WID3004
Semester 2 2021/2022
Assignment 1

This assignment carries 10% of your coursework. Please submit your own work. Plagiarism will not be tolerated at all.

1. Write a Python program to implement the Bisection Method.

2. Use your code to find the roots of $f(x) = x^6 - x - 1$ in the interval $[1, 2]$ accurate to $\varepsilon = 0.01$ where $\varepsilon = b - a$.

3. Write a Python program to implement the Secant method.

4. Use your code to find the roots of the equation in $f(x) = x^4 - 5$ using the Secant method. Hence or otherwise, show that $\sqrt[4]{5} \cong 1.5$

5. Use the Secant method to solve $e^x - (2 - x)^3 = 0$ in the interval $[0, 5]$.

6. Write a Python program to calculate $P_n(x)$ using the Lagrange Method for values of n = 2, 4, 6, 8, 10, 12, 14, 16 in the interval $[-5, 5]$, where

$$x_i = \frac{10i}{n} - 5, i = 1, 2, 3 \ldots n.$$

and

$$y_i = \frac{i}{10} - 5, i = 1, 2, 3 \ldots 100.$$

Estimate the error by finding

$$\varepsilon = \max_{i=0,1,\ldots100} |f(y_i) - P_n(y_i)|$$

$P_n(x)$ interpolates $f(x)$ at $n + 1$. The functions $f(x)$ are as given below.

    a. $f(x) = \frac{1}{1+x^2}$

    b. $f(x) = \sin x$

    c. $f(x) = \frac{\tan x + 1}{\sin(x^2) + 2}$

7. Write a Python program to implement the Jacobi method. Using zero as a starting point solve the equation in the form of $AX = b$ where,

$$A = \begin{pmatrix} 3 & 12 & 0 & -1 & 0 & 0 \\ 4 & 0 & 31 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 17 & -3 \\ 27 & 2 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & 11 \\ 0 & 0 & 0 & 24 & -1 & 0 \end{pmatrix}$$

and

$$b = \begin{pmatrix} 39 \\ 117 \\ 12 \\ 98 \\ 14 \\ 55 \end{pmatrix}$$

8. Write a Python program to implement the Gauss-Seidel method. Using zero as a starting point solve the following equation:

$$\begin{cases} 5x_1 - 2x_2 - x_3 + x_4 = 6 \\ -2x_1 + 4x_2 + x_3 = 0 \\ x_1 + 2x_2 + 6x_3 - x_4 = 6 \\ -x_1 + x_3 + 6x_4 = -14 \end{cases}$$