Measurement-Based Quantum Computing

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Unitary Evolution & Measurement

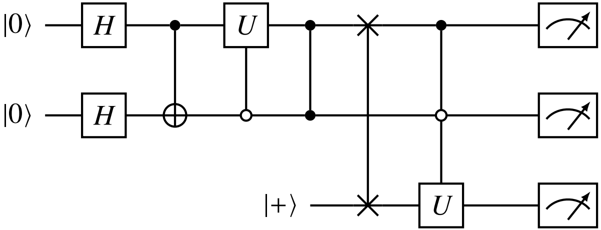


Figure 1: An Example Quantum Circuit

Unitary Evolution & Measurement

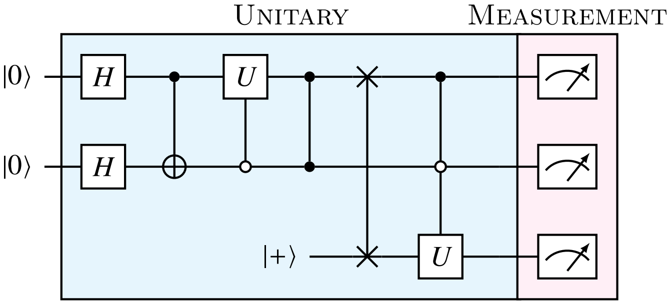


Figure 2: An Example Quantum Circuit

Unitary Evolution & Measurement

Unitary Evolution

- Deterministic
- Reversible
- Intuitive (somewhat)

Measurement

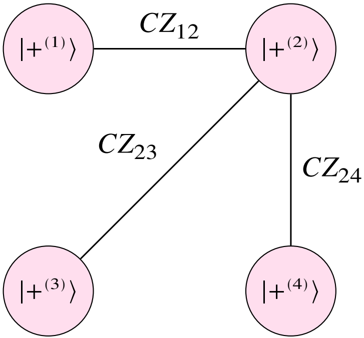
- Probabilistic
- Irreversible

Measurement-Based Quantum Computer

Two componets:

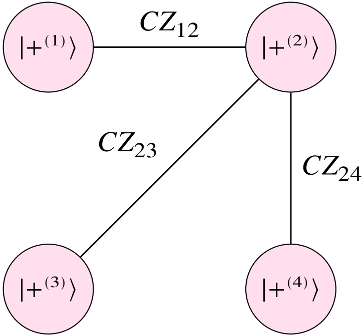
1. Cluster: Holds the qubits. Substrate for universal computation.
2. Measurement Device: Governs the program execution.

The Cluster: Graph States



- A graph has:
  1. A set of vertices $V = \{1, 2, \ldots, N\}$. Represents qubits
  2. A set of edges connecting some of the vertices $E \subseteq [V]^2$ where $|E| = M$. Represents entanglement patterns.

The Cluster: Graph States



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  1. A set of vertices $V = \{1, 2, \ldots, N\}$. Represents qubits
  2. A set of edges connecting some of the vertices $E \subseteq [V]^2$ where $|E| = M$. Represents entanglement patterns.
- Prepare each qubit in $|+\rangle$, apply CZ if they are entangled.

Simulating Circuits with MBQC

- Prove that MBQC is universal
- Reuse existing algorithms and code

Simulating Circuits with MBQC

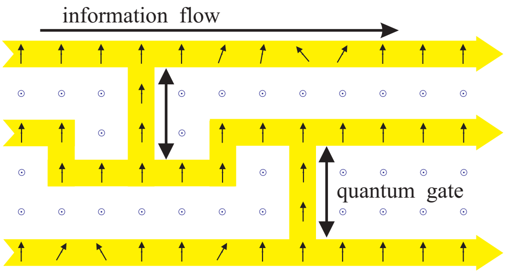
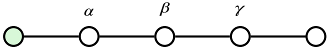


Figure 3: Unitary Gate Simulation Diagram

- Simulate universal gate set $\mathrm{CNOT} = \sigma_x$ and $R(\alpha, \beta, \gamma)$.
- Fix by products after each gate simulation

Simulating Unitary Rotation



- Single qubit measurements on basis

$$\mathscr{B}(\varphi) = \left\{ \frac{|0\rangle + e^{i\varphi}|1\rangle}{2}, \frac{|0\rangle - e^{i\varphi}|1\rangle}{2} \right\}$$

that simulate

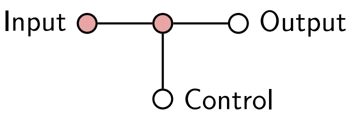
$$\sigma_x^s HP(\varphi) = \sigma_x^s J(\varphi),$$

- MBQC implements

$$\underbrace{\sigma_x^{s_2+s_4}\sigma_z^{s_1+s_3}}_{U_{\Sigma,R}} \underbrace{J(0)J(\gamma)J(\beta)J(\alpha)}_{R(\alpha,\beta,\gamma)}$$

through 4 adaptive measurements.

Simulating CNOT



- Apply σ_x measurements on red vertices.
- MBQC implements

$$U'_{\mathrm{CNOT}} = U_{\Sigma,\mathrm{CNOT}} U_{\mathrm{CNOT}}$$

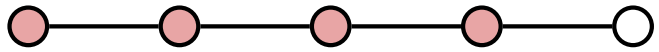
with a by-product

$$U_{\Sigma,\mathrm{CNOT}} = \left(\sigma_x^{(3)}\right)^{s_2} \left(\sigma_z^{(3)}\right)^{s_1} \left(\sigma_z^{(4)}\right)^{s_1}$$

through 4 adaptive measurements.

Additional Patterns

- Measuring a wire of qubits on σ_x basis propagates the information on first qubit to last.



- Measuring a qubit on σ_z basis removes its connection with the cluster

Classical Simulation

- Well known frameworks like Qiskit, Cirq etc. lack MBQC simulators.

Classical Simulation

- Well known frameworks like Qiskit, Cirq etc. lack MBQC simulators.
- Experimental Paddle Quantum backend available.

Classical Simulation of Deutsch's Problem

- Takes binary functions $f : \{0, 1\} \to \{0, 1\}$
- Constant, balanced function classification.
- $\mathcal{O}(1)$ complexity.

Classical Simulation of Deutsch's Problem

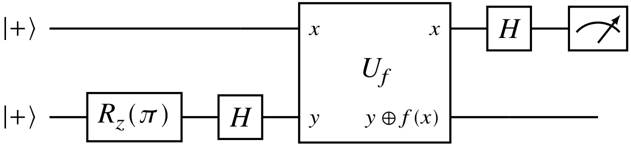


Figure 4: Quantum cicruit implementing Deutsch's algorithm

Classical Simulation of Deutsch's Problem

Type	Function	Oracle Unitary
Constant	$f(x) = 0$	$\mathbb{I}$
Constant	$f(x) = 1$	σ_x^1
Balanced	$f(x) = x$	CNOT
Balanced	$f(x) = \neg x$	$\sigma_x^1\,\mathrm{CNOT}$

Table 1: Deutsch's algorithm unitary oracle implementations

Results

- We discussed a novel model for quantum computation
- Investigate graph states
- Proposed an abstract machine that works by the MBQC principles
- Showed that MBQC is universal
- Small scale verification using classical a simulation