ECE3093 Optimization, Estimation and Numerical Methods. Stochastic Part

Assignment 2 (2013) Part C

This is an individual and not group assignment. This assignment will be released in three separate parts (through Moodle ECE3093 folder Assignments Part2 2013):

Part A Fantasia heart beat data analysis

Part B Gaussian vector simulation

Part C ARMA time and frequency domain simulation

Your work on Part A, Part B and Part C is to be written up and submitted as three separate reports; one for each part. It expected that you will write a MATLAB m-file for each part of this assignment. Advice on how to write these is given to make this as straight forward as possible. If you require further assistance, please seek help as soon as possible.

At Level 3 you are expected to be mature enough to understand that your assignments must be your own work. Accordingly, it is expected that your MATLAB scripts and their output will be your own, original work. All students who submit identical MATLAB scripts or identical output (eg plots of simulations) will all received a mark of zero for these parts of the assessment.

Your reports are to be submitted through the assignment box for Mr Alan Couchman (ground floor, Building 28) at Clayton Campus (Clayton students) or to Dr Nader Kamrani at Sunway Campus (Sunway students).

Deadlines for reports are:

Part A WEEK9, before 6pm Wednesday 8th May 2013

Part B WEEK11, before 6pm Monday 20th May 2013

Part C WEEK12, before 6pm Friday 31st May 2013

NB Include a printout of your m-files for parts A, B and C in the body or as an appendix to your reports.

Part C ARMA time and frequency domain simulation

Q1. Investigation of WN and BM process.

White noise (WN) is a continuous stationary process, and Brownian motion (BM) is its integral. The sequence $\{Z_t\}$ where $Z_t \sim N(0, \sigma^2)$ is discretised WN. A Z_t with $\sigma = 1$ can be simulated by drawing the sequence of from a standard normal distribution. The partial sums of $\{Z_t\}$ generate a discretised Brownian motion sequence called a random walk.

(i) Download the PartC_randomwalk m-file from the Moodle2 ECE3093 Assignments 2013 Part 2 folder and have a look at it. PartC_randomwalk requires the number of time steps as an input; PartC_randomwalk(n).

(ii) Discretiised white noise can be considered an ARMA(0,0) process; $X_t = Z_t$. With reference to the roots of their autogressive polynomials, explain the behaviours you might expect of the three ARMA process:
(1) $X_t = Z_t$ (2) $X_t = Z_t + X_{t-1}$ and (3) $X_t = Z_t + 0.9X_{t-1}$.

(iii) The randomwalk m-file generates the random walk in two ways (these overlay on the MATLAB output plot).

(a) As a cumulative sum of white noise; $Z_t = Z_{t-1} + ... + Z_1$.

(b) As an autoregressive process; $X_t = Z_t + X_{t-1}$.

Change the autoregressive process, so that it implements $X_t = Z_t + 0.9X_{t-1}$.

(iv) Edit randomwalk so that it adds a legend to the plots (see MATLAB Help function 'legend') and make full-page A4 landscape plots of the output of randomwalk(n)
for (a) n = 100 and (b) n = 1000.

(v) Comment on these two plots by answering the following questions:

(1) The theoretical sequence $\{Z_t\} \sim WN(0, \sigma^2)$ is a stationary process. Why?

(2) Would you expect a random walk to be
(a) Gaussian process and, or (b) a stationary process?

(3) Assuming that the Z_t are iid (independent and identically distributed). What would you expect the theoretical mean and variance of the random walk to be at each time step?

(4) What features of your plots support your answers to (1),(2) and (3)?

Q2. Investigation of an ARMA(2,0) process.

(A) Do this first part (a) algebraically. Let $a, b \in \mathbb{R}$ and $\{Z_t\} \sim WN(0, \sigma^2)$.
Consider the following ARMA(2,0)-model (AR(2) process):

$$X_t - aX_{t-1} - 0.5bX_{t-2} = Z_t \,.$$

(i) Show that the theoretical spectral density $f(\lambda) = f_X(\lambda)$ has the following form

$$f(\lambda) = \frac{\sigma^2}{2\pi} \left\| \frac{\theta(e^{-i\lambda})}{\phi(e^{-i\lambda})} \right\|^2 = \frac{\sigma^2}{2\pi} \frac{1}{[1 + a^2 + b^2/4 - a(2-b)\cos(\lambda) - b\cos(2\lambda)]}$$

(B) Obtain your personal values of a and b from the ECE3093 Assignment 2013 Part 2 folder files. eg ECE3093 Clayton Student ID indexed numbers Z U V A a b or ECE3093 Malaysia Student ID indexed numbers Z U V A a b.
$X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2} = Z_t$ where $\phi_1 =$ a and $\phi_2 =$ b/2.

(i) Neatly sketch an argand diagram that shows the location of the roots of the autoregressive polynomial, $\phi(z)$, in relation to the unit circle.

(ii) Explain why your ARMA(2,0) model is stationary.

(iii) Explain why your ARMA(2,0) model is casual.

(C) Download the myDFTspectra13.m-file from the Moodle2 ECE3093 Assignments 2013 Part 2 folder.
This file contains the MATLAB function myDFTspectra13.
The function myDFTspectra13 has the following internal parameters:

(i) varZ = variance of the white noise, $\mathbf{Z_t} \sim \mathrm{WN}(0, \mathrm{varZ})$ eg 1

(ii) a = your Student ID based a = ϕ_1 autoregressive coefficent of X_{t-1}

(iii) b = your Student ID based b = twice the ϕ_2 autoregressive coefficent of X_{t-2}

(iv) n = length of the sample of ARMA $\mathbf{X_t}$ eg 2000 points

(v) maxlags = length of the auto-covariance convolution window for $\gamma_X(h)$
You need to choose maxlags large enough, so that the amplitude of the sample's autocovariance is negligable outside the window; the preset maxlags = 150 should be adequate.

Open and run myDFTspectra13.m in MATLAB. The m-file will pause on the following series of plots:

(i) the ARMA time-series sample overlaying WN(0,varZ).

(ii) sample autocovariance function, SACF.

(iii) the DFT of the sample autocovariance function, PSACF.

(iv) the periodogram of the ARMA sample.

Edit the m-file to do the following:

1 Edit the parameters of the m-file to simulate n points of your ARMA(2,0) model.

2 Calculate the theoretical ACF $\gamma_X(h)$, for $h = [-\text{maxlags} : \text{maxlags}]$.

$\gamma_X(0) = 4\sigma^2(2-b)/[(2+b)((2-b)^2 - 4a^2)]$
$\gamma_X(1) = \gamma_X(0)2a/(2-b)$
$\gamma_X(h) = a\gamma_X(h-1) + \gamma_X(h-2)b/2$ for $h \in \mathbb{N} = 2, 3, 4...$
and is extended to $h \in \mathbb{Z}$ using the symmetry of $\gamma_X(-h) = \gamma_X(h)$

3 Plot the theoretical autocovariances on the sample covariances plot. Use different plot symbols and make an appropriate adjustment to the legend.

4 Calculate the theoretical spectral density $f(\lambda)$ for $\lambda \in [0; +\pi]$ at the frequencies $w \in [0; +\pi]$ returned by MATLAB's periodogram function.

5 Compute the discrete Fourier transform, DFT of the theoretical covariance of the ARMA(2,0) model and do the necessary scaling; as per the PSACF example.

6,7 Semilogy plot the DFT of the the theoretical ACF, PTACF and the theoretical spectral density, TSDAR on the the same plot as the semilogy plot of the DFT of the sample covariance, PSCAF.

8 Plot the theoretical spectral density on the plot of the periodogram of the ARMA time-series. Use different symbols and make an appropriate adjustment to the legend.

(D) Use your edited version of the m-file to obtain A4 landscape hard-copy prints of the four plots it generated:

(1) A plot of a sample of your ARMA process overlayed on a white noise.

(2) A plot of your calculated theoretical autocovariance function, TACF overlayed on the sample covariance function, SACF.

(3) A plot of your calculated spectral density, TSDAR and DFT of your theoretical covariance funcition, PTACF, overlayed on the DFT of the sample covariance, PSACF.

(4) A plot of your calculated spectral density,TSDAR, overlayed on the periodogram of the ARMA sample.

(E) Draw a neat flow-chart diagram indicating clearly how $f(\lambda)$, $\gamma(h)$ and X_t are interrelated by the DFT and periodogram calculation. **Note:** xcorr calculates the autocorrelation of the time-series. An autocorrelation multiplies a time shifted version of the series by itself; and the option "unbiased" corrects for the finite length of the sample. In contrast, a convolution would multiply a shifted and time reversed version of the time series with itself.

(F) Discussion of the behaviour of the plots in (D)

(i) The theoretical autocovariance was obtained by solving the Yule-Walker equations. This method of solution relies on the assumption that white noise, Z is perfectly white; that it is utterly uncorrelated; that, $\gamma_Z(0) = \sigma^2 = varZ$ but otherwise, $\gamma_Z(h) = 0$ for $|h| \in \mathbb{N}$. The autocovariance derived from the time-series sample is derived using a finite sample of a simulated time series. Compare your sample covariance with the theroetical covariance. Does the sample covariance show evidence of the finiteness of the sample that was used to generate it?

(ii) The periodogram estimates the spectral density using the DFT, yet, whereas the DFT of the sample covariance agrees well with the theoretical spectral density, the periodogram does not. Just by looking at your spectral density plot, how do the theoretical spectral density and the periodogram appear to be related?

END OF PART C