Matrix Calculus and Optimization
Statistics 202B
Professor: Mark S. Handcock

Homework 4

Due Wednesday, February 24, 2016

Note: **AMVR** stands for the Brian Everitt and Torsten Hothorn online text "An Introduction to Applied Multivariate Analysis with R" (2011),

MMST stands for the Alan J. Izenman online text "Modern Multivariate Statistical Techniques" (2008).

1) Consider the (exploratory) factor analysis model as expressed in Section 5.3 of **AMVR**.

a) Show how the result $\Sigma = \Lambda^T \Lambda + \Psi$ arises from the assumptions of uncorrelated factors, independence of the specific variates, and independence of common factors and specific variances.
What form does Σ take if the factors are allowed to be correlated?

b) Read Section 5.7 about factor rotation. Show that the communalities in a factor analysis model are unaffected by the transformation $\Lambda^* = \Lambda M$.

2) Problem 17.3 on page 665 of **MMST**. The Eckart-Young Theorem is stated in Chapter 3, page 52 and Chapter 6, page 178. It is a result we have used many times: The best rank r approximation to a matrix A is

$$\tilde{\mathbf{A}} = \mathbf{U}\tilde{\mathbf{\Sigma}}\mathbf{V}^*$$

where the SVD of A is

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^*$$

and $\tilde{\mathbf{\Sigma}}$ is the same matrix as $\mathbf{\Sigma}$ except that it contains only the r largest singular values (the other singular values are replaced by zero). By best we mean the choice that minimizes the Frobenius norm of $\mathbf{A} - \tilde{\mathbf{A}}$.

3) Problem 17.6 on page 665 of **MMST**. To read in the data in Table 17.11 you can use:

```
dget("http://www.stat.ucla.edu/~handcock/202B/datasets/Table17.11.dput")
```