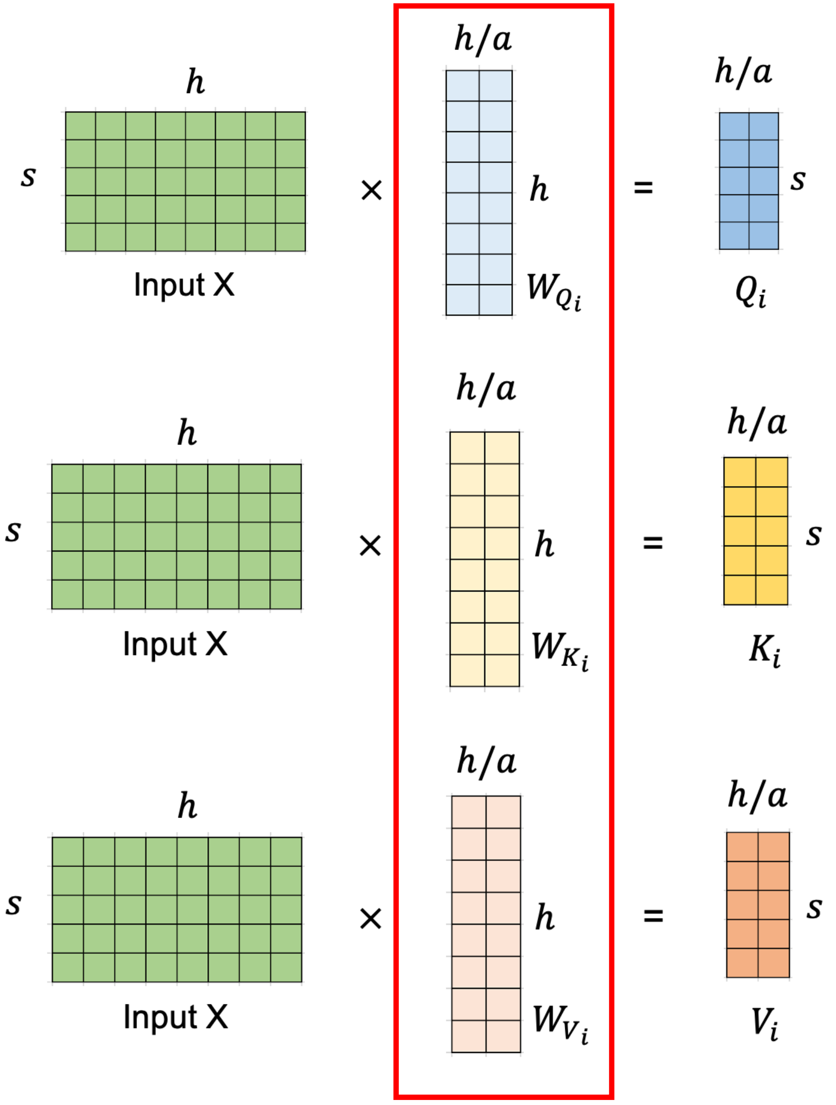
- Word embedding: $2svh$

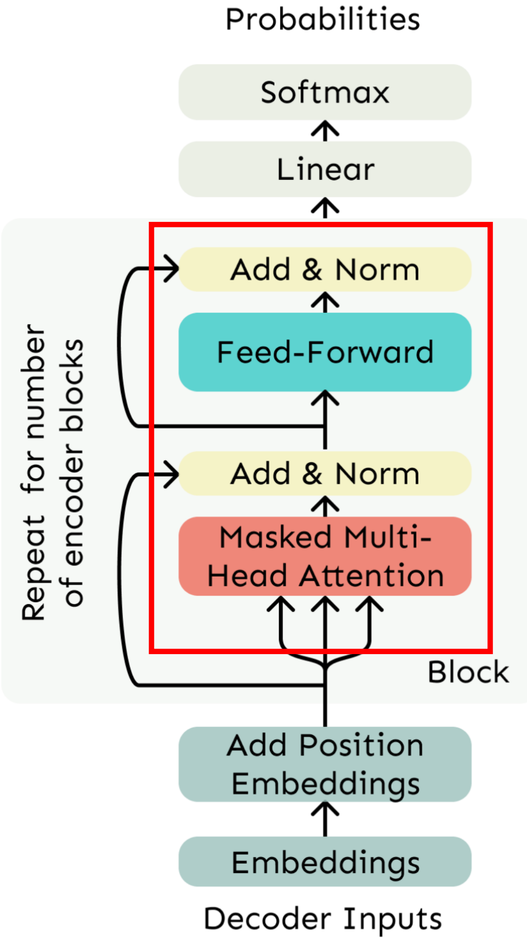


Multi-head attentions

- Multi-head attentions

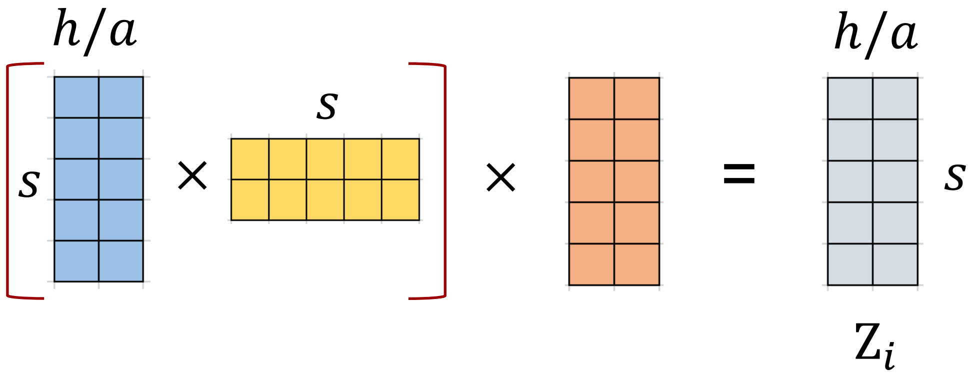
$$6(sh^2/a) \times a = 6sh^2$$





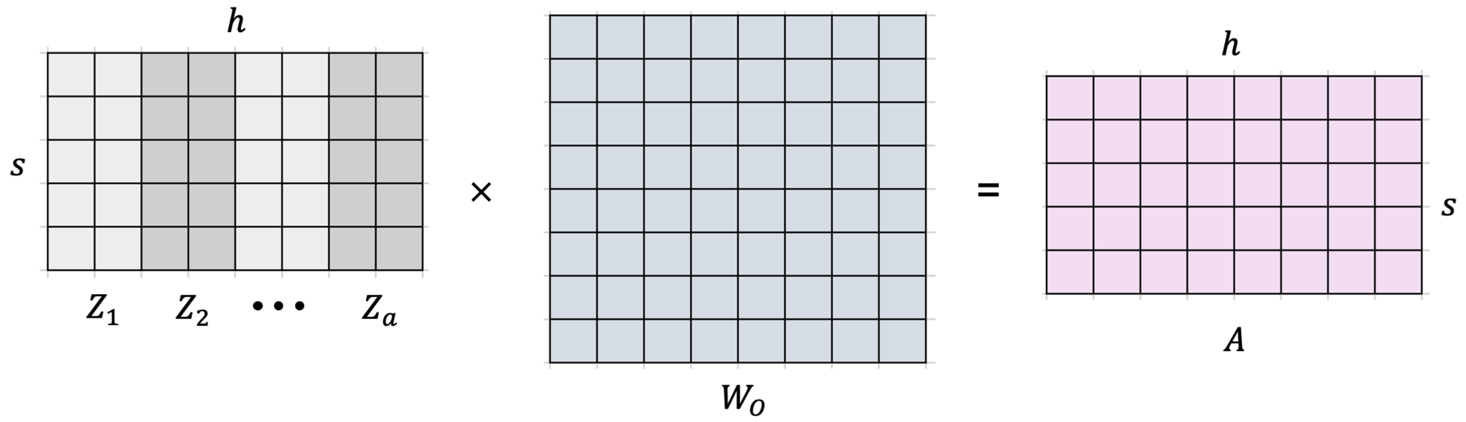
Multi-head attentions

$$\mathrm{softmax}\left(\frac{Q_i K_i^T}{\sqrt{h/a}}\right)V_i \quad = \quad \text{softmax}$$

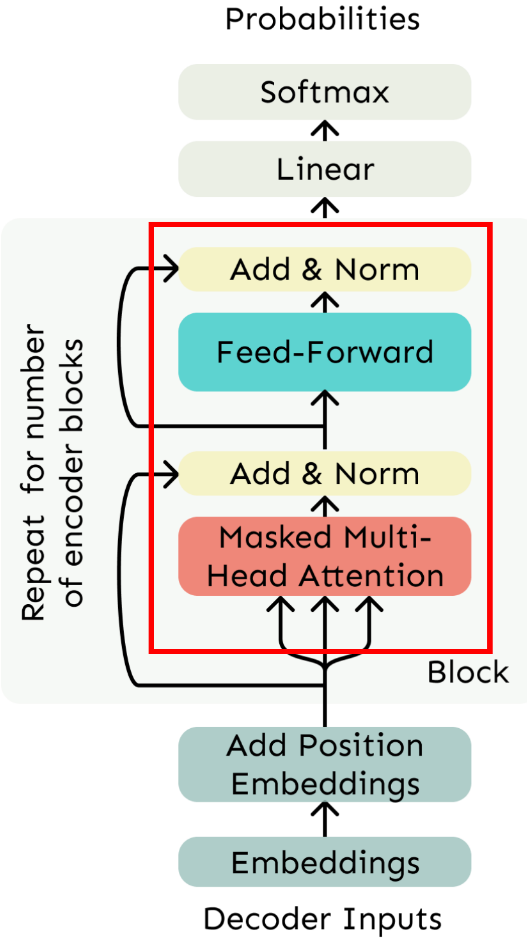


$$(2s^2h/a + 2s^2h/a) \times a = 4s^2h$$

Multi-head attentions



$2sh^2$ Flops

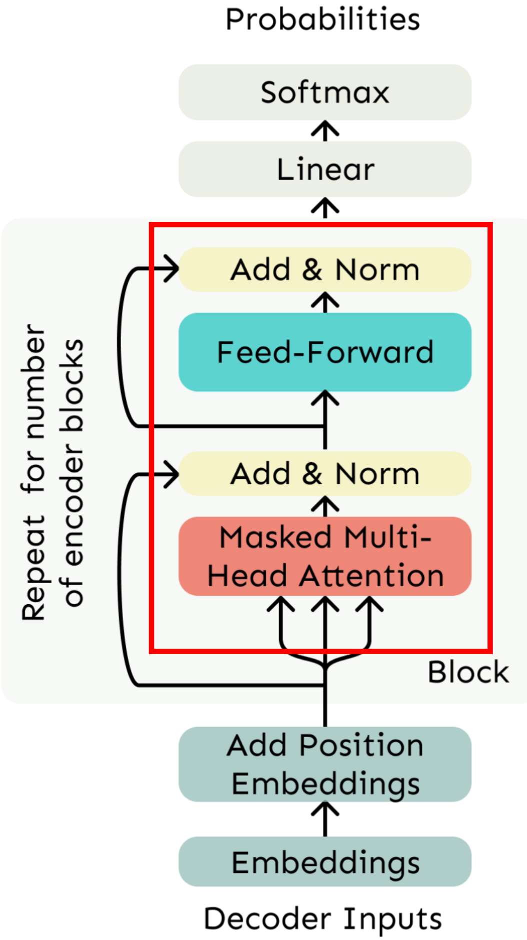


Feed-forward layers

$$X' = \mathrm{ReLU}(A \cdot W_1 + b_1) \cdot W_2 + b_2$$

- Dims of W_1: $h \times 4h$
- Dims of each W_2: $4h \times h$
- $AW_1 + b_1$ needs: $8sh^2$
- $A'W_2 + b_2$ needs: $8sh^2$

$$\left.\begin{matrix} AW_1 + b_1 \text{ needs: } 8sh^2 \\ A'W_2 + b_2 \text{ needs: } 8sh^2 \end{matrix}\right\} 16sh^2$$



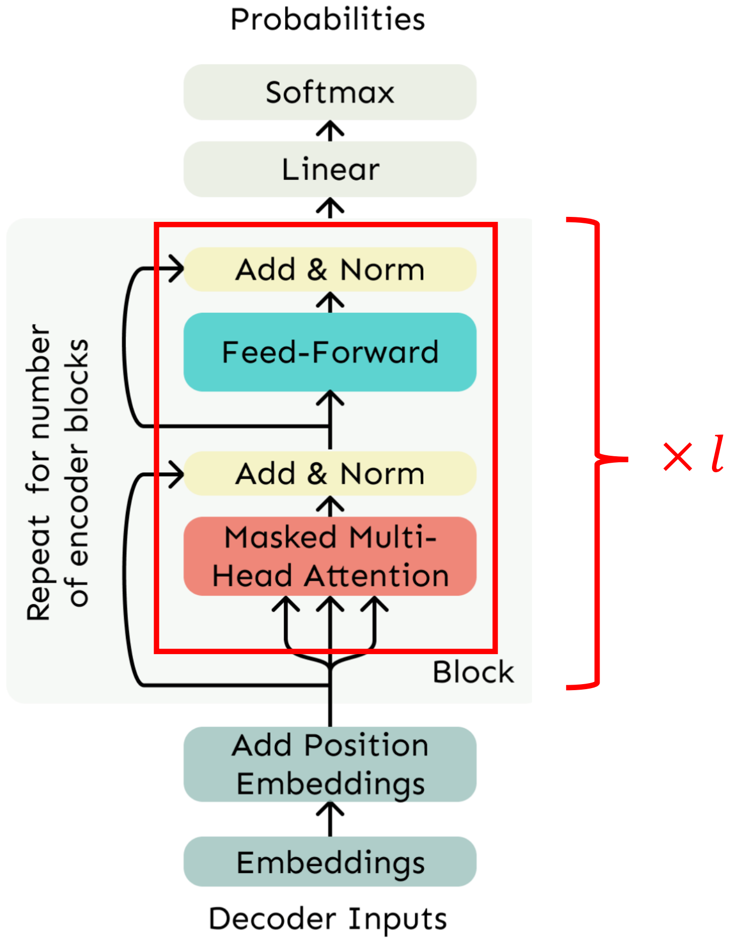
Transformer block

- Multi-head attentions: $8sh^2 + 4s^2\mathrm{h}$

- Feed-forward layers : $16sh^2$

- l layers of attentions :

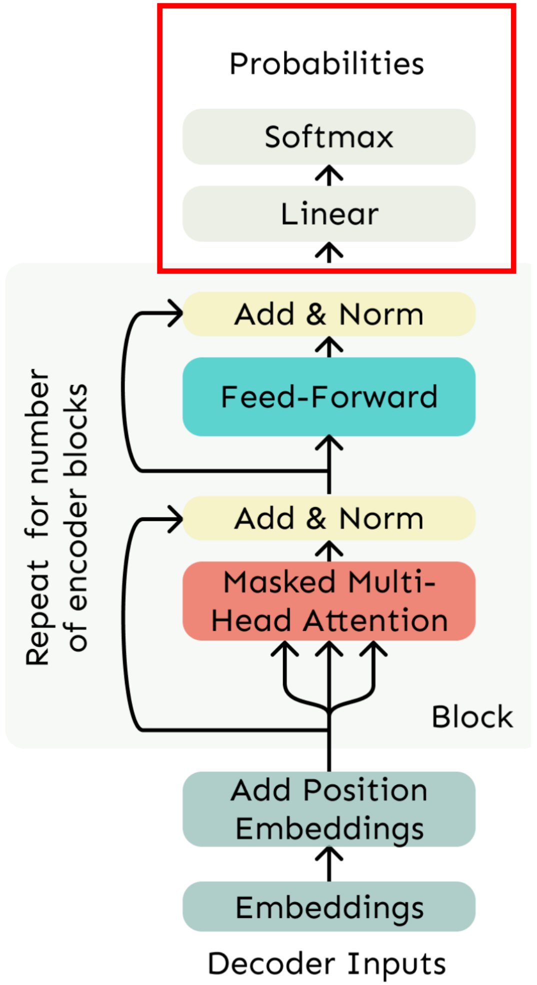
$$(8sh^2 + 16sh^2 + 4s^2h)\times l = 24slh^2 + 4s^2lh$$



Probability predictions

$$p = \text{Softmax}(X \cdot W_v + b_v)$$

- Dims of W_v: $h \times \mathrm{v}$
- We need to : $2shv$ FLOPs



Total forward FLOPs

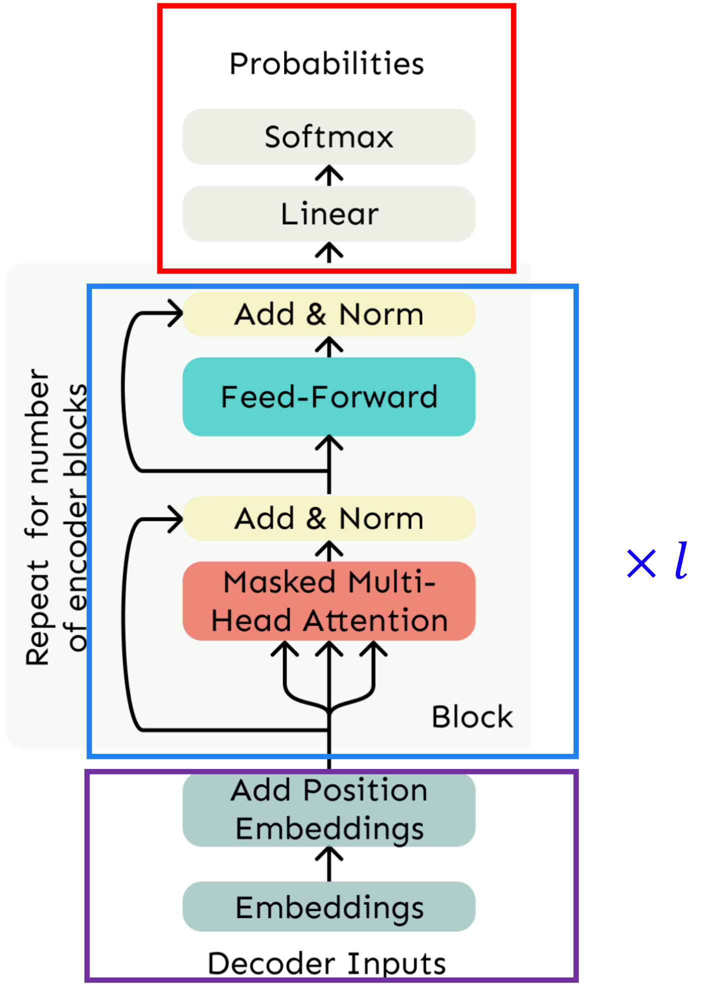
- Embeddings: $2svh$
- Attention blocks: $24lsh^2 + 4s^2lh$
- Probability predictions: $2svh$

Total forward FLOPs:

$$\ell(24sh^2 + 4s^2h) + 4svh$$

When using batch-size b, the total forward FLOPs:

$$b\ell(24sh^2 + 4s^2h) + 4bsvh$$



Total forward-backward FLOPs

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The backward computations are **twice amount** of the forward computations

Total forward-backward FLOPs

$$\Big(b\ell(24sh^2 + 4s^2h) + 4bsvh\Big) \times 3 = 3\Big(b\ell(24sh^2 + 4s^2h) + 4bsvh\Big)$$

When h^2 dominates, the above FLOPs can be simplified as $72bs\ell h^2$

When h^2 dominates, the parameters can be simplified as $P = 12\ell h^2$

Since $T = bs$ is the number of tokens, we thus have $\mathrm{FLOPs} = 6TP$

Example: GPT3-175B

GPT-175B: 175B parameters, 300B tokens

$$6 \times 174600 \times 10^{6} \times 300 \times 10^{9} = 3.1428 \times 10^{23} \, flops$$

Model	Total train compute (PF-days)	Total train compute (flops)	Params (M)	Training tokens (billions)	Flops per param per token	Mult for bwd pass	flops per active param per token	params active for each token
T5-Small	2.08E+00	1.80E+20	60	1,000	3	3	1	0.5
T5-Base	7.64E+00	6.60E+20	220	1,000	3	3	1	0.5
T5-Large	2.67E+01	2.31E+21	770	1,000	3	3	1	0.5
T5-3B	1.04E+02	9.00E+21	3,000	1,000	3	3	1	0.5
T5-11B	3.82E+02	3.30E+22	11,000	1,000	3	3	1	0.5
BERT-Base	1.89E+00	1.64E+20	109	250	6	3	2	1.0
BERT-Large	6.16E+00	5.33E+20	355	250	6	3	2	1.0
RoBERTa-Base	1.74E+01	1.50E+21	125	2,000	6	3	2	1.0
RoBERTa-Large	4.93E+01	4.26E+21	355	2,000	6	3	2	1.0
GPT-3 Small	2.60E+00	2.25E+20	125	300	6	3	2	1.0
GPT-3 Medium	7.42E+00	6.41E+20	356	300	6	3	2	1.0
GPT-3 Large	1.58E+01	1.37E+21	760	300	6	3	2	1.0
GPT-3 XL	2.75E+01	2.38E+21	1,320	300	6	3	2	1.0
GPT-3 2.7B	5.52E+01	4.77E+21	2,650	300	6	3	2	1.0
GPT-3 6.7B	1.39E+02	1.20E+22	6,660	300	6	3	2	1.0
GPT-3 13B	2.68E+02	2.31E+22	12,850	300	6	3	2	1.0
GPT-3 175B	3.64E+03	3.14E+23	174,600	300	6	3	2	1.0