c. At home, you pick up the two 20s, one in each hand. What is the probability that your right hand holds more than 0.25 lb. more than your left hand?

Let $D = R - L$

$R \sim \text{Normal}(20, 0.20)$

$L \sim \text{Normal}(20, 0.20)$

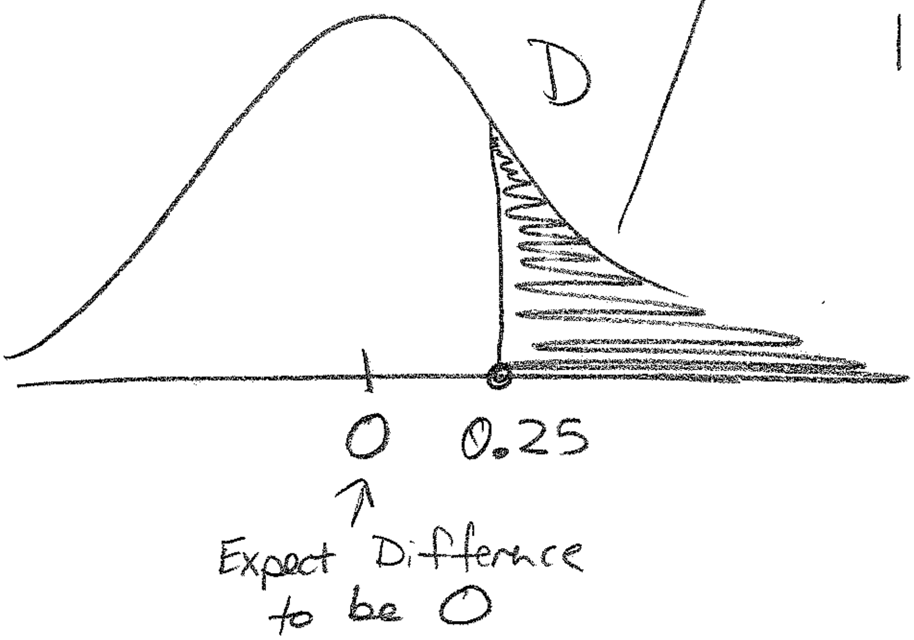
$\sigma_D = \sqrt{0.2^2 + 0.2^2}$

$= 0.283$

$D = R - L \sim \text{Normal}(0, 0.283)$

$P(D > 0.25 \text{ lbs}) = \text{ncdf}(0.25, E99, 0, 0.283)$

$= 0.189$



18.9% chance your right-hand holds at least 0.25 lbs more than left.

* Assuming randomly selected weights

Example: Bags at the flea market labeled "1 Pound" of coffee follow a Normal model with a mean weight of 0.98 pounds and a standard deviation of 0.09 pounds. You buy two bags and go home to weigh them (just to see if you got ripped off).

a. Give the model and its parameters for the sum of the weights of the two bags of coffee.

$X_1 \sim N(0.98, 0.09) \quad X_2 \sim N(0.98, 0.09)$

Let $S = X_1 + X_2 \quad \mu_S = 0.98 + 0.98 = 1.96$

$S \sim N(1.96, 0.1273) \quad \sigma_S = \sqrt{0.09^2 + 0.09^2}$

≈ 0.1273

b. Give the model for the difference in weights of the two bags of coffee.

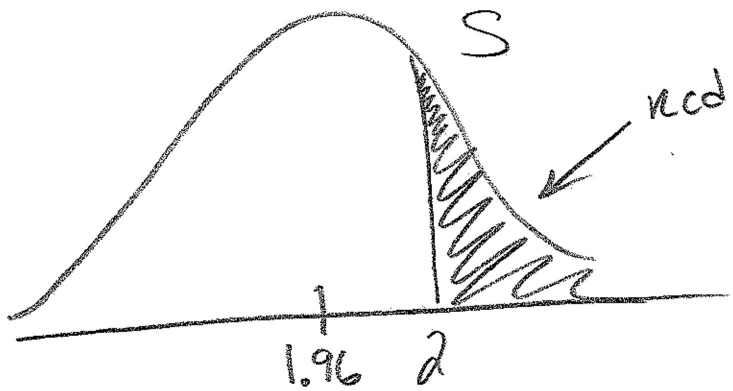
$D = X_1 - X_2 \quad \mu_D = 0.98 - 0.98 = 0$

$D \sim N(0, 0.1273) \quad \sigma_D = \sqrt{0.09^2 + 0.09^2}$

≈ 0.1273

c. Give the probability that you actually got 2 pounds or more of coffee.

$P(\text{Got } 2+ \text{ lbs}) = P(S \geq 2)$



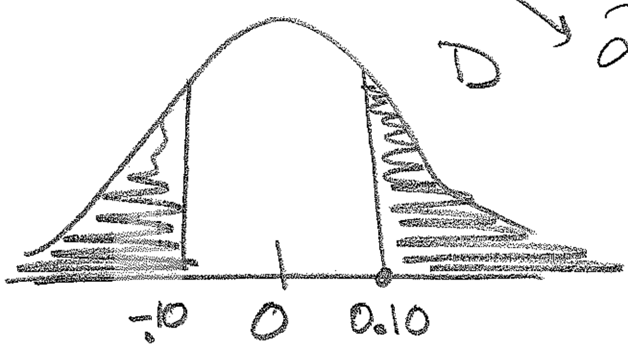
ncdf(2, E99, 1.96, 0.1273)

≈ 0.3767

37.67% chance

d. Give the probability that one bag weighs a tenth of a pound more than the other.

$P(D > 0.10) \times 2$ or shade both ways



$2 \times \text{ncdf}(0.10, E99, 0, 0.1273)$

≈ 0.4321

e. Simulate on StatCrunch your answers from part c and d (to convince us they are correct).

Example: Bowling balls taken to a pro shop for drilling follow a three-step process. The table below summarizes the times and each step follows a Normal model. Steps are independent of each other.

Phase	Mean	Standard Deviation
Measuring	15 minutes	5 minutes
Drilling	12 minutes	7 minutes
Cleaning	~~8~~ 2 minutes	1 minute

a. Give the mean and standard deviation for the total time to get a ball drilled. What's the model?

Let $S = M + D + C$ $\qquad S \sim N(29, 8.66)$

$\mu_S = 15 + 12 + 2 = 29$ minutes

$\sigma_S = \sqrt{5^2 + 7^2 + 1^2} \approx 8.66$

b. Give the mean and standard deviation for the total time to get two balls drilled?

$2S \approx M_1 + M_2 + D_1 + D_2 + C_1 + C_2 \sim$ Normal

$\mu_{2S} = 2 \times 29 = 58$ minutes

$\sigma_{2S} = \sqrt{5^2 + 5^2 + 7^2 + 7^2 + 1^2 + 1^2} \approx 12.25$

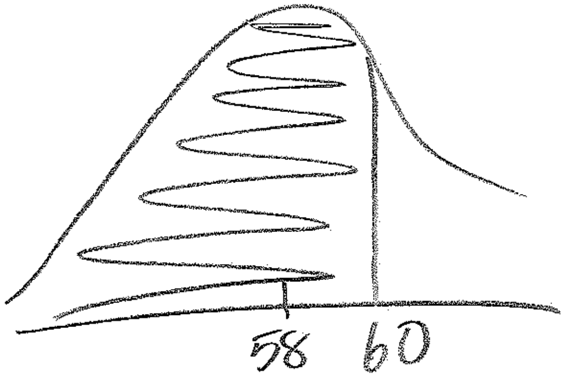
c. You got two bowling balls for Xmas. What's the chance you're in and out in under an hour?

$P(2S < 60 \text{ minutes}) \approx$

ncdf(−E99, 60, 58, 12.25)

≈ 0.565

56.5% chance



Different Probability Models

1. Discrete Uniform

A discrete uniform distribution has a countable number of outcomes, and each outcome is equally likely to occur.

If there are n possible outcomes, each outcome has probability $\frac{1}{n}$

The mean or expected value is:

$$E[x] = \mu = \frac{\text{Minimum Value} + \text{Maximum Value}}{2}$$

The variance and standard deviation are given by:

$$VAR[x] = \frac{n^2 - 1}{12}$$

$$SD[x] = \sqrt{\frac{n^2 - 1}{12}}$$

Example: The Ohio Lottery hosts the Pick 3 game twice a day. One number is drawn (digits 0, 1, …, 9) from three separate containers to construct a number between 000 and 999. Each number is equally likely to be selected each day.

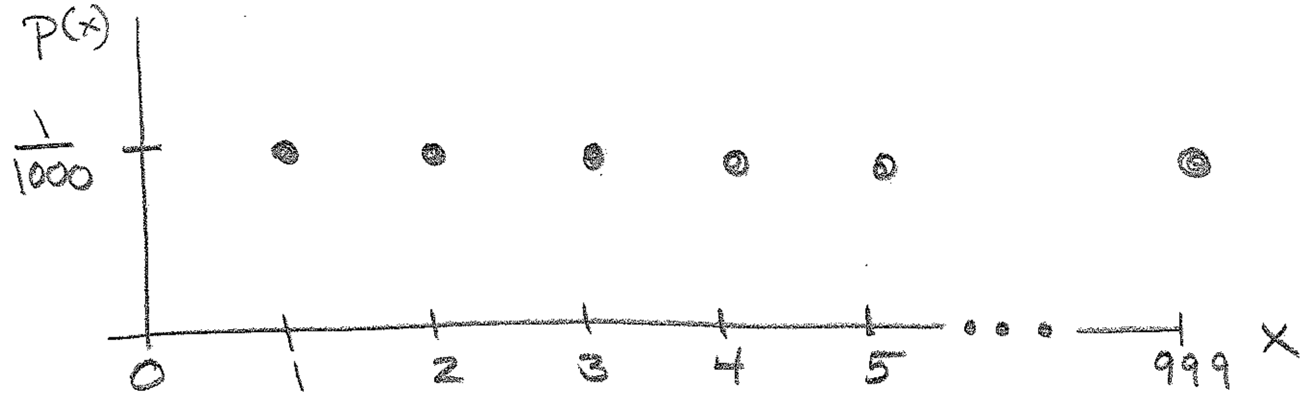
a. Give a table that displays the probability model.

x	000	001	002	etc	…	999
P(X=x)	1/1000	1/1000	1/1000	etc		1/1000

b. Give a function that describes the probability model.

$$P(x) = \frac{1}{1000}, \text{ for } x = 0, 1, 2, \ldots, 999$$

c. Draw a graph of the probability model.



d. What is the expected value or mean of this probability model?

$$E[X] = \frac{0+999}{2} = 499.5$$

e. What is the variance and standard deviation?

$$VAR[X] = \frac{n^2-1}{12} = \frac{1000^2-1}{12} = 83{,}333.25$$

$$SD[X] = \sqrt{83{,}333.25} \approx 288.67$$

f. If your number is selected, the prize is \$500. What is the expected value of each ticket?

$$E[\text{EACH TICKET}] = \$500 \times P(\text{Winning})$$

$$= 500 \times \frac{1}{1000} = \frac{1}{2} \text{ or } 50¢$$

g. If a ticket costs \$1, what is the expected payout from the player's perspective?

$$E[\text{PAYOUT}] = \text{COST} - E[\text{WINNINGS}]$$

$$= \$1.00 - \$0.50$$

$$= \$0.50$$

2. Continuous Uniform

A continuous uniform distribution has a infinite number of outcomes distributed evenly over a fixed range of possible values. All intervals of the same length are equally likely to occur.

The lower bound is denoted __a__ and the upper bound is denoted __b__.

The probability function for this model is:

$$f(x) = \frac{1}{b-a}, \quad a \leq x \leq b$$

The mean or expected value is:

$$E[x] = \frac{1}{2}(a+b)$$

The variance and standard deviation are given by:

$$VAR[x] = \frac{1}{12}(b-a)^2$$

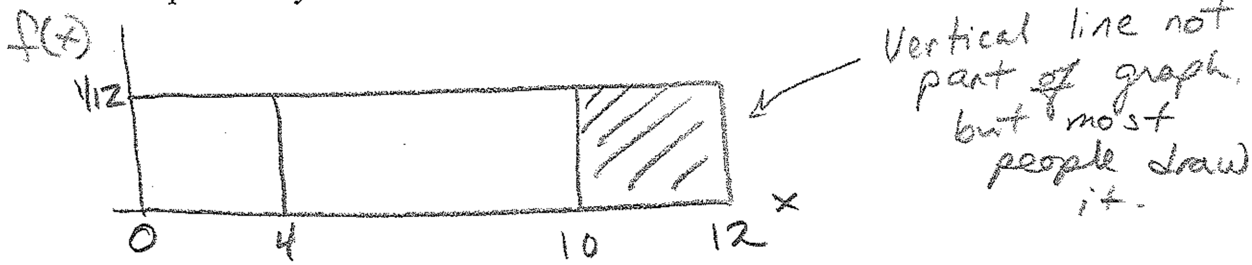
$$SD[x] = \sqrt{VAR[x]}$$

Example: The SEPTA bus system services the Greater Philadelphia area. The Loop through University City arrives at a certain stop every 12 minutes. If you walk up to the stop at a random time during the day, then your wait time is uniformly distributed.

a. Give the probability function for this model.

$$f(x) = \frac{1}{12-0} = \frac{1}{12}, \quad 0 \leq x \leq 12$$

b. Draw the probability function for this model.



c. Probabilities (for continuous models) correspond to areas under the curve (this normal model). What is the chance that you wait at least 10 minutes for the bus?

$P(X \geq 10)$ = Area under curve from 10 to 12

= Length × Height

$= (12-10) \times \frac{1}{12} = \frac{2}{12} = \frac{1}{6} \approx 0.167$

16.7% chance

d. How long do you expect to wait?

$E[X] = \frac{1}{2}(0+12) = \frac{12}{2} = 6$ minutes

e. What is the standard deviation for wait times?

$SD[X] = \sqrt{\frac{1}{12}(12-0)^2} = \sqrt{12} \approx 3.46$ minutes

f. What is the probability you wait exactly 4 minutes?

$P(X=4)$ = Area corresponding to line at $x=4$

$= 0$ (lines have no area)

Since continuous model, infinite number of wait times, each particular wait time has 0 probability.

Can only give probability for intervals.

3. The Geometric Model (Discrete)

Definition: **Bernoulli trials** are outcomes in a probability experiment that have the following three characteristics:

- Each trial has 2 outcomes called S = success and F = failure
- $P(S) = p$, $P(F) = 1 - p$, both are constant trial to trial.
- Trials are independent.

The Geometric probability models how long it will take to get the first success in a series of Bernoulli trials.

- There is one parameter to the model, p.
- The random variable is the number of trials until the first success. Call it X.
- Probability function:

$$P(X = x) = (1-p)^{x-1} p$$ to model the number of trials needed to get that 1st success.

- Expected value / Mean:

$$E[X] = \mu = \frac{1}{p}$$

- Standard Deviation:

$$\sigma = \sqrt{\frac{1-p}{p^2}}$$

- Note: We must have independence in the Bernoulli trials! If you are sampling from a small(ish) population, check the 10% condition.

13%

Example: A certain golfer makes birdie or better on ~~13%~~ of his shots. We'd like to investigate the number of holes he needs to get his first birdie.

a. What assumptions must we make to use Bernoulli trials?

Holes are independent and each hole he has $p = 0.13$ Chance for a Birdie or better.

b. Give the probability model for the number of holes until he makes his first birdie. (or next)

$$P(X=x) = (1-p)^{x-1}p$$
$$= (0.87)^{x-1}(0.13)$$

c. What is the expected hole that he makes his first birdie?

$$E[X] = \frac{1}{p} = \frac{1}{0.13} = 7.69 \text{ Holes}$$

d. What is the standard deviation for the number of holes it takes?

$$SD(X) = \sqrt{\frac{1-p}{p^2}} = \sqrt{\frac{0.87}{(0.13)^2}} \approx 7.17$$

e. What is the probability that he makes it on the first hole?

$$P(X=1) = (0.87)^{1-1}(0.13) = (0.87)^0(.13)$$
$$= \underline{\underline{0.13}}$$

f. What is the probability he makes it on the first or second hole?

$P(X=1 \text{ OR } X=2) = P(X=1) + P(X=2)$

$= \boxed{0.13} + \boxed{(0.87)^{2-1}(0.13)} = 0.2431$

f2.) 1st 2nd or 3rd would be

$0.2431 + (0.87)^{3-1}(0.13) = 0.341$

g. What is the chance he has a birdie-free round?

P(Birdie Free) = P(all 18 holes non-Birdie)

$= (0.87)^{18} = 0.082$

8.2%

h. How many birdies does he expect in the 18 holes he plays?

He expects $18 \times 0.13 = 2.34$ Birdies

i. If we wanted to answer other questions, like "What is the chance he gets exactly three birdies in one round of golf", we cannot use a Geometric model. What model do we need to use?

Binomial Model.