2)

$$tr\left[D_r^{-\frac{1}{2}}(P-\widehat{P})D_c^{-1}(P-\widehat{P})^tD_r^{-\frac{1}{2}}\right] = tr\left[\left(D_r^{-\frac{1}{2}}(P-\widehat{P})D_c^{-\frac{1}{2}}\right)\left(D_r^{-\frac{1}{2}}(P-\widehat{P})D_c^{-\frac{1}{2}}\right)^t\right]$$

By SVD, $D_r^{-\frac{1}{2}}PD_c^{-\frac{1}{2}} = U\Lambda V^t$.
By Eckart-Young Theorem, the best t-rank reduced matrix $D_r^{-\frac{1}{2}}\widehat{P}D_c^{-\frac{1}{2}} = U\Lambda_t V^t$, where the first t diagonal elements of Λ_t are same with those of Λ and other elements are zeros. Thus, $\widehat{P} = D_r^{\frac{1}{2}}U\Lambda_t V^t D_c^{\frac{1}{2}}$.
When $t=1$, $D_r^{-\frac{1}{2}}PD_c^{-\frac{1}{2}} = \left(\frac{n_{ij}}{\sqrt{n_i}\sqrt{n_j}}\right)_{ij}$. Meanwhile we have $D_r^{-\frac{1}{2}}r = (\sqrt{\frac{n_{i+}}{n}})_i$ and $D_c^{-\frac{1}{2}}c = (\sqrt{\frac{n_{+j}}{n}})_j$.

$$\left(D_r^{-\frac{1}{2}}PD_c^{-\frac{1}{2}}\right)\left(D_r^{-\frac{1}{2}}PD_c^{-\frac{1}{2}}\right)^t\left(D_r^{-\frac{1}{2}}r\right) = \left(\sum_{k,j}\frac{n_{ik}}{\sqrt{n_{i+}n_{+k}}}\frac{n_{jk}}{\sqrt{n_{j+}n_{+k}}}\sqrt{\frac{n_{j+}}{n}}\right)_i = \left(\sqrt{\frac{n_{i+}}{n}}\right)_i = D_r^{-\frac{1}{2}}r$$

$$\left(D_r^{-\frac{1}{2}}PD_c^{-\frac{1}{2}}\right)^t\left(D_r^{-\frac{1}{2}}PD_c^{-\frac{1}{2}}\right)\left(D_c^{-\frac{1}{2}}c\right) = \left(\sum_{k,i}\frac{n_{kj}}{\sqrt{n_{k+}n_{+j}}}\frac{n_{ki}}{\sqrt{n_{k+}n_{+i}}}\sqrt{\frac{n_{+i}}{n}}\right)_j = \left(\sqrt{\frac{n_{+j}}{n}}\right)_j = D_c^{-\frac{1}{2}}c$$

So we know $D_r^{-\frac{1}{2}}PD_c^{-\frac{1}{2}}$ have singular value $\lambda=1$. We denote $D_r^{-\frac{1}{2}}PD_c^{-\frac{1}{2}}$ as A and we know its element $\frac{n_{ij}}{\sqrt{n_{i+}n_{+j}}} \leq 1$. So $\lim_n(AA^t)^n = B$ and each row and column of B is either 1 or 0. If A had a singular value greater than 1, this would not happen. So $\lambda = 1$ is the greatest singular value. Namely,

$$D_r^{-\frac{1}{2}}PD_c^{-\frac{1}{2}} = (D_r^{-\frac{1}{2}}r|U_{n*(r-1)})\begin{pmatrix}1 & 0\\ 0 & \Lambda'\end{pmatrix}(D_c^{-\frac{1}{2}}c|V_{n*(s-1)})^t$$

When rank $t=1$, $D_r^{-\frac{1}{2}}\widehat{P}D_c^{-\frac{1}{2}} = D_r^{-\frac{1}{2}}rc^tD_c^{-\frac{1}{2}} \Rightarrow \widehat{P} = rc^t$.