**WOLFGANG DROBETZ**

# HOW TO AVOID THE PITFALLS IN PORTFOLIO OPTIMIZATION? PUTTING THE BLACK-LITTERMAN APPROACH AT WORK

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## 1. Introduction

The MARKOWITZ (1952) formulation of modern portfolio theory combines the two basic objectives of investing: maximizing expected return and at the same time minimizing risk. In a portfolio context, risk is measured as the standard deviation (or volatility) of returns around their expected value. The result of traditional portfolio optimization is a parabolic efficient frontier, indicating the combinations of assets with the highest expected return given a certain level of risk.

Even though modern portfolio theory has stood the test of time within the academic community, and generations of business students have encountered mean-variance portfolio selection as one of the core-concepts of modern investment theory, its impact in the investment community has been surprisingly small.

HE and LITTERMAN (1999) state two reasons for this observation. First, asset managers typically focus on a small segment of the investment universe. They pick stocks they feel are undervalued, looking at fundamental ratios, momentum, and styles. In contrast, the MARKOWITZ approach requires expected returns for all assets in the universe as an input. Second, mean-standard deviation optimization implies a trade-off between risk and expected returns along the efficient frontier. Portfolio weights are a mere result of this relationship. In contrast, asset managers usually think directly in terms of portfolio weights. They find the weights returned by an optimizer extreme, not intuitive and, hence inappropriate for being implemented in a client’s portfolio. Unfortunately, much of this discomfort with the standard approach comes because historical returns are usually used instead of expected returns, and because estimation errors are not taken into account.

These observations were the motivation for the work by FISHER BLACK and ROBERT LITTERMAN (1992). The goal was to reshape modern portfolio theory so as to make it more applicable for investment professionals. Their approach is flexible enough to combine the market equilibrium with subjective views and an investor’s economic reasoning. Since the BLACK-LITTERMAN model starts with neutral portfolio weights that are consistent with market equilibrium, the revised weighting schemes tend to be much less extreme compared to traditional mean-standard deviation optimization.

Unfortunately, even though their paper constitutes a major step forward in bringing academic finance closer to the investment community, the number of professional users seems nevertheless small. The purpose of this paper is to illustrate the BLACK-LITTERMAN approach, putting the emphasis on simple examples rather than the mathematics behind it. The remainder is as follows. Section 2 describes the pitfalls of traditional portfolio optimization in some more detail. Section 3 introduces the reverse-optimization technique. These implicit returns serve as the neutral starting point for the BLACK-LITTERMAN approach. The main contribution of BLACK and LITTERMAN, how to combine equilibrium expected returns with an investor's subjective views, is demonstrated in section 4. This section also includes specific examples using historical returns from European sectors. Section 5 concludes.

## 2. The Deficiencies of Standard Portfolio Theory

Standard portfolio theory has several major deficiencies that prevent a more frequent use among investment professionals. We start by summarizing the major problems faced by practitioners to implement quantitative asset allocation.

### 2.1 Amount of Required Input Data

Traditional MARKOWITZ optimization requires as inputs the expected returns and the expected variance-covariance structure for all assets in the investment universe. Yet, portfolio managers typically have reliable return forecasts for only a small subset of assets. Proper forecasts for the variance-covariance structure are even harder to obtain.[1] It turns out, however, that the ex-post performance of the resulting weighting schemes depends heavily on the quality of input data, especially the vector of expected returns (see MERTON, 1980).

### 2.2 Extreme Portfolio Weights

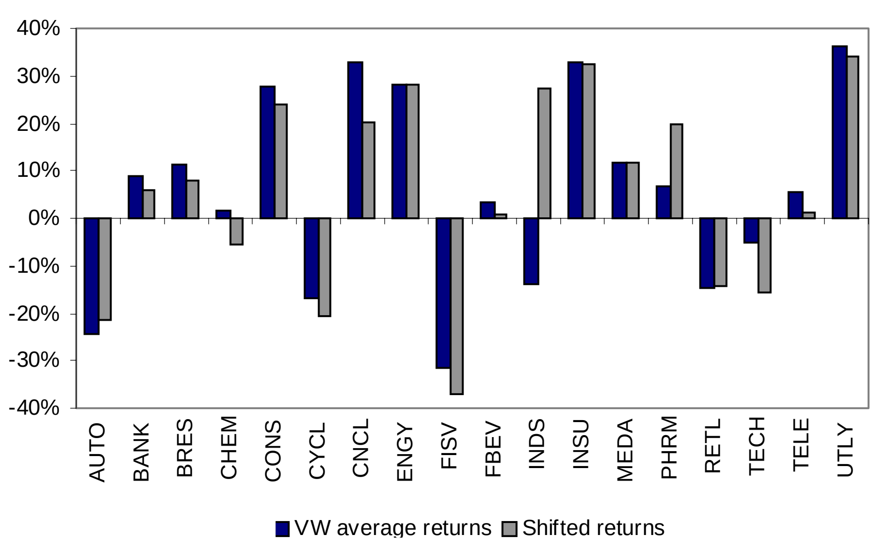
Optimized allocations tend to include large short positions, which cannot be justified by the portfolio manager in a client's portfolio. This is demonstrated in the following example. We use monthly returns in Swiss francs from the Dow Jones STOXX indices for European sectors over the period from June 1993 to November 2000. Descriptive statistics of our data set are shown in the appendix. As already discussed, in most cases the investor does not have a complete set of expected returns for all sectors in the Dow Jones STOXX family. Therefore, we start by setting the expected return for all sectors equal to 13.93%, which is the value-weighted average annual return over the sample period.

The results are shown as the black bars in Figure 1.[2] Using equal returns compensates for the different levels of risk across our sample, but it tends to generate extreme portfolio holdings. For example, mean-standard deviation maximization yields an optimal weight of 36% for utilities (UTLY). This is because utilities had a low volatility of 18.1% per year. In contrast, the much larger bank sector (BANK) receives a weight of only 9%. This is because banks were much more volatile, with an annual standard deviation of 22%. Some of the most volatile European sectors – automobiles (AUTO), cyclical consumption goods (CYCL), financial services (FISV), and the retail sector (RETL) – even receive substantial negative weights. Financial services (FISV) receive the largest negative weight with –31.5%. We also observe that some sectors with small capitalization, such as the construction sector (CONS), receive unreasonably large weights, while some of the larger sectors, such as tech stocks (TECH), receive small (or even negative) weights. Overall, the resulting portfolio hardly constitutes a realistic strategy for an asset manager.

Instead of using equal returns as we do in Figure 1 (for the sake of simplicity), it is often suggested to use historical returns as inputs for expected returns. However, both the vector of expected returns and the covariance structure required for mean-standard deviation optimization must be forward-looking. MERTON (1980) demonstrates that historical returns are bad proxies for future expected returns. This observation makes clear that estimation error is an important issue for putting portfolio theory into practice. MICHAUD (1989) even argues that mean-standard deviation optimizers are estimation-error maximizers. Optimized portfolios tend to overweight (underweight) assets with large (small) expected returns, negative (positive) correlations and small (large) variances. Intuitively, assets with extreme returns tend to be most affected by estimation error. JORION (1985) suggests a BAYES-STEIN shrinkage estimator to alleviate the problem associated with estimation errors.

One remedy often proposed to avoid extreme weights is to introduce constraints in the optimization problem, e.g., non-negativity constraints. Standard constraints come from client-specific and/or legal restrictions. Accordingly, optimized results tend to merely reflect prespecified views, which are not always economically intuitive. Even worse, short-sale constraints typically lead to corner solutions, dropping some assets from 'optimized' portfolios. These constrained weighting schemes are again hard to implement in a client's portfolio because they often imply unreasonable weights in only a small subset of (possibly small-capitalization) assets.

**Figure 1: Optimal Weights Starting from Equal Expected Returns**



### 2.3 Sensitivity of Portfolio Weights

Optimal allocations are particularly sensitive to changes in expected returns. Small changes in input variables can cause dramatic changes in the weighting schemes. Indeed, a major concern of asset managers is that mean-standard deviation optimization leads to extremely unstable portfolio weights.

This property can be demonstrated by extending the example from above involving European sectors. For the moment we assume that an investor has only a single view about European sectors: pharmaceuticals (PHRM) and industrials (INDS) outperform telecom (TELE) and technology (TECH) stocks by 3 % per year. To incorporate this view in a simplistic manner, the expected returns for both industrials and pharmaceuticals are shifted up by 1.5% and the expected returns for telecom and technology stocks are shifted down by 1.5% (starting from the value-weighted average of 13.93%).

The grey bars in Figure 1 show that this small shift in expected returns causes huge swings in the weights for the sectors involved in the investor's view. The weight for industrials soars from –14% to almost 30%. Similarly, the weight for pharmaceuticals jumps from 7% to roughly 20%. In contrast, technology stocks' weight is further reduced from –5% to –16%, the telecom sectors goes down from 5% to 1.5%.

An even more uncomfortable observation is that the weights can change considerably even for sectors without a particular view. For example, the weight for chemicals (CHEM) decreases from 1.5% to –5%, non-cyclical goods (CNCL) drop from 33% to 20%. Yet, the investor has not expressed any views about these sectors. Hence, the strong correlations between European sectors hinder "fine-tuning". Even a small change in the expected return of only one asset can lead to surprising changes in the output weights of other assets (for which the investor has no view) that are difficult to interpret.

### 2.4 Mismatch in Levels of Information

Standard mean-standard deviation optimization does not distinguish strongly held views from vague assumptions. In other words, optimizers generally do not differentiate between the levels of uncertainty associated with input variables. Therefore, the optimal weights associated with revised expected returns often bear no intuitive relation to the views the investor actually wishes to express. Again, one way to alleviate this problem is to apply BAYES-STEIN shrinkage estimators, which shrink expected returns towards a common mean (see JORION, 1985).

To make the problem even worse, in many cases the differences in estimated means may not be statistically significant. This implies that every point on the efficient frontier has a neighborhood that includes an infinite number of statistically equivalent portfolios. Though equivalently optimal, these portfolios may have completely different compositions.[3]

## 3. Neutral Views as the Starting Point

The approach suggested by BLACK and LITTERMAN (1992) grew out of the deficiencies of standard portfolio theory described in the previous section. Their goal was to make mean-standard deviation theory more applicable for investment professionals. The approach allows combining an investor's views about the outlook for his investment universe with some kind of equilibrium returns. Equilibrium returns provide a neutral reference point, leading to more reasonable and more stable optimal portfolios than the traditional mean-standard deviation optimization technique.

BLACK and LITTERMAN argue that the only sensible definition of "neutral" means is the set of expected returns that would clear the market if all investor had identical views.[4] Hence, the natural choice for neutral expected returns is to use the equilibrium expected returns derived from reverse optimization. The BLACK-LITTERMAN model gravitates towards a neutral, i.e. market capitalization weighted, portfolio that tilts in the direction of assets favoured in the views expressed by the investor. The extent of deviation from equilibrium depends on the degree of confidence the investor has in each view. Clearly, another possible starting point would be some strategic asset allocation defined by an investment committee.

It is well known that given the coefficient of relative risk aversion $\delta$, the n-dimensional vector of expected returns $\Pi$, and the covariance matrix $\Omega$, the unconstrained maximization problem faced by an investor with quadratic utility function or assuming normally distributed returns,

$$\max_{\omega} \ \omega'\Pi - \frac{\delta\omega'\Omega\omega}{2}$$

has a solution of optimal portfolio weights

$$\omega^{*} = (\delta\Omega)^{-1}\Pi .$$

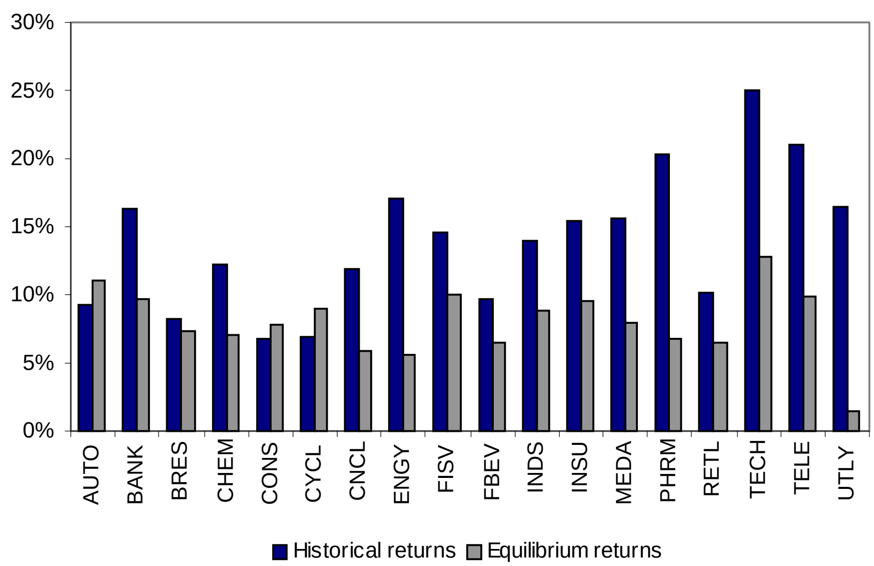
The "reverse optimization" technique suggests working backwards. Assume that a set of portfolio weights $\omega$ is optimal, and solve this equation for the vector of implied returns

$$\Pi = \delta\Omega\omega .$$

By construction, optimization using the implied returns will give the portfolio weights which were used to build the implied returns. Intuitively, this approach is closely linked to the Capital Asset Pricing Model (CAPM). This model predicts that prices will adjust until in market equilibrium the expected returns will be such that the demand for these assets will exactly match the available supply.

Using the relative market capitalizations weights, the gray bars in Figure 2 exhibit the

**Figure 2: Historical Returns and Equilibrium Returns**



implied expected returns for our sample of European sector indices. These implied returns now serve as a neutral starting point to incorporate an investor's view about future returns. The black bars in Figure 2 depict the historical returns over the sample period from June 1993 to November 2000.

Figure 3 shows the resulting optimal portfolio weights, using unconstrained mean-standard deviation optimization. When the historical returns are used, the resulting weights are even more extreme than above when we used equal returns for all sectors. Needless to say, the portfolio represented by the black bars in Figure 3 is impossible to implement for any asset manager.

Again, error-maximization seems to be a problem. Unconstrained optimization leads to extreme short positions in assets with very low past returns, e.g., cyclical goods (CYCL) and retail companies (RETL), and extreme long positions in assets with very high past returns, e.g., energy companies (ENGY). On the other hand, the plausible weight for the technology sector (TECH) indicates that the investor to some extent does trade-off mean against standard deviation. Tech stocks had the highest historical return over the sample period (25.0% per year), but also the highest volatility (31.3% per year).

In contrast, having equilibrium expected returns in the back of the mind, the market portfolio weights are all positive, and they are close to what might be called the "normal" investment behavior of an "average" investor. For this reason, they reflect a hypothetical "passive" manager who tracks the benchmark portfolio. Alternatively, the set of strategic target portfolio weights could represent the benchmark, from which the asset manager can deviate according to his or her economic reasoning in the tactical asset allocation.

**Figure 3: Optimal Portfolio Weights Based on Historical Returns and Equilibrium Returns from Reverse Optimization**

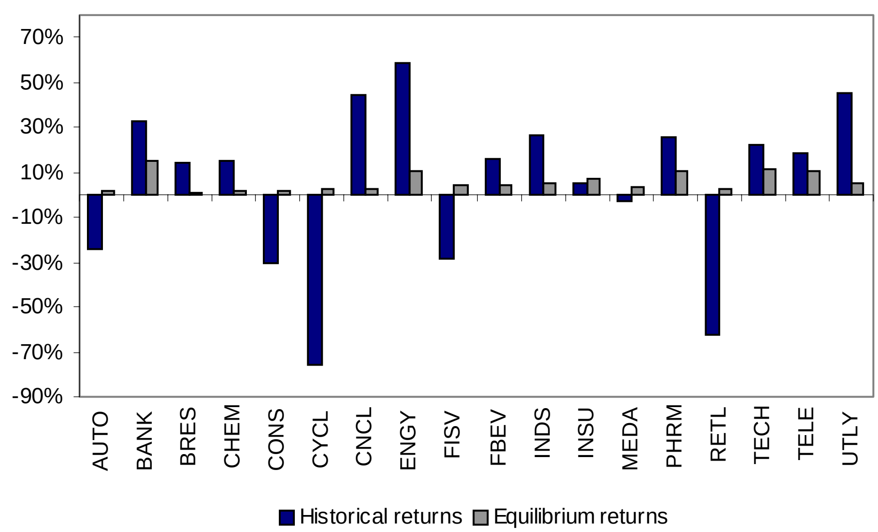
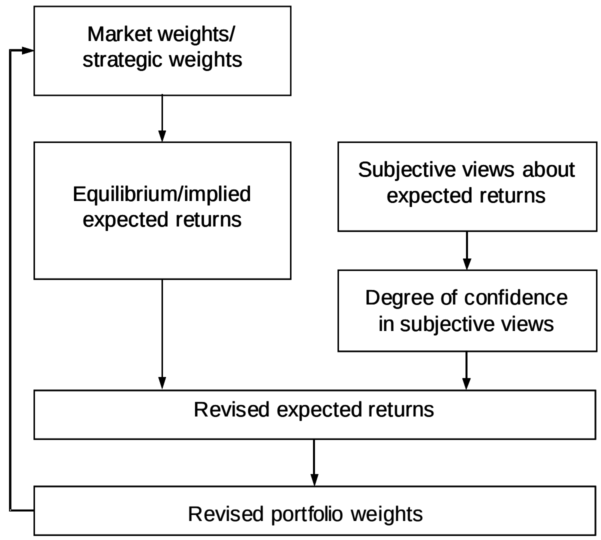


**Figure 4: Major Steps behind the BLACK-LITTERMAN Model**




The major contribution of BLACK and LITTERMAN, however, is to combine equilibrium returns with uncertain views about expected returns to derive realistic portfolio holdings. As BLACK and LITTERMAN state it, the equilibrium concept is interesting but not particularly useful. Its real value is to provide a neutral framework the investor can adjust according to subjective views and constraints.[5] In particular, the market capitalizations weights are tilted in the direction of assets favoured by the investor.

## 4. The BLACK-LITTERMAN Approach

Before we go into the detail of the BLACK-LITTERMAN model, it might be useful to give an intuitive description of the major steps. They are visualized in Figure 4. BLACK and LITTERMAN start with the set of market weights. Alternatively, a set of strategic (i.e. long-term weights) weights can be used. These weights are to represent what we may call the "normal" investment behavior of an "average" investor. In market equilibrium, these weighting schemes imply a corresponding vector of expected returns for the asset universe. However, given the current economic conditions, the asset manager may have expectations about short-term returns that differ from those implied by current market clearing conditions. The BLACK-LITTERMAN approach is very flexible to incorporate many different types of views, and it offers a method to combine the equilibrium (or neutral) view with an investor's subjective views in a consistent way.
In addition to the return expectations, the investor must specify the degree of confidence he or she puts into the stated views. All information is then translated into symmetric confidence bands around normally distributed turns. The most important contribution of BLACK and LITTERMAN is to devise a consistent weighting scheme between the subjective view and the equilibrium view. Intuitively, the higher (lower) the degree of confidence, the more (less) the revised ex-

pected returns are tilted towards the investor's views.
The vector of revised expected returns is then handed over to a portfolio optimizer. As will be shown, even though the view spreads across all asset classes and, hence implies changes for the entire vector of expected returns, the optimal weights change for those asset classes with explicit views only. For all other assets the equilibrium/strategic weights are used. In contrast to ad-hoc methods (see section 2.3 above), this consistently reflects the subjective views of an investor. Equally important, and in contrast to the MARKOWITZ approach, the BLACK-LITTERMAN model approach leads to more stable and less extreme portfolio weights.

### 4.1 Long-term Equilibrium

The following derivation of the BLACK-LITTERMAN approach closely follows their original paper and the recent book on tactical asset allocation by LEE (2000).[6] The vector of equilibrium returns backed out from reverse optimization is again denoted as $\Pi$. BLACK and LITTERMAN (1992) propose that the only meaningful set of equilibrium returns is the set of expected returns that would clear the market. If the Capital Asset Pricing Model holds, the weights based on the market capitalizations are also the optimal weights if the market is in equilibrium. Alternatively, $\Pi$ could denote strategic target weights.
To keep things simple, BLACK and LITTERMAN posit that the covariance matrix of expected returns is proportional to the historical one, rescaled only by a shrinkage factor $\tau$. Since uncertainty of the mean is lower than the uncertainty of the returns themselves, the value of $\tau$ should be close to zero. Hence, a complete specification of the equilibrium distribution of expected returns is

$$E(R) \sim N(\Pi, \tau\Omega)$$

where $\Omega$ is an $n \times n$ covariance matrix of realized historical returns, and $E(R)$ is an $n \times 1$ vector of expected returns. If the investor does not hold a view about expected returns, he or she should simply hold the market portfolio. This reflects the equilibrium state at which supply equals demand. When the investor has some views about expected returns, the difficult task is to combine these views with the market equilibrium in a consistent way.
BLACK and LITTERMAN allow the investor to express his or her degree of confidence about these views. Accordingly, both sources of information, equilibrium expected returns and the views, are expressed as probability distributions. Intuition suggests that the relative weights put on the equilibrium and the views depend on the degrees of confidence in these sources of information. The less certain the investor is about his or her subjective views, the more the resulting portfolio becomes tilted towards equilibrium. In contrast, the weights in a portfolio of an investor who is highly confident in his or her views will deviate more from the equilibrium market portfolio.[7] This notion of the BLACK-LITTERMAN approach is similar to the BAYES-STEIN shrinkage estimator suggested by JORION (1985).

### 4.2 Expressing Views

To implement the BLACK-LITTERMAN approach, the asset manager has to express his or her views in terms of as a probability distribution. BLACK-LITTERMAN assume that the investor has relative views of the form: "I expect that sector A outperforms sector B by V", where V is a given value. Absolute views of the form: "I expect that sector C has an expected return of 10% over the next year", can also be incorporated in the analysis, as we will show in the example below.

For the moment, assume that that the investor has k different views on linear combinations of expected returns of the n assets. These are written in matrix form as

$$P \cdot E(R) = V + e$$

where P is a $k \times n$ matrix, V is a $k \times 1$ vector, and e is a $k \times 1$ vector with error terms of the views. The first view is represented as a linear combination of expected returns in the first row of P. The value of this first view is given by the first element of V, plus an error term as the first element of e. The error term measures the degree of uncertainty about a particular view. More uncertainty is reflected in a higher value of the entry in e. Finally, let the covariance matrix of error terms be denoted as Σ. BLACK and LITTERMAN assume that all views are independently drawn from the future return distribution, implying that Σ is a diagonal matrix. Its diagonal elements are collected in the vector e.
To begin with, assume the investor has only a single view. As before, the investor expects that a value-weighted portfolio of pharmaceuticals and industrials outperforms a value-weighted portfolio of telecom and tech stocks by 3%. This view is represented in matrix form as

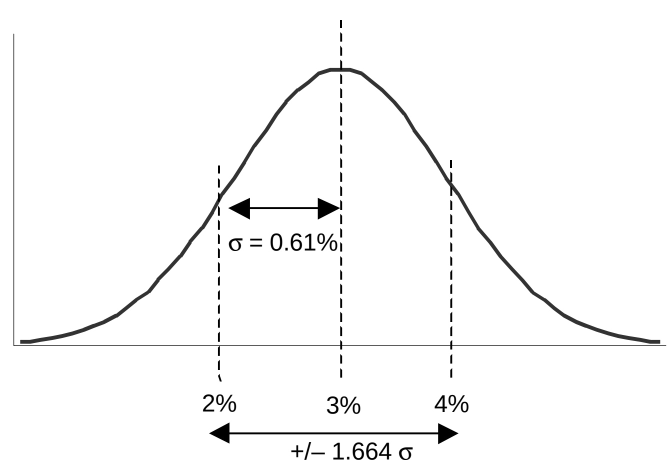
$$P \cdot \begin{pmatrix} E(R_{AUTO}) \\ E(R_{BANK}) \\ \vdots \\ E(R_{UTLY}) \end{pmatrix} = (3\%) + \left(0.61\%^2\right)$$

with

P = (0 … 0 0.34 0 … 0.66 0 –0.51 –0.49 0)

where the single entry in the error vector e indicate the degree of confidence in the view. This requires being a little more precise about the view. For example, the investor could state to be 90% sure that the spread in expected returns between the two portfolios is between 2% and 4% over the next year. Following PITTS (1997b), we interpret this statement as a 90% confidence interval for the return spread with half-width 1% and centered on 3%.[8] Assuming normality, the confidence interval implies 0.61% as the standard deviation. The greater the degree of confidence, the smaller is the interval and/or the greater is the probability mass for an interval. The intuition is shown in Figure 5.

**Figure 5: Interpreting the Degree of Confidence**



The bell-shaped curve in Figure 5 indicates the distribution of stock returns. A normal distribution is completely described with the first two moments, mean and standard deviation.
A 90% interval implies going 1.664 times the standard deviation to the left and the right side of the mean spread at 3%. This results in a range from 2% to 4%.
Since the investor expresses the view as a value-weighted spread, the P vector has a "0.34" for industrial and a "0.66" for pharmaceuticals (both weights summing to one) and a "–0.51" for tech and a "–0.49" for telecom stocks (the weights summing to one again). Hence, the non-zero elements in P are relative market capitalizations weights. The remaining entries in P for all other sectors are zero.
This setup of specifying subjective views is extremely flexible. Virtually all possible (relative and absolute) views can be incorporated into the model. The number of opinions can be smaller or larger than the number of assets in the investment universe. If the asset manager has consistent expected return forecasts for all asset classes, the P vector has as many rows as there are asset classes. In contrast, if the asset manager has useful views for only a small set of assets (which might be the case in the investment practice), the P vector has fewer rows than there are assets. After the equilibrium distribution of expected returns and the views have been specified, the task is to combine these two sources of information. Intuition suggests that the more certain the investor is about his or her views, the more the resulting portfolio should be tilted toward the favoured assets.

### 4.3 Combining Certain Views

For the special case of certain views on behalf of the asset manager, the matrix $\Sigma$ contains only zeros. The distribution of expected returns conditional on the equilibrium and the views can be derived as the solution to the following problem[9]

$$\min_{E(R)} \left[E(R)-\Pi\right]' \tau\Omega^{-1} \left[E(R)-\Pi\right]$$

subject to $P \cdot E(R) = V$.

This is a constrained least-square minimization problem. The sum of square of deviations from expected returns from equilibrium, $[E(R)–\Pi]$, weighted by the covariance matrix, $\tau\Omega$, is minimized, taking the restrictions on expected returns expressed by the asset manager into account. The solution of this problem is given as

$$\text{Mean of } \; E(R) = \Pi + \tau\Omega P'\left(P\tau\Omega P'\right)^{-1}\left(V - P\Pi\right).$$

### 4.4 Combining Uncertain Views

If the investor is not perfectly sure about his or her views, the error terms in the vector e will not be zero. BLACK and LITTERMAN show that the conditional distribution of returns can be derived using BAYES' Law.[10] Without going into the details, Bayesian statistics seeks to characterize the impact of a second source of information on one's belief. The equilibrium distribution of expected returns is called the "prior". If the second source of information, in our case the investor's view, has any value, this leads to a revision of the first source of information. This is usually referred to as "updating" the prior distribution to obtain the "posterior". The magnitude of the revision depends on the degree of certainty expressed in both opinions.
Again, without going into the details of the derivation, the conditional distribution of E(R) is normal with[11]

Mean of $E(R)$ =

$$\left[(t\Omega)^{-1} + P'\Sigma^{-1} P\right]^{-1} \left[(t\Omega)^{-1}\,\Pi + P'\Sigma^{-1}\,V\right]$$

Hence, given the values for P, V, Π, τ, Ω, and Σ we can derive the posterior mean, which can then be handed over to a portfolio optimizer. This procedure results in optimal weights based on revised expected returns and the degree of confidence in the opinions. They are tilted toward the assets most favoured by the investor.

### 4.5 A Simple Example as an Illustration

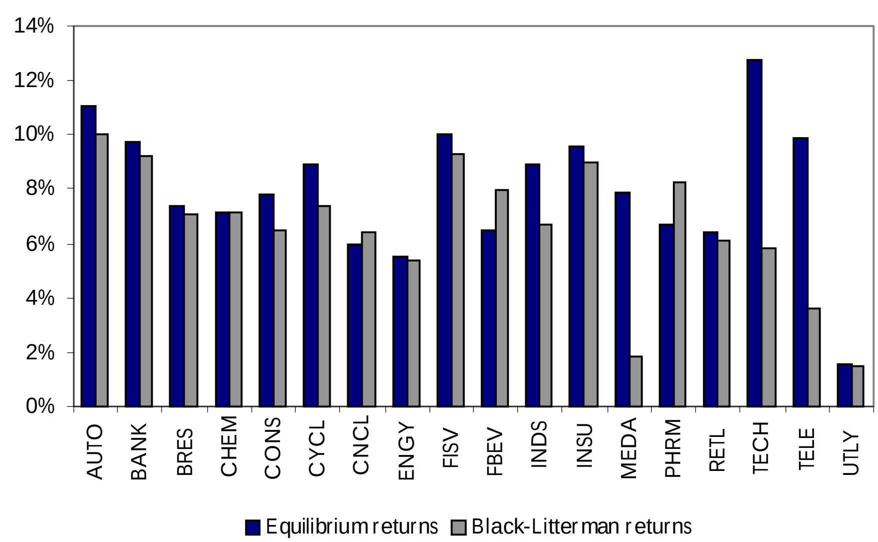
*Incorporating a Relative View*

If the investor does not have any views on expected returns, the equilibrium distribution of returns should be used. In fact, the vector of CAPM-equilibrium returns Π results when setting all entries in the P vector (and the Σ matrix) equal to zero.

The BLACK-LITTERMAN approach does not require the investor to hold views about expected returns on all sectors or asset classes. We continue with the example from above, again assuming that a value-weighted portfolio of pharmaceutical and industrial stocks outperforms a portfolio of telecom and technology stocks by 3% per year. The assumption was that the investor is 90% sure about this 3% return spread (with a half-width of 1%). As demonstrated above, this view is expressed as an expected return of 3% for a value-weighted portfolio with long positions in pharmaceuticals and industrials and short positions in telecom and tech stocks.

The equilibrium expected returns from Figure 2 serve as the reference point. These long-term returns are now combined with the single view on European sectors. Depending on the confidence in the investors view, the optimal portfolio is tilted away from the market portfolio in the direction of the view. The resulting monthly expected returns for our sample of European sectors is shown in Figure 6.[12]

**Figure 6: Expected Returns in the BLACK-LITTERMAN Model**



In contrast to the MARKOWITZ approach, the BLACK-LITTERMAN model adjusts the expected returns for all STOXX sectors away form their equilibrium value towards the view. Because the opinion is expressed as a long position in industrial and pharmaceutical stocks and short positions in telecom and technology stocks, the expected return on this long-short portfolio is substantially raised from an equilibrium value of –3.9% to slightly below the stated 3%. This should come as no surprise, given the high degree of confidence the investor has put in his or her view.

Given this view, decreasing returns for telecom and technology stocks are intuitive. On the other hand, while the expected return for pharmaceutical stocks increases, the expected return for industrials decreases. This is perfectly consistent with the investor's view. Nothing has been said about whether the expected return of a particular sector increases or decreases. All that has been assumed is a 3% spread between the four sectors involved. In other words, the view is not that industrial stocks go up, it only says that they outperform telecom and tech stocks.

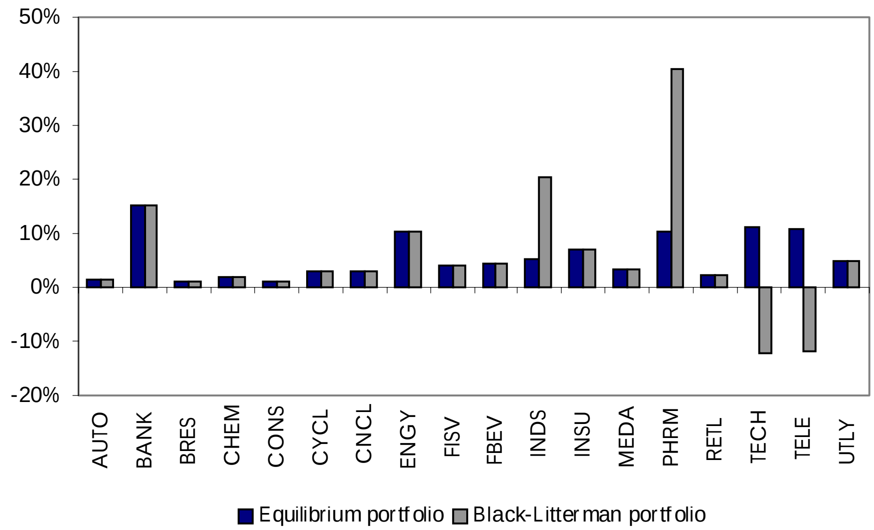
The most interesting observation in Figure 6 is that the expected returns change for those sectors without explicit views as well. Since all European sectors are highly correlated, the investor takes also an implicit view on all the other sectors. This effect occurs because of the matrix multiplications involving P, $\tau\Omega$, and $\Sigma$, expressing the views as a linear combination of expected returns, the covariance matrix of equilibrium expected returns, and the covariance matrix of views. Hence, the BLACK-LITTERMAN model "spreads" the errors inherent in the input values for expected returns through the covariance matrix. This effect reduces the error-maximization problem originally proposed by MICHAUD (1989).

An example for the effect of spreading errors trough the covariance matrix is easy to show. Telecom and technology stocks have been the major drivers of European stock markets over recent years. The correlation structure implies that decreasing returns for these sectors lead to decreasing returns for most of the other sectors as well. However, the changes for those markets without explicit views are generally much smaller than for the markets for which explicit opinions have been stated. The sharply decreasing expected return for the media sector can be explained on the basis of its high correlation with the telecom sector. On the other hand, the slightly higher expected return for chemicals is attributed to their close relationship with the pharmaceutical sector. Overall, the implied changes displayed in Figure 6 seem consistent with common sense.

In a final step, the revised vector of expected returns in Figure 6 is handed over to a portfolio optimizer. Figure 7 displays the resulting weighting scheme (again without short-sale restrictions). Compared to the equilibrium weights, the optimal portfolio increases the weights in industrial and pharmaceutical stocks and decreases the weights in technology and telecom stocks. This exactly represents the investor's view, going long in a value-weighted portfolio of industrials and pharmaceuticals and short in a value-weighted portfolio of tech and telecom equities.

In contrast to standard mean-variance optimization, the sector weights for which no views have been expressed remain unchanged at their relative market capitalization level. Hence, the optimal portfolio weights become much less sensitive to changes in the input variables. Even in unconstrained portfolios extreme positions (both negative and positive) do not frequently occur (unless the spread itself becomes extreme). This is because a neutral weighting serves as the starting point. Admittedly, increasing the tactical exposure in pharmaceuticals from roughly 10% to 40% might not be possible within an asset manager's prespecified strategic bandwidths. On the other hand, the relatively small short positions in

**Figure 7: Optimal Portfolio Weights in the BLACK-LITTERMAN Model**




technology and telecom stocks do not seem unreasonable for an actively managed portfolio.

*Incorporating an Absolute View*

So far we have assumed that the investor has only one view. The BLACK-LITTERMAN approach is flexible enough to incorporate many more views. For example, in addition to the (relative) view on industrials and pharmaceuticals versus telecom and technology stocks, we assume the investor believes that non-cyclical consumption goods perform better than implied by equilibrium conditions. In particular, the expected return for this sector is revised upward from 5.94% to 7.5% per year. Again, the investor is rather certain about his view, expressing a 90% confidence band between 6% and 9%. As explained above, this implies a volatility of 0.91%. The resulting matrix representation of the two views is

$$P \cdot \begin{pmatrix} E(R_{AUTO}) \\ E(R_{BANK}) \\ \vdots \\ E(R_{UTLY}) \end{pmatrix} = \begin{pmatrix} 3\% \\ 7.5\% \end{pmatrix} + \begin{pmatrix} 0.61\%^2 \\ 0.91\%^2 \end{pmatrix}$$

with

$$P = \begin{pmatrix} 0 & \ldots & 0 & 0.34 & 0 & \ldots & 0 & 0.66 & 0 & -0.51 & -0.49 & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 & 1 & 0 & 0 & \ldots & 0 & 0 \end{pmatrix}.$$

The first row of the P matrix expresses the first view on pharmaceuticals and industrials versus telecom and technology stocks. The second row has all zeros expect the “1” for non-cyclical consumption goods.

Again, the different degrees of uncertainty are measured by the value of the corresponding error term. The higher the value of the error term, the more uncertainty the investor puts into the view. Recall, we assumed that the views represent independent draws from the future return distributions. Therefore, the covariance matrix of error terms, denoted as $\Sigma$, is a diagonal matrix. The two elements along the diagonal in $\Sigma$ are collected in the e vector, which is the second part on the right hand side of the matrix representation of the view.

Figure 8 shows the revised expected returns of European sectors, incorporating both views as stated in the P matrix above. Again, the expected returns change for all assets, not only for those sectors for which explicit views have been expressed. The implicit views (e.g., the strong increase in expected returns for food and beverages) are caused by the correlation between the sectors. As expected, since the view is again rather certain, the expected return for non-cyclical consumption goods increases to just below 7.5%. Intuitively, the less confidence the investor had put in his view (e.g., if the confidence level was lower than 90% or $\tau$ smaller than 0.3), the more the revised expected return would end up below 7.5%.

The vector of revised expected returns is again handed over to an optimizer. Figure 9 shows the resulting weights for each sector in the optimal portfolio. Even though the expected returns are revised for the entire set of assets, the optimal weights change only for the sectors with specific views. Starting from neutral weights, this feature of the BLACK-LITTERMANN approach implies much more stable portfolio compositions. In our example with two views, this means that the optimal weight in non-cyclical consumption goods changes from the 2.9% equilibrium value to 16.9%. Compared to Figure 7, the weights for all four sectors incorporated in the first view (pharmaceutical, industrial, technology, and telecom) decrease slightly.

The overall lesson for asset managers is that the portfolio weights in the BLACK-LITTERMAN setup are much less sensitive to changes in expected returns. They do change, but in a consistent way and only to the degree it is intuitive and ultimately embedded in an investor's view. The approach also explicitly incorporates uncertainty about individual views. Recall from the discussion above, it is not

**Figure 8: Expected Returns in the BLACK-LITTERMAN Model**

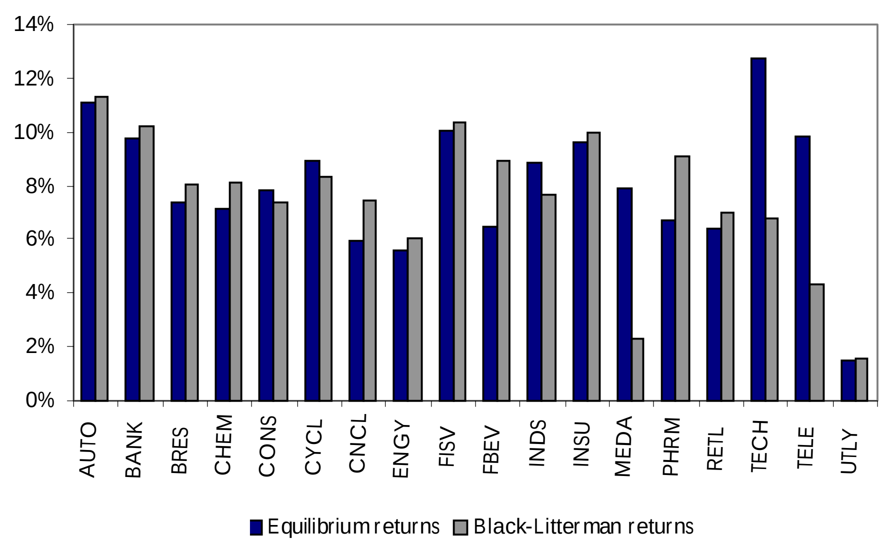
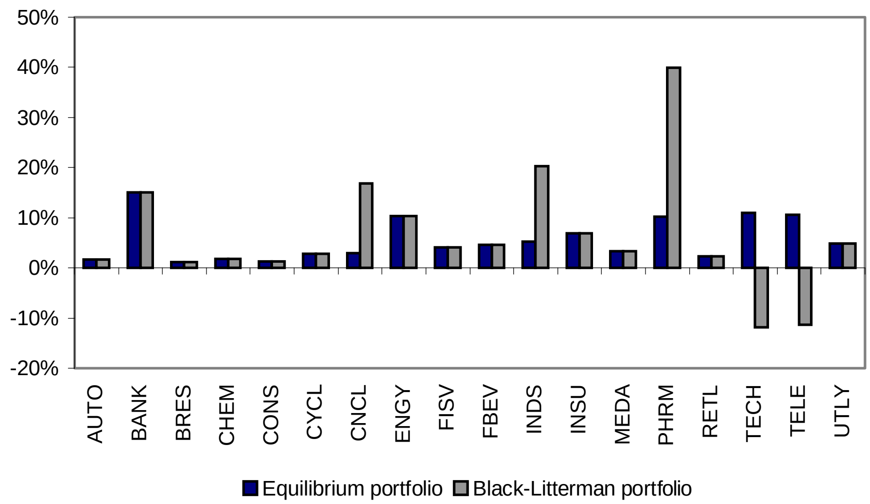


**Figure 9: Optimal Portfolio Weights in the BLACK-LITTERMAN Model**




clear why the optimal weights should change for all sectors, when a view has only been specified for some sectors. In our example, it is not particularly intuitive why increasing the expected return for non-cyclical consumption should affect the optimal weight in, say, automobiles. Consistent with this critique on standard mean-variance optimization, the weights displayed in Figure 9 are different to those in Figure 7 only for those sectors with explicit opinions.

## 5. Summary

In this article we have demonstrated the intuition behind the portfolio optimization model presented by BLACK and LITTERMAN (1992). Their approach helps to alleviate many of the problems associated with the implementation of traditional MARKOWITZ (1952) approach.
Their advice is intuitive and consistent with a "normal" investment behavior of an "average" investor. The asset manager starts from the market portfolio (or some strategic weighting scheme), which constitutes a neutral point of reference. Starting from all positive weights, he or she should then deviate toward the most favoured asset classes by taking appropriate long and short positions. The technique allows to distinguish between strong views and vague assumptions, which is reflected by the optimal amount of deviation from the equilibrium weighting scheme. This technique reduces the problem associated with estimation errors, and leads to more intuitive and less sensitive portfolio compositions. In addition, the BLACK-LITTERMAN approach is very flexible with regards to expressing a variety of possible views.

## APPENDIX
### Data description

**Table A1: Historical Returns, Volatilities, and and Market-Capitalization Weights**

| | | Historical returns | Equilibrium expected returns | Market capitalization weights |
|---|---|---|---|---|
| AUTO | Automobiles | 9,29% | 26,59% | 1,65% |
| BANK | Banks | 16,34% | 21,98% | 15,04% |
| BRES | Basic resources | 8,17% | 24,56% | 1,22% |
| CHEM | Chemicals | 12,14% | 19,39% | 1,80% |
| CONS | Construction | 6,76% | 18,53% | 1,26% |
| CYCL | Cyclical goods & services | 6,95% | 20,11% | 2,85% |
| CNCL | Non-cyclical goods & services | 11,96% | 16,40% | 2,90% |
| ENGY | Energy | 17,06% | 17,43% | 10,30% |
| FISV | Financial services | 14,53% | 22,14% | 4,12% |
| FBEV | Food & beverage | 9,65% | 18,80% | 4,59% |
| INDS | Industrial goods & services | 14,04% | 19,72% | 5,19% |
| INSU | Insurance | 15,51% | 21,81% | 6,89% |
| MEDA | Media | 15,52% | 28,20% | 3,27% |
| PHRM | Pharmaceutical | 20,32% | 19,54% | 10,24% |
| RETL | Retail | 10,12% | 16,34% | 2,27% |
| TECH | Technology | 25,01% | 31,27% | 11,03% |
| TELE | Telecommunication | 20,97% | 29,97% | 10,56% |
| UTLY | Utilities | 16,47% | 18,09% | 4,83% |

**Table A2: Correlation Structure between Dow Jones STOXX Sectors**

| | AUTO | BANK | BRES | CHEM | CONS | CYCL | CNCL | ENGY | FISV | FBEV | INDS | INSU | MEDA | PHRM | RETL | TECH | TELE | UTLY |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AUTO | 1,0000 | | | | | | | | | | | | | | | | | |
| BANK | 0,7901 | 1,0000 | | | | | | | | | | | | | | | | |
| BRES | 0,6921 | 0,5261 | 1,0000 | | | | | | | | | | | | | | | |
| CHEM | 0,8162 | 0,6757 | 0,7961 | 1,0000 | | | | | | | | | | | | | | |
| CONS | 0,8289 | 0,7211 | 0,7865 | 0,8126 | 1,0000 | | | | | | | | | | | | | |
| CYCL | 0,8381 | 0,7759 | 0,7517 | 0,7904 | 0,9219 | 1,0000 | | | | | | | | | | | | |
| CNCL | 0,7273 | 0,6569 | 0,5789 | 0,6911 | 0,7177 | 0,6995 | 1,0000 | | | | | | | | | | | |
| ENGY | 0,5582 | 0,5690 | 0,6043 | 0,7458 | 0,6801 | 0,6453 | 0,5558 | 1,0000 | | | | | | | | | | |
| FISV | 0,8140 | 0,9449 | 0,5757 | 0,6866 | 0,7807 | 0,8345 | 0,6875 | 0,5689 | 1,0000 | | | | | | | | | |
| FBEV | 0,7632 | 0,7459 | 0,5994 | 0,7432 | 0,7263 | 0,7104 | 0,7947 | 0,5729 | 0,7504 | 1,0000 | | | | | | | | |
| INDS | 0,7999 | 0,7456 | 0,7245 | 0,7579 | 0,8954 | 0,9002 | 0,6928 | 0,6367 | 0,7921 | 0,6066 | 1,0000 | | | | | | | |
| INSU | 0,8093 | 0,9279 | 0,4848 | 0,6528 | 0,6869 | 0,7411 | 0,6890 | 0,4898 | 0,9012 | 0,7240 | 0,7203 | 1,0000 | | | | | | |
| MEDA | 0,2674 | 0,2656 | 0,1631 | 0,2309 | 0,4358 | 0,4828 | 0,1622 | 0,2196 | 0,3315 | 0,0335 | 0,5744 | 0,2739 | 1,0000 | | | | | |
| PHRM | 0,5555 | 0,6723 | 0,2979 | 0,4843 | 0,4426 | 0,5206 | 0,6521 | 0,3495 | 0,6859 | 0,6544 | 0,4654 | 0,7346 | 0,1522 | 1,0000 | | | | |
| RETL | 0,7127 | 0,7064 | 0,5407 | 0,6880 | 0,7173 | 0,7282 | 0,8159 | 0,6250 | 0,7041 | 0,7149 | 0,7242 | 0,7039 | 0,2822 | 0,5981 | 1,0000 | | | |
| TECH | 0,5884 | 0,5744 | 0,4513 | 0,4536 | 0,6488 | 0,6996 | 0,3980 | 0,3970 | 0,6162 | 0,2706 | 0,7896 | 0,5980 | 0,7243 | 0,3627 | 0,5010 | 1,0000 | | |
| TELE | 0,4579 | 0,4768 | 0,1772 | 0,2946 | 0,5009 | 0,5529 | 0,3536 | 0,2484 | 0,5213 | 0,1886 | 0,6289 | 0,5296 | 0,7618 | 0,3427 | 0,4602 | 0,7867 | 1,0000 | |
| UTLY | 0,0945 | 0,0841 | 0,0200 | 0,0387 | 0,0792 | 0,1298 | 0,0780 | -0,027 | 0,1271 | 0,1135 | 0,1093 | 0,0330 | 0,1501 | 0,1552 | 0,2259 | 0,0645 | 0,1408 | 1,0000 |

## ENDNOTES

[1] See CHAN, KARCESKI and LAKONISHOK (1999) for a recent attempt to condition the covariance matrix on macroeconomic variables.

[2] Throughout the examples we assume a coefficient of relative risk aversion $\delta$ of 3 for computing portfolio weights.

[3] See DROBETZ (2000) for a discussion and empirical results of spanning tests using volatility bounds for stochastic discount factors.

[4] See BLACK and LITTERMAN (1992), p. 32.

[5] See BLACK and LITTERMAN (1992), p.33.

[6] See BLACK and LITTERMAN (1992), p.35 and the Appendix, and chapter 7 in LEE (2000).

[7] See LEE (2000) for a more in depth analysis.

[8] See PITTS (1997b), p.13.

[9] See LEE (2000), p. 176 and the Appendix in BLACK and LITTERMAN, p.42.

[10] For a discussion of BAYES' Law see for example HAMILTON (1994).

[11] See point 8 of the Appendix in BLACK and LITTERMAN (1992). They suggest that the solution can be derived by the "mixed estimation" of THEIL (1971).

[12] The parameter $\tau$ is set equal to 0.3. Larger values of $\tau$ indicate less confidence in the equilibrium returns.

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