Strategy of Self-driving based on Cellular Automata and Game Theory

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Abstract

This paper mainly discusses the strategy of self-driving cars. We introduce game theory in analyzing the interaction between cars and strategies of cars by establishing a model based on Cellular Automata. Under cooperating assumption, cooperative self-driving car plays cooperator when it has game with a self-driving car and plays betrayer when it has a game with non-self-driving car. Under greedy assumption, non-cooperative self-driving car always plays betrayer in interaction, which means they always do their best response regardless of others. Hand-driving cars behave similar to non-cooperative self-driving cars.After running simulation, we successfully find phase change and self-reorganization phenomenon. Then we apply A^* algorithm to route planning.

1 Introduction

Recently, traffic capacity limitation problems have attracted considerable attention. Knowing that the traffic capacity in many regions is limited due to the number of lanes of roads, government concerned more on traffic issue. Whether the self-driving, cooperating cars save our lives has been a hot problem among people.

Nowadays, there are two kinds of self-driving cars, one is cooperative cars which are expected by the government, the other is non-cooperating cars which use a variety of techniques to detect their surroundings then do some responses.

On the basis of above discussion, we boil the issues:

- the strategy of two kinds of self-driving cars.
- route planning for two kinds of self-driving cars.
- the properties of the strategies
- the effects change as the percentage of self-driving cars increase

To find out those solutions, we should fully understand how the cars interacts with others and how they influence the whole traffic condition. Based on the Cellular Automata model, we construct our model which shows the macroscopic road condition, which contains states of all cars in the road. Then we apply Game Theory to analyze the car behavior and give the strategies. Finally, route planning strategies are given by A^* algorithm.

Our model contains the **driving model** which describes the basic driving behaviors, **the vehicle model** which describes property of the car, basic move principle and move strategies and **route planning model** which illustrates how the self-driving cars plan their route. We implement the model in C++ and analysis the data by Matlab.

2 Assumptions and Hypothesis

- Each road has same speed limit.

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- The width of each lane is only enough for one car.
- Drivers have different risk preferences and personality
- Vehicle can only interact with vehicles that are close to each other.
- Cooperative self-driving cars communicate and exchange data with other self-driving cars.
- Cooperative self-driving cars cooperate with other cooperative self-driving cars.
- Self-driving cars aims to cooperate to increase collective profit.
- Since driving ahead a little (e.g. a cell in cellular automata) won't cost too much, the payment is consider to be zero. That is to say, the valuation equals the utility.
- The position information of all cooperative self-driving cars can be obtained through location technology.

3 Symbols Definition

Symbol	Explanation
P	a set of cars
i	car i
S_i	All possible strategies of car i
s	the vector of strategies selected by some cars
Ah	The decision of driving ahead
Ah_1	The decision of driving ahead 1 cell in a time slot
Ah_2	The decision of driving ahead 2 cell in a time slot
Ch	The decision of changing lane
A	The allocation space
$a \in A$	The decision made at last (allocation)
f	The social choice function
$v_i : A \to R$	The valuation of car i under a
$u_i(s)$	The utility of car i under s
$\alpha_i \in S_i$	The decision of car i
α_i^*	The optimal behavior of car i
$V_i(T)$	velocity of car i in time T
$\beta(\alpha_i)$	current velocity of car i when conducting behavior α_i
$\beta'(\alpha_i)$	expected velocity of car i when conducting behavior α_i next time slot
L_{safe}	Safe breaking distance
L_{real}	Real world distance
V_{model}	Velocity in model
T_{step}	One iteration of the cellular automata.
W_m	Lane width

4 The Model

4.1 Cellular Automata

Cellular automaton model is an effective method to simulate traffic flow. Biham et al [3] constructed a model of traffic flow in two dimensions.T Nagatani [1] represents a simple mean-field theory to analyze the traffic flow of a two-Jane roadway. We construct a method which extends the two-lane model of traffic flow to multi-lanes model and improve the two-lane model proposed by Nagatani to make the simulation more realistic.

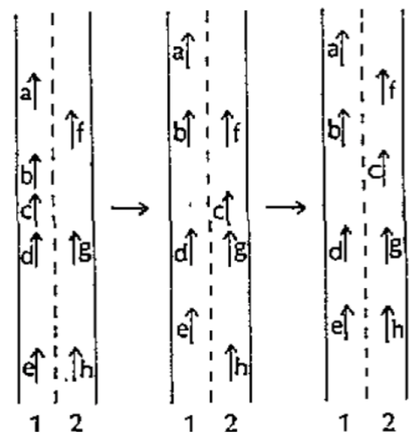

Figure 1: Nagatani's two-lane model

In Nagatani's model, cars in lane 1 move in odd time steps whereas cars in line 2 move in even time steps. If a car is blocked and its neighbor on the other lane is unoccupied, it shifts to the other lane. The iterations of the left and right lanes are not simultaneous, which results in limitation of the selections (freedom) of one lane. In reality, cars in different lanes move at the same time. Decision-making relationships between the vehicle and safety distance problem are also neglected in this model.

In our improved model, we use two kinds of self-driving cars and hand driving cars with three levels of velocities. Multi-lanes are established in our model. Cars may interact in multi-lanes.

4.2 Driving model

Driving straties A car can go straight or change its lane. When going straight, it can be decelerated, accelerated or move at an uniform speed. It may turn left, turn right to another lane.

We denote driving ahead as A and changing the lane as C.The decision space of the car i is $S_i = \{Ah, Ch\}$.

Velocity We simplify the velocity as zero speed, low speed and high speed, noted as 0, 1 and 2. $V_0 = 0$ means the car stops, $V_1 = 1$ means the car will take a step forward and move one cell in next iteration and $V_m = 2$ means the car will take two steps forward and move two cells in the next iteration.

The model is constructed because it can describe all motion states in a simple way.

Since V_m means the maximum velocity, safe distance is $D_{safe} = 1\ cell$ and V_1 is safe velocity.

Basic rules of CA model according to velocity and safety velocity:

- The car with $V_m = 2$ can move forward 1 cell next time, which means decelerate this unit time.
- The car with $V_1 = 1$ can move forward 2 cell next time if and only if there is no car on the forward two cells, which means the car accelerate.
- There is a stochastic deceleration probability which shows the mistake if justice.
- When the distance between the car and the forward car is shorter than safe distance, the car cannot accelerate nor change its lane because of the consideration of safety.
- The car cannot change its lane if the safety distance is not satisfied.

4.3 Vehicle model

Decision making style According to an exploratory postal survey of 711 drivers stratified by age, sex, annual mileage, and accident involvement, decision-making style was measured using a Decision-

Making Questionnaire (DMQ) and driving style was assessed using a Driving Style Questionnaire (DSQ)[2].The results shows that those who have different decision-making style have different behaviors and risks in driving situation. The decision making style is considered as a property of a driver or a car.

We note the decision-making style as η , which is a random number assigned by computer. High η shows that the driver or the setting of the vehicle is more aggressive. Vehicle which has higher η are more likely to occupy the cell (In reality, those people are more desirable to overtake.)

4.3.1 Greedy Principle - Strategy for non-cooperative cars

Drivers (cars) always want to reach the destination in the shortest time but they cannot get the full information of traffic situation. Instead, they only make decision depend on what they see (hand-driving cars) or detect (non-cooperative self-driving car). So they choose the current optimal choice, which means that they use greedy strategy and they aim to do **best response** in each game.

Decision space of the car i is $S_i = \{Ah, Ch\}$, which means the car decide to A (go ahead) or C (changing lane).

Considering arbitrary $i \in All\ P$, which denotes a car.

Due to the greedy strategy, P_i wants to maximum its utility u_i currently. It has the information of its current velocity and the distances from other nearby cars so it will maximum the current moving distance as well as next expected moving distance. In our model, the moving distance in one unit time can be considered as velocity (cell/unit time).

Then the valuation function of i is

$$v_i(a) = \beta_i(a_i) + \beta_i'(a_i) \tag{1}$$

The utility function of i is

$$u_i(s_i', s_{-1}') = \beta_i(s_i) + max_{s_i \in S_i}(\beta_i'(s_i)) \tag{2}$$

where $\beta_i(s_i)$ represents the speed $V_i(T)$ of car i at time T in which the decision $s_i \in S_i$ is made and $max_{s_i \in S_i}(\beta_i(s_i))$ means the optimal expectation speed at T+1.

Non-cooperative cars are similar to the real driver because they just can gain the information similar to the real driver. So Greedy Principle is applied to both hand-driving cars and non-cooperative self-driving cars.

Ah: Driving ahead The conditions of driving ahead (straight) are showed as follows.

Velocity in state 1	Velocity in state 2	Driving behavior
0	1	accelerate
0	2	accelerate
1	2	accelerate
1	0	decelerate
2	0	decelerate
2	1	decelerate
0	0	uniform speed
1	1	uniform speed
2	2	uniform speed

Ch: Changing the lane The condition of changing the lanes (turning left and turning right) are showed as follows.

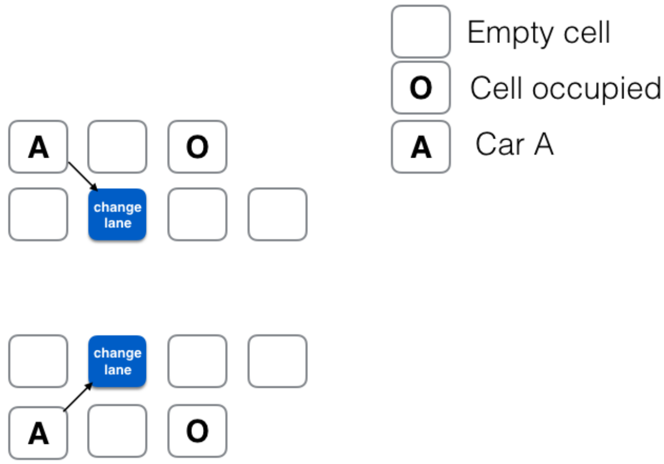

Figure 2: Conditions of changing lane

The car changes its lane when it can get more higher utility through changing the lane. In the figure above, $i = A$ change its lane because $u_A = 1 + max\{1, 2\} = 1 + 2 = 3$, which indicates that it would like to overtake the car in the cell occupied. Only in these two conditions above can a car change its lane.

4.3.2 Cooperation - Strategy for cooperative self-driving car

Each car makes its decision according to greedy strategy. If all cars conduct their decisions, collisions may happen. Since a cell may only be aimed at by no more than three cars, no more than two cars may have collisions when the road has two lanes and no more than three cars may have collisions when the road has more than three lanes. If we don't make cooperative strategy, one car's decision may effect others and block others' way.

This problem can be solved in following method.

Strategy of cooperative self-driving cars According the two assumptions given:

- Cooperative self-driving cars communicate and exchange data with other self-driving cars.
- Cooperative self-driving cars cooperate with other self-driving cars.

To increase capacity of highways, self-driving cars are conducted **cooperative strategy** which means they will choose the global optimal solution if and only if all the cars which aim at the same sell are self-driving cars.

Strategy of other conditions Suppose three cars 1, 2 and 3 which are all cooperative self-driving cars have collisions, the decision space of them is S_1 , S_2 and S_3 are the collections of all decisions of 1,2 and 3 respectively.

The strategy of 3 in allocation is:

$$f : a_3 \equiv \{s_3^* \in S_3 : u_3(s_1, s_2, s_3^*) \geq u_3(s_1, s_2, s_3), \forall s_1 \in S_1, \forall s_2 \in S_2, \forall s_3 \in S_3\} \quad (3)$$

$s_1 \in S_1$ is the decision of 1, $s_2 \in S_2$ is the decision of 2 and $s_3 \in S_3$ is the decision of 3.

Interaction of the cars The collision conditions are showed as follows:

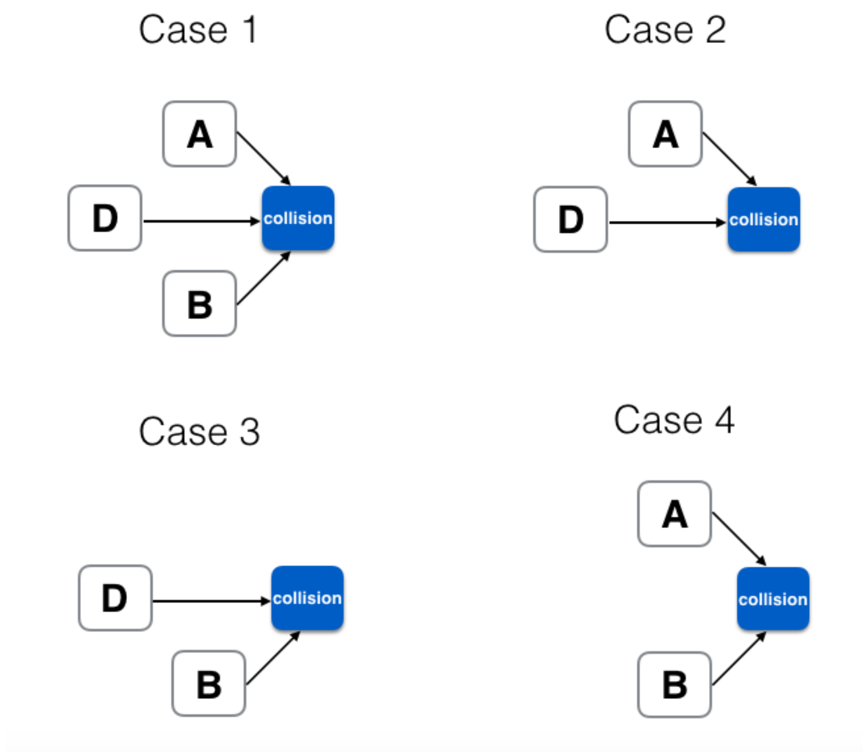

Figure 3: Conditions of collicision (empty cells are ignored)

We represent the utility of a group of decisions as $u(s_A, s_B, s_D)$. Then, we show the utility of global optimal condition and local optimal conditions of decisions.

Case 1:

Utility	Collective utility
$u(Ch, Ah_1, Ah_1) = (3, 2, 1)$	6
$u(Ah_1, Ch, Ah_1) = (2, 3, 1)$	6
$u(Ah_1, Ah_1, Ah_2) = (2, 2, 4)$	8

In case 1, if all cars are cooperative self-driving cars, they will conduct (Ah_1, Ah_1, Ah_2) condition, which means all of them won't change their lanes. They continue driving straight.
When no less than one of the cars is(are) hand-driving car(s) or non-cooperative self-driving cars, they will conduct (Ch, Ah_1, Ah_1) or (Ah_1, Ch, Ah_1). And the car with higher η will occupy the collision cell (High η cars are more initiative.) D is forced to decelerate.

Case 2:

Utility	Collective utility
$u(Ch, Sh_1) = (3, 1)$	4
$u(Sh_1, Sh_2) = (2, 4)$	6

In case 2, if all cars are self-driving cars, they will conduct (Ah_1, Ah_2) condition, which is the global optimal strategy which maximum the collective utility. A and D all maintain previous speed and do not change the lane.
When one or two the cars is(are) hand-driving car(s), they will conduct (Ch, Ah_1) if A has higher η than D.

Case 3:

Utility	Collective utility
$u(Ch, Ah_1) = (3, 1)$	4
$u(Ah_1, Ah_2) = (2, 4)$	6

Case 3 is symmetry to Case 2.

Case 4:

Utility	Collective utility
$u(Ch, Ah_1) = (3, 2)$	5
$u(Ah_1, Ch) = (2, 3)$	5

In case 4, global optimal solution is the local optimal solution. So the high η car may win the cell.

4.3.3 Properties for non-cooperative self-driving cars Mechanism

Truthfulness Since the mechanism is based on the best response, each car will give its true preference. It should be truthful. Since $u_i(s'_i, s'_{-1}) = \beta_i(s_i) + max_{s_i \in S_i}(\beta'_i(s_i))$ and $\beta_i(s_i) = max(\beta_i(s_i))$, we can get $u_i(s'_i, s'_{-1}) \geq v_i(a) - 0 = u_i(s_i, s_{-i})$ for all a. Suppose s'_i is the allocation for car i.

Individual Rationality For each car, it will not go back and its utility will not be less than zero. So, it should be individual rational.

Efficiency Each car will not consider others' utilities. For counter-example, car A and car B are on different lanes. For A, the utility of leaving its own lane is greater than the utility of staying on its own lane. However, it will affect B's utility. If the increased utility is less than the decreased utility, it will make the social welfare lower. In case 1, if the allocation is (Ch, Ah_1, Ah_1) the sum of utility is lower than (Ah_1, Ah_1, Ah_2). It is not efficient.

4.3.4 Properties for cooperative self-driving cars Mechanism

Truthfulness It seems not truthful because a car can choose the strategy which increases its own utility. But when the car achieve the agreement that it can be controlled by Central Management System as an cooperative self-driving car, the valuation could not be modified because it is calculated by the system. What they report is the position information, which can be collected by location system. The cars could not tell the lie.

Individual Rationality For each car, it will not go back and its utility will not be less than zero. So, it should be individual rational.

Efficiency The mechanism is based on maximizing the social welfare. So, it must be efficient.

When there is no collision, choosing the best response won't effect other cars which means they are independent. For all cars without collision, $\sum_i max\ v_i(a) = max \sum v_i(a)$. Then $f \in argmax_{a \in no\ collsion\ A} \sum v_i(a)$

When there is collision, $f \in argmax_{a \in collsion\ A} \sum v_i(a)$. For all a, $f \in argmax_{a \in A} \sum v_i(a)$, the mechanism is efficiency. It maximize social welfare.

4.4 Routing planning model

We will set a series of variables to establish the strategy of route planning.Firstly, the time cost from the start to the end is considered as utility. The road condition may effect the time cost a lot.

4.4.1 Routing planning strategies for Non-cooperative self-driving cars

For Non-cooperative self-driving cars, they will choose the shortest route, which will be decided by using A * algorithm.

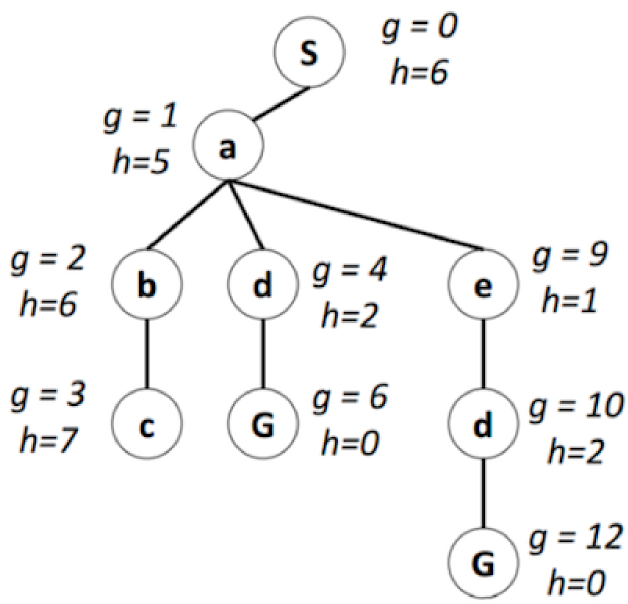

Figure 4: Conditions of changing lane

In A^*, the function should be

$$f(n) = g(n) + h(n) \tag{4}$$

We select the points which minimize $f(n)$ in order to find the shortest route. $g(n)$ is the cumulative distance between the start point and current point, $h(n)$ is the distance between current point and terminal point.

We can proof that if 0<=h(n)<=h*(n), h(n) is admissible. In this case, A* algorithm is optimal and the planning route is optimal. h*(n) means the actual distance between current point and goal point.

4.4.2 Routing planning strategies for cooperative self-driving cars

In the cooperation mode, we can obtain the road condition of city by Central Management System. We also use the A* algorithm and add another term p(n) which means the number of cars. And we finally get f(n)=g(n)+h(n)+p(n). In this case, the route may not the shortest path but the optimal path.

$$f(n) = g(n) + h(n) + p(n) \tag{5}$$

4.4.3 Properties for the Mechanism

For non-cooperative self-driving cars, the route designed is optimal, which is best response since no more information can be obtained. This mechanism is truthful. While in cooperative mechanism, it is still truthful because the result is optimal for the car. In non-cooperation condition, each car choose its shortest route, which may due to traffic jams, which will reduce the utility and social welfare so it is not efficient. But in cooperation, route condition is considered, traffic jams can be reduced. It will maximize the social welfare. It should be efficient.

5 Build real road to the cellular bijection

An important parameter of a real road to consider is V_m, which is the maximum speed allowed by the road. In our model, suppose the speed of a car is v, then $v \in \{0, 1, 2\}$. So naturally we can believe that the $V_{real} = V_m$ and $V_{model} = 2$ in the model is a bijection :$V_m \leftrightarrow 2$. Note that, depending on the nature of the cell, each cell can be occupied by at most 1 car - called the mutual exclusion of vehicles. Considering the distances between vehicles of real road, in order to keep the vehicle at least not rear end on the road, to maintain a certain safe braking distance between vehicles is necessary. According to common sense, relationship between safe braking distance L_{safe} (unit: m) V_{real} and speed the car (unit: m) is approximately $L_{safe} = \frac{V_{real}}{3.6}$. In our model, when a vehicle speed is $V_{model} = 2$, if and only if the distance between the vehicle and the front one is greater or equal to two cells, then after iteration is not possible to have a crash. So we can establish a bijection by assuming $V_{real} = V_m$:

$$L_{real} = \frac{V_m}{3.6} \leftrightarrow 2 \text{ cells} \tag{6}$$

$$\Leftrightarrow 1 \text{ cell} \leftrightarrow \frac{V_m}{6.4} \tag{7}$$

Also consider the mutual exclusion of vehicles, the real situation in a lane with width of Wm can only accommodate a car, so:

$$W_m \leftrightarrow 1 \text{ cell} \tag{8}$$

By (7), (8), a road with length of L_{road} and width of W_{road} can be mapped to a $\left[\frac{V_m}{6.4} \cdot L_{road} \cdot \frac{W_{road}}{W_m}\right.$ cell matrix.

For a further step, consider the usage of **Traffic counts** n cellular automata: we assume that for a period of time the **Traffic counts** in t is N_t. For the main parameters of the road, we have:

$$N(t) \sim Pois(\lambda), \quad P\big(N(t) = k\big) = \frac{e^{-\lambda t}(\lambda t)^k}{k!}, \quad k \geq 0 \tag{9}$$

$$\lambda = \frac{N_t}{t} \tag{10}$$

$$\rho = \frac{N_t}{t \cdot W_{road}} \tag{11}$$

Considering the bijection between real time T_{step} with one iteration of the cellular automata:

$$\frac{1}{2} V_m \cdot T_{step} \leftrightarrow 2 \text{ cells} \tag{12}$$

From formula (6)(12) it can be introduced that:

$$T_{step} = \frac{1}{1.8} s \tag{13}$$

There are two kinds of methods to simulate traffic flow by automatic cellular automata. open boundary condition and periodic boundary condition. Corresponding to different boundary conditions, we have:

$$N(T_{step}) \sim Pois(T_{step}), \quad P(N(T_{step}) = k) = \frac{e^{-\lambda_{model} T_{step} (\lambda_{model} T_{step}) k}}{k!} \quad k \geq 0 \tag{14}$$

$$\lambda_{model} = \frac{N_{T_{step}}}{T_{step}} \tag{15}$$

$$\rho_{model} = \frac{N_{T_{step}} \cdot W_{road}}{T_{step} \cdot W_m} \tag{16}$$

The $N(T_{step})$ = represents the new vehicle in the next iteration under the open boundary conditions, which obey the Poisson distribution with arrival rate λ_{model}; $\rho_{model} = \frac{N_{car}}{N_{cell}}$, N_{car} is the number of vehicle model, N_{cell} cell number, ρ_{model} represents the vehicle density model.

6 The Model Results

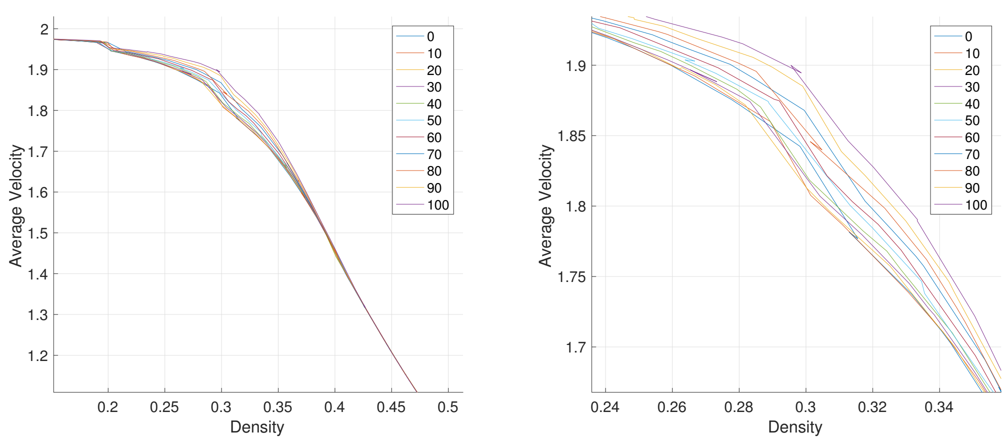

Figure above shows the relationship between average velocities V_{avg} and density of cars ρ with different percentage of cooperative self-driving cars. We can notice that the tipping point of clear-phase (which means the road is clear) and critical-phase will move to right with the increment of percentage of self-driving cars. If car density is high, percentage of cooperative self-driving cars barely has effect.

We can conclude that the strategy of self-driving car is global optimal in this game avoid a worse phenomenon caused by Cut-throat competition to global condition. Bring in cooperative self-driving cars can move the tipping point to right so as to enlarge traffic volume. If density is high, self-organization ability is invalid and no games occurs, so cooperative self-driving car percentage has barely effect on result.

Non-cooperative self-driving cars behave similar to hand-driving cars so it cause no effect.

7 Conclusion

For each car, the non-cooperation mode is much better for their own benefit, but it may do the damage to others' benefits. It may make the whole system unstable. However, in the cooperation mode, sacrificing a small part of cars' benefits will make the system more optimal, which is a long-term and stable choice. Then, how to persuade others to join the cooperation mode is another problem.

8 Strengths and weaknesses

8.1 Strengths

- **Describe car behaviors:** We use gaming model to describe the relationship between cars on the road, which includes many factor of driver.
- **Simplicity and Flexibility:** We describe the behavior of hand-driving cars and self- driving cars in one gaming model. And the model can fit variant hypothesis of driver and self-driving system by changing the utility function.
- **Running time friendly:** The model simplify the velocity to 0,1,2. It can reduce a great mount of extra calculation cost while you can still fully describe the road condition.
- **Extendibility** The application of our model is not limited by the number of lanes, multiple entrance and applying special lanes.
- **Self-organization** All the strategies are given and they are all resonable.

8.2 Weakness

The main weakness of our model is **high density-low speed inaccuracy**:

- 0,1,2 velocity model works not well at low speed when all vehicles has velocity less or equal to 1. That means there no overtaking in such case and this is not true in reality.
- Gaming times under high vehicle density will decrease to zero as no overtaking taking place.
- When construct cells, we make an assumption that 2cell?maximum safe breaking distance. But it will not holds for very low speed.

References

[1] T Nagatan, *Self-organization and phase transition in traffic-flow model of a two-lane roadway* 1993

[2] D. J. FRENOI, R. J. WEST, J. ELANDER and J. M. WILDING *Decisionmaking style, driving style, and self-reported involvement in road traffic accidents* 1993

[3] Biham 0, Middleton A A and Levine D *Phys. Rev.* A 46 R6124 1992