

# Distributed Distributive Justice

---

David Kurka    Jeremy Pitt

SASO 2016, Augsburg, September 12, 2016

**Imperial College**  
**London**

## Introduction

---

The prevalence of open systems – e.g. smart grids, cellular networks and cloud computing – is driving a demand for ‘better’ solutions to a standard **resource allocation problem**:

- How to collectivise and distribute a set of common-pool resources, considering
  - Fairness
  - Inclusivity
  - Sustainability
- How to overcome problems such as
  - No centralised authority
  - No full disclosure
  - Economy of scarcity

- How resources can be distributed in a **fair** manner?
  - How to **prioritise** requests?
  - What **fairness criteria** should be considered when judging merit (equality, equity, proportionality, ...)
  - How to **avoid abuses** in the system (*free-riding*, deceiving, non-compliance)

# Computational Justice

- How resources can be distributed in a **fair** manner?
  - How to **prioritise** requests?
  - What **fairness criteria** should be considered when judging merit (equality, equity, proportionality, ...)
  - How to **avoid abuses** in the system (*free-riding*, deceiving, non-compliance)

## Towards a Computational Justice framework

- Centuries of experience in the context of social organisations can be transferred to new and current technical systems;
- Not only concerned with solution's efficiency, but also aspects such as "correctness", "appropriateness" and "acceptability".

Solutions both generic and yet flexible, and both efficient and yet effective.

## Experimental Setting - LPG' Game

Linear Public Game creating a scenario where agents receive resources independently and can decide to cooperate or not.

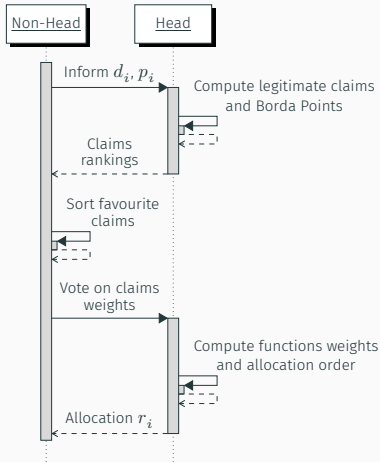
- $n$  agents share an environment with scarce resources and strive to persist on it for as long as possible
- At each timestep (turn)  $t$ , each agent:
  - Determines the resources it has **available**,  $g_i \in [0, 1]$
  - Determines its **need** for resources,  $q_i \in [0, 1]$ 
    - (In an economy of scarcity,  $q_i > g_i$ )
  - Makes a **demand** for resources,  $d_i \in [0, 1]$
  - Makes a **provision** of resources,  $p_i \in [0, 1]$  ( $p_i \leq g_i$ )
  - Receives an **allocation** of resources (defined externally),  $r_i \in [0, 1]$
  - Makes an **appropriation** of resources,  $r'_i \in [0, 1]$
- If  $p_i = g_i \wedge d_i = q_i \wedge r'_i = r_i$  we say that an agent is complying with the game; otherwise it is considered non-compliant.

## Towards a Distributive Justice Framework

---

## Former Solution: *DJ*

- Consider different metrics to evaluate fairness
- Formalisation of Nicholas Rescher's **legitimate claims of justice**



- Four-way algorithm:
  1. Agents inform a 'head' demands and provisions;
  2. Head ranks agents according to different criteria (legitimate claims);
  3. Agents vote on criteria to be prioritized;
  4. Head, considering criteria weights and agents condition, decide an order of allocation.



- Maintain the formalisation of Rescher's legitimate claims as a plural metric for fairness;
- Develop a trust and reputation framework, allowing the computation of priorities and needs in a decentralised and independent way.
- Steps:
  1. Personal opinion formation (legitimate claims of justice)
  2. Comparison to the environment (trust)
  3. Information exchange and trust update
  4. Allocation guided by reputation

## Personal Opinion Formation (legitimate claims of justice)

Canons of equality	$\phi_i^1(S) = \frac{\sum_{t=0}^S r_i(t)}{S}$
	$\phi_i^2(S) = \frac{\sum_{t=0}^S \mathbb{I}(r_i(t) > 0)}{S}$
	$\phi_i^3(S) = \begin{cases} (1 - \alpha) \cdot \phi_i^3(S - 1) + \alpha & \text{if } r_i(S) \geq d_i(S) \\ (1 - \beta) \cdot \phi_i^3(S - 1) & \text{if } r_i(S) < d_i(S) \end{cases}$
Canon of needs	$\phi_i^4(S) = \frac{\sum_{t=0}^S d_i(t)}{S}$
Canon of productivity	$\phi_i^5(S) = \frac{\sum_{t=0}^S p_i(t)}{S}$
Canon of effort	$\phi_i^6(S) = S$
Canon of social utility	$\phi_i^7(S) = \frac{\sum_{t=0}^S \mathbb{I}(\text{head}(t)=i)}{S}$
Canon of supply and demand	$\phi_i^8(S) = \frac{\sum_{t=0}^S \mathbb{I}(p_i(t)=g_i(t) \wedge d_i(t)=q_i(t) \wedge r'_i(t)=r_i(t))}{S}$

- Individual opinion vector:

$$\Phi_i(t) = \{\phi_i^1, \phi_i^2, \phi_i^3, \dots, \phi_i^8\} = \{\phi_i^c : c \in |\mathcal{C}|\}$$

- Individual aggregated opinion:

$$\phi_i(t) = \sum_c \frac{1}{|\mathcal{C}|} \cdot \phi_i^c(t)$$

## Comparison to the Environment (Trust Formulation)

- Accordance index:

$$\tau_{ij}(t) = \text{diff}(\bar{\phi}_{N_i-j}(t), \phi_j^c(t))$$

$$\bar{\phi}_{N_i-j}(t) = \frac{\sum_{n \in N(i) \cap \{i\} - \{j\}} \phi_n(t)}{|N(i)| + 1}$$

- Trust:

$$T_{ij}(t) = \begin{cases} 0.0 & \text{if } j \notin N(j) \\ (1 - \gamma)T_{ij}(t-1) + \gamma\tau_{ij}(t) & \text{if } i \neq j \wedge j \in N(j) \\ 1.0 & \text{if } i = j \end{cases}$$

### Principles

1. Trust more those who say coherent things (according to yourself!)
2. It takes time to change an impression

Update trust, based on common neighbours' trusts

- Iterative process:

$$T'_{ij}(t) = \frac{\sum_{k \in N_{ij}} T_{ik}(t) T_{kj}(t)}{\sum_{k \in N_{ij}} T_{ik}(t)}$$

where:

$$N_{ij} = (N(i) \cap N(j)) \cup \{i\} - \{j\}$$

Define final metrics to perform allocation

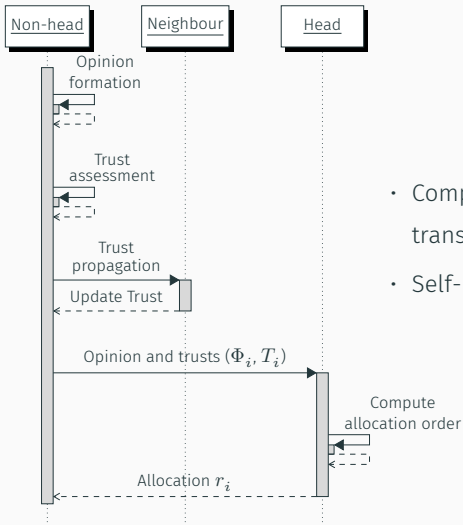
- Reputation index:

$$R_i(t) = \frac{1}{|N_i|} \sum_{j \in N_i} T_{ji}(t)$$

- Urgency index:

$$U_i(t) = R_i(t) * (1 - \Phi_i(t))$$

# Algorithm summary

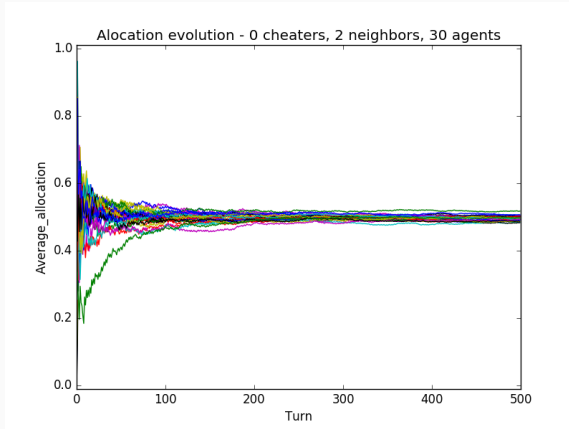


- Computation is made locally, just transmitting a 'verdict' to the head
- Self-moderation of opinions and trusts

## Results and Analysis

---

# Self Organising Allocation

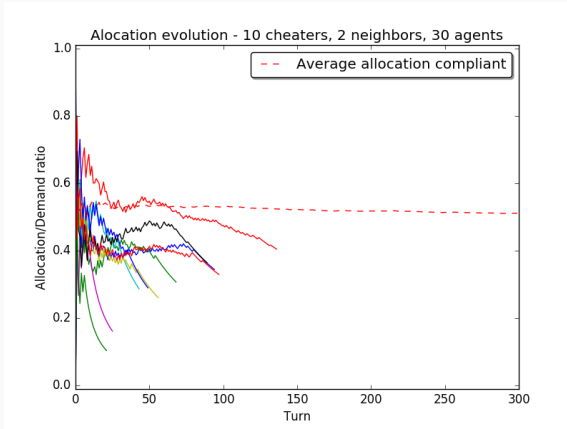


**Figure 1:** Allocation in scenario without non-compliant agents. Each coloured line represents a different agent.

- Agents more in need are prioritized, resulting in a stable and egalitarian distribution.



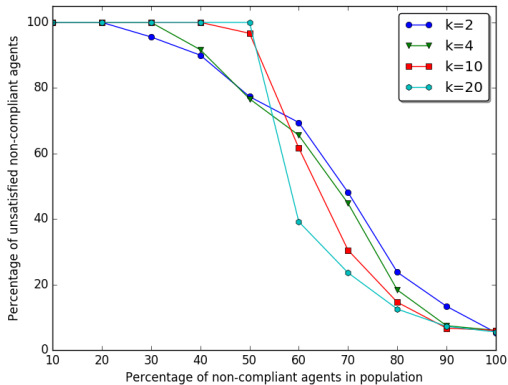
# Dealing With Non-compliant Behaviour



- Non-compliant agents are not trusted, therefore do not receive resources and eventually leave the cluster.

**Figure 2:** Individual allocations for non-compliant agents (solid lines) compared to compliant average (dashed).

# Exploring Effects of Connectivity



- Below 50%:  
non-compliance is  
opposed even with few  
connections;
- Above 50%:  
non-compliance  
becomes the 'rule', but  
can not produce an  
enduring allocation to  
all agents.

**Figure 3:** Ability of non-compliant behaviour detection, by size of neighbourhood ( $k$ ).

## Conclusion

---

## Final Remarks

- Presented solution demonstrate that it is possible to define **fair policies of resource allocation** in environments characterized by decentralisation, scarcity and no full information disclosure, using independent agents' accounts.
- Lessons learned:
  - Independent subjective assessment of justice can be considered, if inserted into a context of **influence** and **trust**;
  - Authority emerges from **reputation** autonomously;
  - Ability to aggregate individual, subjective assessments, into **collective, objective facts**.
- Future steps:
  - Explore asynchronous game;
  - Distributed pool of resources;
  - Collective mechanisms of verification.

# Acknowledgements

- National Council for Scientific and Technological Development (CNPq), Brazil;
- Diverse collaborators.



Thank you!