HOMEWORK 3. PROJECTED GD AND PROXIMAL GD

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Attention: Turn in your homework at the beginning of our lecture on Oct.17, 2023

1 Convergence Analysis of Projected GD

Use Lemma 2.3 in our notes for Projected gradient descent to prove the following theorem.

> **Theorem 1.1.** Let $\mathcal{X} \subseteq \mathbb{R}^d$ be a closed convex set, $f : \mathbb{R}^d \to \mathbb{R}$ be differentiable, L-smooth and μ-strongly convex. If $\gamma = \frac{1}{L}$, projected gradient descent with arbitrary $x_0 \in \mathcal{X}$ satisfies
> $$\|x_K - x^\star\| \leq (1 - \frac{\mu}{L})^K \|x_0 - x^\star\|, \quad K > 0.$$

2 Proximity Operator of ℓ_1-norm

Show that

$$\forall x \in \mathbb{R}^d, h(x) = \|x\|_1, \left(\text{prox}_{th}(x)\right)_i = \text{sign}(x_i)\max\{|x_i| - t, 0\}, i = 1, \ldots, d.$$