Math2413 Exam2 Review

sec2.4-2.6,3.1-3.5,3.7

Find the derivatives of the following functions and simplify.

1. Find the derivative of the function.

   $$g(x) = \left(\frac{x+1}{x^2+7} \right)^3$$

2. Find the derivative of the function.

   $$f(x) = x^8(5+7x)^3$$

3. Find $\frac{dy}{dx}$ by implicit differentiation.

   $$x^2 + 5x + 9xy - y^2 = 4$$

4. Find $\frac{dy}{dx}$ by implicit differentiation.

   $$\sin x + 7\cos 14y = 2$$

5. Use implicit differentiation to find an equation of the tangent line to the ellipse $\frac{x^2}{2} + \frac{y^2}{98} = 1$ at $\left(1, 7\right)$.

6. Find the points at which the graph of the equation has a vertical or horizontal tangent line.

   $$5x^2 + 4y^2 - 10x + 24y + 8 = 0$$

7. Assume that x and y are both differentiable functions of t. Find $\frac{dx}{dt}$ when $x = 11$ and $\frac{dy}{dt} = -8$ for the equation $xy = 132$.

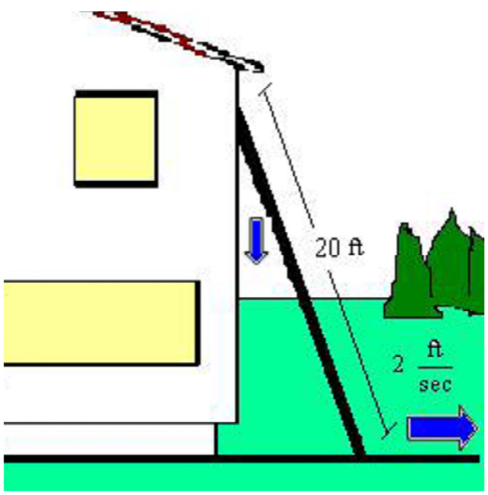
8. A point is moving along the graph of the function $y = \dfrac{1}{9x^2 + 4}$ such that $\dfrac{dx}{dt} = 2$ centimeters per second. Find $\dfrac{dy}{dt}$ when $x = 2$.

9. The radius r of a sphere is increasing at a rate of 6 inches per minute. Find the rate of change of the volume when $r = 11$ inches.

10. A spherical balloon is inflated with gas at the rate of 300 cubic centimeters per minute. How fast is the radius of the balloon increasing at the instant the radius is 70 centimeters?

11. A conical tank (with vertex down) is 12 feet across the top and 18 feet deep. If water is flowing into the tank at a rate of 18 cubic feet per minute, find the rate of change of the depth of the water when the water is 10 feet deep.

12. A ladder 20 feet long is leaning against the wall of a house (see figure). The base of the ladder is pulled away from the wall at a rate of 2 feet per second. How fast is the top of the ladder moving down the wall when its base is 13 feet from the wall? Round your answer to two decimal places.



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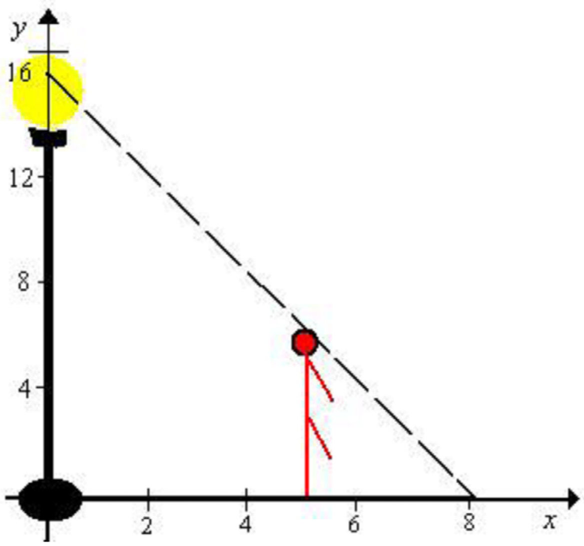
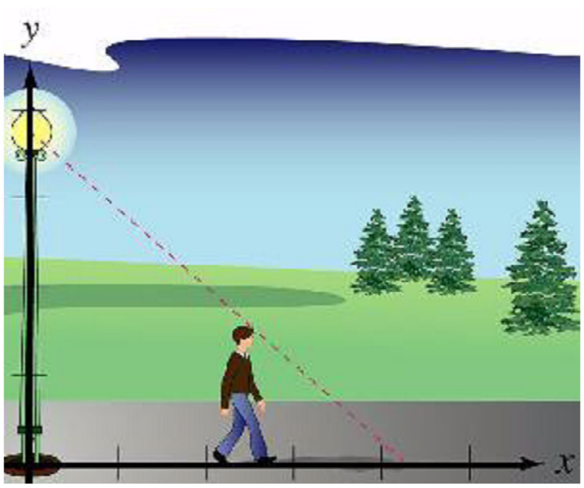
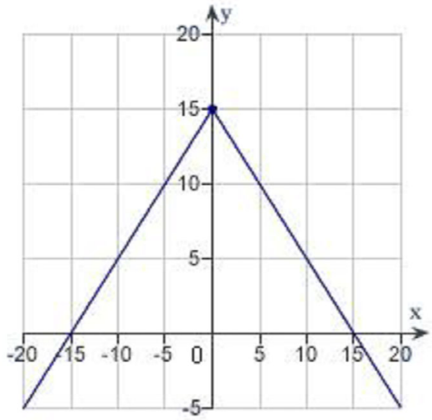
13. A man 6 feet tall walks at a rate of 13 feet per second away from a light that is 15 feet above the ground (see figure). When he is 5 feet from the base of the light, at what rate is the length of his shadow changing?



14. A man 6 feet tall walks at a rate of 2 ft per second away from a light that is 16 ft above the ground (see figure). When he is 8 ft from the base of the light, find the rate.at which the tip of his shadow is moving.



15. Find the value of the derivative (if it exists) of the function $f(x) = 15 - |x|$ at the extremum point $\left(0,\ 15\right)$.



16. Find all critical numbers of the function $g(x) = x^4 - 4x^2$.

17. Find any critical numbers of the function $g(t) = t\sqrt{2-t}$, $t < 2$.

18. Locate the absolute extrema of the function $f(x) = x^3 - 12x$ on the closed interval $\left[\,0,\ 4\,\right]$.

19. Locate the absolute extrema of the given function on the closed interval $[-10,\ 10]$.

$$f(x) = \frac{10x}{x^2 + 1}$$

20. Determine whether Rolle's Theorem can be applied to the function $f(x) = (x-5)(x-6)(x-7)$ on the closed interval $\left[\,5, 7\,\right]$. If Rolle's Theorem can be applied, find all numbers c in the open interval $\left(5, 7\right)$ such that $f'(c) = 0$.

21. The height of an object t seconds after it is dropped from a height of 250 meters is $s(t) = -4.9t^2 + 250$. Find the time during the first 8 seconds of fall at which the instantaneous velocity equals the average velocity.

22. Identify the open intervals where the function $f(x) = x\sqrt{30 - x^2}$ is increasing or decreasing.

23. For the function $f(x) = 4x^3 - 48x^2 + 6$:

(a) Find the critical numbers of f (if any);
(b) Find the open intervals where the function is increasing or decreasing; and
(c) Apply the First Derivative Test to identify all relative extrema.

24. For the function $f(x) = (x-1)^{\frac{2}{3}}$:

(a) Find the critical numbers of f (if any);
(b) Find the open intervals where the function is increasing or decreasing; and
(c) Apply the First Derivative Test to identify all relative extrema.

25. Determine the open intervals on which the graph of $y = -6x^3 + 8x^2 + 6x - 5$ is concave downward or concave upward.

26. Find the second derivative of the function $f(x) = \sin 5x^6$.

27. Find the derivative of the function.

$$f(\theta) = \frac{7}{5}\sin^2 2\theta$$

28. Find the derivative of the function.

$$f(t) = (1+8t)^{\frac{5}{9}}$$

29. Find the cubic function of the form $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$ and the coefficients a, b, c, d are real numbers, which satisfies the conditions given below.

Relative maximum: $(3, 0)$

Relative minimum: $(5, -2)$

Inflection point: $(4, -1)$

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30. Find the limit.

$$\lim_{x \to \infty} \left(5 + \frac{3}{x^2} \right)$$

31. Find the limit.

$$\lim_{x \to \infty} \frac{3x + 2}{-6x - 6}$$

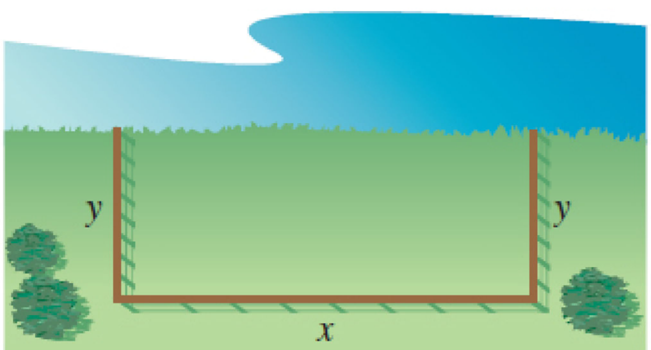
32. Find the limit.

$$\lim_{x \to -\infty} \frac{-6x}{\sqrt{64x^2 - 5}}$$

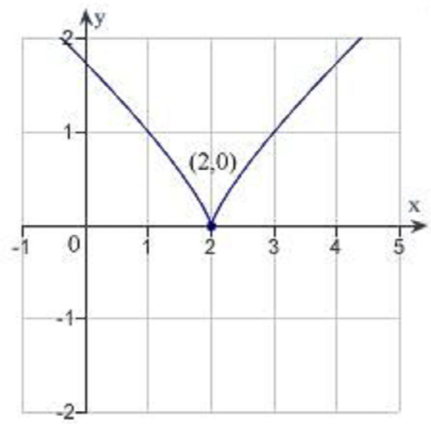
33. Find the point on the graph of the function $f(x) = \sqrt{x}$ that is closest to the point $\left(18, 0 \right)$.

34. A rectangular page is to contain 144 square inches of print. The margins on each side are 1 inch. Find the dimensions of the page such that the least amount of paper is used.

35. A farmer plans to fence a rectangular pasture adjacent to a river (see figure). The pasture must contain 720,000 square meters in order to provide enough grass for the herd. No fencing is needed along the river. What dimensions will require the least amount of fencing?



36. Find the value of the derivative (if it exists) of $f(x) = (x-2)^{4/5}$ at the indicated extremum.



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Answer Section

SHORT ANSWER

1. $g'(x) = \dfrac{3\left(7 - 2x - x^2\right)(1+x)^2}{\left(7 + x^2\right)^4}$
2. $f'(x) = x^7(5+7x)^2(40+77x)$
3. $\dfrac{dy}{dx} = \dfrac{2x+5+9y}{2y-9x}$
4. $\dfrac{dy}{dx} = \dfrac{\cos x}{98\sin 14y}$
5. $y = -2x+9$
6. There is a horizontal tangent at $x = 1$ and a vertical tangent at $y = -3$.
7. $\dfrac{dx}{dt} = \dfrac{22}{3}$
8. $\dfrac{dy}{dt} = -\dfrac{9}{200}$
9. $\dfrac{dV}{dt} = 2904\pi \text{ in}^3 / \text{min}$
10. $\dfrac{dr}{dt} = \dfrac{3}{196\pi}$ cm/min
11. $\dfrac{81}{50\pi}$ ft/min
12. -1.71 ft/sec
13. $\dfrac{26}{3}$ ft/sec
14. $\dfrac{16}{5}$ ft per minute
15. does not exist
16. critical numbers: $x = 0$, $x = \sqrt{2}$, $x = -\sqrt{2}$
17. $\dfrac{4}{3}$
18. absolute max: $\left(4,\ 16\right)$; absolute min: $\left(2,\ -16\right)$
19. absolute max: $\left(1,\ 5\right)$; absolute min $\left(-1,\ -5\right)$
20. Rolle's Theorem applies; $c = 6 + \dfrac{\sqrt{3}}{3}$, $6 - \dfrac{\sqrt{3}}{3}$
21. 4 seconds

22. increasing: $\left(-\sqrt{15}, \sqrt{15}\right)$; decreasing: $\left(-\sqrt{30}, -\sqrt{15}\right) \cup \left(\sqrt{15}, \sqrt{30}\right)$
23. (a) $x = 0$, 8
    (b) increasing: $\left(-\infty, 0\right) \cup \left(8, \infty\right)$; decreasing: $\left(0, 8\right)$
    (c) relative max: $f(0) = 6$; relative min: $f(8) = -1018$
24. (a) $x = 1$
    (b) decreasing: $\left(-\infty, 1\right)$; increasing: $\left(1, \infty\right)$
    (c) relative min: $f(1) = 0$
25. concave upward on $\left(-\infty, \dfrac{4}{9}\right)$; concave downward on $\left(\dfrac{4}{9}, \infty\right)$
26. $f''(x) = 150x^4 \cos 5x^6 - 900x^{10} \sin 5x^6$
27. $f'(\theta) = \dfrac{28 \sin 2\theta \cos 2\theta}{5}$
28. $f'(t) = \dfrac{40}{9}(1 + 8t)^{\frac{-4}{9}}$
29. $f(x) = \dfrac{1}{2}x^3 - 6x^2 + \dfrac{45}{2}x - 27$
30. 5
31. $-\dfrac{1}{2}$
32. $\dfrac{3}{4}$
33. $\left(\dfrac{35}{2}, \sqrt{\dfrac{35}{2}}\right)$
34. 14, 14
35. $x = 1000$ and $y = 720$
36. $f'(2)$ is undefined.