

Best Linear Prediction, Multivariate Situation, formulae

- ▶ Best linear prediction:

$$\mathcal{P}(\mathbf{X}|\mathbf{Y}) = E[\mathbf{X}] + \text{Cov}(\mathbf{X}, \mathbf{Y})\text{Cov}(\mathbf{Y})^{-1}(\mathbf{Y} - E[\mathbf{Y}])$$

$$\mathcal{P}(\mathbf{X}|\mathbf{Y} = \mathbf{y}) = E[\mathbf{X}] + \text{Cov}(\mathbf{X}, \mathbf{Y})\text{Cov}(\mathbf{Y})^{-1}(\mathbf{y} - E[\mathbf{Y}])$$

- ▶ Prediction error:

$$\text{Cov}(\mathbf{X} - \mathcal{P}(\mathbf{X}|\mathbf{Y})) = \text{Cov}(\mathbf{X}) - \text{Cov}(\mathbf{X}, \mathbf{Y})\text{Cov}(\mathbf{Y})^{-1}\text{Cov}(\mathbf{X}, \mathbf{Y})^T$$

- ▶ Here we define

$$\text{Cov}(\mathbf{X}, \mathbf{Y}) := \text{Cov}(\mathbf{Y}, \mathbf{X})^T := E[(\mathbf{X} - E[\mathbf{X}])(\mathbf{Y} - E[\mathbf{Y}])^T] \in \mathbb{R}^{m \times n}$$