# Homework 4

Computational Astrophysics (ASTR660)

(Due at the start of class on November 24, 2022)

## Exercise 1

**[Angry bird simulations – initial value problems (1 pt)**

In Lecture 7 we implemented the Euler's method and the RK2 method for solving initial value problems (IVPs) of ordinary differential equations (ODEs) and applied it to simulate the trajectory of the angry bird. Please also complete the implementation of the RK4 method if you have not done so.

(1) Overplot the trajectories of the three methods (Euler's, RK2, and RK4) along with the analytical solution for time step $dt = 0.1$ seconds.

(2) For each method, obtain the results for varied time steps: $dt = 10^{-3}, 10^{-2}, 10^{-1}, 10^{0}$ seconds. Let's define the error of the result being the difference between the numerical and analytical solutions in absolute values averaged over all data points, i.e., $\mathcal{E} = (\sum_i |y_{\rm i} - y_{\rm i,anal}|)/N$, where $N$ is the number of data points. Make a log-log plot of $\mathcal{E}$–$dt$ for each method.

(3) From the results you obtained in (1) and (2), describe how the three methods compare and explain the trends that you see.

## Exercise 2

**[Orbits of binary systems (2.5 pt)]**

Another important IVP in astrophysics is to simulate the orbits of stars. Let's complete in-class Exercise #2 in Lecture 7 to simulate the orbit of a binary star system. Please follow the instructions on p.48-p.54 of the lecture slides.

(1) To start, use the Euler's method for updating the positions and velocities of the two stars. Plot the trajectories of the two stars for time steps $dt = 0.00001$ year and $dt = 0.01$ year and see if you could reproduce the results on p.53-54 of the lecture slides.

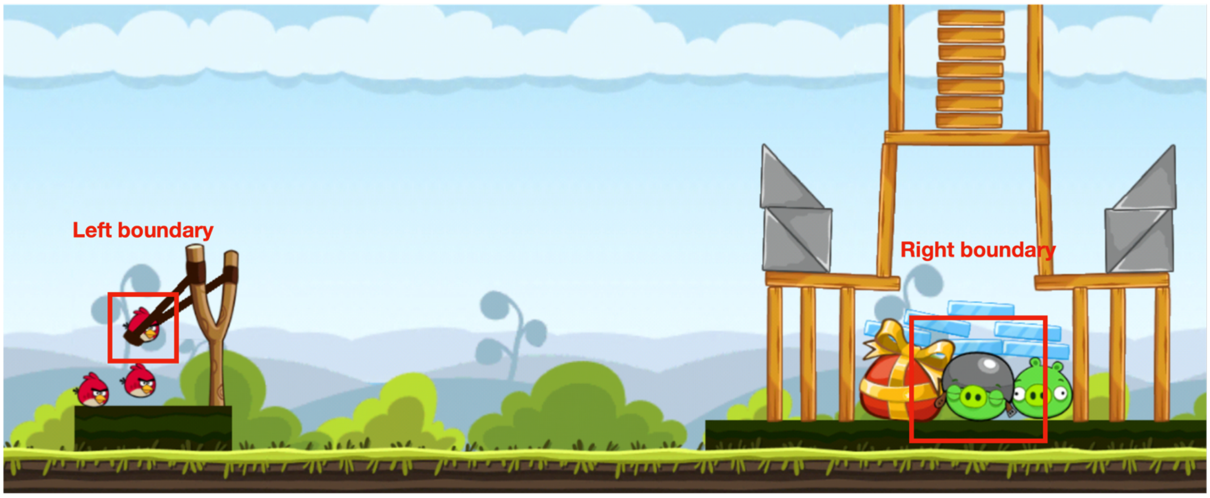
(2) Implement the RK4 method and see how the accuracy of the results are improved.

## Exercise 3

**[Angry bird simulations – boundary value problems (1 pt)**

In Lecture 8 we discussed about the shooting method for solving boundary value problems (BVPs) and how we could combine it with the bisection method to speed up the search for the initial guess. Let's apply the code you developed during class to the angry bird simulation, as illustrated in Figure 3.

Let's assume the initial position of the angry bird is at $(x, y) = (0, 0)$ with an initial velocity of 30 m/s. The target (the "unfortunate pig") is located at coordinates $(x, y) = (50, 0)$ in units of meters. Please use the shooting + bisection method for finding the optimal angle for the initial velocity, $\theta \equiv \arctan(v_{y,0}/v_{x,0})$, that could hit the target. Use the RK4 method in this exercise to obtain accurate results.




# Exercise 4

### [Solving BVPs using the finite-difference method (1.5 pt)

Another algorithm for solving BVPs is the finite-difference method, i.e., to replace derivatives with finite-difference approximations and solve the algebraic difference equations on fixed mesh points between boundary $t = [a, b]$, i.e., solving for $y(t_i)$ at $t_i = a + ih$ where $i = 0, 1...N-1$, $N$ is the number of mesh points, and $h$ is the separation between adjacent mesh points. Let's use this methods for solving the ODE

$$y'' = 6t, \tag{4.1}$$

with boundary conditions, $y(0) = 1$ and $y(1) = 1$.

After replacing the second derivative in the equation with finite-difference approximations, we have

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} = 6t_i \tag{4.2}$$

for $i = 1, 2, ..., N-2$, and we know $y_0 = 1$ and $y_{N-1} = 1$ from the boundary conditions. Written in matrix form, this is then equivalent to solving a linear system of equations.

Use $h = 0.05$, please write a python program to solve this linear system. Overplot the results $(y_i(t_i))$ with the analytical solution.