# HOMEWORK 7. GRADIENT CLIPPING

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**Attention**: Turn in your homework at the beginning of our lecture on Nov. 28, 2023

## 1 Backpropagation in RNN

Consider the following RNN formulation

$$h_t = w_x x_t + w_h h_{t-1} \tag{1}$$

$$\hat{y}_t = w_o h_t \tag{2}$$

for $t = 1, \cdots, T$, where $w_x \in \mathbb{R}, w_h \in \mathbb{R}$, and $w_o \in \mathbb{R}$ are parameters to learn, $x_t \in \mathbb{R}$ is the input data at iteration $t$, $h_t \in \mathbb{R}$ is the hidden state at iteration $t$ with initialization $h_0$, and $\hat{y}_t \in \mathbb{R}$ is the output at iteration. Given the samples $\{x_t, y_t\}_{t=1}^T$, we consider the following loss function

$$L(w_x, w_h, w_o; \{x_t, y_t\}_{t=1}^T) = \frac{1}{2T}\sum_{t=1}^{T}(\hat{y}_t - y_t)^2 \tag{3}$$

Please derive $\frac{\partial L}{\partial w_x}$ and $\frac{\partial L}{\partial w_o}$.

## 2 $(L_0, L_1)$-smooth condition

Prove the following statement: Let $f$ be the univariate polynomial $f(x) = \sum_{i=1}^d a_i x^i$. When $d \geq 3$, then $f(x)$ is $(L_0, L_1)$-smooth for some $L_0$ and $L_1$ but not $L$-smooth.

**Hint:** Since $f(x)$ is twice differentiable, the $(L_0, L_1)$-smooth condition can be simplified as $\|\nabla^2 f(x)\| \leq L_0 + L_1\|\nabla f(x)\|$.