Black-Litterman Model

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Overview

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2. How does the BLM work?
   - Introduction to BL Model
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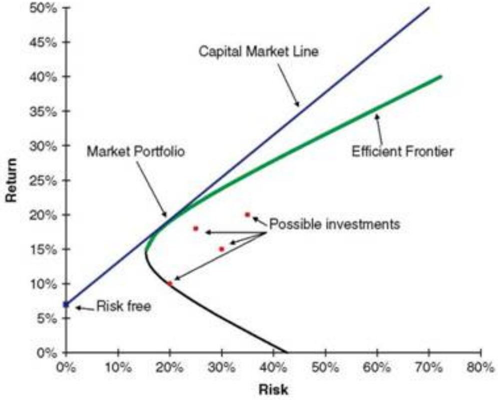
The Markowitz framework and its problems



The Markowitz framework and its problems

Highly-concentrated portfolios

- expected returns should be specified for every component of the relevant portfolio

Input-sensitivity

- Estimation error maximization

Unintuitive

- No way to incorporate investors view
- No way to incorporate confidence level
- No intuitive starting point for expected return
- Complete set of expected returns is required

CAPM

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f)$$

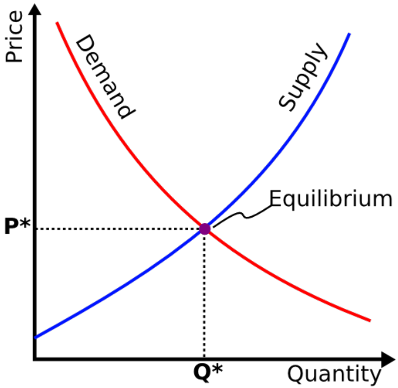
R_f – risk-free rate

$\beta_i = \dfrac{Cov(R_i, R_m)}{Var(R_m)}$ – Beta of the security(a measure of the risk arising from exposure to general market movements)

R_m – expected market return

General equilibrium

Market situation where demand and supply requirements of all decision makers (buyers and sellers) have been satisfied without creating surpluses or shortages.



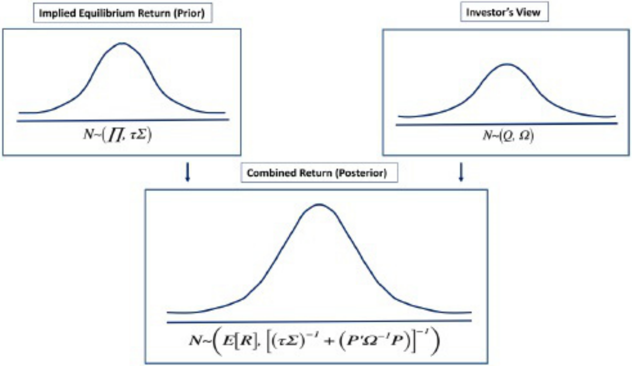
How does the BLM work?

B-L model uses a Bayesian approach to combine **the subjective views** of an investor regarding the expected returns of one or more assets with **the market equilibrium vector of expected returns (the prior distribution)** to form a new mixed estimate of expected returns **(the posterior distribution)**

Introduction to BL Model

- start with the market returns using **reverse optimization** and **CAPM**;
- apply your own **unique views** of how certain markets are going to behave;
- the end result includes both a set of expected returns of assets as well as the optimal portfolio weights.

Introduction to BL Model



Computing equilibrium returns

Reverse Optimization Framework

quadratic utility function

$$U(w) = w\Pi - (\frac{\delta}{2})w\Sigma w \rightarrow max$$

Π – vector of equilibrium excess returns

w – vector of weights

$\delta = \dfrac{\text{risk premium}}{\text{variance}}$ – risk aversion parameter

Σ – covariance matrix of the excess returns

Computing equilibrium returns

Vector of implied excess equilibrium returns

$$\Pi = \delta \Sigma w$$

$$\delta = \frac{E(R) - R_f}{\sigma^2}$$

CAPM formula

$$E(R_i) - R_f = \beta(E(R_m) - R_f)$$

$$\beta(E(R_m) - R_f) = \frac{cov(R_i, R_m w)}{\sigma^2}(E(R_m) - R_f) = \delta \Sigma w = \Pi$$

Investor's views

- Each view is unique and uncorrelated with another
- The sum of weights in the views is either 1 **(absolute view)** or 0 **(relative view)**
- There is no requirement that all assets have a view. It is possible that views conflict

Investor's views.Inputs

- P- a $k \times n-$ matrix of the assets weights within each view.
- $\Omega-$ a $k \times k-$ diagonal covariance matrix with entries of the uncertainty within each view.
- Q- the $k \times 1-$ vector with expected returns of the portfolio from the views described in matrix P.

Investor's views.Inputs

- Asset A has an absolute return of 5%
- Asset B will outperform Asset C by 1%

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}; Q = \begin{bmatrix} 5 \\ 1 \end{bmatrix}; \Omega = \begin{bmatrix} w_{11} & 0 \\ 0 & w_{22} \end{bmatrix}$$

$$\Omega = diag(P(\tau\Sigma)P^T)$$

Black Litterman Formula

Bayes Theorem

$$P(E[R]/\Pi) = \frac{P(E[R])P(\Pi/E[R])}{P(\Pi)}$$

- E[R] - expected return
- Π - equilibrium return
- personal views are used for **the prior distribution** $E[R] \sim N(\Pi, \tau\Sigma)$
- **the observation distribution** is $\Pi/E[R] \sim N(Q, \Omega)$
- Then observed data is used to generate **a posterior distribution** $P(E[R]/\Pi)$

Black Litterman Formula

$$\Longrightarrow$$

$$\frac{1}{\sqrt{(2\pi)^n \det(\tau\Sigma)}} \exp(-\frac{1}{2}(E[R] - \Pi)'(\tau\Sigma)^{-1}(E[R] - \Pi))$$

$$\times$$

$$\frac{1}{\sqrt{(2\pi)^n \det(\Omega)}} \exp(-\frac{1}{2}(PE[R] - Q)'(\Omega)^{-1}(PE[R] - Q))$$

$$\sim$$

$$\exp(-\frac{1}{2}(E[R] - E[\hat{R}])'(M)(E[R] - E[\hat{R}]))$$

Black Litterman Formula

the posterior expected return

$$E[R]/\Pi \sim N(E[\hat{R}], M)$$

mean of the posterior expected return

$$E[\hat{R}] = \Pi + \tau\Sigma P^T(P\tau\Sigma P^T + \Omega)^{-1}(Q - P\Pi)$$

variance of the posterior expected return

$$M = ((\tau\Sigma)^{-1} + P^T\Omega^{-1}P)^{-1}$$

Conclusion

Advantages

- investors can insert their views
- control over the confidence level of views
- more intuitive interpretation, less extreme shifts in portfolio weights

Disadvantages

- does not give the best possible portfolio, merely the best portfolio given the views stated
- sensitive to assumptions
- normal distribution

BLM example. S&P BSE 500 Index by Industry Sectors

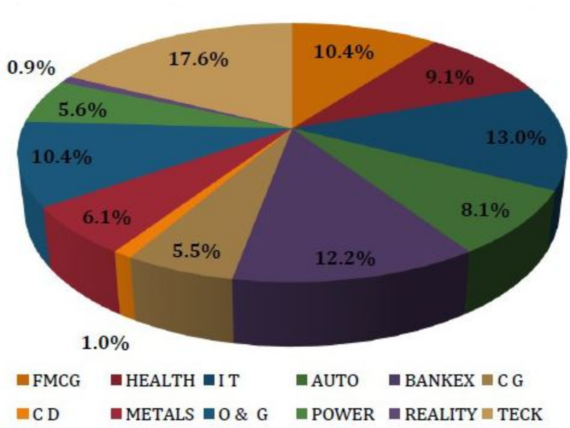


Figure: Equilibrium Weights (w) calculated on Market Capitalization

source:Bloomberg

BLM example. S&P BSE 500 Index by Industry Sectors

	FMCG	HEALTH	IT	AUTO	BANKEX	CG	CD	METALS	O & G	POWER	REALITY	TECK
FMCG	0.0227	0.0130	0.0074	0.0174	0.0238	0.0237	0.0203	0.0222	0.0204	0.0219	0.0324	0.0096
HEALTH	0.0130	0.0207	0.0122	0.0169	0.0204	0.0216	0.0199	0.0207	0.0173	0.0200	0.0300	0.0132
IT	0.0074	0.0122	0.0363	0.0109	0.0088	0.0101	0.0118	0.0100	0.0087	0.0081	0.0150	0.0312
AUTO	0.0174	0.0169	0.0109	0.0368	0.0415	0.0414	0.0297	0.0354	0.0331	0.0350	0.0469	0.0147
BANKEX	0.0238	0.0204	0.0088	0.0415	0.0706	0.0608	0.0406	0.0502	0.0462	0.0541	0.0750	0.0149
CG	0.0237	0.0216	0.0101	0.0414	0.0608	0.0769	0.0422	0.0552	0.0489	0.0548	0.0825	0.0164
CD	0.0203	0.0199	0.0118	0.0297	0.0406	0.0422	0.0489	0.0386	0.0334	0.0382	0.0546	0.0155
METALS	0.0222	0.0207	0.0100	0.0354	0.0502	0.0552	0.0386	0.0709	0.0479	0.0527	0.0748	0.0159
O & G	0.0204	0.0173	0.0087	0.0331	0.0462	0.0489	0.0334	0.0479	0.0461	0.0439	0.0624	0.0137
POWER	0.0219	0.0200	0.0081	0.0350	0.0541	0.0548	0.0382	0.0527	0.0439	0.0622	0.0760	0.0138
REALITY	0.0324	0.0300	0.0150	0.0469	0.0750	0.0825	0.0546	0.0748	0.0624	0.0760	0.1358	0.0222
TECK	0.0096	0.0132	0.0312	0.0147	0.0149	0.0164	0.0155	0.0159	0.0137	0.0138	0.0222	0.0290

Figure: Variance-Covariance matrix(Σ) over historical method

Parameters

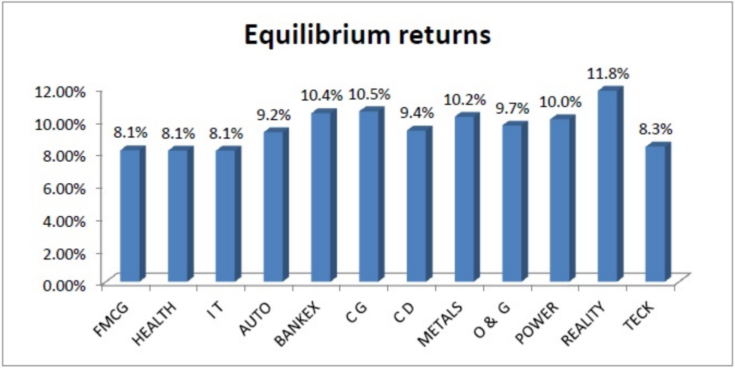
- risk free rate $R_f = 6\%$
- mean return $E(R) = 9.49\%$
- standart deviation $\sigma = 16.71\%$

$$\delta = \frac{E(R) - R_f}{\sigma^2} = 1.25$$

(based on monthly return series 2011 to 2016)

BLM example. S&P BSE 500 Index by Industry Sectors

$$\Pi = \delta \Sigma w$$



BLM example. S&P BSE 500 Index by Industry Sectors

Investor's views

- **absolute:**
  - IT sector will give the excess return of 10%
  - Metal Sector will give the excess return of -6%
- **relative:**
  - Telecoms Sector will underperform Real Estate Sector by 20%

	Q matrix
View 1	10.00%
View 2	-6.00%
View 3	20.00%

BLM example. S&P BSE 500 Index by Industry Sectors

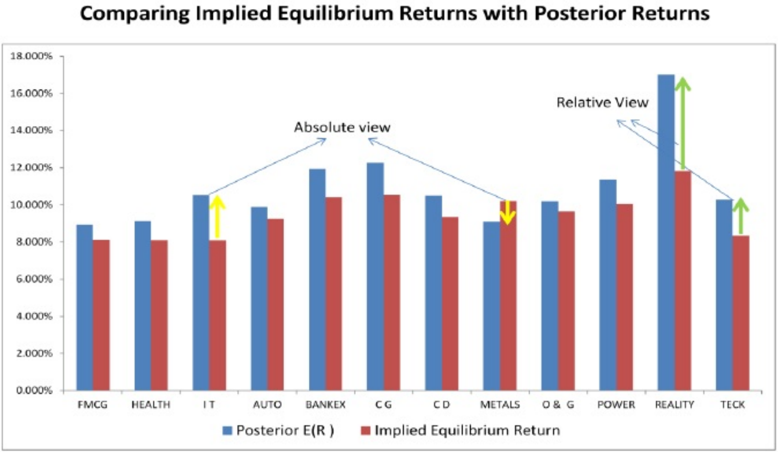
P matrix	FMCG	HEALTH	IT	AUTO	BANKEX	C G	C D	METALS	O & G	POWER	REALTY	TECK
View 1	0	0	1	0	0	0	0	0	0	0	0	0
View 2	0	0	0	0	0	0	0	1	0	0	0	0
View 3	0	0	0	0	0	0	0	0	0	0	1	-1

Ω matrix	View 1	View 2	View 3
View 1	0.015	-	-
View 2	-	0.028	-
View 3	-	-	0.048

$$\Omega = diag(P(\tau\Sigma)P^T)$$

with $\tau = 0.4$

BLM example. S&P BSE 500 Index by Industry Sectors



Thank you for your attention!