# Quick Review of Stat I - Simple Linear Regression in Two Pages

- We will investigate the relationship between two quantitative variables.
- The x variable is the independent variable. In statistics, it's the  $\bigcirc \times \bigcirc$ variable.
- The y variable is the dependent variable. In statistics, it's the  $\underline{\text{response}}$ variable.
- The correct graph to make is a Scatterplot
- The number one requirement is that the relationship looks linear. If it doesn't, stop the analysis and do something else (details to come).
- We can measure the strength and direction of the linear relationship between two quantitative variables by calculating the <u>Correlation</u> coefficient,
- Correlation goes from \_\_\_\_\_, which is a perfect negative linear relationship, to , which is no linear relationship, to + which is a perfect positive linear relationship.
- Correlations beyond the ±0.8 are considered strong.
- If the data look linearly related and the correlation is worthwhile (context dependent), the next step is to create a linear model to predict \_\_\_\_\_\_ values based on \_\_\_\_\_\_ values.
- The equation of a straight line is:

equation of a straight line is:

$$y = m \times + b$$
(Math class)

 $y = b_0 + b_1 \times (s + a + c + a + s)$ 

The slope has a formula, though we don't use it much:

So does the *y*-intercept:

- The slope tells us how much \_\_\_\_\_ changes for a one-unit change in \_\_\_\_\_. Put it in context.

- The whole linear modeling idea is based on minimizing the <u>residuals</u>

  A residual is the vertical distance between a data point and the regression line. In words and symbols:

  Residual = Observed y Expected y
- A key summary number is the standard deviation of the residuals, \_\_\_\_\_\_\_, also known as the standard deviation of the errors. We want this to be \_\_\_\_\_\_\_,
- We can compare the standard deviation of the residuals to the standard deviation of the y-variable to get a sense of how much variation the model accounted for.
- Also, we can look at standardized residuals. The 68-95-99.7% rule kicks in, and residuals larger than ± 2 are unusual and should be looked at.
- Don't <u>extraplate</u>. We build regression models to do prediction in other words, plug in an <u>x</u> value and see what the model says for the value of <u>y</u>. We must only do this for the range of x-values we have data for.
- Finally, don't infer that the x-variable <u>Causes</u> the y-variable to change, even if there is a strong linear relationship. There is always the chance for lurking variables.

Open up the "Stat II Wal-Mart Supermarket" dataset on StatCrunch run an entire linear regression analysis, hitting all points. We use the Wal-Mart price to predict the supermarket price. Go.

X = Walmart price, y = Supernarket Price

Scatterplot: Linear, Positive, strong, about

5 products much more expensive

than the rest.

Correlation: F=0.903 (Strong Positive)

Model: Super = 0.191 + 1.133 (Walmart)

e.g. for prediction >

If something costs \$2 at Wal MART,

we predict it to cost

Super = 0.191+ 1.133 (2) = \$2.457

at the Supermarket.

b,= 1.133

For every \$1.00 increase at Walmaret,

we expect the supermarket

Price to increase by ~ \$1.13

y-intercept: If something cost

X=0 at Walmart, the model predicts

G=\$60.191 at the Supermaket.

Meaningless answer!

R<sup>2</sup>: 81.5% of the variation in So 81.5% of the Supermarket (y) prices at the Supermarket (y) is explained by the Wal-Mart price (x).

18.5% is explained by other x-variables

Standard deviation of the errors: Se = 0.608 Look at Sy = 1.407

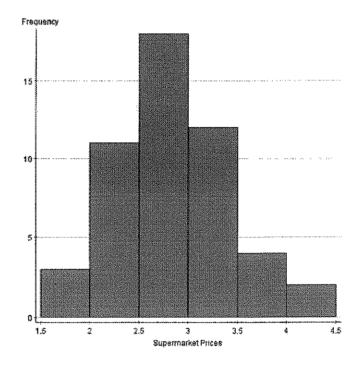
## Stat II Time - Inference for Regression

#### Questions:

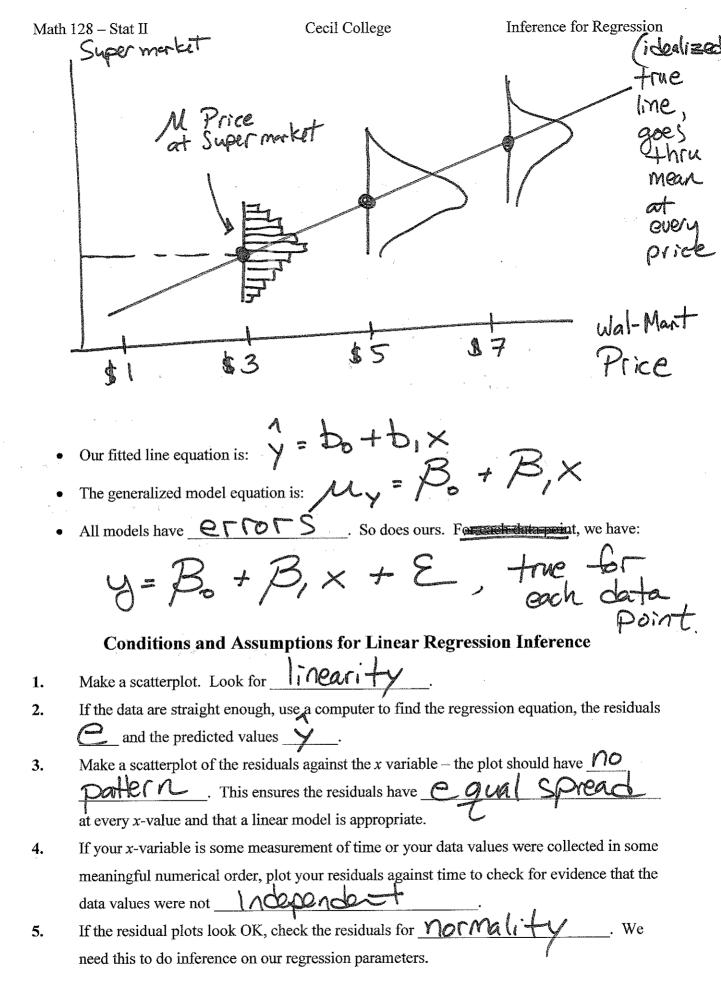
- Could the positive relationship we see between the Wal-Mart price and the supermarket price just be due to chance?
  - We can estimate the slope, but how reliable is our estimate? Probably

## Wal-Mart Example: Our Sample versus the Entire Population

- Understand that even if we knew prices of every single product at Wal-Mart and the supermarket, our data would not line up perfectly on a straight line.
- Rather, we want to Mode | the relationship between the two variables and we can imagine an idealized regression line.
- This idealized regression assumes the <u>Mean</u> supermarket price falls exactly on the line for every price point at Wal-Mart.
- For a product at Wal-Mart (x) that costs \$2.49, there is a distribution of supermarket prices:



• This is true at every possible Wal-Mart price point. Sketch a graph of this on the next page:



For the Wal-Mart and supermarket problem, check all conditions, using StatCrunch. Also note how to automatically calculate the residuals and the predicted values.

Boxes for residuals, fitted values.

Scatterplot: X = WALMART

y = Residuals

No pattern V Equal Spread v

Histo/QQ Residuals

Unimodal & Symmetric V QQ off a bit ....

n = 95 so not too concerned (n=30 is the magic number for inference)

Regression Inference - A List of Things We Can Test

First, test if the slope of the regression equation is 1. If this test has statistically significant results, it means your regression is technically meaningful.

- 2. Calculate a confidence interval for the true population slope. We know our slope is just one example from the actual data we happened to collect. A confidence interval will give us a range of plausible values and deepen our understanding.
- For a particular x-value of interest, we can find a confidence interval for the mean 3. predicted y-value.
- Also, we can find a prediction interval for an individual new observation at a particular x-4. value.

# 1. Testing if the Slope Differs From Zero

We will never calculate this by hand, but here is the formula for the standard error of the slope:

SE(bi) = Se  

$$\sqrt{n-1}$$
 Sx

It clearly depends on three things →

- 1. The spread about the regression line, \_\_\_\_\_ More spread about the regression line results in \_\_\_\_\_ variation in your slope, sample-to-sample.
- 2. The spread of your  $\times$  values. The more spread out your x-values are, the your slope will vary, sample-to-sample.
- 3. The sample size n. If you have a large sample size, your slope should vary esc. sample-to-sample.

**Example:** For fun, verify the standard error of the slope.

Summary statistics:

Column		Mean	Std. Dev.		•	_	Variance
Walmart Price	95		1.1208996	•	3	3.18	1.2564158
Supermarket Price			1.4066174	2.99	2	3	1.9785725

Simple linear regression results:

Dependent Variable: Supermarket Price Independent Variable: Walmart Price

Supermarket Price = 0.1907152 + 1.1329474 Walmart Price

Sample size: 95

R (correlation coefficient) = 0.9028

R-sq = 0.81508136

Estimate of error standard deviation: 0.6081193

Parameter estimates:

Parameter		Std. Err.	Alternative	DF	T-Stat	P-Value
Intercept	0.1907152		≠ 0	93	1.2196608	0.2257
Slope	1.132947 <b>4</b>	0.055957485	<b>→</b> 0	93	20.246574	

 $SE(b) = \frac{0.6081193}{\sqrt{95.1'(1.1208996)}} \approx 0.05596$ 

Question:

Why test if the slope equals zero?

Answer:

The regression model is:

If the slope equal zero, the regression model becomes:

With no x in the model, y doesn't depend on x at all!

The intercept  $b_0$  would turn out to be  $\checkmark$ 

In fact, for any x value, the predicted y value would be

Always quess y for

Official Steps for Testing the Slope

any value

1. The hypotheses are:

Can do <, > tests

Can test against a non-zero number.

2. The test statistic is:

test statistic is:
$$\frac{1}{L} = \frac{D_1 - Usually}{SE(D_1)} = \frac{1}{N-2}$$

- 3. The *P*-value would be found by shading in both directions (for our general  $\neq$  test) under the Student's *t* model with n-2 degrees of freedom.
- 4. When running regressions using the computer, this test is automatically performed, even if you didn't want to:

	Parameter	Estimate			; :		P-Value	į.
	Intercept	0.1907152	0.1563674	≠0	93	1.2196608	0.2257	
STATEMENT	Slope	ž	0.055957485	≠0		20.246574		7
					<b>/</b>		The state of the s	A STANSON AND A

5. We can calculate a confidence interval for the true slope of the population using:

**Example:** Back to Wal-Mart, calculate by hand a 99% confidence interval for the true population slope. Interpret. Then demonstrate on technology.

$$b_1 = 1.133$$
  $df = 95-2=93$ 
 $t^* = 0.99$ 
 $5E(b_1) = 0.05596$ 
 $1.133 \pm 2.6297(0.05596)$ 
 $(0.986) = 1.280$ 

Were 99% confident the true slope falls inside this interval.

A Quick Note on the y-Intercept

• On computerized output, the intercept is also always tested – check for Wal-Mart.

The Honshu Earthquake that hit Japan a few years ago caused massive destruction. Is there value in trying to predict the magnitude based on the depth of the earthquake? Run a preliminary linear regression analysis, checking the conditions and testing for the slope.

X=Depth, y=magn; tide

Scatterptot: Not really linear,  $\Gamma = -0.088$ , very weak negative. A few outliers, namely the trig one > depth = 24.4 mag = 8.9

EQUATION: MAG = 5.32 - 0.0054 (Depth) R<sup>2</sup> = 0.77%

\* Residuals US. X = depth, no pattern, pretty equal spread v

See "by one" is almost 4 SD above predicted value.

\* Data is in chronological order, so check residuals in time order (index)

- Upward trend near "big one", suggest data values not independent.

\* Residuals not normal, suggesting caution.

Data set has n= 446 values so CLT
takes over and normality not crucial.

\* Ho: b,=0 vs. Ha: b, ≠0 (test slope)

Plalue = 0.0638, fail to reject Ho.

Since we're not convinced the slope

differs from [72]0, this regression has

limited value.

Back in the Fall 2012 semester, Cecil College students randomly selected a book from the college library, and recorded the number of pages and the weight (lbs). Open the "Library Data – Quiz 2" dataset.

Check the conditions to predict a book's weight based on its number of pages. Test if the model is useful. Create a 95% confidence interval for the true slope. Interpret.

Scatterplot > Linear, positive, a few large books,

Residual Plot - No Pattern, suggesting linearity
OK. Large residuals for large books

(funneling) so we must exercise caution.

Histogram of Residuals: Unimodal & Symmetric

and n=115 (large) so OK here.

Model: Weight = 0.126 + 0.00165 (Pages)

Test slape: Ho: b, =0

HA: D, +0

Test Stat: 6=11.33

P-Value < 0.0001

Réject Ho and conclude slope is not O.

This means that # of pages does

have predictive value for weight of

[73] a 500 K

95% CI for true slope B, is b, ± t\* (SE of slope)

0.00165 ± 1.981 (0.000146)

0.00165 ± 0.000289

(0.001361 to 0.001939)

95% confident that, on average, between 0.001361 and 0.001939 1 bs. are added

to a book's weight for each additional page.

STAT OR WICH:

Select CI in the regression menus. Default 15 HT.

## **Confidence Intervals for Prediction**

- When we plug in an x value into our regression equation, the y value we solve for is at best an educated guess.
- There are two questions we can answer with predictions:
  - $\circ$  For a given x value, give a confidence interval for the mean y value.
    - **Example:** When predicting the weight of a book based on the number of pages, suppose you're interested in 600 page books. We can create a 95% confidence interval for the *mean* weight of 600 page books.
  - $\circ$  For a given x value, give a prediction interval for a particular y value.
    - **Example:** We pull a 600 page book off the shelves. We can create a 95% prediction interval for the weight of that *particular* book.
- Confidence intervals for mean y values at a particular x value are much skinmer than the same prediction interval. It is much harder to predict the y value of the next observation.

## Formulas

Both intervals have this form, with n-2 degrees of freedom for the t critical value:

Yu = t + x SE

The SE = Standard error is

different depending on which
interval we are calculating.

Confidence Interval Formula for the Mean y value at a Particular x Value

$$\hat{y}_{\nu} \pm t_{\nu 2}^{*}(SE) \quad \text{with}$$

$$SE(\hat{\mu}_{\nu}) = \sqrt{SE^{2}(b_{1}) \cdot (x_{\nu} - x)^{2} + \frac{Se^{2}}{n}}$$

Prediction Interval Formula for the y value at a Particular x Value

$$SE(\hat{\gamma}_{0}) = \sqrt{SE^{2}(h) \cdot (h-x)^{2} + \frac{Se^{2}}{n} + Se^{2}}$$

$$SE(\hat{\gamma}_{0}) = \sqrt{SE^{2}(h) \cdot (h-x)^{2} + \frac{Se^{2}}{n} + Se^{2}}$$

- The standard error of a single predicted value has extra variability when compared to the standard error for the mean. This is because individuals vary around the predicted mean.
- In the formula, there is an extra term in our prediction intervals:
- This extra term makes prediction intervals wide, as expected.
- StatCrunch will output these values, but first, one example by hand.

For the Library dataset, calculate a 95% confidence interval for the mean weight of a book with 600 pages. Interpret. Then calculate a 95% prediction interval for the same book. Compare.

N = 115, df = 113 and  $t^* = 1.981$   $SE(b_1) = 0.000146$ , Se = 0.355  $X_{U} = 600$ , X = 402.096 $Y_{U} = 0.126 + 0.00165(600) = 1.116$ 

95% CI for mean weight at x = 600 pages: 1.116 ± 1.981  $(0.000146)^2(600-400.096)^2 + \frac{(0.355)^2}{115}$ 

(1.116 ± 0.087 => (1.029, 1.203)

We're 95% confidut the interval contains the mean weight of a 600-page book.

95% PI for weight of the next 600 page 1.116  $\pm$  1.981  $\sqrt{(0.000146)^2(600-402.096)^2+\frac{0.355^2}{1.15}}+0.355^2$ 1.116  $\pm$  0.709  $\Rightarrow$  (0.407, 1.825)

We're 95% confident the next 600 page book with weigh between 0.407 1b.

and 1.825 1bs.

For the Wal-Mart dataset, use StatCrunch to compute a 99% confidence interval for the mean price at the supermarket if something costs \$3.49 at Wal-Mart. Then determine a 99% prediction interval for the same item. Interpret.

Under Regression menus, 121 Predict

y for x = 3.49

and 9970

We are 99% confident that the mean price at the Supermarket is between a \$3.98 and a \$4.34 if something costs \$3.49 at Ual MART.

We are 99% conf. Heat He next

\$3.49 ifem at WalMART WILL

cost Between \$2.53 and

\$5.76 at the Supr Mannet

#### **Regression Problems**

Example:

How does the *Zillow Value* of a house depend on its *Assessed Value*? Data from Professor Kupe's neighborhood produced the following regression output and

graphs.

Simple linear regression results:

Dependent Variable: Zillow Value Independent Variable: Assessed

Zillow Value = -22913.75 + 1.038321 Assessed

Sample size: 30

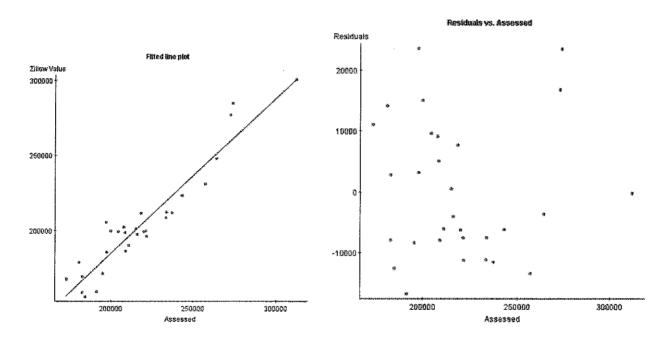
R (correlation coefficient) = 0.9478

R-sq = 0.8983507

Estimate of error standard deviation: 11460.4795

#### Parameter estimates:

Parameter	ii 3		Alternative	:		P-Value
		14591.321			-1.5703685	
Slope	1.038321	0.06600575	≠0	28	15.730766	<0.0001



**a.** The intercept is negative. Discuss its value, taking note of its *P*-value.

**b.** Is the regression meaningful? Why?

c. The output reports  $s_e = 11,460.4795$ . What does this number mean in this context?

**d.** What's the value of the standard error of the slope of the regression line? What's it mean in context.

**Example:** In the Chicken Sandwiches dataset, we'd like to predict *Calories* based on *Fat*.

a. To use a linear model, we must check many conditions. Check them all.

Scatterplat > Linear, positive 
Residuals vs. Fat -> Random Scatter 
Equal spread 
Residuals -> Unimodal, Symmetric -

**b.** Burger King has been toying with a new sandwich that would have 59 grams of fat. Predict its calories or explain why you would be hesitant to do so.

X=59g fat is extrapolation.

Biggest x-value we have is

Sonic Club Toaster x= 46g

Predictions, beware.

**c.** Give the value of the residual for the Subway Oven Roasted and explain what it means in context.

e=37.37 calories

Sandwich has 37.37 more calories than predicted, based on having 5 grams of fat. **d.** Predict, with a 95% interval, the number of calories the next newly-invented chicken sandwich would have if we knew it was going to have 34 grams of fat.

95% prediction interval if x = 34 g.
(503,4 to 759,5 calories)

**e.** Is there evidence of a relationship between fat grams and number of calories? Back up your answer.

We test if slope is zero

Ho: B. = 0 Prvalue < 0.0001

Hh: B. #0 Reject to

There is evidence that fat grams usefully can predict # of cabries (

**f.** Give a 95% confidence interval for the mean number of calories for sandwiches with 19 grams of fat.

We're 95% confident that 19g fit sandwicks average between 428.0 and 463.7 calories.

g. What percentage of the variation in calories is explained by grams of fat?

R2 - 82.5%

# Before ANOVA, Review Inference One Last Time

Example:

Some college professors make bound lecture notes available to their students in an effort to improve teaching effectiveness. At one college, two groups of students were surveyed – 86 students enrolled in a promotional strategy class that required the purchase of lecture notes and 35 students enrolled in a sales / retailing elective that did not offer the notes. At the end of the semester, students were asked to respond: "Having a copy of the lecture notes was [would be] helpful in understanding the material". Responses were measured on a 9-point scale, where 1 = strongly disagree and 9 = strongly agree.

Summarized results are found in the table:

	Classes Buying Lecture Notes	Classes Not Buying Lecture Notes
Sample Size	86	35
Sample Mean	8.48	7.80
Sample Variance	0.94	2.99

- a. Do the samples provide sufficient evidence to conclude there is a difference in the mean responses of the two groups of students? Test using  $\alpha = 0.01$ .
- **b.** If we made an error, what kind of error? What would this mean in the context of the problem?
- **c.** Interpret the *P*-value in context.
- **d.** Construct a 99% confidence interval for the difference in sample means. Interpret the result.

a.) 2-Sample T Test for a difference
in means.

Ho: M NOTES = MNO NOTES

HA: M NOTES + MNO NOTES

TI- 2 Sample T Test

t= 1.32 P-Value = 0.195

Fail to Reject Ho, conclude there
is not enough evidence to say

there is a difference in

Mean responses

- b) Since we failed to reject the,

  could have made a Type II ever.

  This means that the means do differ

  but our collected date did not show it.
- C.) P-Value = 0.195. If the two population means were actually equal, we'd get sample means of Thores

  YNO HOTES = 7.80 or ones even more different 19.5% of the time.
  - (195% is not unusual enough to convince us the 75 are different)
  - d.) 2- SampT Fint (-0.72 to 2.08)

99% confident this interval captures
the true difference in means.
As O is in the interval, no
difference is plausible.

In the Journal of Genetic Psychology, 13 male college students were asked to complete the following survey about each of his parents: "My relationship with my father (mother) can best be described as (1) Awful! (2) Poor, (3) Average, (4) Good, or (5) Great!

The data obtained are in the table:

Student	1	2	3	4	5	6	7	8	9	10	11	12	13
Attitude Toward Father	2	5	4	4	3	5	4	2	4	5	4	5	3
Attitude Toward Mother	3	5	3	5	4	4	5	4	5	4	5	4	3
		0	1				eso	2	-	and grades	\$2.00°	Sections	0

- a. Specify the appropriate hypotheses for testing whether male students' attitudes towards their fathers differs from their attitude towards their mothers, on average.
- **b.** Conduct the test, using a 5% level of significance.
- **c.** What assumptions about the sample and the population did you have to make in order to ensure the validity of the test?

a) Dependent samples so just run a TTest on the differences.

Find the Peject Ho, not convinced Students attitudes differ on father on father vs. mother.

C.) n=13 so differences must be normal Also, these guys are unbiased, represent all guys.

Obstructive sleep apnea is a disorder that causes a person to stop breathing momentarily and then awaken briefly. These sleep interruptions, which can occur hundreds of times in a night, can drastically reduce the quality of rest and cause fatigue during the waking hours.

Researchers at Stanford University studied 159 commercial truck drivers and found that 124 of them suffered from obstructive sleep apnea (*Chest*, May 1995).

**a.** Use the study results, to estimate, with 90% confidence, the fraction of truck drivers who suffer from the sleep disorder.

b. Sleep researchers believe that about 25% of the general population suffer from obstructive sleep apnea. Comment on whether or not this value represents the true percentage of truck drivers who suffer from the sleep disorder.

a) 1-Prop Z interval on TI

Assume unbiased sample, have 10+ S/F/V

P = 124 = 0.7799

Were 90% confidet that between

72.6% and 83.4% of all truckers have slows aprea.

b) We'd test Ho: P=75% vs.

HA: P=75% and

clearly reject Ho.

I would ausotion the aprea testing methods! Are truckers really that different?

One study of gambling newsletters that purport to improve a bettor's odds of winning bets on NFL football games indicates that the newsletters' betting schemes were not profitable. Suppose a random sample of 50 games is selected to test one gambling newsletter. Following the newsletter's recommendations, 30 of the 50 games produced winning wagers.

Do we have evidence that the newsletter can be said to significantly increase the odds of winning over what one could expect by selecting the winner at random? Interpret the *P*-value in context.

If we picked our teams at random (say flip a coin) we'd get 30 right or more 7.86% of the time.

Our data is not super-convincing that the newsletter's picks do better but there is some evidence they do.

Proceed with caution.

(25,600 customers week 7) Joe, notes

Census data for NYC indicate that 29.2% of the under-18 population is white, 28.2% black, 31.5% Latino, 9.1% Asian, and 2% other ethnicities.

The New York Civil Liberties Union points out that of 26,181 police officers, 64.8% are white, 14.5% are black, 19.1% are Hispanic, and 1.4% are Asian. Do the police officers reflect the ethnic composition of the city's youth? Test an appropriate hypothesis and state your conclusion.

Observed: 16965 3796 Hispanic Asian 367

Expected: 7644.852 7383,042

8247-015

2382,471 523.62

CHI-SQUARE, GOODNESS OF

Ho: NYPD distribution of cops matches the City's youth.

II=4, run on Tl

X2 = 16512.75

P. Value = 0

Reject Ho, conclude distribution of cops doesn't match

distribution of NY youth ethnicities

( Poes it have to ?)