ME 254 Computational Fluid Dynamics
Homework 7

Due on 05/10/19 at 11:59 pm (through Catcourses)
Maximum points: 210

Instructions: You are required to attempt Problem 1 and can choose either Problem 2 or 3 as your other problem. If you have time, you can do both Problem 2 and 3 to get bonus points. Your performance in this assignment will be used to determine the outcome of the Comprehensive exam of the ME Graduate Program (if applicable to you).

Jose A. Millan
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1. (**50 points**) The quasi 1-D Euler equations are given by

$$\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho u A)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial(\rho u A)}{\partial t} + \frac{\partial(\rho u^2 A)}{\partial x} + A\frac{\partial p}{\partial x} = 0 \tag{2}$$

$$\frac{\partial(E_t A)}{\partial t} + \frac{\partial(E_t u A)}{\partial x} + \frac{\partial(p u A)}{\partial x} = 0 \tag{3}$$

where A is the cross-sectional area

(a) Write the above equations in the form

$$\mathbf{U}_t + [B]\mathbf{U}_x = \mathbf{S} \tag{4}$$

(b) Find the eigenvalues of $[B]$.

(c) Determine the left eigenvectors.

(d) Determine $[T]^{-1}$

(e) Find $[|B|] = [T][|\Lambda|][T]^{-1}$ for $0 < u < a$.

2. (**160 points**) Consider a long shock tube with a diaphragm that separates the high-pressure region in the left and the low-pressure region in the right. The high-pressure region is at a pressure of 1 atm and the low-pressure region at 0.1 atm. Both regions are at a temperature of 300 K and zero velocity initially. The diaphragm is ruptured at $t = 0$. You can assume perfect gas law to be valid.

(a) (**120 points**) Write a program to solve the the 1-D Euler's equations (not the quasi 1-D) for the shock tube problem using the first-order Roe-scheme. Use suitable length of computational domain, cell size and timestep.

(b) (**40 points**) Plot the density, velocity, pressure and temperature variation in the shock tube at four different instants of time suitable enough to show the solution evolution.

3. (**160 points**) Consider the quasi 1-D Euler's equations for the isentropic flow of a calorically perfect gas with $\gamma = 1.4$ through a nozzle having an area distribution

$$\frac{A}{A_t} = 1.0 + 4.95(x-1)^2 \tag{5}$$

with $0 \le x \le 2$. The inlet is at stagnation conditions (zero velocity) with pressure of 10^5 Pa and 300 K. The exit pressure is 0.58×10^5 Pa. The inlet and exit boundaries should both be subsonic. At the inlet, two parameters need to be specified and this can be the pressure and density with the velocity used as the first cell value. At the exit, only the pressure is set with other quantities to be taken from the last cell values.

(a) (**120 points**) Write a program to solve quasi 1-D flow through the nozzle using the first-order Roe scheme. Use suitable cell size and timestep.

(b) (**40 points**) Plot the steady-state p, V, T, and M as a function of x.

Quasi 1D euler equation:

$$(1) \quad \frac{\partial(\rho A)}{\partial x} + \frac{\partial(\rho u A)}{\partial x} = 0$$

$$(2) \quad \frac{\partial(\rho u A)}{\partial t} + \frac{\partial(\rho u^2 A)}{\partial x} + \overset{*}{\frac{\partial(p A)}{\partial x}} = 0$$

$$(3) \quad \frac{\partial(E_t A)}{\partial t} + \frac{\partial(E_t u)}{\partial x} + \frac{\partial(\rho u A)}{\partial x} = 0$$

we need the following

$$* \quad \frac{\partial(p A)}{\partial x} = \frac{\partial(pA)}{\partial x} - \frac{\partial\left(\int p\, dA\right)}{\partial x}$$

$$= \frac{\partial}{\partial x}(pA) - \frac{p\, dA}{\partial x}$$

Knowing this...

$\longrightarrow$

$$\frac{\partial}{\partial t}\begin{Bmatrix} \rho A \\ \rho u A \\ E_t A \end{Bmatrix} + \frac{\partial}{\partial x}\begin{Bmatrix} \rho u A \\ \rho u^2 A + A p \\ E_t u A + p u A \end{Bmatrix} = \begin{Bmatrix} 0 \\ p\, dA/dx \\ 0 \end{Bmatrix}$$

now, this can be re-expressed as : (Used Kursen, i, as reference)

$$\partial_t(\vec{U}) + \partial_x(\vec{F}) = \vec{G}$$

$$\vec{G} = \partial_x(\ln A)\,\vec{F} - \frac{1}{A}\vec{S} = \partial_x(\ln A)\,\rho u \begin{bmatrix} 1 \\ u \\ H \end{bmatrix}$$

Now this can be expanded as...

$$\frac{\partial}{\partial t}\begin{bmatrix} \rho \\ \rho u \\ E_t \end{bmatrix} + \frac{\partial}{\partial x}\begin{bmatrix} \rho u \\ \rho u^2 + p \\ u(E_t + p) \end{bmatrix} = \frac{\partial}{\partial x}\ln(A)\begin{bmatrix} \rho \\ \rho u \\ E_t \end{bmatrix}$$

Now, this can be wrote with the Jacobian in mind...

$U_t + [Q]\,U_x = S \longrightarrow$ wolfram code will take care of the rest...

$E_1 = U_2$

E_2 has a p term, so if we assume ...

$$p = (\gamma - 1)\left[E_t - \tfrac{1}{2}\rho u^2\right]$$

$$E_2 = \frac{U_2^2}{U_1} + (\gamma - 1)\left[U_3 - \frac{1}{2}\frac{U_2^2}{U_1}\right]$$

$$E_3 = (E_t + p)\,u$$

$$= \left[U_3 + (\gamma-1)\left(U_3 - \tfrac{1}{2}U_2^2/U_1\right)\right]\frac{U_2}{U_1}$$

lets derive everything and then put it in the Jacobian

$$\frac{\partial E_1}{\partial U_1} = 0 \qquad \frac{\partial E_1}{\partial U_2} = 1 \qquad \frac{\partial E_1}{\partial U_3} = 0$$

$$\frac{\partial E_2}{\partial U_1} = \frac{U_2^2(-3+\gamma)}{2U_1^2} \qquad \frac{\partial E_2}{\partial U_2} = -\frac{U_2(-3+\gamma)}{U_1}$$

$$\frac{\partial E_2}{\partial U_3} = -1 + \gamma$$

(b) find the eigen values of [B]

$$B = \frac{\partial\{E\}}{\partial\{U\}} = \begin{bmatrix} \frac{\partial E_1}{\partial U_1} & \frac{\partial E_1}{\partial U_2} & \frac{\partial E_1}{\partial U_3} \\ \frac{\partial E_2}{\partial U_1} & \frac{\partial E_2}{\partial U_2} & \frac{\partial E_2}{\partial U_3} \\ \frac{\partial E_3}{\partial U_1} & \frac{\partial E_3}{\partial U_2} & \frac{\partial E_2}{\partial U_3} \end{bmatrix}$$

get reminded that ...

$$U = \begin{Bmatrix} \rho \\ \rho u \\ E_t \end{Bmatrix} \qquad E = \begin{Bmatrix} \rho u \\ \rho u^2 + p \\ (E_t + p)\, u \end{Bmatrix}$$

hence...

$U_1 = \rho \qquad E_1 = \rho U$

$U_2 = \rho u \qquad E_2 = \rho U^2 + p$

$U_3 = E_t \qquad E_3 = (E_t + p)\, u$

$$\frac{\partial E_3}{\partial U_1} = \frac{U_2^3(-1+\gamma) - U_1 U_2 U_3 \gamma}{U_1^3}$$

$$\frac{\partial E_3}{\partial U_2} = \frac{-3U_2^3(-1+\gamma) + 2U_1 U_3 \gamma}{2U_1^2}$$

$$\frac{\partial E_3}{\partial U_3} = \frac{U_2 \gamma}{U_1}$$

$\longrightarrow$ $H = (E+p)/\rho = \dfrac{a^2}{\gamma - 1} + \dfrac{1}{2}U^2$

$$B = \begin{bmatrix} 0 & 1 & 0 \\ \dfrac{U_2^2(-3+\gamma)}{2U_1^2} & -\dfrac{U_2(-3+\gamma)}{U_1}, & -1+\gamma \\ \overset{①}{\dfrac{U_2^3(-1+\gamma) - U_1 U_2 U_3 \gamma}{U_1^3}} & \overset{②}{\dfrac{-3U_2^2(-1+\gamma) + 2U_1 U_3 \gamma}{2U_1^2}} & \overset{③}{\dfrac{\gamma U_2}{U_1}} \end{bmatrix}$$

$\rightarrow$ now lets replace this by U_1, U_2, U_3

① $\left(\dfrac{U_2}{U_1}\right)^3(\gamma - 1) - \gamma \dfrac{U_2 U_3}{U_1^2}$; ② $\dfrac{U_3}{U_1}\gamma - \dfrac{3}{2}\left(\dfrac{U_2}{U_1}\right)^2(\gamma - 1)$

③ $\gamma \dfrac{U_2}{U_1}$

ok, lets continue

① If the total specific enthalpy H

$$H = (E + p)/\rho = \frac{a^2}{\gamma - 1} + \frac{1}{2}u^2 = \frac{E}{\rho} + \frac{p}{\rho}$$

① $\rightarrow$ $\frac{-\gamma E u}{\rho} + (\gamma - 1)u^3$ $\qquad E_t = \rho\left(e + \frac{u^2}{2}\right)$

② $\rightarrow$ $\gamma\frac{E}{\rho} - \frac{3}{2}(\gamma - 1)u^2$

learning the matrix

now I need to solve for E...

$$\left(\frac{E}{\rho}\right) + \left(\frac{p}{\rho}\right) = \frac{a^2}{\gamma - 1} + \frac{1}{2}u^2$$

$$\left(\frac{E}{\rho}\right) = \left(\frac{a^2}{\gamma - 1}\right) + \frac{1}{2}u^2 - \left(\frac{p}{\rho}\right)$$

$$\left(\frac{E}{\rho}\right) = \left(\frac{a^2}{\gamma - 1}\right) + \frac{1}{2}u^2 - \left(\frac{1}{\rho}\right)\left[(\gamma - 1)\left[E_t - 1/2\,\rho u^2\right]\right]$$

$$\left(\frac{E}{\rho}\right) = \left(\frac{a^2}{\gamma - 1}\right) + \frac{1}{2}u^2 - \frac{(\gamma - 1)}{\rho}E_t + \frac{1}{2}\left(\frac{(\gamma - 1)}{\rho}\right)(\rho u^2)$$

$$\left(\frac{E}{\rho}\right) = \frac{u^2}{2} + \frac{a^2}{(-1 + \gamma)\gamma}$$ Now replacing that ...

replacing into ①

$$① \rightarrow \frac{1}{2} U^3(-2+\gamma) - \frac{a^2 U}{-1+\gamma}$$

replacing into ②

$$② \quad U^2\left(\frac{3}{2}-\gamma\right) + \frac{a^2}{-1+\gamma}$$

Now we can build the entire matrix

$$B = \begin{bmatrix} 0 \, , & 1 \, , & 0 \\ \frac{1}{2} U^2(\gamma-3) \, , & U(3-\gamma) \, , & \gamma - 1 \\ \frac{1}{2} U^3(-2+\gamma) - \frac{a^2 U}{-1+\gamma} \, , & U^2\left(\frac{3}{2}-\gamma\right) + \frac{a^2}{-1+\gamma} \, , & U\gamma \end{bmatrix}$$

Now, getting the eigen-values...

$$\left.\begin{aligned} \lambda_1 &= U - a \\ \lambda_2 &= U \\ \lambda_3 &= U + a \end{aligned}\right] \qquad \underline{\text{Solution}} \qquad \begin{aligned} \lambda_1^{(-)} &= U - a \\ \lambda_2^{(0)} &= U \\ \lambda^{(+)} &= U + a \end{aligned}$$

In[1]:= E1[U1_, U2_, U3_] := U2

$$E2[U1\_, U2\_, U3\_] := \frac{U2^2}{U1} + (\gamma - 1) * \left(U3 - \frac{U2^2}{2 * U1}\right)$$

$$E3[U1\_, U2\_, U3\_] := \left(U3 + (\gamma - 1) * \left(U3 - \frac{U2^2}{2 * U1}\right)\right) * \left(\frac{U2}{U1}\right)$$

In[4]:=
$$B = \left\{\{0, 1, 0\}, \left\{\frac{U2^2 (-3+\gamma)}{2\, U1^2}, \frac{2\, U2}{U1} - \frac{U2 (-1+\gamma)}{U1}, -1+\gamma\right\},\right.$$
$$\left.\left\{\frac{U2^3 (-1+\gamma) - U1 * U2 * U3 * \gamma}{U1^3}, \frac{U3 + \left(-\frac{U2^2}{2\, U1} + U3\right)(-1+\gamma)}{U1} - \frac{U2^2 (-1+\gamma)}{U1^2}, \frac{U2\, \gamma}{U1}\right\}\right\};$$

MatrixForm[Simplify[
B]]

Out[5]//MatrixForm=
$$\begin{pmatrix} 0 & 1 & 0 \\ \frac{U2^2 (-3+\gamma)}{2\, U1^2} & -\frac{U2 (-3+\gamma)}{U1} & -1+\gamma \\ \frac{U2^3 (-1+\gamma) - U1\, U2\, U3\, \gamma}{U1^3} & \frac{-3\, U2^2 (-1+\gamma) + 2\, U1\, U3\, \gamma}{2\, U1^2} & \frac{U2\, \gamma}{U1} \end{pmatrix}$$

In[6]:=
$$\text{Flatten}\left[\text{FullSimplify}\left[\text{Solve}\left[(W) == \left(\frac{a^2}{\gamma - 1}\right) + \left(\frac{1}{2} * u^2\right) - (\gamma - 1) * W + \left(\frac{1}{2}\right)\left(\frac{\gamma - 1}{\rho}\right)(\rho * u^2), W\right]\right]\right]$$

$$\text{FullSimplify}\left[-\gamma \left(\frac{u^2}{2} + \frac{a^2}{(-1+\gamma)\, \gamma}\right)(u) + (\gamma - 1)\, u^3\right]$$

$$\text{FullSimplify}\left[\gamma \left(\frac{u^2}{2} + \frac{a^2}{(-1+\gamma)\, \gamma}\right) - \frac{3}{2} (\gamma - 1)\, u^2\right]$$

Out[6]=
$$\left\{W \to \frac{u^2}{2} + \frac{a^2}{(-1+\gamma)\, \gamma}\right\}$$

Out[7]=
$$\frac{1}{2} u^3 (-2+\gamma) - \frac{a^2\, u}{-1+\gamma}$$

Out[8]=
$$u^2 \left(\frac{3}{2} - \gamma\right) + \frac{a^2}{-1+\gamma}$$

```
B1 = {{0, 1, 0}, {1/2 * u^2 * (γ - 3), u * (3 - γ), γ - 1},
   {1/2 * u^3 * (-2 + γ) - (a^2 * u)/(-1 + γ), u^2 (3/2 - γ) + a^2/(-1 + γ), u * γ}};
MatrixForm[Simplify[B1]]
```

Out[13]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} u^2 (-3+\gamma) & -u(-3+\gamma) & -1+\gamma \\ \frac{1}{2} u^3 (-2+\gamma) - \frac{a^2 u}{-1+\gamma} & u^2 \left(\frac{3}{2}-\gamma\right) + \frac{a^2}{-1+\gamma} & u\gamma \end{pmatrix}$$

In[26]:=
```
FullSimplify[Eigenvalues[B1]]
(* this is for part b*)
```

Out[26]= $\{a+u, -a+u, u\}$

```
Lvector = FullSimplify[Eigenvectors@Transpose@B1];
MatrixForm[Lvector]
(*These are the left eigenvectors, in matrix form, row, part c*)
```

Out[16]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} u \left(u - \frac{2a}{-1+\gamma}\right) & -u + \frac{a}{-1+\gamma} & 1 \\ \frac{1}{2} u \left(u + \frac{2a}{-1+\gamma}\right) & -u - \frac{a}{-1+\gamma} & 1 \\ \frac{u^2}{2} - \frac{a^2}{-1+\gamma} & -u & 1 \end{pmatrix}$$

```
iLvector = FullSimplify[Inverse[Lvector]];
MatrixForm[iLvector]
(*Inverse of left eigenvector, ppart d*)
```

Out[18]//MatrixForm=

$$\begin{pmatrix} \frac{-1+\gamma}{2a^2} & \frac{-1+\gamma}{2a^2} & \frac{1-\gamma}{a^2} \\ \frac{(a+u)(-1+\gamma)}{2a^2} & -\frac{(a-u)(-1+\gamma)}{2a^2} & \frac{u-u\gamma}{a^2} \\ \frac{2a^2+2au(-1+\gamma)+u^2(-1+\gamma)}{4a^2} & \frac{2a^2-2au(-1+\gamma)+u^2(-1+\gamma)}{4a^2} & -\frac{u^2(-1+\gamma)}{2a^2} \end{pmatrix}$$

```
check = FullSimplify[Lvector.B1.iLvector];
MatrixForm[check]
(*Need this in order to get the λ matrix*)
```

Out[20]//MatrixForm=

$$\begin{pmatrix} a+u & 0 & 0 \\ 0 & -a+u & 0 \\ 0 & 0 & u \end{pmatrix}$$

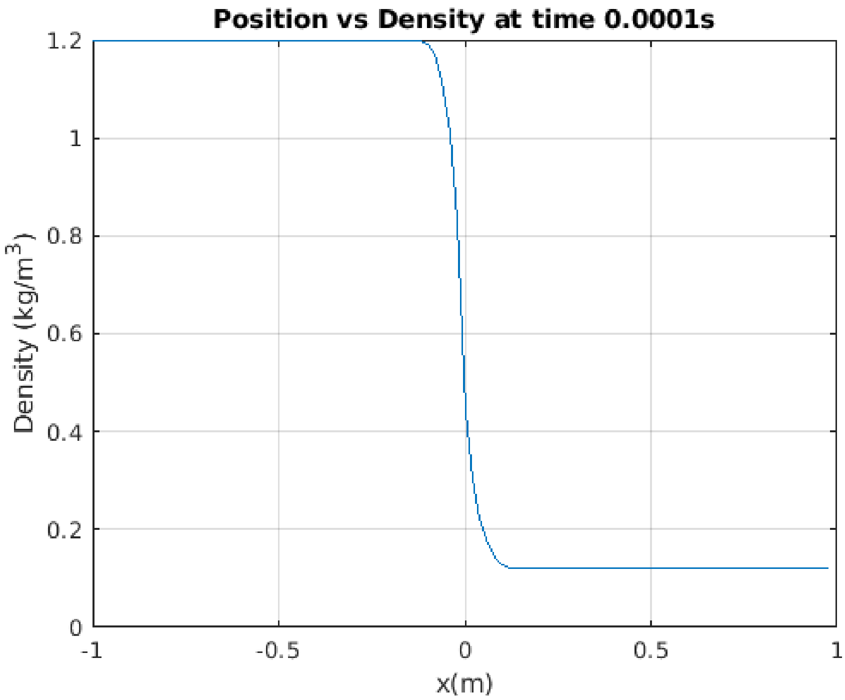
```
check2 = FullSimplify[iLvector.check.Lvector];
MatrixForm[check2]
(*This gets me part e, which should return me the jacobian again*)
```

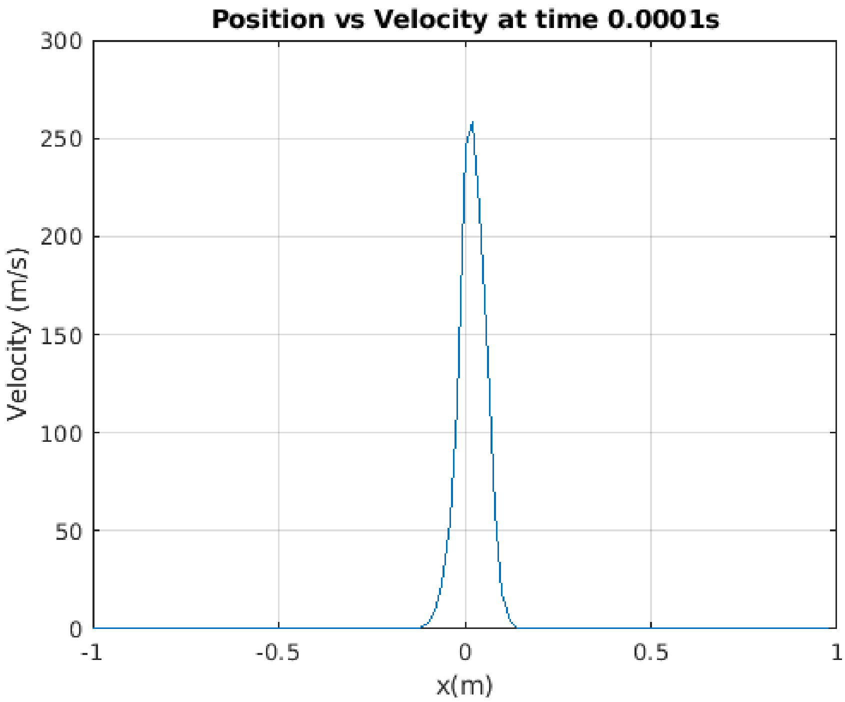
Out[25]//MatrixForm=

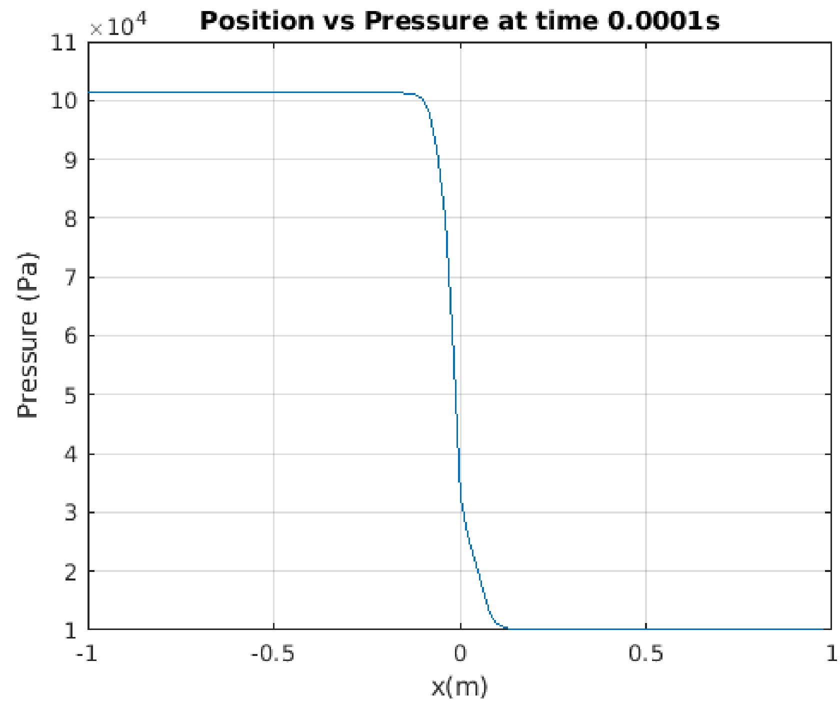
$$\begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} u^2 (-3+\gamma) & -u(-3+\gamma) & -1+\gamma \\ \frac{1}{2} u^3 (-2+\gamma) - \frac{a^2 u}{-1+\gamma} & u^2 \left(\frac{3}{2}-\gamma\right) + \frac{a^2}{-1+\gamma} & u\,\gamma \end{pmatrix}$$

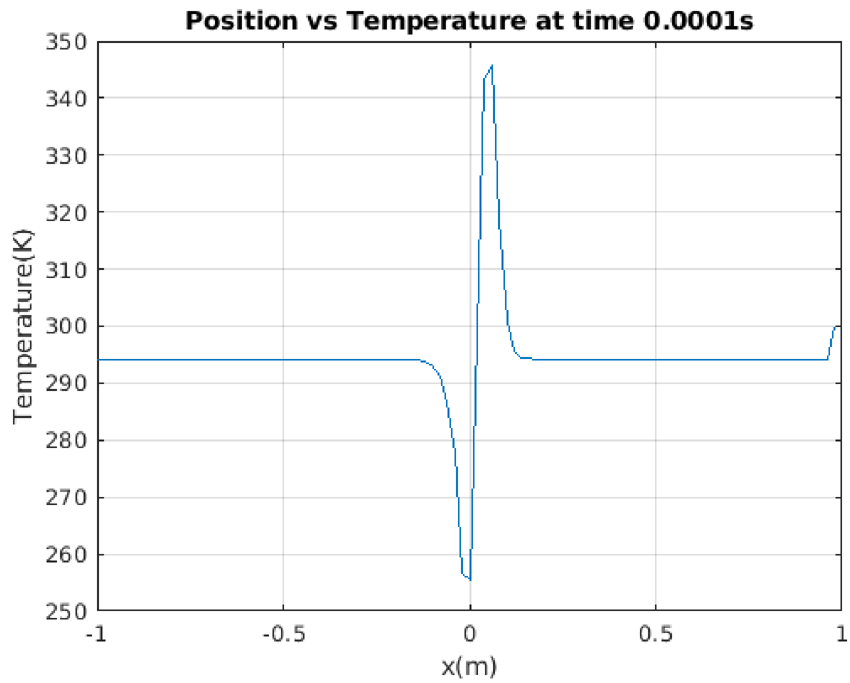
(b) Plots of the long shock tubes at different times (density, velocity, pressure and temperature)

At time $1(10^{-4})$s

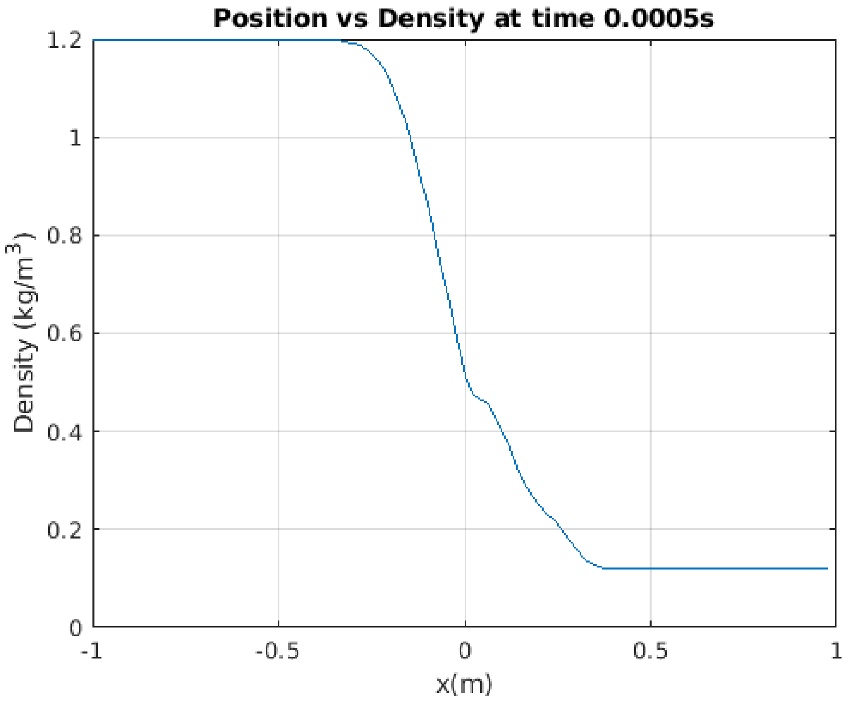


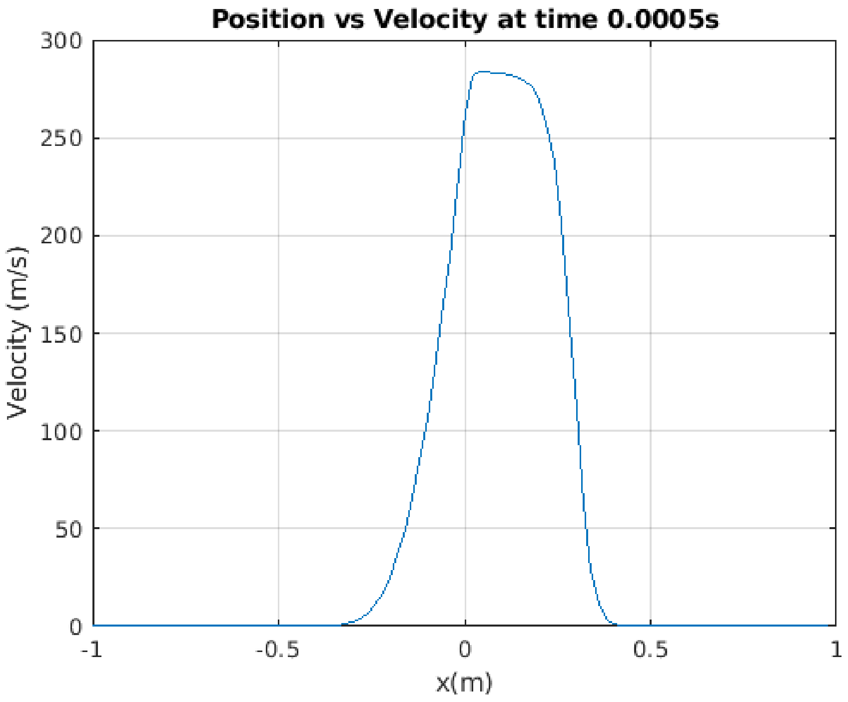


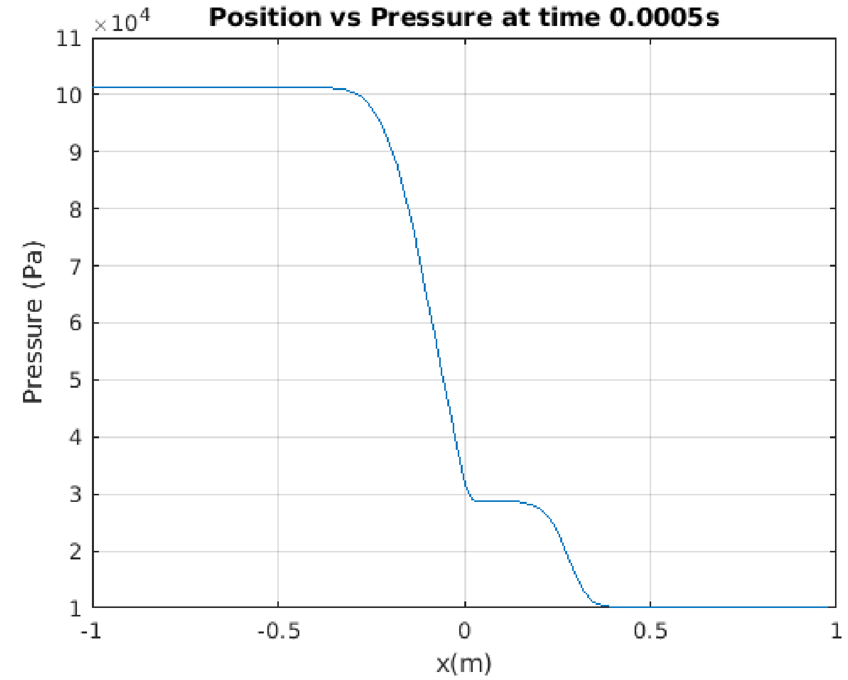

Position vs Pressure at time 0.0001s
×10^4
11
10
9
8
7
6
5
4
3
2
1
Pressure (Pa)
-1
-0.5
0
0.5
1
x(m)


Position vs Temperature at time 0.0001s
350
340
330
320
310
300
290
280
270
260
250
Temperature(K)
-1
-0.5
0
0.5
1
x(m)

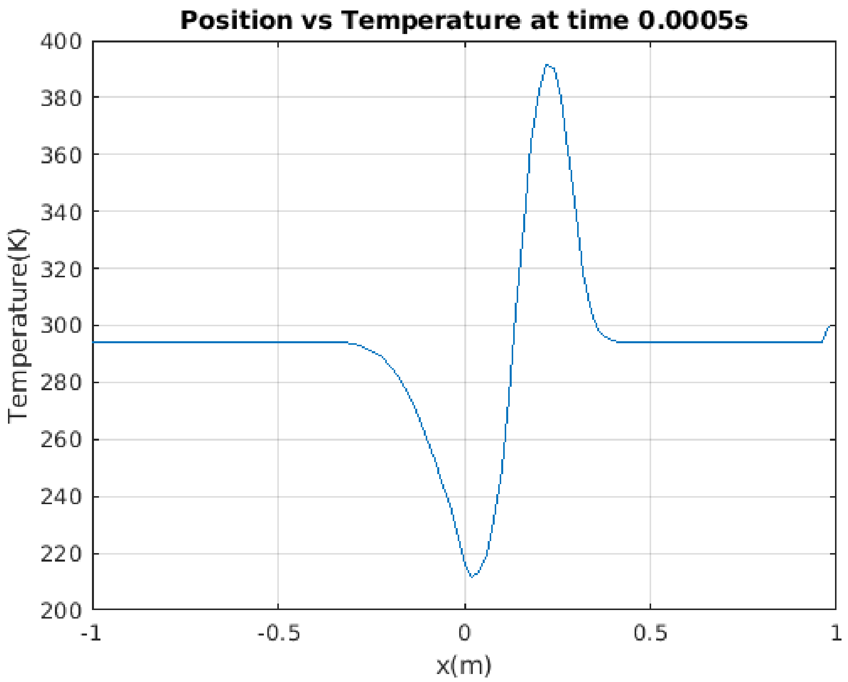
At time $5(10^{-4})$s





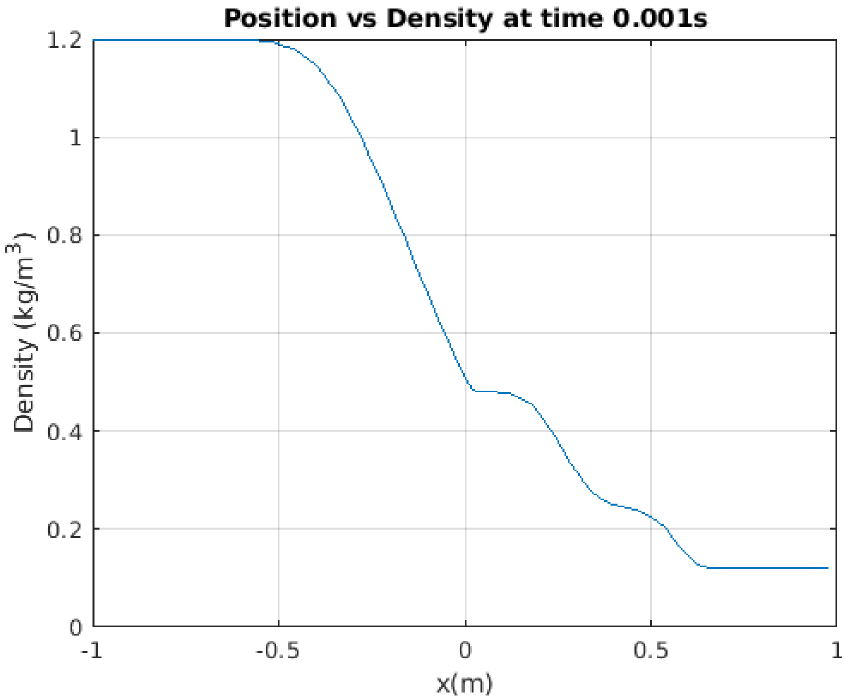


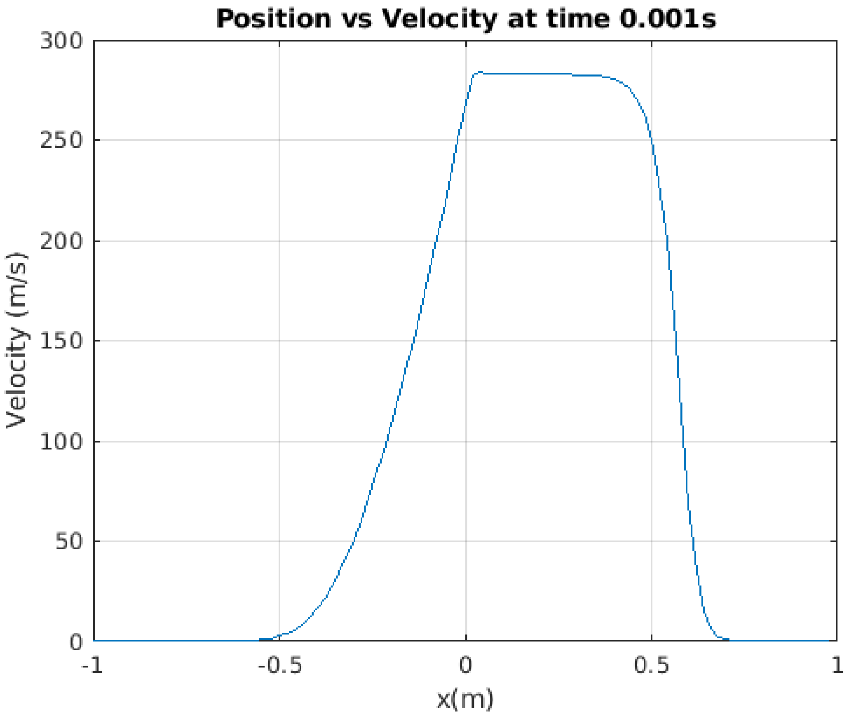
Position vs Pressure at time 0.0005s
×10^4
11
10
9
8
7
6
5
4
3
2
1
Pressure (Pa)
-1
-0.5
0
0.5
1
x(m)

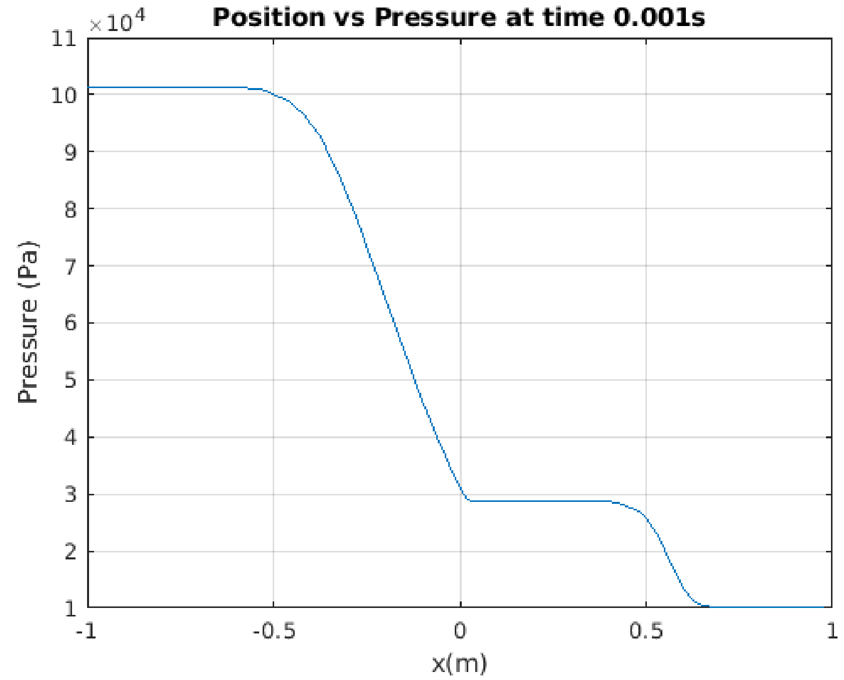


Position vs Temperature at time 0.0005s
400
380
360
340
320
300
280
260
240
220
200
Temperature(K)
-1
-0.5
0
0.5
1
x(m)

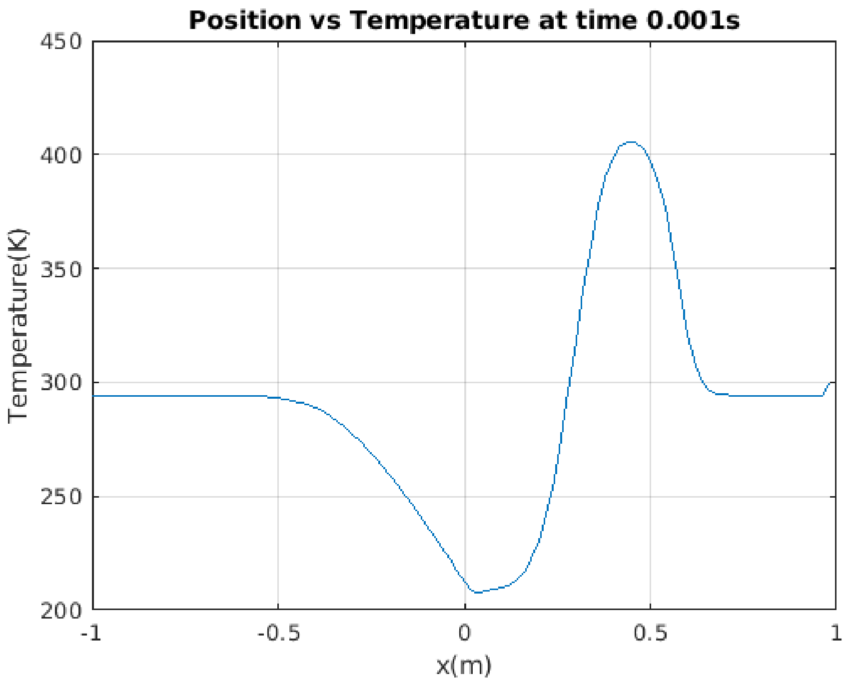
At time $1(10^{-3})$s





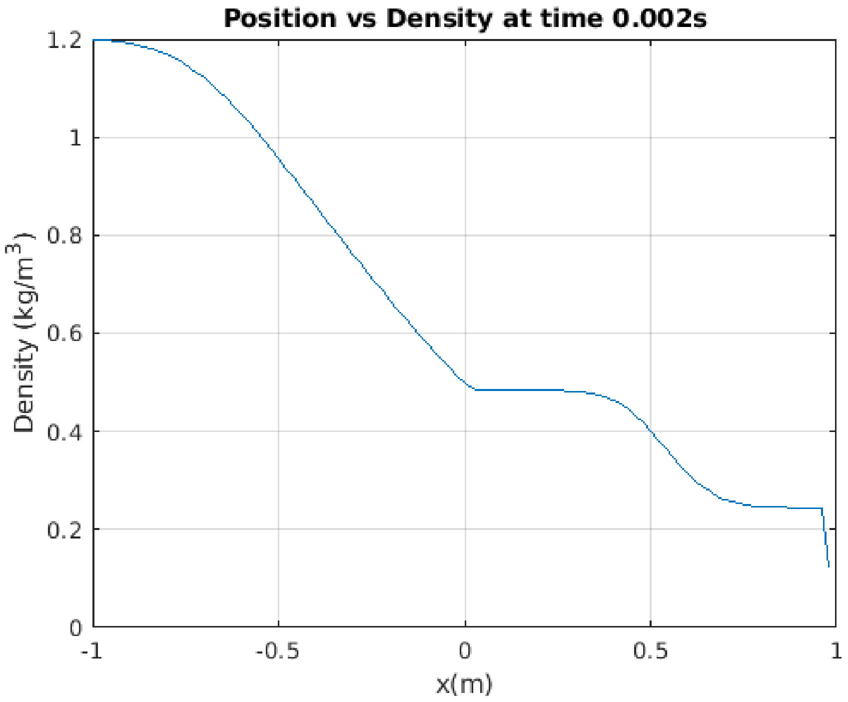


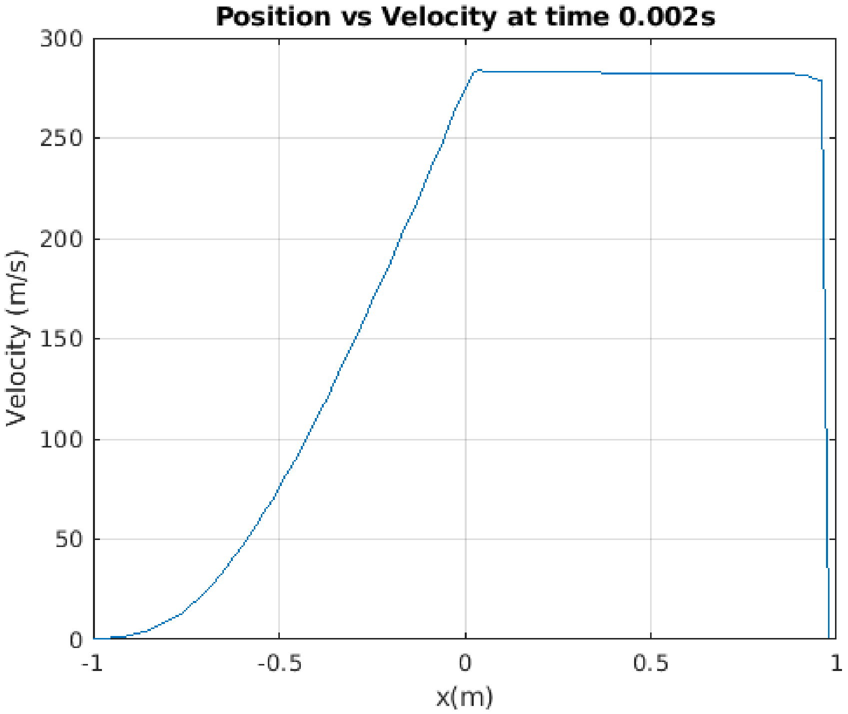
Position vs Pressure at time 0.001s
$\times 10^4$
Pressure (Pa)
x(m)

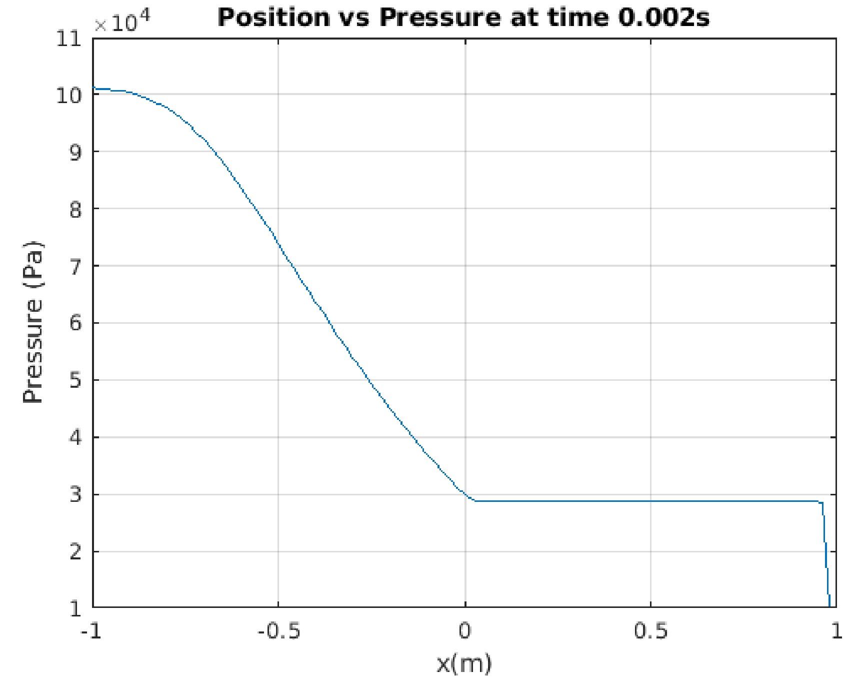


Position vs Temperature at time 0.001s
Temperature(K)
x(m)

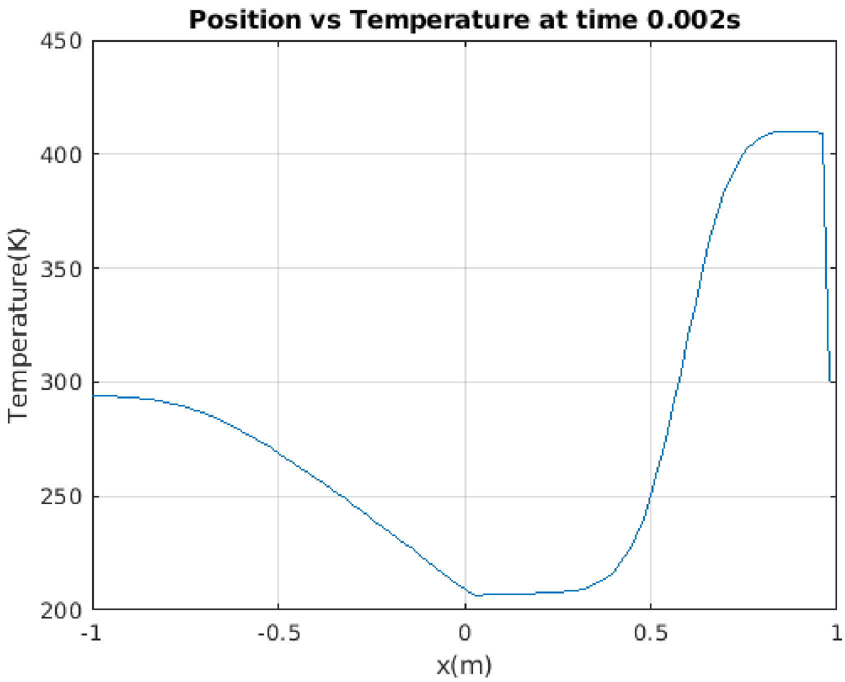
At time $2(10^{-3})$s







Position vs Pressure at time 0.002s
×10^4
11
10
9
8
7
6
5
4
3
2
1
Pressure (Pa)
-1
-0.5
0
0.5
1
x(m)



Position vs Temperature at time 0.002s
450
400
350
300
250
200
Temperature(K)
-1
-0.5
0
0.5
1
x(m)

```
clc;
close all;
clear all;

global gamma
gamma = 1.4;

% initializing the mesh
x_left = -1;
x_right = 1;
n = 100;
dx = (x_right-x_left)/n;
dt = 1E-6;

% initializing the variable memory to be used
Ti = 0;                 % initial time
Te = 1E-4;              % final time
T = zeros(n,1);         % temperature
rho = zeros(n,1);       % density
p = zeros(n,1);         % pressure
E = zeros(n,1);         % entalphy
uu = zeros(n,1);        % velocity
a = zeros(n,1);         % mach number
x = zeros(n,1);         % position

U = zeros(3,n);         % velocity vector
F = zeros(3,n);         % fluxes

% now lets define the variables
rho1 = 1.20;            % density (1)
uu1 = 0;                % velocity (1)
p1 = 101325;            % pressure (1)
rho2 = 0.120;           % density (2)
p2 = 10132.5;           % pressure (2)
uu2 = 0;                % velocity (2)

% Temperatures
T1 = 300;
T2 = 300;

% This will get my U vector
for i = 1:n
    x(i) = x_left+(i-1/2)*dx;
    if x(i) <= 0
        rho(i) = rho1;
        uu(i) = uu1;
        p(i) = p1;
        T(i) = T1;
    else
        rho(i) = rho2;
        uu(i) = uu2;
        p(i) = p2;
```

```
        T(i) = T2;
    end

    E(i) = p(i)/(gamma-1)+1/2*rho(i)*uu(i)^2;
    U(:,i) = [rho(i); rho(i)*uu(i); E(i)];
end

while Ti<Te

    %fluxes at i+1/2
    F(:,1) = roe(U(:,1),U(:,2));

    for i = 2:n-1
        F(:,i) = roe(U(:,i),U(:,i+1));
        U(:,i) = U(:,i)+dt/dx*(F(:,i-1)-F(:,i));
    end

    for i = 1:n-1
        rho(i) = U(1,i);
        uu(i) = U(2,i)/U(1,i);
        E(i) = U(3,i);
        p(i) = (E(i)-1/2*rho(i)*uu(i)^2)*(gamma-1);
        T(i) = p(i)/287/rho(i);
    end

    Ti=Ti+dt;

end

% So.. the final solution is
rhof = rho;
uuf = uu;
pf = p;
Tf = T;

% figure(1)
% plot(x_left:dx:x_right-dx,rhof)
% xlabel('x(m)')
% ylabel('Density (kg/m^3)')
% titlename = ['Position vs Density at time ',num2str(Te), 's'];
% title(titlename)
% grid on
%
% figure(2)
% plot(x_left:dx:x_right-dx,uuf)
% xlabel('x(m)')
% ylabel('Velocity (m/s)')
% titlename = ['Position vs Velocity at time ',num2str(Te), 's'];
% title(titlename)
% grid on
%
% figure(3)
% plot(x_left:dx:x_right-dx,pf)
% xlabel('x(m)')
```

```
% ylabel('Pressure (Pa)')
% titlename = ['Position vs Pressure at time ',num2str(Te), 's'];
% title(titlename)
% grid on
%
% figure(4)
% plot(x_left:dx:x_right-dx,Tf)
% xlabel('x(m)')
% titlename = ['Position vs Temperature at time ',num2str(Te), 's'];
% ylabel('Temperature(K)')
% title(titlename)
% grid on


% The following function will solve the fluxes in the Roe's scheme
function flux = roe(U_l,U_r)

 global gamma

 % density
 pL = U_l(1);
 pR = U_r(1);

 % density*u_x velocity
 uuL = U_l(2)/U_l(1);
 uuR = U_r(2)/U_r(1);

 % Entalphy
 El = U_l(3);
 Er = U_r(3);

 % Pressure
 ppL = (El-1/2*pL*uuL^2)*(gamma-1);
 ppR = (Er-1/2*pR*uuR^2)*(gamma-1);

 % I will need the averages
 [u_bar, a_bar, h_bar] = roe_mean(pL,pR,ppL,ppR,uuL,uuR);

 % having all this set up, lets go and define the eigenvalues
 lambda_1 = u_bar;
 lambda_2 = u_bar+a_bar;
 lambda_3 = u_bar-a_bar;

 lambda = [lambda_1 lambda_2 lambda_3];


 % Entropy fix
 ee = 10^-6;
 abs_lamda = zeros(3,1);

 for i = 1:3
     if abs(lambda(i)) >= ee
         abs_lamda(i) = abs(lambda(i));
     else
```

```
        abs_lamda(i) = (lambda(i)^2+ee^2)/2/ee;
    end
end

% Eigenvectors
S1 = [1; u_bar; 1/2*u_bar^2];
S2 = [1; u_bar+a_bar; h_bar+u_bar*a_bar];
S3 = [1; u_bar-a_bar; h_bar-u_bar*a_bar];

S = [S1 S2 S3];

delta_u=U_r-U_l;

% Inverse of Eigenvectors
Si1 = (gamma-1)/a_bar^2*(delta_u(1)*(h_bar-u_bar^2)+u_bar*delta_u(2)-
delta_u(3));
Si2 = 1/2/a_bar*(delta_u(1)*(-u_bar+a_bar)+delta_u(2)-a_bar*Si1);
Si3 = delta_u(1)-Si1-Si2;

Si = [Si1; Si2 ;Si3];

% fluxes
FL = [pL*uuL;pL*uuL^2+ppL;(El+ppL)*uuL];
FR = [pR*uuR;pR*uuR^2+ppR;(Er+ppR)*uuR];
flux = 1/2*(FL+FR)-1/2*S*(abs_lamda.*Si);
end

function [u_bar a_bar h_bar]  = roe_mean(pL,pR,ppL,ppR,uuL,uuR)

    global gamma

    u_bar = (sqrt(pL)*uuL+sqrt(pR)*uuR)/(sqrt(pL)+sqrt(pR));

    % average of energy
    EL = ppL/(gamma-1)+1/2*pL*uuL^2;
    ER = ppR/(gamma-1)+1/2*pR*uuR^2;

    % entalphy
    H_L = (EL+ppL)/pL;
    H_R = (ER+ppR)/pR;

    % mean of entalphy
    h_bar = (sqrt(pL)*H_L+sqrt(pR)*H_R)/(sqrt(pL)+sqrt(pR));

    % mean of mach number
    a_bar = sqrt((gamma-1)*(h_bar-1/2*u_bar^2));

end
```

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