$$X^* = \begin{pmatrix} X \\ \sqrt{k}I_r \end{pmatrix} \qquad Y^* = \begin{pmatrix} Y \\ 0 \end{pmatrix}$$

$$\widehat{\beta} = (X^{*t}X^*)^{-1}X^{*t}Y^* = \left[\begin{pmatrix} X^t & \sqrt{k}I_r \end{pmatrix} \begin{pmatrix} X \\ \sqrt{k}I_r \end{pmatrix} \right]^{-1} \begin{pmatrix} X^t & \sqrt{k}I_r \end{pmatrix} \begin{pmatrix} Y \\ 0 \end{pmatrix} = (X^tX + kI_r)^{-1}X^tY$$
 proved.