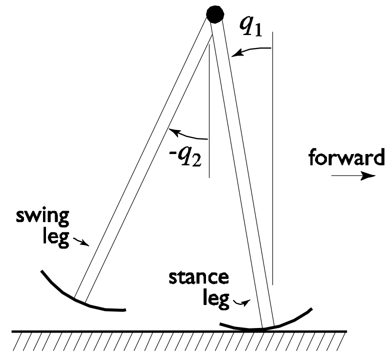
Equations of Motion for 2-D Anthropomoprhic Walking Model



The two-dimensional passive dynamic walking model, shown at right, has equations of motion

$$\mathbf{M}(q)\dot{u} + V(q,\dot{q}) + G(q) = 0$$

where the generalized coordinates q_1 and q_2 are the stance and swing leg angles, respectively, measured counter-clockwise from the ground normal. The generalized speeds u_1 and u_2 are the time-derivatives of q_1 and q_2, respectively. This model descends a gentle slope with no external power or control. The entries to the equations are given below.

$$\begin{aligned}
M_{11} &= I_l + MC^2 + ML^2 + M_pL^2 - 2MCR - 2MLR - 2M_pLR + 4MR^2 \\
&\quad + 2M_pR^2 + 2R\left(MC + \left(M + M_p\right)L - \left(2M + M_p\right)R\right)c_1 \\
M_{12} &= M\left(C-L\right)\left(\left(L-R\right)c_{12} + Rc_2\right),\ M_{22} = I_l + M\left(C-L\right)^2 \\
G_1 &= g\left(\left(2M + M_p\right)R\sin\gamma + \left(MC + \left(M + M_p\right)L - \left(2M + M_p\right)R\right)\sin\left(\gamma - q_1\right)\right),\ G_2 = M\left(C-L\right)\left(g\sin\left(\gamma - q_2\right)\right) \\
V_1 &= -R\left(MC + \left(M + M_p\right)L - \left(2M + M_p\right)R\right)s_1u_1^2 - \left(M\left(C-L\right)\left(R-L\right)s_{12} + Rs_2\right)u_2^2,\ V_2 = -M\left(C-L\right)\left(L-R\right)s_{12}u_1^2
\end{aligned}$$

The parameter values are L (leg length), I_l (moment of inertia of leg), C (distance from bottom of leg to leg center of mass), M (mass of leg), R (radius of curvature of foot), M_p (pelvis mass), I_p (pelvis moment of inertia), g (gravitational constant), γ (slope). Trigonometric terms are $c_1 \triangleq \cos q_1$, $c_{12} \triangleq \cos\left(q_1 - q_2\right)$, etc.

The foot hits the ground when the height above the ground is zero, i.e. $\left(L-R\right)c_1 - \left(L-R\right)c_2 = 0$ (note that it is sufficient just to check if the two angles are equal). The state after impact can be determined using conservation of angular momentum, with H_1 being the momentum of the entire system about the foot contact point, and H_2 being the momentum of the trailing leg about the hip. With - and + superscripts denoting the instant just before and after impact, respectively,

$$\begin{aligned}
H_1^- &= \begin{bmatrix} I_l + MC^2 - MLC - MRC + MLR + 2MR^2 + M_pR^2 + \\ R\left(2MC + M_pL - \left(2M + M_p\right)R\right)c_1 + \left(L-R\right)\left(2MC + M_pL - \left(2M + M_p\right)R\right)c_{12} + \\ \left(MRC + MLR + M_pLR - 2MR^2 - M_pR^2\right)c_2 \end{bmatrix} u_1^- \\
&\quad + \left[I_l + M\left(C-L\right)\left(C - R + Rc_2\right)\right]u_2^- \\
H_2^- &= \left(I_l + \left(C-L\right)M\left(C-R\right) + \left(C-L\right)MRc_1\right)u_1^- \\
H_1^+ &= \begin{bmatrix} I_l + MC^2 + ML^2 + M_pL^2 - 2MCR - 2MLR - 2M_pLR + 4MR^2 + \\ 2M_pR^2 + 2R\left(MC + \left(M + M_p\right)L - \left(2M + M_p\right)R\right)c_1 + \\ M\left(C-L\right)\left(L-R\right)c_{12} + MR\left(C-L\right)c_2 \end{bmatrix} u_1^+ \\
&\quad + \left[I_l + M\left(C-L\right)\left(C - L + \left(L-R\right)c_{12} + Rc_2\right)\right]u_2^+ \\
H_2^+ &= \left[\left(C-L\right)M\left(L-R\right)c_{12} + \left(C-L\right)MRc_2\right]u_1^+ + \left(I_l + M\left(C-L\right)^2\right)u_2^+
\end{aligned}$$

These equations may be expressed as $M^+u^+ = H^-$ and solved for u^+. Note that each of the angular momenta are dependent on the state of the system before or after impact, as appropriate.