Seminar 31-32

Multivariate Time Series models

Problem 1

What is "spurious regression"?

Using STATA simulate two independent RW-processes x and y. Then try to estimate a regression of y on x. Comment you results after several attempts.

Problem 2 *(Exercise 9.3 from Verbeek M. A guide to modern Econometrics, 2004)*

In the file mts_income.dta you find quarterly data on UK nominal consumption and income, for 1971:1 to 1985:2 ($T = 58$).

a) Test for a unit root in the income and consumption series using several augmented Dickey–Fuller tests.

b) Investigate the correlation of income and consumption.

c) Perform a regression by OLS explaining consumption from income. Test for cointegration using two different tests.

d) Perform a regression by OLS explaining income from consumption. Test for cointegration.

e) Compare the estimation results and R^2 from the last two regressions.

f) Discuss an error-correction (ECM) model. Interpret the components of these model (what are the long-term and short-term effects).

g) Estimate an error-correction model for the change in consumption. Pay attention to the adjustment parameter. Test whether the adjustment coefficient is zero.

h) Repeat the last question for the change in income. What do you conclude?

i) Discuss a vector autoregressive (VAR) model using these two time series and a VECM representation for this model.

Table 9.2 Asymptotic critical values residual unit root tests for cointegration (with constant term) (Davidson and MacKinnon, 1993)

Number of variables (incl. Y_t)	Significance level 1%	5%	10%
2	−3.90	−3.34	−3.04
3	−4.29	−3.74	−3.45
4	−4.64	−4.10	−3.81
5	−4.96	−4.42	−4.13

Table 9.3 5% Critical values CRDW tests for co-integration (Banerjee *et al.*, 1993)

Number of variables (incl. Y_t)	Number of observations 50	100	200
2	0.72	0.38	0.20
3	0.89	0.48	0.25
4	1.05	0.58	0.30
5	1.19	0.68	0.35

Problem 3

a) Download the data file balance2.dta.
b) Investigate y, i and c for stationarity:
   - y is ln(GDP)
   - i is ln(income)
   - c is ln(consumption)
c) Investigate these time series for cointegration using Durbin-Watson test.
d) Discuss the Johansen test. Use it to test for cointegration and number of cointegrating relations.
e) Estimate a vector error-correction (VECM) model, interpret the results and provide tests for specification of the model.

Problem 4 *(9.6 Illustration "Money Demand and Inflation" from Verbeek M. A guide to modern Econometrics, 2004)*

Read attentively this chapter and try to repeat calculations and interpret the results.

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VECM is STATA

$$\Delta \mathbf{y}_t = \alpha(\beta \mathbf{y}_{t-1} + \mu + \rho t) + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathbf{y}_{t-i} + \gamma + \tau t + \epsilon_t$$

Five different trend specifications are available:

Option in `trend()`	Parameter restrictions	Johansen (1995) notation
`trend`	none	$H(r)$
`rtrend`	$\tau = 0$	$H^*(r)$
`constant`	$\rho = 0$, and $\tau = 0$	$H_1(r)$
`rconstant`	$\rho = 0$, $\gamma = 0$ and $\tau = 0$	$H_1^*(r)$
`none`	$\mu = 0$, $\rho = 0$, $\gamma = 0$, and $\tau = 0$	$H_2(r)$

Johansen procedure

$$\Delta\vec{Y}_t = \delta + \Gamma_1\Delta\vec{Y}_{t-1} + \cdots + \Gamma_{p-1}\Delta\vec{Y}_{t-p+1} + \Pi\vec{Y}_{t-1} + \vec{\varepsilon}_t, \qquad \Pi = \gamma\beta'$$

Where β is a matrix of cointegration vectors and γ represents the matrix of weights with which each cointegrating vector enters each of the ΔY_t equations.

$r_k(\Pi) = 0$ if $\Pi=0$, i.e. all elements of Y_t are I(1) processes without cointegration

$r_k(\Pi) = (max\ rank\ if\ \Pi)$ is invertible, i.e. all elements of Y_t are I(0) processes

the hypothesis H_0: $r \leq r_0$ versus the alternative H_1: $r_0 < r \leq k$, can be tested using the statistic

$$\lambda_{trace}(r_0) = -T \sum_{j=r_0+1}^{k} \log(1-\hat{\lambda}_j). \tag{9.52}$$

This test is the so-called **trace test**. It checks whether the smallest $k - r_0$ eigenvalues are significantly different from zero. Furthermore, we can test H_0: $r \leq r_0$ versus the more restrictive alternative H_1: $r = r_0 + 1$ using

$$\lambda_{max}(r_0) = -T\log(1-\hat{\lambda}_{r_0+1}). \tag{9.53}$$

This alternative test is called the **maximum eigenvalue test**, as it is based on the estimated $(r_0 + 1)$th largest eigenvalue.

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From Enders "Applied Econometric Time Series" (pp. 132-133)

Dolado, Jenkinson and Sosvilla-Rivero (1990) suggested the following procedure to test for a unit root when the form of the data-generating process is unknown. The following is a straightforward modification of their method:

STEP 1: As shown in Figure 4.7, start with the least restrictive of the plausible models (which will generally include a trend and drift) and use the τ_τ statistic to test the null hypothesis $\gamma = 0$. Unit root tests have low power to reject the null hypothesis; hence, if the null hypothesis of a unit root is *rejected*, there is no need to proceed. Conclude that the $\{y_t\}$ sequence does not contain a unit root.

STEP 2: If the null hypothesis is *not rejected*, it is necessary to determine whether too many deterministic regressors were included in Step 1 above.[11] Test for the significance of the trend term under the null of a unit root (e.g., use the $\tau_{\beta\tau}$ statistic to test the significance of a_2). You should try to gain additional confirmation for this result by testing the hypothesis $a_2 = \gamma = 0$ using the ϕ_3 statistic. If the trend is not significant, proceed to Step 3. Otherwise, if the trend is significant, retest for the presence of a unit root (i.e., $\gamma = 0$) using the standardized normal distribution. After all, if a trend is inappropriately included in the estimating equation, the limiting distribution of a_2 is the standardized normal. If the null of a unit root is rejected, proceed no further; conclude that the $\{y_t\}$ sequence does not contain a unit root. Otherwise, conclude that the $\{y_t\}$ sequence contains a unit root.

STEP 3: Estimate (4.35) without the trend [i.e., estimate a model in the form of (4.13)]. Test for the presence of a unit root using the τ_μ statistic. If the null is rejected, conclude that the model does not contain a unit root. If the null hypothesis of a unit root is not rejected, test for the significance of the constant (e.g., use the $\tau_{\alpha\mu}$ statistic to test the significance of a_0 given $\gamma = 0$). Additional confirmation of this result can be obtained by testing the hypothesis $a_0 = \gamma = 0$ using the ϕ_1 statistic. If the drift is not significant, estimate an equation in the form of (4.12) and proceed to Step 4. If the drift is significant, test for the presence of a unit root using the standardized

Figure 4.7 A procedure to test for unit roots.

Estimate $\Delta y_t = a_0 + \gamma y_{t-1} + a_2 t + \Sigma\beta_i \Delta y_{t-i} + \varepsilon_t$