

Quadratic Formula

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Proof. Notice the following derivation of the quadratic formula for any given quadratic equation.

Notice the standard form of a quadratic equations:

$$ax^2 + bx + c = 0$$

We now divide the entire equation by its leading coefficient, a :

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Now we add zero by adding and subtracting $\frac{b^2}{4a^2}$:

$$x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2 = 0$$

Now we can convert the first three terms into a square of a binomial and also combine the last two terms:

$$\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a^2} = 0$$

Now we work to isolate the variable, x , through simple algebra:

$$\begin{aligned}\left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\ \sqrt{\left(x + \frac{b}{2a}\right)^2} &= \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \\ x + \frac{b}{2a} &= \frac{\pm \sqrt{b^2 - 4ac}}{2a}\end{aligned}$$

Thus, we get the quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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