1)Let Z be binomial random variable so that if $Z = 1$, deaths follows Poisson(μ_1) and if $Z = 2$, deaths follows Poisson(μ_2). Obviously, Z is the latent variable which we cannot observe. First we calculate the conditional distribution $(Z|X, \theta)$:

$$P(Z_i = 1|X = x_i, \theta_m) = \frac{\alpha_m \frac{\mu_{m,1} e^{-\mu_{m,1}}}{x_i!}}{\alpha_m \frac{\mu_{m,1} e^{-\mu_{m,1}}}{x_i!} + (1-\alpha_m)\frac{\mu_{m,2} e^{-\mu_{m,2}}}{x_i!}} = z_{x_i}(\theta_m)$$

$$P(Z_i = 2|X = x_i, \theta_m) = \frac{(1-\alpha_m)\frac{\mu_{m,2} e^{-\mu_{m,2}}}{x_i!}}{\alpha_m \frac{\mu_{m,1} e^{-\mu_{m,1}}}{x_i!} + (1-\alpha_m)\frac{\mu_{m,2} e^{-\mu_{m,2}}}{x_i!}} = 1 - z_{x_i}(\theta_m)$$

Now, we calculate the $Q(\theta|\theta_m)$.

$$Q(\theta|\theta_m) = \mathbb{E}_{Z|X,\theta_m}\left[\log L(\theta; X, Z)\right] = \sum_i^n \mathbb{E}_{Z|X,\theta_m}\left[\log L(\theta; x_i, z_i)\right]$$

$$= \sum_i^n \left[P(Z_i = 1|X = x_i, \theta_m) \log L(\theta; x_i, z_i) + P(Z_i = 2|X = x_i, \theta_m) \log L(\theta; x_i, z_i)\right]$$

$$= \sum_i^n \left[z_{x_i}(\theta_m) * \log(\alpha \frac{\mu_1^{x_i} e^{-\mu_1}}{x_i!}) + (1 - z_{x_i}(\theta_m)) * \log((1-\alpha)\frac{\mu_2^{x_i} e^{-\mu_2}}{x_i!})\right]$$

$$= \sum_i^n \left[z_{x_i}(\theta_m) * (\log\alpha + x_i \log\mu_1 - \mu_1 - \log x_i!) + (1 - z_{x_i}(\theta_m)) * (\log(1-\alpha) + x_i \log\mu_2 - \mu_2 - \log x_i!)\right]$$

Calculate the maximum point θ_{m+1}.

$$\frac{\partial Q(\theta|\theta_m)}{\partial \mu_1} = \sum_i^n z_{x_i}(\theta_m)\left[\frac{x_i}{\mu_1} - 1\right] = \sum_{i=0}^9 n_i z_i(\theta_m)\left[\frac{i}{\mu_1} - 1\right] = 0$$

$$\Rightarrow \mu_{m+1,1} = \frac{\sum_{i=0}^9 z_i(\theta_m) i \theta_m}{\sum_{i=0}^9 z_i(\theta_m)\theta_m}$$

$$\frac{\partial Q(\theta|\theta_m)}{\partial \mu_2} = \sum_i^n (1 - z_{x_i}(\theta_m))\left[\frac{x_i}{\mu_2} - 1\right] = \sum_{i=0}^9 n_i(1 - z_i(\theta_m))\left[\frac{i}{\mu_2} - 1\right] = 0$$

$$\Rightarrow \mu_{m+1,2} = \frac{\sum_{i=0}^9 (1 - z_i(\theta_m)) i \theta_m}{\sum_{i=0}^9 (1 - z_i(\theta_m))\theta_m}$$

$$\frac{\partial Q(\theta|\theta_m)}{\partial \alpha} = \sum_i^n \left[\frac{z_{x_i}(\theta_m)}{\alpha} - \frac{1 - z_{x_i}(\theta_m)}{1-\alpha}\right] = \sum_{i=0}^9 n_i\left[\frac{z_i(\theta_m)}{\alpha} - \frac{1 - z_i(\theta_m)}{1-\alpha}\right] = 0$$

$$\Rightarrow \alpha_{m+1} = \frac{\sum_{i=0}^9 n_i z_i(\theta_m)}{\sum_{i=0}^9 n_i}$$

Proof is finished.